Association Rule Mining

- Frequent Itemsets, Association Rules
- Apriori Algorithm
- Compact Representation of Frequent Itemsets
- FP-Growth Algorithm: An Alternative Frequent Itemset Generation Algorithm
- Evaluation of Association Patterns

Frequent Pattern

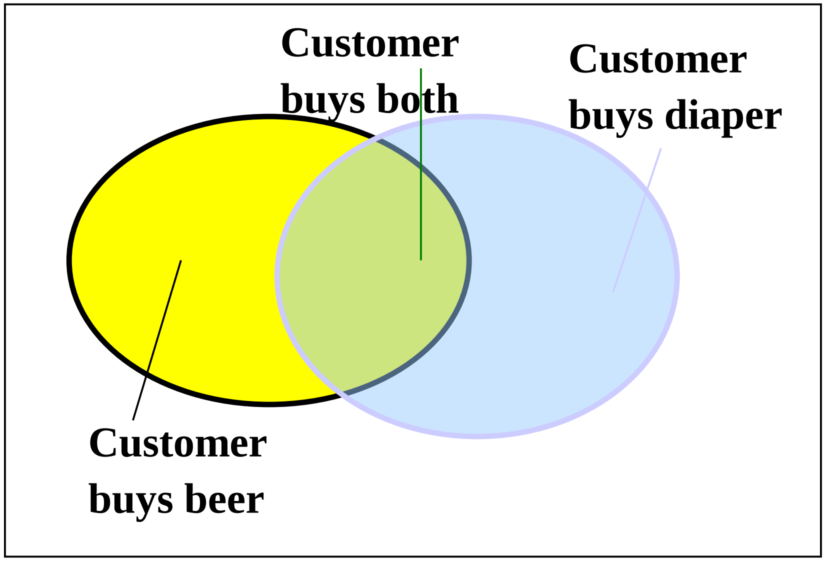
- **Frequent Pattern**: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set.
- For example, a set of items, such as milk and bread, that appear frequently together in a transaction data set is a ***frequent itemset***.
- A subsequence, such as buying first a PC, then a digital camera, and then a memory card, if it occurs frequently in a shopping history database, is a **(*frequent*) *sequential pattern***.
- A substructure can refer to different structural forms, such as subgraphs, subtrees, or sublattices, which may be combined with itemsets or subsequences. If a substructure occurs frequently, it is called a **(*frequent*) *structured pattern***.

Frequent Pattern
Market Basket Analysis

- **Frequent Pattern**: a pattern that occurs frequently in a data set.
  - A set of items that appear frequently together in a transaction data set is called as a ***frequent itemset***.
- An example of ***frequent itemset mining*** is **market basket analysis**.
  - This process analyzes customer buying habits by finding associations between the different items that customers place in their “shopping baskets”.
  - If we think of the universe as the set of items available at the store, then each item has a Boolean variable representing the presence or absence of that item.
  - Each basket can then be represented by a Boolean vector of values assigned to these variables.
  - The Boolean vectors can be analyzed for buying patterns that reflect items that are frequently associated or purchased together.
  - These ***patterns*** can be represented in the form of ***association rules***.

Basic Concepts: Frequent Patterns

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



- **itemset**: A set of one or more items
  - **k-itemset** $X = \{x_1, \ldots, x_k\}$
- **(absolute) support of X**: Frequency of an itemset X.
  - Absolute Support of {Beer} is 3
- **(relative) support of X** is the fraction of transactions that contains X (i.e., the probability that a transaction contains X).
  - Relative Support of {Beer} is 3/5
- An itemset X is **frequent** if **X's support** is no less than a **minsup** threshold.

Basic Concepts: Association Rules

Association Rule

- **An implication expression of the form X $\rightarrow$ Y, where X and Y are itemsets**

Association Rule Mining:

- Find all the rules X $\rightarrow$ Y with **minimum support** and **minimum confidence**
  - **support**, probability that a transaction contains $X \cup Y$: $P(X \cup Y)$
    - Fraction of transactions that contain both X and Y
  - **confidence**, conditional probability that a transaction having X also contains *Y* : $P(Y/X) = support(X \cup Y) / support(X)$
    - Measures how often items in Y appear in transactions that contain X

Basic Concepts: Association Rules

Association Rule

– **An implication expression of the form X → Y, where X and Y are itemsets**

Association Rule Mining:

- Find all the rules X → Y with **minimum support** and **minimum confidence**

Let minsup = 50%, minconf = 50%

Frequent Pattterns:

{Beer}:3, {Nuts}:3, {Diaper}:4, {Eggs}:3,

{Beer, Diaper}:3

Association Rules:

– { Beer } → { Diaper } (60%, 100%)
– { Diaper } → { Beer } (60%, 75%)

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

Why Use Support and Confidence?

- **Support** is an important measure because a rule that has very low support may occur simply by chance.
  - A low support rule may be uninteresting from a business perspective because it may not be profitable to promote items that customers seldom buy together
  - For these reasons, support is often used to eliminate uninteresting rules

- **Confidence** measures the reliability of the inference made by a rule.
  - For a given rule X → Y, the higher the confidence, the more likely it is for Y to be present in transactions that contain X.

- Association analysis results should be interpreted with caution.
  - The inference made by an association rule does not necessarily imply causality.
  - Instead, it suggests a strong co-occurrence relationship between items in the antecedent and consequent of the rule.

Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - **support** ≥ ***minsup*** threshold
  - **confidence** ≥ ***minconf*** threshold

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the ***minsup*** and ***minconf*** thresholds

  ⇒ Computationally not feasible!

Mining Association Rules

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

Observations:

- All the above rules are binary partitions of the same itemset: {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Association Rule Mining

- The problem of mining association rules can be reduced to that of mining frequent itemsets.

- In general, association rule mining can be viewed as a *two-step process*:

1. **Find all frequent itemsets**: By definition, each of these itemsets will occur at least as frequently as a predetermined minimum support count, **minsup**.
   - Generate all itemsets whose **support ≥ minsup**
2. **Generate strong association rules from the frequent itemsets**: By definition, these rules must satisfy minimum support and minimum confidence.
   - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

– Frequent itemset generation is still computationally expensive

Association Rules - Example

Transactions
A,B,D
A,B,C,D
A
A,B,C
B,C
B

minsup = 0.5 minconf=0.7

- Find frequent itemsets and association rules satisfying minsup and minconf.

Association Rules - Example

Transactions
A,B,D
A,B,C,D
A
A,B,C
B,C
B

minsup = 0.5 minconf=0.7

- Find frequent itemsets and association rules satisfying minsup and minconf.

Frequent Itemsets:

1-itemsets: **{A}** **support({A}) = 4/6**
{B} **support({B}) = 5/6**
{C} **support({C}) = 3/6**

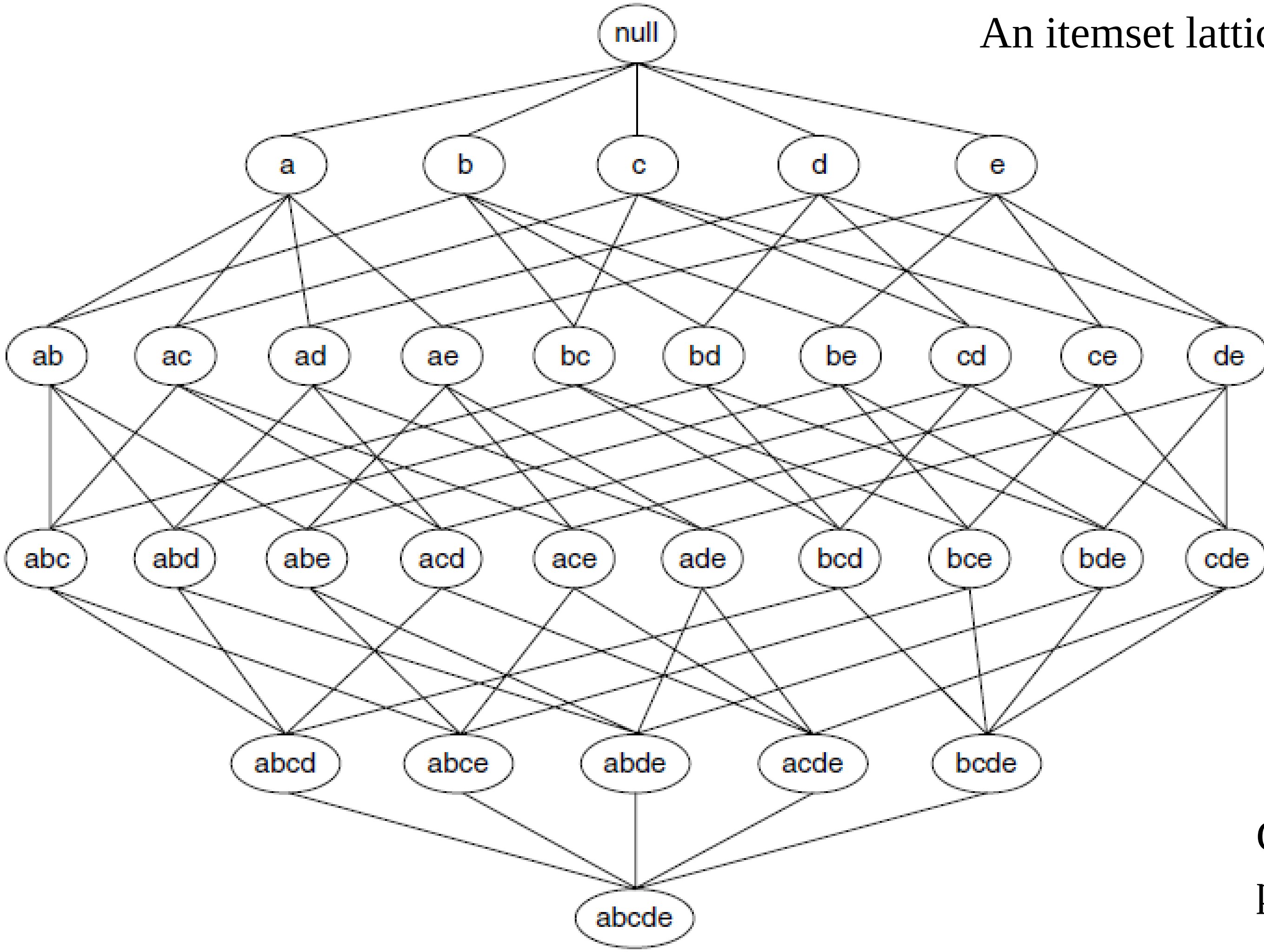
2-itemsets: **{A,B}** **support({A,B}) = 3/6**
{B,C} **support({B,C}) = 3/6**

Association Rules:

A→B **conf(A→B) = 3/4**
C→B **conf(C→B) = 3/3**

Frequent Itemset Generation

An itemset lattice



Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- **Brute-force approach:**
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database

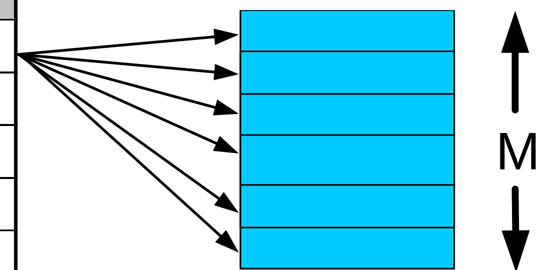
Transactions

N

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

w

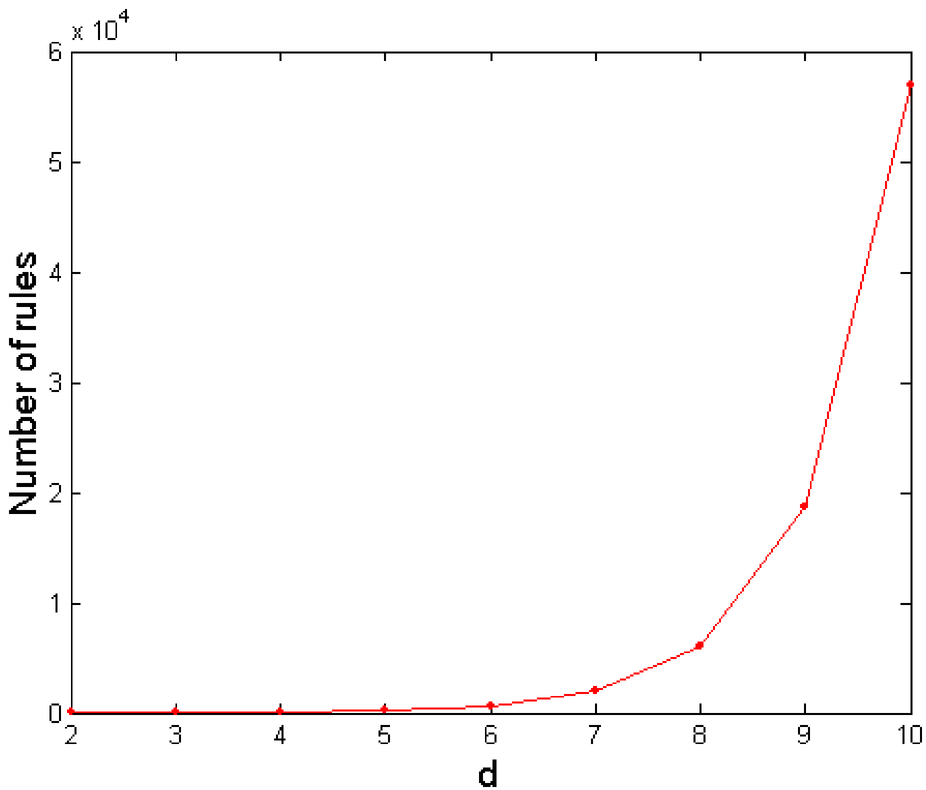
List of Candidates



  - Match each transaction against every candidate
  - Complexity ~ O(NMW) => Expensive since $M = 2^d$!!!

