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# Derivation of a parameter stabilizing training criterion for adaptive neuro-fuzzy inference systems in motion control

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*This paper presents a novel training algorithm for adaptive neuro-fuzzy inference systems. The algorithm combines the error back-propagation algorithm with the variable structure systems approach. Expressing the parameter update rule as a dynamic system in continuous time and applying sliding mode control (SMC) methodology to the dynamic model of the gradient based training procedure results in the parameter stabilizing part of training algorithm. The proposed combination therefore exhibits a degree of robustness to the unmodelled multivariable internal dynamics of the gradient-based training algorithm. With conventional training procedures, the excitation of this dynamics during a training cycle can lead to instability, which may be difficult to alleviate owing to the multidimensionality of the solution space and the ambiguities concerning the environmental conditions. This paper shows that a neuro-fuzzy model can be trained such that the adjustable parameter values are forced to settle down (parameter stabilization) while minimizing an appropriate cost function (cost optimization), which is based on state tracking performance. In the application example, trajectory control of a two degrees of freedom direct drive robotic manipulator is considered. As the controller, an adaptive neuro-fuzzy inference mechanism is used and, in the parameter tuning, the proposed algorithm is utilized.*

**Nomenclature**

| | |
|---|---|
| $c_{ij}$ | centre of the membership function $\mu_{ij}$ |
| $d$ | desired output |
| $e$ | observed output error |
| $F$ | output of the computationally intelligent architecture |
| $J$ | cost function for tracking performance |
| $J_S$ | cost of stability |
| $K_\phi$ | proportional rate component parameter of the switching scheme |
| $\tilde{N}$ | normalization operator in the adaptive neuro-fuzzy inference system structure |
| $N_\phi$ | change prescribed by the error back-propagation algorithm |
| $Q_\phi$ | constant rate component parameter of the switching scheme |
| $s_\phi$ | switching function for the parameter |
| $T_S$ | sampling interval of update dynamics |
| $u_j$ | $j$th input of the fuzzy inference system |
| $V_\phi$ | Lyapunov function for the parameter $\phi$ |
| $w$ | vector of firing strengths |
| $w_n$ | vector of normalized firing strengths |
| $\alpha_i$ | weighting factor |
| $\beta$ | scaling factor for parameter stabilizing law |
| $\varepsilon$ | boundary layer parameter |
| $\zeta$ | constant learning rate |
| $\eta_\phi$ | variable learning rate for parameter $\phi$ |
| $\mu_{ij}$ | membership function of the $j$th input of the $i$th rule |
| $\sigma_{ij}$ | width of the membership function |
| $\phi$ | generic parameter of a computationally intelligent system |
| $\phi^*$ | optimal value of the generic parameter |
| $\Delta\phi$ | change in the parameter $\phi$ |

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## 1. Introduction

Contrary to what is known in the realm of predicate logic, representation of knowledge by fuzzy quantities can provide extensive degrees of freedom if the task to be achieved can be expressed better in words than in numbers. The concept of fuzzy logic in this sense can be viewed as a generalization of binary logic and refers to the manipulation of knowledge with sets, whose boundaries are unsharp. In the application domain, the innovations in data mining, data fusion, sensor technology, recognition technology and fast microprocessors are ever increasingly encouraging the use of fuzzy logic controllers, whose operating philosophy is suitable to incorporate the expert knowledge into the design procedure. In conjunction with this, the interpretation of the information content of fuzzy logic control (FLC) is based on the subjective judgements, intuitions and the experience of an expert. From this point of view, a suitable way of expressing the expert knowledge is the use of IF antecedent, THEN consequent rules, which can easily evaluate the necessary action to be executed for the current state of the system under investigation. Therefore the paradigm offers a possibility of designing intelligent controllers operating in an environment, in which the conditions are inextricably intertwined, subject to uncertainties and impreciseness.

One of the major problems in the training of fuzzy logic controllers is the lack of stabilizing forces, the existence of which prevents the unbounded growth in the adjustable parameters. This is intimately related to the analytic explanation of the internal dynamics of the training strategy, which typically concern several tens of variables even for the simple structures. Strictly speaking, a method violating the stability requirements is a potential danger from the safety point of view. In this paper, the problem of training stability is elaborated and an adaptive neuro fuzzy inference system is considered.

