

# **Introduction**

Erkut Erdem

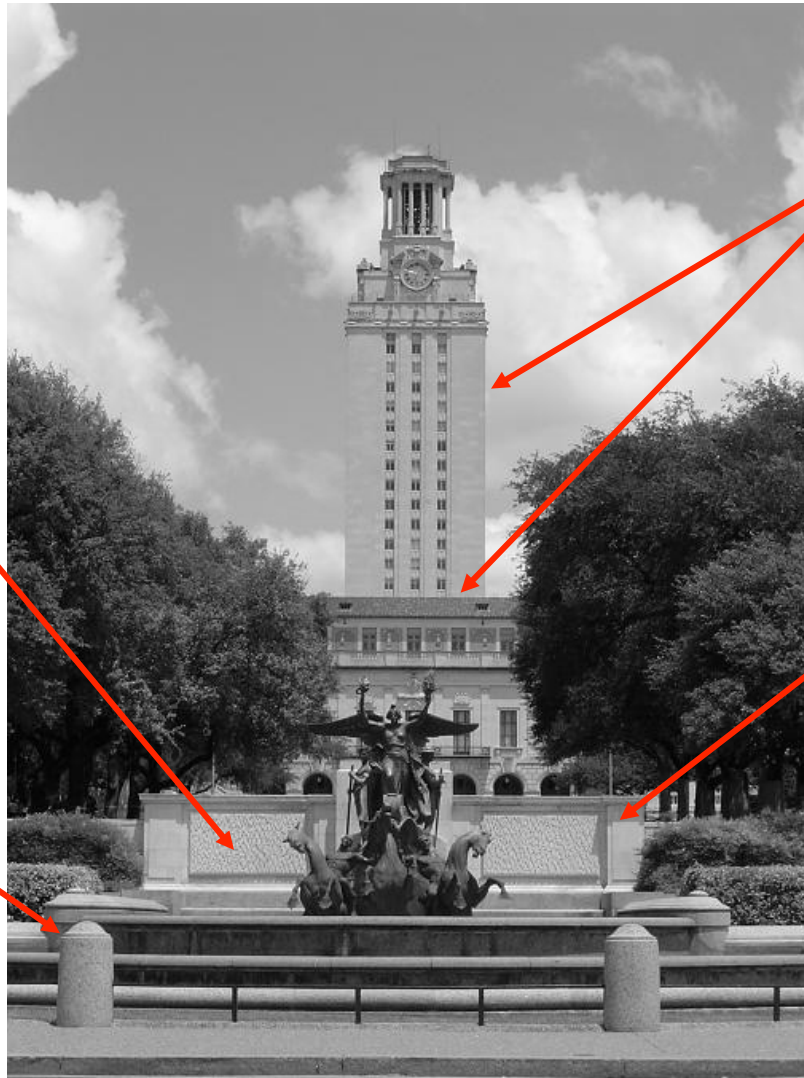
# What causes an edge?

