1. Determine the truth value of each of these statements if the domain consists of real numbers. **Give a short explanation for each answer to receive full credit.**

   (a) (3 points) $\exists x (x^3 = -1)$

   (b) (3 points) $\exists x (x^4 < x^2)$

   (c) (3 points) $\forall x ((-x^2) = x^2)$

   (d) (3 points) $\forall x (2x > x)$
2. (4 points) What is the cardinality of each of these sets?
   a) \{a\}
   b) \{\{a\}\}
   c) \{a, \{a\}\}
   d) \{a, \{a\}, \{a, \{a\}\}\}

3. Determine whether each of these arguments is valid. If it is valid, show the steps of your conclusion. If it is not valid, give a logical error.
   a) (4 points)
      \[
      \begin{align*}
      \text{If } n \text{ is a real number such that } n > 2, \text{ then } n^2 > 4. \\
      n \leq 2 \\
      \hline
      n^2 \leq 4
      \end{align*}
      \]
   b) (4 points)
      \[
      \begin{align*}
      \text{If it snows today, the university will close.} \\
      \text{The university is not closed today.} \\
      \hline
      \text{Therefore, it did not snow today.}
      \end{align*}
      \]
4. Answer the following questions. Write your answer clearly and do not calculate the any number (for example, leave $2^{10}$ as it is instead of calculating $2^{10} = 1024$).

(a) (4 points) How many ways are there to travel in $xyz$ space from the origin $(0,0,0)$ to the point $(12,8,4)$ by taking steps **two units** in the positive $x$, positive $y$, or positive $z$ direction?

(b) (4 points) How many ways are there to pack twelve **identical** DVD’s into four **distinguishable** boxes so that each box contains at least one DVD?

(c) (4 points) Find the coefficient of $x^4$ in $(2 + 5x)^{13}$.

(d) (4 points) How many functions are there $f : A \rightarrow B$, where $|A| = 3$, $|B| = 7$?

(e) (4 points) How many one-to-one functions are there $f : A \rightarrow B$, where $|A| = 3$, $|B| = 7$?

(f) (4 points) How many subsets with an odd number of elements does a set with 10 elements have?
5. Determine the truth value of the statement \( \exists x \forall y (x \leq y^2) \) and if the domain for \( x \) and \( y \) consists of the following sets. Give a short explanation for your answer.

(a) (2 points) the positive real numbers

(b) (2 points) the integers

(c) (2 points) the nonzero real numbers

6. (12 points) Determine whether each of these statements is true or false.

a) \( 0 \in \emptyset \)  \hspace{1cm} b) \( \emptyset \in \{0\} \)

c) \( \{0\} \subset \emptyset \)  \hspace{1cm} d) \( \emptyset \subset \{0\} \)

e) \( \{0\} \in \{0\} \)  \hspace{1cm} f) \( \emptyset \in \{\emptyset\} \)
7. (9 points) Show that if $f$ is a function from $S$ to $T$, where $S$ and $T$ are finite sets with $|S| > |T|$, then there are elements $s_1$ and $s_2$ in $S$ such that $f(s_1) = f(s_2)$.

8. How many bit strings of length 10 contain
   (a) (4 points) an equal number of 0’s and 1’s?
   (b) (5 points) at most four 1’s?
   (c) (6 points) at least four 1’s, where the number of 1’s is even?
9. How many permutations of the letters ABCDEFGH contain
   (a) (2 points) the string ED?
   
   (b) (2 points) the strings BA and FGH?
   
   (c) (2 points) the strings CAB and BED?
   
   (d) (2 points) the strings BCA and ABF?
   
   (e) (2 points) the strings AB, DE and GH?