In the domain of fuzzy logic, behaviour of a system is modelled through the use of linguistic descriptions. Although the earliest work by Zadeh on fuzzy systems has not received as much attention as it deserved in the early 1960s, since then the methodology has become a well-developed framework. The typical architectures of fuzzy inference systems are those introduced by Wang (1994, 1997), Takagi and Sugeno (1985) and Jang *et al.* (1997). Wang (1994) constructed a fuzzy system having Gaussian membership functions, a product inference rule and a weighted average defuzzifier. This architecture is accepted as the standard method in most applications. Takagi and Sugeno (1985) changed the defuzzification procedure where dynamic systems were used in the defuzzification procedure. The potential advantage of the method is that, under certain constraints, the stability of the system can be studied (Passino and Yurkovich, 1998). Jang *et al.* (1997), proposed an adaptive neuro-fuzzy inference system (ANFIS), in which a polynomial was used as the defuzzifier; see also the related literature (Nauck *et al.* 1997, Efe and Kaynak 1999). The choice concerning the order of the polynomial and the variables to be used in the defuzzifier are left to the designer.

In control engineering practice, stability and robustness are of crucial importance. Because of this, the implementation-oriented control engineering expert is always in pursuit of a design, which provides accuracy as well as insensitivity to environmental disturbances and structural uncertainties. At this point, it must be emphasized that these ambiguities can never be modelled accurately. When the designer tries to minimize the ambiguities by the use of a detailed model, then the design becomes so tedious that its cost increases dramatically. A suitable way of tackling with uncertainties without the use of complicated models is to introduce components based on variable structure systems (VSS) theory into the system structure.

Variable structure control has successfully been applied to a wide variety of systems having uncertainties in the representative system models. The philosophy of the control strategy is simple, being based on two goals. First, the system is forced towards a desired dynamics and, second, the system is maintained on that differential geometry. In the literature, the former is named the reaching mode, while the latter is called the sliding mode. The control strategy borrows its name from the latter dynamic behaviour, and is called *sliding mode control (SMC)*.

The earliest notion of SMC strategy was constructed on a second-order system in the late 1960s by Emelyanov (1967). The work stipulated that a special line could be defined on the phase plane, such that any initial state vector can be driven towards the plane and then can be maintained on it, while forcing the error dynamics towards the origin. Since then, the theory has greatly been improved and the sliding line has taken the form of a multidimensional surface, called the *sliding surface*, around which a switching control action takes place.

Numerous contributions to VSS theory have been made during the last decade, some of them are as follows. Hung *et al.* (1993) have reviewed the control strategy for linear and nonlinear systems. In that study, the switching schemes, putting the differential equations into canonical forms and generating simple SMC-based systems, are considered in detail. Application of the SMC scheme to robotic manipulators and discussion on the quality of the scheme have been presented in another study of Gao and Hung (1993).

One of the crucial points in SMC is the selection of the parameters of the sliding surface. Some studies devoted to the adaptive design of sliding surfaces have shown that the performance of control system can be refined by interfacing it with an adaptation mechanism, which regularly redesigns the sliding surface (Kaynak *et al.* 1984; Bekiroglu 1996). This eventually results in a robust control system. The performance of the SMC scheme is proven to be satisfactory in the face of external disturbances and uncertainties in the system model representation. The latest studies consider this robustness property by equipping the system with computationally intelligent methods. In some recent studies (Erbatur *et al.* 1996, Byungkook and Ham 1998), fuzzy inference systems are proposed for the SMC scheme. The standard fuzzy system was studied, and the relevant robustness analyses carried out. In particular, the work presented by Byungkook and Ham (1998) emphasized that the robustness and stability properties of soft-computing-based control strategies can be analysed through the use SMC theory. It is shown in the paper in this way that the approach is robust, that is it can compensate the deficiencies caused by poor modelling of plant dynamics and external disturbances.

The objective of this paper is to develop a stable training procedure for ANFISs, which will enforce the adjustable parameters to settle down to a steady-state solution while minimizing an appropriate cost function. This is achieved by performing a suitable mixture of gradient-based parametric displacements (Rumelhart *et al.*) and VSS-based stabilizing parametric displacements.

This paper is organized as follows. In § 2 the conventional method followed in the gradient-based optimization techniqueis summarized. In § 3 the derivation of the SMC-based parameter stabilizing law is presented. In § 4, the ANFIS architecture is considered and the relevant formulation for the approach is given, and § 5 is devoted to the plant to be controlled in this study. This is followed by the simulation studies in § 6. Conclusions constitute § 7.

## 2. Training of neuro-fuzzy systems using gradient descent

In this section, a widely used technique of parameter adjustment is briefly reviewed. The method was first formulated by Rumelhart *et al.* (1986) and is known as error backpropagation in the related literature. The approach has successfully been applied to a wide variety of optimization problems. Using the nomenclature given at the beginning of the paper, the algorithm can be stated as follows:

$$e = d - F(\phi, u), \tag{1}$$

$$J = \tfrac{1}{2}e^2, \tag{2}$$

$$\Delta\phi = -\eta_\phi \frac{\partial J}{\partial \phi}. \tag{3}$$

The observation error in (1) is used to minimize the cost function in (2) by utilizing the rule described by (3):

$$\Delta\phi = \eta_\phi e \frac{\partial F(\phi, u)}{\partial \phi}. \tag{4}$$

The minimization proceeds iteratively as given in (4), for which the sensitivity derivative with respect to the generic parameter $\phi$ is needed. It is apparent that the method is applicable to the architectures in which the outputs are differentiable with respect to the subject of optimization.

