# Filters

CMP 719 – Computer Vision
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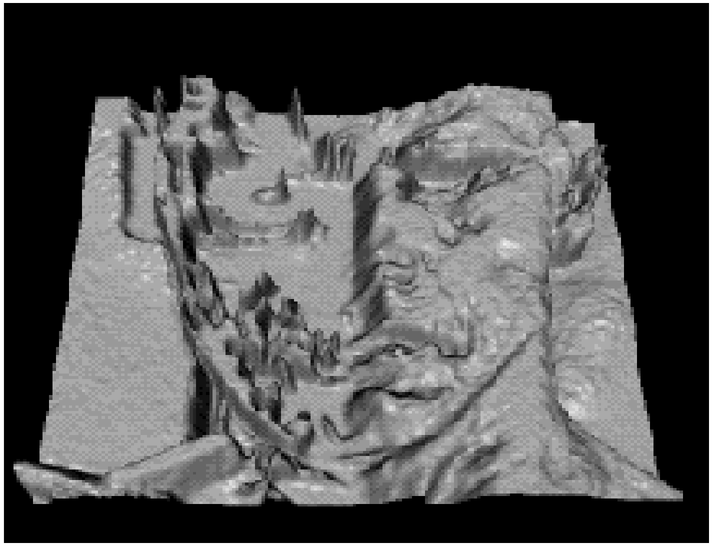
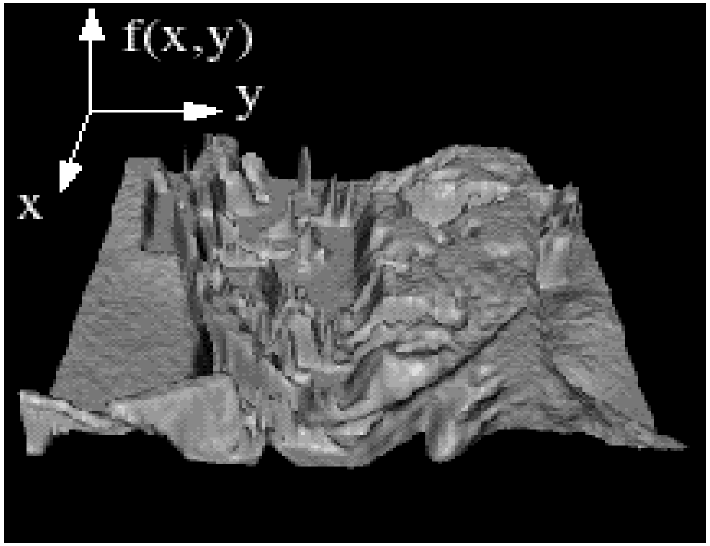
Hacettepe University

# Today's topics

- Image Formation
- Image filters in spatial domain
  - Filter is a mathematical operation of a grid of numbers
  - Smoothing, sharpening, measuring texture
- Image filters in the frequency domain
  - Filtering is a way to modify the frequencies of images
  - Denoising, sampling, image compression
- Templates and Image Pyramids
  - Filtering is a way to match a template to the image
  - Detection, coarse-to-fine registration

# Images as functions






Source: S. Seitz

# Images as functions

- We can think of an image as a function, *f*, from $\mathrm{R}^2$ to R:
  - $f(x, y)$ gives the intensity at position $(x, y)$
  - Realistically, we expect the image only to be defined over a rectangle, with a finite range:
    - $f$: $[a,b] \times [c,d] \rightarrow [0, 255]$

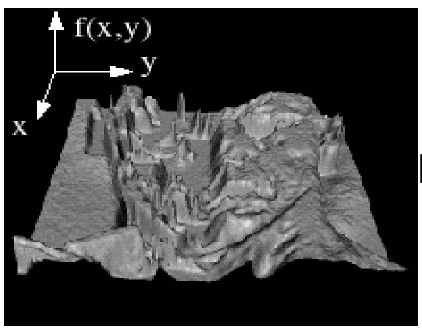
- A color image is just three functions pasted together. We can write this as a “vector-valued” function:

$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$

Source: S. Seitz

# Digital images

- In computer vision we operate on **digital** (**discrete**) images:
  - **Sample** the 2D space on a regular grid
  - **Quantize** each sample (round to nearest integer)
- Image thus represented as a matrix of integer values.

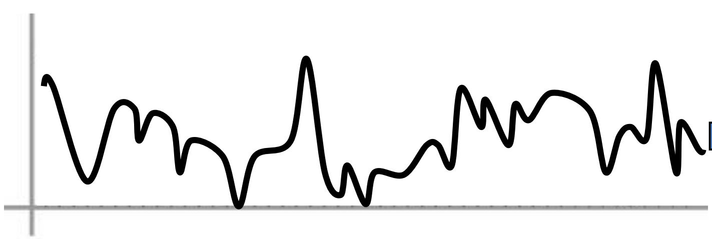


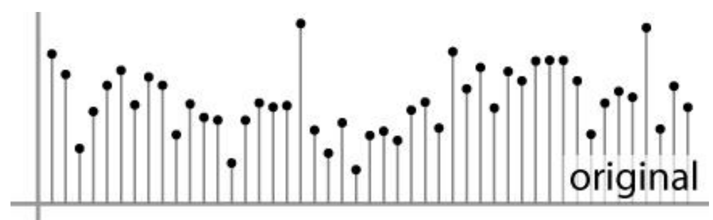

$j$

$i$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 62 | 79 | 23 | 119 | 120 | 105 | 4 | 0 |
| 10 | 10 | 9 | 62 | 12 | 78 | 34 | 0 |
| 10 | 58 | 197 | 46 | 46 | 0 | 0 | 48 |
| 176 | 135 | 5 | 188 | 191 | 68 | 0 | 49 |
| 2 | 1 | 1 | 29 | 26 | 37 | 0 | 77 |
| 0 | 89 | 144 | 147 | 187 | 102 | 62 | 208 |
| 255 | 252 | 0 | 166 | 123 | 62 | 0 | 31 |
| 166 | 63 | 127 | 17 | 1 | 0 | 99 | 30 |

2D






1D

Adapted from S. Seitz

# Images as discrete functions

- Cartesian Coordinates

$$f[n,m] = \begin{bmatrix} \ddots & & \vdots & & \\ & f[-1,1] & f[0,1] & f[1,1] & \\ \dots & f[-1,0] & \underline{f[0,0]} & f[1,0] & \dots \\ & f[-1,-1] & f[0,-1] & f[1,-1] & \\ & & \vdots & & \ddots \end{bmatrix}$$

Source: Fei Feli Li, Stanford University

# Today’s topics

- Image Formation

- Image filters in spatial domain
  - Filter is a mathematical operation of a grid of numbers
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- Image filters in the frequency domain
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# Zebras vs. Dalmatians

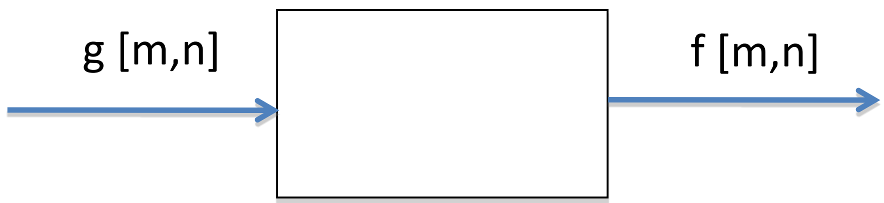
Both zebras and dalmatians have black and white pixels in about the same number

– if we shuffle the images point-wise processing is not affected

Need to measure properties relative to small *neighborhoods* of pixels

- find different image patterns

# Filtering




We want to remove unwanted sources of variation, and keep the information relevant for whatever task we need to solve


Source: Torralba, MIT

# Filters

- **Filtering:**

  – Form a new image whose pixels are a combination of original pixel values

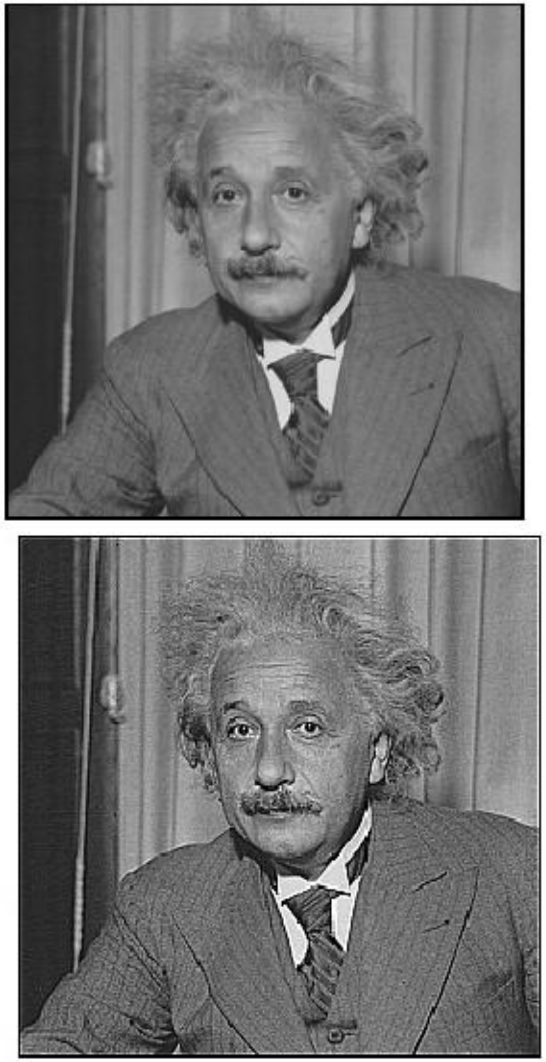
  - compute function of local neighborhood at each position
- **Goals:**
- Extract useful information from the images

  Features (textures, edges, corners, distinctive points, blobs…)
- Modify or enhance image properties:

  super-resolution; in-painting; de-noising, resizing
- Detect patterns

  Template matching

Source: Fei Feli Li, Stanford University; James Hays, Brown



***Smooth/Sharpen Images...***

***Find edges...***

***Find waldo...***

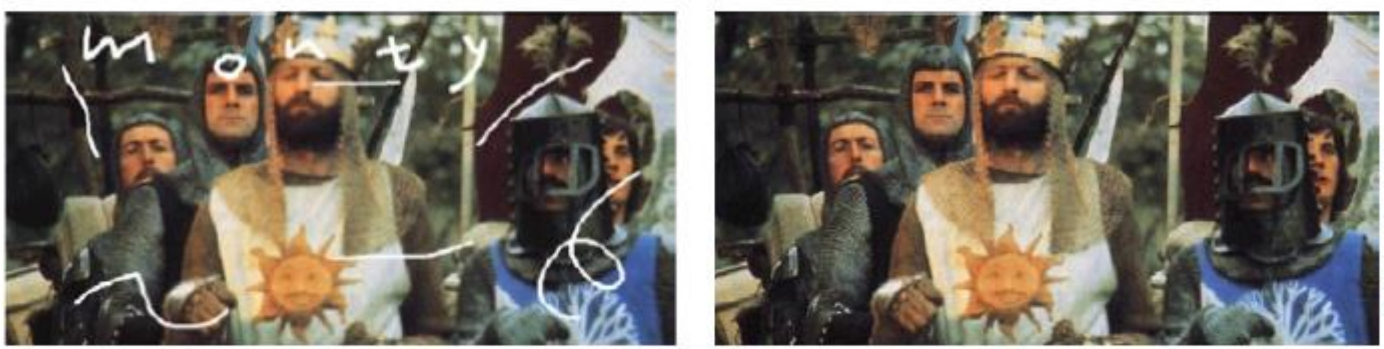
Source: Darrell, Berkeley

De-noising

Salt and pepper noise

Super-resolution

In-painting




Bertamio et al

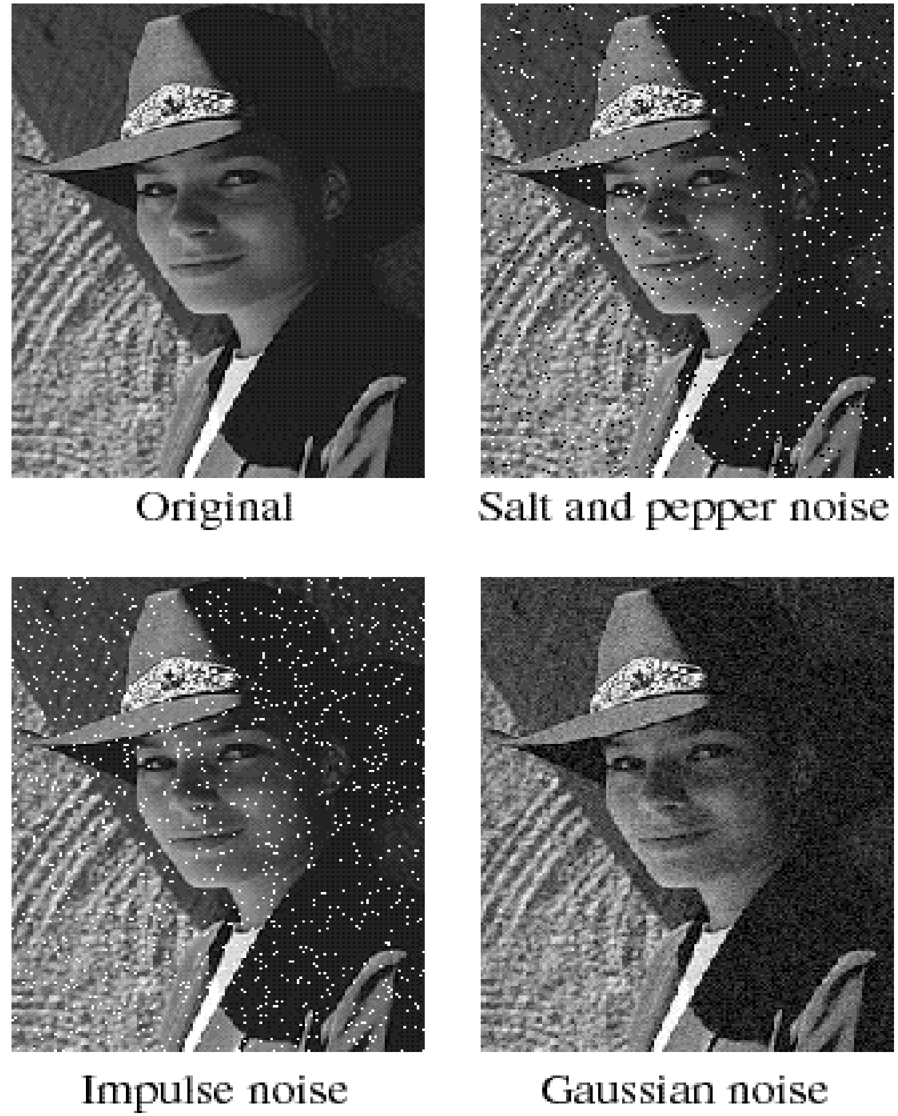
Source: Fei Feli Li, Stanford University

# Common types of noise

– **Salt and pepper noise**: random occurrences of black and white pixels

– **Impulse noise:** random occurrences of white pixels

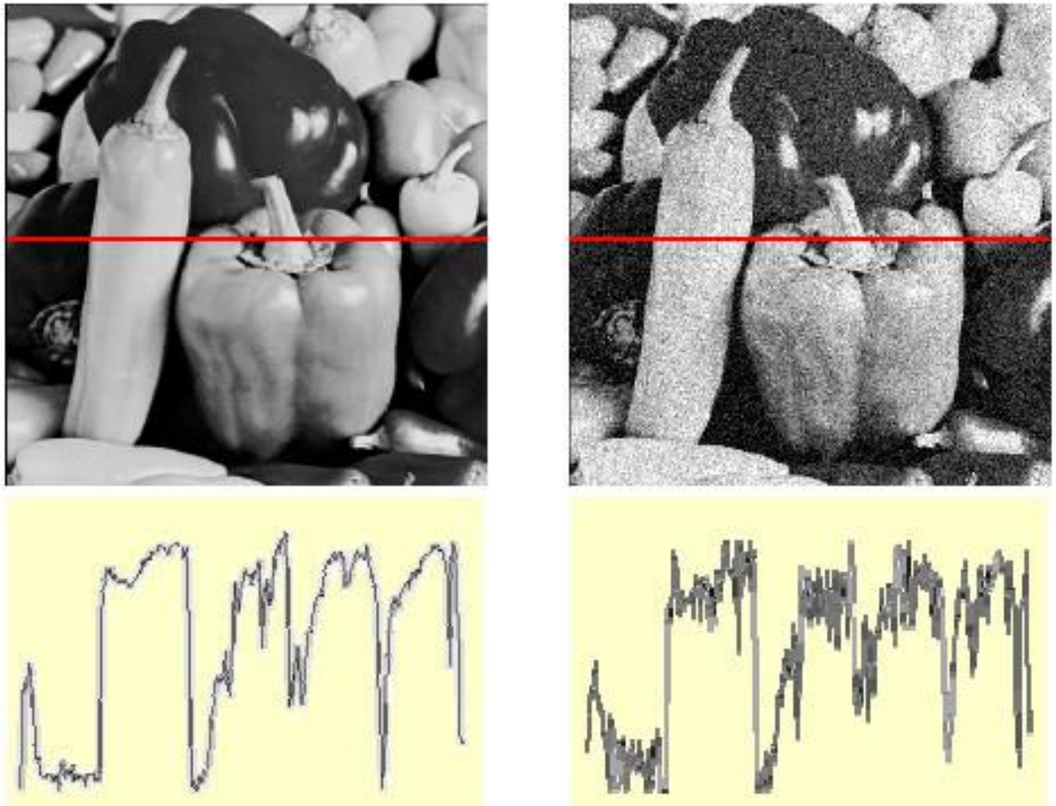
– **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution




Source: Darrell, Berkeley

Source: S. Seitz

# Gaussian noise



$$f(x,y) = \overbrace{\bar{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

Gaussian i.i.d. ("white") noise:
$\eta(x,y) \sim \mathcal{N}(\mu,\sigma)$

```
>> noise = randn(size(im)).*sigma;

>> output = im + noise;
```

Source: Darrell, Berkeley

Fig: M. Hebert

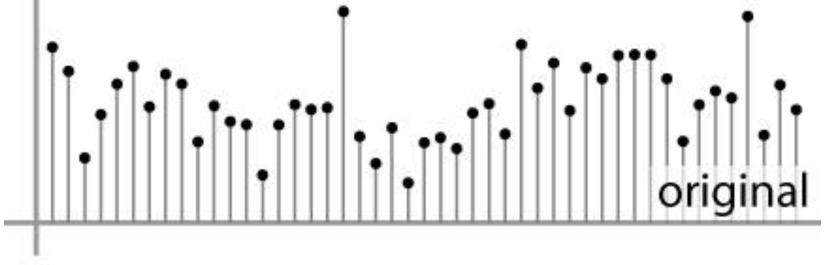
# First attempt at a solution

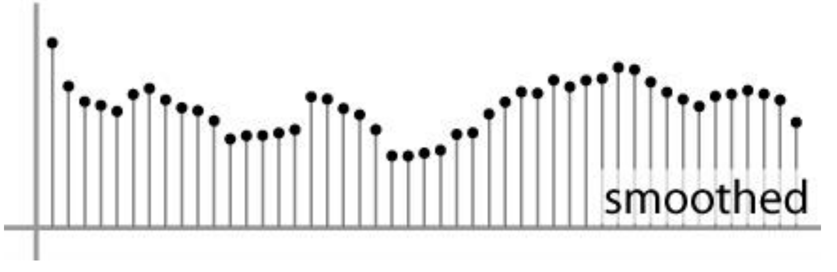
- Let’s replace each pixel with an average of all the values in its neighborhood
- Assumptions:
  - Expect pixels to be like their neighbors
  - Expect noise processes to be independent from pixel to pixel

Source: Darrell, Berkeley

# First attempt at a solution

- Let's replace each pixel with an average of all the values in its neighborhood
- Moving average in 1D:



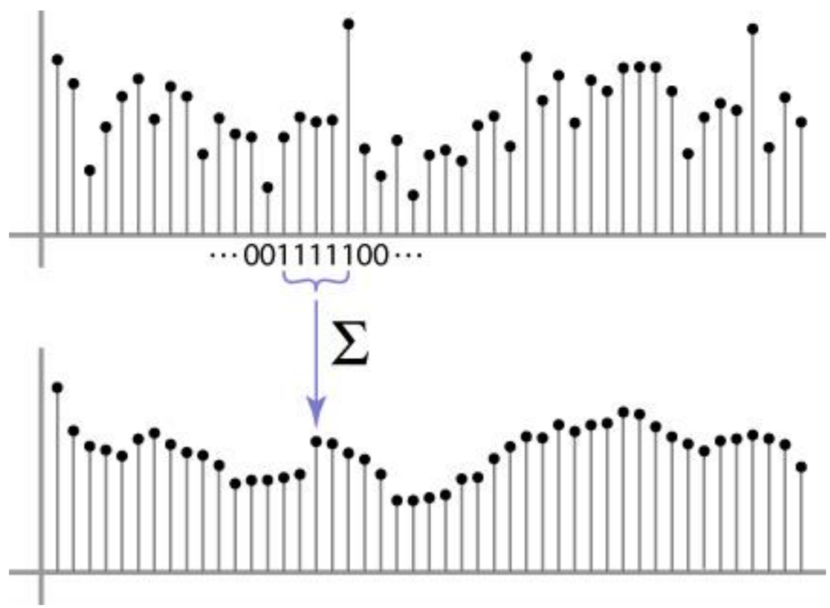




Source: Darrell, Berkeley

Source: S. Marschner

# Weighted Moving Average

- Can add weights to our moving average
- *Weights* [1, 1, 1, 1, 1] / 5

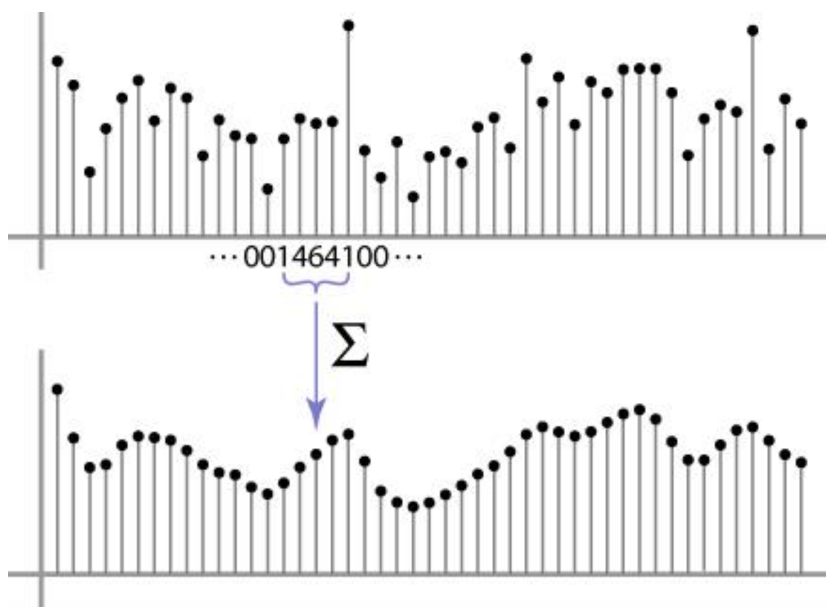



Source: Darrell, Berkeley

Source: S. Marschner

# Weighted Moving Average

- Non-uniform weights [1, 4, 6, 4, 1] / 16




Source: Darrell, Berkeley

Source: S. Marschner

# Moving Average In 2D

$F[x, y]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$G[x, y]$

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Source: Darrell, Berkeley

Source: S. Seitz

# Moving Average In 2D

$F[x, y]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$G[x, y]$

| | | | | | | | | | |
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| | | | | | | | | | |
| | 0 | 10 | | | | | | | |
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Source: Darrell, Berkeley

Source: S. Seitz

# Moving Average In 2D

$F[x, y]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$G[x, y]$

| | | | | | | | | | |
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| | | | | | | | | | |
| | 0 | 10 | 20 | | | | | | |
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Source: Darrell, Berkeley

Source: S. Seitz

# Moving Average In 2D

$F[x, y]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$G[x, y]$

| | | | | | | | | | |
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| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | | | | | |
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Source: Darrell, Berkeley

Source: S. Seitz

# Moving Average In 2D

$F[x, y]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$G[x, y]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
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Source: Darrell, Berkeley

Source: S. Seitz

# Moving Average In 2D

$F[x, y]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$G[x, y]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | 60 | 90 | 90 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 10 | 20 | 30 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 0 | 0 | 0 | 0 | 0 | |
| | | | | | | | | | |

Source: Darrell, Berkeley

Source: S. Seitz

# Correlation filtering

Say the averaging window size is 2k+1 x 2k+1:

$$G[i,j] = \underbrace{\frac{1}{(2k+1)^2}}_{\text{Attribute uniform weight to each pixel}} \underbrace{\sum_{u=-k}^{k} \sum_{v=-k}^{k} F[i+u, j+v]}_{\text{Loop over all pixels in neighborhood around image pixel } F[i,j]}$$

Now generalize to allow different weights depending on neighboring pixel's relative position:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} \underbrace{H[u,v]}_{\text{Non-uniform weights}} F[i+u, j+v]$$

Source: Darrell, Berkeley

# Correlation filtering

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

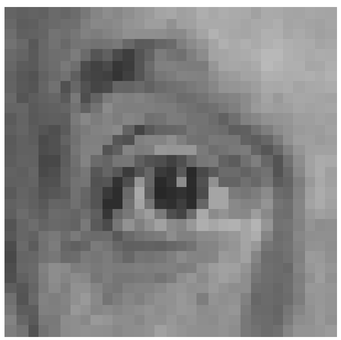
This is called cross-correlation, denoted $G = H \otimes F$

Filtering an image: replace each pixel with a linear combination of its neighbors.

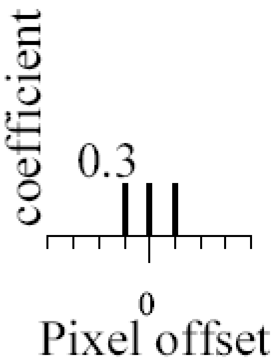
The filter “kernel” or “mask” *H*[*u,v*] is the prescription for the weights in the linear combination.

Source: Darrell, Berkeley

# Averaging Filter



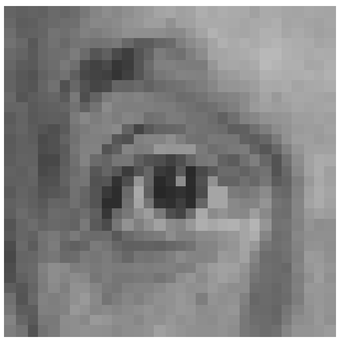
original



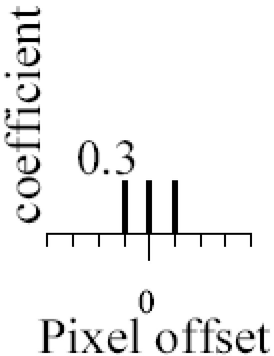

?

