BBM 413 Fundamentals of Image Processing

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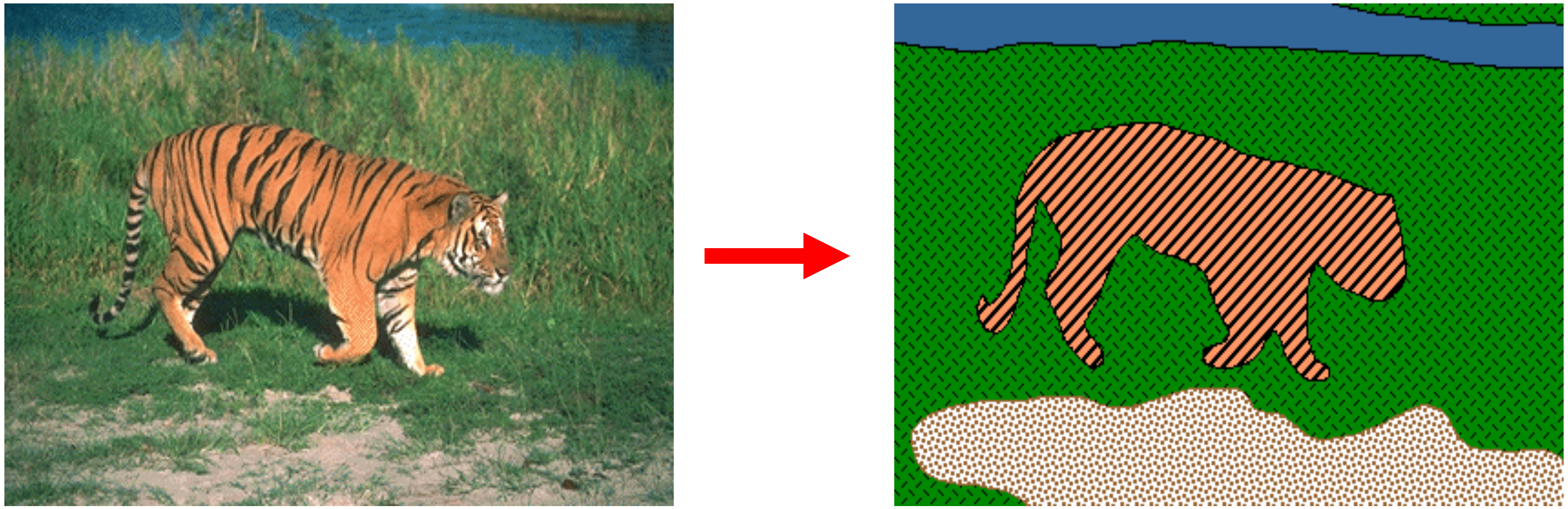
Segmentation – Part 2

Review- Image segmentation

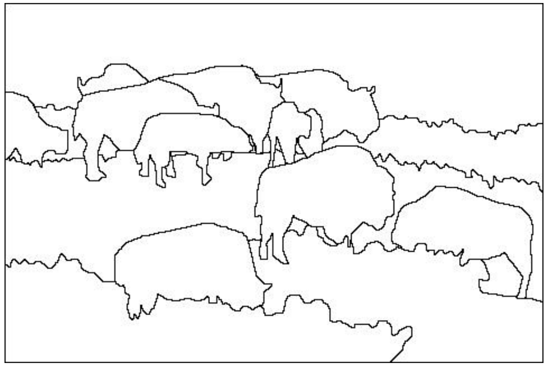
- Goal: identify groups of pixels that go together



Slide credit: S. Seitz, K. Grauman

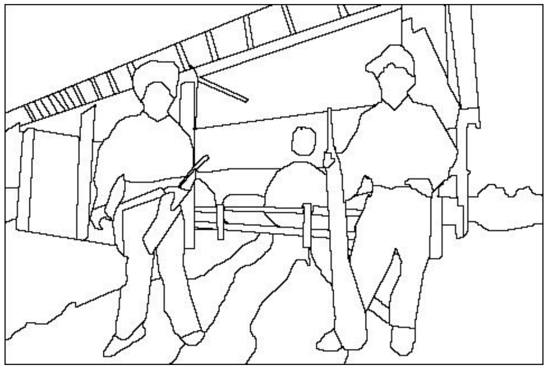
Review- The goals of segmentation

- Separate image into coherent “objects”

image

human segmentation

http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/

Slide credit: S. Lazebnik

Review- What is segmentation?

- Clustering image elements that "belong together"

  - Partitioning
    - Divide into regions/sequences with coherent internal properties
  - Grouping
    - Identify sets of coherent tokens in image

Slide credit: Fei-Fei Li

Review- K-means clustering

- Basic idea: randomly initialize the *k* cluster centers, and iterate between the two steps we just saw.

1. Randomly initialize the cluster centers, $c_1, \ldots, c_K$
2. Given cluster centers, determine points in each cluster
   - For each point p, find the closest c_i. Put p into cluster i
3. Given points in each cluster, solve for c_i
   - Set c_i to be the mean of points in cluster i
4. If c_i have changed, repeat Step 2

Properties

- Will always converge to *some* solution
- Can be a "local minimum"
  - does not always find the global minimum of objective function:

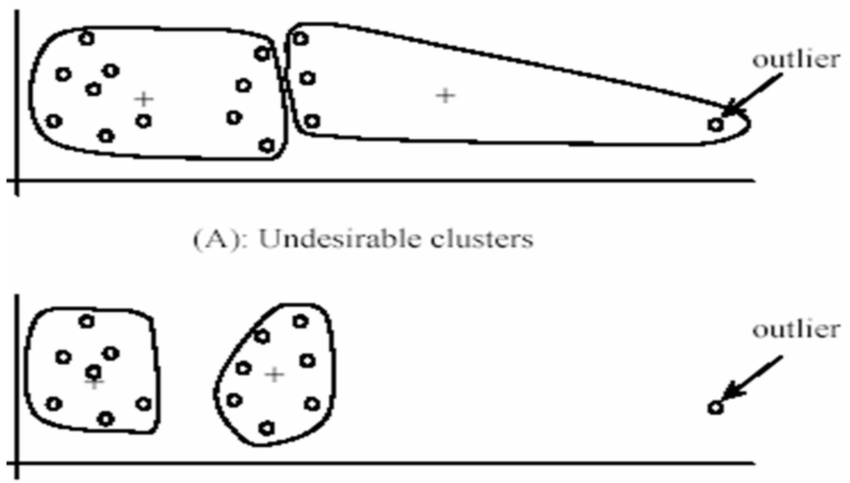
$$\sum_{\text{clusters } i} \; \sum_{\text{points p in cluster } i} \|p - c_i\|^2$$

Slide credit: S. Seitz

Review - K-means: pros and cons

Pros

- Simple, fast to compute
- Converges to local minimum of within-cluster squared error

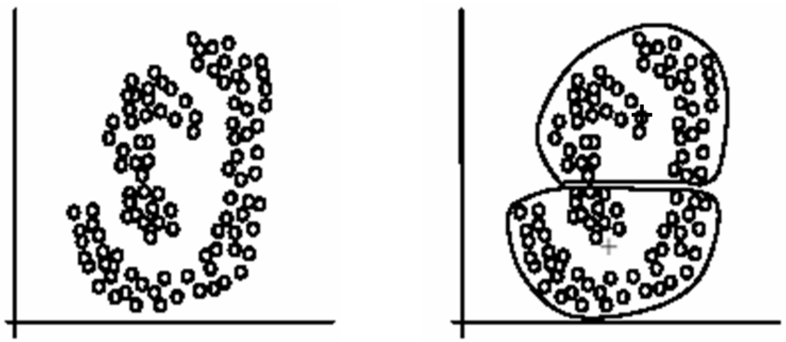


(A): Undesirable clusters

(B): Ideal clusters

Cons/issues

- Setting k?
- Sensitive to initial centers
- Sensitive to outliers
- Detects spherical clusters
- Assuming means can be computed



(A): Two natural clusters (B): *k*-means clusters

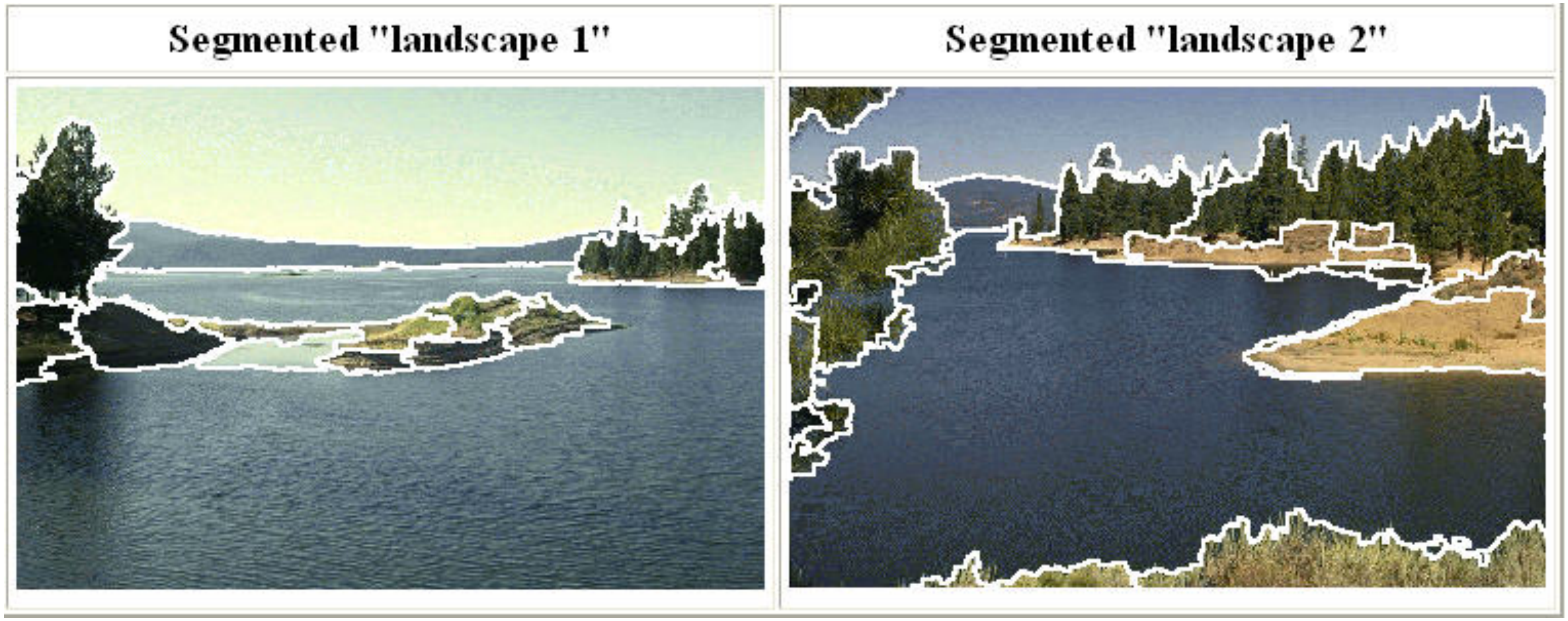
Slide credit: K Grauman

Segmentation methods

- Segment foreground from background
- Histogram-based segmentation
- Segmentation as clustering
  - K-means clustering
  - Mean-shift segmentation
- Graph-theoretic segmentation
  - Min cut
  - Normalized cuts
- Interactive segmentation

Mean shift clustering and segmentation

- An advanced and versatile technique for clustering-based segmentation

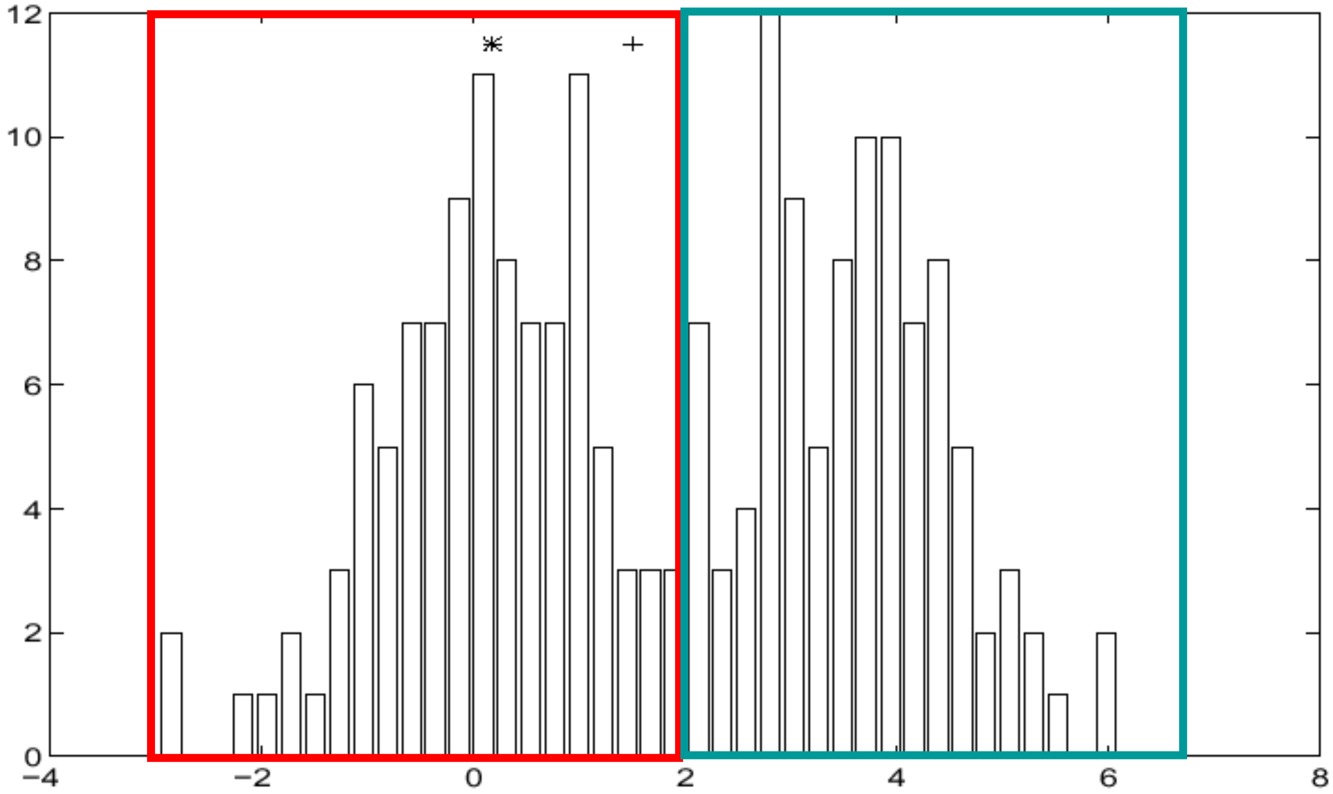


http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Slide credit: S. Lazebnik

Finding Modes in a Histogram



- How Many Modes Are There?
  - Easy to see, hard to compute

Slide credit: S. Seitz

Mean shift algorithm

- The mean shift algorithm seeks *modes* or local maxima of density in the feature space

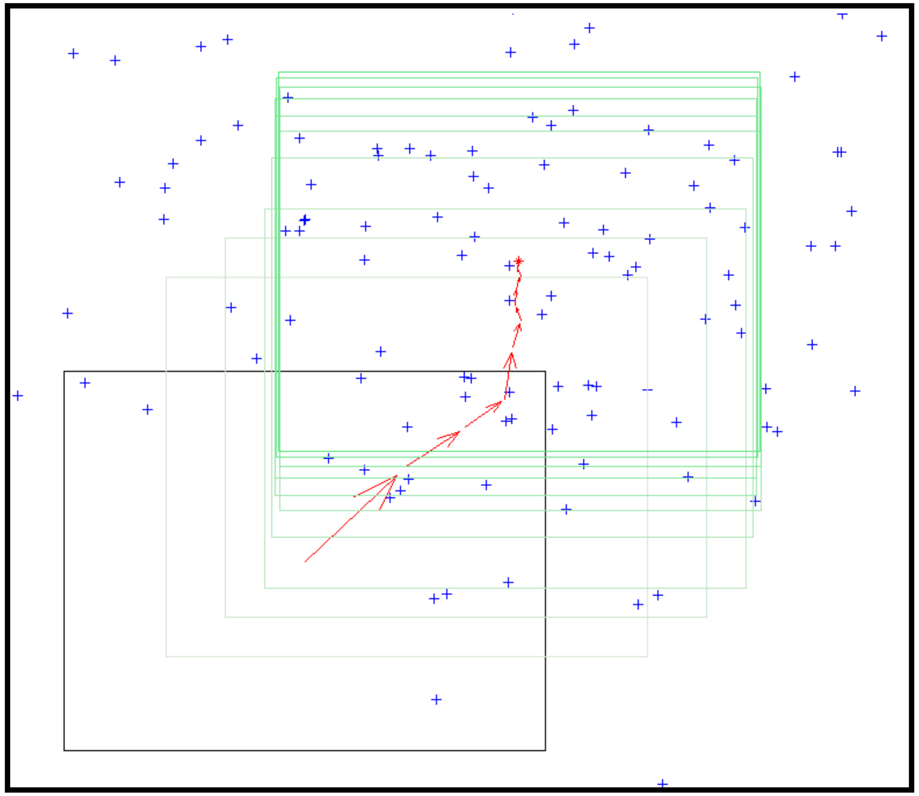
Slide credit: S. Lazebnik

Mean shift algorithm

Mean Shift Algorithm

1. Choose a search window size.
2. Choose the initial location of the search window.
3. Compute the mean location (centroid of the data) in the search window.
4. Center the search window at the mean location computed in Step 3.
5. Repeat Steps 3 and 4 until convergence.

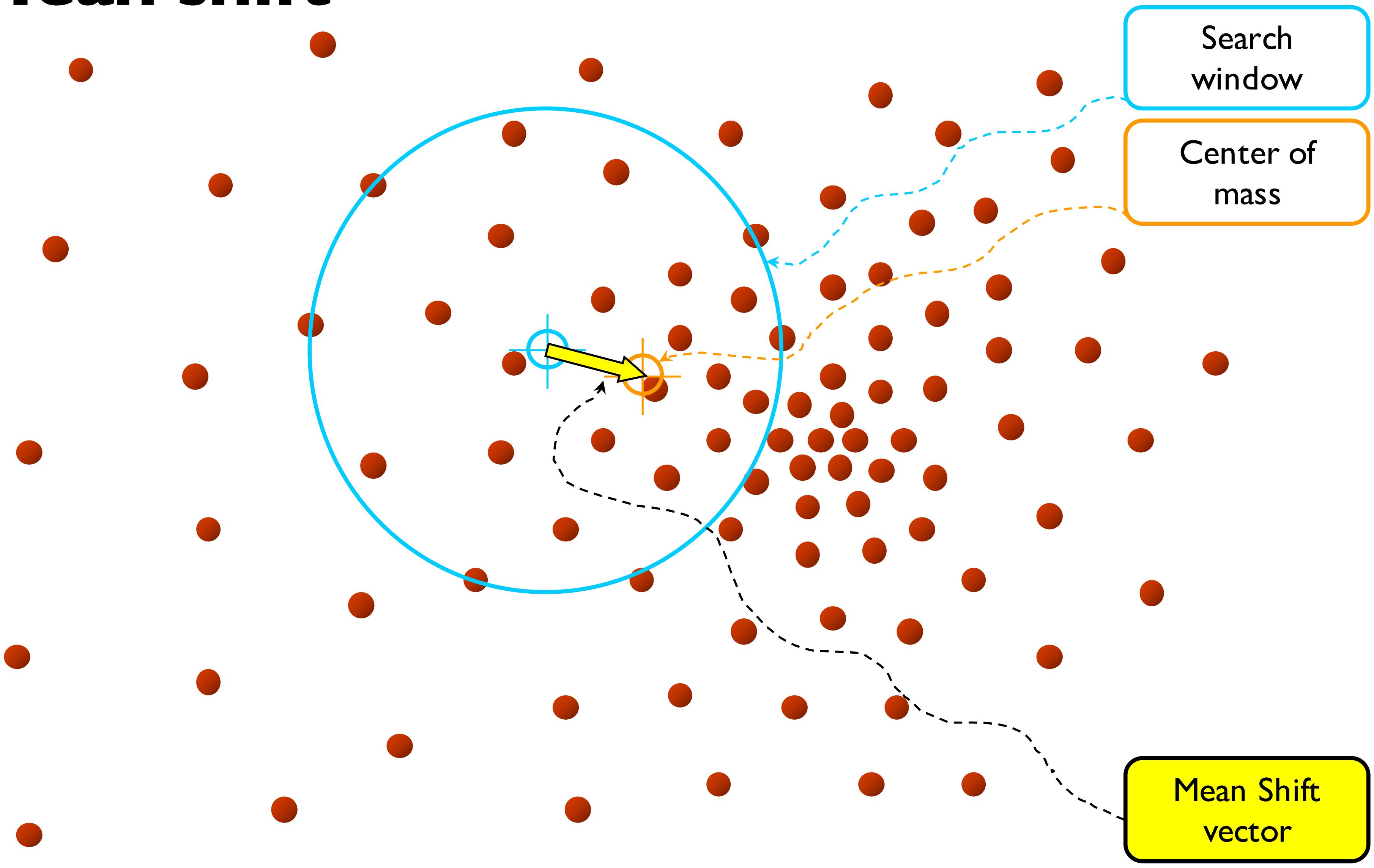
The mean shift algorithm seeks the "mode" or point of highest density of a data distribution:



Two issues:
(1) Kernel to interpolate density based on sample positions.
(2) Gradient ascent to mode.