Reflectance change:  
appearance  
information, texture

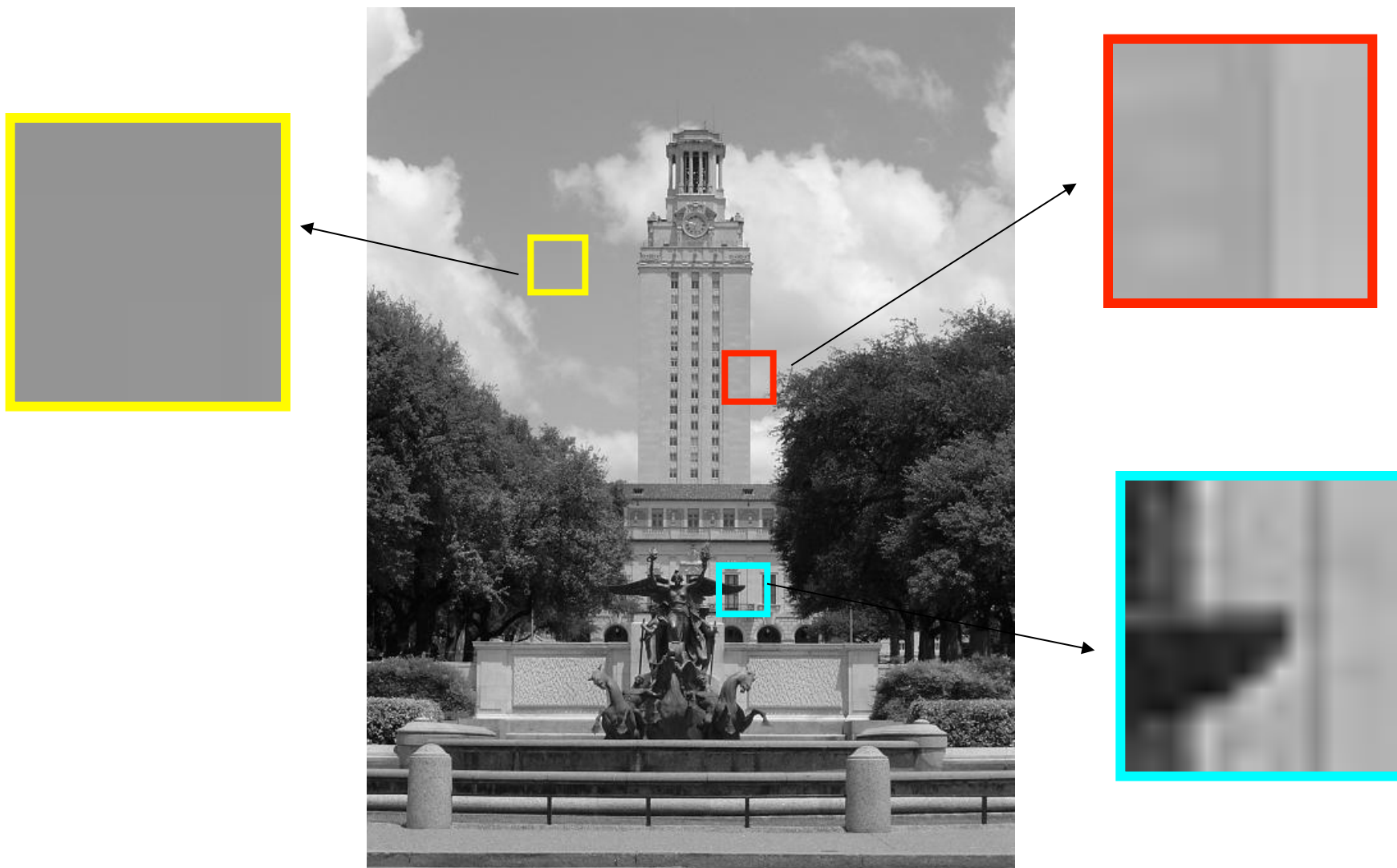
Change in surface  
orientation: shape

Depth discontinuity:  
object boundary

Cast shadows



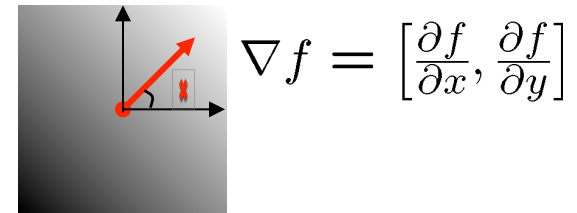
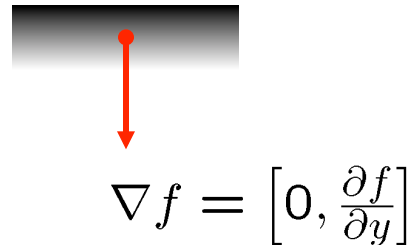
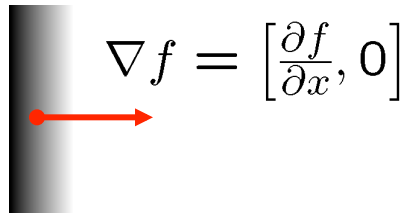
# Edges/gradients and invariance



Source: K. Grauman

# Image Gradient

$$\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$



The gradient direction (orientation of edge normal) is given by:

$$\theta = \tan^{-1} \left( \frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}} \right)$$

The *edge strength* is given by the gradient magnitude

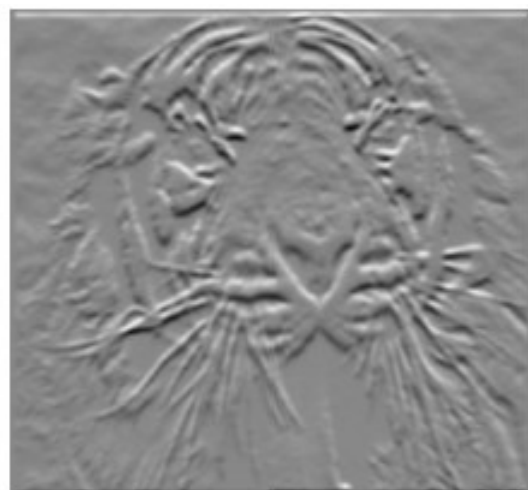
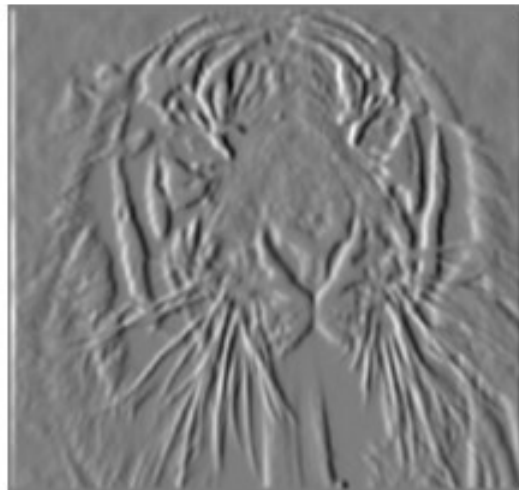
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