Computational Complexity

- Given d unique items:
  - Total number of itemsets = 2^d
  - Total number of possible association rules:



$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k} \times \sum_{j=1}^{d-k}\binom{d-k}{j}\right]$$
$$= 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M
  - The **Apriori principle** is an effective way to eliminate some of the candidate itemsets without counting their support values.

- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases

- **Frequent Itemsets, Association Rules**
- **Apriori Algorithm**
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Reducing Number of Candidates
Apriori Principle

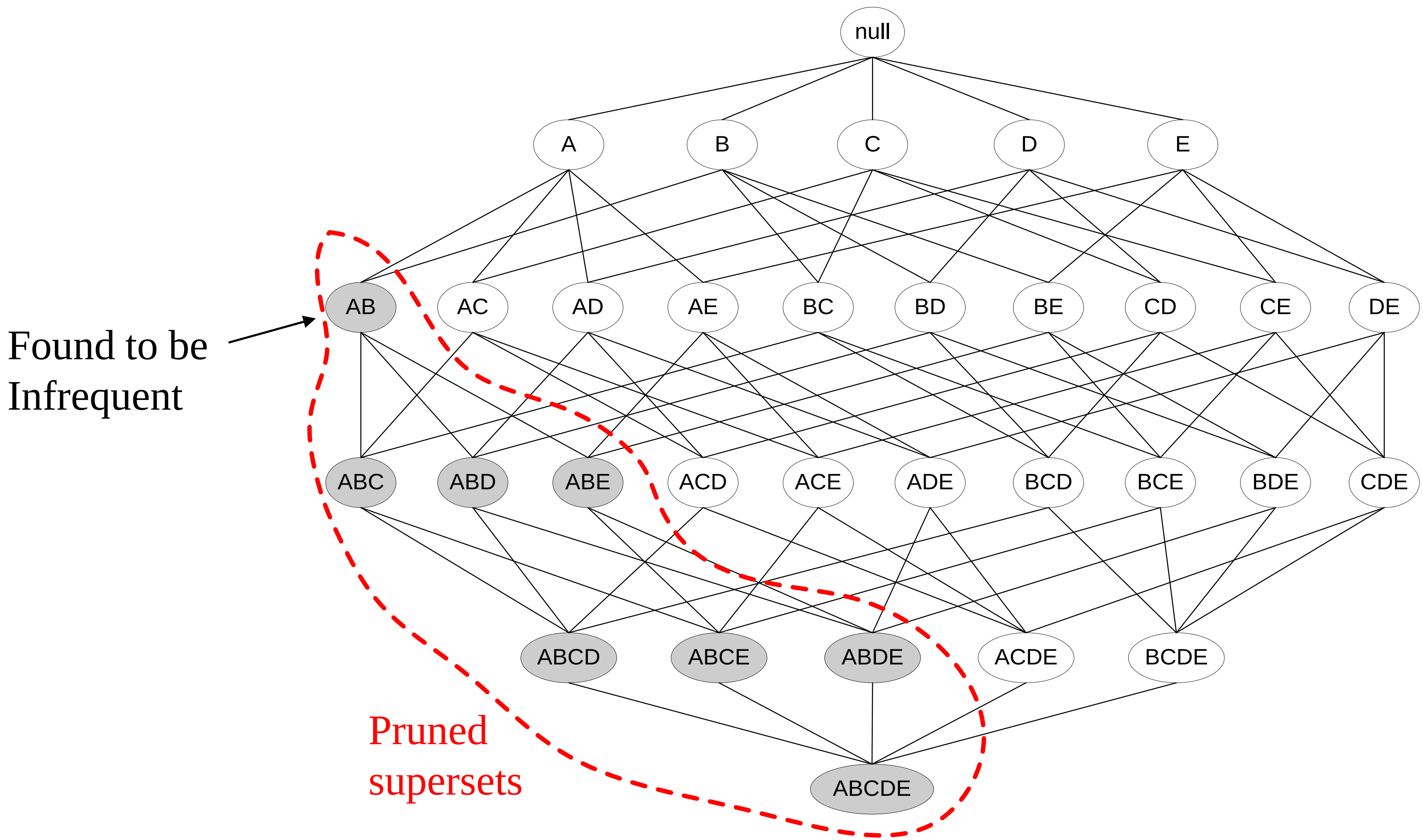
- **Apriori Principle: If an itemset is frequent, then all of its subsets must also be frequent.**

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  – Support of an itemset never exceeds the support of its subsets
  – This is known as the anti-monotone property of support

Illustrating Apriori Principle



Illustrating Apriori Principle

TID	Items
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Minimum Support = 3

Generate 1-itemset candidates

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

$6 + 15 + 20 = 41$

With support-based pruning,

$6 + 6 + 4 = 16$

Illustrating Apriori Principle

TID	*Items*
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Items (1-itemsets)

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Minimum Support = 3

Eliminate infrequent 1-itemset candidates

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

$6 + 15 + 20 = 41$

With support-based pruning,

$6 + 6 + 4 = 16$

Illustrating Apriori Principle

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Items (1-itemsets)

Itemset
{Bread,Milk}
{Bread, Beer }
{Bread,Diaper}
{Beer, Milk}
{Diaper, Milk}
{Beer,Diaper}

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

$6 + 15 + 20 = 41$

With support-based pruning,

$6 + 6 + 4 = 16$

Generate 2-itemset candidates

Illustrating Apriori Principle

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	**3**
{Beer, Bread}	**2**
{Bread,Diaper}	**3**
{Beer,Milk}	**2**
{Diaper,Milk}	**3**
{Beer,Diaper}	**3**

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Eliminate infrequent 2-itemset candidates

If every subset is considered,

$$^6C_1 + {}^6C_2 + {}^6C_3$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

Illustrating Apriori Principle

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	**3**
{Bread,Beer}	**2**
{Bread,Diaper}	**3**
{Milk,Beer}	**2**
{Milk,Diaper}	**3**
{Beer,Diaper}	**3**

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

Itemset
{ Beer, Diaper, Milk}
{ Beer,Bread,Diaper}
{Bread, Diaper, Milk}
{ Beer, Bread, Milk}

If every subset is considered,

$$^6C_1 + {}^6C_2 + {}^6C_3$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

Generate 3-itemset candidates

Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

Itemset	Count
{ Beer, Diaper, Milk}	2
{ Beer,Bread, Diaper}	2
{Bread, Diaper, Milk}	2
{Beer, Bread, Milk}	1

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

$$6 + 6 + 1 = 13$$

Prune 3-itemset candidates with infrequent 2-itemsets

Eliminate infrequent 3-itemset candidates

Apriori Algorithm:
Finding Frequent Itemsets Using Candidate Generation

- **Apriori pruning principle**: If there is any itemset which is infrequent, its superset should not be generated/tested!

Apriori Algorithm: F_k: frequent k-itemsets L_k: candidate k-itemsets

- Let k=1
- Generate F_1 = {frequent 1-itemsets}
- Repeat until F_k is empty
  - **Candidate Generation**: Generate L_{k+1} from F_k
  - **Candidate Pruning**: Prune candidate itemsets in L_{k+1} containing subsets of length k that are infrequent
  - **Support Counting**: Count the support of each candidate in L_{k+1} by scanning the DB
  - **Candidate Elimination**: Eliminate candidates in L_{k+1} that are infrequent, leaving only those that are frequent => F_{k+1}

Apriori Algorithm:
Candidate Generation: F_{k-1} x F_{k-1} Method

- Merge two frequent (k-1)-itemsets if their first (k-2) items are identical

- F_3 = {ABC,ABD,ABE,ACD,BCD,BDE,CDE}
  - Merge(**AB**C, **AB**D) = **AB**CD
  - Merge(**AB**C, **AB**E) = **AB**CE
  - Merge(**AB**D, **AB**E) = **AB**DE

  - Do not merge(**A**BD,**A**CD) because they share only prefix of length 1 instead of length 2

- L_4 = {ABCD,ABCE,ABDE} is the set of candidate 4-itemsets **generated**

Apriori Algorithm:
Candidate Pruning

- Let F_3 = {ABC,ABD,ABE,ACD,BCD,BDE,CDE} be the set of frequent 3-itemsets

- L_4 = {ABCD,ABCE,ABDE} is the set of **candidate 4-itemsets generated**

- Candidate pruning
  – Prune ABCE because ACE and BCE are infrequent
  – Prune ABDE because ADE is infrequent

- After **candidate pruning**: L_4 = {ABCD}

Apriori Algorithm:
Support Counting of Candidate Itemsets

- Scan the database of transactions to determine the support of each candidate itemset
  - Must match every candidate itemset against every transaction, which is an expensive operation

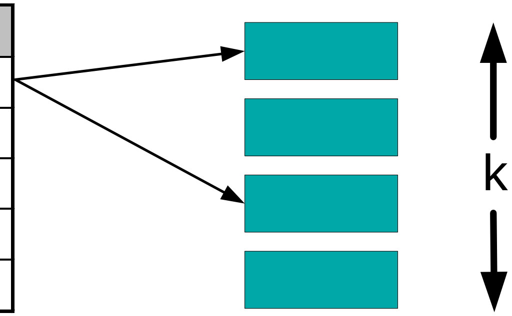
- To reduce the number of comparisons, store the candidates in a hash structure
  - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets

Transactions

N

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Hash Structure



Buckets

Apriori Algorithm

Algorithm: Apriori. Find frequent itemsets using an iterative level-wise approach based on candidate generation.

Input:

- D, a database of transactions;
- *min_sup*, the minimum support count threshold.

Output: L, frequent itemsets in D.

Method:

L_1 = find_frequent_1-itemsets(D);
for $(k = 2; L_{k-1} \neq \phi; k{+}{+})$ {
  C_k = apriori_gen(L_{k-1});
  for each transaction $t \in D$ { // scan D for counts
    C_t = subset(C_k, t); // get the subsets of t that are candidates
    for each candidate $c \in C_t$
      c.count++;
  }
  $L_k = \{c \in C_k | c.count \geq min\_sup\}$
}
return $L = \cup_k L_k$;

Apriori Algorithm

```
procedure apriori_gen(L_{k-1}:frequent (k-1)-itemsets)
    for each itemset l_1 ∈ L_{k-1}
        for each itemset l_2 ∈ L_{k-1}
            if (l_1[1] = l_2[1]) ∧ (l_1[2] = l_2[2])
                ∧ ... ∧ (l_1[k-2] = l_2[k-2]) ∧ (l_1[k-1] < l_2[k-1]) then {
                c = l_1 ⋈ l_2; // join step: generate candidates
                if has_infrequent_subset(c, L_{k-1}) then
                    delete c; // prune step: remove unfruitful candidate
                else add c to C_k;
            }
    return C_k;
```

```
procedure has_infrequent_subset(c: candidate k-itemset;
            L_{k-1}: frequent (k-1)-itemsets); // use prior knowledge
    for each (k-1)-subset s of c
        if s ∉ L_{k-1} then
            return TRUE;
    return FALSE;
```

Apriori Algorithm - An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1^{st} scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2^{nd} scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3^{rd} scan

L_3

Itemset	sup
{B, C, E}	2

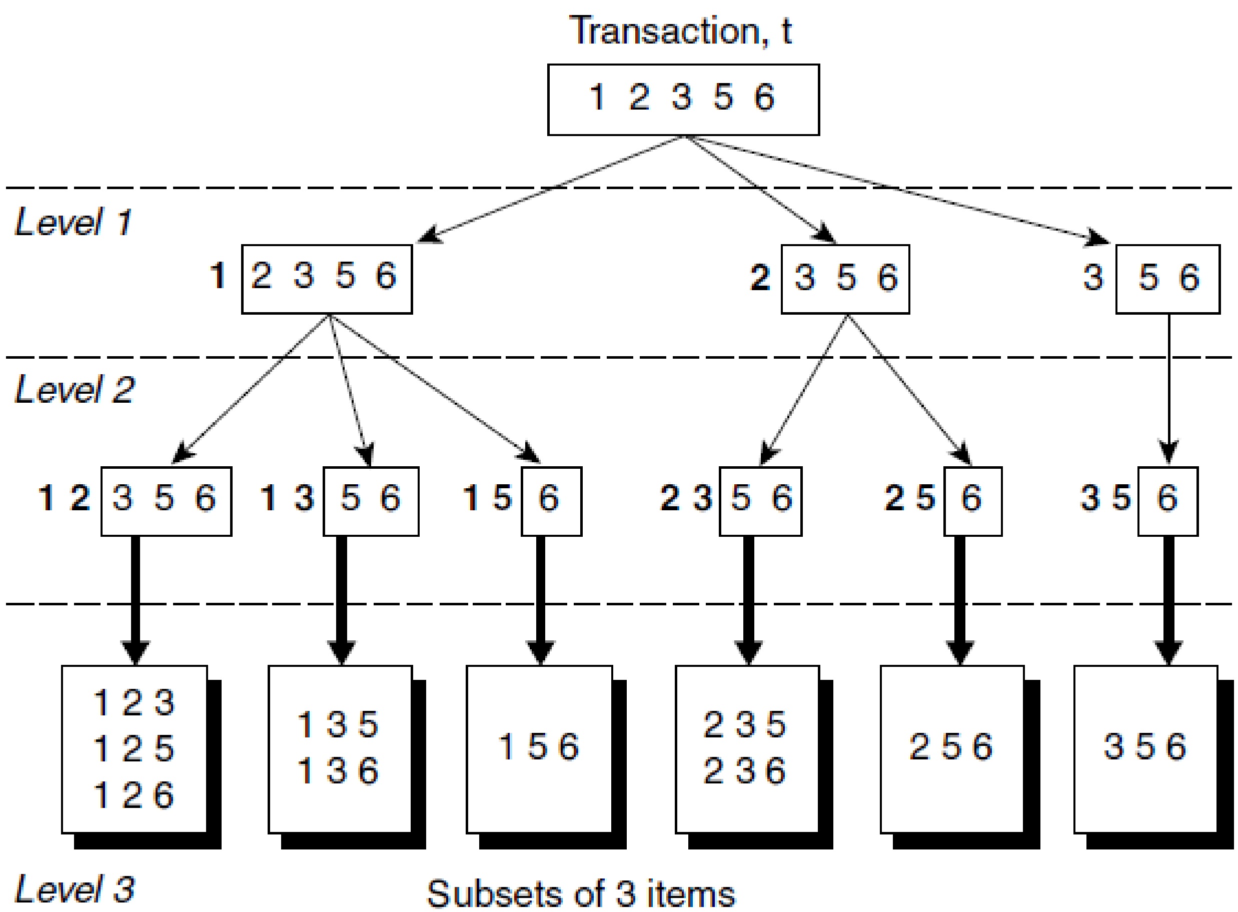
Support Counting Using Hash Tree

- Why counting supports of candidates a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
  - Must match every candidate itemset against every transaction, which is an expensive operation