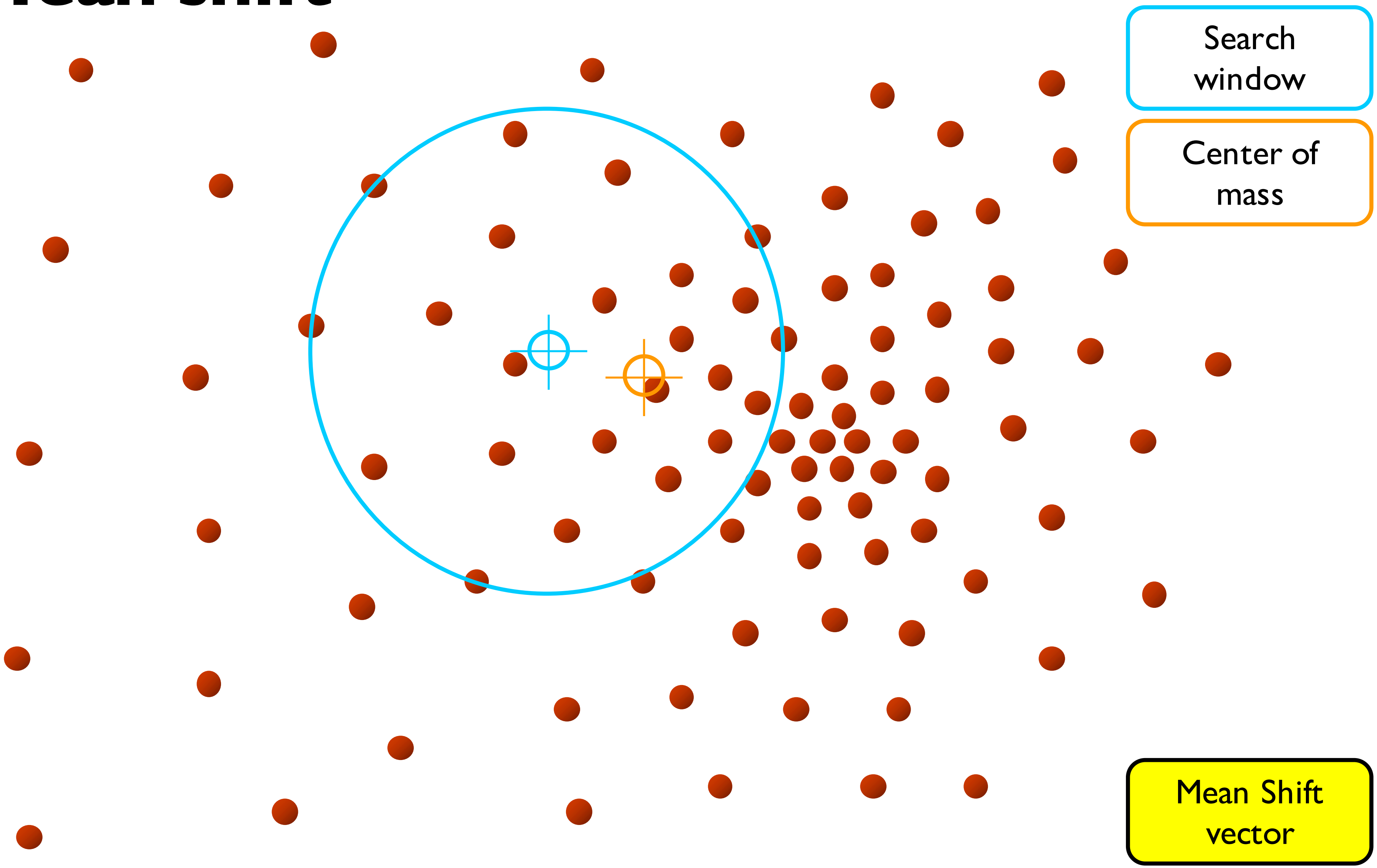
Slide credit: B. Freeman and A. Torralba

Mean shift



Slide credit: Y. Ukrainitz & B. Sarel

Mean shift



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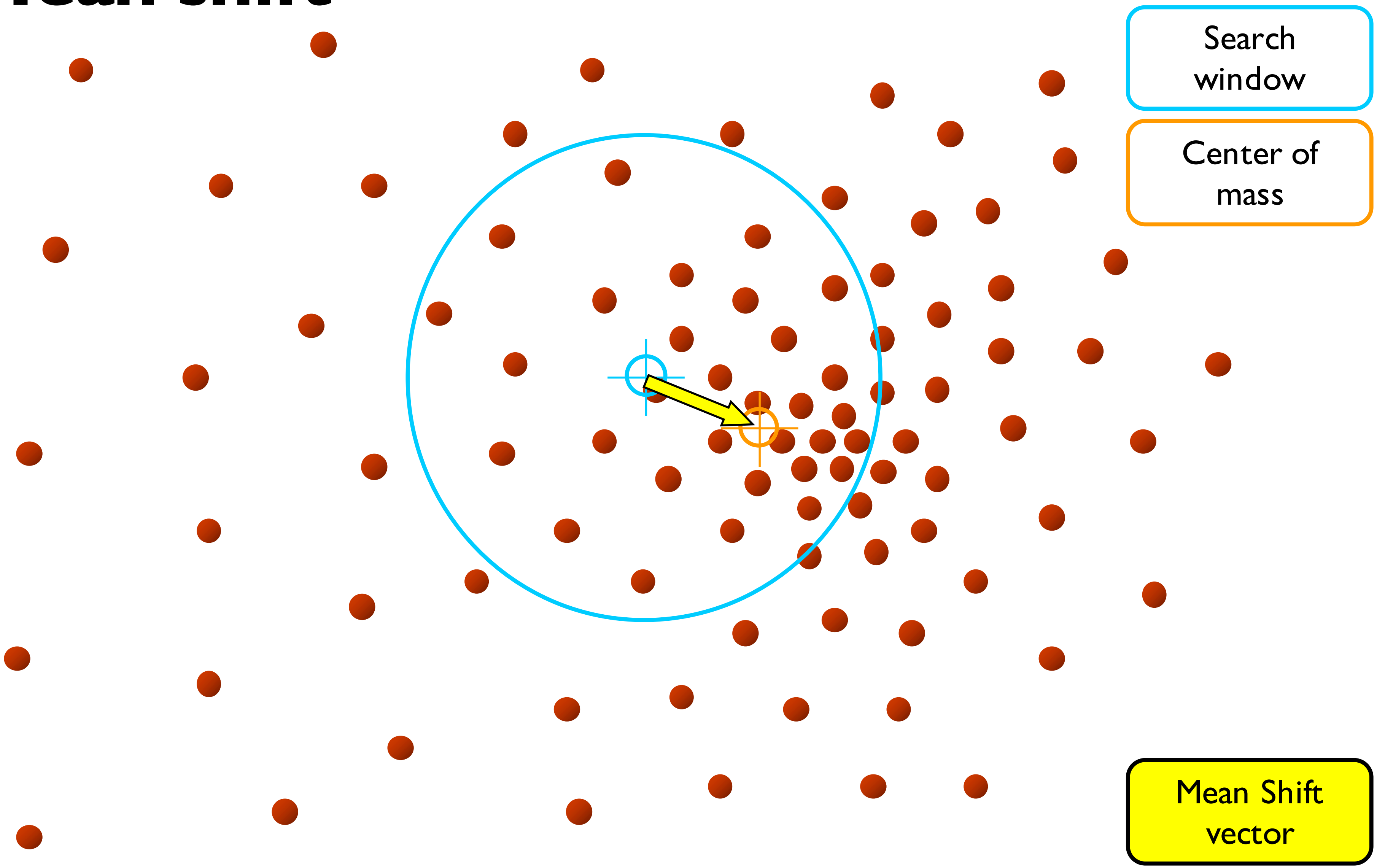
Mean shift



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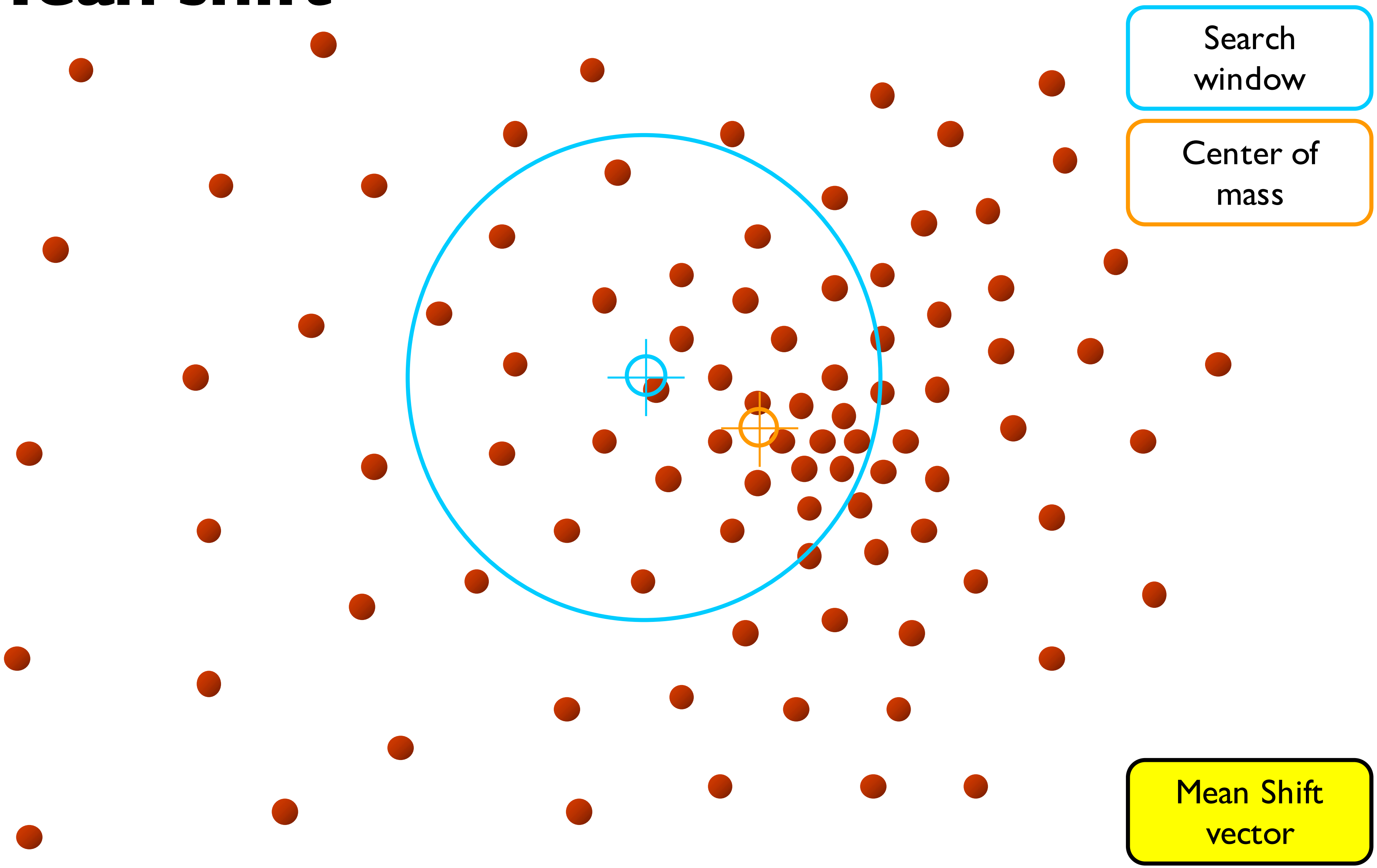
Mean shift



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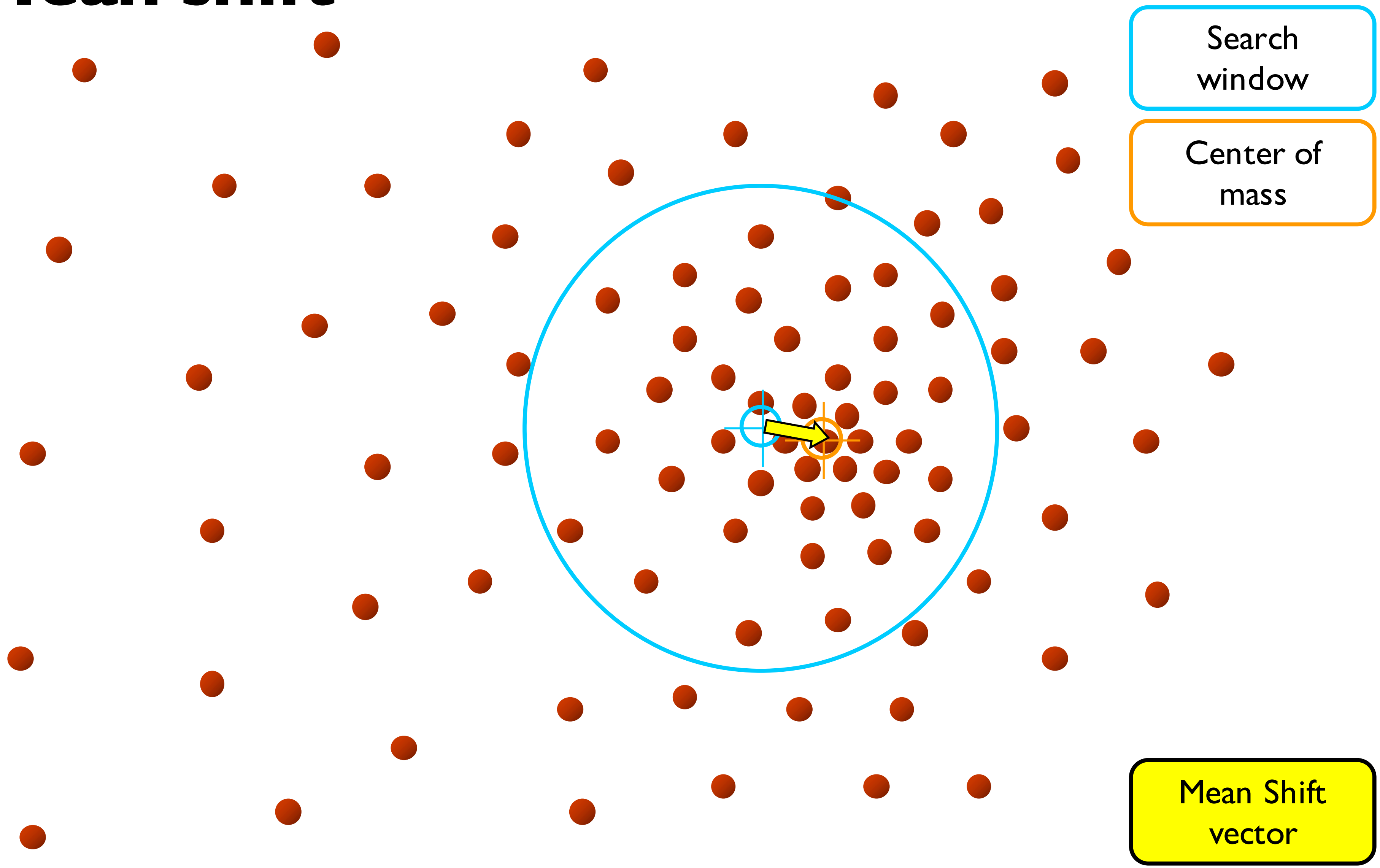
Mean shift



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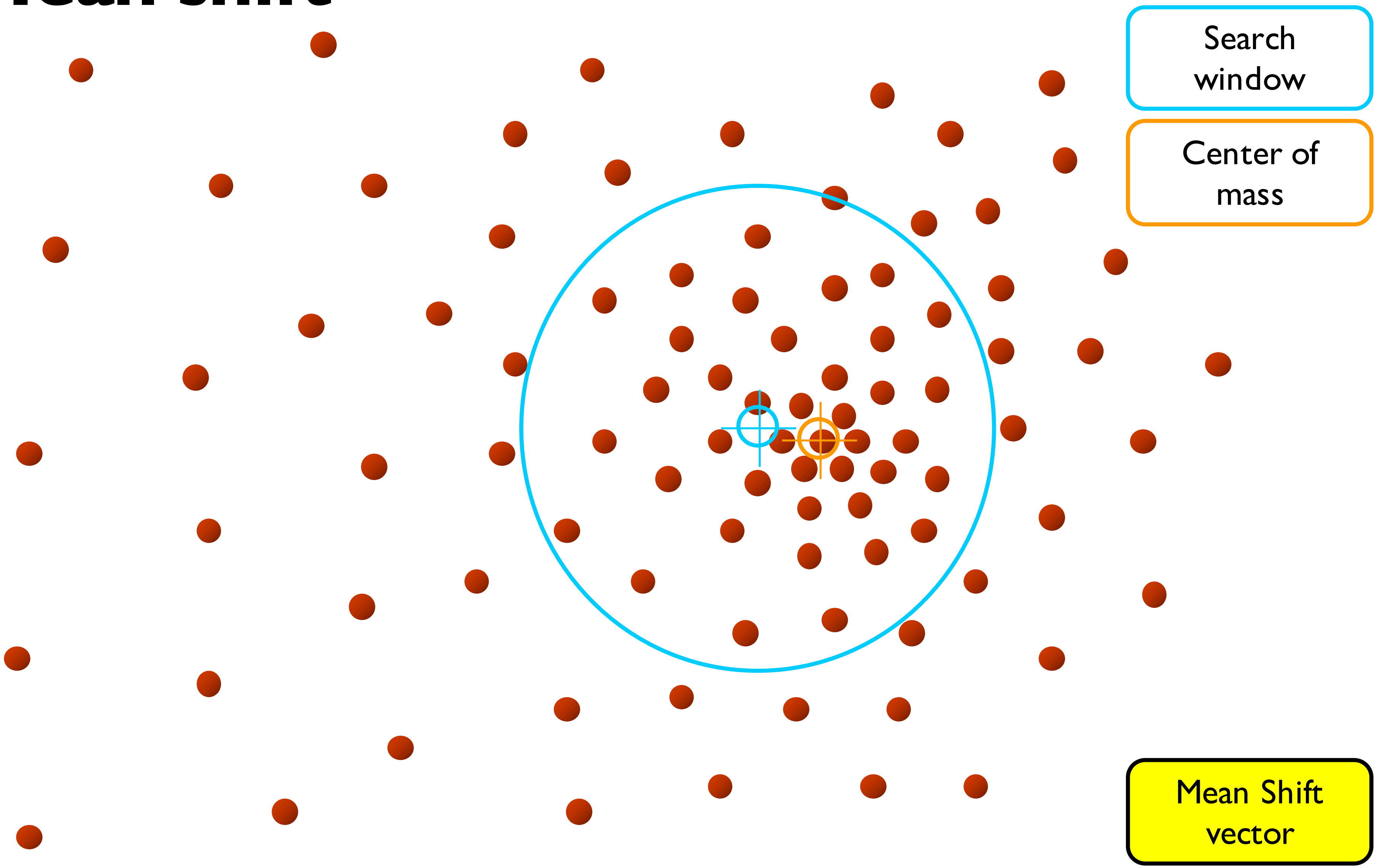
Mean shift