# Partial derivatives of an image

$$\frac{\partial f(x, y)}{\partial x}$$



$$\frac{\partial f(x, y)}{\partial y}$$



-1	1
----	---

-1
1

# Original Image



Source: K. Grauman

# Gradient magnitude image



Source: K. Grauman

# Thresholding gradient with a lower threshold



Source: K. Grauman



# Thresholding gradient with a higher threshold

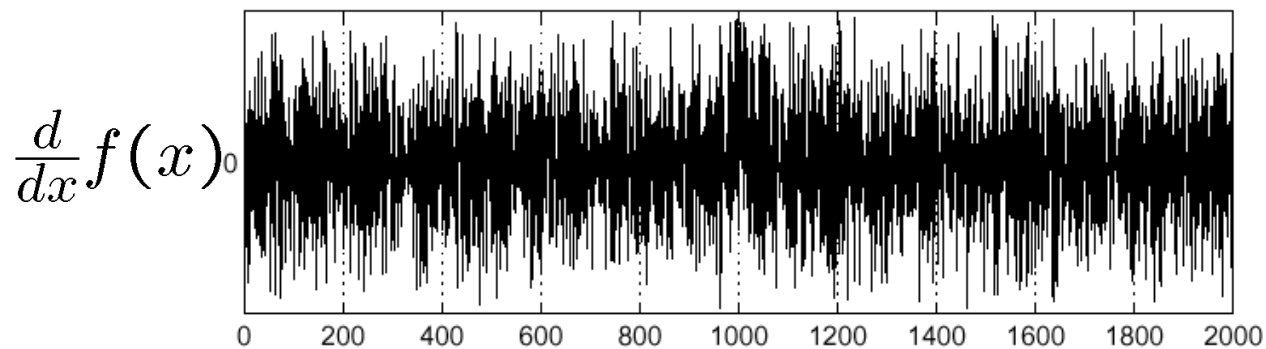
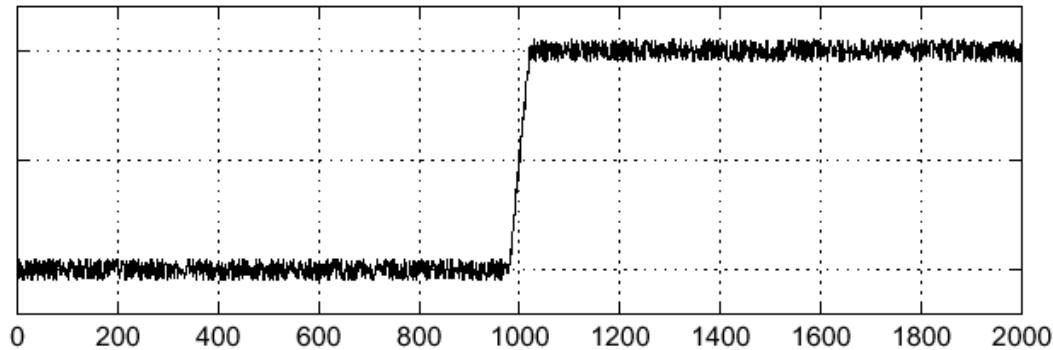


Source: K. Grauman

# Effects of noise

Consider a single row or column of the image

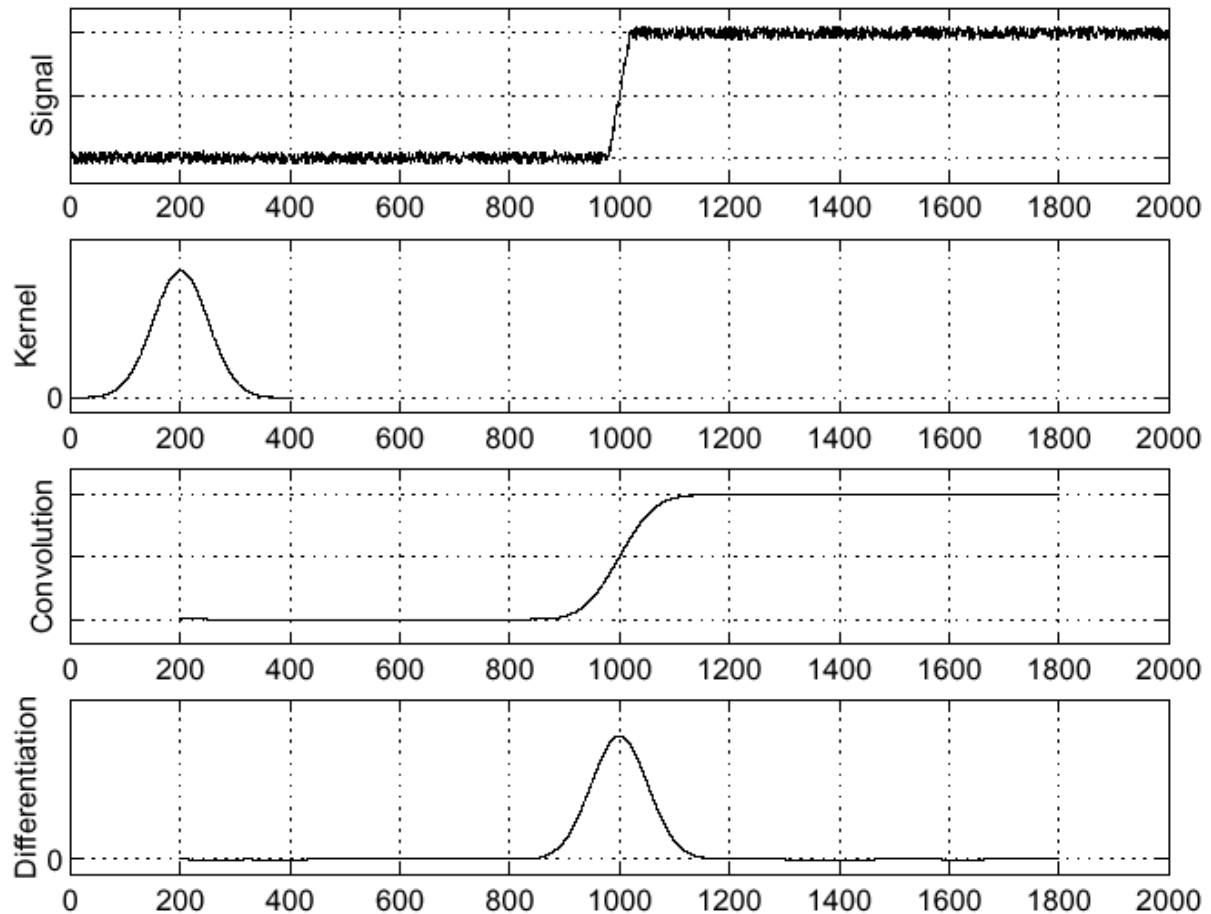
- Plotting intensity as a function of position gives a signal



