Lecture 4: Review of Discrete Probability

BBM205

Exercises

1. What is the probability of these events when we randomly choose a permutation \(\{1, 2, \ldots, n\}\), where \(n \geq 4\)?
   a) 2 precedes \(n\).
   b) 1 precedes 2 and 2 precedes \(n\).
   c) 1 immediately precedes 3.
   d) 1 immediately precedes 3, and \(n\) immediately precedes 2.

2. Suppose that \(E_1\) and \(E_2\) are events such that \(p(E_1) = \frac{3}{4}\) and \(p(E_2) = \frac{4}{5}\). Show that \(\frac{11}{20} \leq p(E_1 \cap E_2) \leq \frac{3}{4}\).

3. Suppose that E and F are events in a sample space and that \(p(E) = \frac{1}{3}\), \(p(F) = \frac{1}{2}\), and \(p(E|F) = \frac{2}{5}\). Find \(p(F|E)\).

4. What is the probability that a five-card poker hand contains exactly one ace?

5. What is the probability that a five-card poker hand contains at least one ace?

6. What is the probability that a five-card poker hand contains a flush, that is, five cards of the same suit?

7. For each of the following pairs of events, which are subsets of the set of all possible outcomes when a coin is tossed three times, determine whether or not they are independent.
   a) \(E_1\): the first coin comes up tails; \(E_2\): the second coin comes up heads
   b) \(E_1\): the first coin comes up tails; \(E_2\): two, and not three, heads come up in a row.
   c) \(E_1\): the second coin comes up tails; \(E_2\): two, and not three, heads come up in a row.

8. A student takes a random card from a deck of 52 cards, looks at it and puts it back. If he does this \(k\) times, what is the probability that he/she took a card of some kind at least twice? What is the smallest \(k\) such that this probability is greater than \(\frac{1}{2}\)?