## 3. Synthesis of the parameter stabilizing criteria by using the variable structure systems approach

A continuous-time dynamic model of the parameter update rule prescribed by the gradient descent technique can be written as follows (in the analysis presented, the dot over a parameter should be understood as the time derivative):

$$\Delta\dot{\phi} = -\frac{1}{T_S}\Delta\phi + \frac{\eta_\phi}{T_S}N_\phi. \tag{5}$$

The above model is composed of the sampling time denoted by $T_S$, the gradient-based non-scaled parameter change denoted by $N_\phi = e\,(F(\phi, u)/)\phi$ and a scaling factor denoted by $\eta_\phi$, for the selection of which a detailed analysis is presented in the subsequent discussion. Using the Euler first-order approximation for the derivative term, one obtains the following relation, which obviously validates the constructed model in (5):

$$\frac{\Delta\phi(k+1) - \Delta\phi(k)}{T_S} = -\frac{\Delta\phi(k)}{T_S} + \frac{1}{T_S}\eta_\phi N_\phi(k). \tag{6}$$

This leads to the following representation:

$$\Delta\phi(k+1) = \eta_\phi N_\phi(k). \tag{7}$$

If (4) and (7) are compared, the equivalence between the continuous and discrete forms of the update dynamics is seen. The value of the sampling period to be used is a determining factor in validating the dynamic model in (5). Assuming that the signal exciting the system in (5) has smooth characteristics between successive sampling instants, the dynamic model in (5) can be used as an approximate model. This necessitates a sufficiently small sampling period. However, there is a trade-off because the cost of reducing the sampling time causes an increase in the total number of arithmetic operations to be performed during a training course.

The synthesis of the parameter stabilizing component is based on the integration of the system in (5) with VSS methodology. In the design of variable structure controllers, one method that can be followed is the reaching law approach (Hung *et al.* 1993). For the use of this theory in the stabilization of the training dynamics, let us define the switching function as follows

$$s_\phi = \Delta\phi - \Delta\phi_{\mathrm{d}} = \Delta\phi. \tag{8}$$

Since the order of the system in (5) is one, the switching function in (8) is selected as of zero order (Young *et al.* 1999), and it does not use any differentiated quantity. The design strategy in VSS technique requires the desired values of the system state, which can be denoted by $\Delta\phi_{\mathrm{d}}$. However, since the aim of the design is based on the minimization of parametric displacements in time, the desired value of the $\Delta\phi$ quantity is zero. Therefore the switching function in (8) suitably fulfils the design requirements of VSS strategy. The adopted reaching law is described by

$$\dot{s}_\phi = -\frac{Q_\phi}{T_{\mathrm{S}}}\tanh\left(\frac{s_\phi}{\varepsilon}\right) - \frac{K_\phi}{T_{\mathrm{S}}}\, s_\phi = \Delta\dot{\phi}. \tag{9}$$

This selection corresponds to the constant plus proportional rate reaching mode dynamics. Details of the selection of reaching laws have been given by Hung *et al.* (1993).

In (9), $Q_\phi$ and $K_\phi$ are the gains, and $\varepsilon$ is the width of the boundary layer. Equating (9) and (5) and solving for $\Delta\phi$ yields the following:

$$\Delta\phi = \eta_\phi N_\phi + Q_\phi \tanh\left(\frac{s_\phi}{\varepsilon}\right) + K_\phi s_\phi. \tag{10}$$

With the solution given in (10), the update dynamics are forced to behave as defined by (9), which is actually stable dynamics defined by the adopted switching function. In the derivations presented below, a key point is the fact that the system described by (5) is also driven by $\eta_\phi$, which is known as the learning rate in the related literature. Now we demonstrate that some special selection of this quantity leads to a rule that minimizes the magnitude of parametric displacement. Let us define the following quantity in order to keep analytic comprehensibility:

$$A_\phi = Q_\phi \tanh\left(\frac{\Delta\phi}{\varepsilon}\right) + K_\phi\, \Delta\phi. \tag{11}$$

Now we have a model described by (5), and a solution formulated by (10). If one chooses a positive definite Lyapunov function as given by

$$V_\phi = \tfrac{1}{2} s_\phi^2 = \tfrac{1}{2}(\Delta\phi)^2, \tag{12}$$

the time derivative of this function must be negative definite for stability in the parameter change ($\Delta\phi$) dynamics:

$$\dot{V}_\phi = (\Delta\phi)(\Delta\dot{\phi}). \tag{13}$$

Clearly the stability in parametric change space implies convergence in the system parameters.

