## Spelling Correction and the Noisy Channel

## Spelling Tasks

- Spelling Error Detection
- Spelling Error Correction:
- Autocorrect
- hte $\rightarrow$ the
- Suggest a correction
- Suggestion lists


## Types of Spelling Errors

- Non-word Errors: Non-word spelling correction is the detection and correction of spelling errors that result in non-words
- graffe $\rightarrow$ giraffe
- Real-word Errors: Real word spelling correction is the task of detecting and correcting spelling errors even if they accidentally result in an actual word.
- Typographical errors
- three $\rightarrow$ there
- Cognitive Errors (homophones)
- piece $\rightarrow$ peace,
- too $\rightarrow$ two


## Non-word Spelling Errors

- Non-word spelling error detection:
- Any word not in a dictionary is an error
- The larger the dictionary the better
- Non-word spelling error correction:
- Generate candidates: real words that are similar to error
- Choose the one which is best:
- Shortest weighted edit distance
- Highest noisy channel probability


## Real Word Spelling Errors

- For each word $w$, generate candidate set:
- Find candidate words with similar pronunciations
- Find candidate words with similar spelling
- Include $w$ in candidate set
- Choose best candidate
- Noisy Channel
- Classifier


## Noisy Channel Model of Spelling



- We see an observation x of a misspelled word
- Find the correct word w


## Applying Bayes to a Noisy Channel

- In applying probability theory to a noisy channel, what we are looking for is the most probable source given the observed signal. We can denote this:


## mostprobable-source $^{\boldsymbol{m}} \operatorname{argmax}_{\text {Source }} \mathbf{P}($ Source|Signal $)$

- Unfortunately, we don't usually know how to compute this.
- We cannot directly know : what is the probability of a source given an observed signal?
- We will apply Bayes’ rule


## Applying Bayes to a Noisy Channel

- From Bayes rule, we know that:

$$
P(\text { Source } \mid \text { Signal })=\frac{P(\text { Signal } \mid \text { Source }) P(\text { Source })}{P(\text { Signal })}
$$

- So, we will have:

$$
\arg \max \text { source } \frac{P(\text { Signal } \mid \text { Source }) P(\text { Source })}{P(\text { Signal })}
$$

- For each Source, $P$ (Signal) will be same. So we will have:

$$
\operatorname{argmax}_{\text {Source }} P(\text { Signal } \mid \text { Source }) \mathbf{P ( S o u r c e ) ~}
$$

## Applying Bayes to a Noisy Channel to Spelling

- We have some word that has been misspelled and we want to know the real word.
- In this problem, the real word is the source and the misspelled word is the signal.
- We are trying to estimate the real word.
- Assume that

| V | is the space of all the words we know |
| :--- | :--- |
| s | denotes the misspelling (signal) |
| $\omega$ | denotes the correct word (estimate) |

- So, we will have the following equation:

$$
\varpi=\operatorname{argmax}_{w \in V} P(s \mid w) P(w)
$$

## Getting Numbers

- We need a corpus to compute: $\mathbf{P}(\mathbf{w})$ and $\mathbf{P}(\mathbf{s} \mid \mathbf{w})$
- Computing $\mathrm{P}(\mathrm{w})$
- We will count how often the word w occurs in the corpus.
- $\mathrm{So}, \mathrm{P}(\mathrm{w})=\mathrm{C}(\mathrm{w}) / \mathrm{N}$ where $\mathrm{C}(\mathrm{w})$ is the number of w occurs in the corpus, and N is the total number of words in the corpus.
- What happens if $\mathrm{P}(\mathrm{w})$ is zero.
- We need a smoothing technique (getting rid of zeroes).
- A smoothing technique: $\mathrm{P}(\mathrm{w})=(\mathrm{C}(\mathrm{w})+0.5) /(\mathrm{N}+0.5 * \mathrm{VN})$ where VN is the number of words in V (our dictionary).
- Computing $\mathrm{P}(\mathrm{s} \mid \mathrm{w})$
- It is fruitless to collect statistics about the misspellings of individual words for a given dictionary. We will likely never get enough data.
- We need a way to compute $\mathrm{P}(\mathrm{s} \mid \mathrm{w})$ without using direct information.
- We can use spelling error pattern statistics to compute $\mathrm{P}(\mathrm{s} \mid \mathrm{w})$.


## Spelling Error Patterns

- There are four patterns:

| Insertion | -- ther for the |
| :--- | :--- |
| Deletion | -- ther for there |
| Substitution | -- noq for now |
| Transposition | -- hte for the |

- For each pattern we need a confusion matrix.
- del $[\mathbf{x}, \mathbf{y}]$ contains the number of times in the training set that characters $x y$ in the correct word were typed as $x$.
- ins[x,y] contains the number of times in the training set that character $x$ in the correct word were typed as $x y$.
$-\mathbf{s u b}[\mathbf{x}, \mathbf{y}]$ contains the number of times that x was typed as y .
- trans[x,y] contains the number of times that xy was typed as yx.