Where is the edge?

# Solution: Smooth first

Sigma = 50



$$\frac{\partial}{\partial x}(h \star f)$$

Where is the edge?

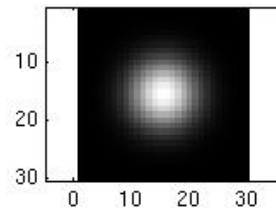
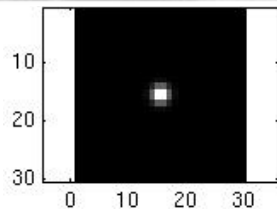
Look for peaks in

$$\frac{\partial}{\partial x}(h \star f)$$

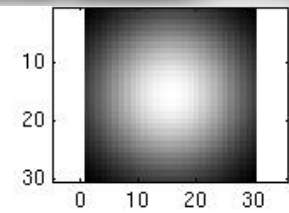
Source: K. Grauman

# Smoothing with a Gaussian

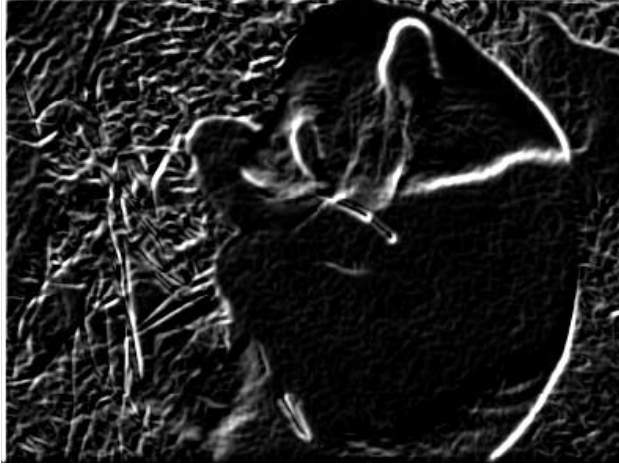
Recall: parameter  $\sigma$  is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.



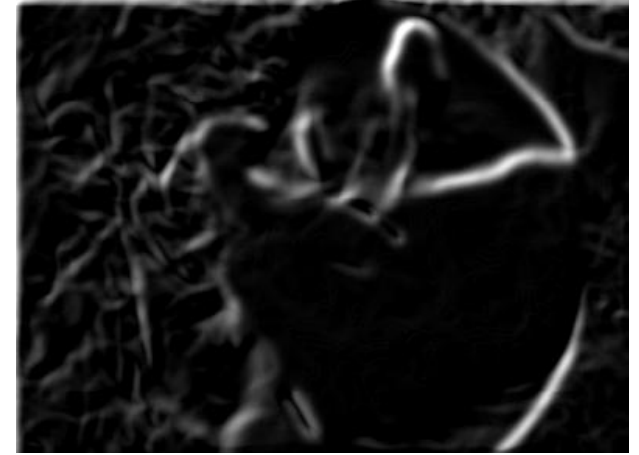
...



# Effect of $\sigma$ on derivatives



$\sigma = 1$  pixel



$\sigma = 3$  pixels

The apparent structures differ depending on Gaussian's scale parameter.