If (5) and (10) are substituted into (13), the following constraint is obtained for stability in the Lyapunov sense.

$$\eta_\phi^2 + \frac{1}{N_\phi}(A_\phi - \Delta\phi)\eta_\phi - \frac{1}{N_\phi^2} A_\phi\, \Delta\phi < 0. \tag{14}$$

Equation (14) can be rewritten in a more tractable form as follows:

$$\left(\eta_\phi + \frac{1}{N_\phi} A_\phi\right)\left(\eta_\phi - \frac{1}{N_\phi}\Delta\phi\right) < 0. \tag{15}$$

Since $A_\phi$ and $\Delta\phi$ have the same signs, the roots of (15) clearly have opposite signs. The expression on the left-hand side assumes negative values between the roots. Therefore, in order to satisfy the inequality in (15), the learning rate must satisfy the constraint given by

$$0 < \eta_\phi < \min\left(\left|\frac{1}{N_\phi}\Delta\phi\right|, \left|-\frac{1}{N_\phi} A_\phi\right|\right). \tag{16}$$

In (16), the interval of learning rate is restricted to positive values. This is because compatibility between the gradient-based approaches and the proposed approach must be preserved. An appropriate selection of $\eta_\phi$ could be as follows:

$$\eta_\phi = \beta \min\left(\left|\frac{1}{N_\phi}\Delta\phi\right|, \left|-\frac{1}{N_\phi} A_\phi\right|\right), \quad 0 < \beta < 1. \tag{17}$$

By substituting the learning rate formulated in (17) into the stabilizing solution given in (10), the stabilizing component $\Delta\phi_{\mathrm{VSS}}$ of the parameter change formula is obtained as

$$\Delta\phi_{\mathrm{VSS}} = \beta \min(|\Delta\phi|, |A_\phi|)\,\mathrm{sgn}(N_\phi) + A_\phi, \tag{18}$$

where $\Delta\phi$ on the right-hand side is the final update value yet to be obtained. The law introduced in (18) minimizes the cost of stability, which is the Lyapunov function defined by (12). The question now reduces to the following: can this law minimize the cost defined by (2)? The answer is obviously not, because the stabilizing criteria in (18) is derived from the displacement of the parameter vector denoted by $\Delta\phi$, whereas the minimization of (2) is achieved when $\phi$ tends to $\phi^*$ regardless of what the displacement is. In order to minimize (2), the parameter change anticipated by gradient-based optimization technique, which is reviewed in § 2, should somehow be integrated into the final form of parameter update mechanism. As introduced in § 2, the error back-propagation (EBP) algorithm evaluates a parameter change as given by

$$\Delta\phi_{\mathrm{EBP}} = \zeta N_{\phi}, \tag{19}$$

where $\zeta$ is the constant learning rate in the conventional sense. Combining the laws (18) and (19) in a weighted average, the following eventual parameter update law is obtained.

$$\Delta\phi = \frac{\alpha_1\,\Delta\phi_{\mathrm{VSS}} + \alpha_2\,\Delta\phi_{\mathrm{EBP}}}{\alpha_1 + \alpha_2}. \tag{20}$$

The parameter update formula given by (20) carries mixed information containing both the parametric convergence, which is introduced by VSS part, and the cost minimization, which is due to the EBP technique. The balancing in this mixture is left to the designer by an appropriate selection of $\alpha_1$ and $\alpha_2$, which are non-negative weight values. If the value of $\alpha_1$ is increased, the VSS based update rule is given more importance. On the other hand, increasing $\alpha_2$ causes the EBP part to dominate the mixed displacement value. In the extreme cases, in which $\alpha_1 = 0$ or $\alpha_2 = 0$, the effect of one of the components disappears. More explicitly, setting $\alpha_1$ to zero leads to the ordinary EBP technique and the problems of unbounded parameter growth arise, on the other hand, setting $\alpha_2$ to zero eliminates the effect of EBP part, that is the learning ability of the algorithm is inactivated. Therefore, learning with small parameter change effort can be achieved by suitably setting the weight parameters $\alpha_1$ and $\alpha_2$.