## Estimating P(s|w)

## Noisy Channel Model for Spelling Correction

- Assuming a single spelling error, $\mathrm{P}(\mathrm{s} \mid \mathrm{w})$ will be computed as follows.

$$
\begin{array}{ll}
\mathrm{P}(\mathrm{~s} \mid \mathrm{w})=\operatorname{del}\left[\mathrm{w}_{\mathrm{i}-1}, \mathrm{w}_{\mathrm{i}}\right] / \operatorname{count}\left[\mathrm{w}_{\mathrm{i}-1} \mathrm{w}_{\mathrm{i}}\right] & \text { if deletion } \\
\mathrm{P}(\mathrm{~s} \mid \mathrm{w})=\operatorname{ins}\left[\mathrm{w}_{\mathrm{i}-1}, \mathrm{~s}_{\mathrm{i}}\right] / \operatorname{count}\left[\mathrm{w}_{\mathrm{i}-1}\right] & \text { if insertion } \\
\mathrm{P}(\mathrm{~s} \mid \mathrm{w})=\operatorname{sub}\left[\mathrm{s}_{\mathrm{i}}, \mathrm{w}_{\mathrm{i}}\right] / \operatorname{count}\left[\mathrm{w}_{\mathrm{i}}\right] & \text { if substitution } \\
\mathrm{P}(\mathrm{~s} \mid \mathrm{w})=\operatorname{trans}\left[\mathrm{w}_{\mathrm{i}}, \mathrm{w}_{\mathrm{i}+1}\right] / \operatorname{count}\left[\mathrm{w}_{\mathrm{i}} \mathrm{w}_{\mathrm{i}+1}\right] & \text { if transposition }
\end{array}
$$

# Words within 1 edit distance of misspelled word acress 

| Error | Candidate <br> Correction | Correct <br> Letter | Error <br> Letter | Type |
| :--- | :--- | :--- | :--- | :--- |
| acress | actress | t | - | deletion |
| acress | cress | - | a | insertion |
| acress | caress | ca | ac | transposition |
| acress | access | c | r | substitution |
| acress | across | o | e | substitution |
| acress | acres | - | s | insertion |
| acress | acres | - | $s$ | insertion |

- $80 \%$ of errors are within edit distance 1
- Almost all errors within edit distance 2


## Unigram Prior Probability

- Counts from 404,253,213 words in Corpus of Contemporary English (COCA)

| word | Frequency of word | P(word) |
| :--- | ---: | :--- |
| actress | 9,321 | .0000230573 |
| cress | 220 | .0000005442 |
| caress | 686 | .0000016969 |
| access | 37,038 | .0000916207 |
| across | 120,844 | .0002989314 |
| acres | 12,874 | .0000318463 |

## Noisy Channel Model for acress

| Candidate <br> Correction | Correct <br> Letter | Error <br> Letter | x\|w | $\mathbf{P ( x \| w o r d )}$ |
| :--- | :--- | :--- | :--- | :--- |
| actress | t | - | c\|ct | .000117 |
| cress | - | a | a\|\# | $\mathbf{. 0 0 0 0 0 1 4 4}$ |
| caress | ca | ac | ac\|ca | $\mathbf{. 0 0 0 0 0 1 6 4}$ |
| access | c | r | r\|c | .000000209 |
| across | o | e | e\|o | $\mathbf{. 0 0 0 0 0 9 3}$ |
| acres | - | s | es \|e | $\mathbf{. 0 0 0 0 3 2 1}$ |
| acres | - | s | ss \|s | $\mathbf{. 0 0 0 0 3 4 2}$ |

## Noisy Channel Probability for acress

| Candidate Correction | Correct <br> Letter | Error <br> Letter | x\|w | $\mathbf{P}(\mathrm{x} \mid$ word $)$ | P(word) | $10^{9}$ * $\mathrm{P}(\mathrm{x} \mid \mathrm{w}) \mathrm{P}(\mathrm{w})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| actress | t | - | $c \mid c t$ | . 000117 | .0000231 | 2.7 |
| cress | - | a | a \\| \# | .00000144 | . 000000544 | .00078 |
| caress | ca | ac | ac\|ca | .00000164 | .00000170 | . 0028 |
| access | C | r | $r \mid c$ | .000000209 | .0000916 | . 019 |
| across | $\bigcirc$ | e | elo | .0000093 | .000299 | 2.8 |
| acres | - | S | es/e | . 0000321 | . 0000318 | 1.0 |
| acres | - | S | SS $\mid ~ S$ | . 0000342 | . 0000318 | 1.0 |