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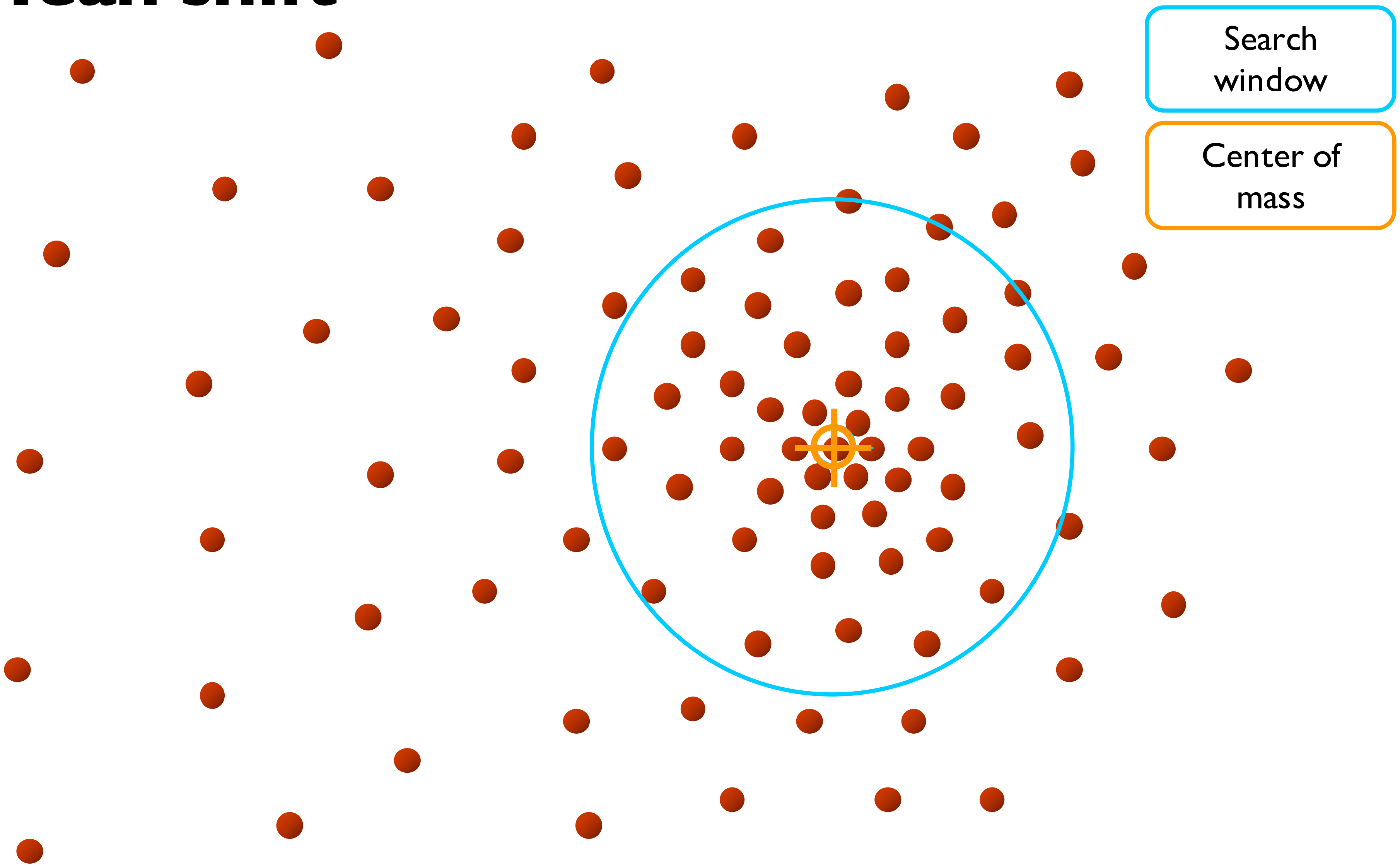
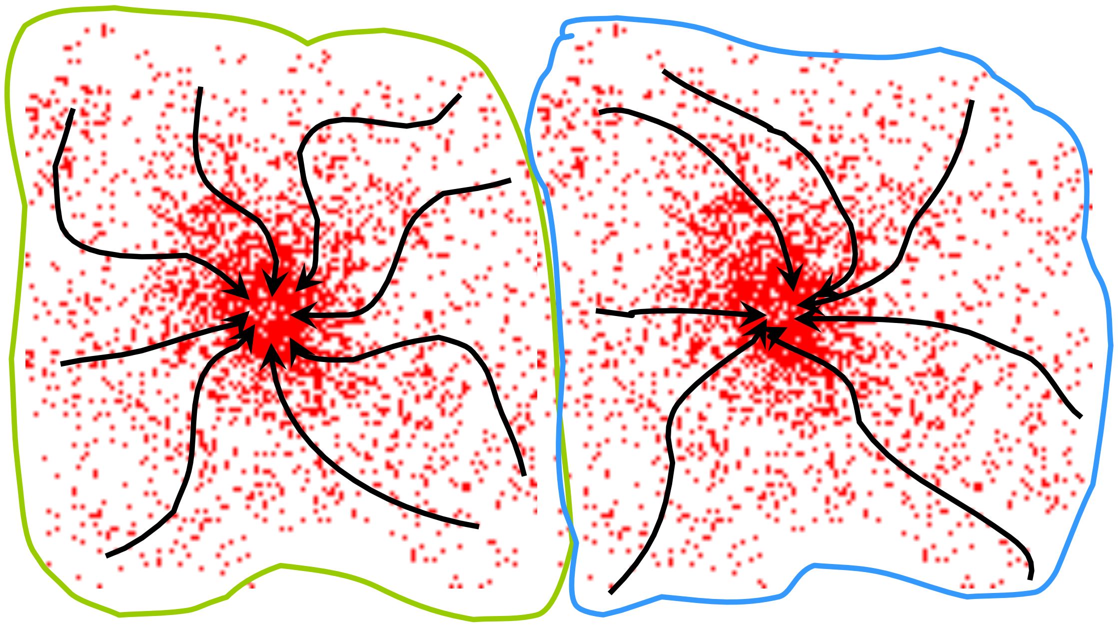
Mean shift



Slide credit: Y. Ukrainitz & B. Sarel

Mean shift clustering

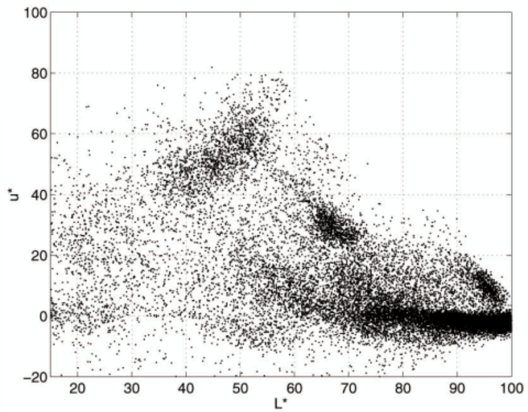
- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode



Slide credit: Y. Ukrainitz & B. Sarel

Mean shift clustering/segmentation

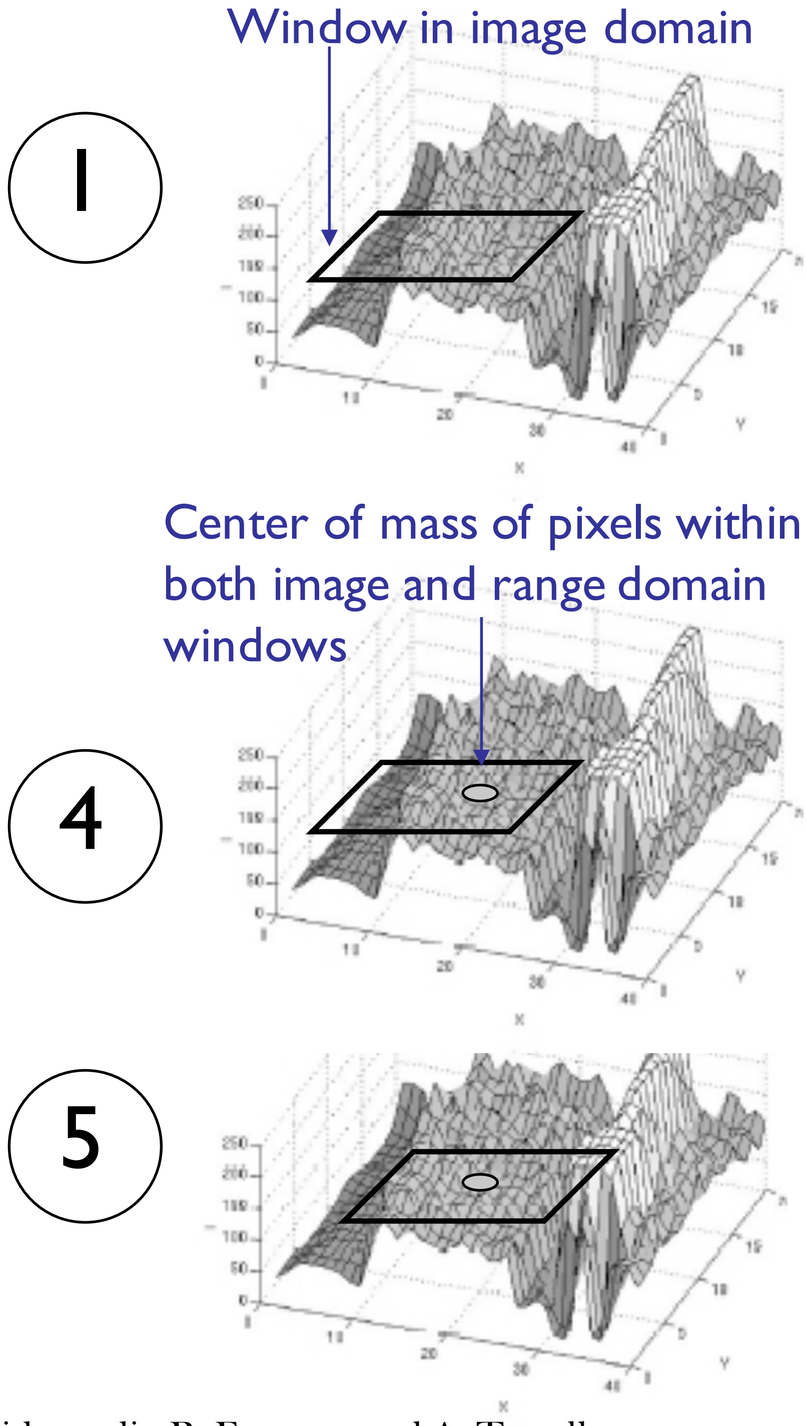
- Find features (color, gradients, texture, etc)
- Initialize windows at individual feature points
- Perform mean shift for each window until convergence
- Merge windows that end up near the same "peak" or mode

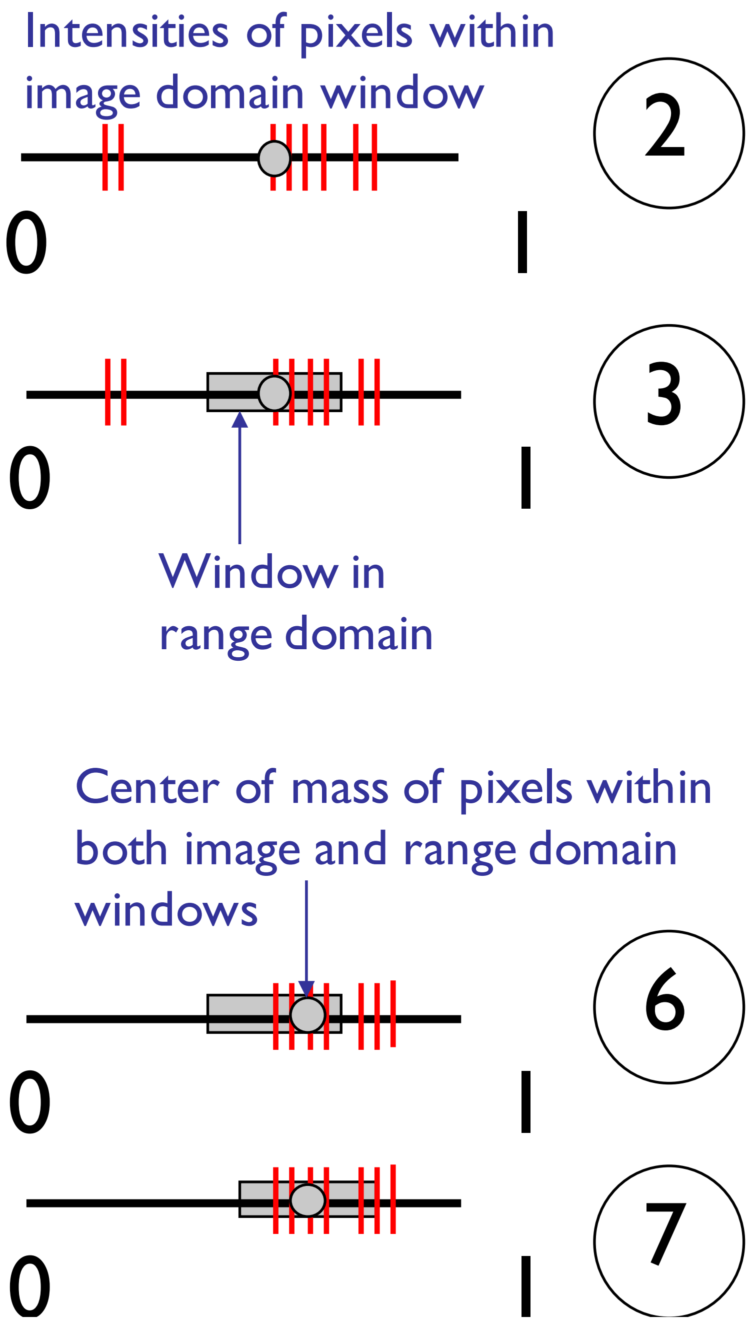


(a)

Slide credit: S. Lazebnik

Apply mean shift jointly in the image (left col.) and range (right col.) domains





Slide credit: B. Freeman and A. Torralba

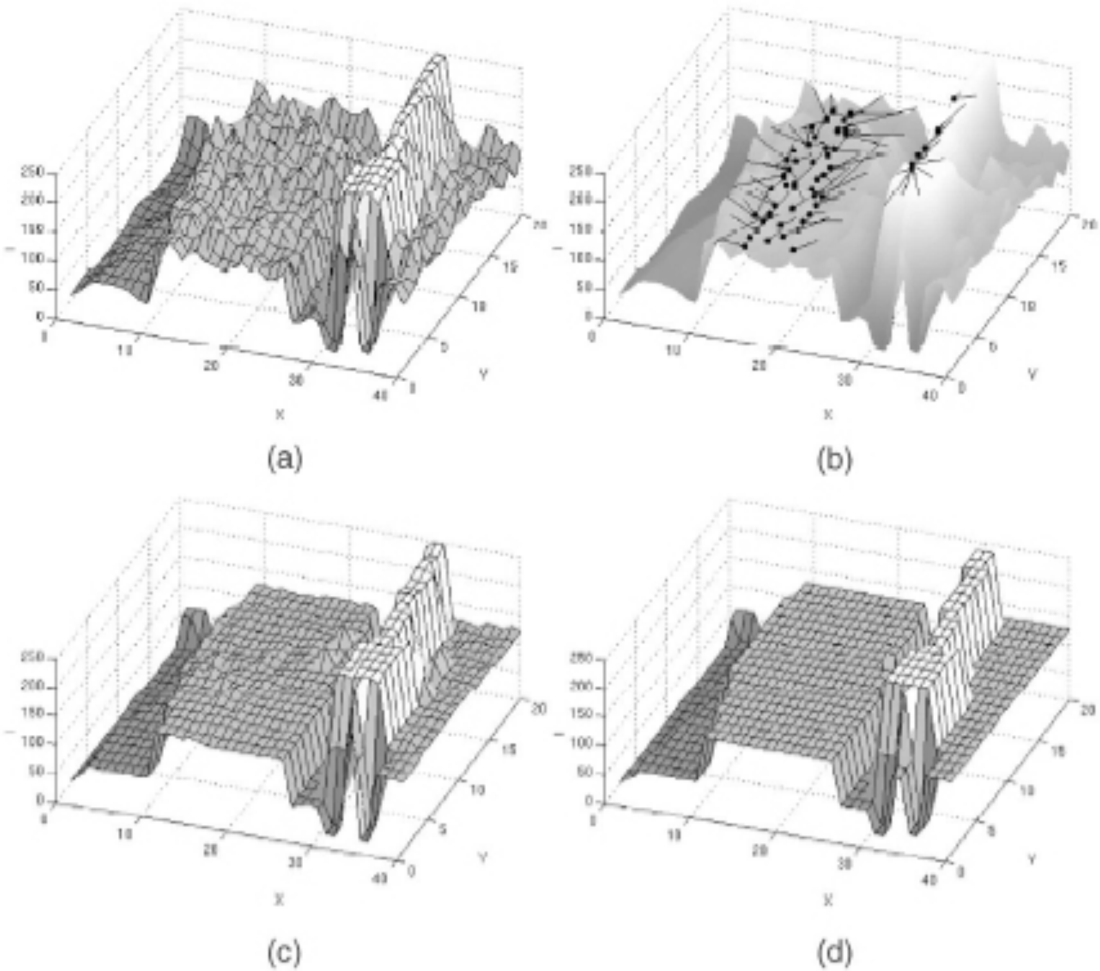
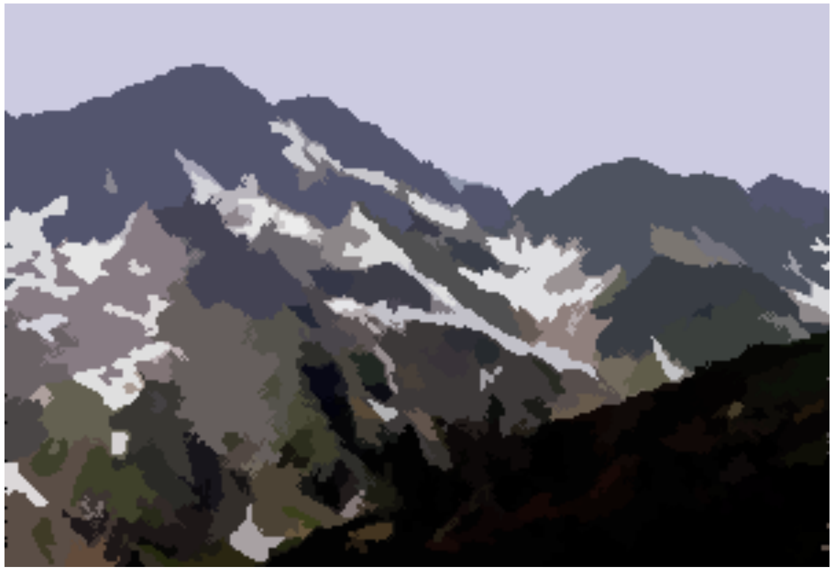


Fig. 4. Visualization of mean shift-based filtering and segmentation for gray-level data. (a) Input. (b) Mean shift paths for the pixels on the plateau and on the line. The black dots are the points of convergence. (c) Filtering result $(h_s, h_r) = (8, 4)$. (d) Segmentation result.