## 4. Application to adaptive neuro-fuzzy inference systems

ANFISs are realized by an appropriate combination of neural and fuzzy systems. This hybrid combination enables one to utilize both the verbal and the numeric power of intelligent systems. As is known from the theory of fuzzy systems, different fuzzification and defuzzification mechanisms with different rule-based structures can result in various solutions to a given task. This paper considers the ANFIS structure with the first-order Sugeno model containing nine rules. Gaussian membership functions with the product inference rule are used at the fuzzification level. Fuzzifier outputs are the firing strengths for each rule. The vector of the firing strengths is normalized and the resulting vector is defuzzified by utilizing the first order Sugeno model. The structure for two inputs and one output is illustrated in figure 1. The rule structure is as follows for a system having $m$ inputs and one output:

$$\text{IF } u_1 \text{ is } U_{i,1} \text{ and } u_2 \text{ is } U_{i,2} \text{ and} \ldots\ u_m \text{ is } U_{i,m},$$

$$\text{THEN } f_i = q_{i,1}u_1 + \cdots + q_{i,m}u_m + q_{i,m} + 1.$$

In the IF part of this representation, lower-case variables denote the inputs, and capital variables stand for the fuzzy sets corresponding to the domain of each linguistic label. The ANFIS output is clearly a linear function of the adjustable defuzzifier parameters denoted by $q_{i,j}$. The system that is considered in this study uses Gaussian membership functions as described by

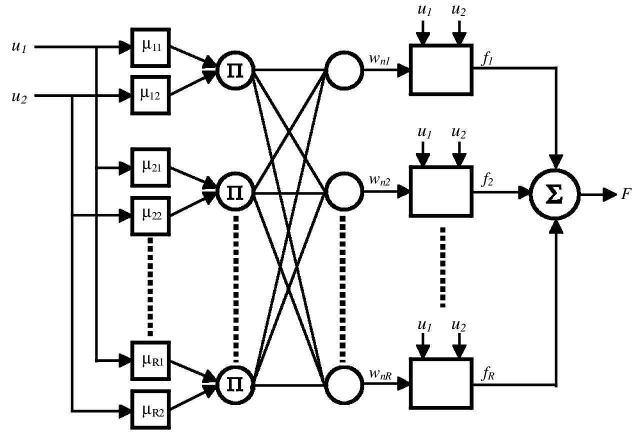


**Figure 1. Architecture of the ANFIS.**

$$\mu_{ij}(u_j) = \exp\left[-\left(\frac{u_j - c_{ij,}}{\sigma_{ij}}\right)^2\right], \tag{21}$$

where $c_{ij}$ and $\sigma_{ij}$ characterize the centre and width respectively of the $j$th membership function of the $i$th rule. The initial values of the membership functions are selected such that the region of interest is covered appropriately. The overall realization performed by the system considered is given by

$$F = \frac{\sum_{i=1}^{\#\mathrm{rules}} f_i \prod_{j=1}^{\#\mathrm{inputs}} \mu_{ij}(u_j)}{\sum_{i=1}^{\#\mathrm{rules}} \prod_{j=1}^{\#\mathrm{inputs}} \mu_{ij}(u_j)} = \sum_{i=1}^{\mathrm{rules}} f_i w_{ni}, \tag{22}$$

where linear functions of input variables are used for defuzzification with the algebraic product aggregation method. In (22), the vector of firing strengths denoted by $w$ is normalized and the resulting vector is represented by $w_n$:

$$w_{ni} = \frac{\prod_{j=1}^{\#\mathrm{inputs}} \mu_{ij}(u_j)}{\sum_{k=1}^{\#\mathrm{rules}} \prod_{j=1}^{\#\mathrm{inputs}} \mu_{kj}(u_j)}. \tag{23}$$

With the definition given in (23), and the realization described by (22), the adjustable parameter set is selected as follows.

$$\phi = \{c_{ij}, \sigma_{ij}, q_{i,j}\}_{i=1,\ldots,\mathrm{rules};\, j=1,\ldots,\mathrm{inputs}} \tag{24}$$

The relevant back-propagated error values are given by

$$N_{q_{i,j}} = \begin{cases} ew_{ni}u_j, & 1 \leqslant j \leqslant m+1, \\ wq_{ni}, & j = m+1, \end{cases} \tag{25}$$

$$N_{c_{i,j}} = e(f_i - F)w_{ni}2\,\frac{u_j - c_{ij}}{\sigma_{ij}^2}, \tag{26}$$

$$N_{\sigma_{ij}} = e(f_i - f)w_{ni}^2\,\frac{(u_j - c_{ij})^2}{\sigma_{ij}^3}. \tag{27}$$

Using the quantities formulated in (25)–(27), the part of the update value that is responsible for the minimization of the tracking error can be formulated as given in (19). The parameter stabilizing part of the training signal is evaluated by the use of (11) and (18). The final form of the mixed training criteria can now be constructed as a weighted average of the prescribed values described by (20).