- Method:
  - Candidate itemsets are stored in a ***hash-tree***
  - ***Leaf* node** of hash-tree contains a list of itemsets and counts
  - ***Interior* node** contains a hash table
  - ***Subset function*:** finds all the candidates contained in a transaction

Support Counting Using Hash Tree
Subset Operation

- Enumerating subsets of three items from a transaction t



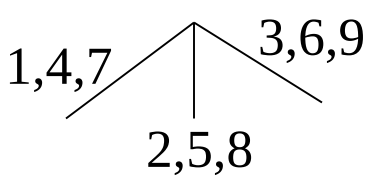
Support Counting Using Hash Tree
Generate Candidate Hash Tree

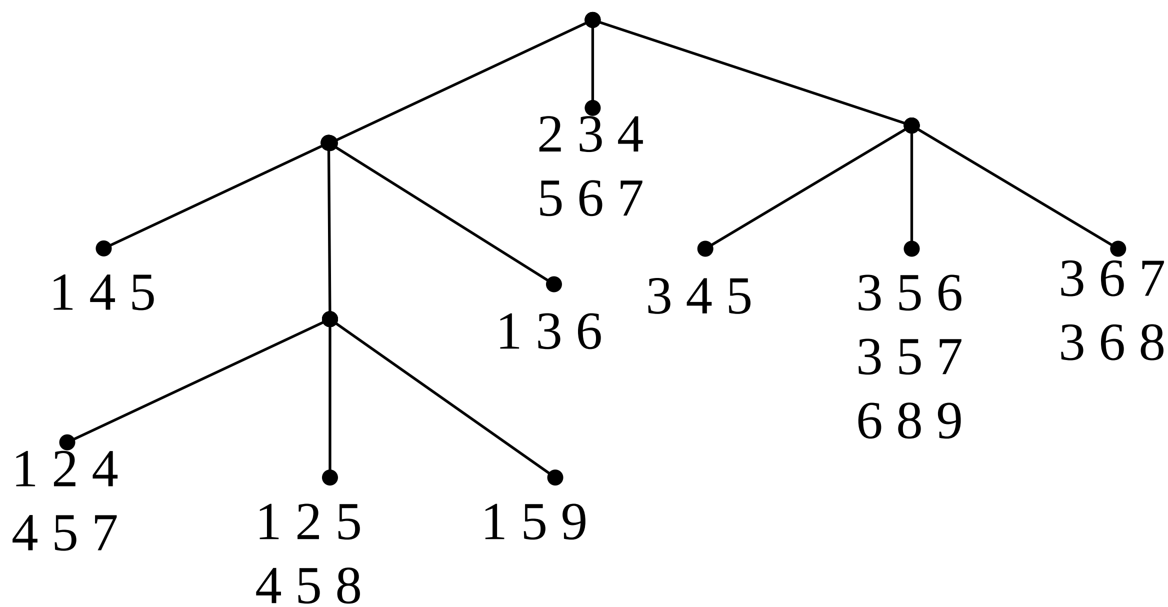
- Suppose you have 15 candidate itemsets of length 3:

  {1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

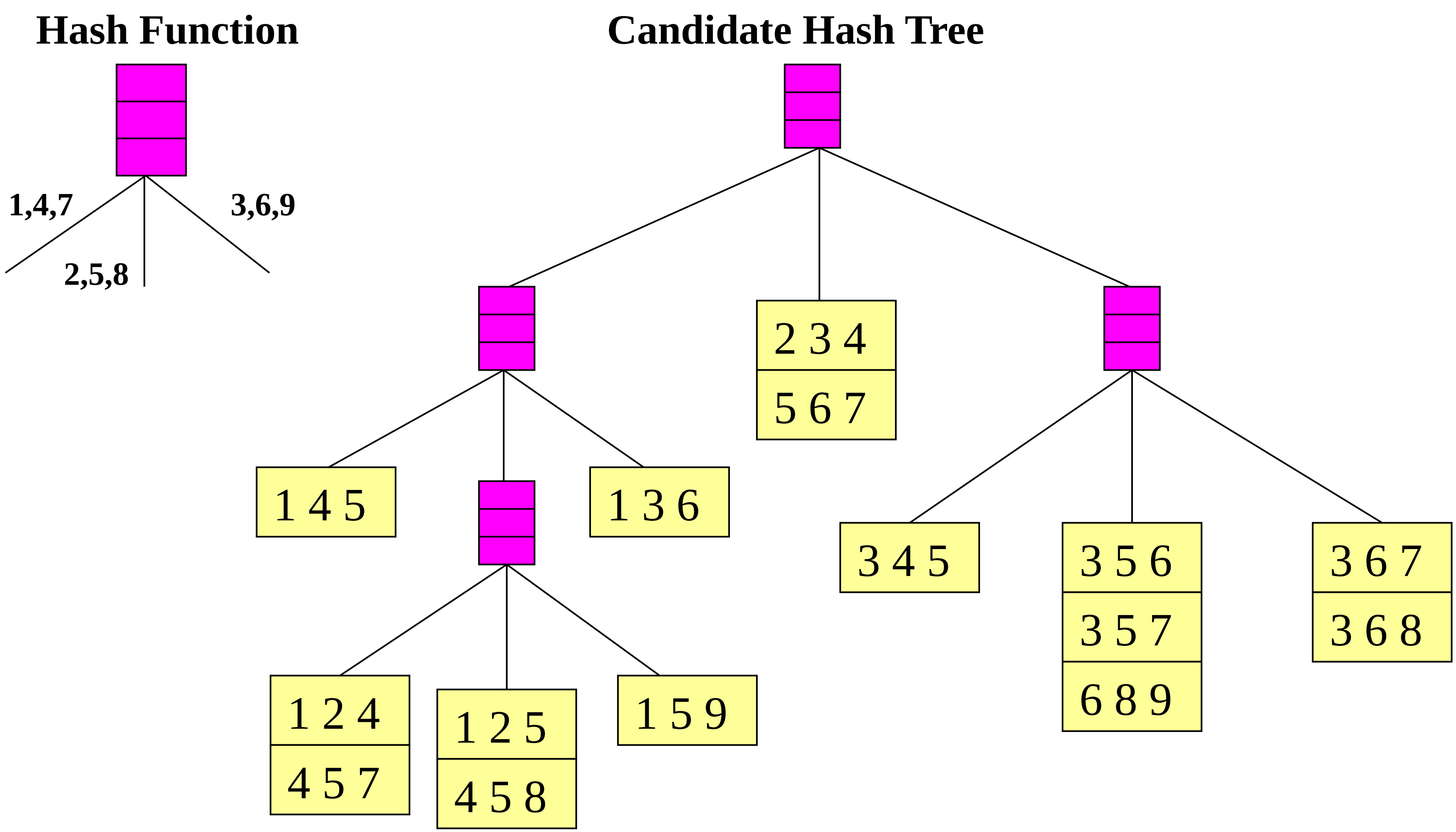
- We need: Hash function
  - HashFunc: **mod 3**