## Noisy Channel Probability for acress

| Candidate Correction | Correct <br> Letter | Error <br> Letter | x/w | $\mathbf{P}(\mathrm{x} \mid$ word $)$ | P(word) | $10^{9}$ * $\mathrm{P}(\mathrm{x} \mid \mathrm{w}) \mathrm{P}(\mathrm{w})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| actress | t | - | $c \mid c t$ | . 000117 | .0000231 | 2.7 |
| cress | - | a | a\|\# | . 00000144 | . 000000544 | . 00078 |
| caress | ca | ac | ac\|ca | .00000164 | . 00000170 | . 0028 |
| access | C | r | $r \mid c$ | .000000209 | .0000916 | . 019 |
| across | 0 | e | elo | .0000093 | .000299 | 2.8 |
| acres | - | S | es\|e | .0000321 | .0000318 | 1.0 |
| acres | - | S | SS\|S | . 0000342 | .0000318 | 1.0 |

## Using a Bigram Language Model

- "a stellar and versatile acress whose "
- Counts from the Corpus of Contemporary American English with add-1 smoothing

```
P(actress|versatile)=.000021
P(whose|actress) = .0010
P(across|versatile) =.000021
P(whose|across) = .000006
```

$P(" v e r s a t i l e ~ a c t r e s s ~ w h o s e ") ~=~ .000021 * .0010 ~=~ 210 ~ \times 10^{-10}$
P("versatile across whose") =.000021*.000006 = 1 x10-10

## Real-Word Spelling Correction

Real-word spelling errors
...leaving in about fifteen minuets to go to her house.
The design an construction of the system...
Can they lave him my messages?
The study was conducted mainly be John Black.

- $25-40 \%$ of spelling errors are real words.


## Solving Real-world Spelling Errors

- For each word in sentence
- Generate candidate set
- the word itself
- all single-letter edits that are English words
- words that are homophones
- Choose best candidates
- Noisy channel model
- Task-specific classifier


## Noisy Channel for Real-word Spell Correction

- Given a sentence $\mathrm{w}_{1}, \mathrm{w}_{2}, \mathrm{w}_{3}, \ldots, \mathrm{w}_{\mathrm{n}}$
- Generate a set of candidates for each word $w_{i}$
- Candidate $\left(\mathrm{w}_{1}\right)=\left\{\mathrm{w}_{1}, \mathrm{w}^{\prime}{ }_{1}, \mathrm{w}^{\prime}{ }_{1}, \mathrm{w}^{\prime}{ }^{\prime}{ }_{1}, \ldots\right\}$
- Candidate $\left(\mathrm{w}_{2}\right)=\left\{\mathrm{w}_{2}, \mathrm{w}^{\prime}{ }_{2}, \mathrm{w}^{\prime}{ }_{2}, \mathrm{w}^{\prime}{ }^{\prime}{ }_{2}, \ldots\right\}$
- Candidate $\left(\mathrm{w}_{\mathrm{n}}\right)=\left\{\mathrm{w}_{\mathrm{n}}, \mathrm{w}_{\mathrm{n}}, \mathrm{w}^{\prime}{ }_{\mathrm{n}}, \mathrm{w}^{\prime}{ }^{\prime}{ }_{\mathrm{n}}, \ldots\right\}$
- Choose the sequence W that maximizes $\mathrm{P}(\mathrm{W})$


## Noisy Channel for Real-word Spell Correction



## Noisy Channel for Real-word Spell Correction



## Simplification: One Error Per Sentence

- Out of all possible sentences with one word replaced
$-\mathrm{w}_{1}, \mathbf{w},{ }_{2}, \mathrm{w}_{3}, \mathrm{w}_{4} \quad$ two off thew
$-\mathrm{w}_{1}, \mathrm{w}_{2}, \mathbf{w}^{\prime}{ }_{3}, \mathrm{w}_{4} \quad$ two of the
$-\mathbf{w}^{\prime}{ }^{\prime}{ }_{1}, \mathrm{w}_{2}, \mathrm{w}_{3}, \mathrm{w}_{4} \quad$ too of thew
$-\ldots$
- Choose the sequence W that maximizes $\mathrm{P}(\mathrm{W})$


## Where to Get Probabilities

- Language model: Unigram, Bigram, ...
- Channel model
- Same as for non-word spelling correction
- Plus need probability for no error, $\mathrm{P}(\mathrm{w} \mid \mathrm{w})$
- Probability of no error
- What is the channel probability for a correctly typed word?
- P("the"|"the")
- Obviously this depends on the application
- . 90 (1 error in 10 words)
- . 95 (1 error in 20 words)
- . 99 (1 error in 100 words)
- . 995 (1 error in 200 words)