## 5. Plant model

In this study, a two-degrees-of-freedom direct drive robotic manipulator, which is illustrated in figure 2, is used as the test bed. Since the dynamics of such a mechatronic system are modelled by nonlinear and coupled differential equations, precise output tracking becomes a difficult objective owing to the strong interdependence between the variables involved. Furthermore, the ambiguities concerning the friction-related dynamics in the plant model make the design much more complicated. Therefore the methodology adopted must use the methods of computational intelligence in some sense.

The general form of the robot dynamics is described by

$$M\ddot{\theta} + V(\theta, \dot{\theta}) = \tau - f_c \tag{28}$$

where $M(\theta)$, $V(\theta, \dot{\theta})$, $\tau(t)$ and $f_c$ stand for the state varying inertia matrix, the vector of Coriolis terms, the applied torque inputs and the friction terms respectively.

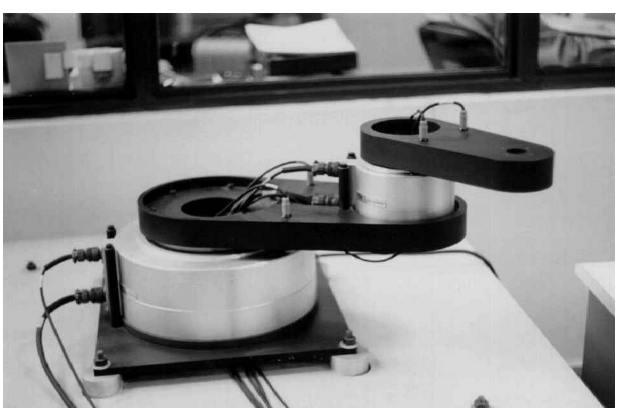

**Figure 2. Physical view of the direct-drive robotic manipulator.**

**Table 1. Manipulator parameters.**

| | |
|---|---|
| Motor 1 rotor inertia $I_1$ | 0.2670 |
| Arm 1 inertia $I_2$ | 0.3340 |
| Motor 2 rotor inertia $I_3$ | 0.0075 |
| Motor 2 stator inertia $I_3$ | 0.0400 |
| Arm 2 inertia $I_4$ | 0.0630 |
| Motor 1 mass $M_1$ | 73.0000 |
| Arm 1 mass $M_2$ | 9.7800 |
| Motor 2 mass $M_3$ | 14.0000 |
| Arm 2 mass $M_4$ | 4.4500 |
| Arm 1 length $L_1$ | 0.3590 |
| Arm 2 length $L_2$ | 0.2400 |
| Arm 1 centre of gravity $L_3$ | 0.1360 |
| Arm 2 centre of gravity $L_4$ | 0.1020 |
| Axis 1 friction $f_{c1}$ | 5.3000 |
| Axis 2 friction $f_{c2}$ | 1.1000 |
| Torque limit 1 $\tau_{sat1}$ | 245.0000 |

The plant parameters are given in table 1 in standard units.

If the angular positions and angular velocities are defined as the state variables of the system, four coupled and first order differential equations can define the model in state space. The terms seen in (28) are given explicitly by

$$M(\theta) = \begin{bmatrix} p_1 + 2p_3\cos(\theta_2) & p_2 + p_3\cos(\theta_2) \\ p_2 + p_3\cos(\theta_2) & p_2 \end{bmatrix}, \tag{29}$$

$$V(\theta, \dot{\theta}) = \begin{bmatrix} -\dot{\theta}_2(2\dot{\theta}_1 + \dot{\theta}_2)p_3\sin(\theta_2) \\ \dot{\theta}_1^2 p_3 \sin(\theta_2) \end{bmatrix}. \tag{30}$$

In the above, $p_1 = 2.0857$, $p_2 = 0.1168$ and $p_3 = 0.1630$. The details of the plant model have been presented in the *Direct Drive Manipulator R&D Package User Guide* (IMI 1992).

## 6. Simulation studies

In the simulation studies presented, the plant introduced in § 5 is controlled by the ANFIS analysed in § 4. The main objective is to keep the update dynamics in a stable region. This is achieved through a suitable combination of gradient based optimization technique and the strategy based on the VSS theory. For this purpose, the control system structure is as illustrated in figure 3.

The reference velocity trajectory, described by

$$\dot{\theta}_{d1,2} = \sin\left(\frac{2\pi t}{5}\right) \tag{31}$$

and depicted in figure 4, is used in all simulations with zero initial errors. The settings used in the simulations are given in table 2.