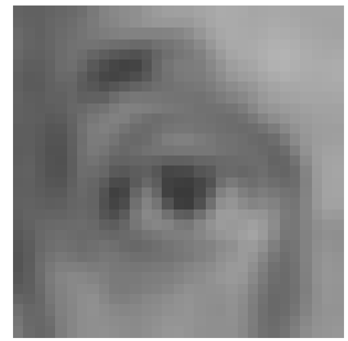
adapted from Darrell and Freeman, MIT

# Averaging Filter



original

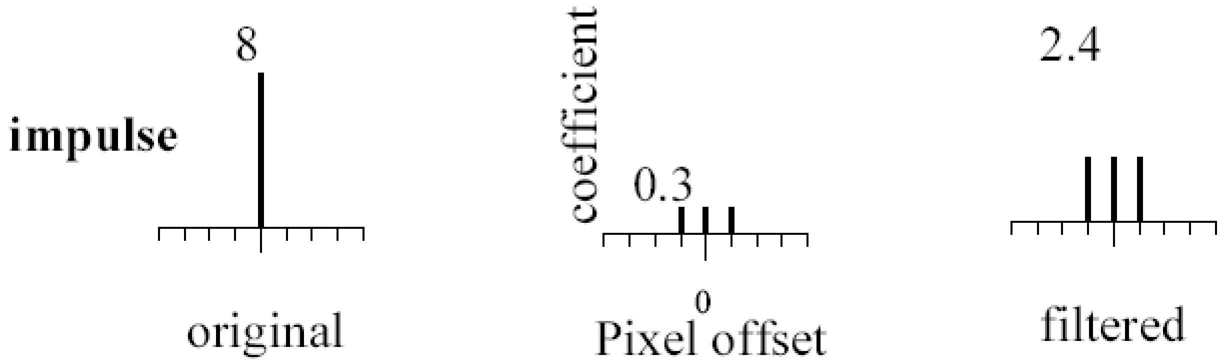


Blurred (filter applied in both dimensions).

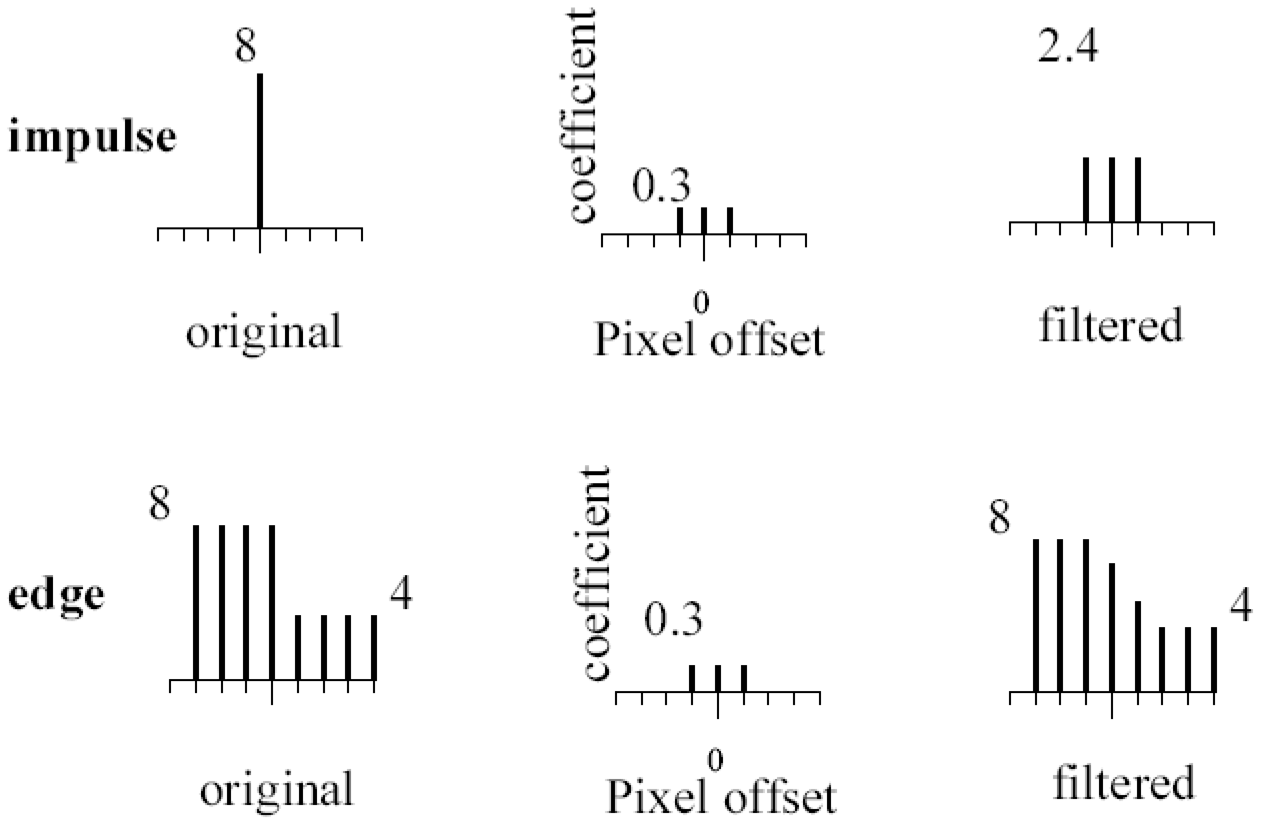
adapted from Darrell and Freeman, MIT

# Averaging Filter




adapted from Darrell and Freeman, MIT

# Averaging Filter




adapted from Darrell and Freeman, MIT

# Averaging filter

- What values belong in the kernel *H* for the moving average example?

$F[x, y]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$\otimes$

$H[u, v]$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & ? & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

“box filter”

$G[x, y]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
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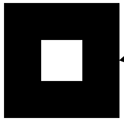
$$G = H \otimes F$$

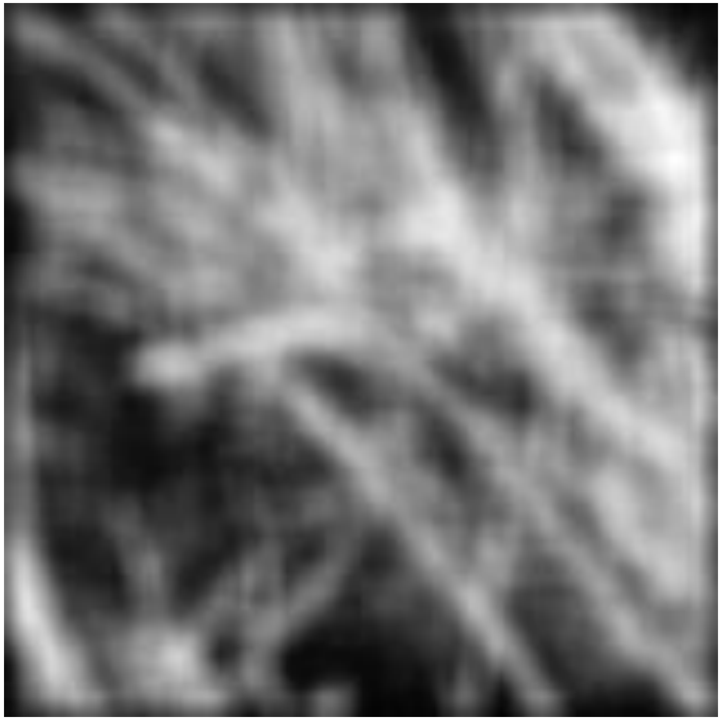
Source: Darrell, Berkeley

# Smoothing by averaging



depicts box filter:
white = high value, black = low value

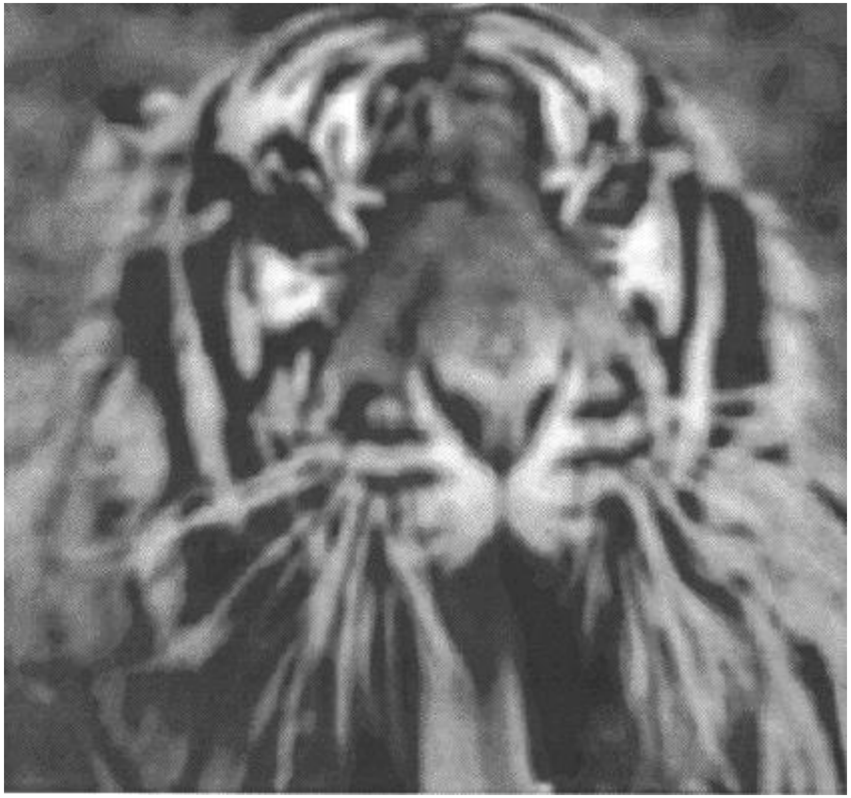
original



filtered

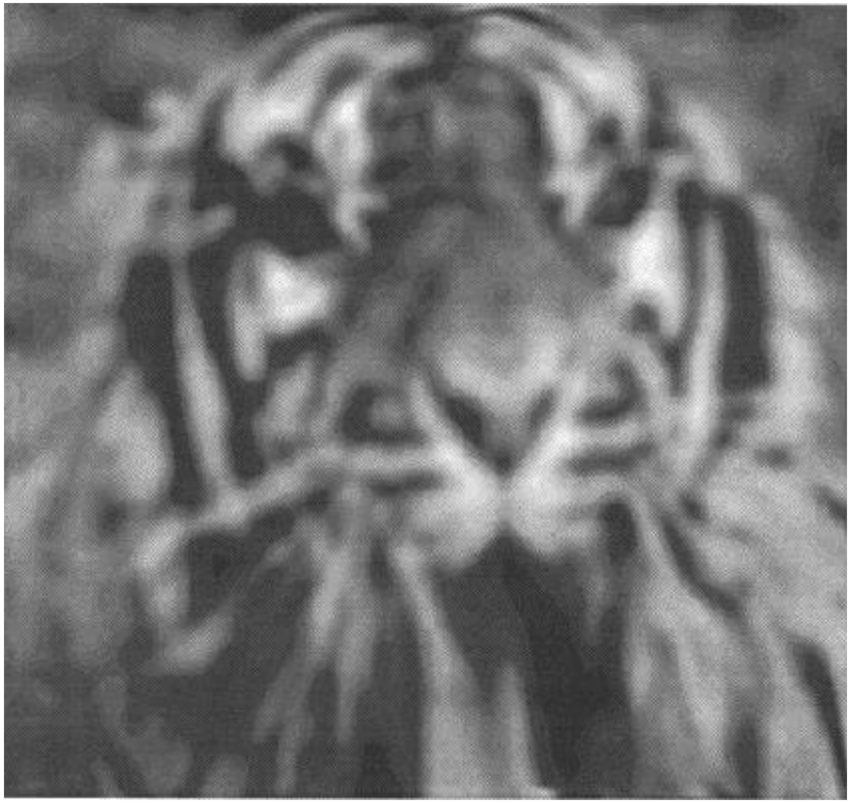
Source: Darrell, Berkeley

# Example



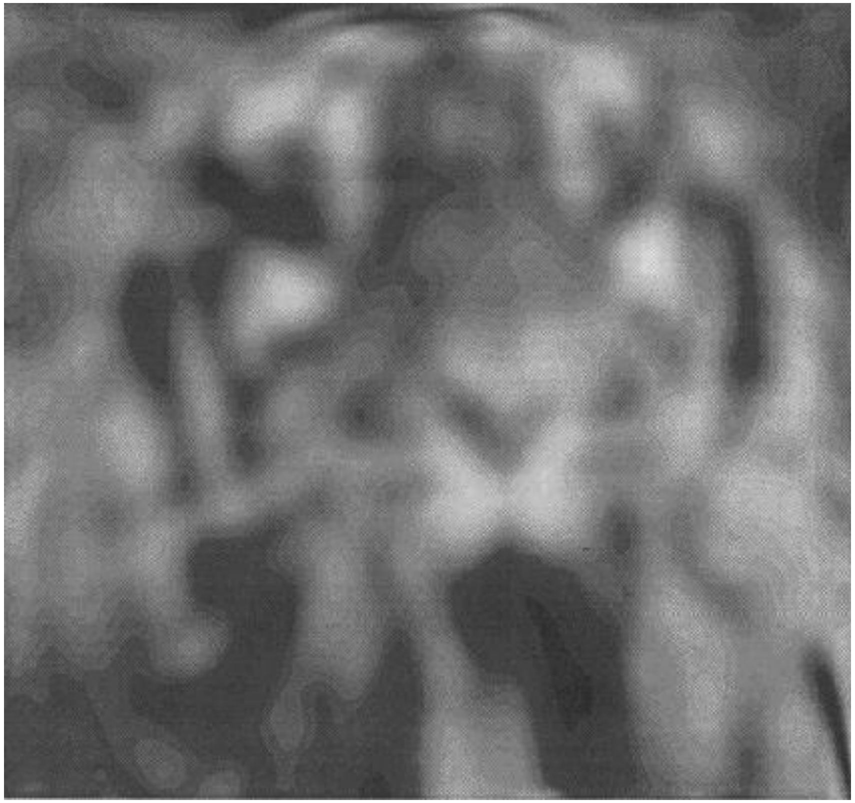
Source: Martial Hebert, CMU

# Example



Source: Martial Hebert, CMU

# Example



Source: Martial Hebert, CMU

# Smoothing by averaging

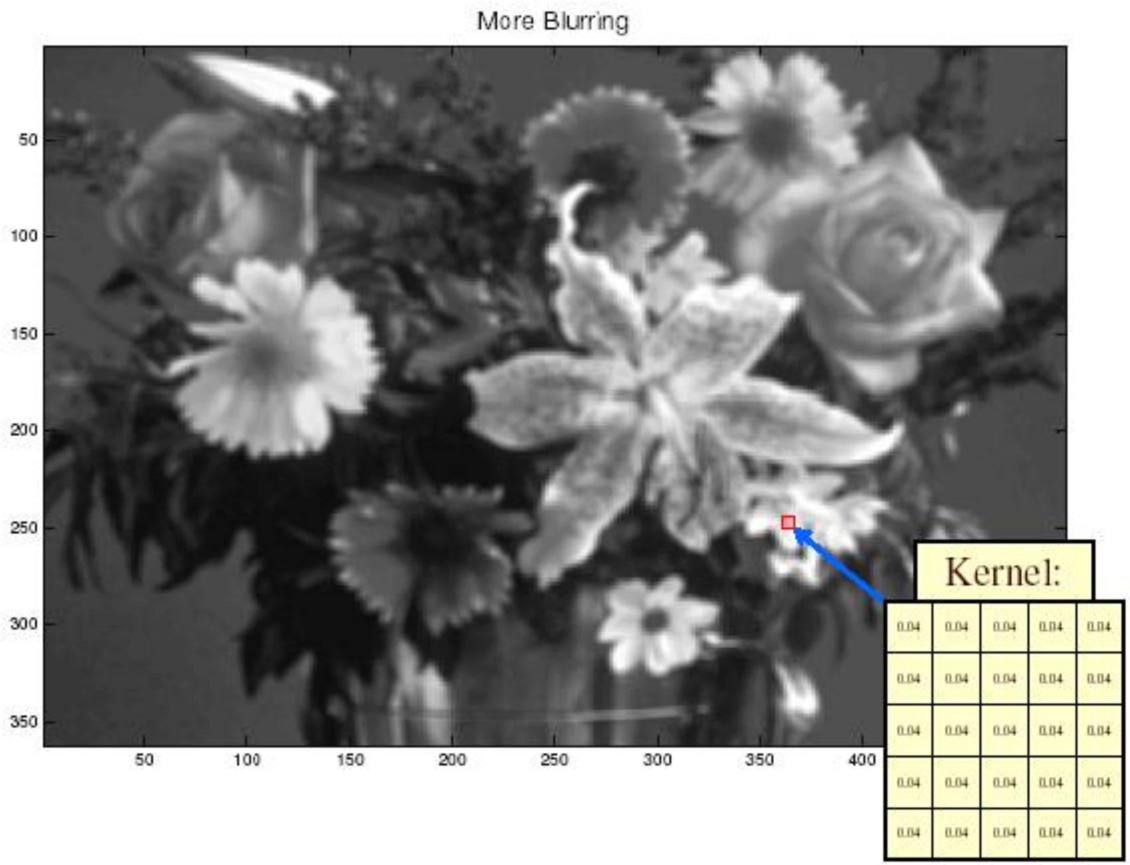

Source: Martial Hebert, CMU

# Smoothing by averaging


| Kernel: | | |
|---|---|---|
| 1/9 | 1/9 | 1/9 |
| 1/9 | 1/9 | 1/9 |
| 1/9 | 1/9 | 1/9 |

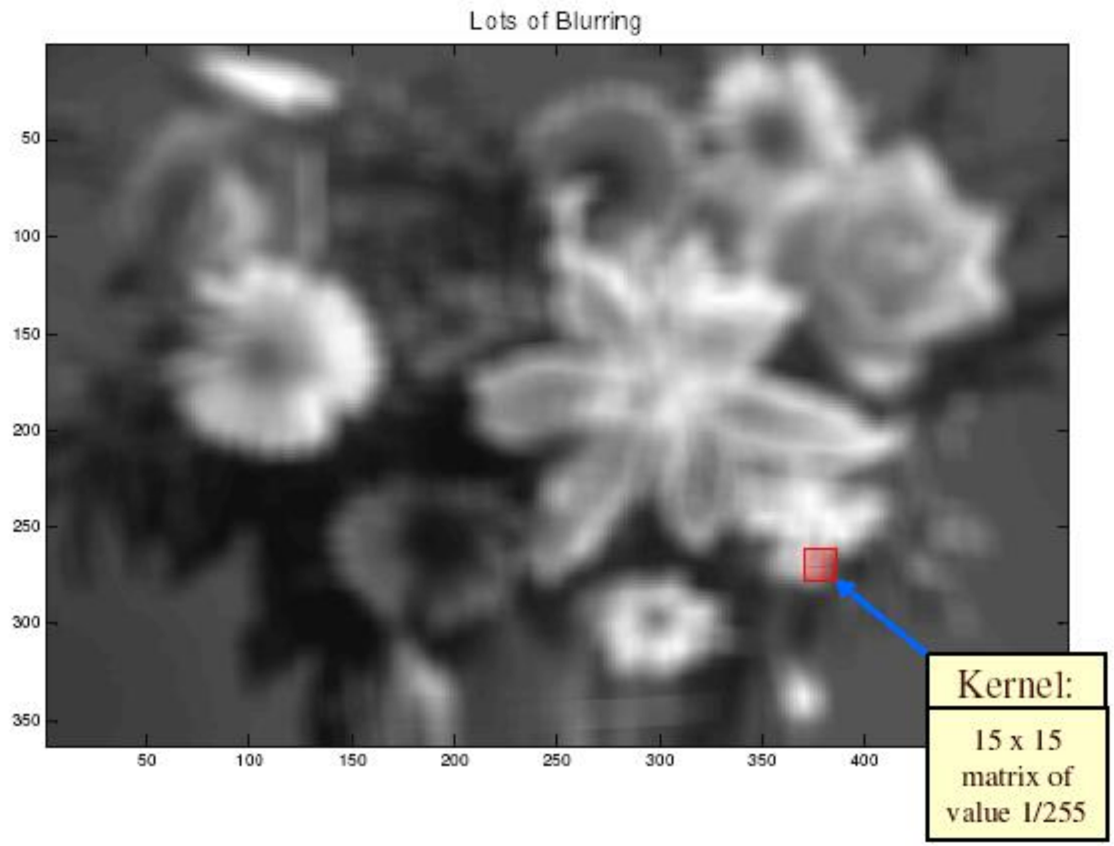
Source: Martial Hebert, CMU

# Smoothing by averaging




Source: Martial Hebert, CMU

# Smoothing by averaging




Source: Martial Hebert, CMU

# Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$F[x, y]$

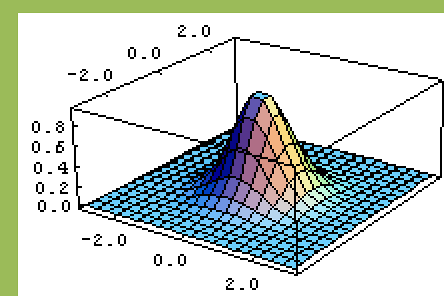
$\frac{1}{16}$

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 4 | 2 |
| 1 | 2 | 1 |

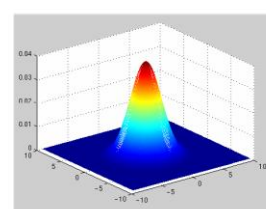
$H[u, v]$

This kernel is an approximation of a Gaussian function:

$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



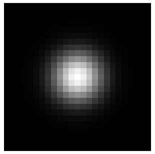

A weighted average that weights pixels at its center much more strongly than its boundaries