Comaniciu and Meer, IEEE PAMI vol. 24, no. 5, 2002

Slide credit: B. Freeman and A. Torralba

Mean shift segmentation results



http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

Slide credit: S. Lazebnik

More results

Slide credit: S. Lazebnik

More results

Slide credit: S. Lazebnik

Mean shift pros and cons

- Pros
  - Does not assume spherical clusters
  - Just a single parameter (window size)
  - Finds variable number of modes
  - Robust to outliers
- Cons
  - Output depends on window size
  - Computationally expensive
  - Does not scale well with dimension of feature space

Slide credit: S. Lazebnik

Segmentation methods

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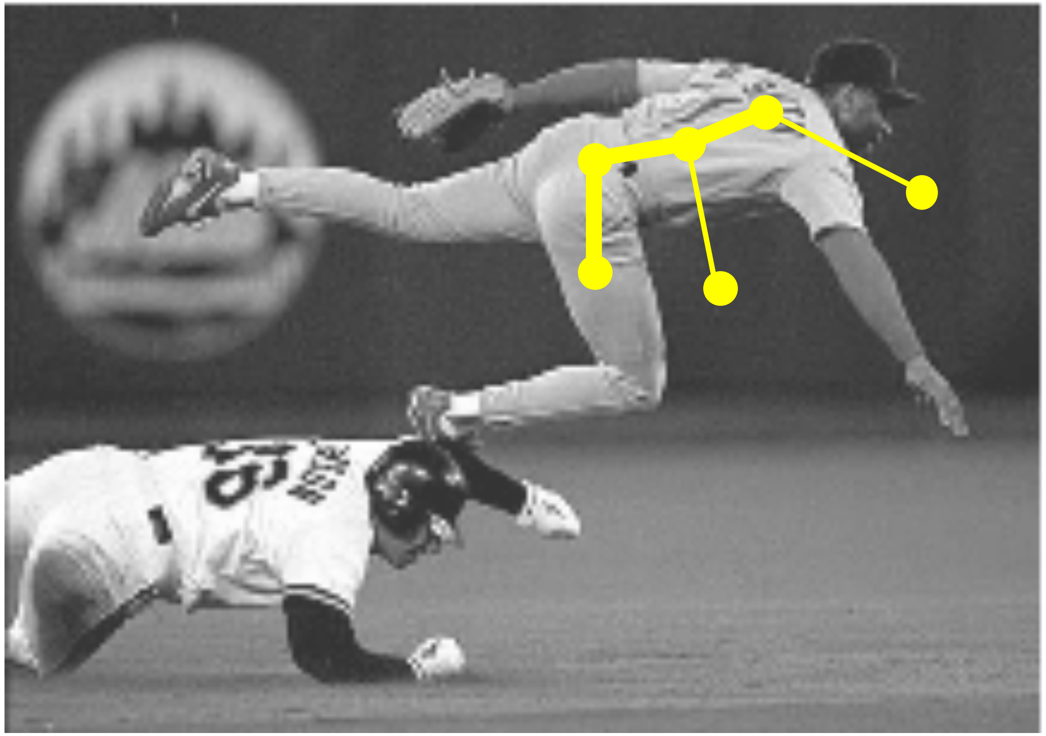
Graph-Theoretic Image Segmentation

Build a weighted graph G=(V,E) from image



V: image pixels

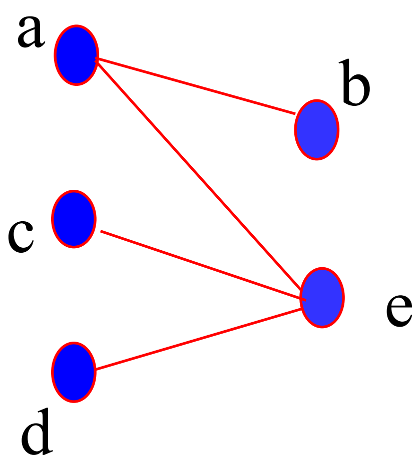
E: connections between pairs of nearby pixels

W_{ij} : probability that i &j belong to the same region

Segmentation = graph partition

Slide credit: B. Freeman and A. Torralba

Graphs Representations



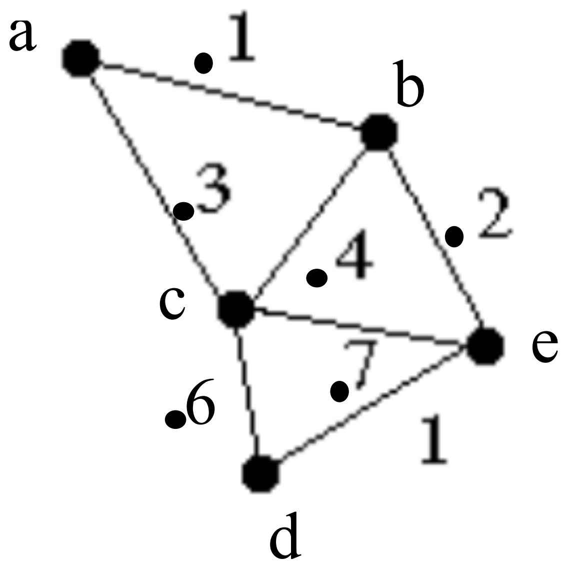
$$
\begin{array}{c c}
 & \begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} &
\begin{bmatrix}
0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

Adjacency Matrix

Slide credit: B. Freeman and A. Torralba

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

A Weighted Graph and its Representation



Affinity Matrix

$$W = \begin{bmatrix} 1 & .1 & .3 & 0 & 0 \\ .1 & 1 & .4 & 0 & .2 \\ .3 & .4 & 1 & .6 & .7 \\ 0 & 0 & .6 & 1 & 1 \\ 0 & .2 & .7 & 1 & 1 \end{bmatrix}$$

W_{ij} : probability that i &j belong to the same region

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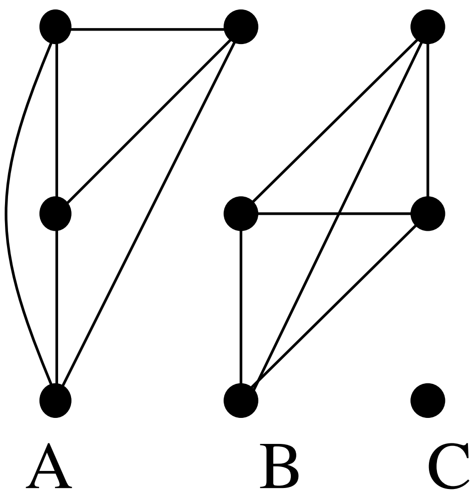
* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

Segmentation by graph partitioning



- Break graph into segments
  - Delete links that cross between segments
  - Easiest to break links that have low affinity
    - similar pixels should be in the same segments
    - dissimilar pixels should be in different segments

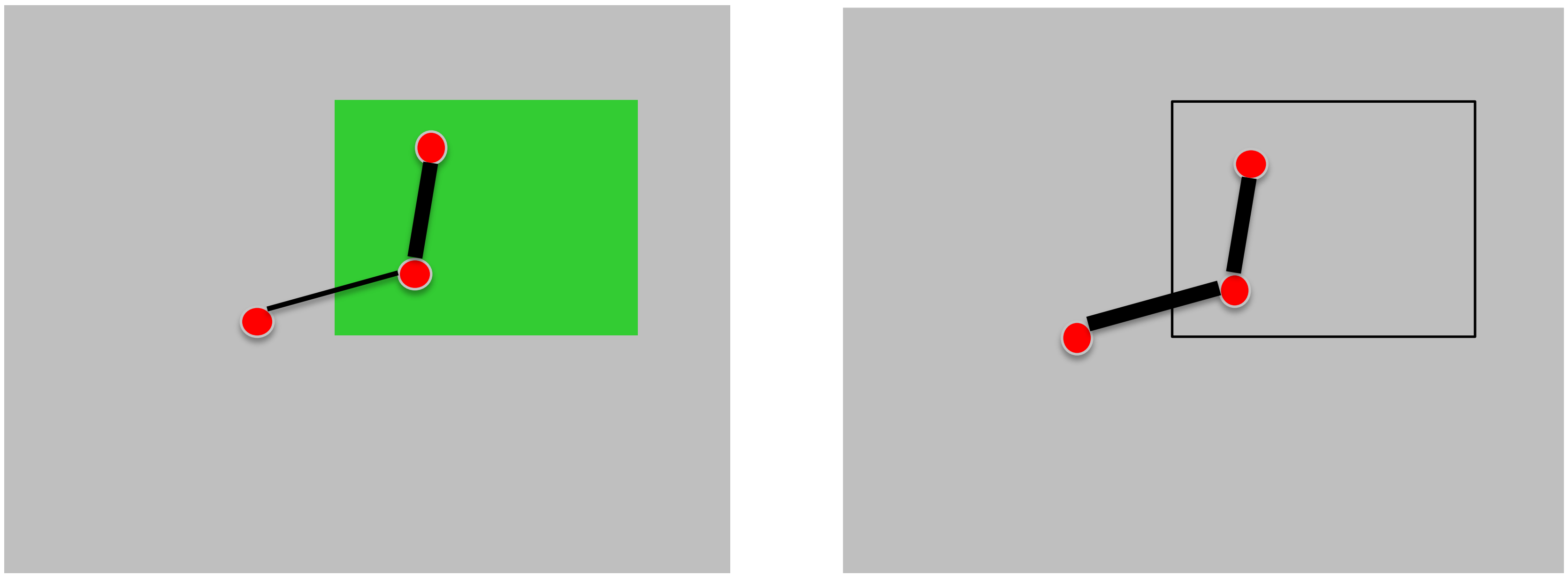
Slide credit: S. Seitz

Affinity between pixels

Similarities among pixel descriptors

$$W_{ij} = \exp(-\| z_i - z_j \|^2 / \sigma^2)$$

σ = Scale factor…
it will hunt us later



Slide credit: B. Freeman and A. Torralba

Affinity between pixels

Similarities among pixel descriptors

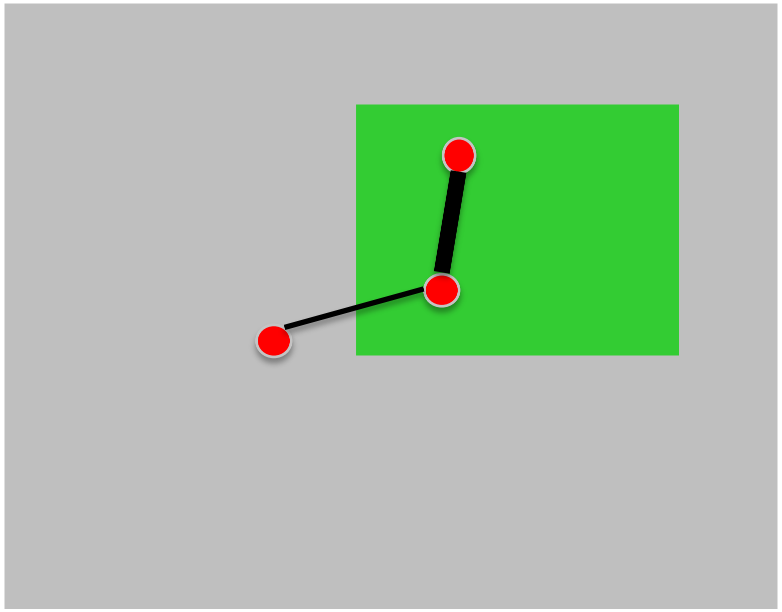
$$W_{ij} = \exp(-\| z_i - z_j \|^2 / \sigma^2)$$

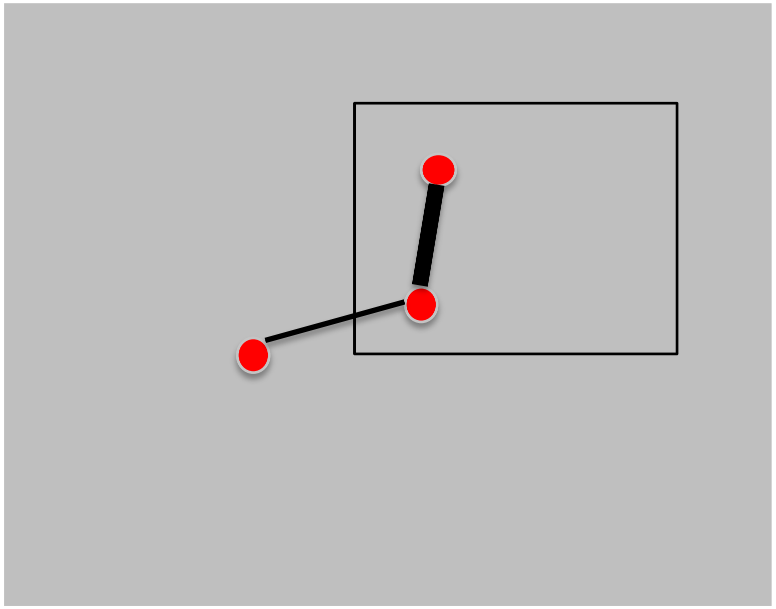
σ = Scale factor…
it will hunt us later

Interleaving edges

$$W_{ij} = 1 - \max_{\text{Line between i and j}} Pb$$

With Pb = probability of boundary

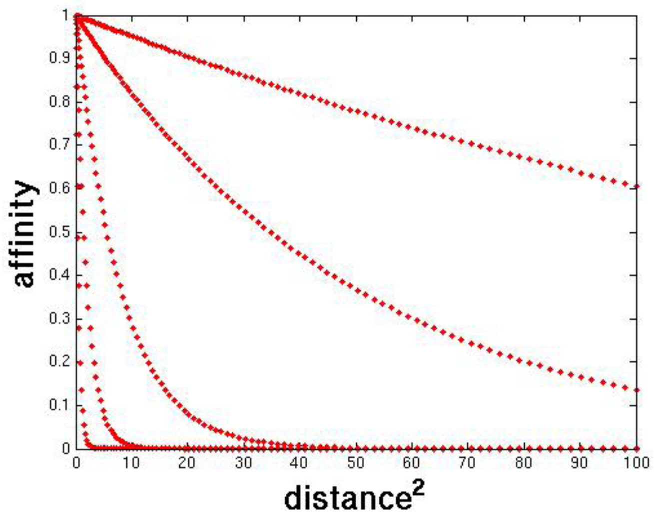


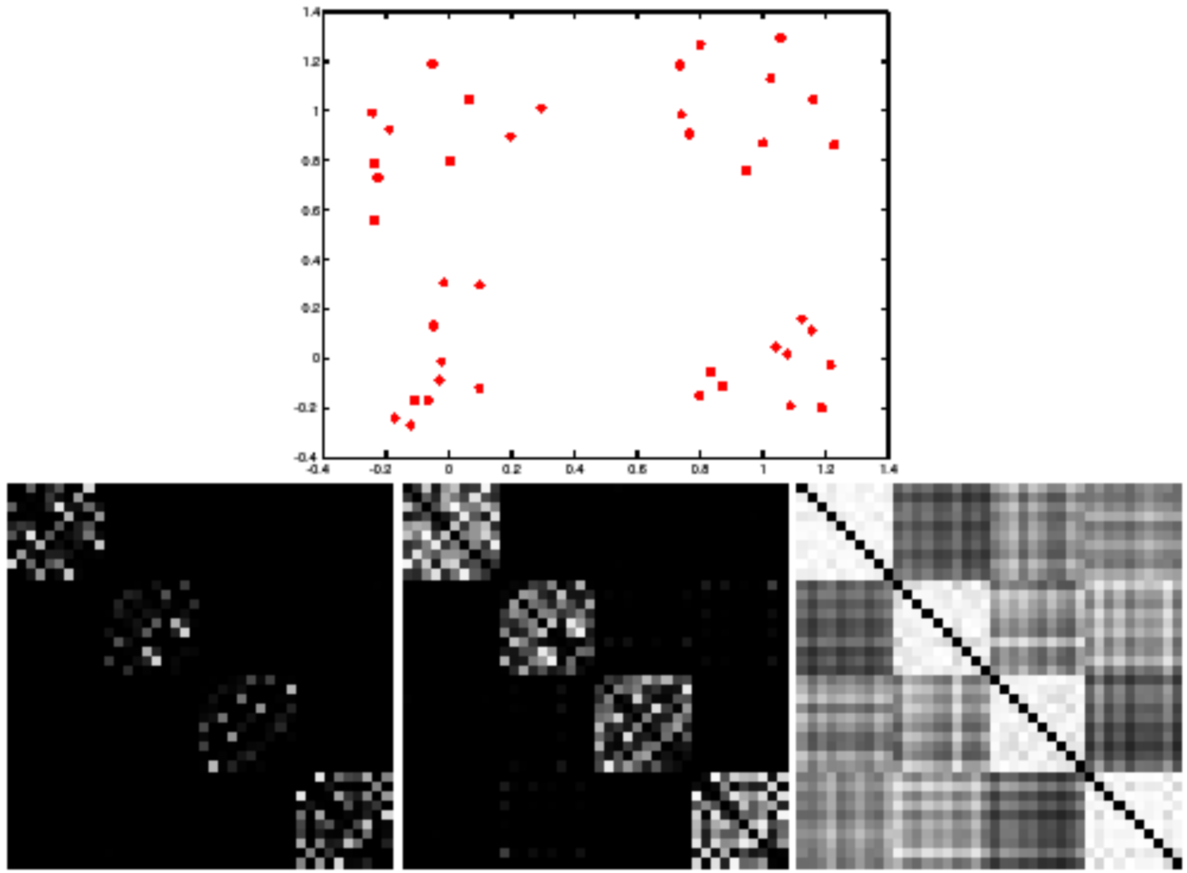


Slide credit: B. Freeman and A. Torralba

Scale affects affinity

- Small σ: group only nearby points
- Large σ: group far-away points





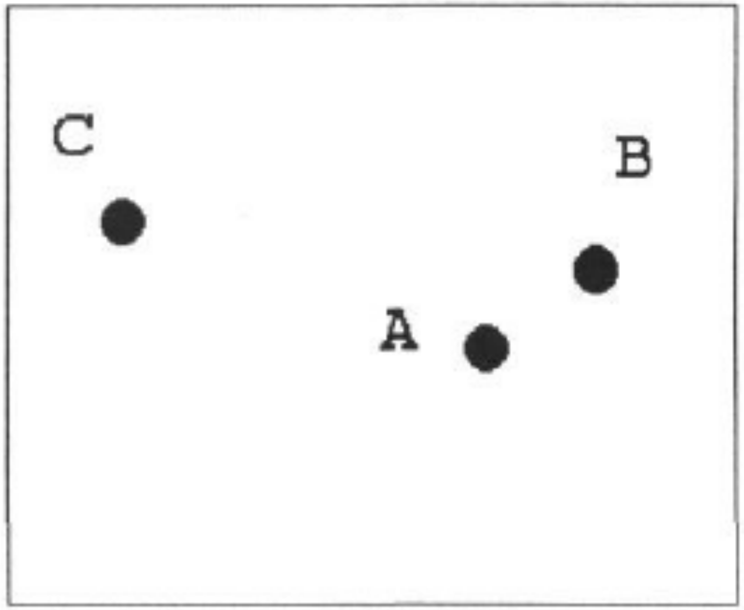
Slide credit: S. Lazebnik

Feature grouping by "relocalisation" of eigenvectors of the proximity matrix

British Machine Vision Conference, pp. 103-108, 1990

Guy L. Scott
Robotics Research Group
Department of Engineering Science
University of Oxford