Hash function



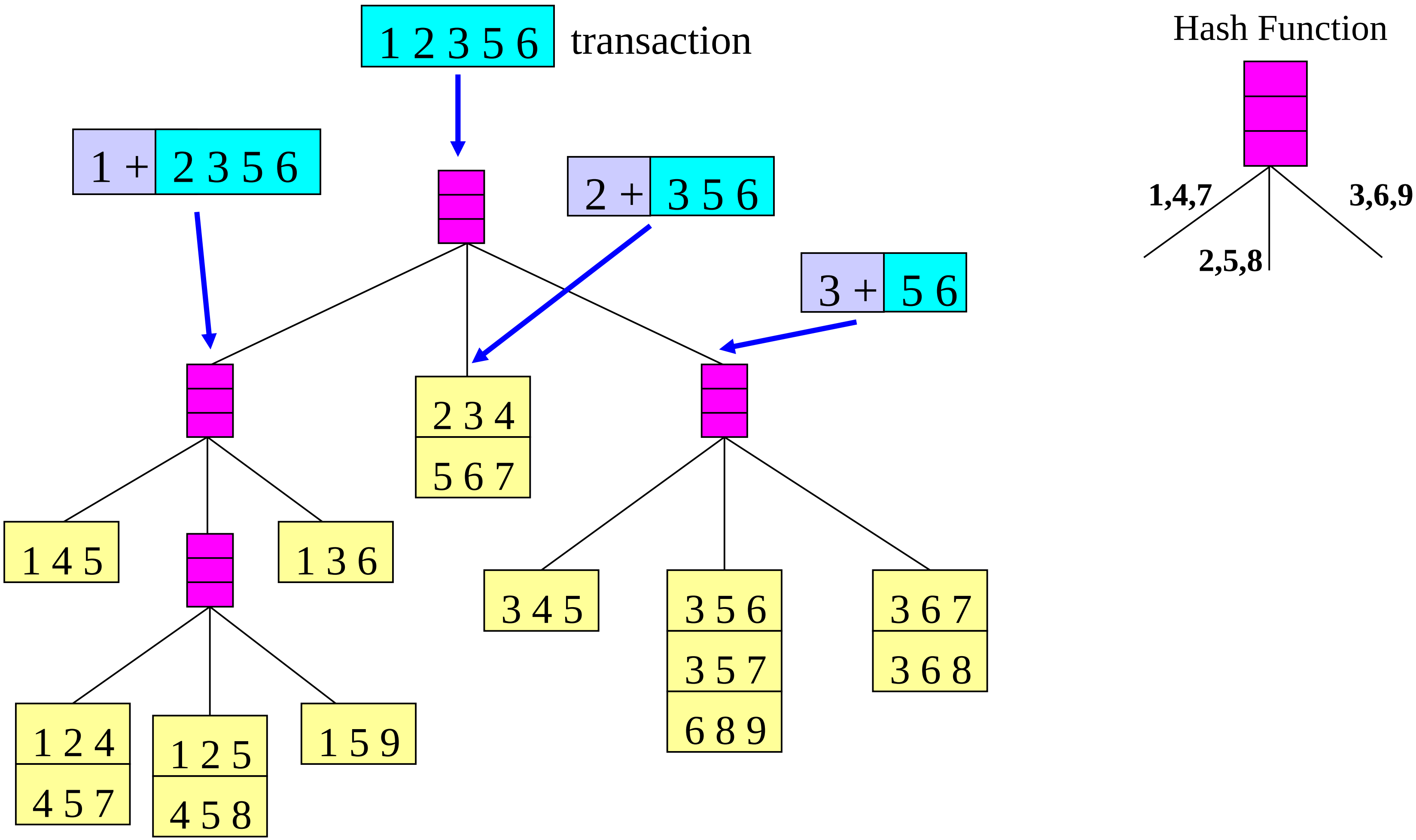


Support Counting Using Hash Tree
Generate Candidate Hash Tree



Support Counting Using Hash Tree

Traverse Candidate Hash Tree to Update Support Counts



Support Counting Using Hash Tree

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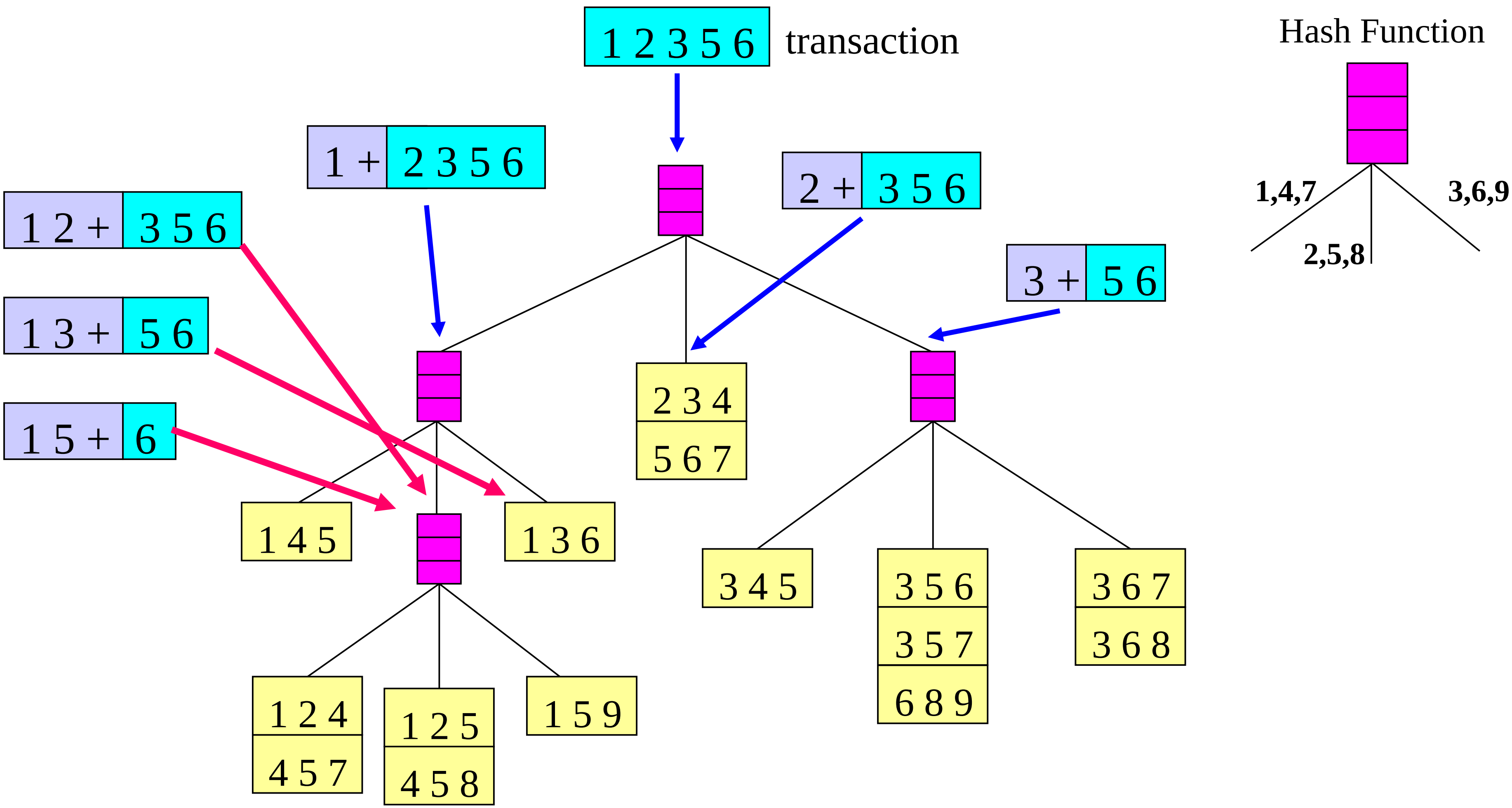
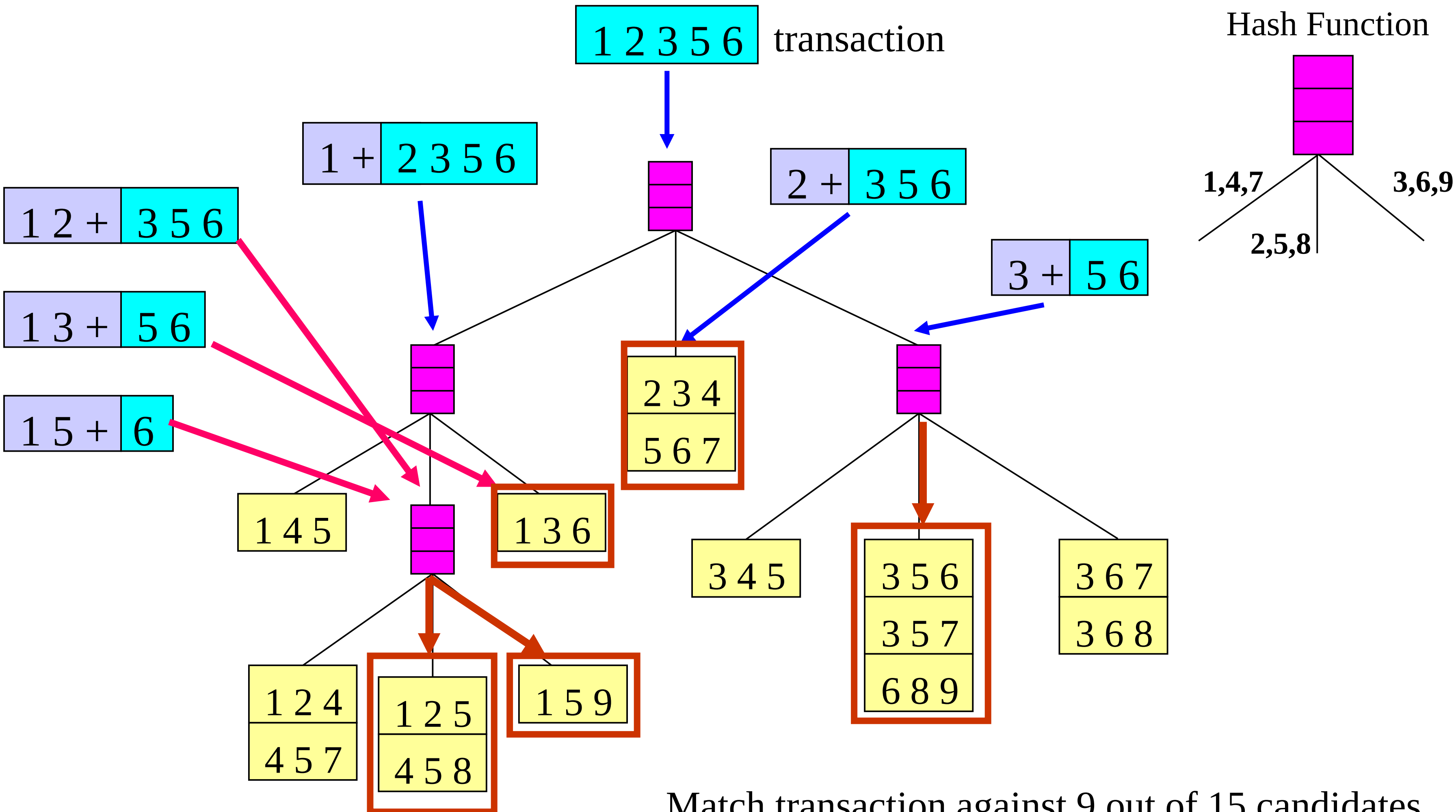


Support Counting Using Hash Tree

Traverse Candidate Hash Tree to Update Support Counts



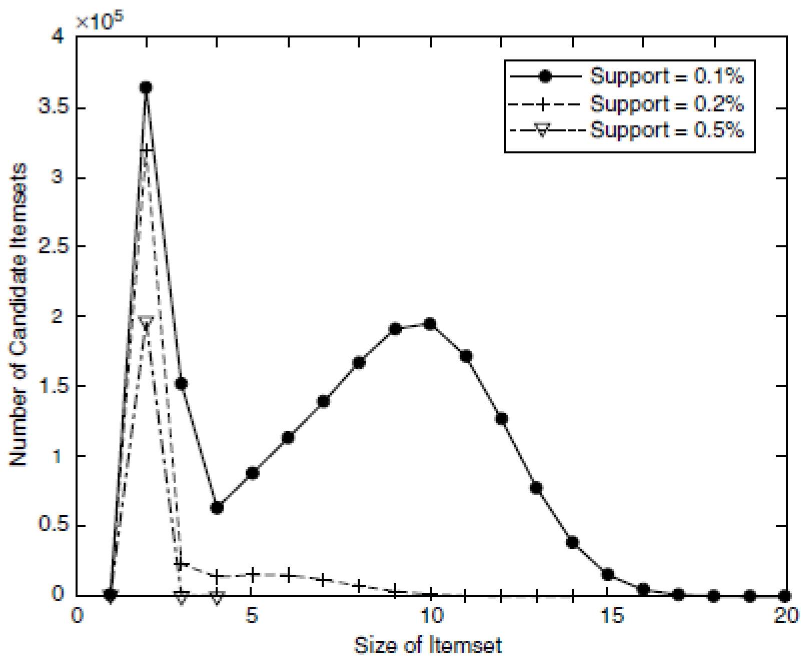
Match transaction against 9 out of 15 candidates

Factors Affecting Complexity of Apriori Algorithm

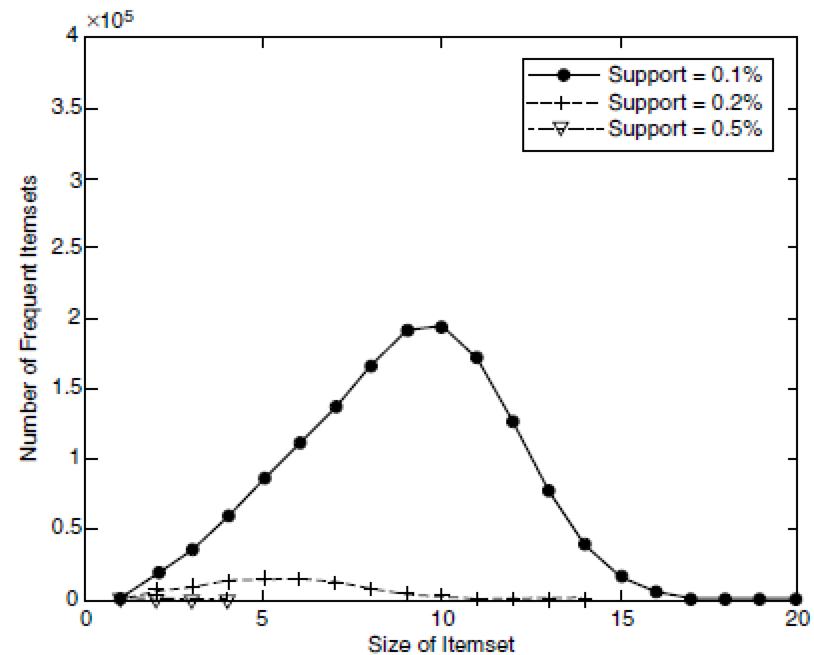
- **Choice of minimum support threshold**
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- **Dimensionality (number of items) of the data set**
  - more space is needed to store support count of each item
  - if number of frequent items also increases, both computation and I/O costs may also increase
- **Size of database**
  - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- **Average transaction width**
  - transaction width increases with denser data sets
  - This may increase max length of frequent itemsets and number of subsets in a transaction increases with its width

Effect of Support Threshold

- Effect of support threshold on the number of candidate and frequent itemsets



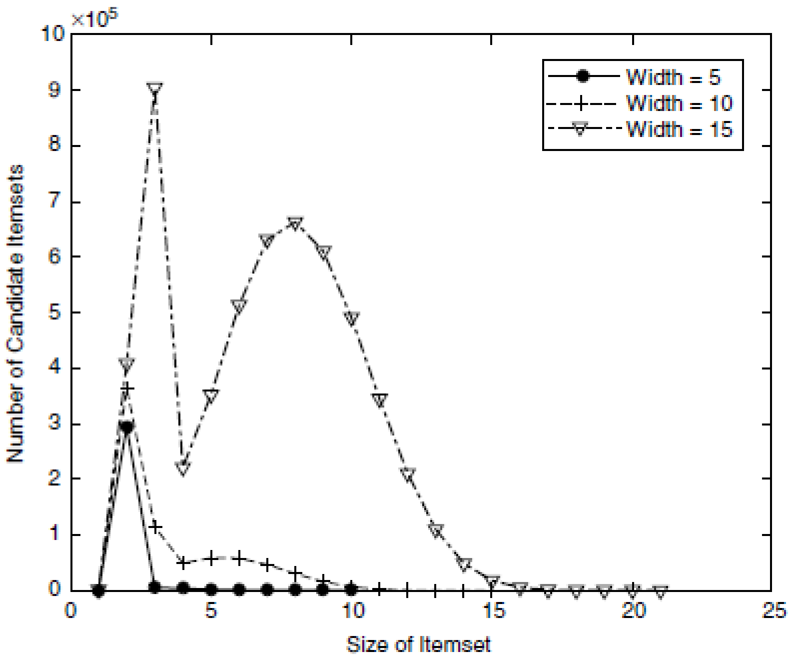
Number of candidate itemsets



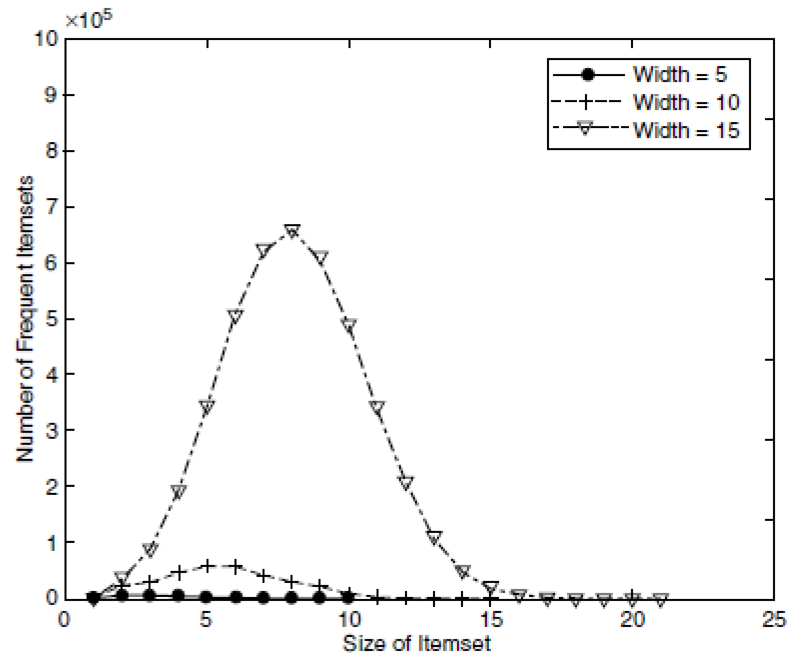
Number of frequent itemsets

Effect of Average Transaction Width

- Effect of average transaction width on the number of candidate and frequent itemsets



Number of candidate itemsets



Number of frequent itemsets

Effect of Support Distribution

- How to set the appropriate *minsup* threshold?
  - If *minsup* is set too high, we could miss itemsets involving interesting rare items (e.g., expensive products)
  - If *minsup* is set too low, it is computationally expensive and the number of itemsets is very large
- Using a single minimum support threshold may not be effective

Multiple Minimum Support

- How to apply multiple minimum supports?
  - MS(i): minimum support for item i
  - e.g.: MS(Milk)=5%, MS(Coke) = 3%,
    MS(Broccoli)=0.1%, MS(Salmon)=0.5%
  - MS({Milk, Broccoli}) = min (MS(Milk), MS(Broccoli))
    = 0.1%
  - Challenge: Support is no longer anti-monotone
    - Suppose: Support(Milk, Coke) = 1.5% and
      Support(Milk, Coke, Broccoli) = 0.5%
    - {Milk,Coke} is infrequent but {Milk,Coke,Broccoli} is frequent

Multiple Minimum Support

- Order the items according to their minimum support (in ascending order)
  - e.g.: MS(Milk)=5%, MS(Coke) = 3%, MS(Broccoli)=0.1%, MS(Salmon)=0.5%
  - Ordering: Broccoli, Salmon, Coke, Milk

- Need to modify Apriori such that:
  - L_1 : set of frequent items
  - F_1 : set of items whose support is ≥ MS(1) where MS(1) is $\min_i$(MS(i))
  - C_2 : candidate itemsets of size 2 is generated from F_1 instead of L_1

Multiple Minimum Support

- Modifications to Apriori:
  - In traditional Apriori,
    - A candidate (k+1)-itemset is generated by merging two frequent itemsets of size k
    - The candidate is pruned if it contains any infrequent subsets of size k
  - Pruning step has to be modified:
    - Prune only if subset contains the first item
    - e.g.: Candidate={Broccoli, Coke, Milk} (ordered according to minimum support)
    - {Broccoli, Coke} and {Broccoli, Milk} are frequent but {Coke, Milk} is infrequent
      - Candidate is not pruned because {Coke,Milk} does not contain the first item, i.e., Broccoli.