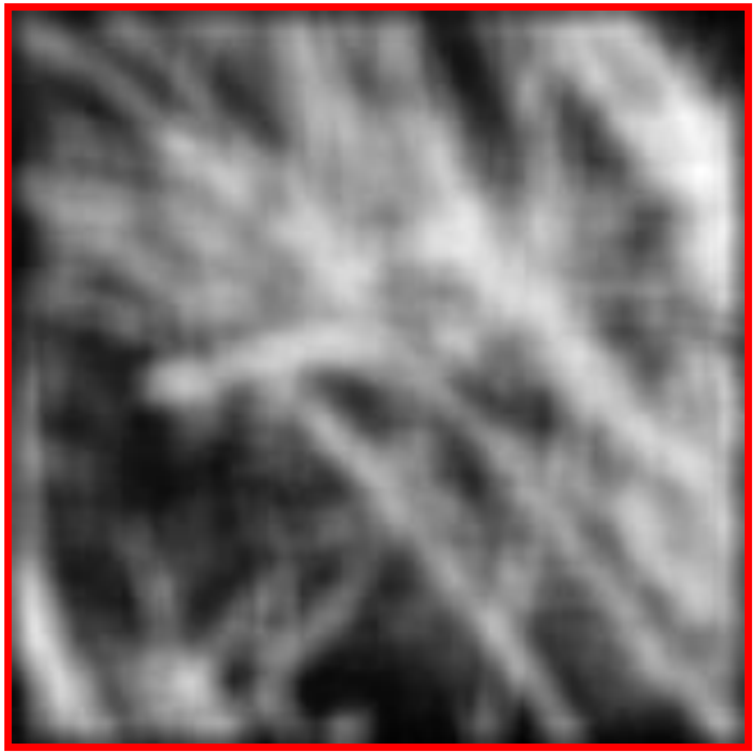


Source: Darrell, Berkeley

Source: S. Seitz

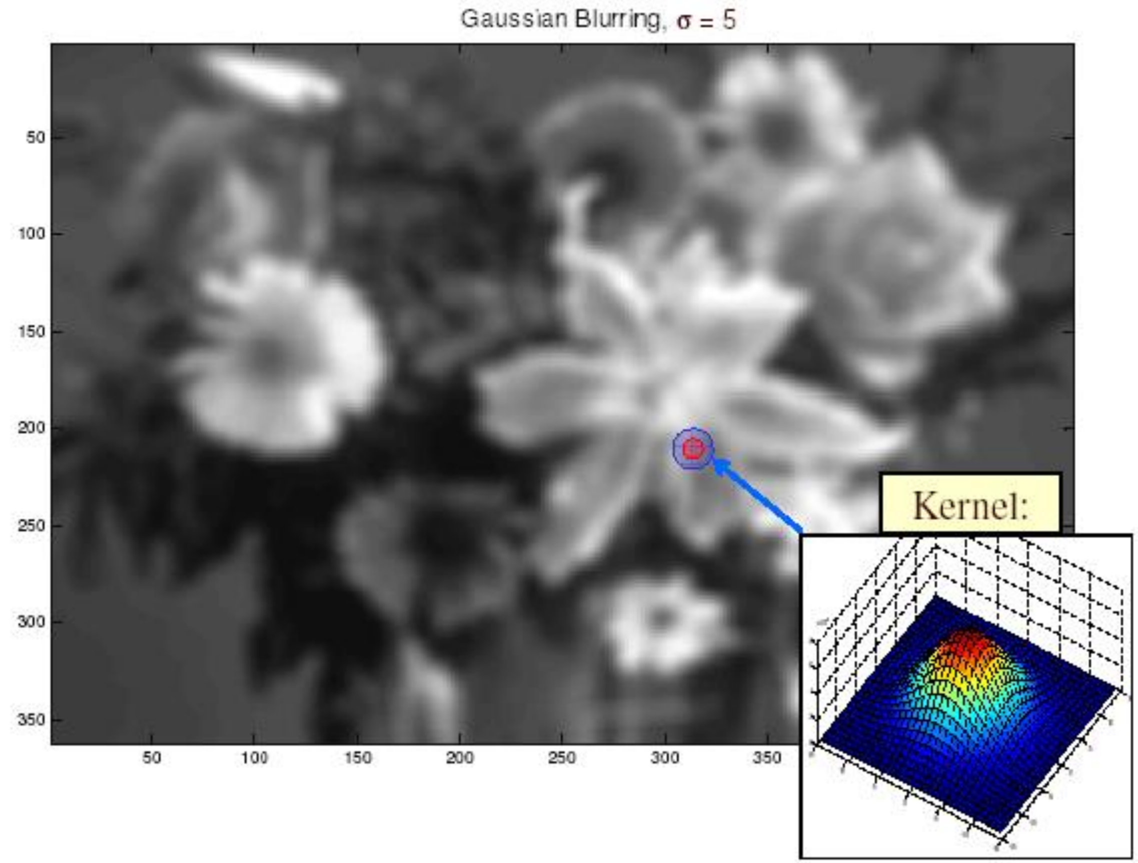
# Smoothing with a Gaussian





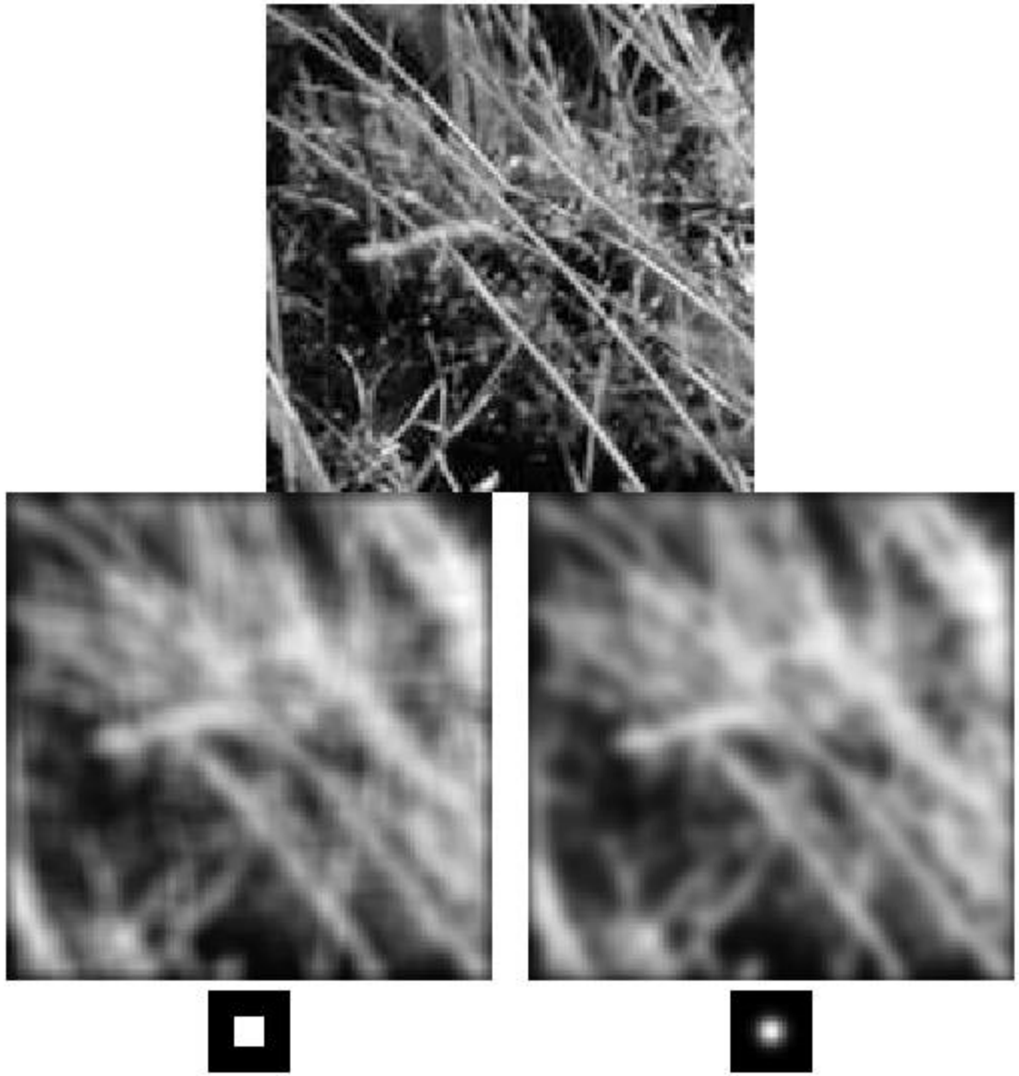
Source: Darrell, Berkeley

# Smoothing with a Gaussian




Source: Martial Hebert, CMU

# Smoothing with a Gaussian



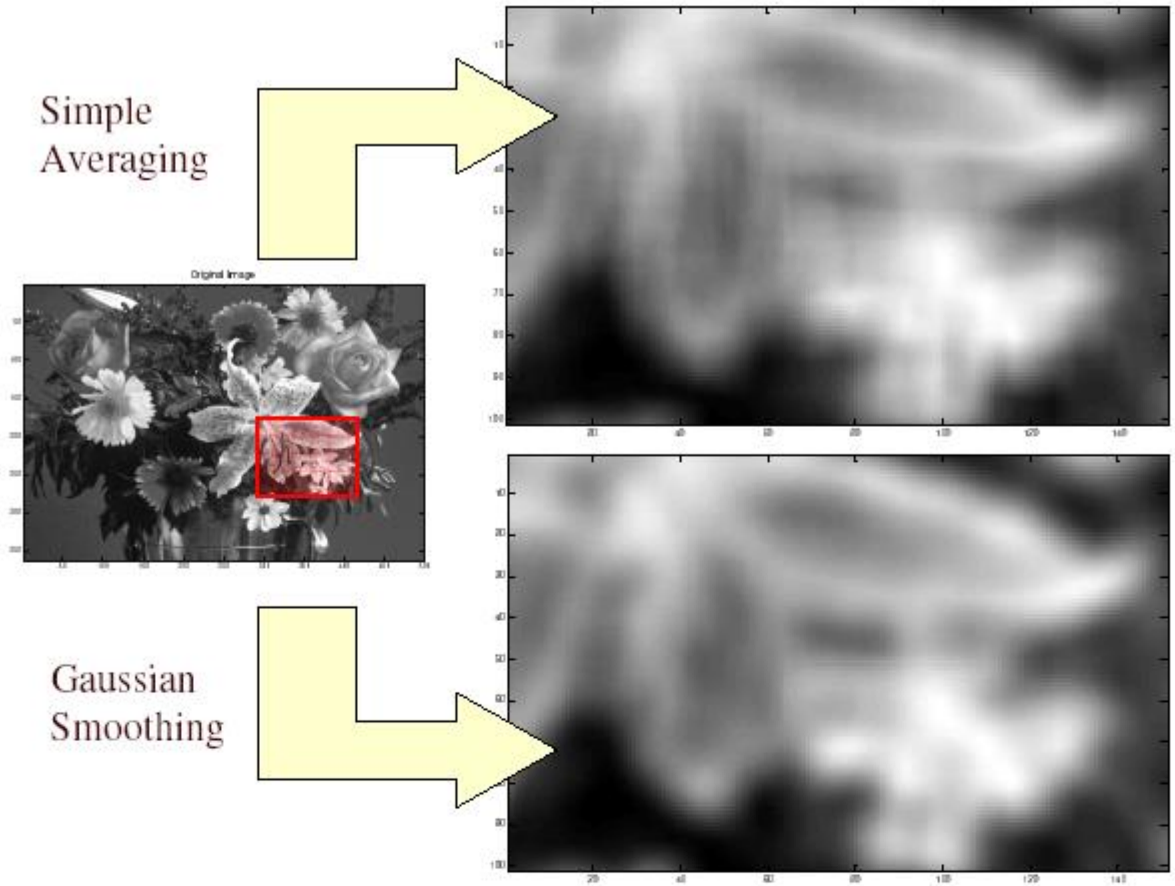
Result of blurring using a uniform local model

Produces a set of narrow vertical horizontal and vertical bars – ringing effect

Result of blurring using a set of Gaussian weights

Source: David Forsyth, UIUC

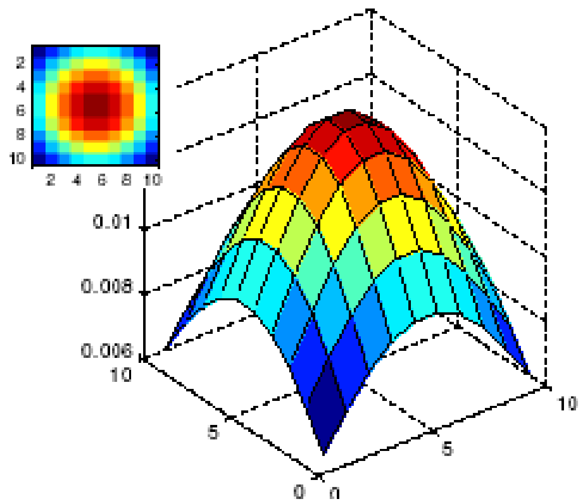
# Smoothing with a Gaussian



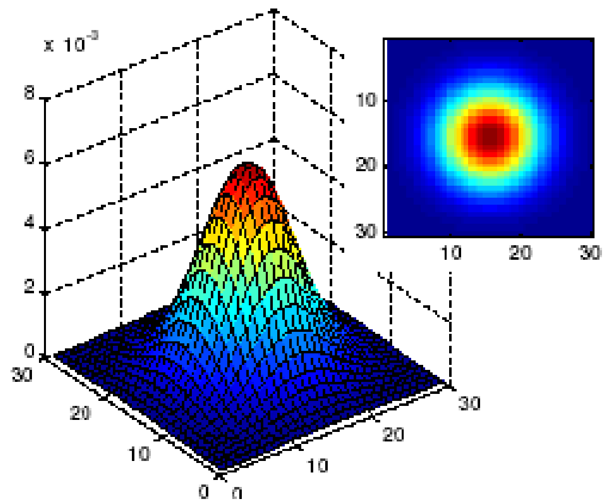

Source: Martial Hebert, CMU

# Gaussian filters

- What parameters matter here?
- **Size** of kernel or mask
  - Note, Gaussian function has infinite support, but discrete filters use finite kernels



σ = 5 with 10 x 10 kernel

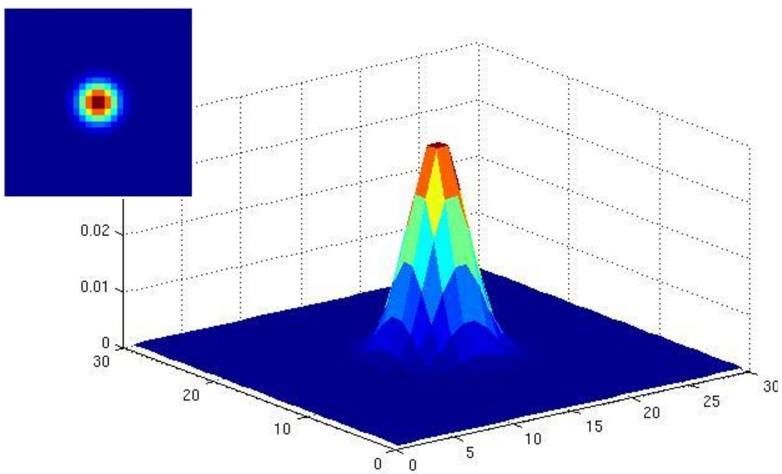


σ = 5 with 30 x 30 kernel

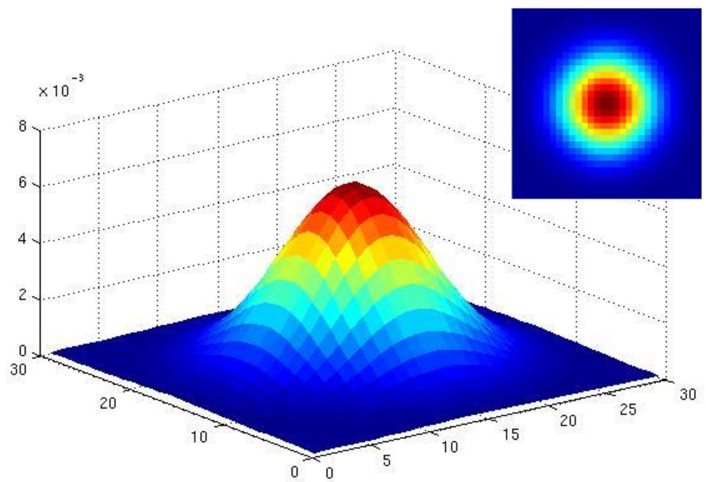
Source: Darrell, Berkeley

# Gaussian filters

- What parameters matter here?
- **Variance** of Gaussian: determines extent of smoothing




σ = 2 with 30 x 30 kernel




σ = 5 with 30 x 30 kernel

Source: Darrell, Berkeley

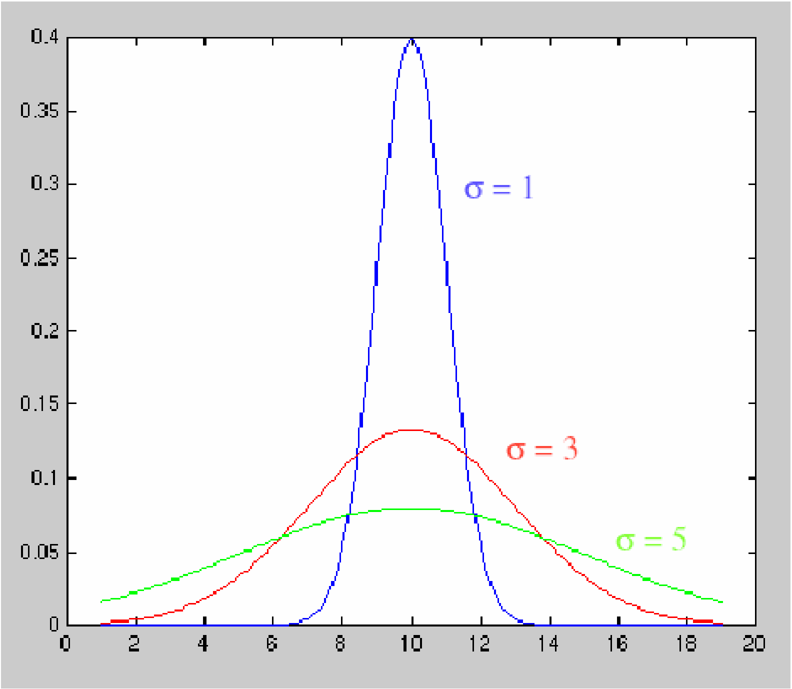
# Smoothing with a Gaussian

If σ is small : the smoothing will have little effect

If σ is larger : neighboring pixels will have larger weights resulting in consensus of the neighbors

If σ is very large : details will disappear along with the noise

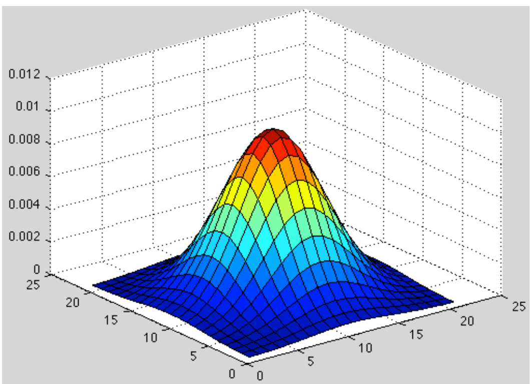
Effect of σ

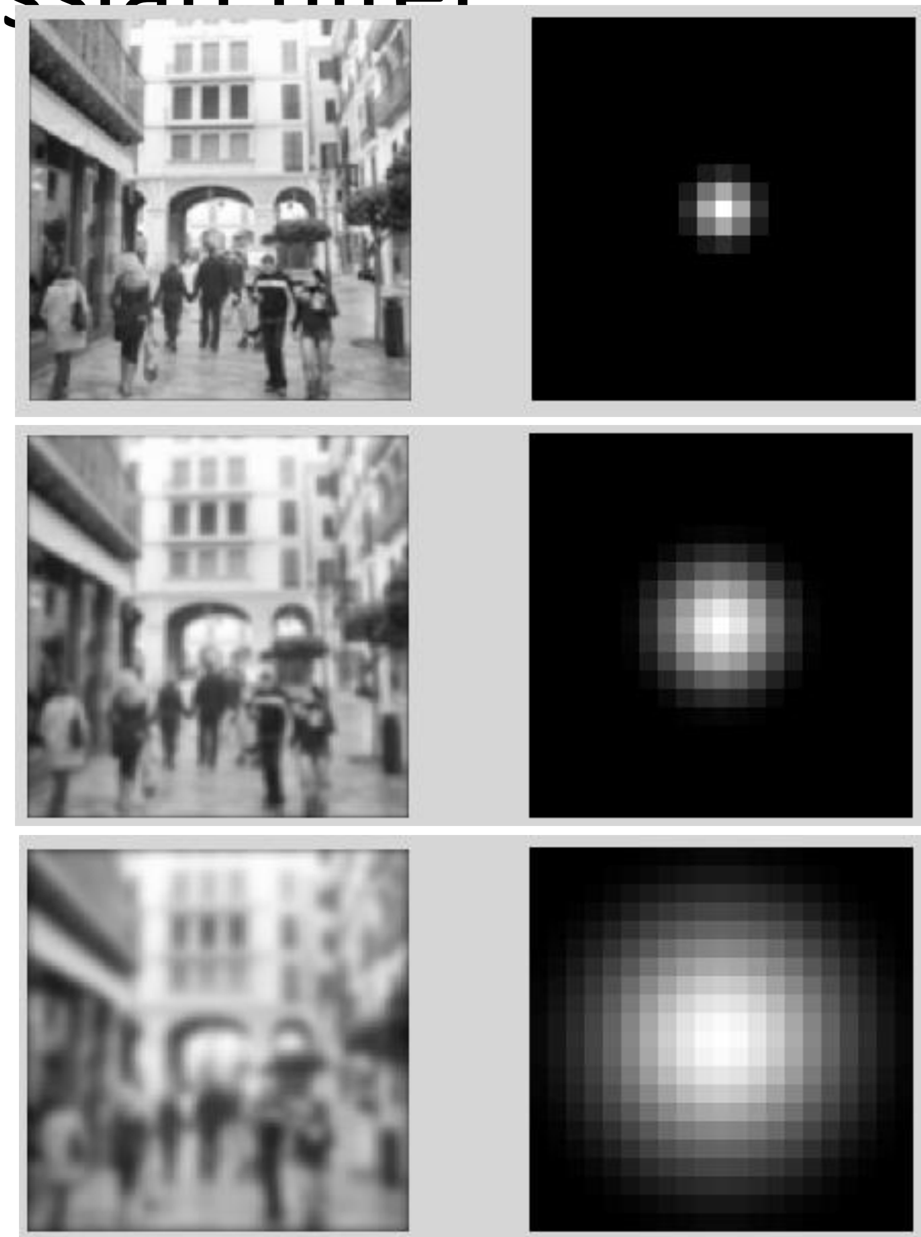



Source: Martial Hebert, CMU

# Gaussian filter

$$G(x, y; \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



σ=1

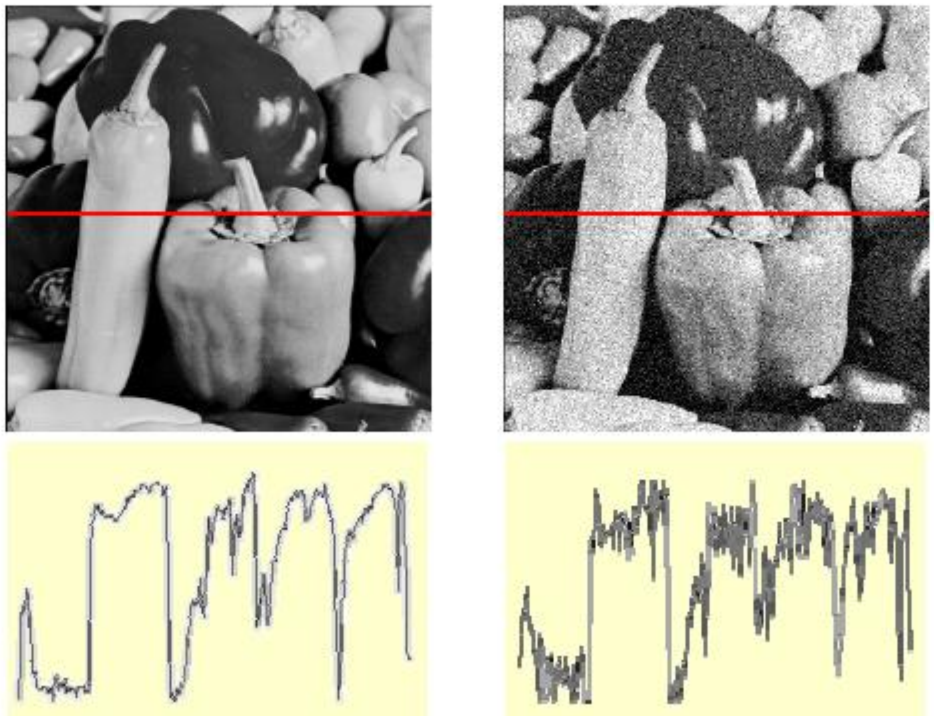
σ=2

σ=4

Source: Torralba, MIT

# Gaussian smoothing to remove noise

Image Noise

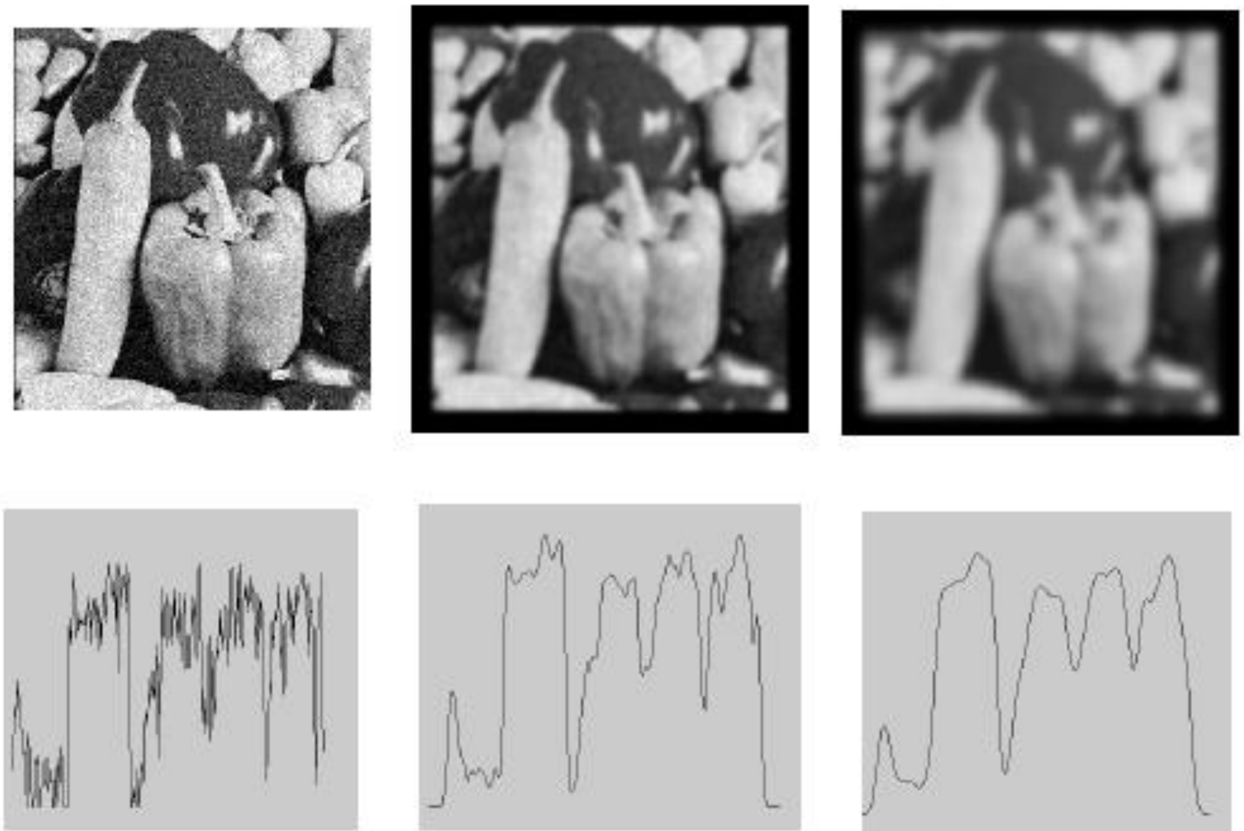


$$f(x,y) = \overbrace{\bar{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

Gaussian i.i.d. ("white") noise:
$\eta(x,y) \sim \mathcal{N}(\mu, \sigma)$

Source: Martial Hebert, CMU

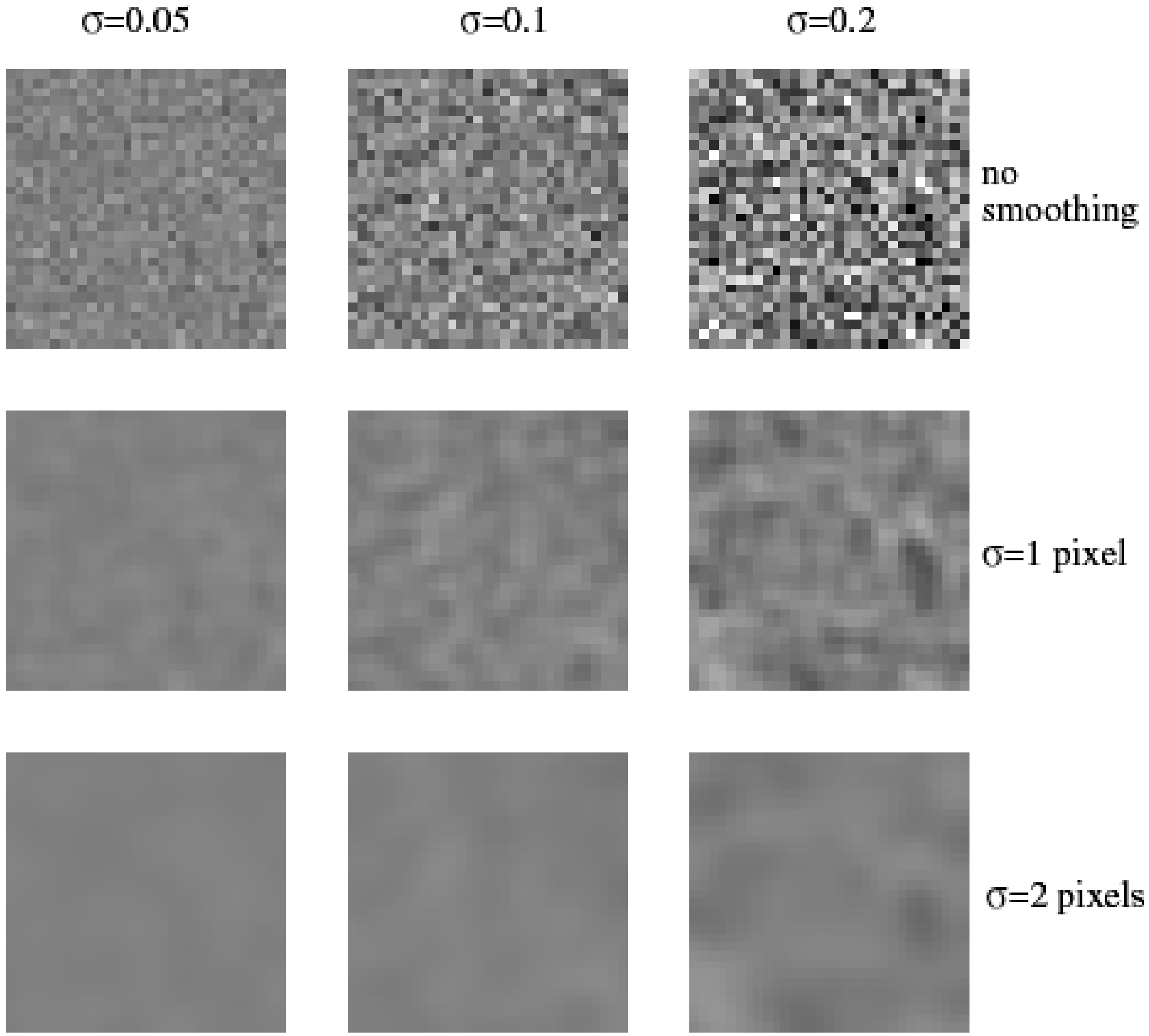
# Gaussian smoothing to remove noise



No smoothing $\sigma = 2$ $\sigma = 4$

Source: Martial Hebert, CMU

# Smoothing with a Gaussian

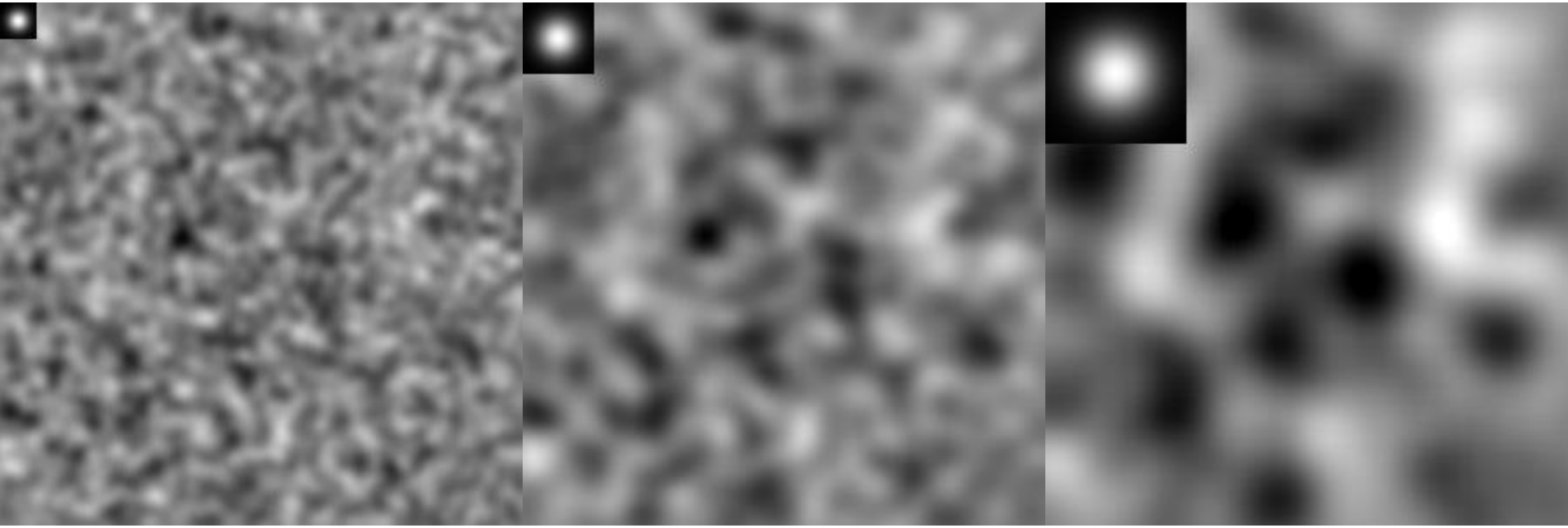



**The effects of smoothing**
Each row shows smoothing with gaussians of different width; each column shows different realizations of an image of gaussian noise.