H. Christopher Longuet-Higgins
University of Sussex
Falmer
Brighton



Three points in feature space

$$W_{ij} = \exp(-\| z_i - z_j \|^2 / \sigma^2)$$

With an appropriate σ

W=

	A	B	C
A	1.00	0.63	0.03
B	0.63	1.00	0.0
C	0.03	0.0	1.00

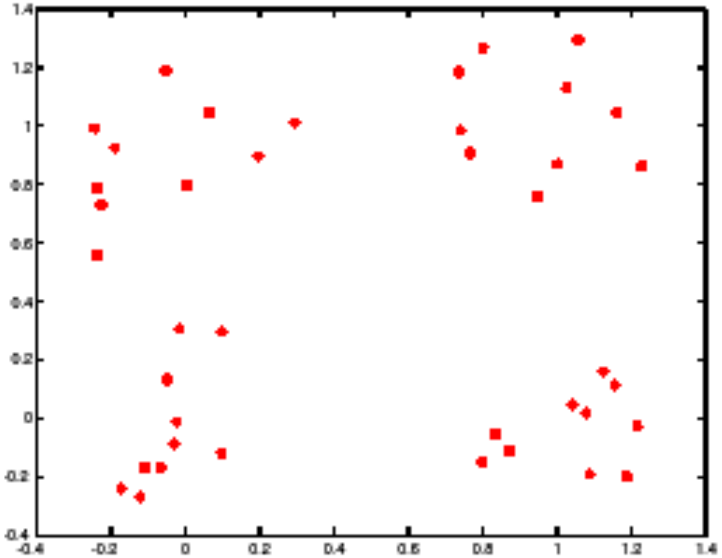
The eigenvectors of W are:

	E_1	E_2	E_3
Eigenvalues	1.63	1.00	0.37
A	-0.71	-0.01	0.71
B	-0.71	-0.05	-0.71
C	-0.04	1.00	-0.03

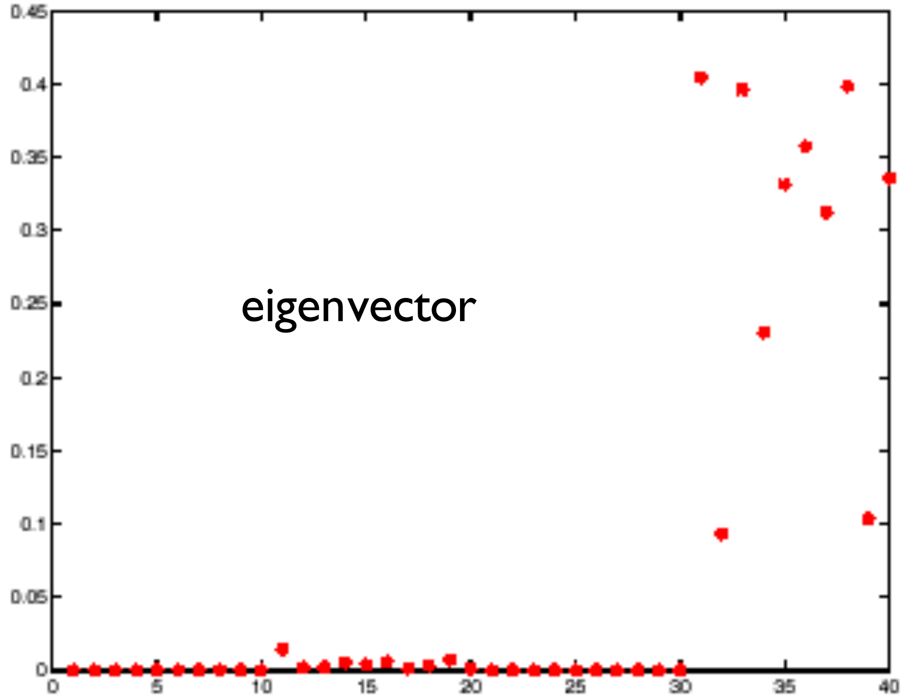
The first 2 eigenvectors group the points as desired…

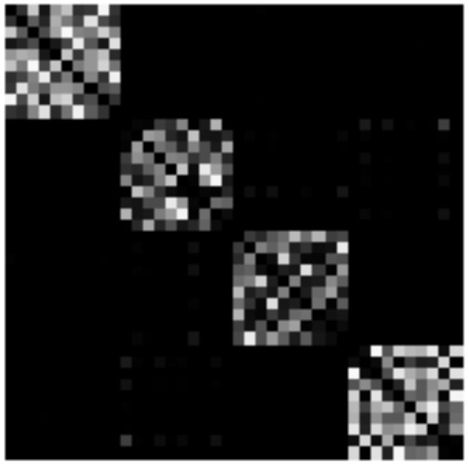
Slide credit: B. Freeman and A. Torralba

Example eigenvector



points

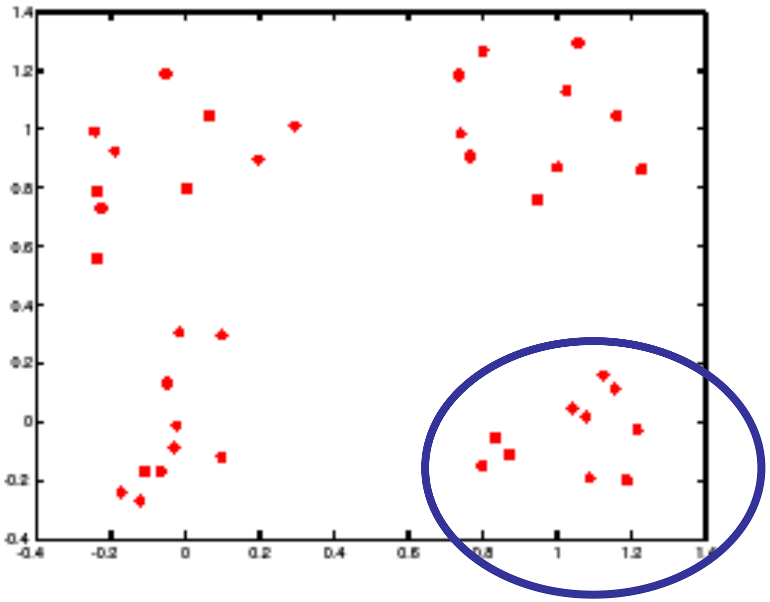




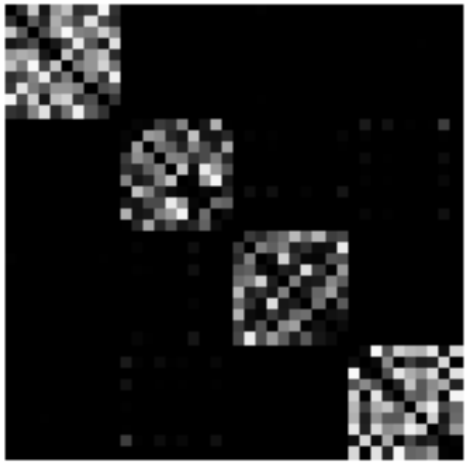
Affinity matrix

Slide credit: B. Freeman and A. Torralba

Example eigenvector



points



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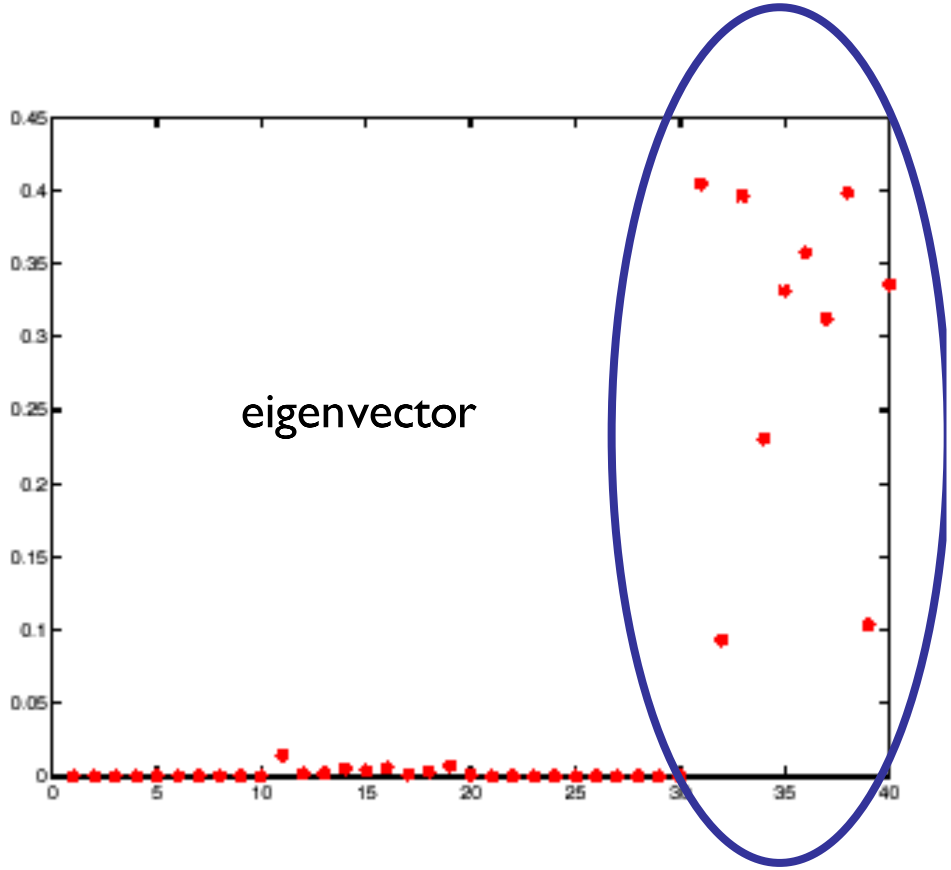
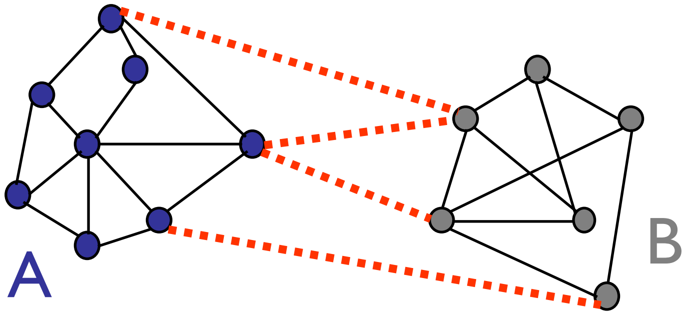


eigenvector

Slide credit: B. Freeman and A. Torralba

Graph cut



- Set of edges whose removal makes a graph disconnected
- Cost of a cut: sum of weights of cut edges
- A graph cut gives us a segmentation
  - What is a "good" graph cut and how do we find one?

Slide credit: S. Seitz

Segmentation methods

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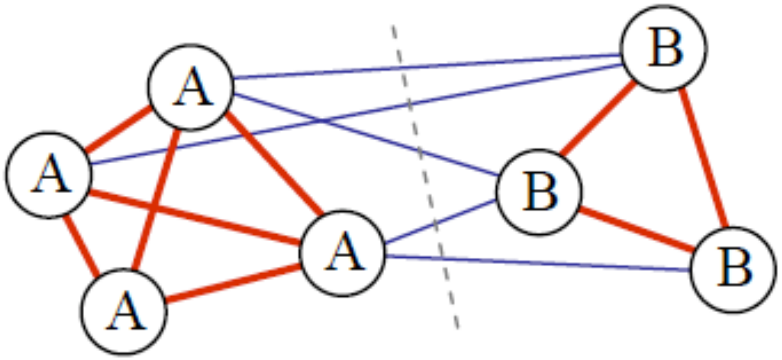
Minimum cut

A cut of a graph *G* is the set of edges *S* such that removal of *S* from *G* disconnects *G*.



Cut: sum of the weight of the cut edges:

$$cut(\mathrm{A},\mathrm{B}) = \sum_{u \in \mathrm{A}, v \in \mathrm{B}} \mathrm{W}(u,v),$$

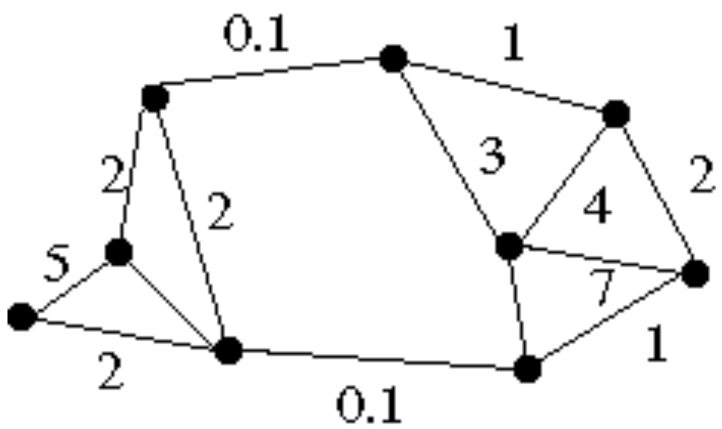
with $\mathrm{A} \cap \mathrm{B} = \varnothing$

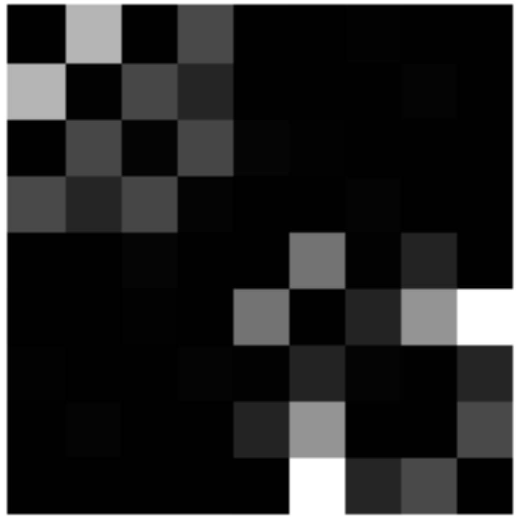
Slide credit: B. Freeman and A. Torralba

Minimum cut

- We can do segmentation by finding the *minimum cut* in a graph
  - Efficient algorithms exist for doing this

Minimum cut example



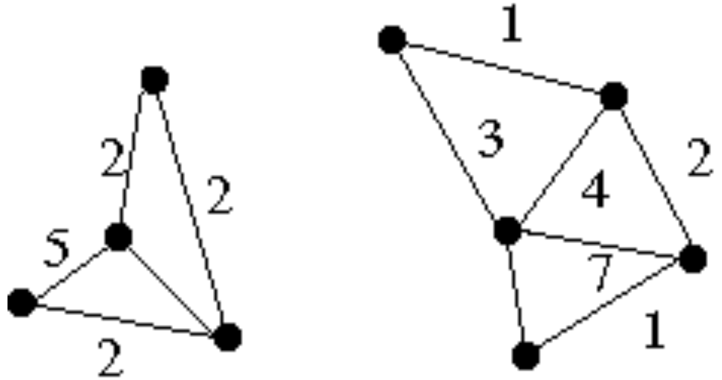


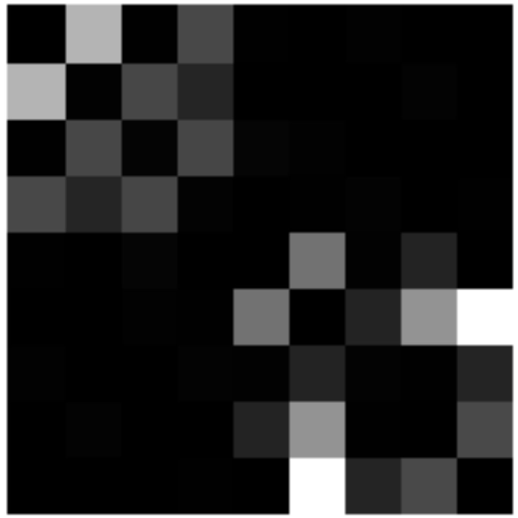
Slide credit: S. Lazebnik

Minimum cut

- We can do segmentation by finding the *minimum cut* in a graph
  - Efficient algorithms exist for doing this

Minimum cut example

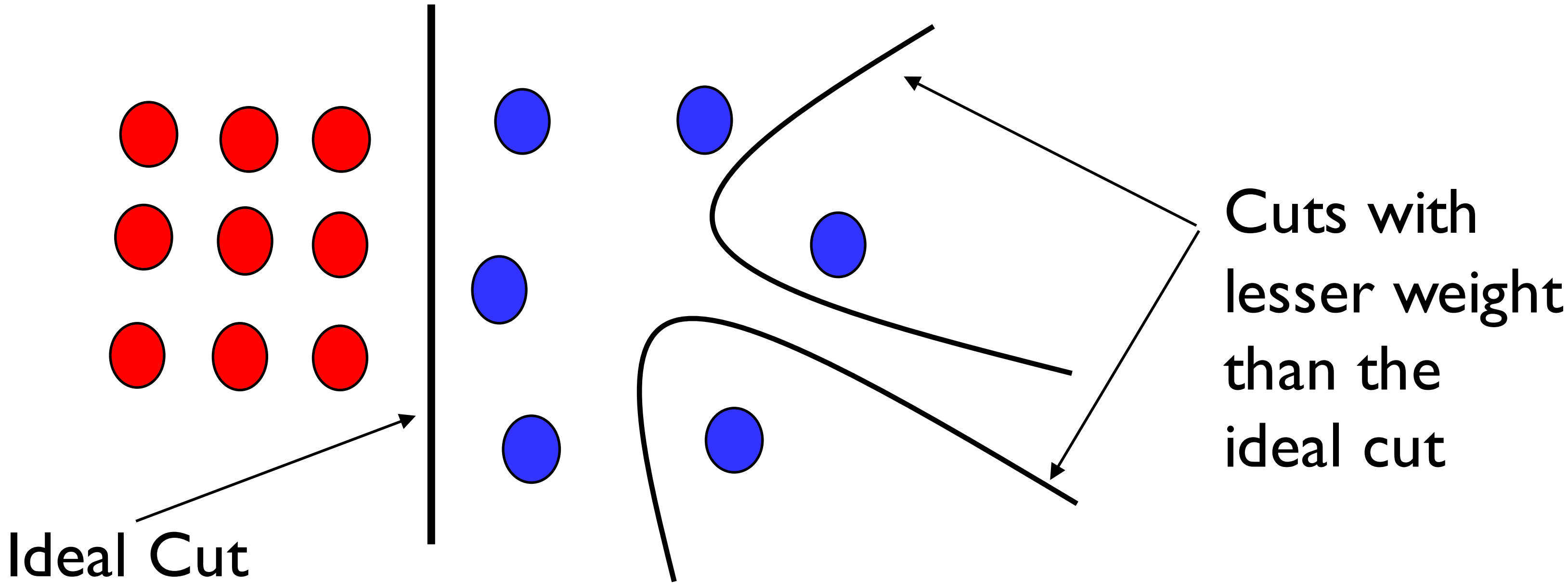




Slide credit: S. Lazebnik

Drawbacks of Minimum cut

- Weight of cut is directly proportional to the number of edges in the cut.



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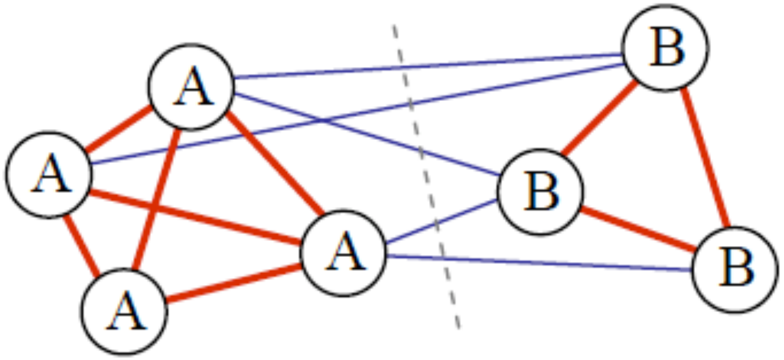
Normalized cuts

Write graph as V, one cluster as A and the other as B



$$\text{Ncut(A,B)} = \frac{\text{cut(A,B)}}{\text{assoc(A,V)}} + \frac{\text{cut(A,B)}}{\text{assoc(B,V)}}$$

cut(A,B) is sum of weights with one end in A and one end in B

$$cut(\mathrm{A,B}) = \sum_{u \in \mathrm{A}, v \in \mathrm{B}} \mathrm{W}(u,v),$$

with $\mathrm{A} \cap \mathrm{B} = \varnothing$

assoc(A,V) is sum of all edges with one end in A.

$$assoc(\mathrm{A,B}) = \sum_{u \in \mathrm{A}, v \in \mathrm{B}} \mathrm{W}(u,v)$$

A and B not necessarily disjoint

J. Shi and J. Malik. Normalized cuts and image segmentation. PAMI 2000

Slide credit: B. Freeman and A. Torralba

Normalized cut

- Let W be the adjacency matrix of the graph
- Let D be the diagonal matrix with diagonal entries
  $D(i, i) = \Sigma_j W(i, j)$
- Then the normalized cut cost can be written as

$$\frac{y^T (D - W) y}{y^T D y}$$

where y is an indicator vector whose value should be 1 in the ith position if the ith feature point belongs to A and a negative constant otherwise

J. Shi and J. Malik. Normalized cuts and image segmentation. PAMI 2000

Slide credit: S. Lazebnik

Normalized cut

- Finding the exact minimum of the normalized cut cost is NP-complete, but if we *relax* y to take on arbitrary values, then we can minimize the relaxed cost by solving the *generalized eigenvalue problem* $(D - W)y = \lambda Dy$
- The solution y is given by the generalized eigenvector corresponding to the second smallest eigenvalue
- Intitutively, the ith entry of y can be viewed as a "soft" indication of the component membership of the ith feature
  - Can use 0 or median value of the entries as the splitting point (threshold), or find threshold that minimizes the Ncut cost

J. Shi and J. Malik. Normalized cuts and image segmentation. PAMI 2000

Slide credit: S. Lazebnik

Normalized cut algorithm

1. Given an image or image sequence, set up a weighted graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$, and set the weight on the edge connecting two nodes being a measure of the similarity between the two nodes.