Larger values: larger scale edges detected

Smaller values: finer features detected

# Derivative theorem of convolution

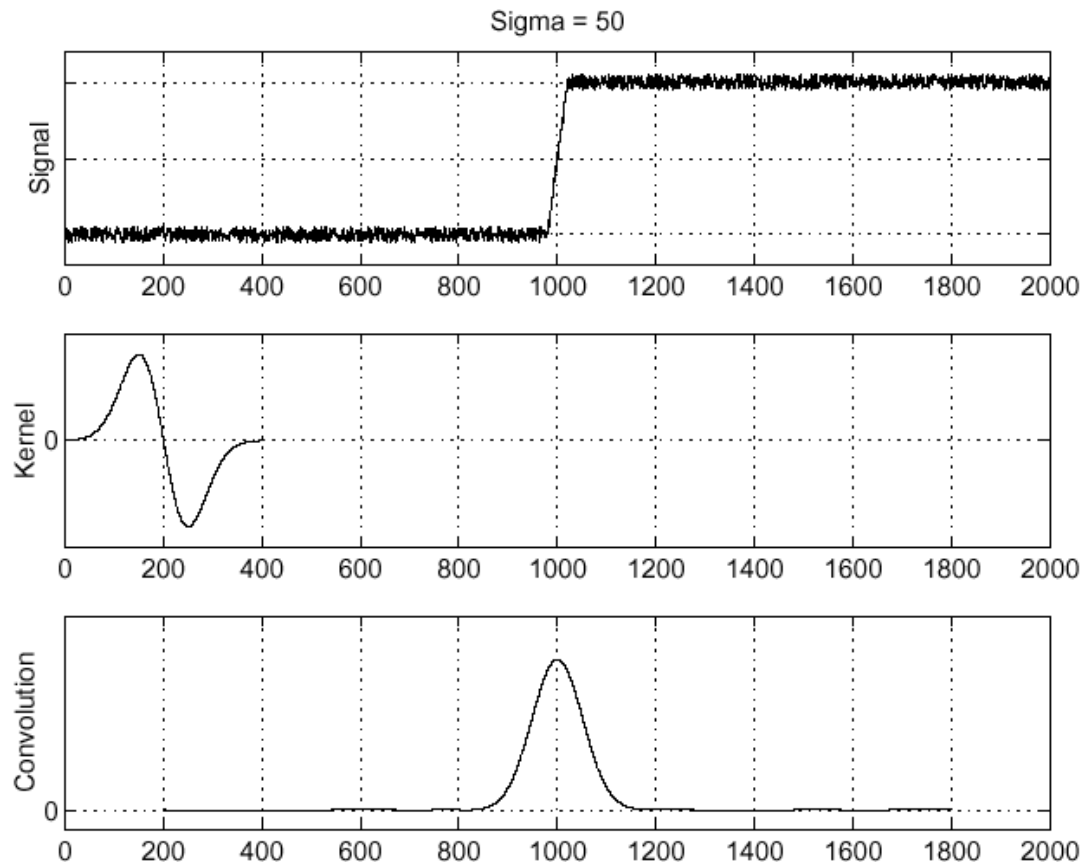
$$\frac{\partial}{\partial x}(h \star f) = \left(\frac{\partial}{\partial x}h\right) \star f$$

Differentiation property of convolution.