Source: David Forsyth, UIUC

# Smoothing with a Gaussian

- Filtered noise is sometimes useful
  - looks like some natural textures, can be used to simulate fire, etc.



Source: David Forsyth, UIUC

# Gaussian kernel

$$g(x,y)=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)$$

| | | |
|---|---|---|
| 0.0751 | 0.1238 | 0.0751 |
| 0.1238 | 0.242 | 0.1238 |
| 0.0751 | 0.1238 | 0.0751 |

Gaussian is an approximation to the binomial distribution.

Can approximate Gaussian using binomial coefficients.

$$a_{nr}\equiv\frac{n!}{r!(n-r)!}\equiv\binom{n}{r}$$

1X3 filter: n=(3-1)=2, r=0,1,2

| | | |
|---|---|---|
| 1 | 2 | 1 |

$n$ = number of elements in the 1D filter minus 1

$r$ = position of element in the filter kernel (0, 1, 2…)

g = 1/4

| | | |
|---|---|---|
| 1 | 2 | 1 |

g' g =

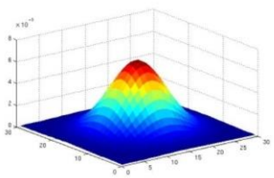
| | | |
|---|---|---|
| 0.0625 | 0.1250 | 0.0625 |
| 0.1250 | 0.2500 | 0.1250 |
| 0.0625 | 0.1250 | 0.0625 |

Source: from Michael Black

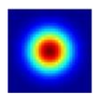
# Matlab

```
>> hsize = 10;
>> sigma = 5;
>> h = fspecial('gaussian' hsize, sigma);
```

```
>> mesh(h);
```



```
>> imagesc(h);
```



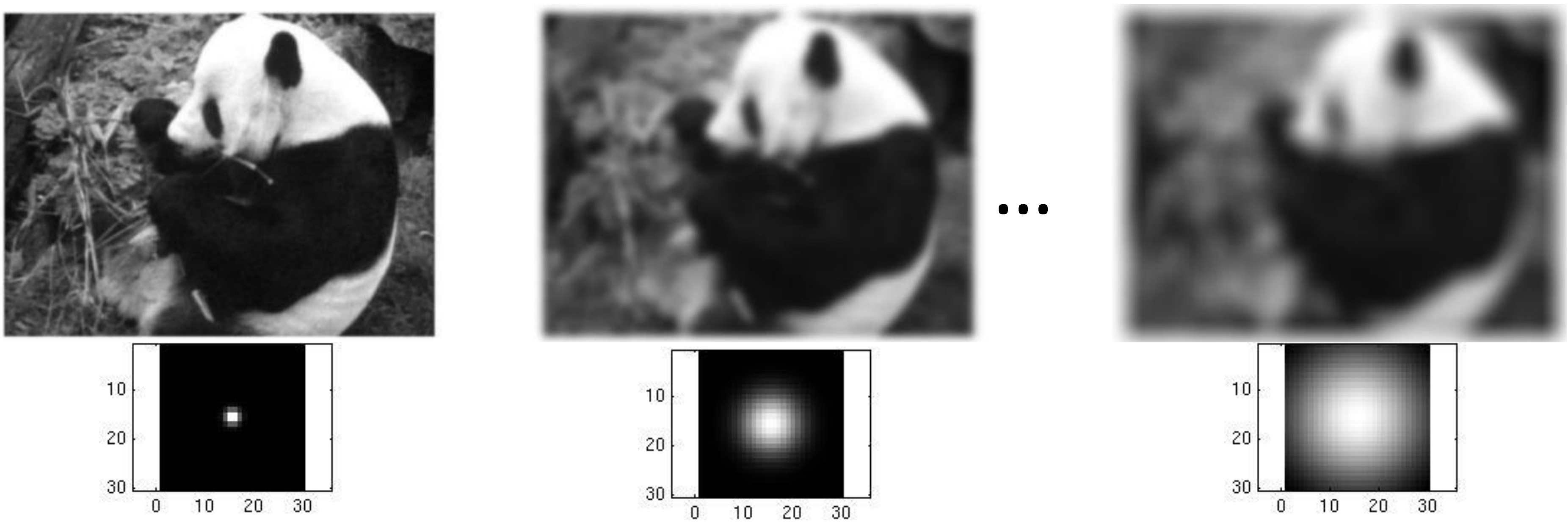
```
>> outim = imfilter(im, h);
>> imshow(outim);
```

outim

Source: Darrell, Berkeley

# Smoothing with a Gaussian

Parameter σ is the "scale" / "width" / "spread" of the Gaussian kernel, and controls the amount of smoothing.




```
for sigma=1:3:10
   h = fspecial('gaussian`, fsize, sigma);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
```

Source: Darrell, Berkeley

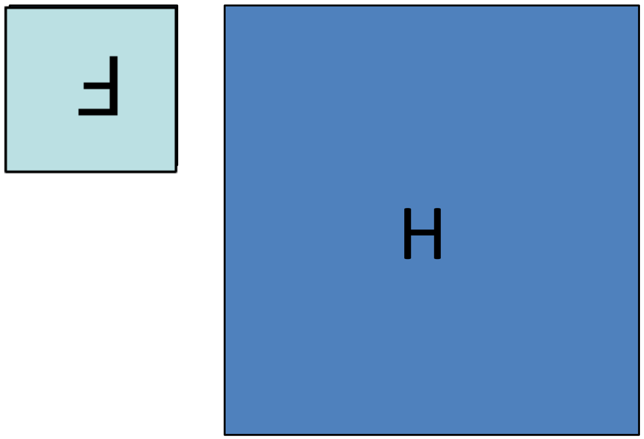
# Convolution

- Convolution:
  - Flip the filter in both dimensions (bottom to top, right to left)
  - Then apply cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H \star F$$

*Notation for convolution operator*




Source: Darrell, Berkeley

# Convolution vs. correlation

Convolution

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

$$G = H \star F$$

Cross-correlation

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

$$G = H \otimes F$$

For a Gaussian or box filter, how will the outputs differ?

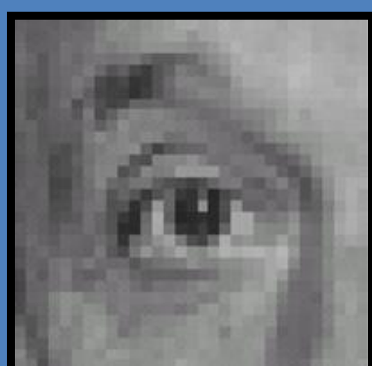
If the input is an impulse signal, how will the outputs differ?

Source: Darrell, Berkeley

# Predict the filtered outputs

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = ?$$

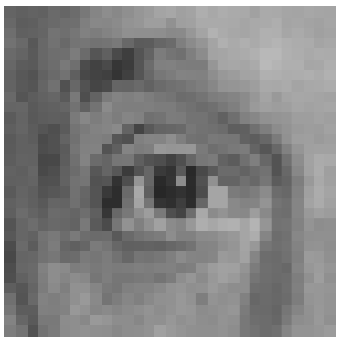
$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = ?$$



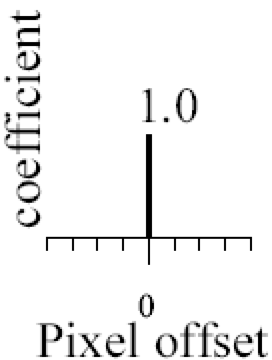
$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = ?$$

Source: Darrell, Berkeley

# Practice with linear filters



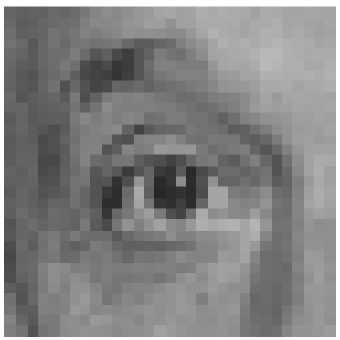
original



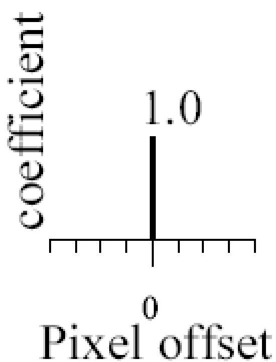

?

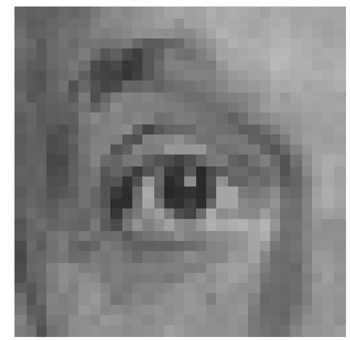
adapted from Darrell and Freeman, MIT

# Practice with linear filters



original



Filtered
(no change)

adapted from Darrell and Freeman, MIT

# Practice with linear filters

Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

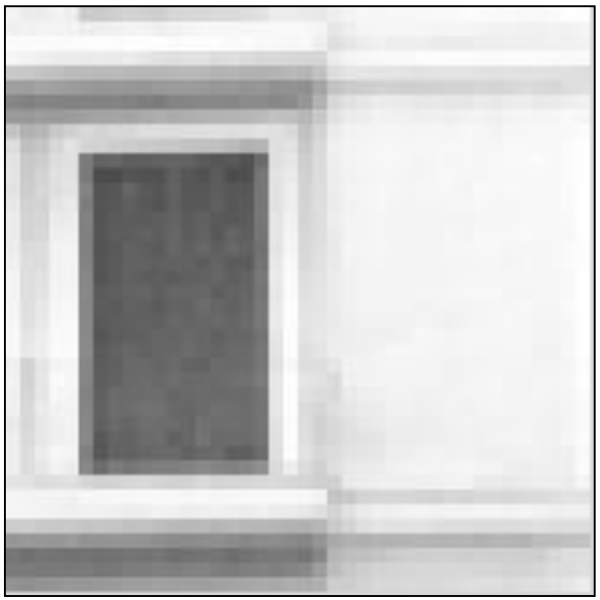
Filtered
(no change)

Source: Darrell, Berkeley

Source: D. Lowe

# Impulse

$$f[m,n] = I \otimes g = \sum_{k,l} h[m-k, n-l]\, g[k,l]$$



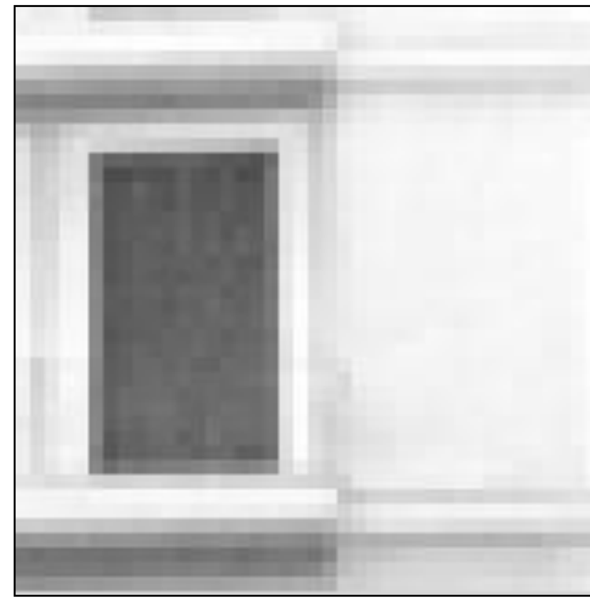
g[m,n]

$\otimes$

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

h[m,n]

=



f[m,n]

Source: Torralba, MIT

# Practice with linear filters

Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

?

Source: Darrell, Berkeley

Source: D. Lowe

# Practice with linear filters

Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

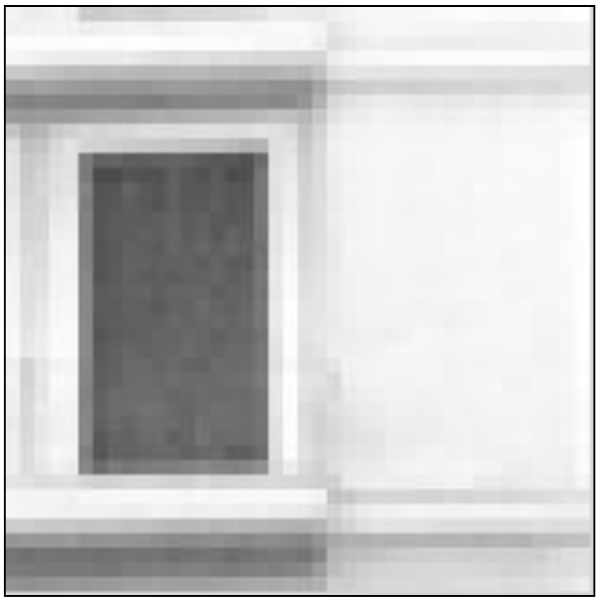
Shifted left by 1 pixel with correlation

Source: Darrell, Berkeley

Source: D. Lowe

# Shifts

$$f[m,n] = I \otimes g = \sum_{k,l} h[m-k, n-l] g[k,l]$$



g[m,n]

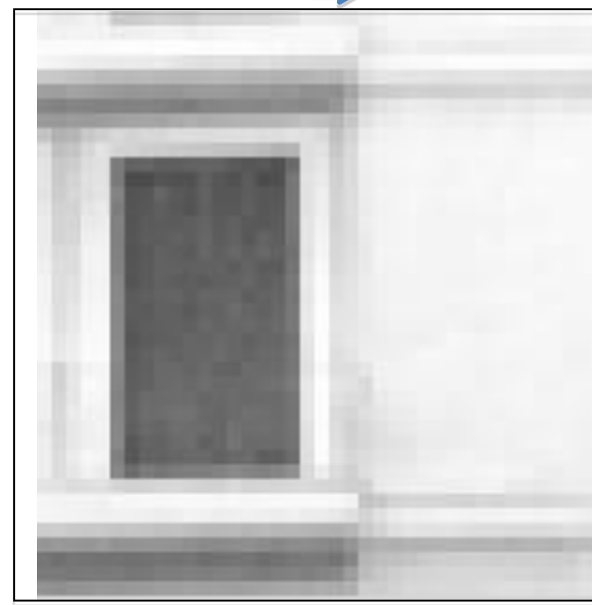
$\otimes$

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

h[m,n]

=

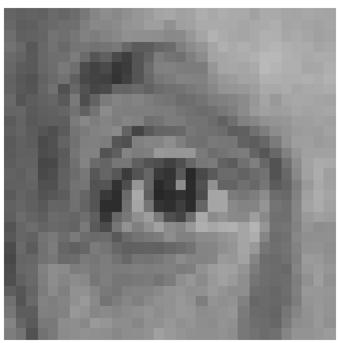
2pixels →



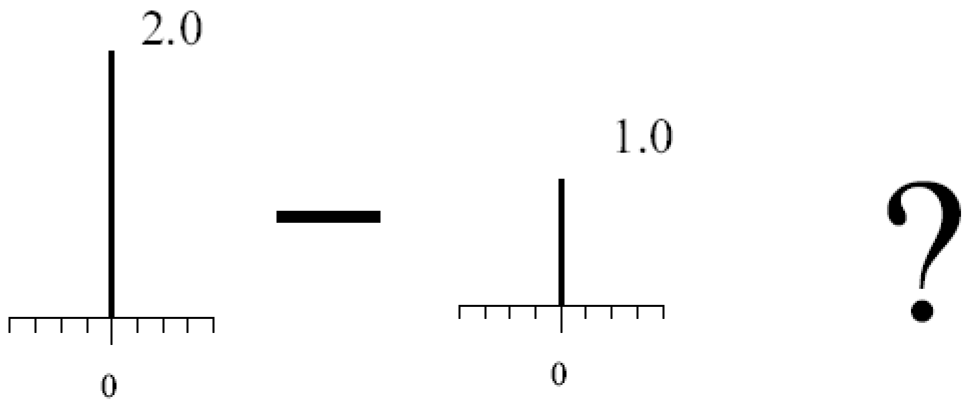
f[m,n]

Source: Torralba, MIT

# Practice with linear filters

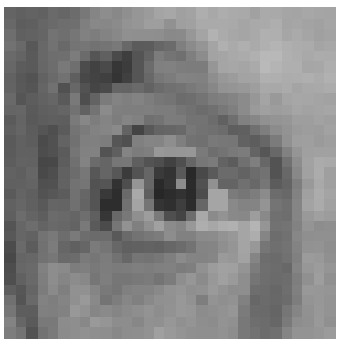


original

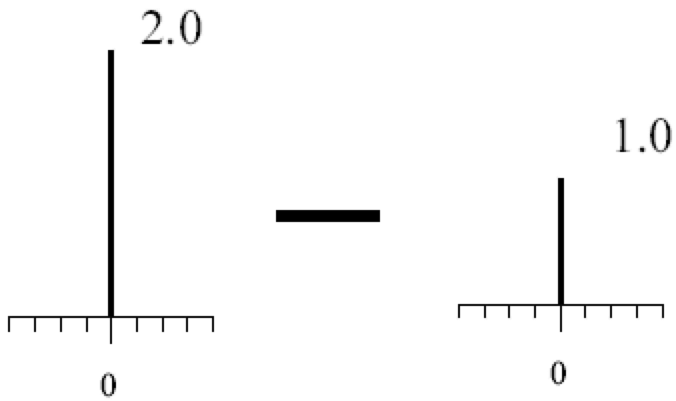


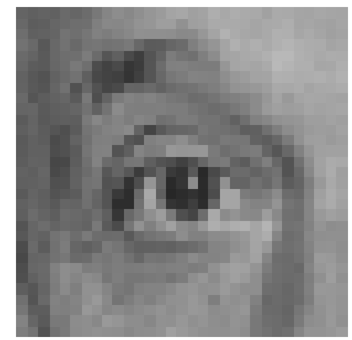

adapted from Darrell and Freeman, MIT

# Practice with linear filters



original

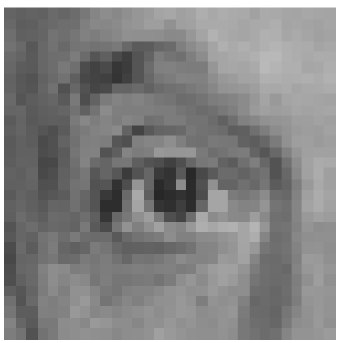


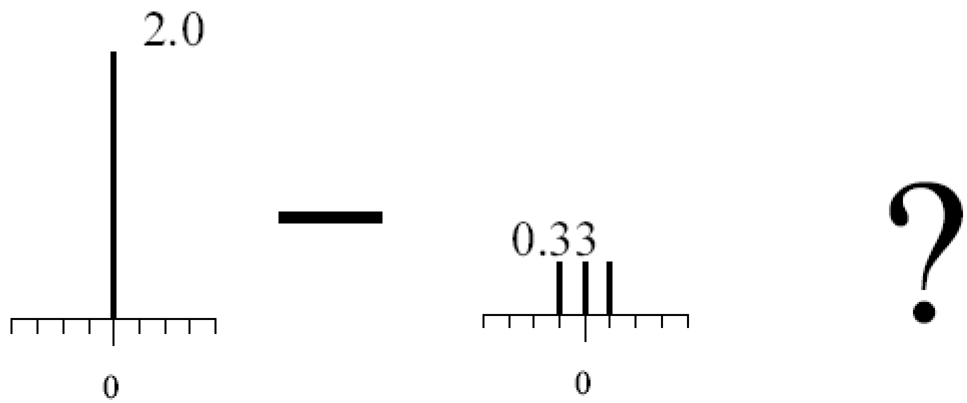
Filtered
(no change)

adapted from Darrell and Freeman, MIT

# Practice with linear filters

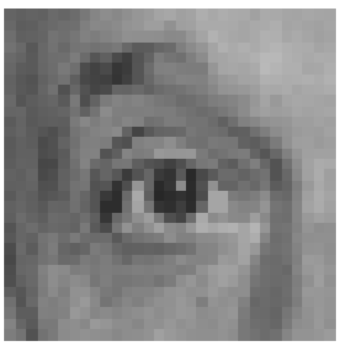


original

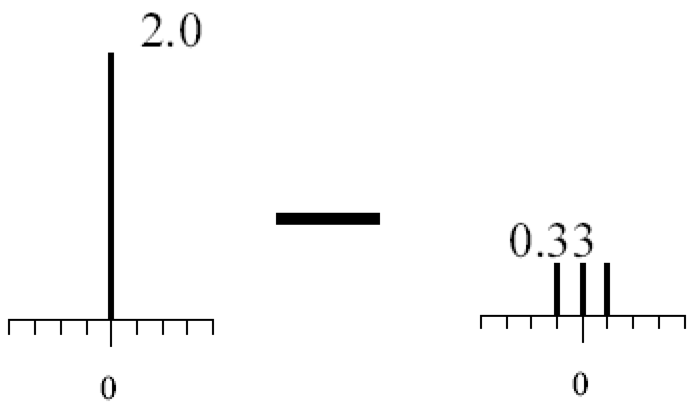


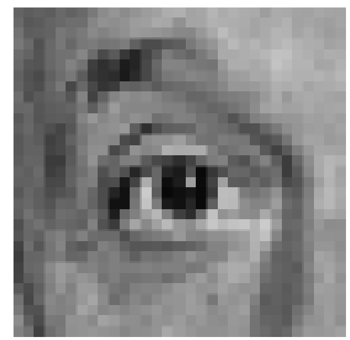

adapted from Darrell and Freeman, MIT

# Sharpening



original

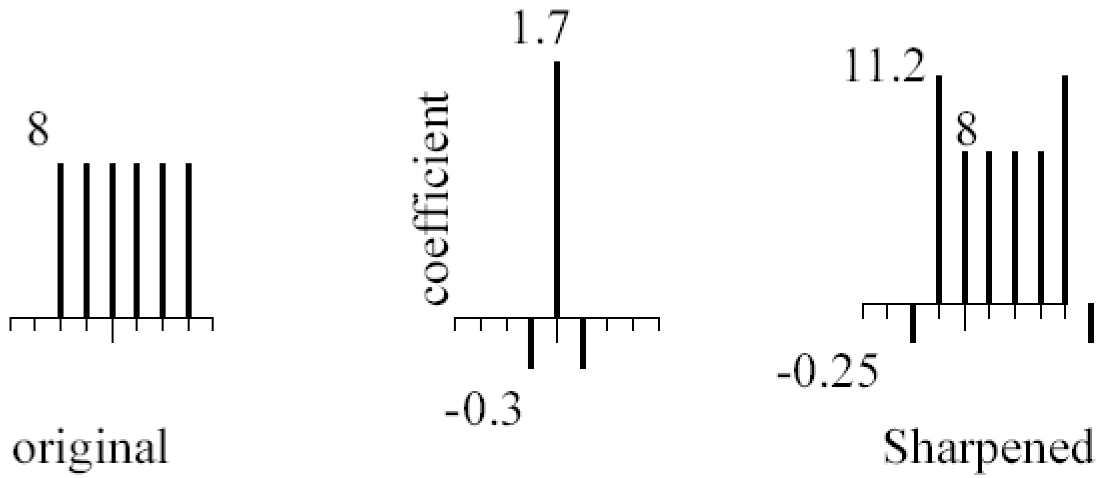


Sharpened
original

adapted from Darrell and Freeman, MIT

# Sharpening



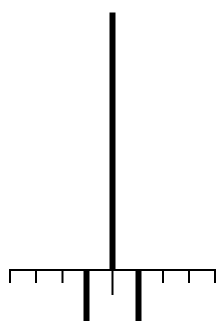

Sharpened
(differences are accentuated; constant areas are left untouched).

adapted from Darrell and Freeman, MIT

# Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



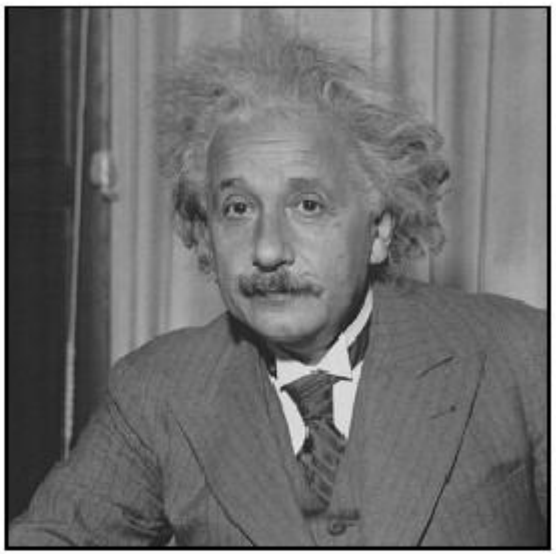
Sharpening filter

- Accentuates differences with local average

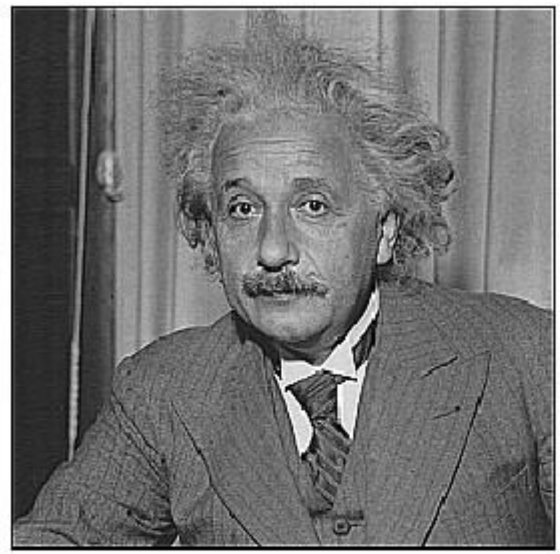
Source: Darrell, Berkeley

Source: D. Lowe

# Filtering examples: sharpening



before



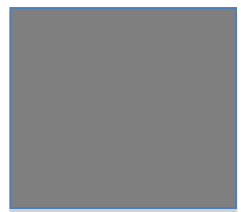
after

Source: Darrell, Berkeley

# Rectangular filter

g[m,n]

