BBM 205

Problem Set 4a:

Sets, Functions, Introduction to Counting

1. For each of the following sets, determine whether 2 is an element of the set.

    (a) $\{x \in \mathbb{R} | x$ is an integer greater than $1\}$

    (b) $\{x \in \mathbb{R} | x$ is the square of an integer$\}$

    (c) $\{2, \{2\}\}$

    (d) $\{\{2\}, \{\{2\}\}\}$

    (e) $\{\{2\}, \{2, \{2\}\}\}$

    (f) $\{\{\{2\}\}\}$

2. How many different elements does $A \times B$ have if A has m elements and B has n elements?

3. What is the cardinality of each of these sets?
    a) $\{a\}$ b) $\{\{a\}\}$
    c) $\{a, \{a\}\}$ d) $\{a, \{a\}, \{a, \{a\}\}\}$

4. (Spring 2015) What is the cardinality of each of these sets.

$$\emptyset, \qquad \{\emptyset\}, \qquad \{\emptyset, \{\emptyset\}\}, \qquad \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}.$$

5. (Spring 2015) Determine whether each of these statements is true or false.

$$x \in \{x\}, \qquad \{x\} \subset \{x\}, \qquad \{x\} \in \{x\},$$
$$\{x\} \in \{\{x\}\}, \qquad \emptyset \subseteq \{x\}, \qquad \emptyset \in \{x\}.$$

6. Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find
    a) $A \cap B \cap C$ b) $A \cup B \cup C$
    c) $(A \cup B) \cap C$ d) $(A \cap B) \cup C$

7. How many license plates can be made using either three letters followed by three digits or four letters followed by two digits?

8. How many different functions are there from a set with 8 elements to a set with 3 elements?

9. How many bit strings of length 10 contain
   a) exactly four 1's?
   b) at least four 1's?
   c) at most four 1's?
   d) an equal number of 0's and 1's?

10. (Spring 2015)

    (a) How many bit strings of length seven either begin with two 0's or end with three 1's?

    (b) How many subsets with more than two elements does a set with 100 elements have?

    (c) How many ways are there to select three **unordered** elements from a set with five (different) elements when **repetition is allowed**?

11. (Fall 2016) Determine whether each of these statements is true or false.
    a) $0 \in \emptyset$ b) $\emptyset \in \{0\}$
    c) $\{0\} \subset \emptyset$ d) $\emptyset \subset \{0\}$
    e) $\{0\} \in \{0\}$ f) $\{0\} \in \{0\}$
    g) $\{\emptyset\} \subseteq \{\emptyset\}$

12. Let $A = \{a, b, c\}$, $B = \{x, y\}$ and $C = \{0, 1\}$. Find
    a) $A \times B \times C$ b) $C \times B \times A$
    c) $C \times A \times B$ d) $B \times B \times B$