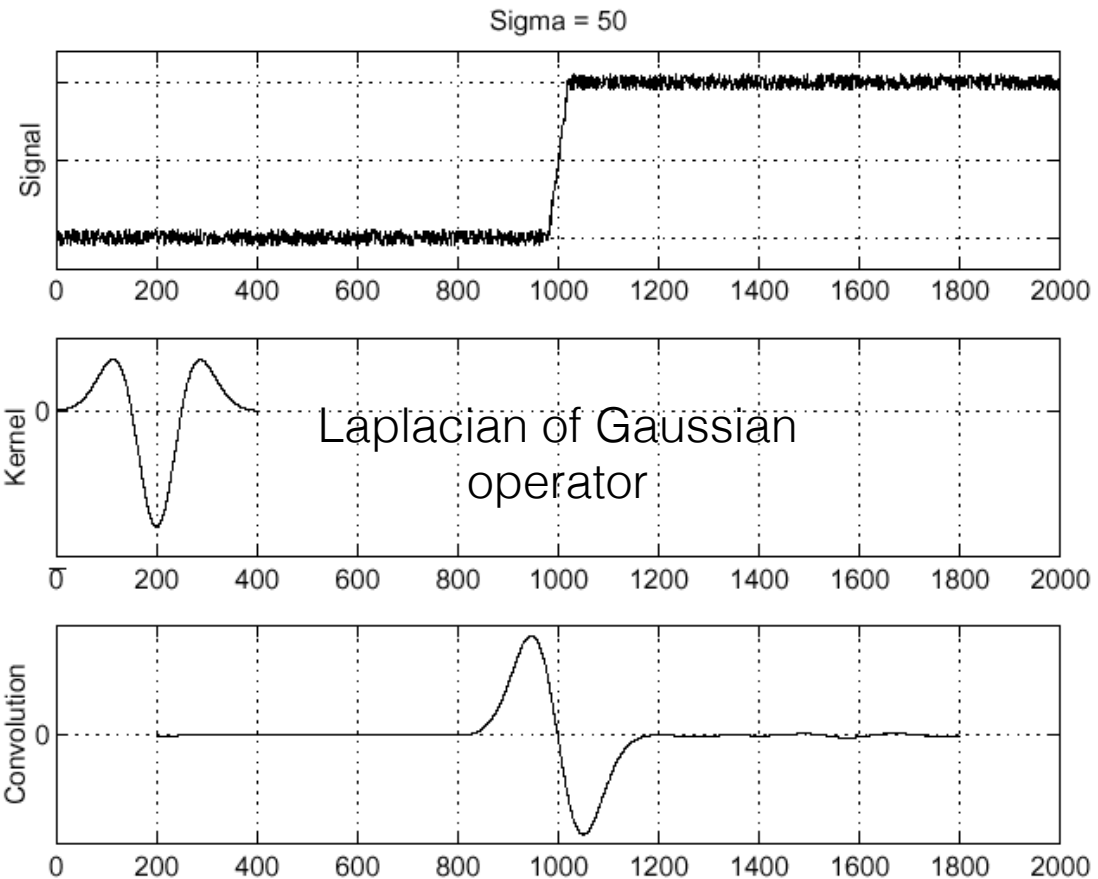
$$\frac{\partial}{\partial x}h$$

$$\left(\frac{\partial}{\partial x}h\right) \star f$$



# Laplacian of Gaussian

Consider  $\frac{\partial^2}{\partial x^2}(h \star f)$



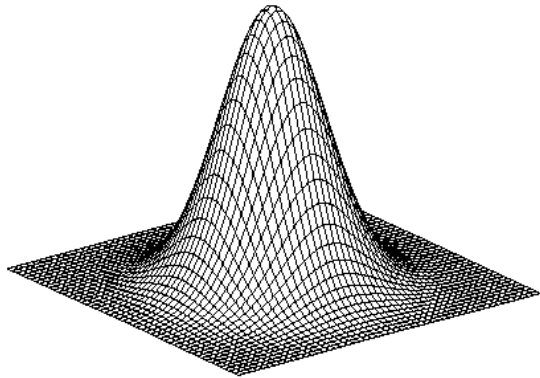
$$\frac{\partial^2}{\partial x^2}h$$

$$\left(\frac{\partial^2}{\partial x^2}h\right) \star f$$

Where is the edge?

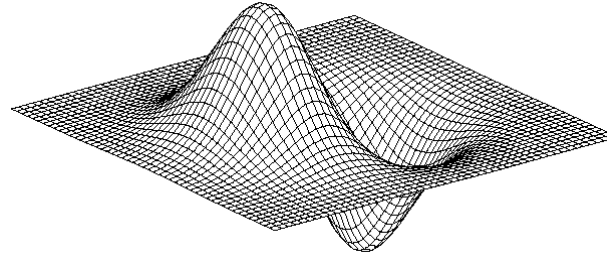
Zero-crossings of bottom graph

# 2D edge detection filters



Gaussian

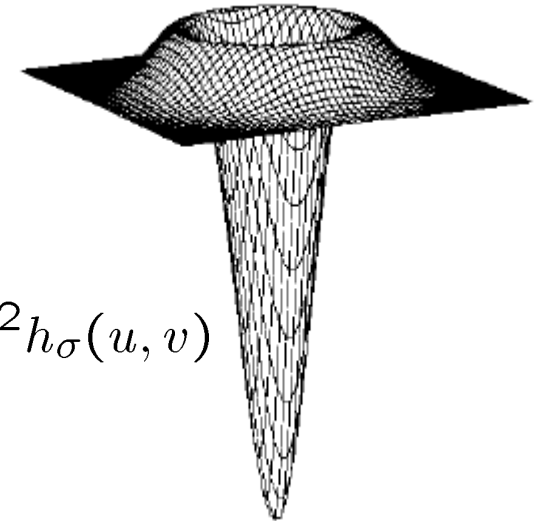
$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$