Rule Generation in Apriori Algorithm

- Given a frequent itemset **L**, find all non-empty subsets $\mathbf{f \subset L}$ such that **candidate rule $\mathbf{f \rightarrow L - f}$** satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

$$
\begin{array}{llll}
ABC \rightarrow D & ABD \rightarrow C & ACD \rightarrow B & BCD \rightarrow A \\
D \rightarrow ABC & C \rightarrow ABD & B \rightarrow ACD & A \rightarrow BCD \\
 & & & \\
AB \rightarrow CD & AC \rightarrow BD & AD \rightarrow BC & \\
CD \rightarrow AB & BD \rightarrow AC & BC \rightarrow AD &
\end{array}
$$

- If $|L| = k$, then there are $2^k - 2$ candidate association rules
  - (ignoring $L \rightarrow \varnothing$ and $\varnothing \rightarrow L$)

Rule Generation in Apriori Algorithm

- How to efficiently generate rules from frequent itemsets?

- In general, confidence does not have an anti-monotone property

  $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$

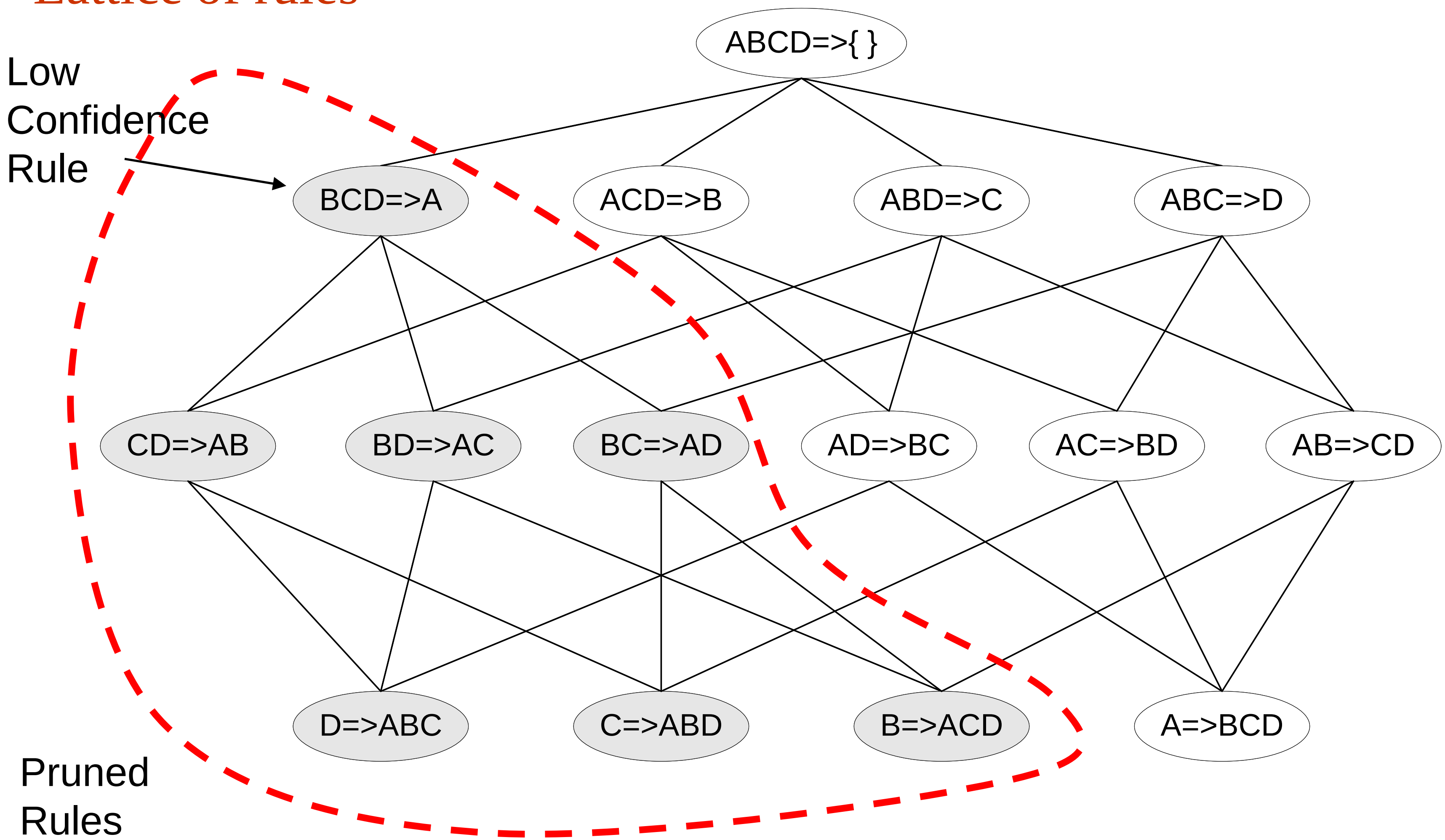
- But confidence of rules generated from the same itemset has an anti-monotone property
  - E.g., Suppose {A,B,C,D} is a frequent 4-itemset:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

  - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation in Apriori Algorithm

Lattice of rules



- **Frequent Itemsets, Association Rules**
- **Apriori Algorithm**
- **Compact Representation of Frequent Itemsets**
- **FP-Growth Algorithm: An Alternative Frequent Itemset Generation Algorithm**
- **Evaluation of Association Patterns**

Compact Representation of Frequent Itemsets

- The number of frequent itemsets produced from a transaction data set can be very large.
- Some produced itemsets can be redundant because they have identical support as their supersets

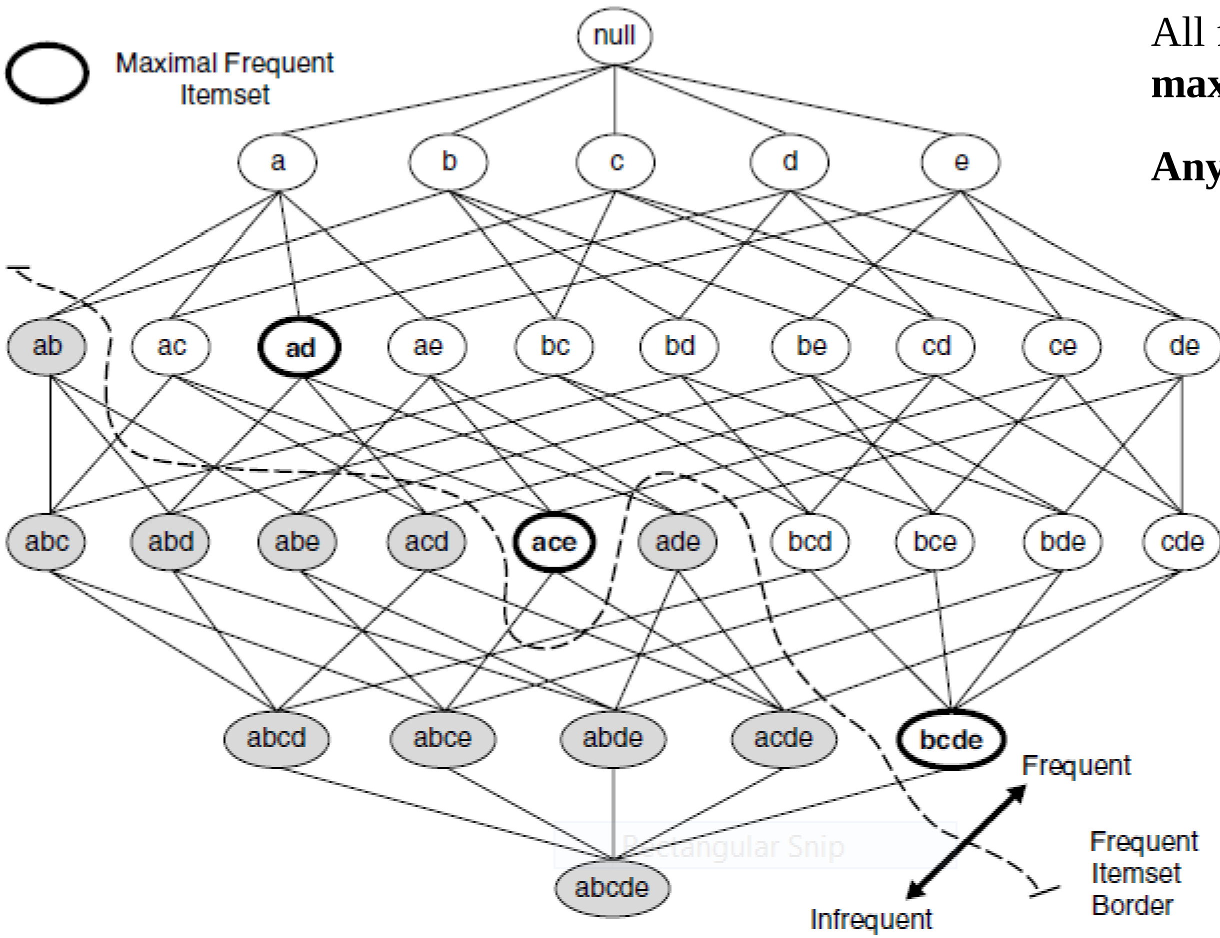
- It is useful to identify a small representative set of itemsets from which all other frequent itemsets can be derived. ➔ Need a compact representation
  - **Maximal Frequent Itemsets** and
  - **Closed Frequent Itemsets**

Maximal Frequent Itemsets

Maximal Frequent Itemset: A maximal frequent itemset is defined as a frequent itemset for which none of its immediate supersets are frequent.

- Maximal frequent itemsets effectively provide a compact representation of frequent itemsets.
- Maximal frequent itemsets form the smallest set of itemsets from which all frequent itemsets can be derived.

Maximal Frequent Itemsets



All frequent itemsets can be derived from **maximal frequent itemsets** **ad**, **ace**, **bcde**.

Any frequent itemset $\subseteq$
a maximal frequent itemset

Maximal Frequent Itemsets

- Despite providing a compact representation, maximal frequent itemsets do not contain the support information of their subsets.
- For example, the support of the maximal frequent itemsets {a, c, e}, {a, d}, and {b,c,d,e} do not provide any hint about the support of their subsets.
- An additional pass over the data set is therefore needed to determine the support counts of the non-maximal frequent itemsets.

- It might be desirable to have a minimal representation of frequent itemsets that preserves the support information. ➔ **Closed Frequent Itemsets**

Closed Frequent Itemsets

Closed Itemset: An itemset X is closed if none of its immediate supersets has exactly the same support count as X.

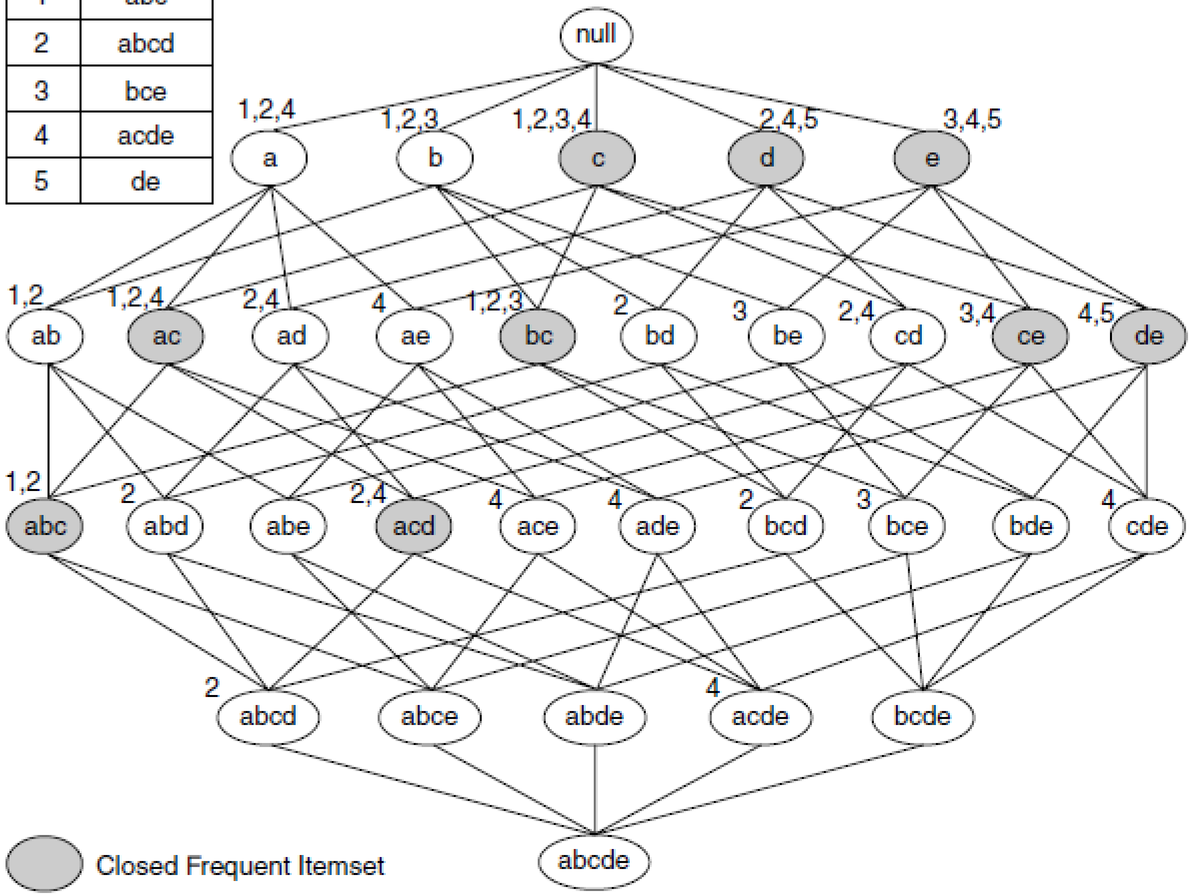
- **Closed itemsets** provide a minimal representation of itemsets without losing their support information.
- Put another way, X is not closed if at least one of its immediate supersets has the same support count as X.