$\otimes$



h[m,n]

=

f[m,n]

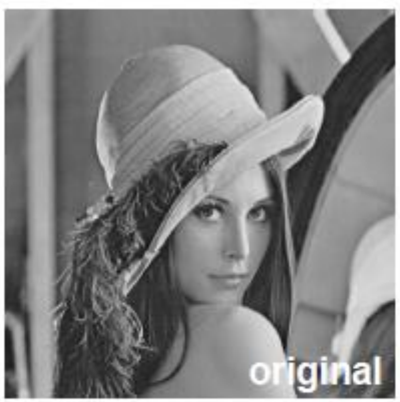
Source: Torralba, MIT

# What does blurring take away?


–

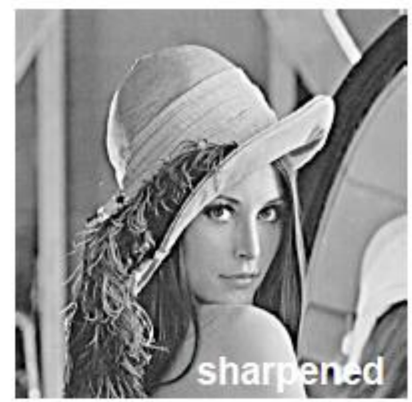

=


- Let’s add it back:




+ a


=




Source: Fei Fei Li, Stanford

# Rectangular filter

$\otimes$

$h[m,n]$

=

$f[m,n]$

$g[m,n]$

Source: Torralba, MIT

# Rectangular filter

$\otimes$

h[m,n]

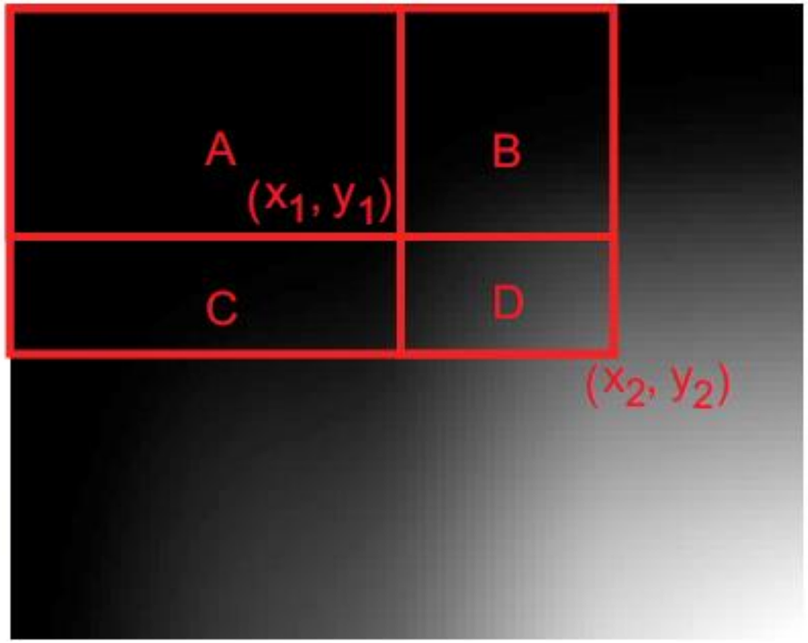
=

f[m,n]

g[m,n]

Source: Torralba, MIT

# Integral image




Source: Torralba, MIT

# Shift invariant linear system

- **Shift invariant:**
  - Operator behaves the same everywhere, i.e. the value of the output depends on the pattern in the image neighborhood, not the position of the neighborhood.

- **Linear:**
  - Superposition: $h * (f1 + f2) = (h * f1) + (h * f2)$
  - Scaling: $h * (k f) = k (h * f)$

Source: Darrell, Berkeley

# Properties of convolution

- Linear & shift invariant
- Commutative:

  f * g = g * f
- Associative

  (f * g) * h = f * (g * h)
- Identity:

  unit impulse e = [..., 0, 0, 1, 0, 0, ...].  f * e = f
- Differentiation:

$$\frac{\partial}{\partial x}(f * g) = \frac{\partial f}{\partial x} * g$$

Source: Darrell, Berkeley

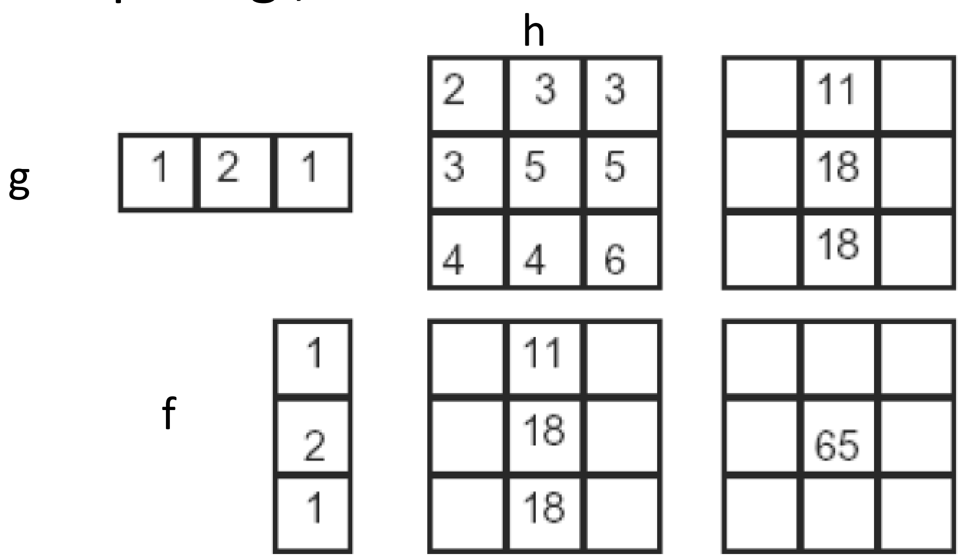
# Separability

- In some cases, filter is separable, and we can factor into two steps:
  - Convolve all rows
  - Convolve all columns

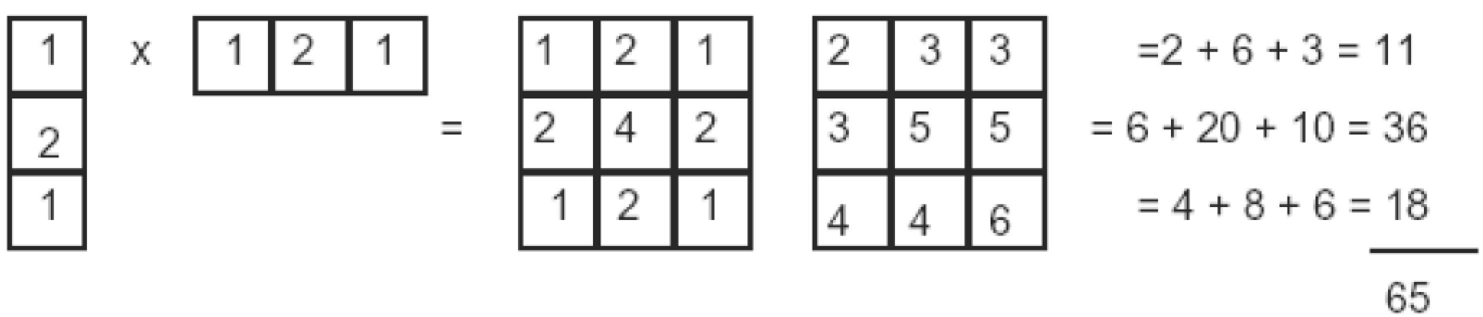
Source: Darrell, Berkeley

# Separability

- In some cases, filter is separable, and we can factor into two steps: e.g.,




What is the computational complexity advantage for a separable filter of size k x k, in terms of number of operations per output pixel?




Source: Darrell, Berkeley $f * (g * h) = (f * g) * h$

# Advantages of separability

First convolve the image with a one dimensional horizontal filter

Then convolve the result of the first convolution with a one dimensional vertical filter

For a kxk Gaussian filter, 2D convolution requires $k^2$ operations per pixel

But using the separable filters, we reduce this to 2k operations per pixel.

adapted from Larry Davis, University of Maryland

# Seperable Gaussian

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-x^2 / (2\sigma^2))$$

$$g(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-y^2 / (2\sigma^2))$$

Product?

$$g(x, y) = \frac{1}{2\pi\sigma^2} \exp(-(x^2 + y^2) / (2\sigma^2))$$

# Advantages of Gaussians

- Convolution of a Gaussian with itself is another Gaussian
  - so we can first smooth an image with a small Gaussian
  - then, we convolve that smoothed image with another small Gaussian and the result is equivalent to smoother the original image with a larger Gaussian.
  - If we smooth an image with a Gaussian having sd $\sigma$ twice, then we get the same result as smoothing the image with a  Gaussian having standard deviation $(2\sigma)^{1/2}$

adapted from Larry Davis, University of Maryland

# Effect of smoothing filters

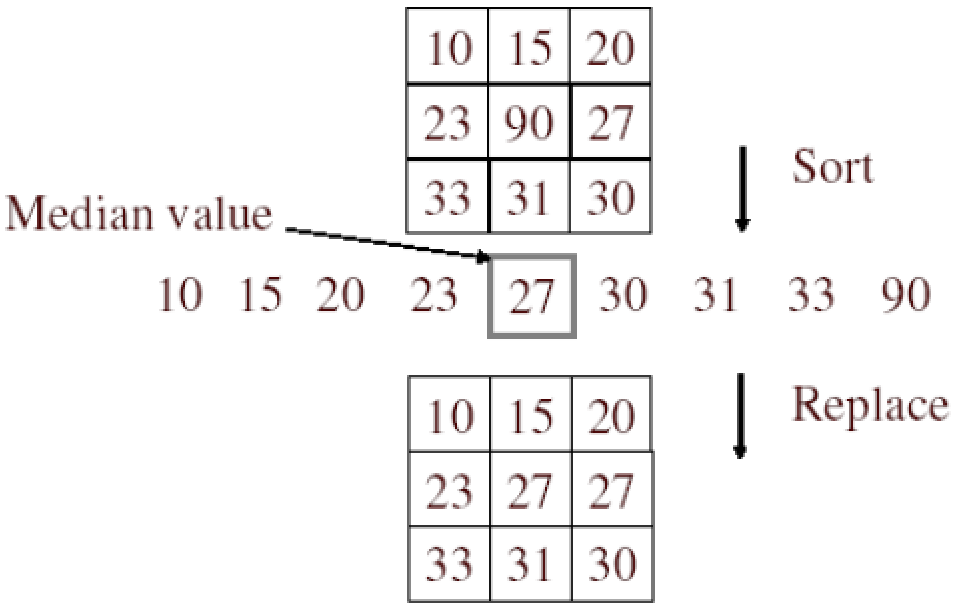
5x5

Additive Gaussian noise

Salt and pepper noise

Source: Darrell, Berkeley

# Median filter

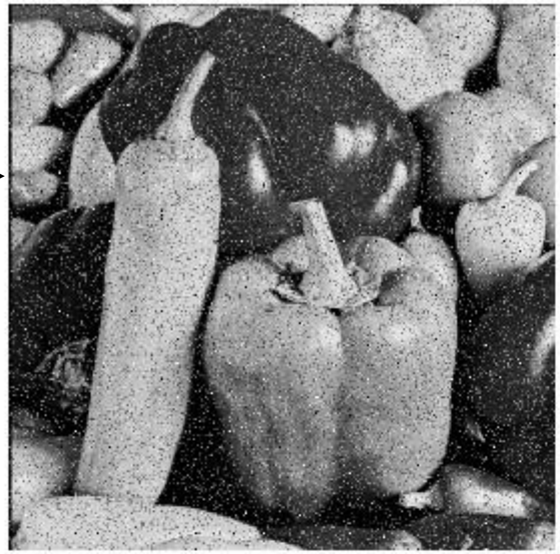



- No new pixel values introduced
- Removes spikes: good for impulse, salt & pepper noise

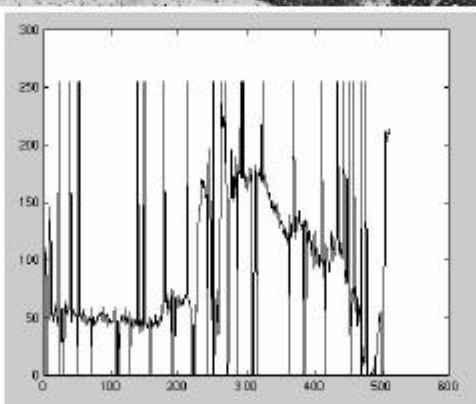
Source: Darrell, Berkeley

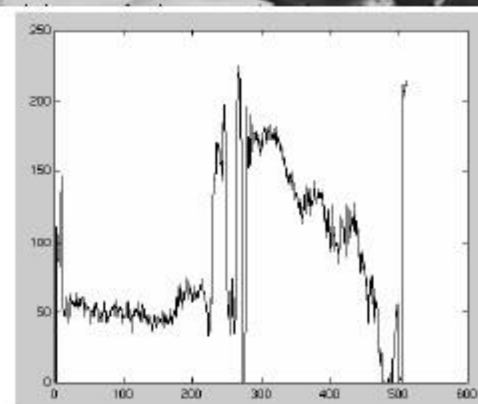
# Median filter

Salt and pepper noise



Median filtered



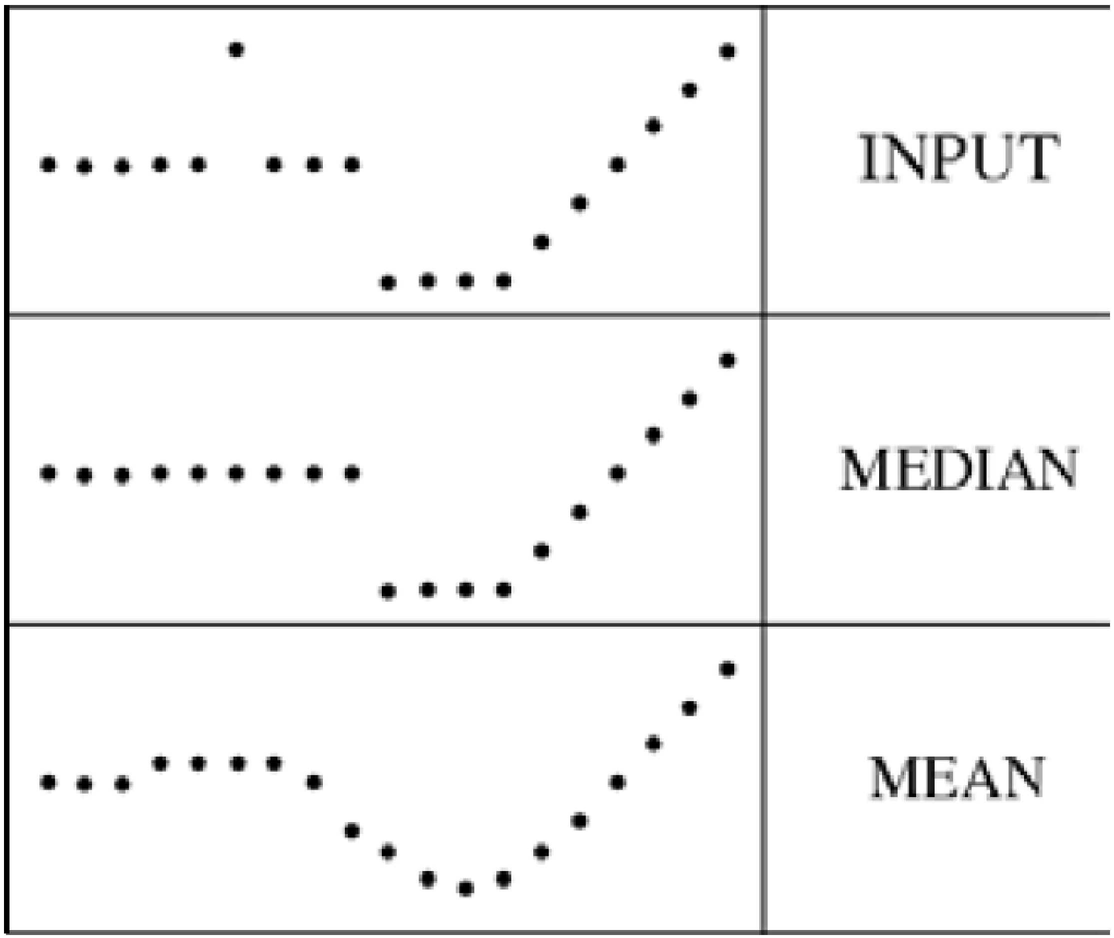

Plots of a row of the image

Source: Darrell, Berkeley

Source: M. Hebert

# Median filter

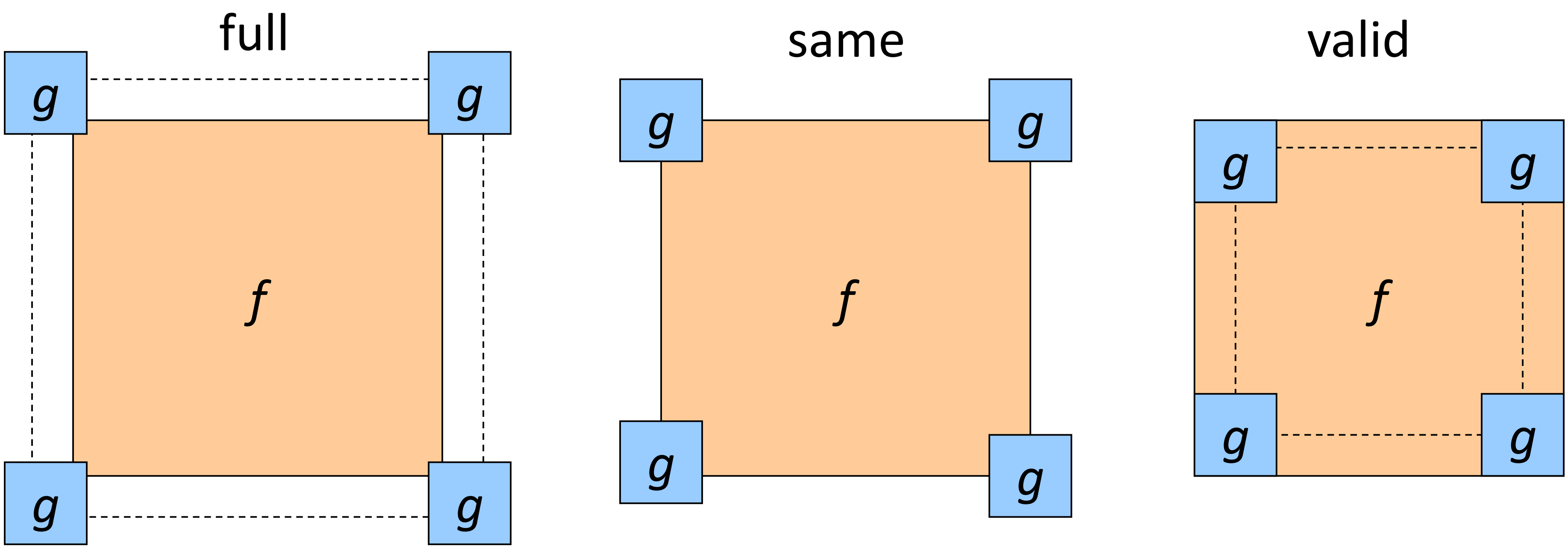
- Median filter is edge preserving




Source: Darrell, Berkeley

# Boundary issues

- What is the size of the output?
- MATLAB: filter2(g, f, *shape*)
  - *shape* = ‘full’: output size is sum of sizes of f and g
  - *shape* = ‘same’: output size is same as f
  - *shape* = ‘valid’: output size is difference of sizes of f and g

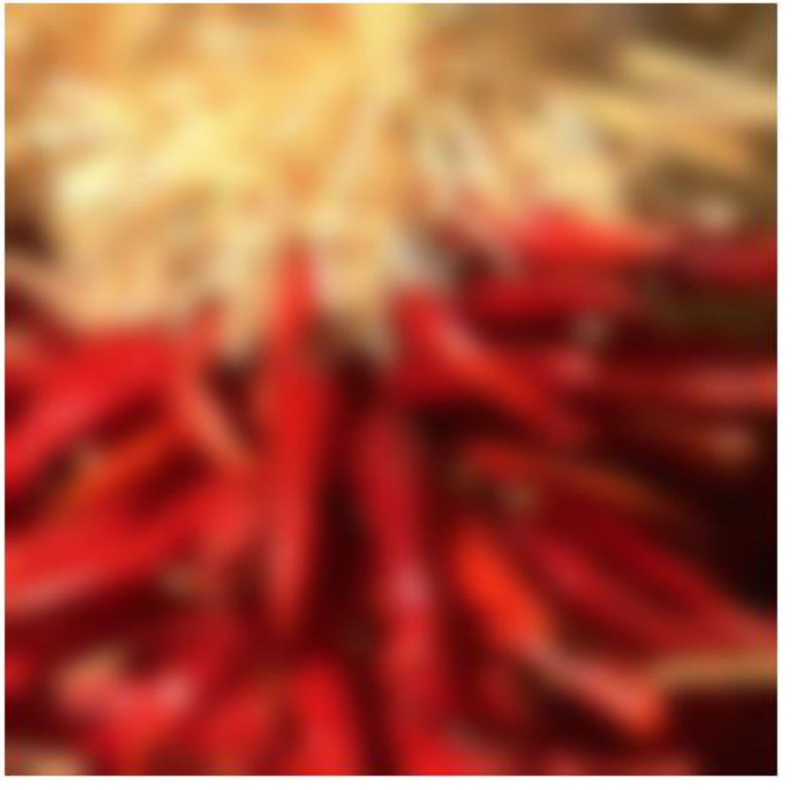



Source: Darrell, Berkeley

Source: S. Lazebnik

# Boundary issues

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge



Source: Darrell, Berkeley

Source: S. Marschner

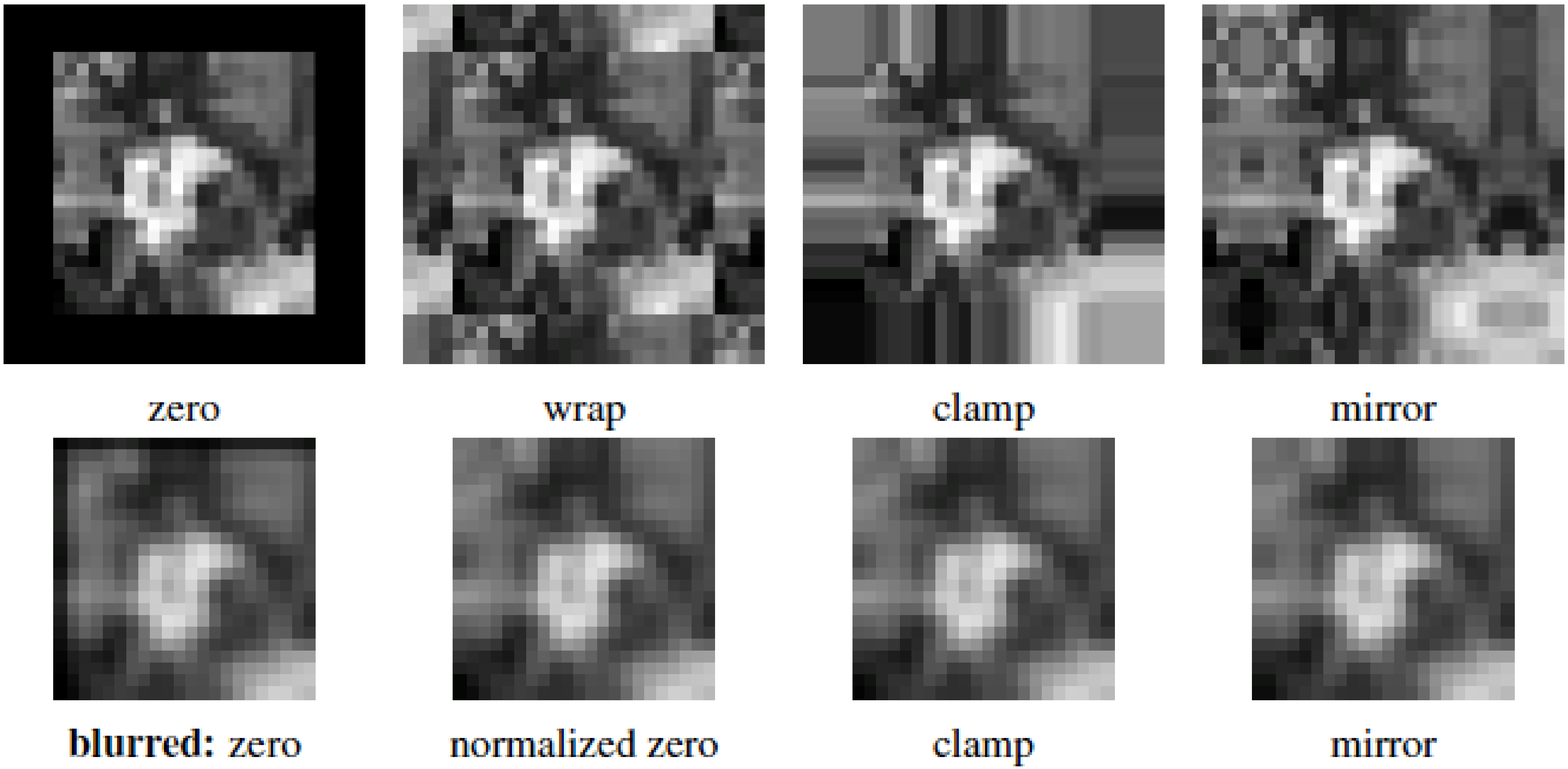
# Boundary issues

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods (MATLAB):
    - clip filter (black): imfilter(f, g, 0)
    - wrap around: imfilter(f, g, ‘circular’)
    - copy edge: imfilter(f, g, ‘replicate’)
    - reflect across edge: imfilter(f, g, ‘symmetric’)

Source: Darrell, Berkeley

Source: S. Marschner

# Borders




Source: Torralba, MIT

From Rick's book

# Today’s topics

- Image Formation

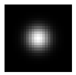
- Image filters in spatial domain
  - Filter is a mathematical operation of a grid of numbers
  - Smoothing, sharpening, measuring texture

