BBM 205
Problem Set 4b:
Pigeonhole Principle, Permutations and Combinations, Binomial Theorem

1. How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other?
2. How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that $a_1 \equiv a_2 \pmod 5$ and $b_1 \equiv b_2 \pmod 5$.
3. How many different ways are there to choose 6 donuts from the 21 varieties at a donut shop?
4. How many different strings can be made from the letters in MASSACHUSETTS, using all letters?
5. A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels, and plain bagels. How many ways are there to choose
   (a) six bagels?
   (b) a dozen bagels?
   (c) two dozen bagels?
   (d) a dozen bagels with at least one of each kind?
   (e) a dozen bagels with at least three egg bagels and no more than two salty bagels?
6. A club has 25 members.
   (a) How many ways are there to choose four members of the club to serve on an executive committee?
   (b) How many ways are there to choose a president, vice president, secretary, and treasurer of the club, where no person can hold more than one office?

7. (Spring 2015) Let d be a positive integer. Use pigeonhole principle to show that among any group of $d+1$ integers there are two with exactly the same remainder when they are divided by d.

8. (Spring 2015) Use Pascal's identity to prove the following whenever n and r are positive integers.

$$\sum_{k=0}^{k=r} \binom{n+k}{k} = \binom{n+r+1}{r}$$

9. (Spring 2015) How many solutions are there to the inequality

$$x_1 + x_2 + x_3 \leq 11,$$

where x_1, x_2, and x_3 are nonnegative integers?

10. (Spring 2015) Seven women and nine men are on the faculty in the computer science department at a school.

    (a) How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?

    (b) How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be on the committee?

11. (Spring 2015) How many ways are there to distribute five balls into seven boxes if each box must have at most one ball in it if

    (a) both the balls and boxes are labelled?

    (b) the balls are labelled, but the boxes are unlabelled?

    (c) the balls are unlabelled, but the boxes are labelled?

    (d) both the balls and boxes are unlabelled?

12. (Spring 2015)

    (a) How many license plates can be made using either three letters followed by three digits or four letters followed by two digits?

    (b) How many different functions are there from a set with 10 elements to a set with 5 elements?

(c) How many permutations of the letters ABCDEFG contain
a) the string BCD?
b) the strings ABC and CDE?
c) the strings CBA and BED?

(d) Show that if n and k are integers with $1 \leq k \leq n$, then $\binom{n}{k} \leq n^k/2^{k-1}$.

(e) How many different ways are there to choose 6 donuts from the 21 varieties at a donut shop?

(f) How many different strings can be made from the letters in ABRACADABRA, using all letters?

(g) A bowl contains 10 red balls and 10 blue balls. A person selects balls at random without looking at them. How many balls must be selected to be sure of having at least three balls of the same color?

13. (Spring 2015)

(a) Find the coefficient of x^5 in $(4-3x)^{21}$.

(b) Find the coefficient of $x^3y^2z^5$ in $(x+y+z)^{10}$.

14. (Fall 2016) Answer the following questions.

(a) How many ways are there to travel in xyz space from the origin $(0,0,0)$ to the point $(12,8,4)$ by taking steps **two units** in the positive x, positive y, or positive z direction?

(b) How many ways are there to pack twelve **identical** DVD's into four **distinguishable** boxes so that each box contains at least one DVD?

(c) Find the coefficient of x^4 in $(2+5x)^{13}$.

(d) How many functions are there $f : A \to B$, where $|A| = 3$, $|B| = 7$?

(e) How many one-to-one functions are there $f : A \to B$, where $|A| = 3$, $|B| = 7$?

(f) How many subsets with an odd number of elements does a set with 10 elements have?

15. (Fall 2016) Show that if f is a function from S to T, where S and T are finite sets with $|S| > |T|$, then there are elements s_1 and s_2 in S such that $f(s_1) = f(s_2)$.

16. (Fall 2016) How many bit strings of length 10 contain

    (a) an equal number of 0's and 1's?

    (b) at most four 1's?

    (c) at least four 1's, where the number of 1's is even?