Closed Frequent Itemset: An itemset is a closed frequent itemset if it is closed and its support is greater than or equal to *minsup*.

Closed Frequent Itemsets

TID	Items
1	abc
2	abcd
3	bce
4	acde
5	de

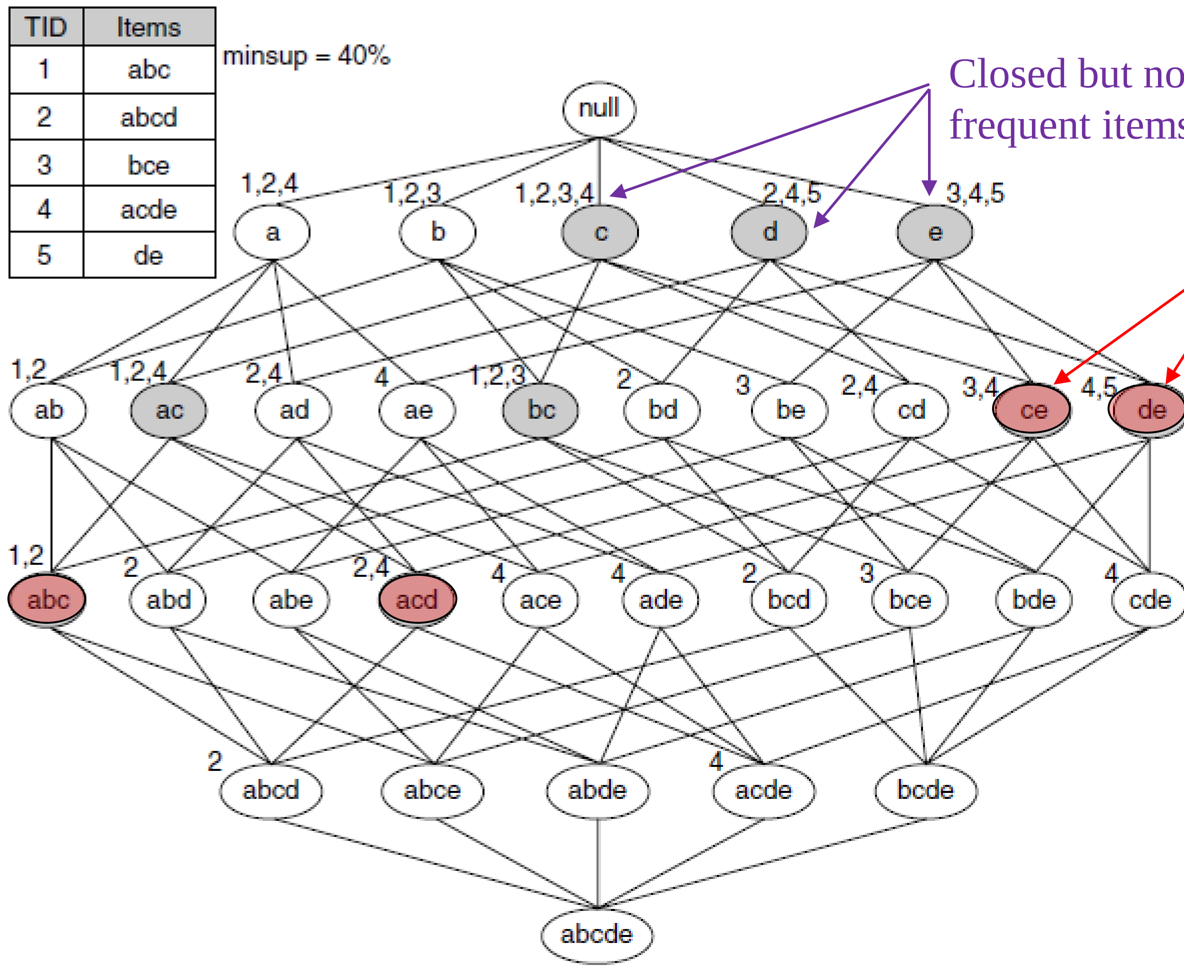
minsup = 40%



All subsets of a closed frequent itemset are frequent and their supports is greater than or equal to the support of that closed frequent itemset.

For example, all subsets of a closed frequent itemset **abc** are frequent and their supports ≥ support of **abc**.

Maximal vs Closed Itemsets



TID	Items
1	abc
2	abcd
3	bce
4	acde
5	de

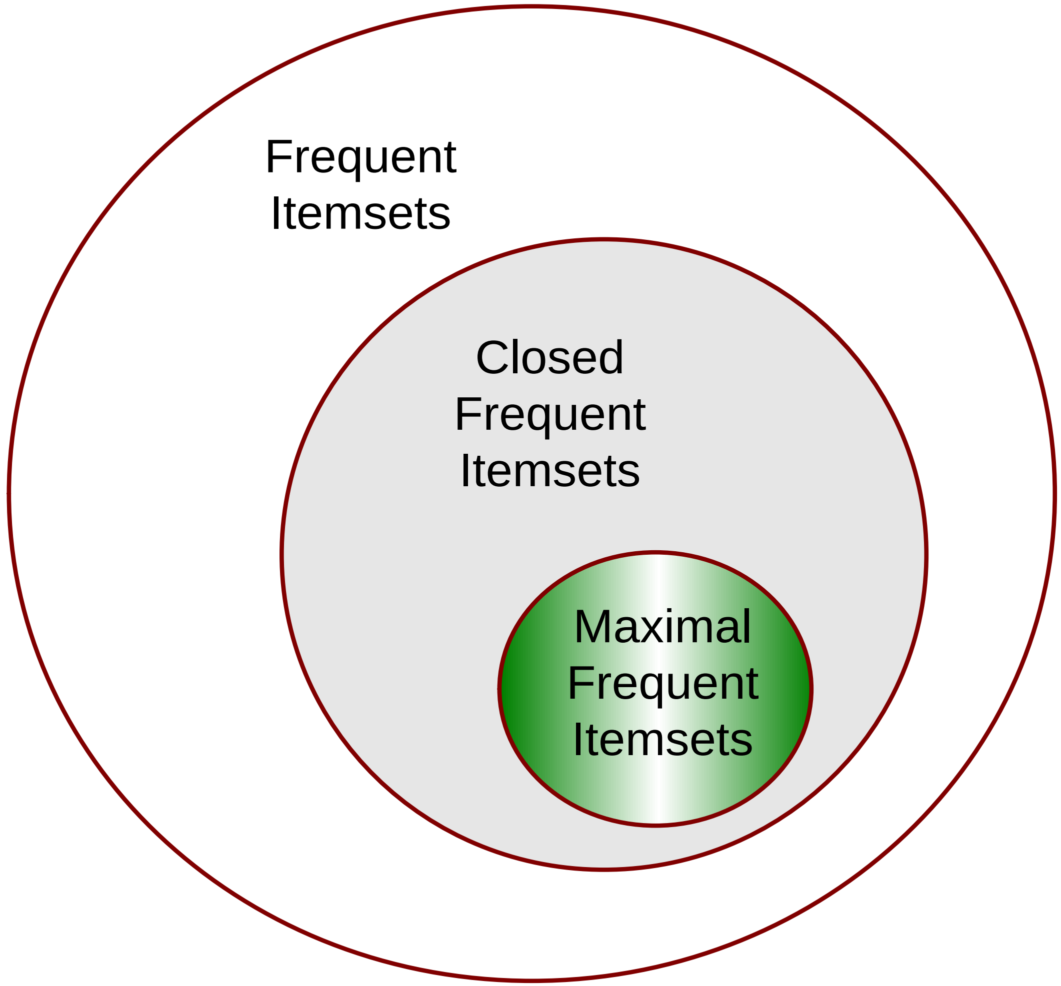
Closed but not maximal frequent itemsets

Closed and maximal frequent itemsets

Closed = 9

Maximal = 4

Maximal vs Closed Itemsets



- **Frequent Itemsets, Association Rules**
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FP-Growth (Frequent Pattern Growth) Algorithm

- **FP-growth algorithm** that takes a radically different approach to discovering frequent itemsets.
  - The algorithm does not subscribe to the generate-and-test paradigm of Apriori

- **FP-growth algorithm** encodes the data set using a compact data structure called an **FP-tree** and extracts frequent itemsets directly from this structure.
  - Use a compressed representation of the database using an FP-tree
  - Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets

FP-Tree Construction

- An FP-tree is a compressed representation of the input data.

- It is constructed by reading the data set one transaction at a time and mapping each transaction onto a path in the FP-tree.
  - Different transactions can have several items in common, their paths may overlap.
  - The more the paths overlap with one another, the more compression we can achieve using the FP-tree structure.

FP-Tree Construction

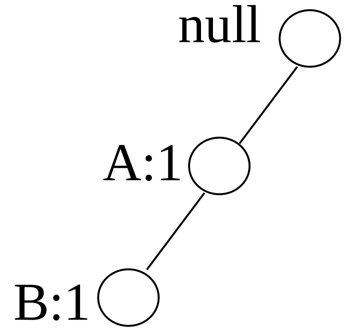
- Each node in the tree contains the label of an item along with a counter that shows the number of transactions mapped onto the given path.
  - Initially, the FP-tree contains only the root node represented by the null symbol.
  - Every transaction maps onto one of the paths in the FP-tree.

- The size of an FP-tree is typically smaller than the size of the uncompressed data because many transactions in market basket data often share a few items in common.
  - best-case scenario, all transactions have same set of items
    ➔ FP-tree contains only a single branch.
  - worst-case scenario happens when every transaction has a unique set of items
    ➔ FP-tree is effectively the same as the size of the original data.
  - physical storage requirement for FP-tree is higher because it requires additional space to store pointers between nodes and counters for each item.

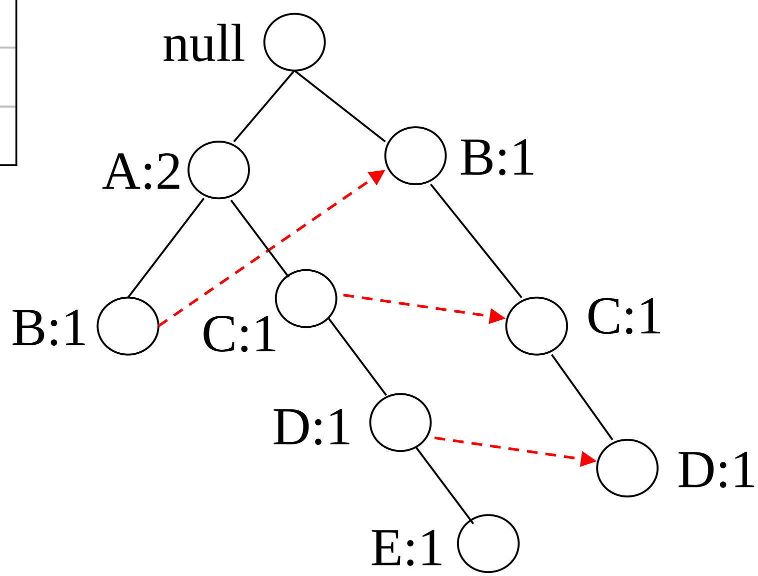
FP-Tree Construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

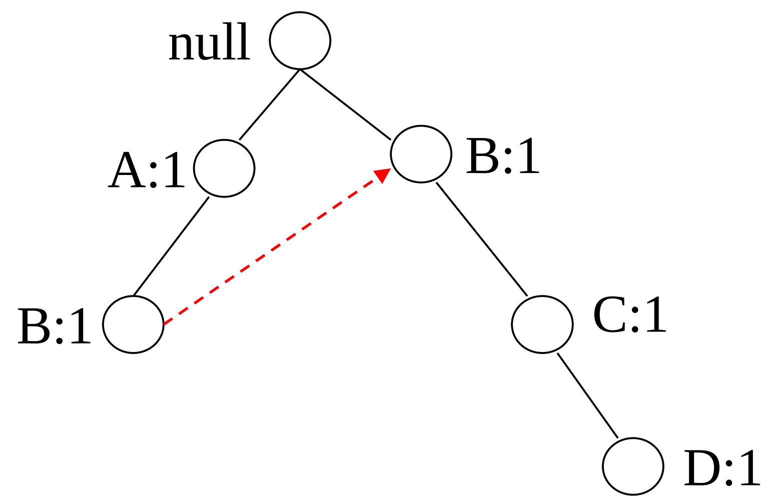
After reading TID=1:



After reading TID=3:



After reading TID=2:

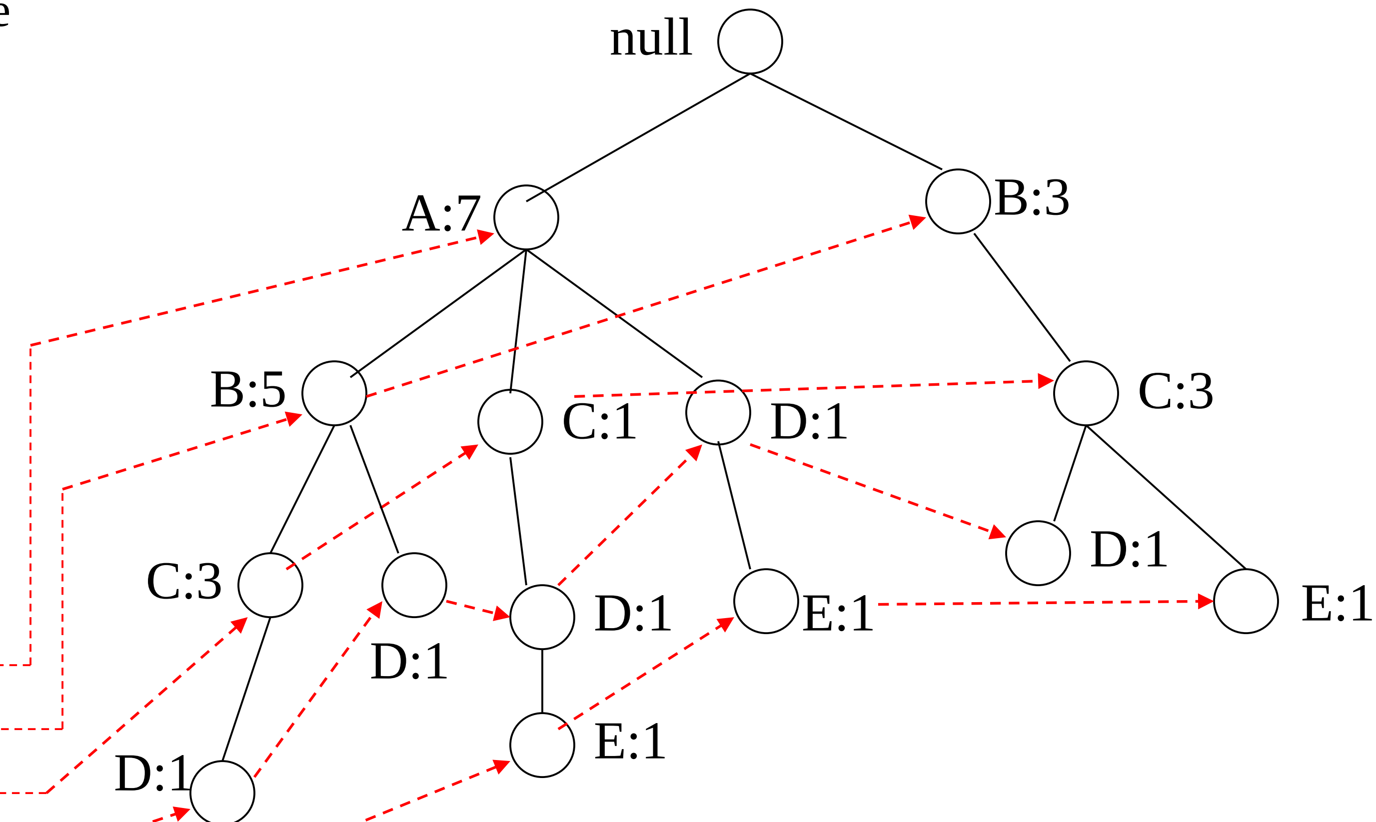


FP-Tree Construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

Transaction Database

After reading all transactions:



Header table

Item	Pointer
A	
B	
C	
D	
E	

sorted

Pointers are used to assist frequent itemset generation

Frequent Itemset Generation in FP-Growth Algorithm

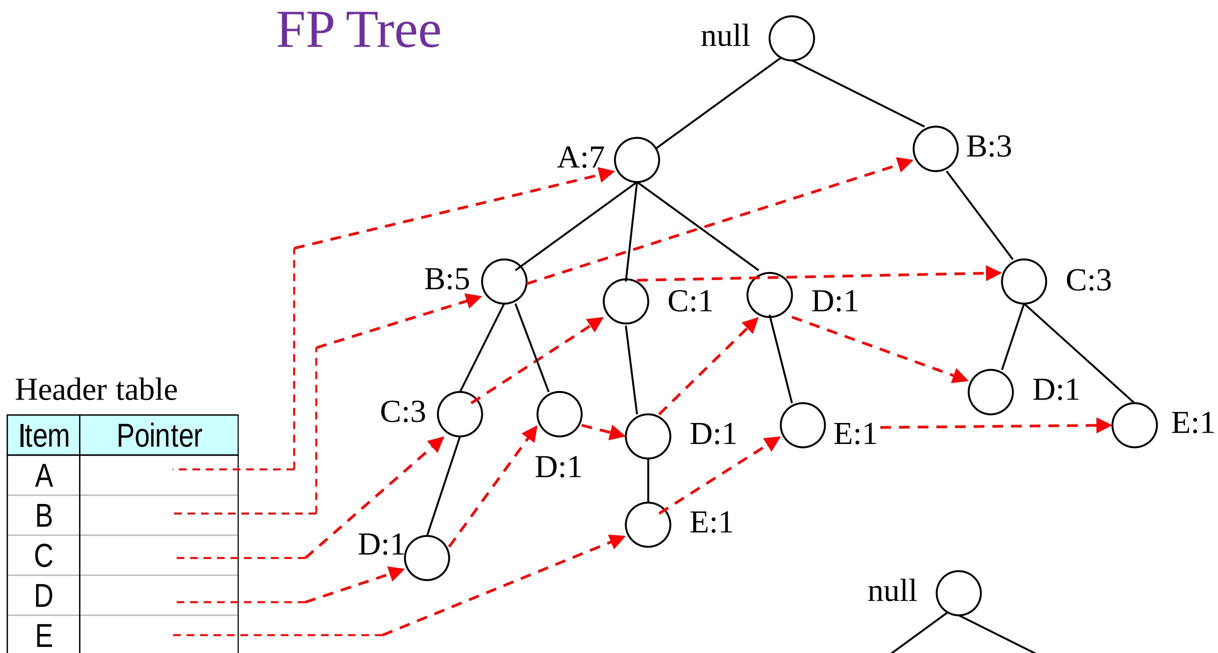
- FP-growth is an algorithm that generates frequent itemsets from an FP-tree by exploring the tree in a bottom-up fashion.
  - This bottom-up strategy for finding frequent itemsets ending with a particular item is equivalent to the suffix-based approach
  - Since every transaction is mapped onto a path in the FP-tree, we can derive the frequent itemsets ending with a particular item, say **e**, by examining only the paths containing node **e**.
  - The algorithm looks for frequent itemsets ending in **e** first, followed by **d**, **c**, **b**, and finally, **a**.
- FP-growth finds all the frequent itemsets ending with a particular suffix by employing a divide-and-conquer strategy to split the problem into smaller subproblems.
  - To find all frequent itemsets ending in **e**, we must first check whether the itemset **{e}** itself is frequent.
  - If it is frequent, we consider the subproblem of finding frequent itemsets ending in **de**, followed by **ce**, **be**, and **ae**.
  - In turn, each of these subproblems are further decomposed into smaller subproblems.
  - By merging the solutions obtained from the subproblems, all the frequent itemsets ending in **e** can be found.

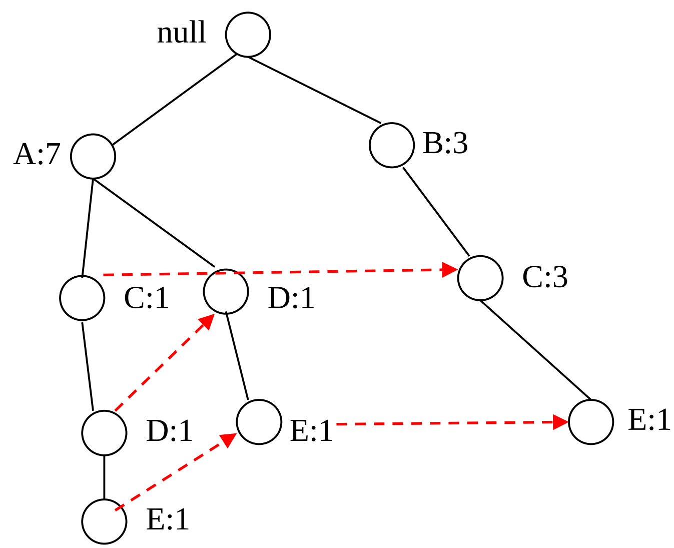
Finding Frequent Itemsets Ending with e

1. The first step is to gather all the paths containing node **e**. These initial paths are called ***prefix paths***
2. From the prefix paths, the support count for **e** is obtained by adding the support counts associated with node **e**. Assuming that the minimum support count is 2, **{e}** is declared a frequent itemset because its support count is 3.
3. Because **{e}** is frequent, the algorithm has to solve the subproblems of finding frequent itemsets ending in **de**, **ce**, **be**, and **ae**. Before solving these subproblems, it must first convert the prefix paths into a ***conditional FP-tree***, which is structurally similar to an FP-tree, except it is used to find frequent itemsets ending with a particular suffix.
   - First, the support counts along the prefix paths must be updated because some of the counts include transactions that do not contain item **e**.
   - The prefix paths are truncated by removing the nodes for **e**.
   - After updating the support counts along the prefix paths, some of the items may no longer be frequent
     - the node b appears only once and has a support count equal to 1, which means that there is only one transaction that contains both b and e. Item b can be safely ignored from subsequent analysis because all itemsets ending in be must be infrequent.
4. FP-growth uses the conditional FP-tree for **e** to solve the subproblems of finding frequent itemsets ending in **de**, **ce**, and **ae**.

Prefix Paths Ending with e

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}





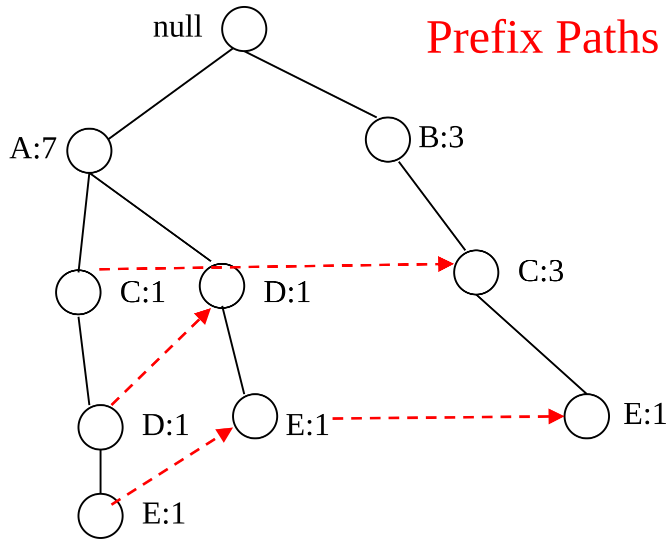
Prefix Paths Ending with e

Conditional FP-Tree for e

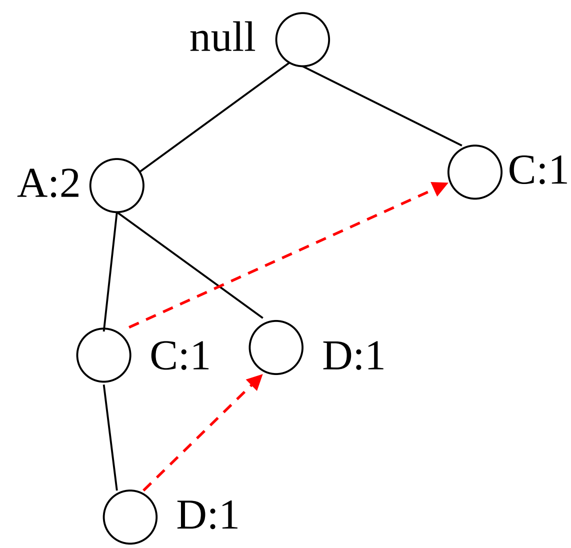
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup=2

Prefix Paths Ending with e



Conditional FP-Tree for e



To create Conditional FP-Tree for e

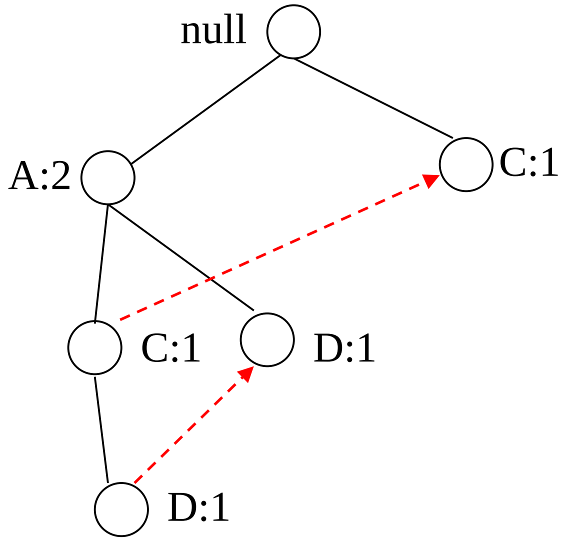
- Update support counts because paths without e are removed
- e is frequent (support=3), Remove e nodes from prefix paths
- Remove infrequent nodes

Conditional FP-Tree for de

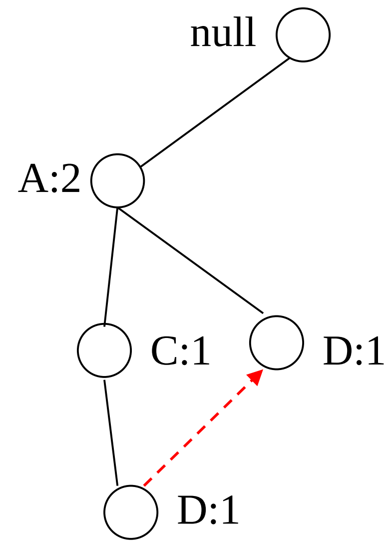
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup=2