2. Solve $(\mathbf{D} - \mathbf{W})\boldsymbol{x} = \lambda \mathbf{D}\boldsymbol{x}$ for eigenvectors with the smallest eigenvalues.

3. Use the eigenvector with second smallest eigenvalue to bipartition the graph.

4. Decide if the current partition should be sub-divided, and recursively repartition the segmented parts if necessary.

Slide credit: B. Freeman and A. Torralba

Global optimization

- In this formulation, the segmentation becomes a global process.
- Decisions about what is a boundary are not local (as in Canny edge detector)

Slide credit: B. Freeman and A. Torralba

Boundaries of image regions defined by a number of attributes

- Brightness/color
- Texture
- Motion
- Stereoscopic depth
- Familiar configuration

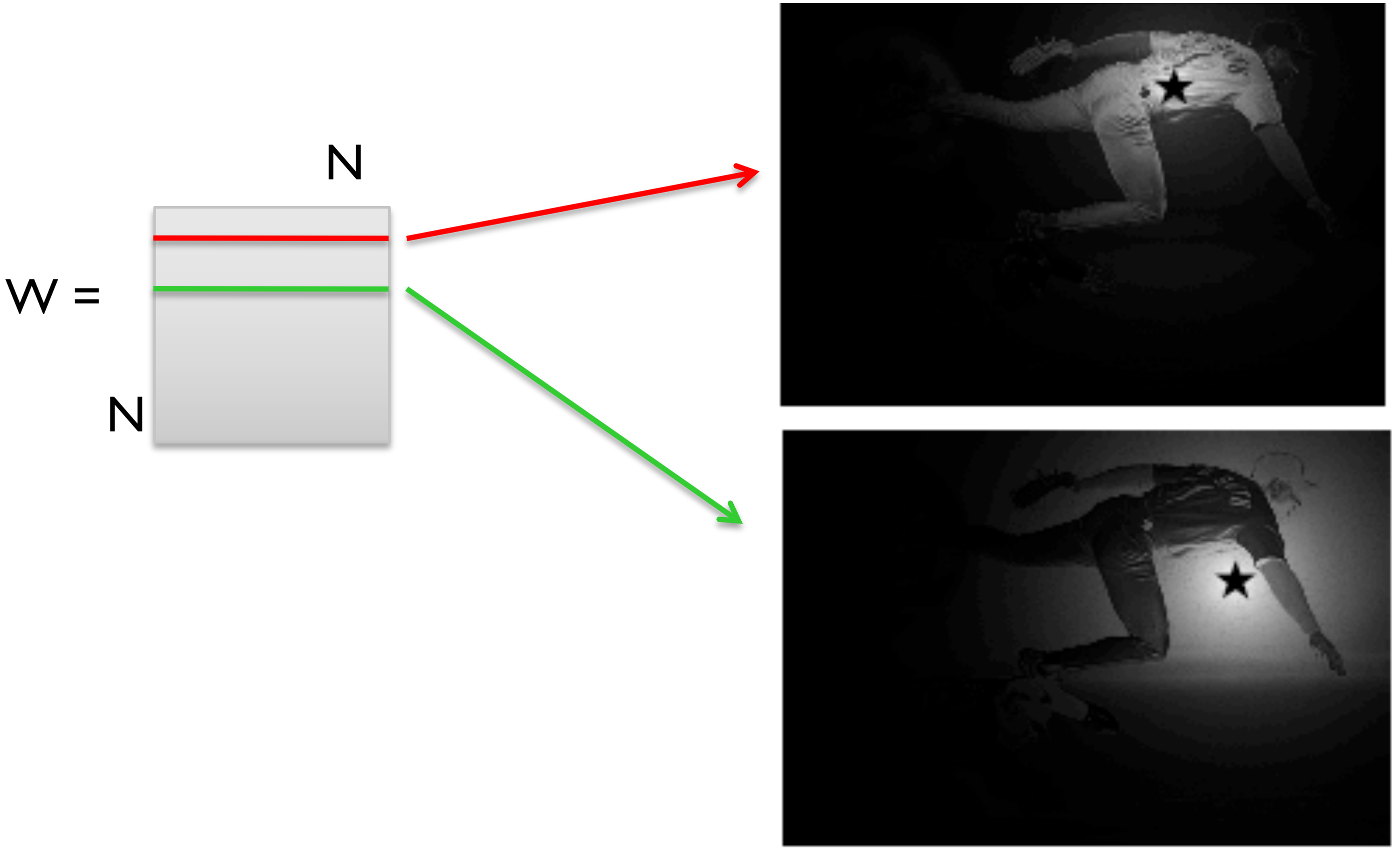
[Malik]

Example

Affinity:

$$w_{ij} = \underbrace{e^{\frac{-\|F_{(i)}-F_{(j)}\|_2^2}{\sigma_I}}}_{\text{brightness}} * \underbrace{\begin{cases} e^{\frac{-\|X_{(i)}-X_{(j)}\|_2^2}{\sigma_X}} & \text{if } \|X(i)-X(j)\|_2 < r \\ 0 & \text{otherwise} \end{cases}}_{\text{Location}}$$

N pixels = ncols * nrows



Slide credit: B. Freeman and A. Torralba

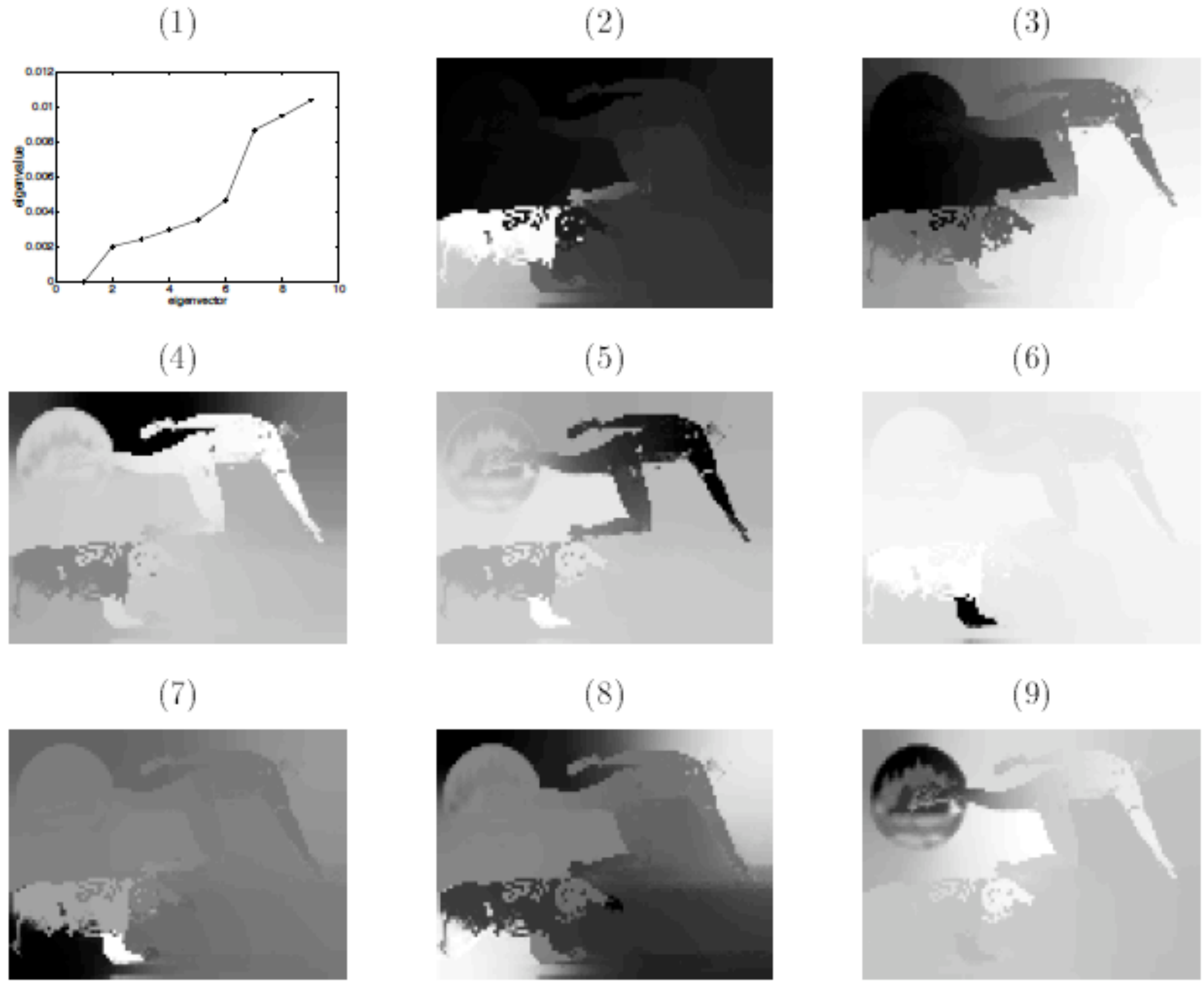


Figure 12: Subplot (1) plots the smallest eigenvectors of the generalized eigenvalue system (11). Subplot (2) - (9) shows the eigenvectors corresponding the 2nd smallest to the 9th smallest eigenvalues of the system. The eigenvectors are reshaped to be the size of the image.

Slide credit: B. Freeman and A. Torralba

Brightness Image Segmentation

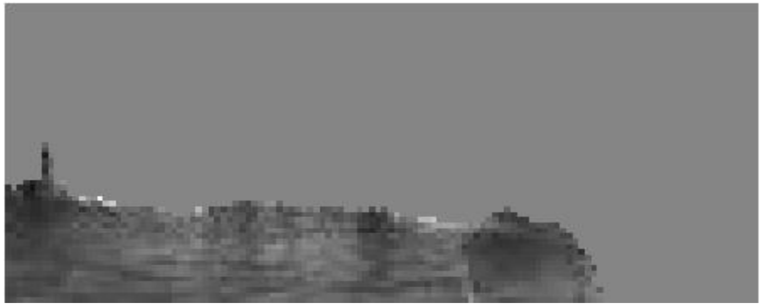
converge. On the 100×120 test images shown here, the normalized cut algorithm takes about 2 minutes on Intel Pentium 200MHz machines.

A multiresolution implementation can be used to reduce this running time further on larger images. In our current experiments, with this implementation, the running time on a 300×400 image can be reduced to about 20 seconds on Intel Pentium 300MHz machines. Furthermore, the bottle-neck of the computation, a sparse matrix-vector

http://www.cs.berkeley.edu/~malik/papers/SM-ncut.pdf

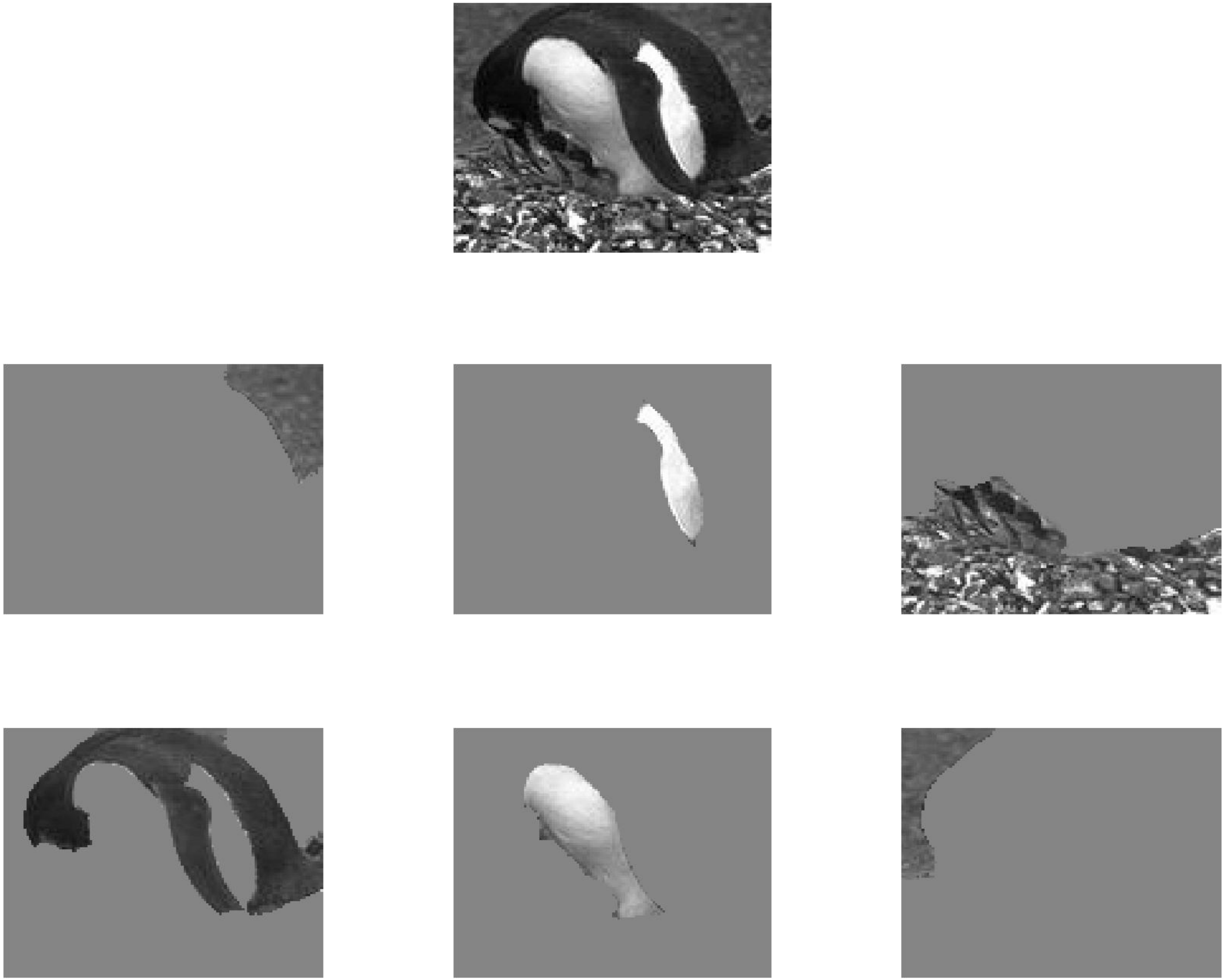
Slide credit: B. Freeman and A. Torralba

Brightness Image Segmentation



http://www.cs.berkeley.edu/~malik/papers/SM-ncut.pdf

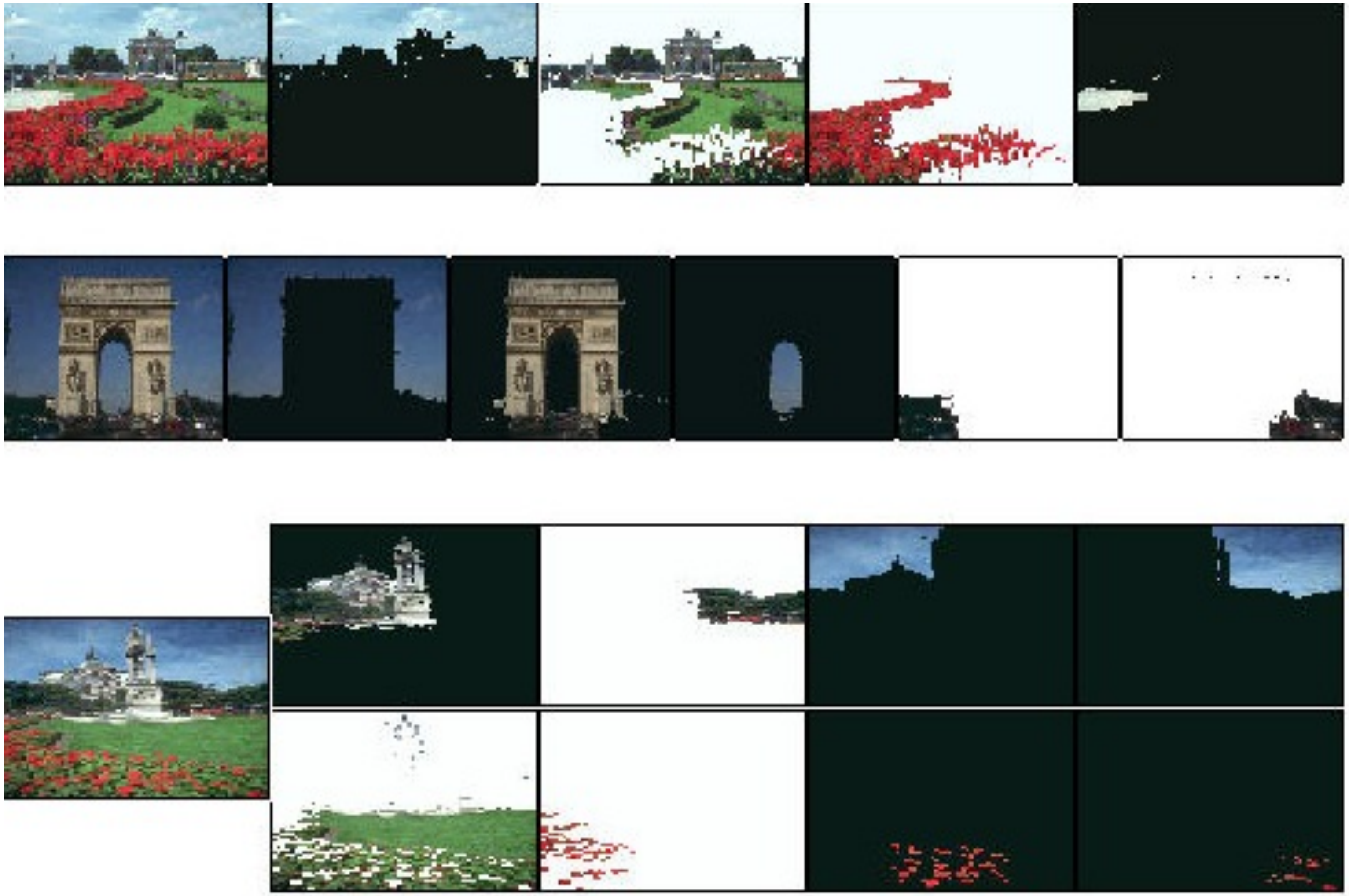
Slide credit: B. Freeman and A. Torralba