The results presented concern the tuning of all adjustable parameters of the ANFIS structure during the

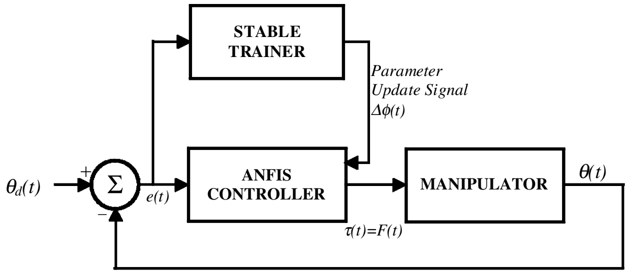


**Figure 3. Control of the manipulator using the proposed training method.**

**Table 2. The settings used in the simulations.**

| | |
|---|---|
| $T_S$ | 2.5 ms |
| $\beta$ | 0.1 |
| $\zeta$ | 0.02 |
| $\alpha_1$ | 3.0 for all $i$ |
| $\alpha_2$ | 2.0 for all $i$ |
| $Q$ | 0.1 |
| $K$ | 0.1 |
| $\varepsilon$ | 1.0 |
| #rules | (9 for each link) |
| #ANFIS inputs | 2 (for each link) |

learning process. The choice of the initial values of the membership function parameters is made by trial and error. In figures 5 and 6, the state tracking errors are illustrated. In the former, VSS-based stabilizing criteria are incorporated into the gradient technique while in the latter, the results are obtained solely from the gradient-based training procedure. Clearly, a comparison of the error magnitudes suggests the use of the proposed technique. The second emphasis on the assessment of these figures is on the required time for observing a periodicity in the error trends. This point is closely related to the stabilizing property of the VSS-based information. As can easily be seen from figure 5, the use of mixed update values results in fast convergence with high tracking precision. In figure 7, the produced control signals for both links are illustrated. The smoothness observed in the applied torques is another prominent feature of the approach, which clearly prevents the excitation of the high-frequency dynamics and produces physically admissible control sequences whose limits are determined by the dynamics of the actuators.