Conditional FP-Tree for e

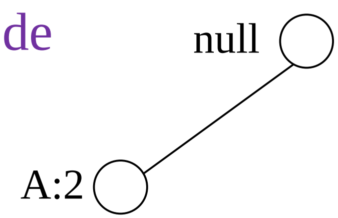


Prefix Paths Ending with de



de is frequent (support=2)

Conditional FP-Tree for de

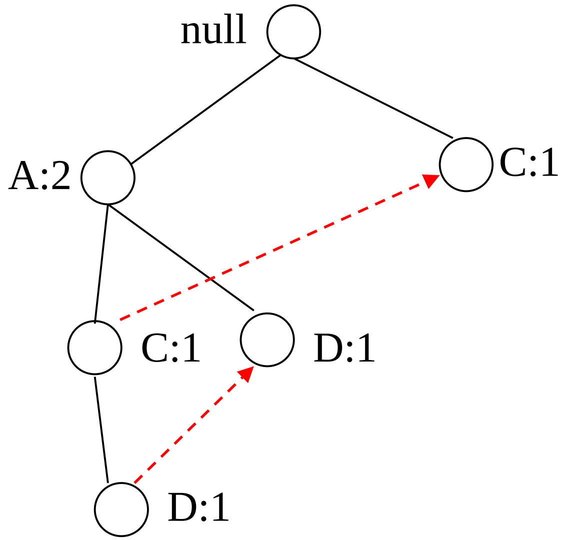


Conditional FP-Tree for ce

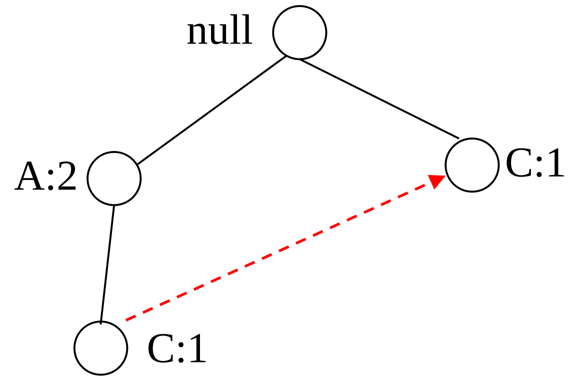
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup=2

Conditional FP-Tree for e

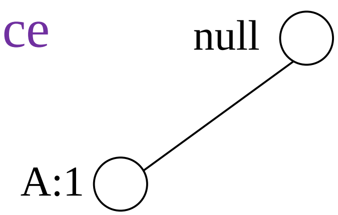


Prefix Paths Ending with ce



ce is frequent (support=2)

Conditional FP-Tree for ce

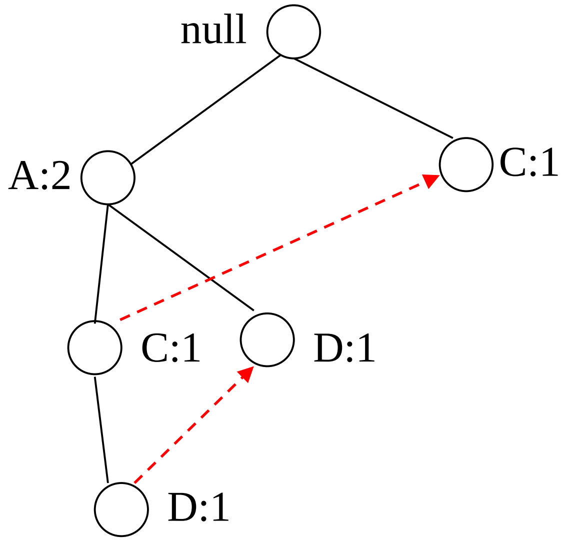


Conditional FP-Tree for ae

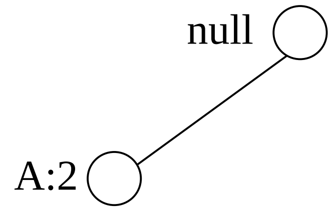
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup=2

Conditional FP-Tree for e



Prefix Paths Ending with ae



ae is frequent (support=2)

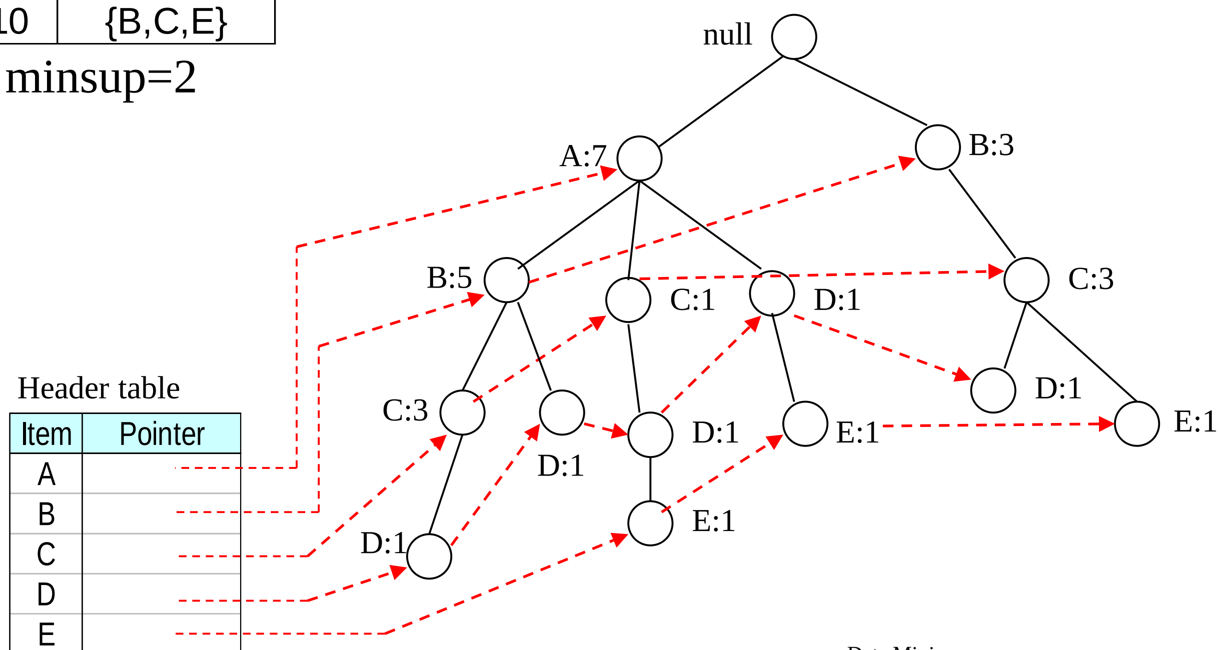
Conditional FP-Tree for ae

Frequent Itemsets Ordered by Suffixes

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup=2

Suffix	Frequent Itemsets
E	{E}, {D,E}, {A,D,E}, {C,E}, {A, E},
D	{D}, {C,D}, {B,C,D}, {A,C,D}, {B,D}, {A,B,D}, {A,D}
C	{C}, {B,C}, {A,B,C}, {A,C}
B	{B}, {A,B}
A	{A}



FP Tree

- **Frequent Itemsets, Association Rules**
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Evaluation of Association Patterns

- Association rule algorithms tend to produce too many rules
  - many of them are ***uninteresting*** or ***redundant***
  - {A,B} $\rightarrow$ {D} is ***Redundant*** if {A,B,C} $\rightarrow$ {D} and {A,B} $\rightarrow$ {D} have same support & confidence
  - An association rule X −→ Y is **redundant** if there exists another rule X'→ Y', where X is a subset of X' and Y is a subset of Y', such that the support and confidence for both rules are identical.

- ***Interestingness measure*** can be used to prune/rank the derived patterns

- In the original formulation of association rules, support & confidence are the only measures used

Computing Interestingness Measure

- Given a rule X → Y, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

	Y	$\overline{Y}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{X}$	f_{01}	f_{00}	f_{0+}
	f_{+1}	f_{+0}	$\|T\|$

f_{11}: support of X and Y
f_{10}: support of X and $\overline{Y}$
f_{01}: support of $\overline{X}$ and Y
f_{00}: support of $\overline{X}$ and $\overline{Y}$

Drawback of Confidence

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 0.75 = support({Tea,Coffee}) / support({Tea})

but P(Coffee) = 0.9

$\Rightarrow$ Although confidence is high, rule is misleading

$\Rightarrow$ $P(\text{Coffee}|\overline{\text{Tea}}) = 0.9375$

Measure for Association Rules

- So, what kind of rules do we really want?
  - Confidence($X \rightarrow Y$) should be sufficiently high
    - To ensure that people who buy X will more likely buy Y than not buy Y
  - Confidence($X \rightarrow Y$) > support(Y)
    - Otherwise, rule will be misleading because having item X actually reduces the chance of having item Y in the same transaction
    - Is there any measure that capture this constraint?
      - Answer: Yes. There are many of them.

Statistical Independence

- Population of 1000 students
  - 600 students know how to swim (S)
  - 700 students know how to bike (B)
  - 420 students know how to swim and bike (S,B)

  - $P(S \wedge B) = 420/1000 = 0.42$
  - $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$

  - $P(S \wedge B) = P(S) \times P(B)$ => Statistical independence
  - $P(S \wedge B) > P(S) \times P(B)$ => Positively correlated
  - $P(S \wedge B) < P(S) \times P(B)$ => Negatively correlated

Statistical-Based Measures for Interestingness

- Statistical-Based Measures use statistical dependence information.
- Two of them are **Lift** and **Interest** (they are equal).

$$\text{Lift} = P(Y|X) \,/\, P(Y)$$

$$\text{Interest} = P(X,Y) \,/\, P(X)\,P(Y)$$

$$\text{Lift}(A,B) = \text{conf}(A \rightarrow B) \,/\, \text{support}(B)$$

$$= \text{support}(A \cup B) \,/\, \text{support}(A)\ \text{support}(B)$$

$$\text{Interest}(A,B) = \text{support}(A \cup B) \,/\, \text{support}(A)\ \text{support}(B)$$

$$\text{Interest}(A,B) \begin{cases} = 1 & \mathit{if\ A\ and\ B\ are\ independent} \\ > 1 & \mathit{if\ A\ and\ B\ are\ positively\ correlated} \\ < 1 & \mathit{if\ A\ and\ B\ are\ negatively\ correlated} \end{cases}$$

Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 0.75 = support({Tea,Coffee}) / support({Tea})

but P(Coffee) = 0.9

➔ Lift = 0.75/0.9 = 0.8333 (< 1, therefore is negatively correlated)

Example: Lift/Interest

- *play basketball* $\Rightarrow$ *eat cereal* [40%, 66.7%] is misleading
  - The overall % of students eating cereal is 75% > 66.7%.
- *play basketball* $\Rightarrow$ *not eat cereal* [20%, 33.3%] is more accurate, although with lower support and confidence

	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

$$lift(B,C) = \frac{2000/5000}{3000/5000*3750/5000} = 0.89$$

$$lift(B,\neg C) = \frac{1000/5000}{3000/5000*1250/5000} = 1.33$$

Limitations of Interest Factor

- We expect the words *data* and *mining* to appear together more frequently than the words *compiler* and *mining* in a collection of computer science articles.

	p	$\overline{p}$	
q	880	50	930
$\overline{q}$	50	20	70
	930	70	1000

	r	$\overline{r}$	
s	20	50	70
$\overline{s}$	50	880	930
	70	930	1000

Contingency tables for word pairs { p,q} and { r,s}.

- The interest factor for {p,q} is 1.02 and for {r, s} is 4.08.
  - Although p and q appear together in 88% of the documents, their interest factor is close to 1, which is the value when p and q are statistically independent.
  - On the other hand, the interest factor for {r, s} is higher than {p, q} even though r and s seldom appear together in the same document.
  - Confidence is perhaps the better choice in this situation because it considers the association between p and q (94.6%) to be much stronger than that between r and s (28.6%).

Different Measures

- There are lots of measures proposed in the literature
- Some measures are good for certain applications, but not for others
- What criteria should we use to determine whether a measure is good or bad?

#	Measure	Formula
1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j) \log \frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i) \log P(A_i), -\sum_j P(B_j) \log P(B_j))}$
8	J-Measure (J)	$\max\Big(P(A,B)\log(\frac{P(B\mid A)}{P(B)}) + P(A\overline{B})\log(\frac{P(\overline{B}\mid A)}{P(\overline{B})}),$ $P(A,B)\log(\frac{P(A\mid B)}{P(A)}) + P(\overline{A}B)\log(\frac{P(\overline{A}\mid B)}{P(\overline{A})})\Big)$
9	Gini index (G)	$\max\Big(P(A)[P(B\mid A)^2 + P(\overline{B}\mid A)^2] + P(\overline{A})[P(B\mid \overline{A})^2 + P(\overline{B}\mid \overline{A})^2]$ $-P(B)^2 - P(\overline{B})^2,$ $P(B)[P(A\mid B)^2 + P(\overline{A}\mid B)^2] + P(\overline{B})[P(A\mid \overline{B})^2 + P(\overline{A}\mid \overline{B})^2]$ $-P(A)^2 - P(\overline{A})^2\Big)$
10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B\mid A), P(A\mid B))$
12	Laplace (L)	$\max\Big(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2}\Big)$
13	Conviction (V)	$\max\Big(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})}\Big)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B) - P(A)P(B)$
17	Certainty factor (F)	$\max\Big(\frac{P(B\mid A)-P(B)}{1-P(B)}, \frac{P(A\mid B)-P(A)}{1-P(A)}\Big)$
18	Added Value (AV)	$\max(P(B\mid A) - P(B), P(A\mid B) - P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\mid A) - P(B), P(A\mid B) - P(A))$

Properties of A Good Measure

3 properties a good measure M must satisfy:

- $M(A,B) = 0$ if A and B are statistically independent
- $M(A,B)$ increase monotonically with $P(A,B)$ when $P(A)$ and $P(B)$ remain unchanged
- $M(A,B)$ decreases monotonically with $P(A)$ [or $P(B)$] when $P(A,B)$ and $P(B)$ [or $P(A)$] remain unchanged