http://www.cs.berkeley.edu/~malik/papers/SM-ncut.pdf

Slide credit: B. Freeman and A. Torralba

Results on color segmentation



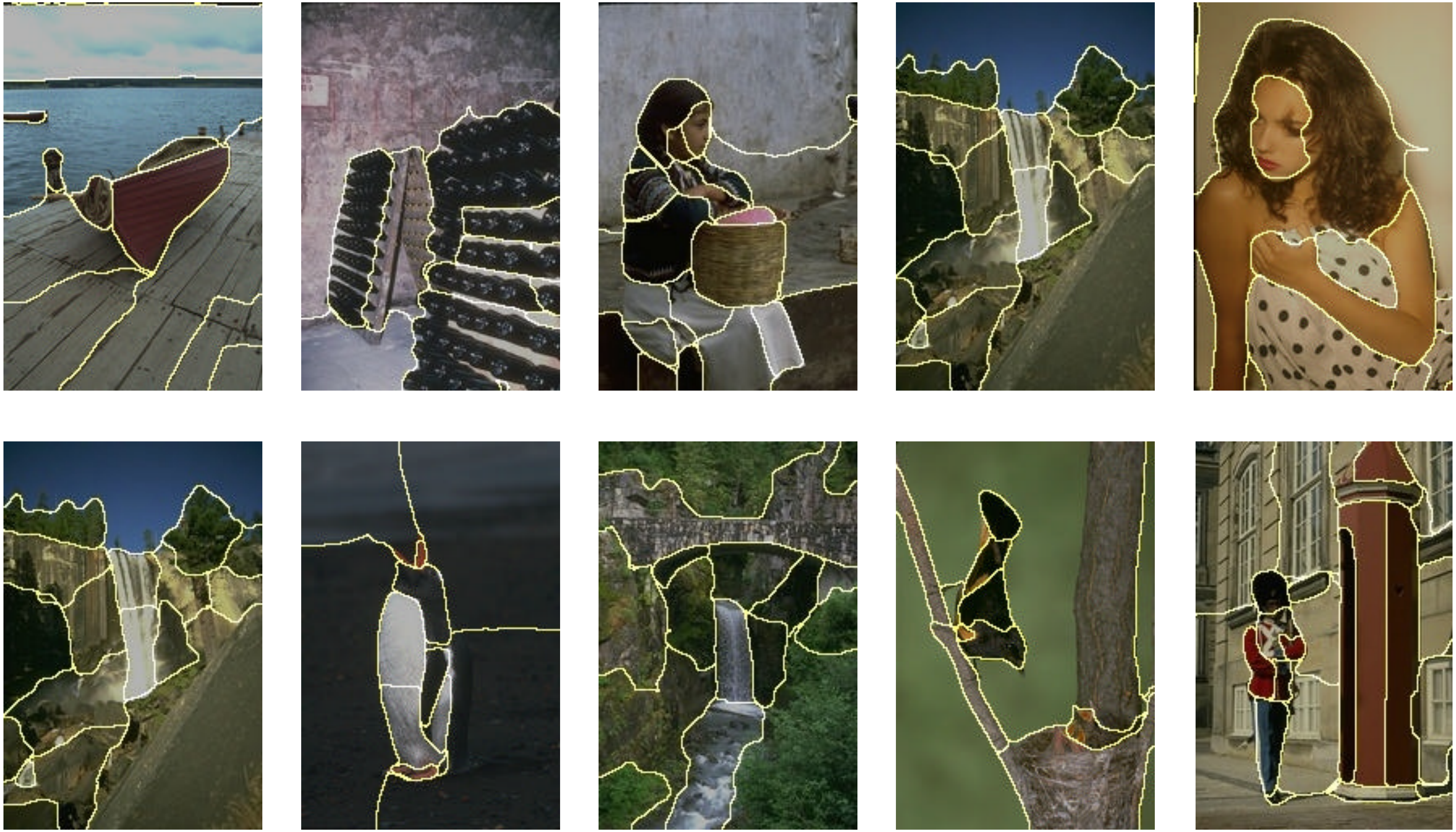
http://www.cs.berkeley.edu/~malik/papers/SM-ncut.pdf

Slide credit: B. Freeman and A. Torralba

Example results

Slide credit: S. Lazebnik

Results: Berkeley Segmentation Engine



http://www.cs.berkeley.edu/~fowlkes/BSE/

Slide credit: S. Lazebnik

Normalized cuts: Pro and con

- Pros
  - Generic framework, can be used with many different features and affinity formulations
- Cons
  - High storage requirement and time complexity
  - Bias towards partitioning into equal segments

Slide credit: S. Lazebnik

Segmentation methods

- Segment foreground from background
- Histogram-based segmentation
- Segmentation as clustering
  - K-means clustering
  - Mean-shift segmentation
- Graph-theoretic segmentation
    - Min cut
    - Normalized cuts
- Interactive segmentation

Intelligent Scissors [Mortensen 95]

- Approach answers a basic question
  - Q: how to find a path from seed to mouse that follows object boundary as closely as possible?

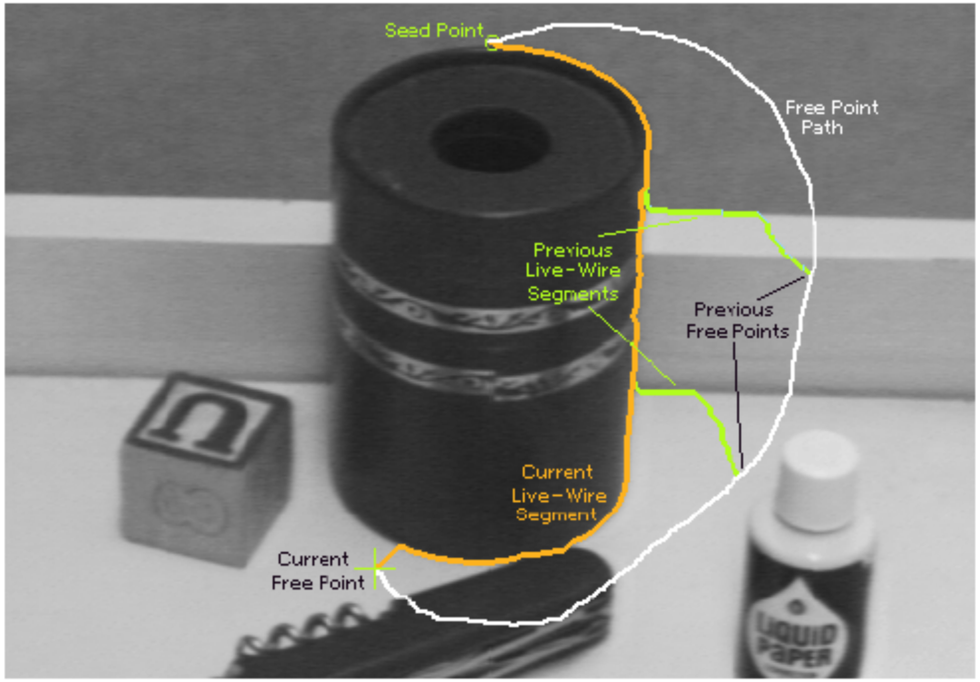


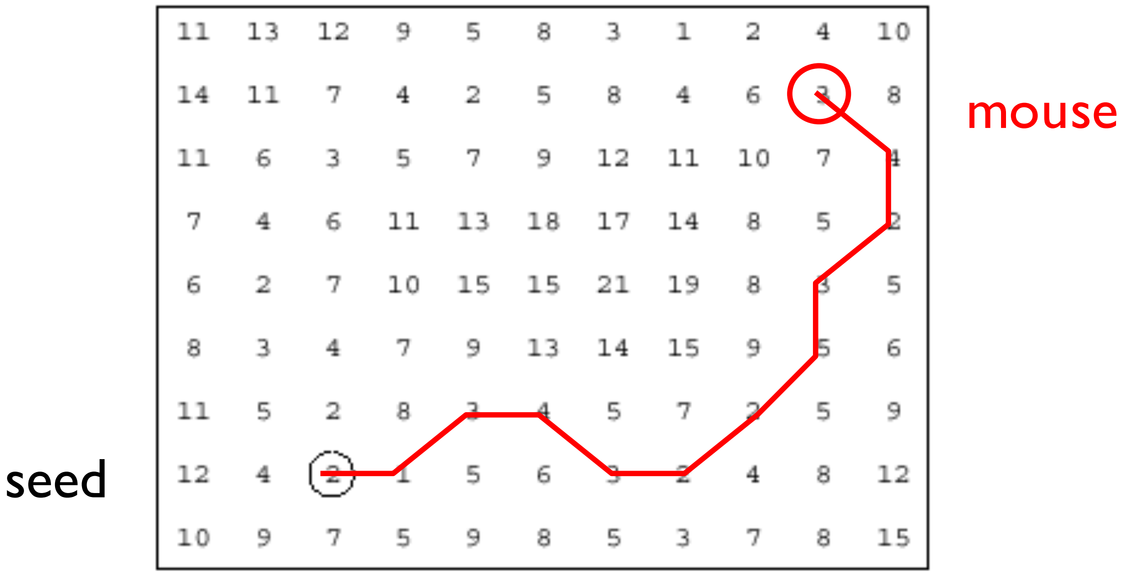
Figure 2: *Image demonstrating how the live-wire segment adapts and snaps to an object boundary as the free point moves (via cursor movement). The path of the free point is shown in white. Live-wire segments from previous free point positions (t_0, t_1, and t_2) are shown in green.*

Mortensen and Barrett, Intelligent Scissors for Image Composition, Proc. 22nd annual conference on Computer graphics and interactive techniques, 1995

Slide credit: S. Seitz

Intelligent Scissors

- Basic Idea
  - Define edge score for each pixel
    - edge pixels have low cost
  - Find lowest cost path from seed to mouse



Questions

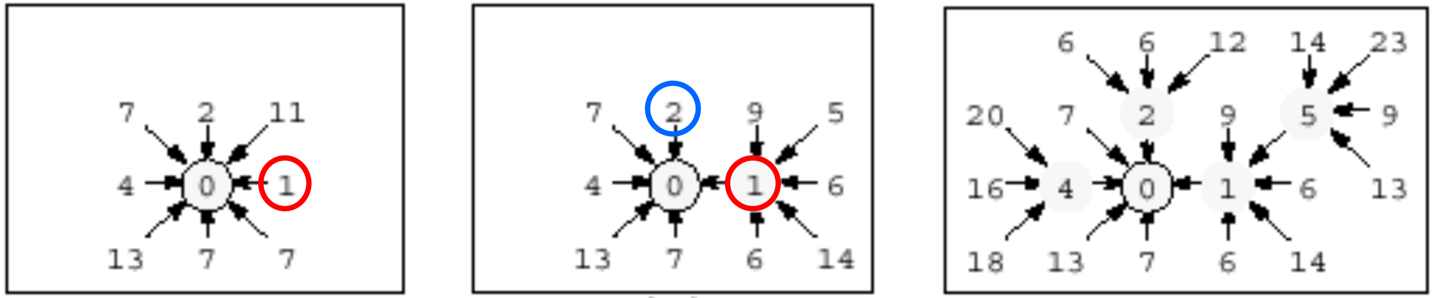
- How to define costs?
- How to find the path?

Slide credit: S. Seitz

Path Search (basic idea)

- Graph Search Algorithm
  - Computes minimum cost path from seed to *all other pixels*

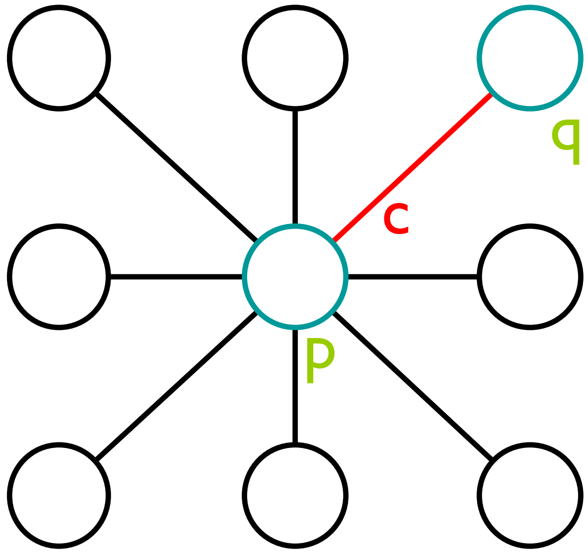
11	13	12	9	5	8	3	1	2	4	10
14	11	7	4	2	5	8	4	6	3	8
11	6	3	5	7	9	12	11	10	7	4
7	4	6	11	13	18	17	14	8	5	2
6	2	7	10	15	15	21	19	8	3	5
8	3	4	7	9	13	14	15	9	5	6
11	5	2	8	3	4	5	7	2	5	9
12	4	2	1	5	6	3	2	4	8	12
10	9	7	5	9	8	5	3	7	8	15



Slide credit: S. Seitz

How does this really work?

- Treat the image as a graph



Graph

- node for every pixel **p**
- link between every adjacent pair of pixels, **p**,**q**
- cost **c** for each link

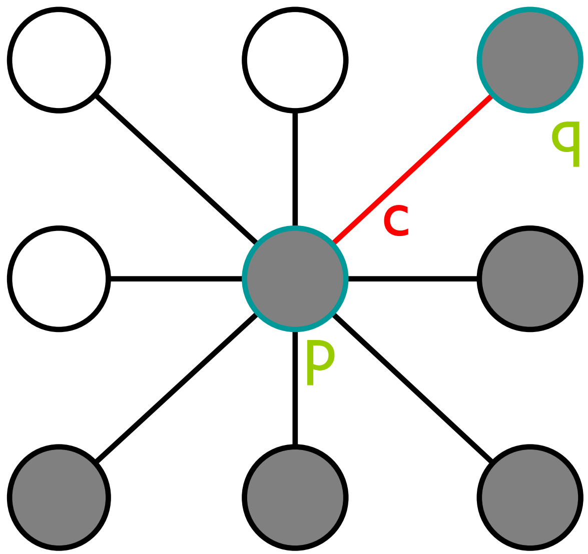
Note: each *link* has a cost

- this is a little different than the figure before where each pixel had a cost

Slide credit: S. Seitz

Defining the costs

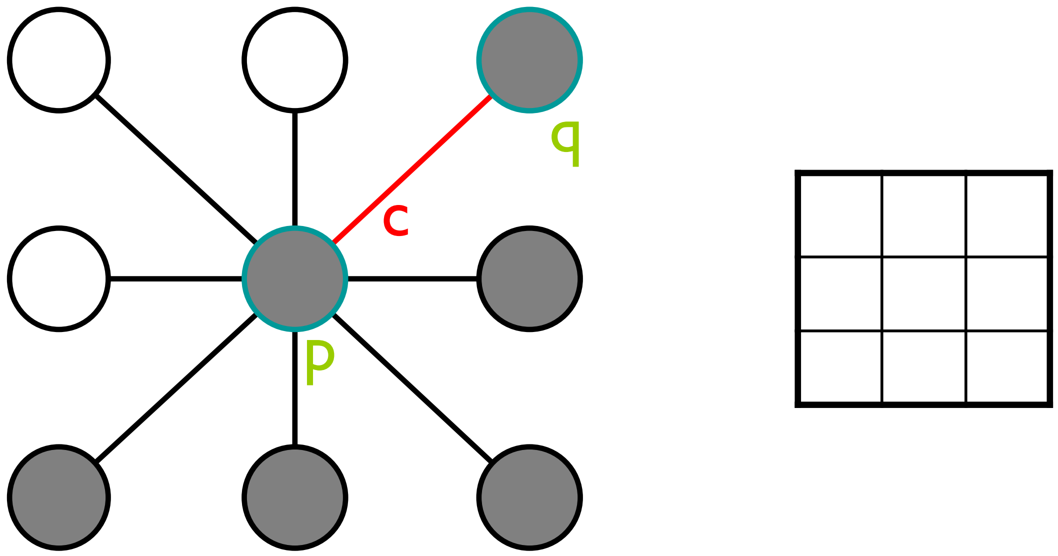
- Treat the image as a graph



Want to hug image edges: how to define cost of a link?

- the link should follow the intensity edge
  - want intensity to change rapidly $\perp$ to the link
- c ≈ - |difference of intensity $\perp$ to link|

Defining the costs

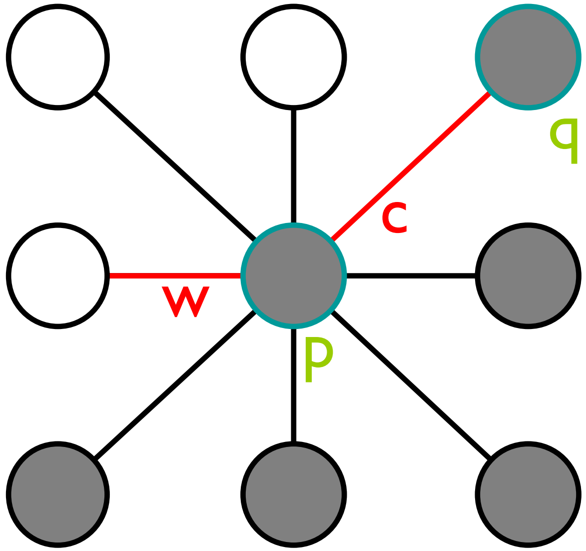


- c can be computed using a cross-correlation filter
  - assume it is centered at p
- Also typically scale c by its length
  - set c = (max-|filter response|)
    - where max = maximum |filter response| over all pixels in the image

Slide credit: S. Seitz

Defining the costs

$$\frac{1}{4}\begin{bmatrix} 1 & 1 & \\ & & \\ -1 & -1 & \end{bmatrix}$$

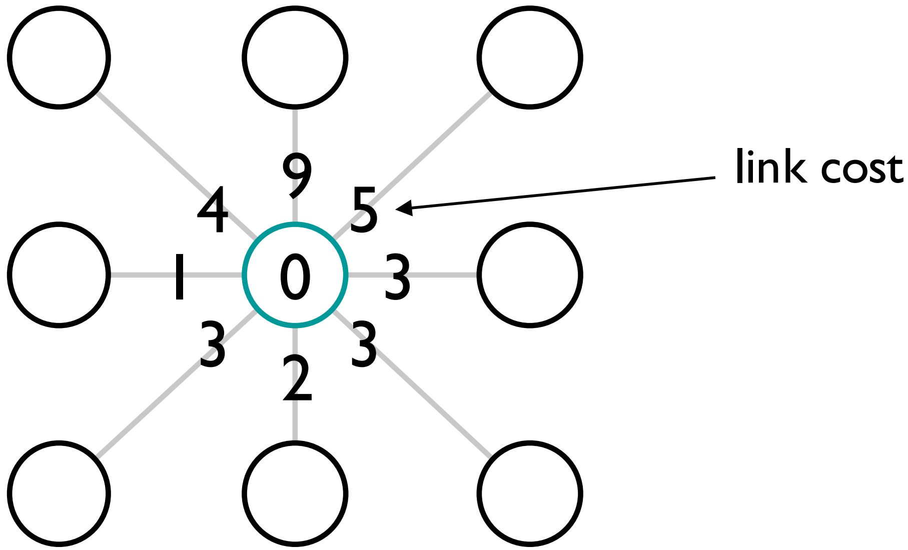


$$\frac{1}{\sqrt{2}}\begin{bmatrix} & 1 & \\ & & -1 \\ & & \end{bmatrix}$$

- c can be computed using a cross-correlation filter
  - assume it is centered at p
- Also typically scale c by its length
  - set c = (max-|filter response|)
    - where max = maximum |filter response| over all pixels in the image

Slide credit: S. Seitz

Dijkstra's shortest path algorithm



Algorithm

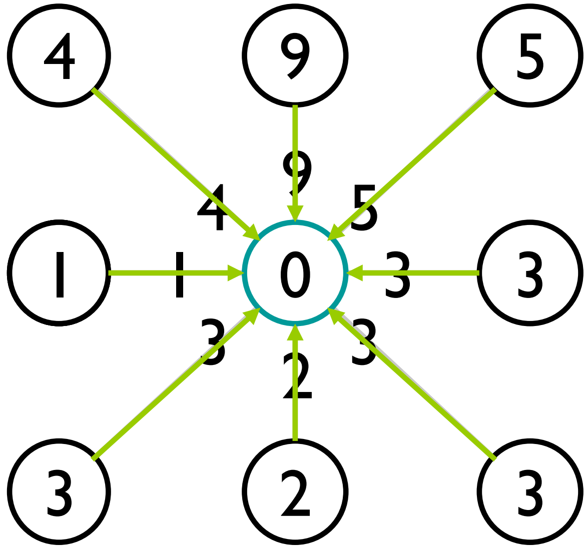
1. init node costs to ∞, set p = seed point, cost(p) = 0
2. expand p as follows:

   for each of p's neighbors q that are not expanded

   » set cost(q) = min(cost(p) + c_{pq}, cost(q))