Laplacian of Gaussian



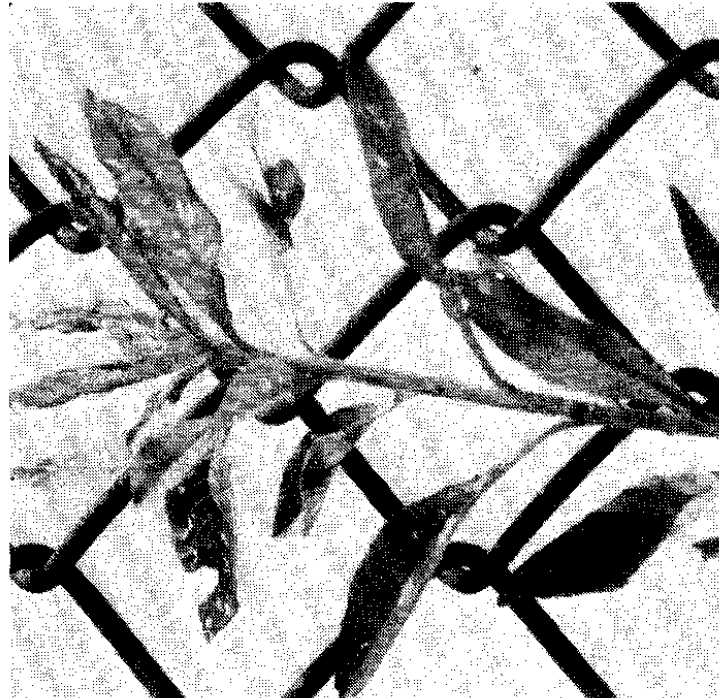
$$\nabla^2 h_{\sigma}(u, v)$$

-  The Laplacian operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



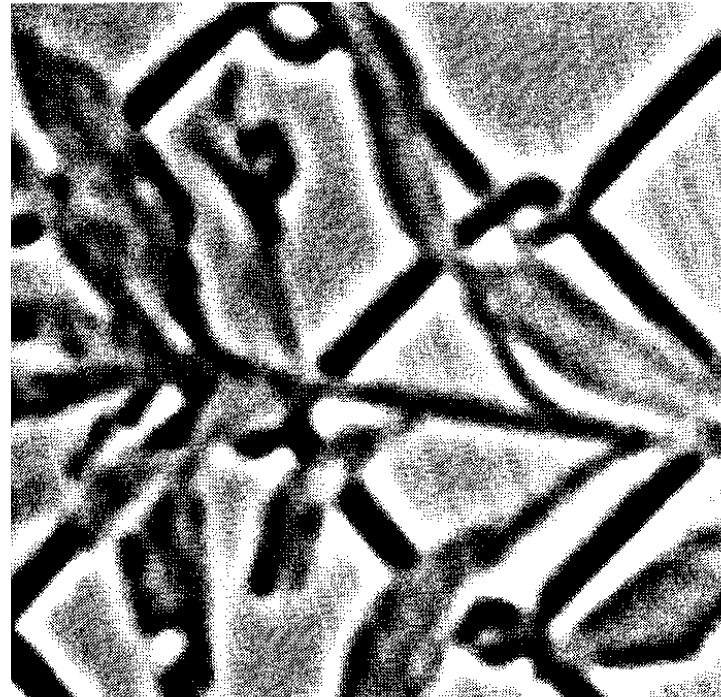
# Laplacian of Gaussian



original image

Source: D. Marr and  
E. Hildreth (1980)

# Laplacian of Gaussian



convolution with  
 $\nabla^2 h_\sigma(u, v)$

Source: D. Marr and  
E. Hildreth (1980)

# Laplacian of Gaussian



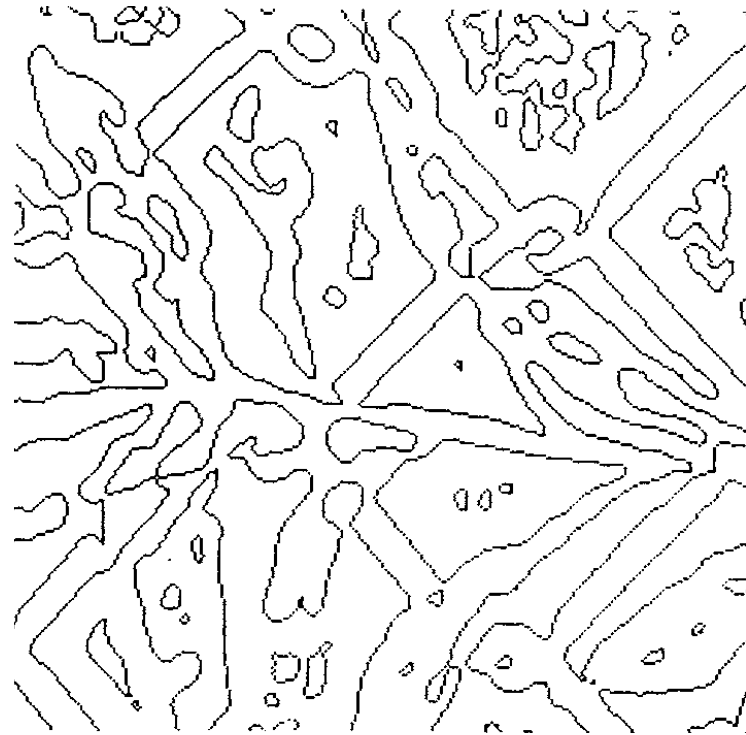
convolution with

$$\nabla^2 h_\sigma(u, v)$$

(pos. values – white, neg. values – black)

Source: D. Marr and  
E. Hildreth (1980)

# Laplacian of Gaussian



zero-crossings

Source: D. Marr and  
E. Hildreth (1980)

# The Canny edge detector



original image (Lena)

Source: K. Grauman

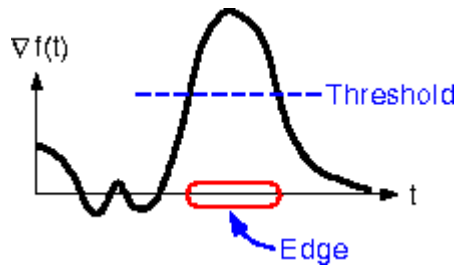
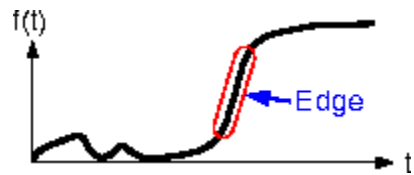
# The Canny edge detector