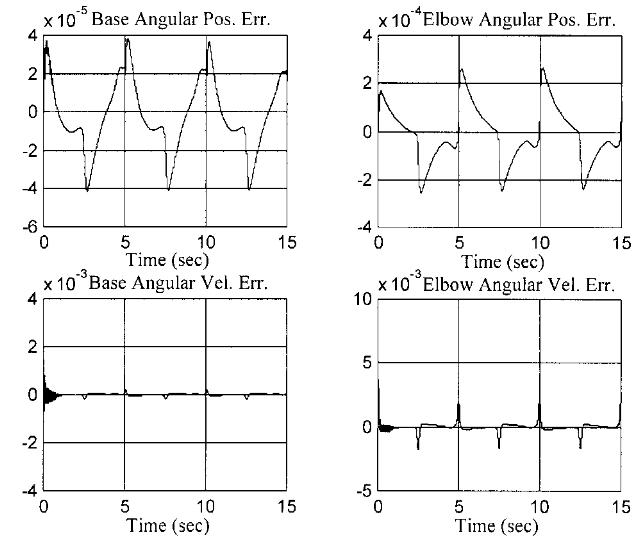


**Figure 5. State tracking errors with the VSS-based criteria.**

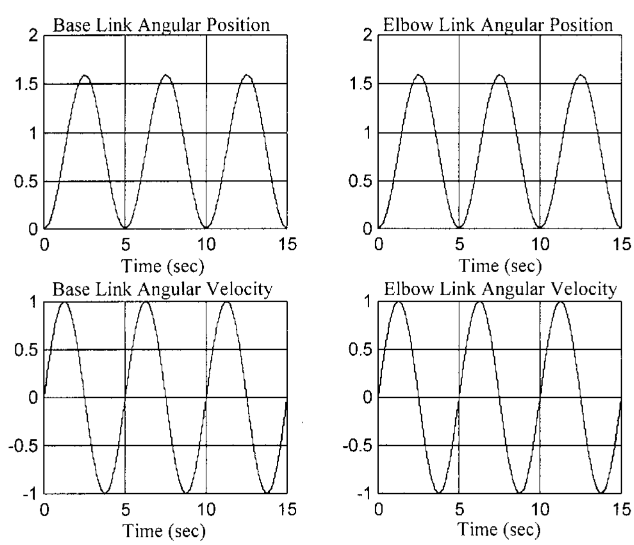


**Figure 4. Reference position and velocity trajectories.**

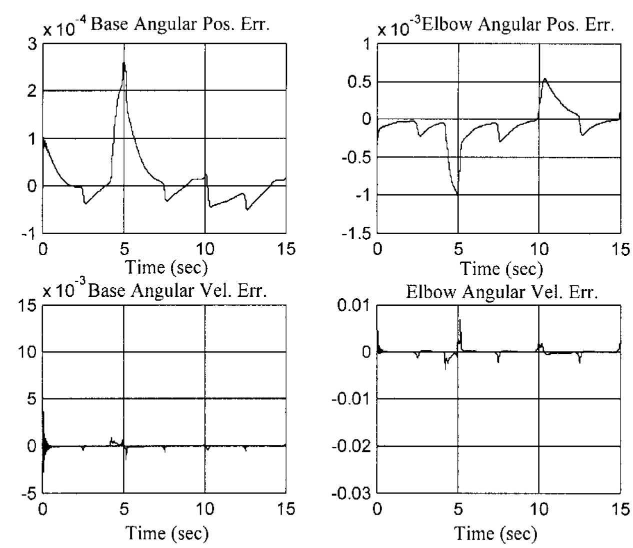


**Figure 6. State tracking errors without the VSS-based criteria.**

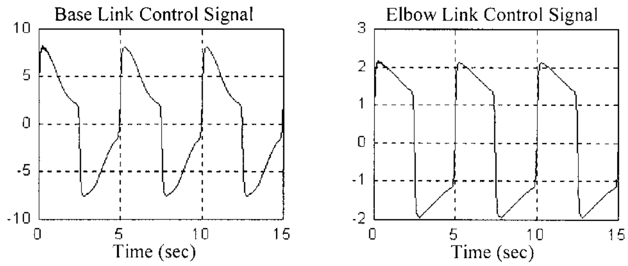


**Figure 7. Applied torque inputs.**

In the training of the controllers, the sum of the squared error values are defined to be the cost of tracking whereas the squared sum of parametric changes are defined to be the total cost of stability. These cost functions are described by

$$J(t) = \tfrac{1}{2}(e_1^2 + e_2^2) \qquad (32)$$

and

$$J_S(t) = \sum_{\phi}[\Delta\phi(t)]^2, \qquad (33)$$

and their time behaviours are illustrated in figure 7.

As can be inferred from figure 8, the parametric stabilization performance of the proposed methodology is highly promising. A remarkable property of the algorithm presented is the fact that it operates on line. Therefore, the difficulties that are likely to occur in on-line learning and control are alleviated by the robustness provided by the VSS technique. However, the use of the proposed technique increases the computational complexity compared with the cases in which the ordinary EBP technique is utilized. For the applications equipped with high-speed processors, the computational burden can be alleviated and the above-mentioned contribution of the proposed technique can be observed.

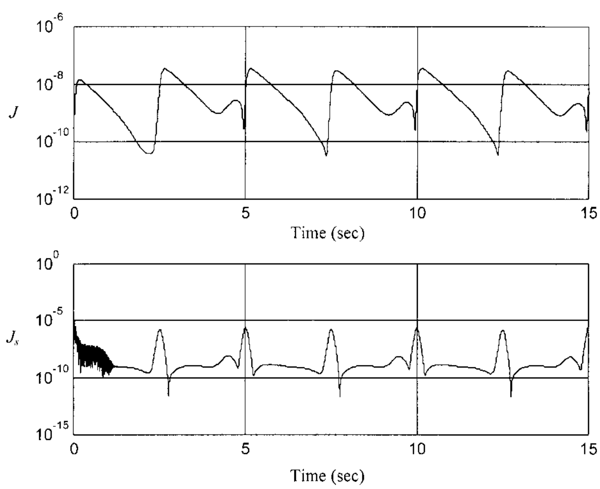


**Figure 8. Time behaviour of the tracking cost and stability cost with the VSS-based criteria.**

## 7. Conclusions

In this paper, a novel technique for improving the learning performance of ANFIS is presented. An approximate continuous-time dynamic model of the EBP procedure is constructed and the VSS approach is incorporated into the model of the parameter update law. In this procedure, the gradient descent method is responsible for the minimization of squared error in (32) while the VSS-based law is responsible for reducing the tendencies towards instability caused by the possible instantaneous large displacements evaluated by the gradient technique.

The conventional approaches suffer from some handicaps, such as imperfect modelling, noisy observations or time varying parameters. If the effects of these factors are transformed to the cost hypersurface, whose dimensionality is determined by the adjustable design parameters, it is evident that the surface may have directions along which the sensitivity derivatives assume large values. In these cases, gradient based optimization procedures are likely to evaluate large parametric displacements, which can eventually lead to a locally divergent behaviour. In control engineering practice, such a behaviour constitutes a potential danger from a safety point of view. The approach presented in this paper takes care of the instantaneous fluctuations in parameter space. Since the VSS technique is well known for its robustness to structural uncertainties and environmental disturbances, an appropriate combination of the EBP technique and the VSS theory can eliminate the handicaps stated above. The fluctuations that are most likely to occur in the parameter space during training are dampened out. The combination is therefore a good candidate for safe parameter tuning.

In the application example presented, the results confirm the prominent features of the approach, which are discussed in the previous section. The algorithm is applicable to any neuro-fuzzy system model provided that the model output is differentiable with respect to the parameter of interest.

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