Slide credit: S. Seitz

Dijkstra’s shortest path algorithm



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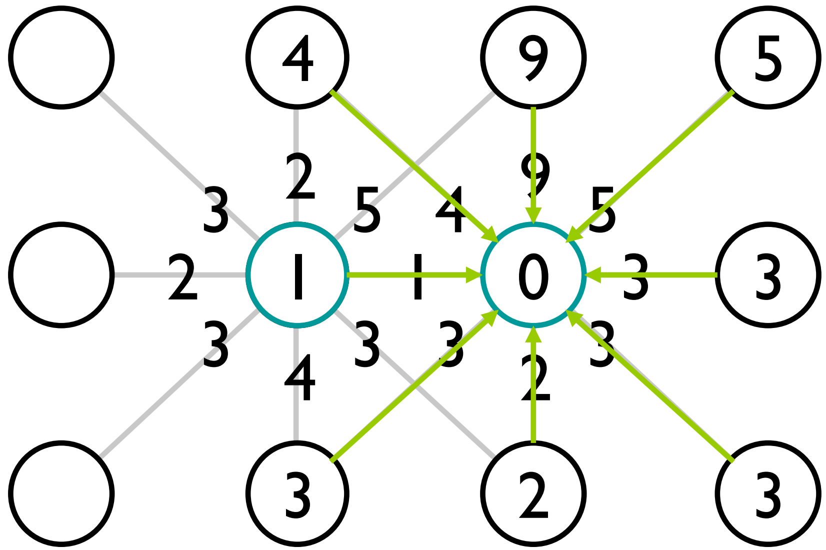
   » set cost(q) = min(cost(p) + c_{pq}, cost(q))

      » if q’s cost changed, make q point back to p

   » put q on the ACTIVE list (if not already there)

Slide credit: S. Seitz

Dijkstra's shortest path algorithm



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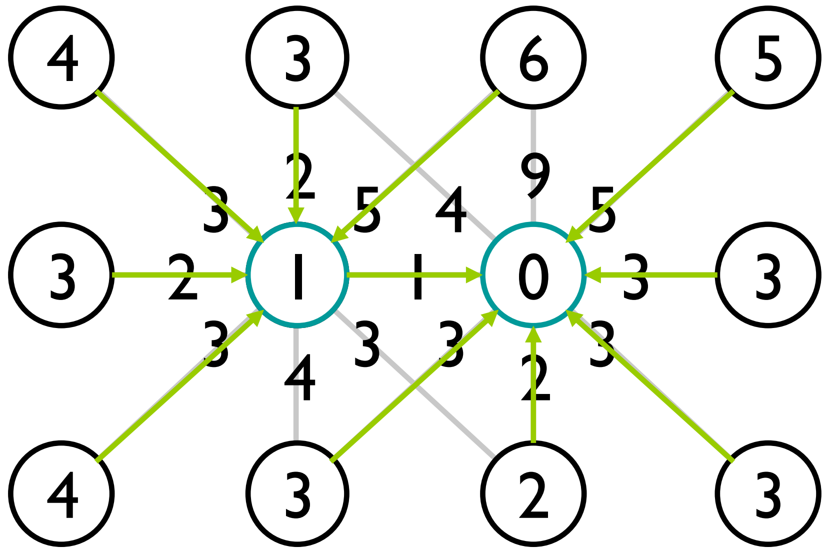
   » if q's cost changed, make q point back to p

   » put q on the ACTIVE list (if not already there)
3. set r = node with minimum cost on the ACTIVE list
4. repeat Step 2 for p = r

Slide credit: S. Seitz

Dijkstra's shortest path algorithm



Algorithm

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   » put q on the ACTIVE list (if not already there)
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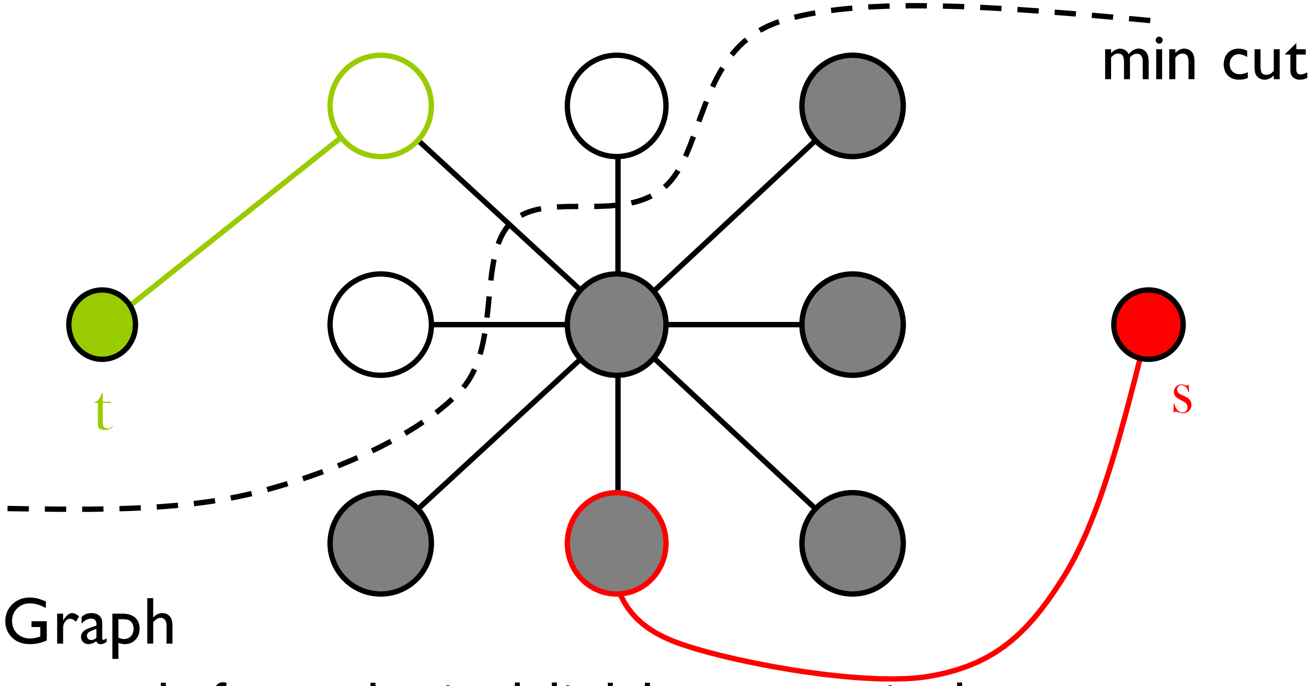
Slide credit: S. Seitz

Segmentation by min (s-t) cut



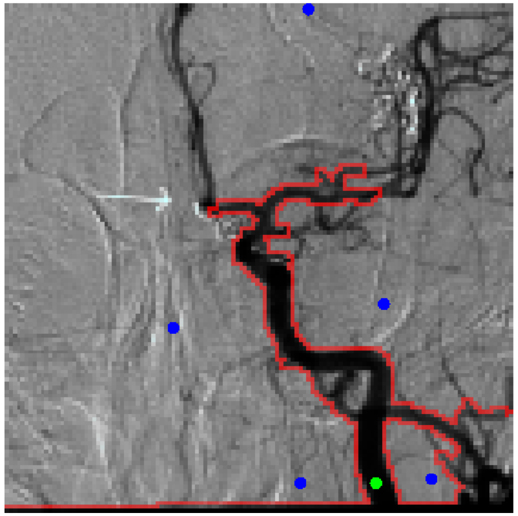
- Graph
  - node for each pixel, link between pixels
  - specify a few pixels as foreground and background
    - create an infinite cost link from each bg pixel to the "t" node
    - create an infinite cost link from each fg pixel to the "s" node
  - compute min cut that separates s from t
  - how to define link cost between neighboring pixels?

Y. Boykov and M-P Jolly, Interactive Graph Cuts for Optimal Boundary & Region Segmentation of Objects in N-D images, ICCV, 2001.

Slide credit: S. Seitz

Random Walker

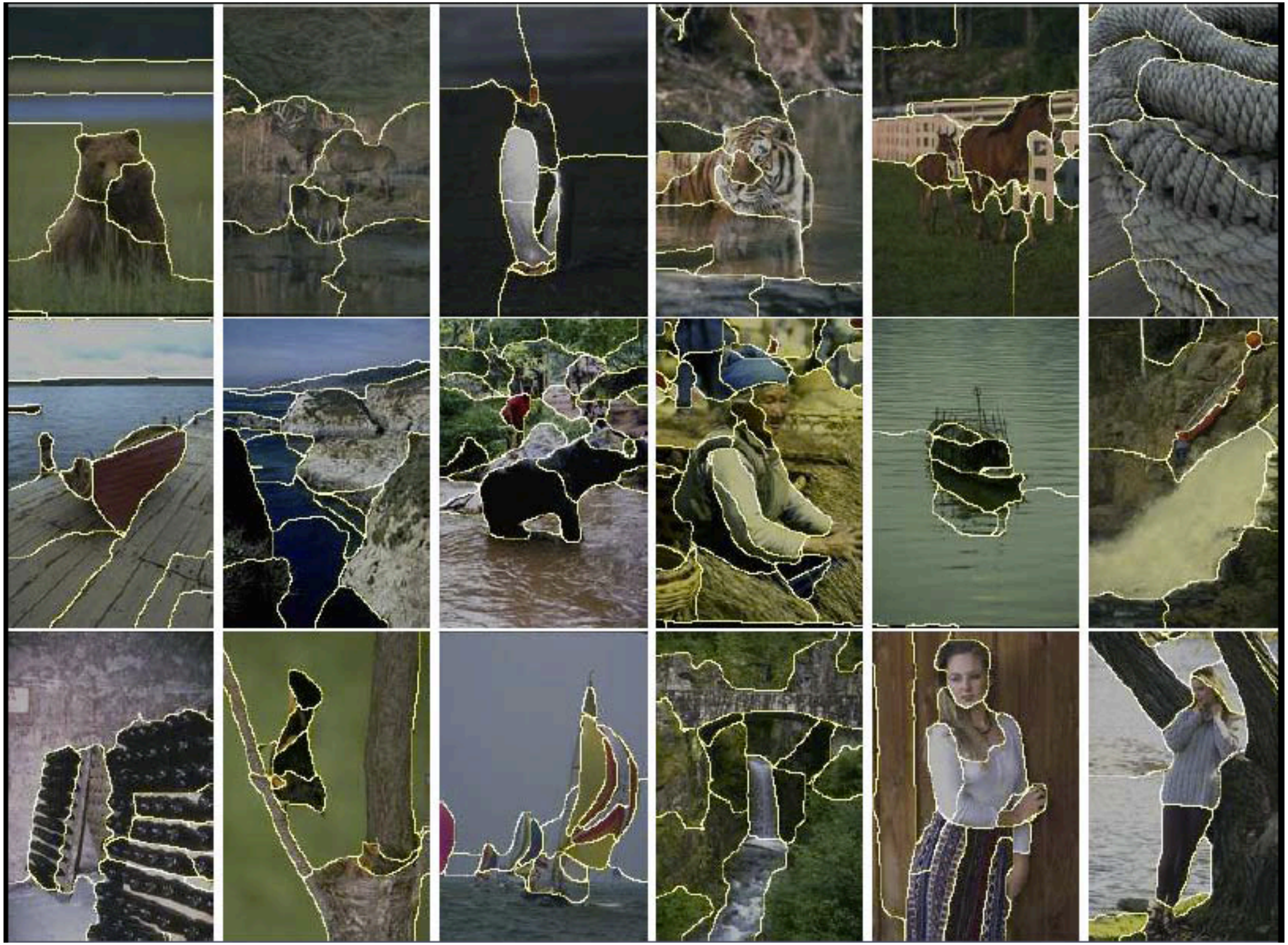
- Compute probability that a random walker arrives at seed



L. Grady, [Random Walks for Image Segmentation](), IEEE T-PAMI, 2006

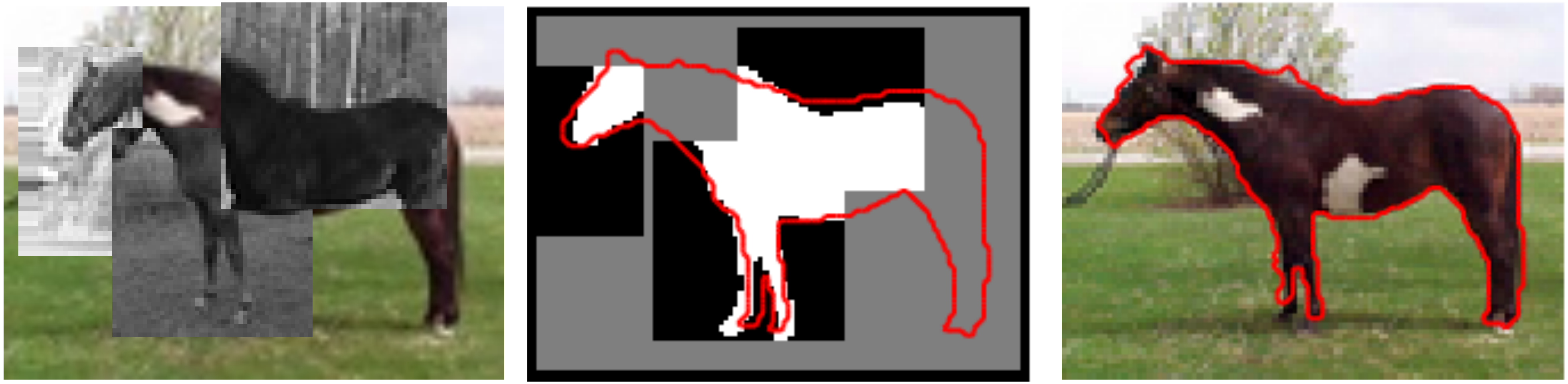
http://cns.bu.edu/~lgrady/Random_Walker_Image_Segmentation.html

Do we need recognition to take the next step in performance?



Slide credit: B. Freeman and A. Torralba

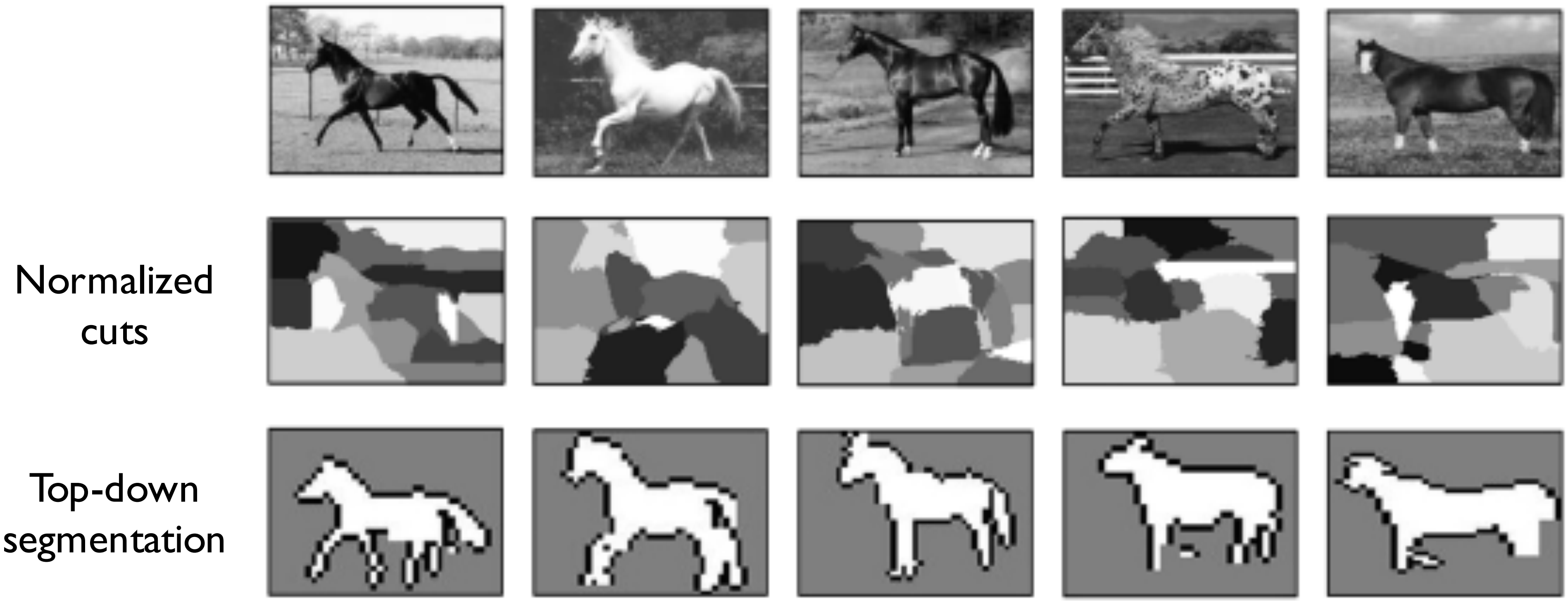
Top-down segmentation



- E. Borenstein and S. Ullman, Class-specific, top-down segmentation, ECCV 2002
- A. Levin and Y. Weiss, Learning to Combine Bottom-Up and Top-Down Segmentation, ECCV 2006.

Slide credit: S. Lazebnik

Top-down segmentation



- E. Borenstein and S. Ullman, [Class-specific, top-down segmentation,](#) ECCV 2002
- A. Levin and Y. Weiss, [Learning to Combine Bottom-Up and Top-Down Segmentation](#), ECCV 2006.

Slide credit: S. Lazebnik

Motion segmentation


Input sequence

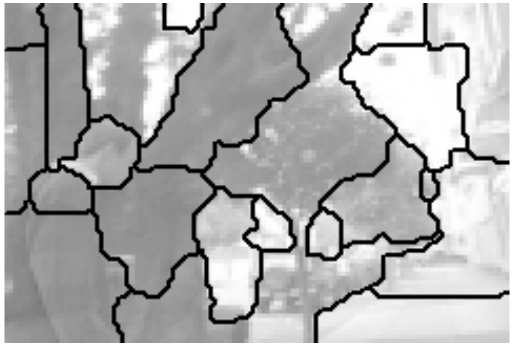
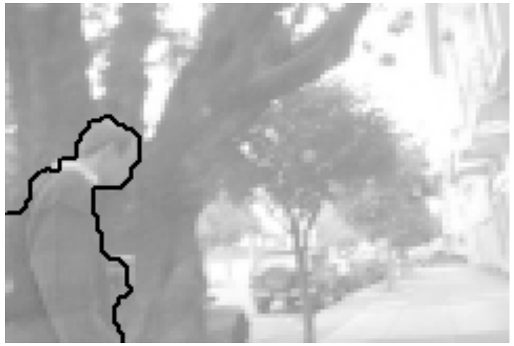


Image Segmentation


Motion Segmentation


Input sequence

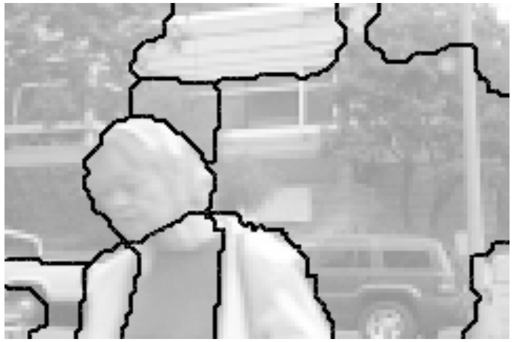


Image Segmentation


Motion Segmentation

A. Barbu, S.C. Zhu. Generalizing Swendsen-Wang to sampling arbitrary posterior probabilities, IEEE TPAMI, 2005.

Slide credit: K. Grauman