thresholding

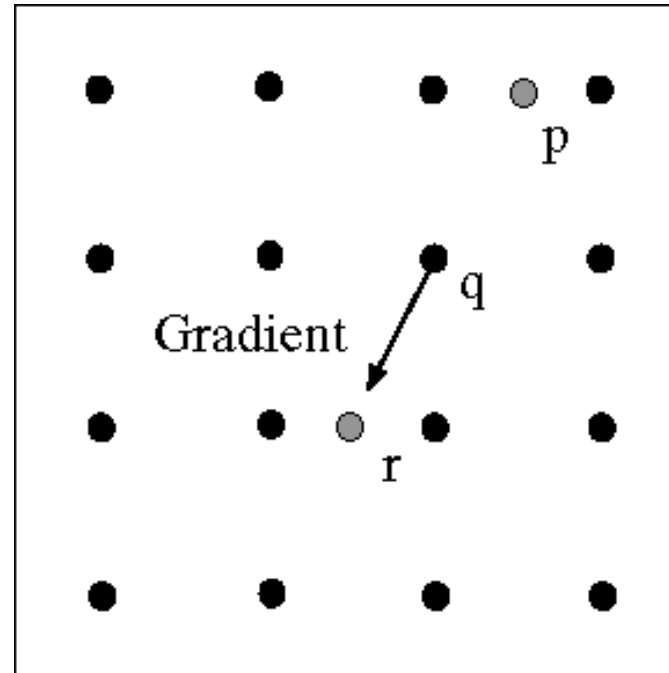
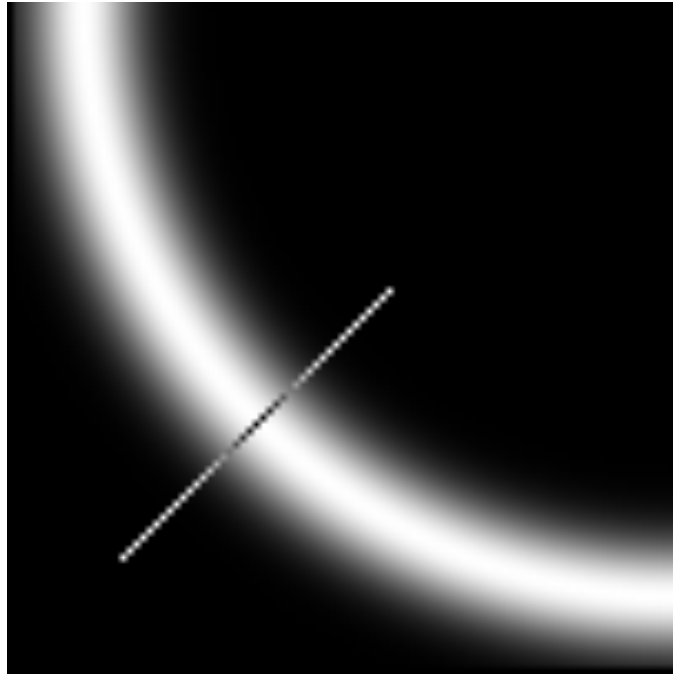
Source: K. Grauman

# The Canny edge detector



How to turn these thick regions of the gradient into curves?

# Non-maximum suppression



Check if pixel is local maximum along gradient direction,  
select single max across width of the edge  
– requires checking interpolated pixels  $p$  and  $r$



# The Canny Edge Detector



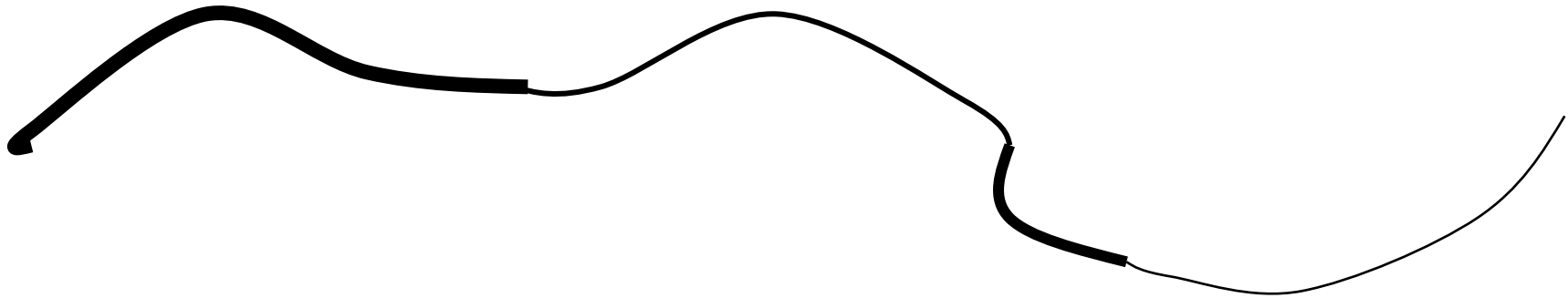
Problem: pixels along this edge didn't survive the thresholding

thinning  
(non-maximum suppression)

Source: K. Grauman

# Hysteresis thresholding

- Check that maximum value of gradient value is sufficiently large
  - drop-outs? use **hysteresis**
    - use a high threshold to start edge curves and a low threshold to continue them.



# Hysteresis thresholding



original image



high threshold  
(strong edges)



low threshold  
(weak edges)



hysteresis threshold