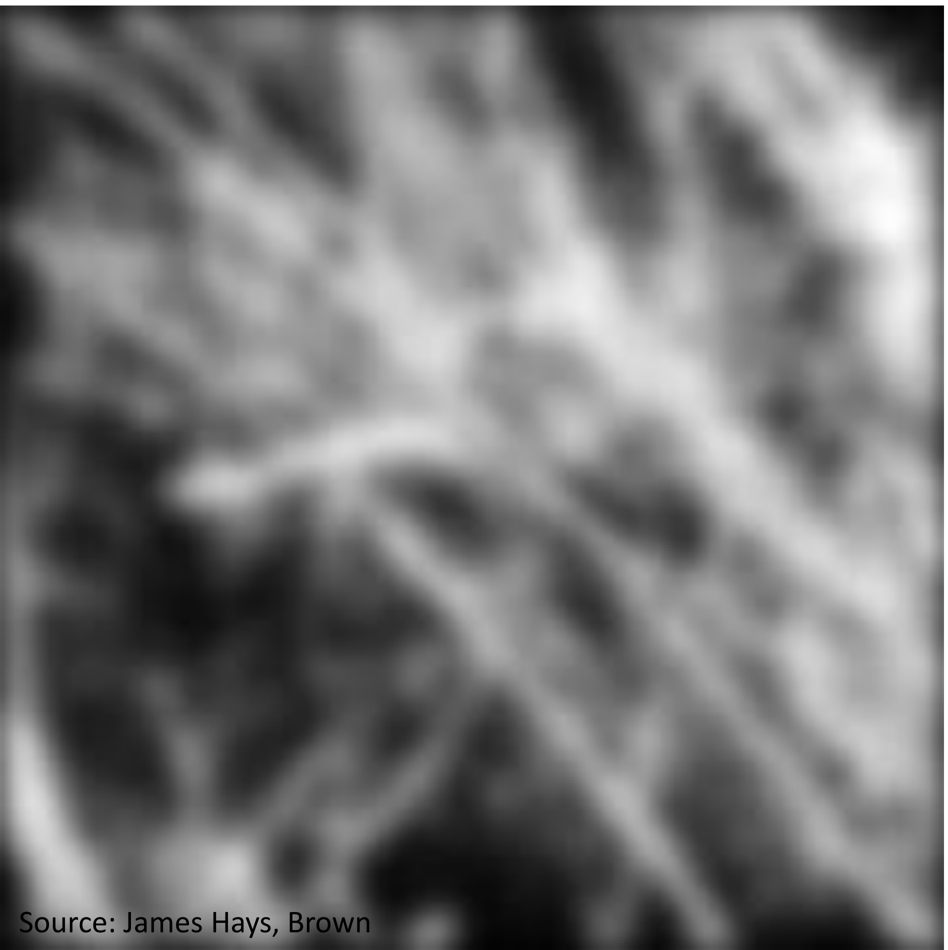
- Image filters in the frequency domain
  - Filtering is a way to modify the frequencies of images
  - Denoising, sampling, image compression

- Templates and Image Pyramids
  - Filtering is a way to match a template to the image
  - Detection, coarse-to-fine registration

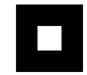
# Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

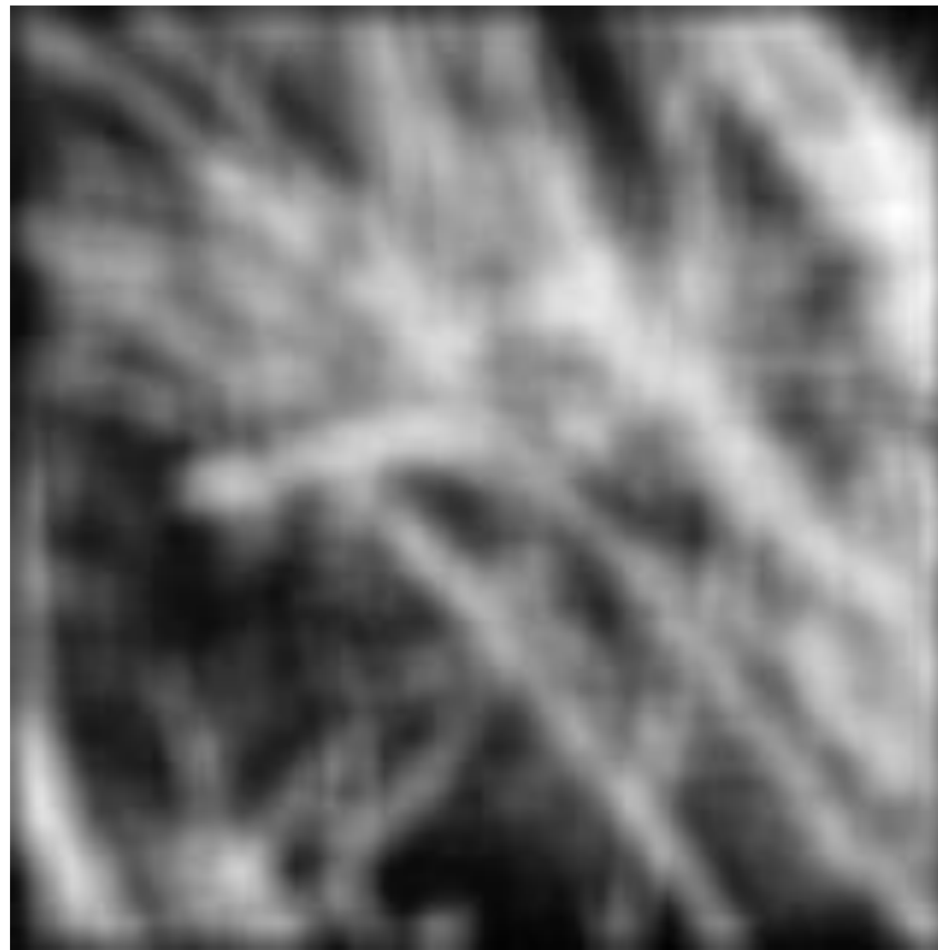
Gaussian





Box filter





Source: James Hays, Brown

# Why does a lower resolution image still make sense to us? What do we lose?

Source: James Hays, Brown

Image: http://www.flickr.com/photos/igorms/136916757/

Slide: Hoiem

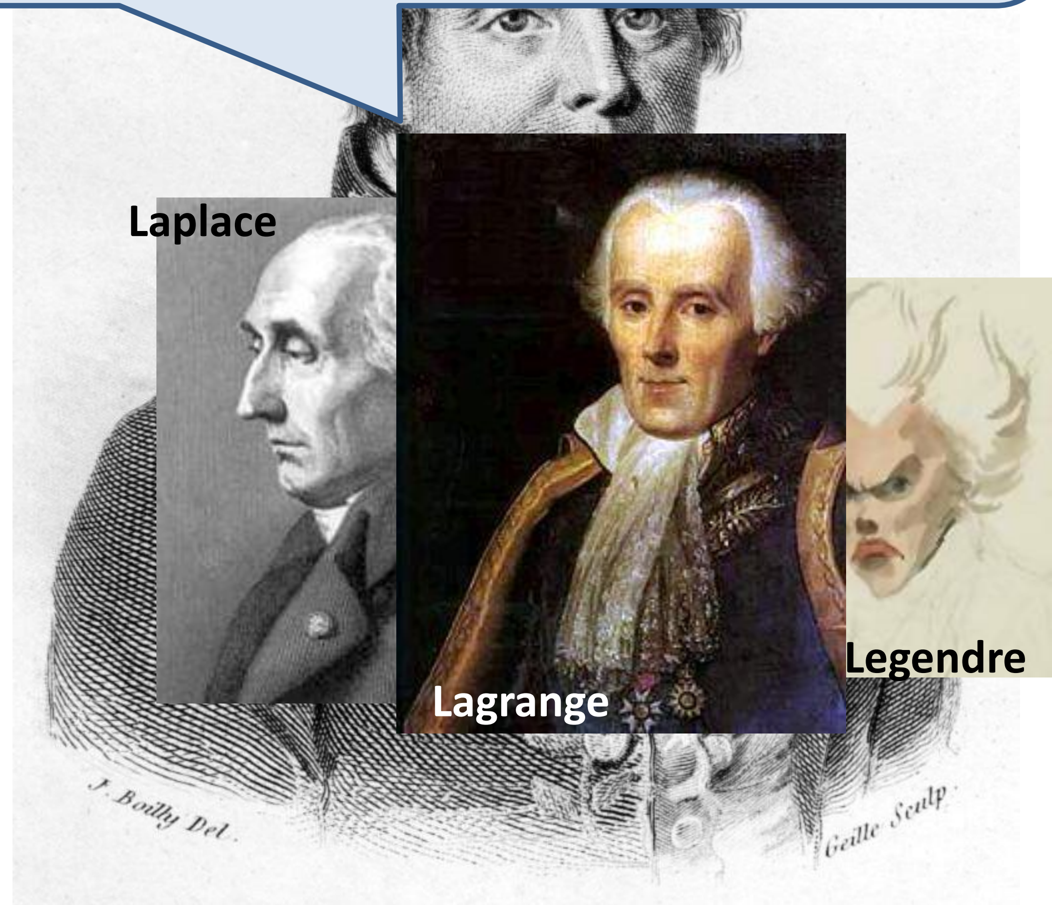
# Jean Baptiste Joseph Fourier (1768-1830)

had crazy idea (1807):

> ***Any** univariate function can rewritten as a weighted sum sines and cosines of different frequencies.*

*...the manner in which the author arrives at these equations is not exempt of difficulties and...his analysis to integrate them still leaves something to be desired on the score of generality and even rigour.*

- Don't believe it?
  - Neither did Lagrange, Laplace, Poisson and other big wigs
  - Not translated into English until 1878!
- But it's (mostly) true!
  - called Fourier Series
  - there are some subtle restrictions



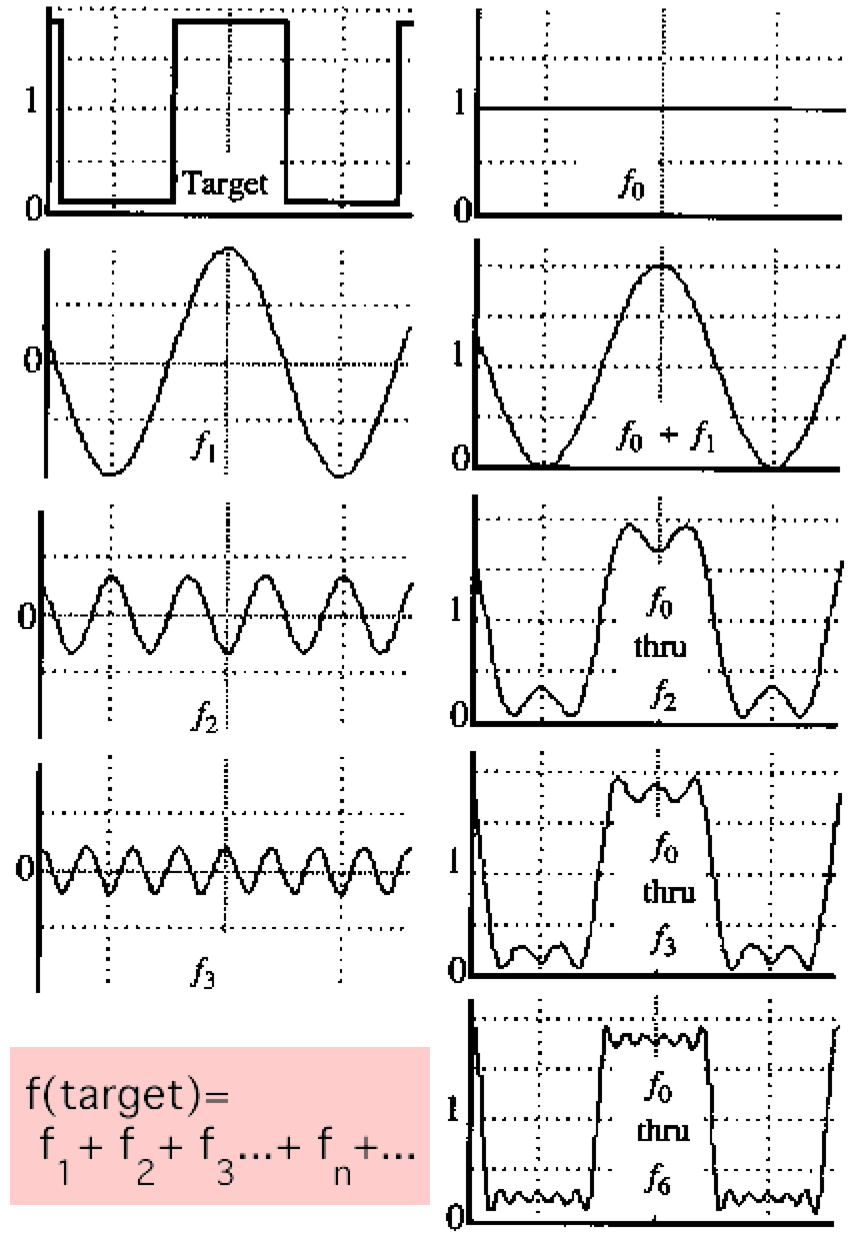

Source: James Hays, Brown

# A sum of sines

Our building block:

$$A\sin(\omega x+\phi)$$

Add enough of them to get any signal *f(x)* you want!

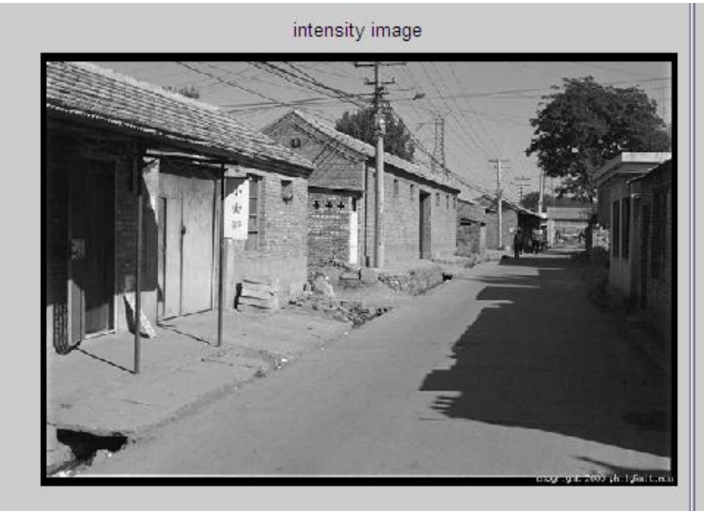



f(target)=
$f_1 + f_2 + f_3 ... + f_n + ...$

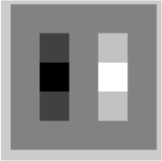
Source: James Hays, Brown

# Filtering in spatial dom

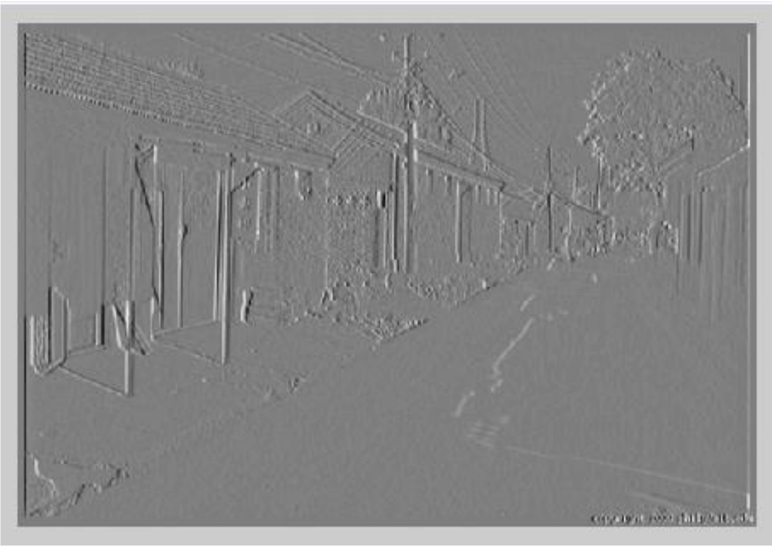
| | | |
|---|---|---|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |




