1. (a) Find a recurrence relation and the initial conditions for $c_n$, that is, the minimum number of moves in which the $n$-disk Tower of Hanoi puzzle can be solved.
   
   (b) Solve this recurrence relation.

2. Let $S_n$ denote the number of $n$-bit strings that do not contain the pattern 000. Find a recurrence relation and initial conditions for the sequence $\{S_n\}$.

3. Let $S_n$ denote the number of $n$-bit strings that do not contain the pattern 00.
   
   (a) Find a recurrence relation and initial conditions for the sequence $\{S_n\}$.
   
   (b) Show that $S_n = f_{n+2}$ for $n = 2, 3, \ldots$, where $f_n$ denotes $n$th Fibonacci number.

4. Derive a recurrence relation for $C(n, k) = \binom{n}{k}$, the number of $k$-element subsets of an $n$-element subset. Specifically, write $C(n + 1, k)$, in terms of $C(n, i)$ for appropriate $i$.

5. Let $S(n, k)$ denote the number of functions from $\{1, \ldots, n\}$ onto $\{1, \ldots, k\}$. Show that $S(n, k)$ satisfies the recurrence relation

$$S(n, k) = k^n - \sum_{i=1}^{k-1} C(k, i)S(n, i).$$

6. Solve the recurrence relation

$$S_n = 2S_{n-1},$$

subject to the initial condition $S_0 = 1$. 

7. Solve the recurrence relations with the given initial conditions below.

(a) \( a_n = 6a_{n-1} - 8a_{n-2}, \ a_0 = 1, \ a_1 = 0. \)

(b) \( a_n = 7a_{n-1} - 10a_{n-2}, \ a_0 = 5, \ a_1 = 16. \)

(c) \( a_n = 2a_{n-1} + 8a_{n-2}, \ a_0 = 4, \ a_1 = 10. \)

(d) \( a_n = -3a_{n-1}, \ a_0 = 2. \)

(e) \( a_n = 2na_{n-1}, \ a_0 = 1. \)

(f) \( a_n = a_{n-1} + n, \ a_0 = 0. \)

8. Solve the recurrence relation \( \sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}} \) with initial conditions \( a_0 = a_1 = 1 \) by making the substitution \( b_n = \sqrt{a_n}. \)

9. Solve the recurrence relation

\[
a_n = \sqrt{\frac{a_{n-2}}{a_{n-1}}}
\]

with initial conditions \( a_0 = 8, \ a_1 = 1/(2\sqrt{2}) \) by taking the logarithm of both sides and making the substitution \( b_n = \log a_n. \)