*

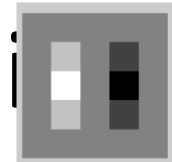


=

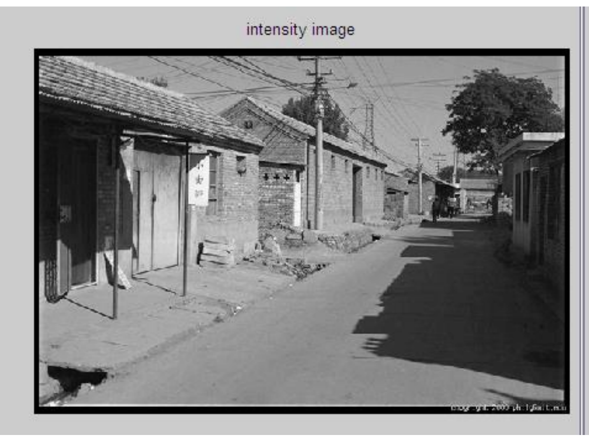


Source: James Hays, Brown

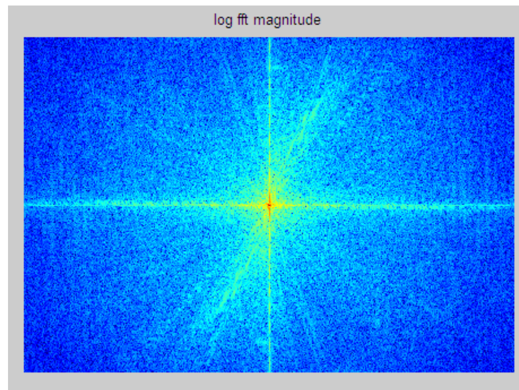
# Filtering in frequency domai

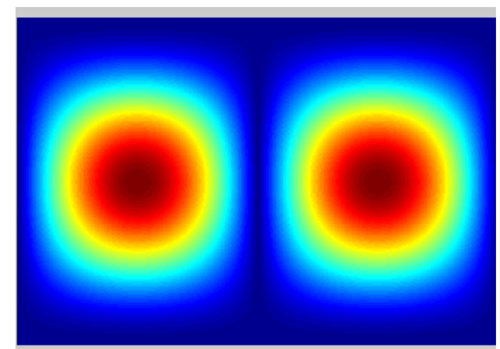


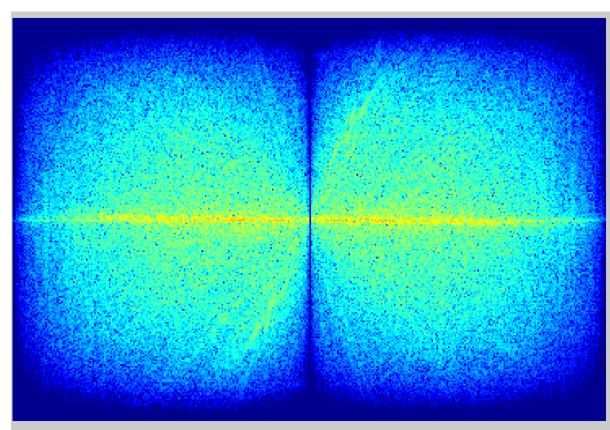
FFT



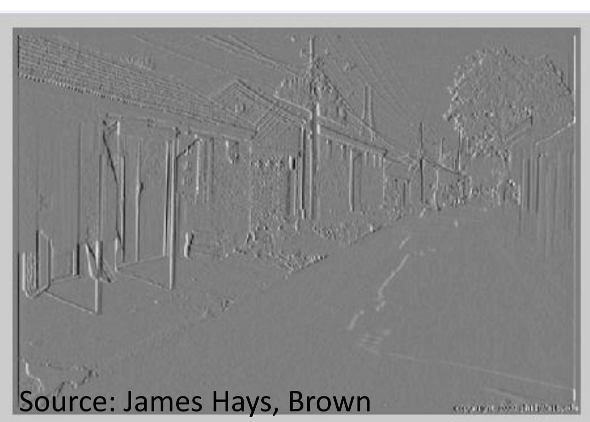

FFT





Inverse FFT

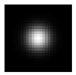


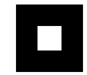
Source: James Hays, Brown

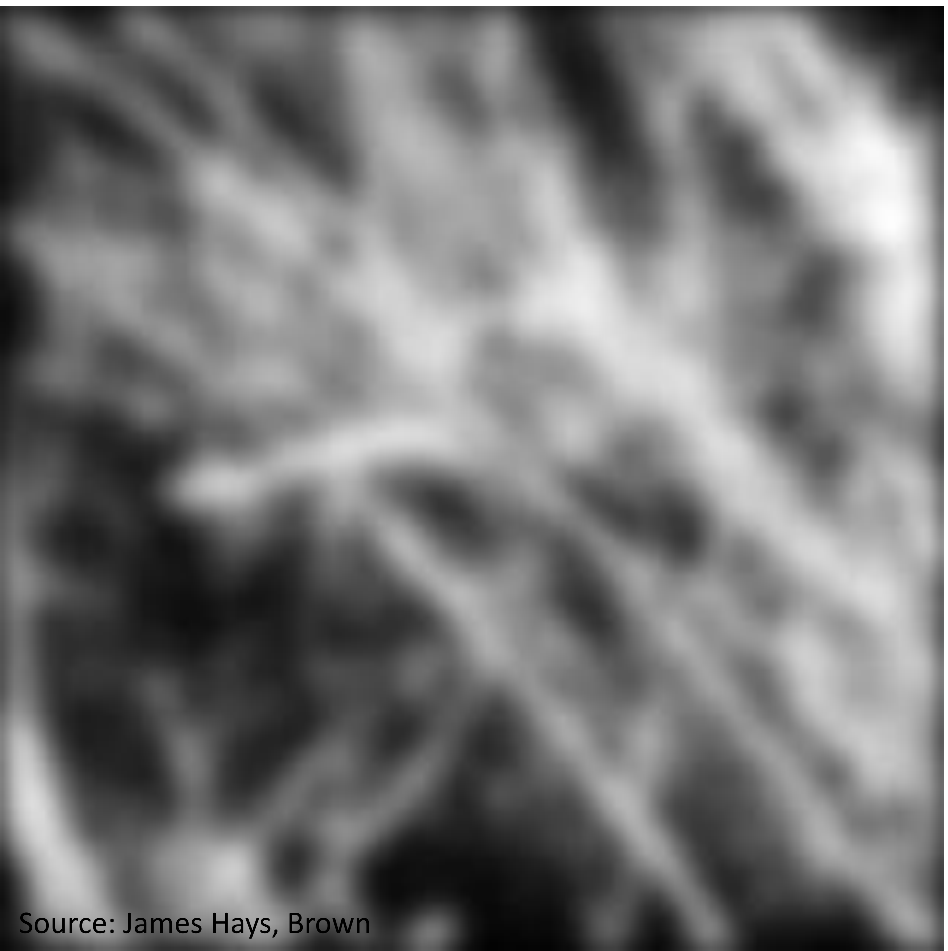
Slide: Hoiem

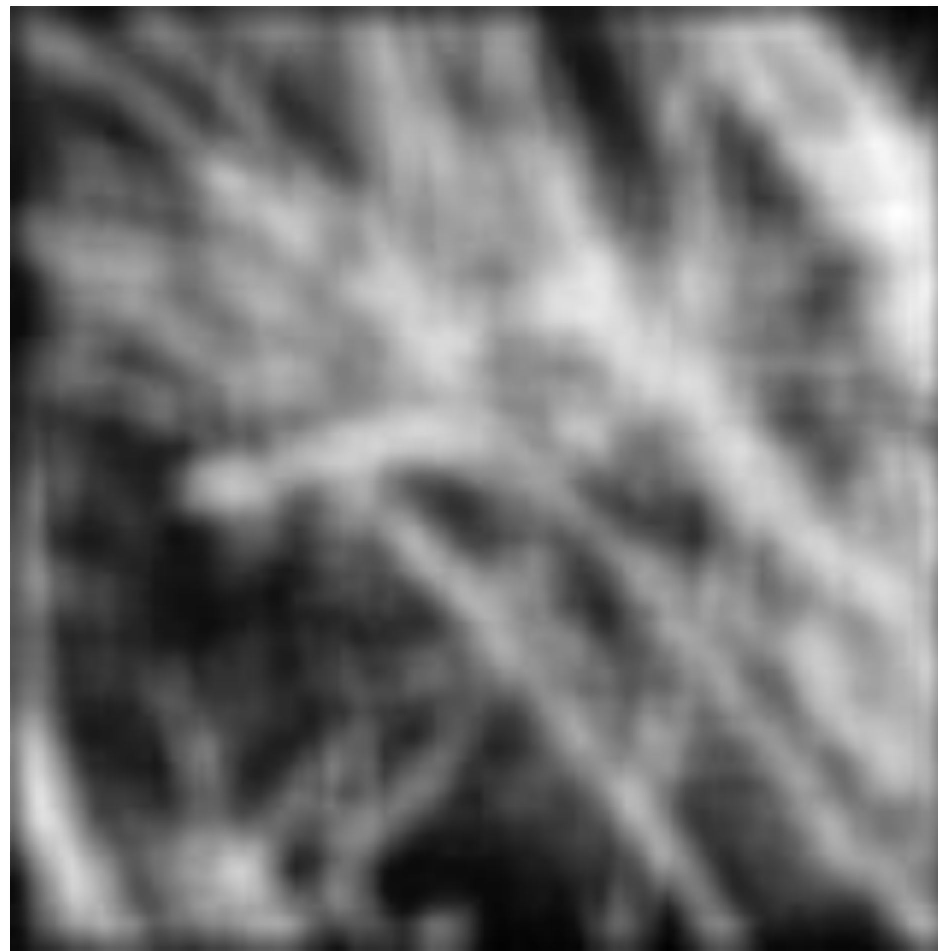
# Filtering

## Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

Gaussian 

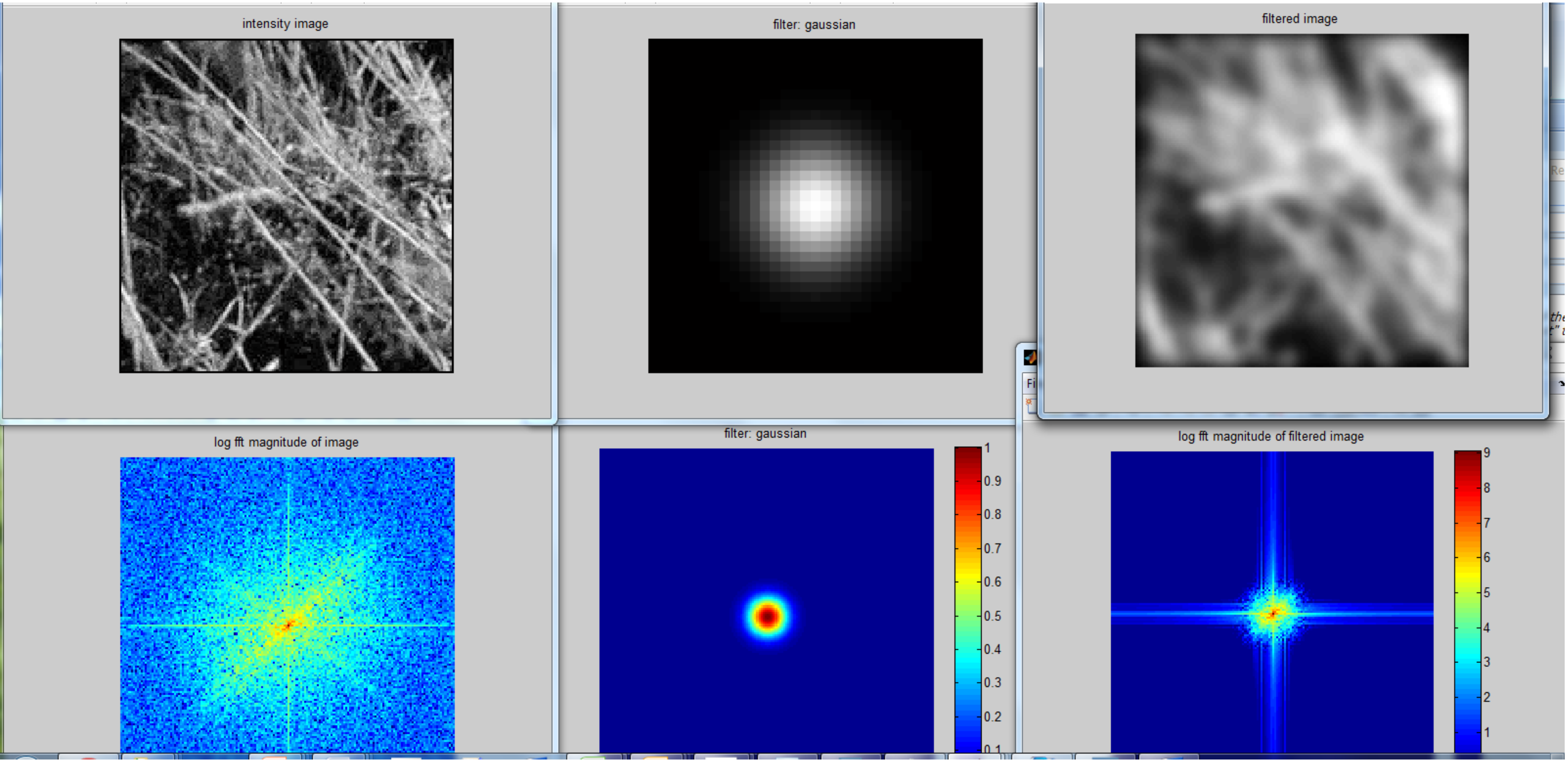
Box filter 





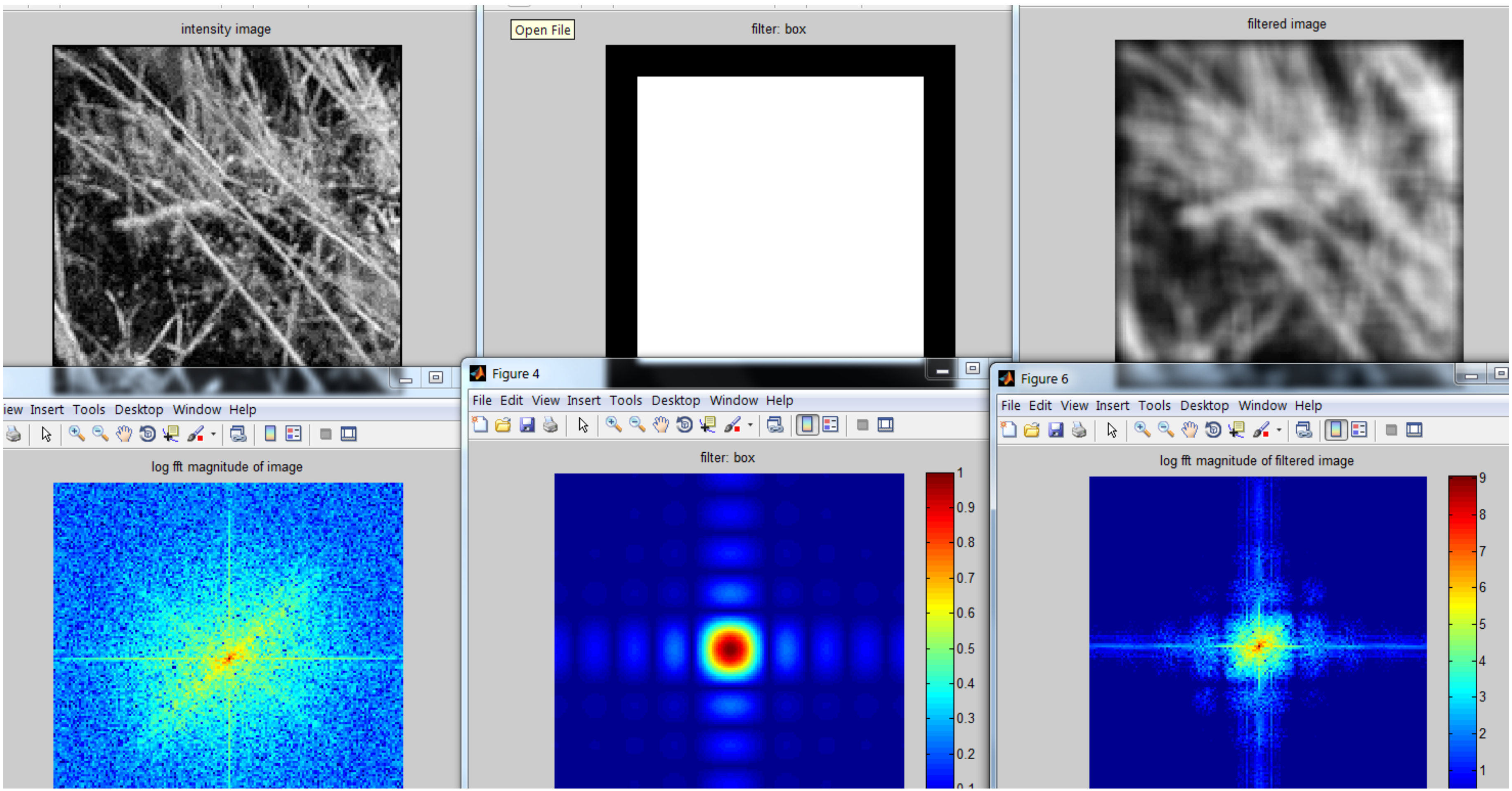
Source: James Hays, Brown

# Gaussian



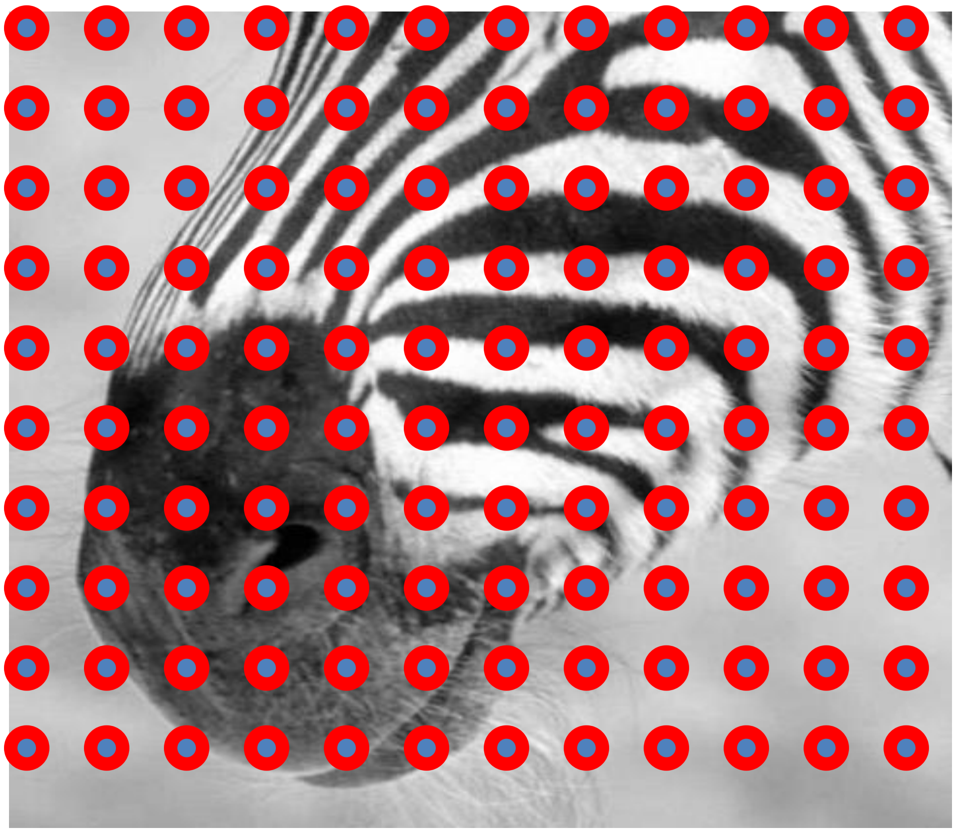

Source: James Hays, Brown

# Box Filter




Source: James Hays, Brown

# Subsampling by a factor of 2

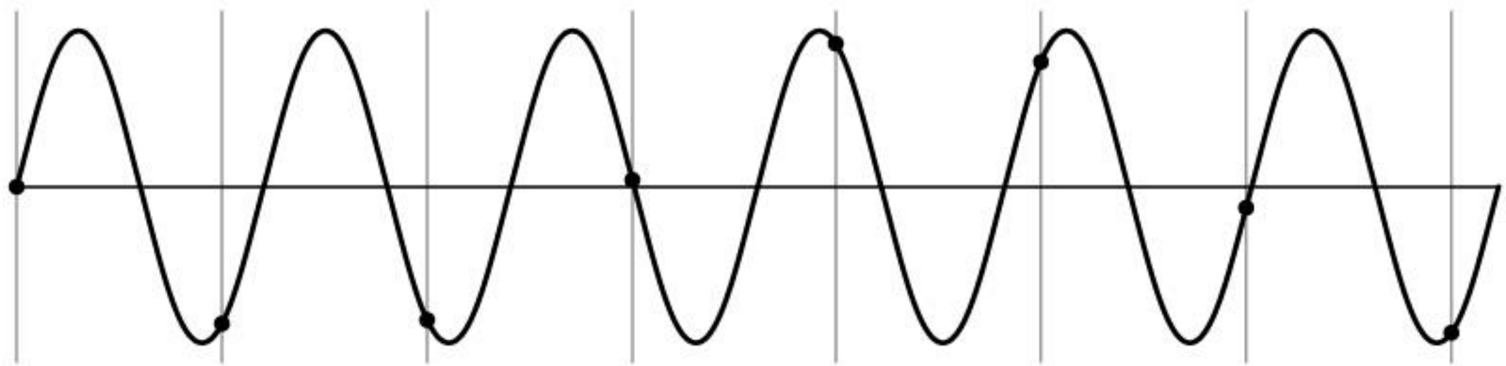


Throw away every other row and column to create a 1/2 size image

Source: James Hays, Brown

# Aliasing problem

- 1D example (sinewave):

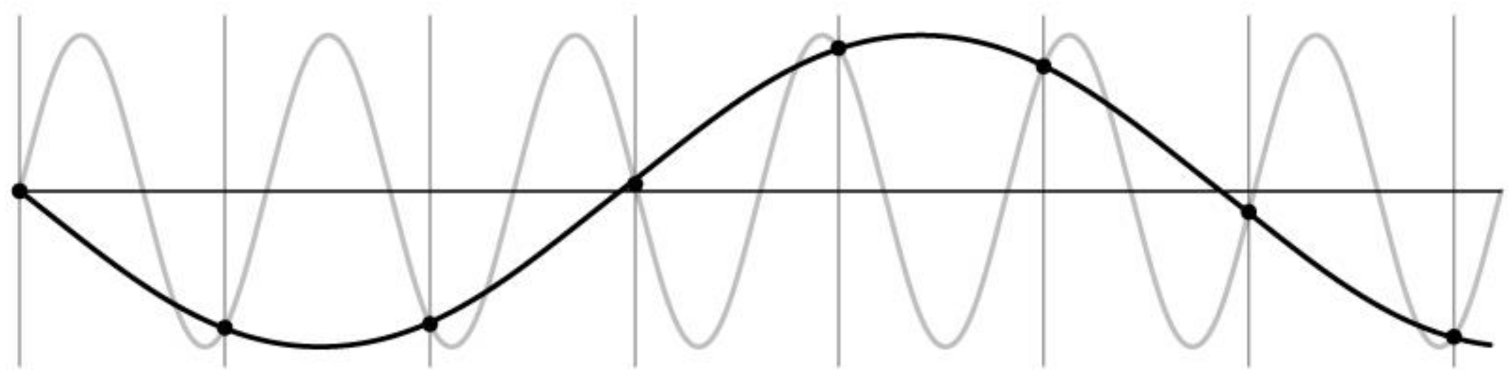


Source: S. Marschner

Source: James Hays, Brown

# Aliasing problem

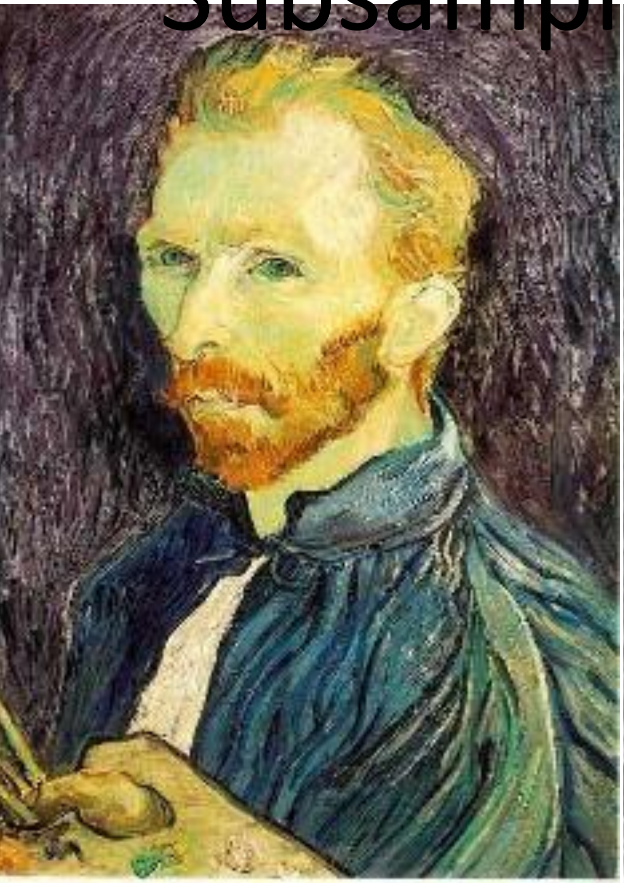
- 1D example (sinewave):



Source: S. Marschner

Source: James Hays, Brown

# Subsampling without pre-filtering



1/2

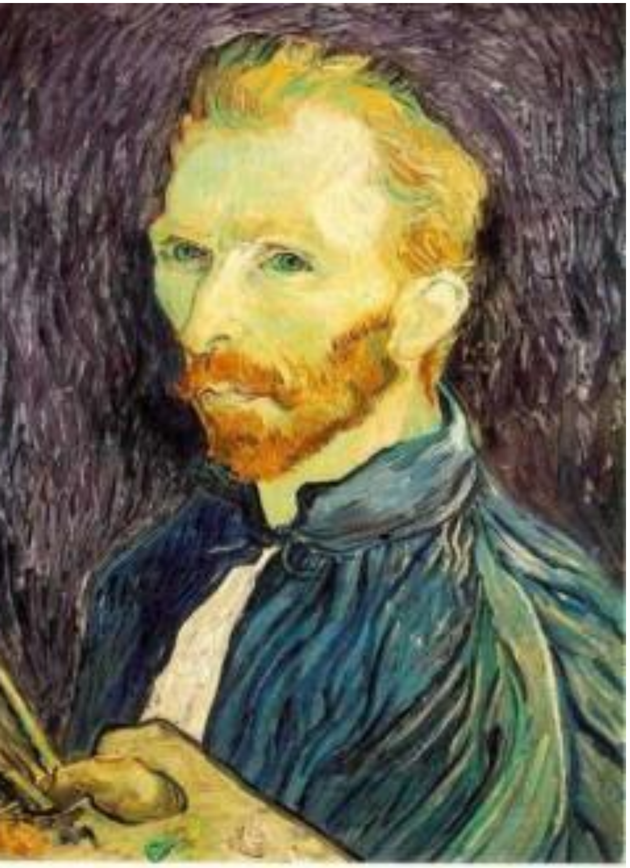
1/4 (2x zoom)

1/8 (4x zoom)

Slide by Steve Seitz

Source: James Hays, Brown

# Subsampling with Gaussian pre-filtering



Gaussian 1/2

G 1/4

G 1/8

Slide by Steve Seitz

Source: James Hays, Brown

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  - Smoothing, sharpening, measuring texture
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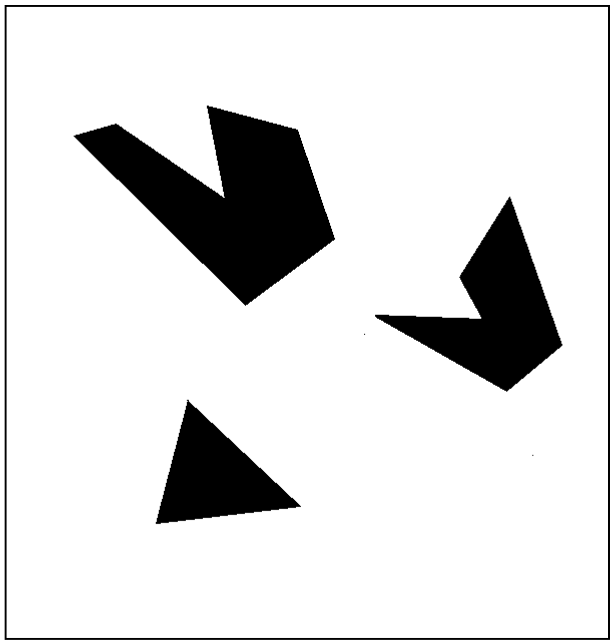
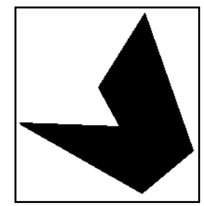
  - Denoising, sampling, image compression
- Templates and Image Pyramids
  - Filtering is a way to match a template to the image
  - Detection, coarse-to-fine registration

# Template matching



Scene

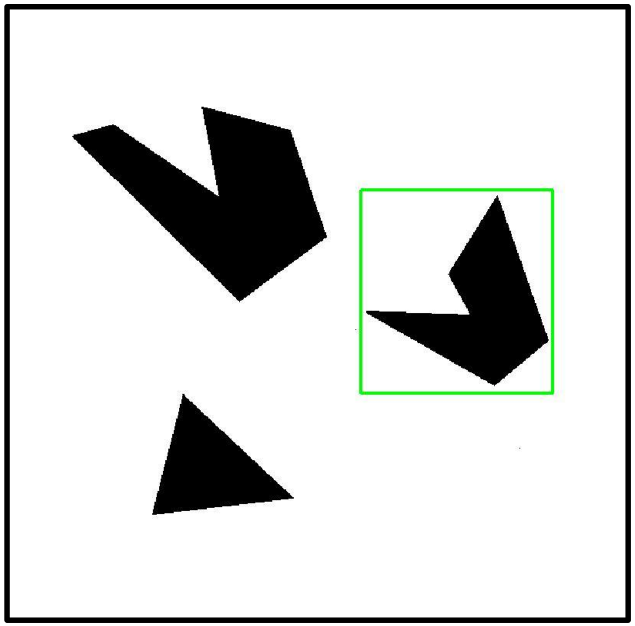


Template (mask)

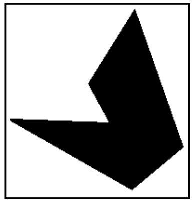
A toy example

Source: Darrell, Berkeley

# Template matching



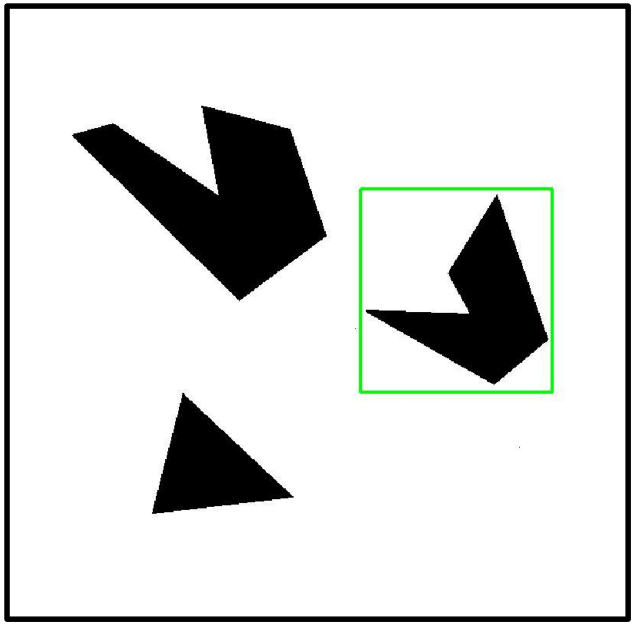
Detected template



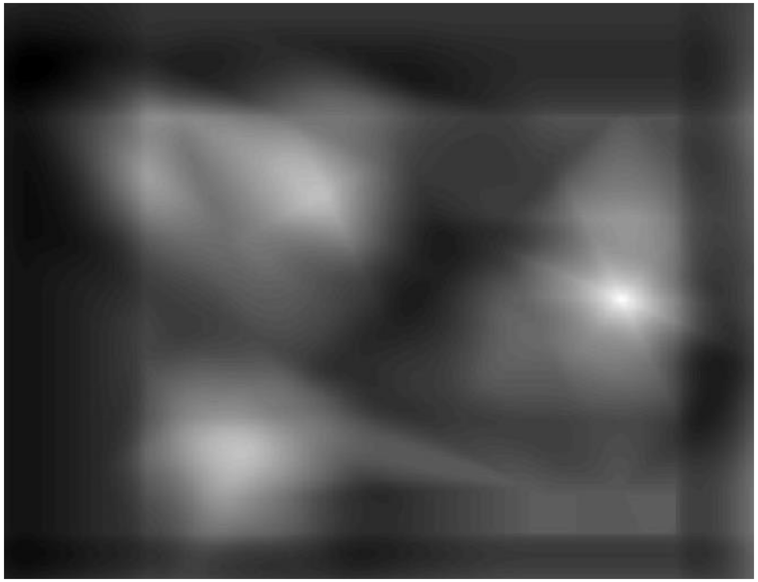
Template

Source: Darrell, Berkeley

# Template matching



Detected template

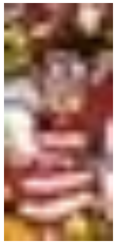


Correlation map

Source: Darrell, Berkeley

# Where's Waldo?

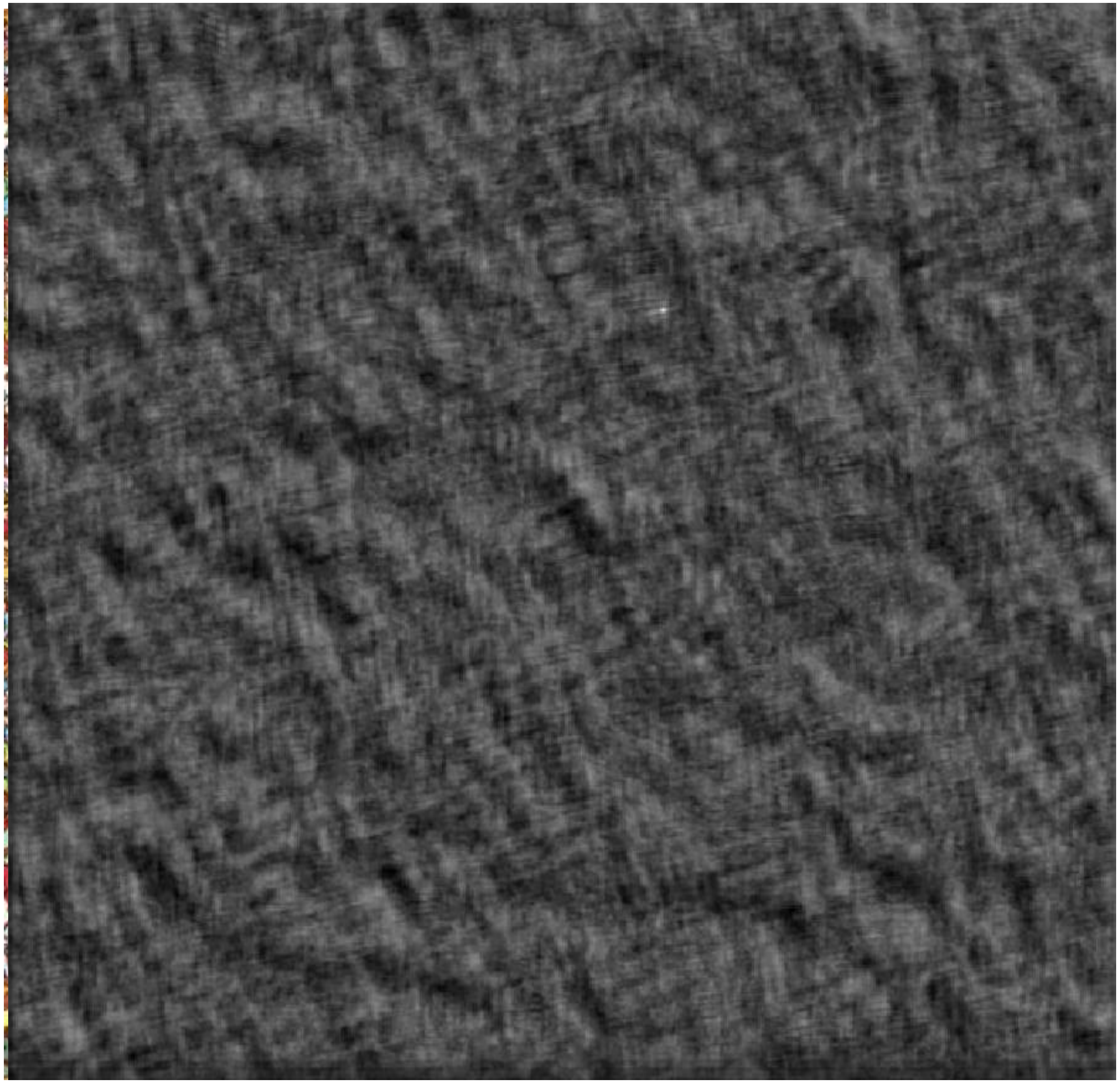
Scene



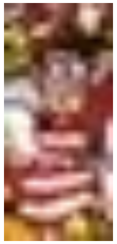
Template

Source: Darrell, Berkeley

# Where’s Waldo?



Scene

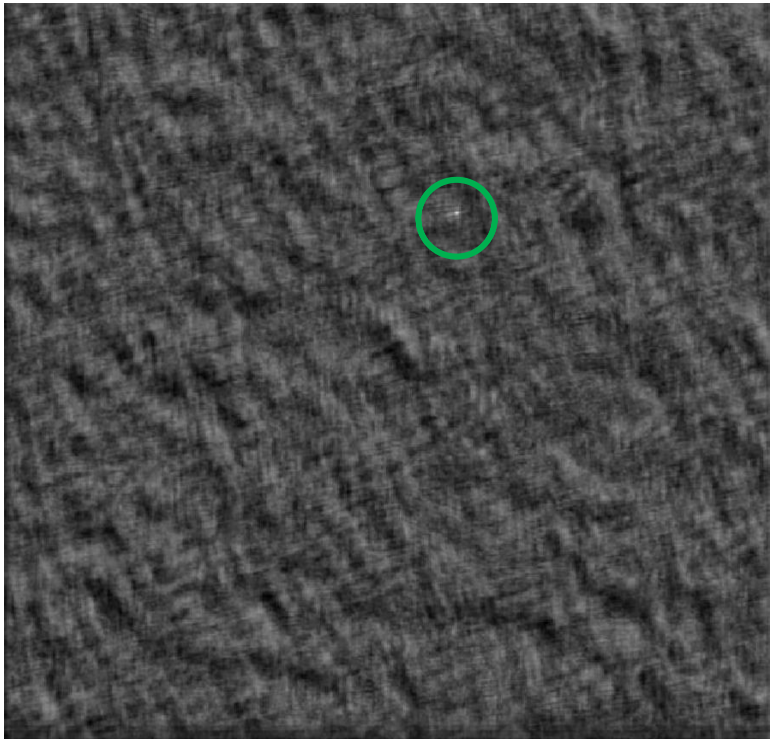


Template

Source: Darrell, Berkeley

# Where’s Waldo?

Detected template



Correlation map

Source: Darrell, Berkeley

# Template matching

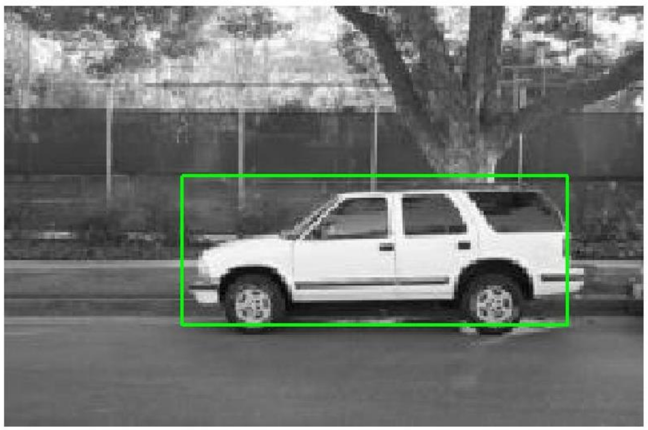
Scene

Template

What if the template is not identical to some subimage in the scene?

Source: Darrell, Berkeley

# Template matching



Detected template

Template

Match can be meaningful, if scale, orientation, and general appearance is right.

Source: Darrell, Berkeley

# Application

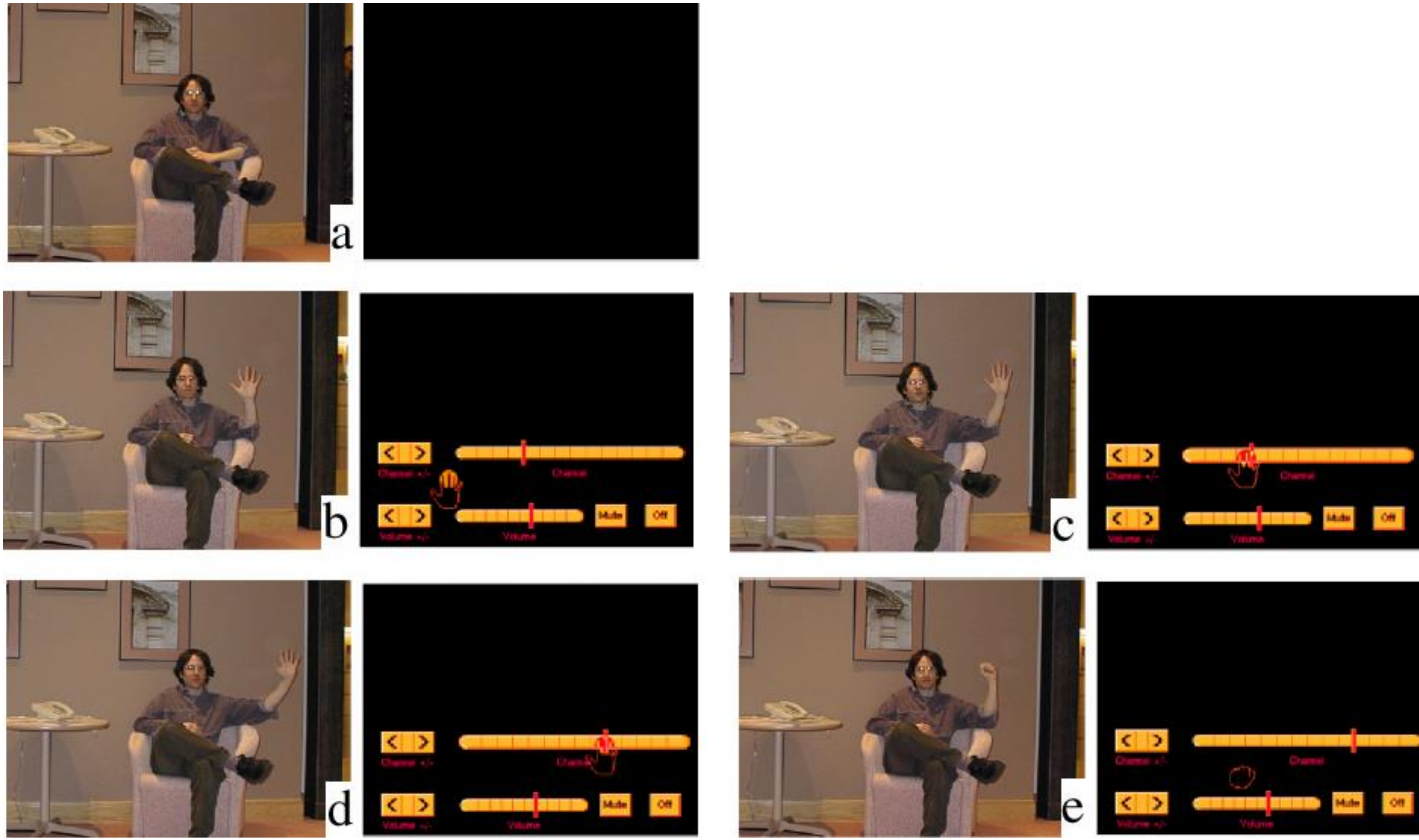
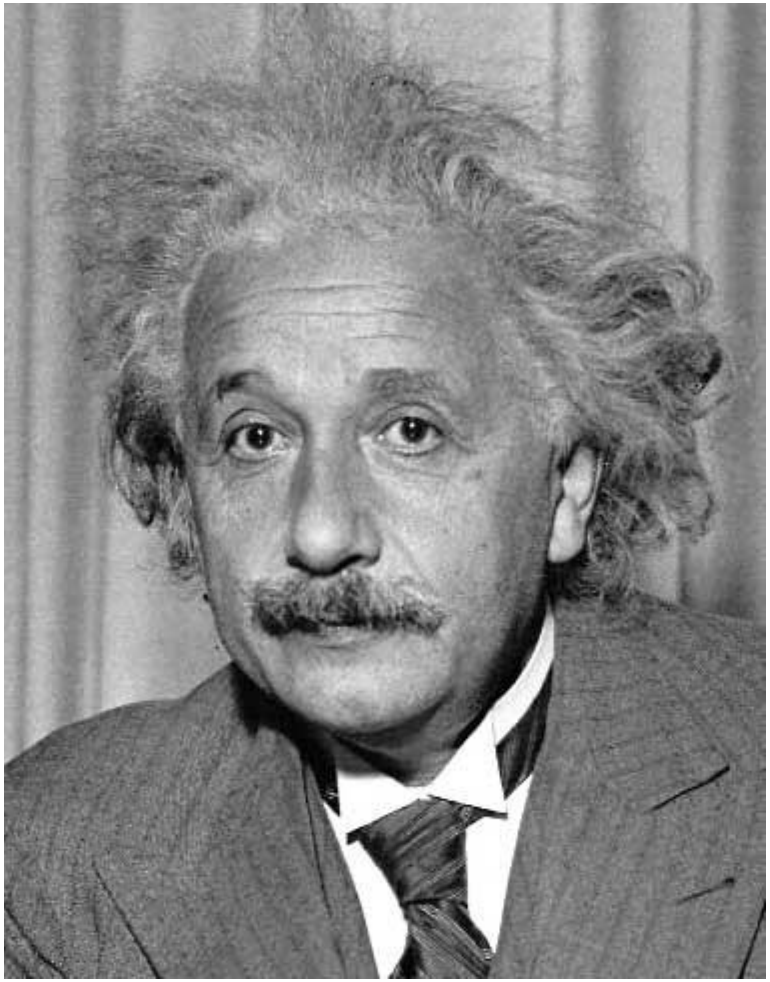



Figure from “Computer Vision for Interactive Computer Graphics,” W.Freeman et al, IEEE Computer Graphics and Applications, 1998

# Template matching

- Goal: find  in image

- Main challenge: What is a good similarity or distance measure between two patches?
  - Correlation
  - Zero-mean correlation
  - Sum Square Difference
  - Normalized Cross Correlation

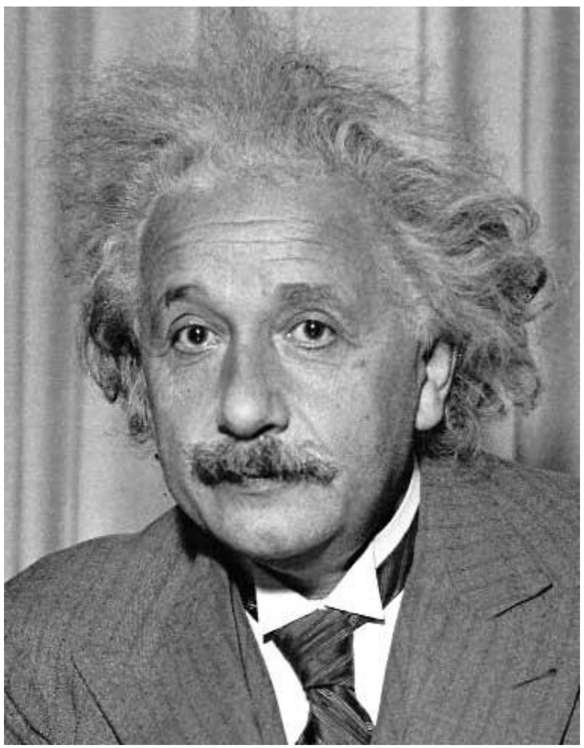


# Matching with filters

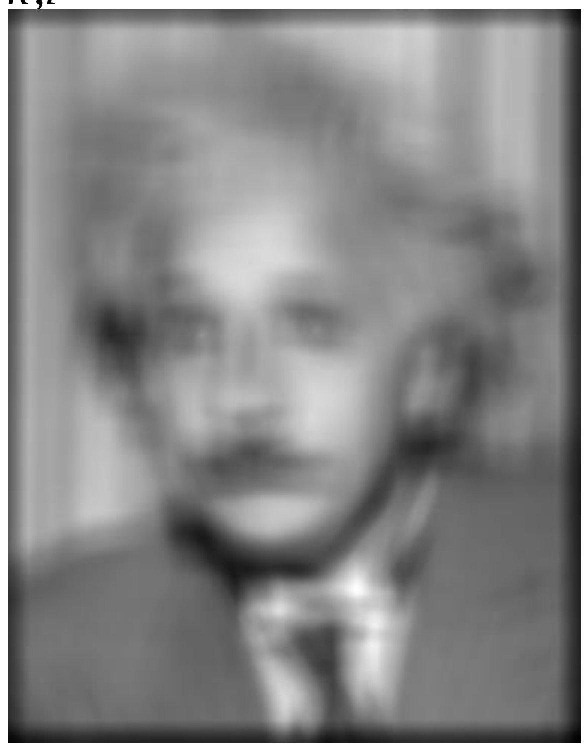
- Goal: find  in image
- Method 0: filter the image with eye patch

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

f = image
g = filter



Input



Filtered Image

What went wrong?

response is stronger for higher intensity

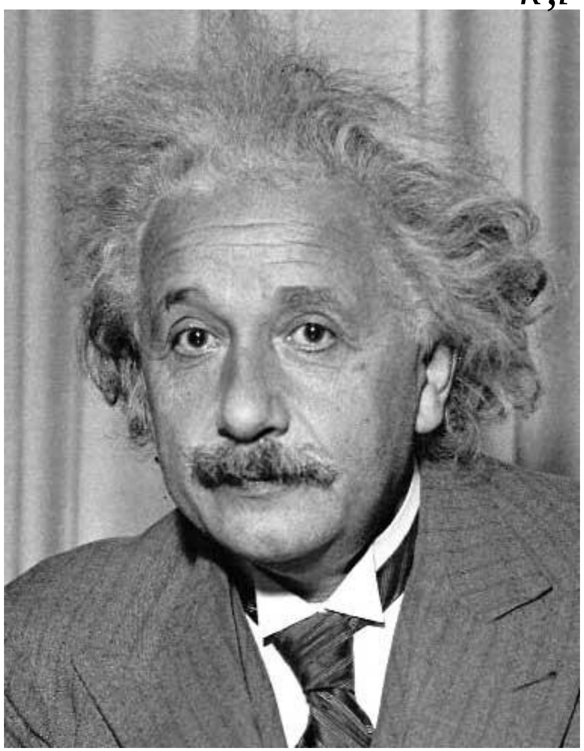
Source: Hays, Brown

# Matching with filters

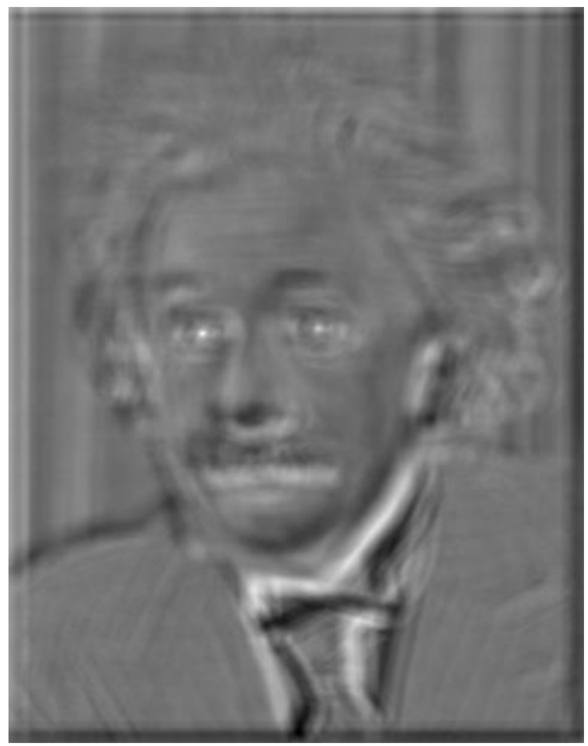
- Goal: find  in image
- Method 1: filter the image with zero-mean eye

$$h[m,n]=\sum_{k,l}(f[k,l]-\bar{f})(g[m+k,n+l])$$

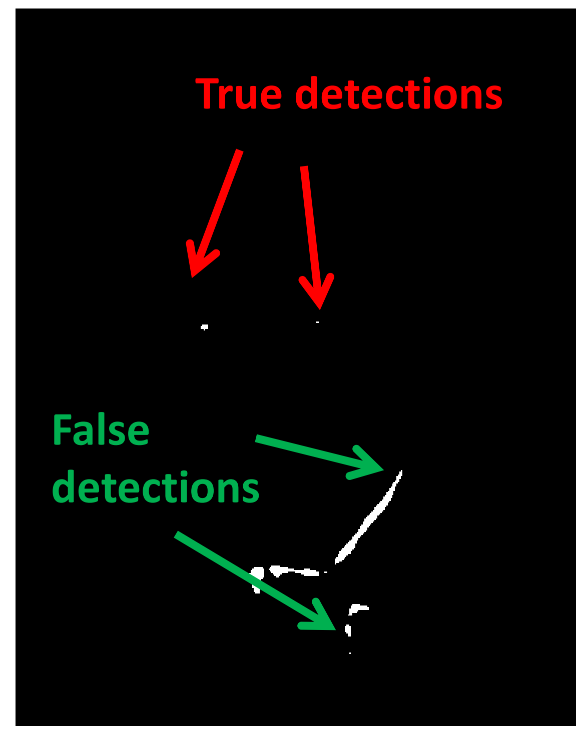
mean of f



Input



Filtered Image (scaled)




Thresholded Image

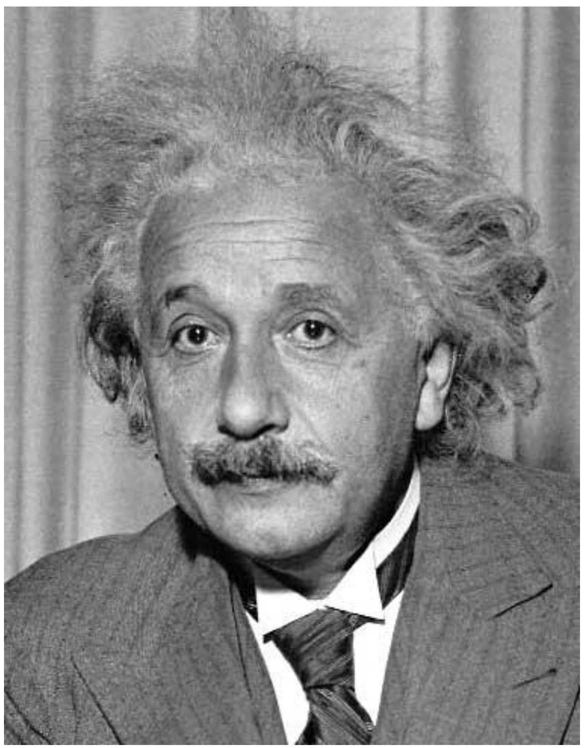
Source: Hays, Brown

Slide: Hoiem

# Matching with filters

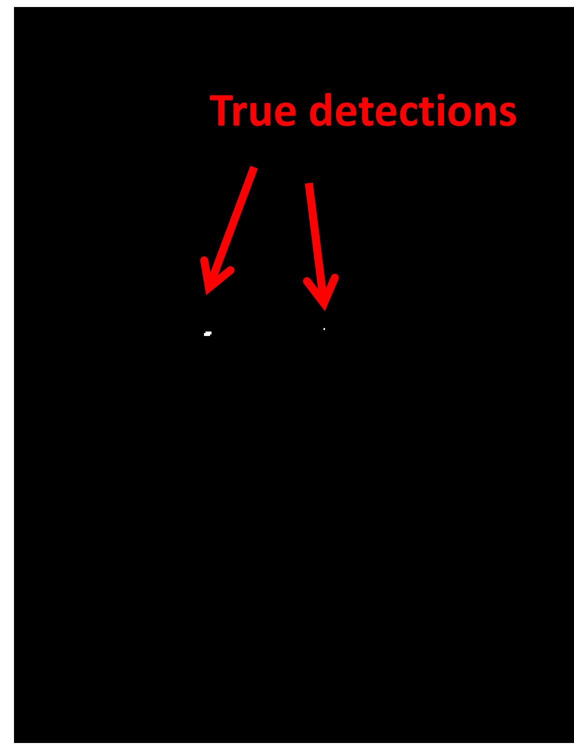
- Goal: find  in image
- Method 2: SSD

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k, n+l])^2$$



Input

1- sqrt(SSD)




Thresholded Image

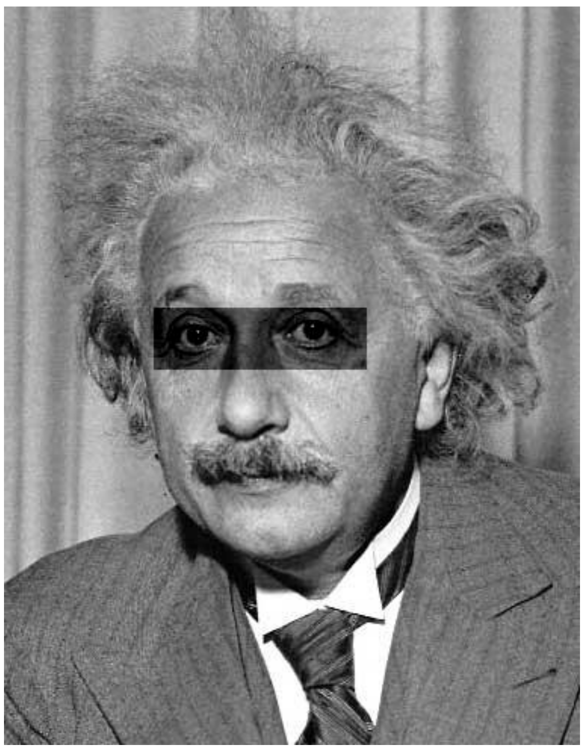
Source: Hays, Brown

Slide: Hoiem

# Matching with filters

- Goal: find  in image
- Method 2: SSD

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k, n+l])^2$$



Input

1- sqrt(SSD)

**What's the potential downside of SSD?**

SSD is sensitive to average intensity

Source: Hays, Brown

Slide: Hoiem

# Matching with filters

- Goal: find  in image
- Method 3: Normalized cross-correlation

mean template

mean image patch

$$h[m,n] = \frac{\sum_{k,l} (g[k,l] - \bar{g})(f[m-k,n-l] - \bar{f}_{m,n})}{\left( \sum_{k,l} (g[k,l] - \bar{g})^2 \sum_{k,l} (f[m-k,n-l] - \bar{f}_{m,n})^2 \right)^{0.5}}$$

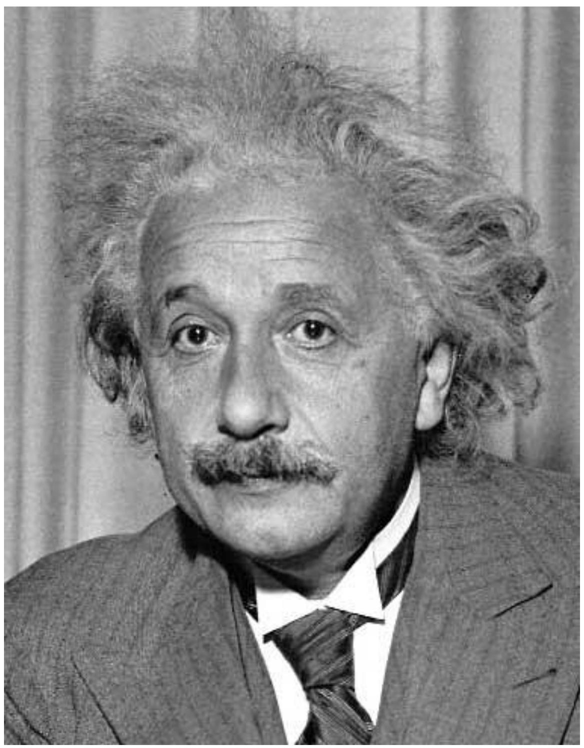
Matlab: `normxcorr2(template, im)`

Source: Hays, Brown

Slide: Hoiem

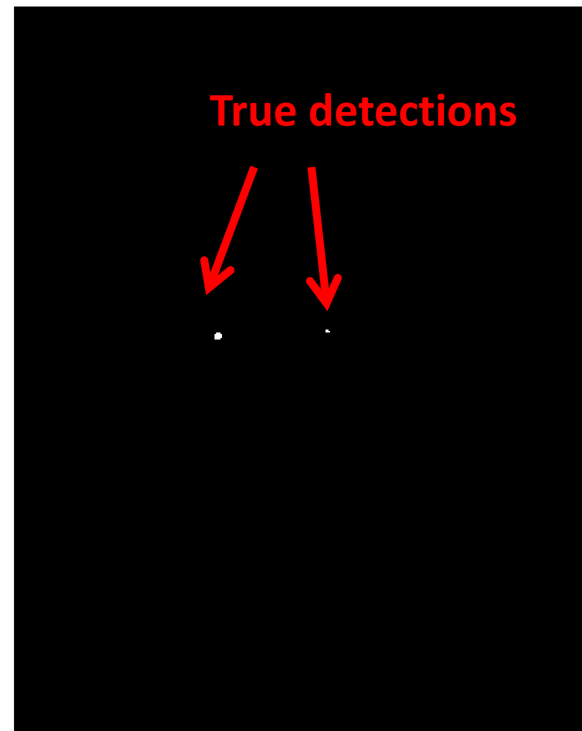
# Matching with filters

- Goal: find  in image
- Method 3: Normalized cross-correlation



Input

Normalized X-Correlation



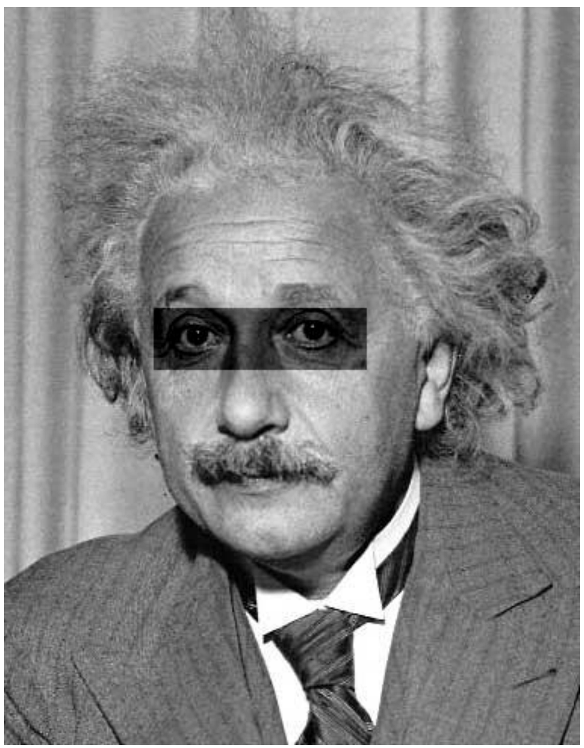

Thresholded Image

Source: Hays, Brown

Slide: Hoiem

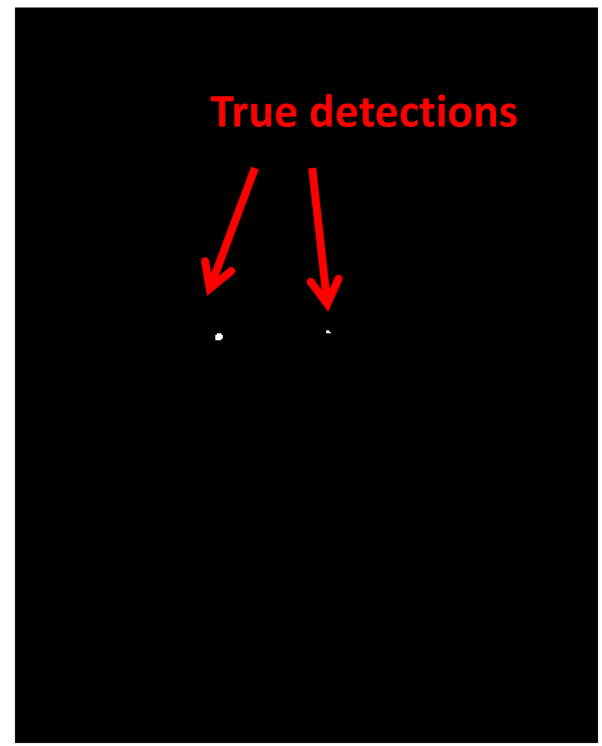
# Matching with filters

- Goal: find  in image
- Method 3: Normalized cross-correlation



Input

Normalized X-Correlation




Thresholded Image

Source: Hays, Brown

Slide: Hoiem

# Q: What is the best method to use?

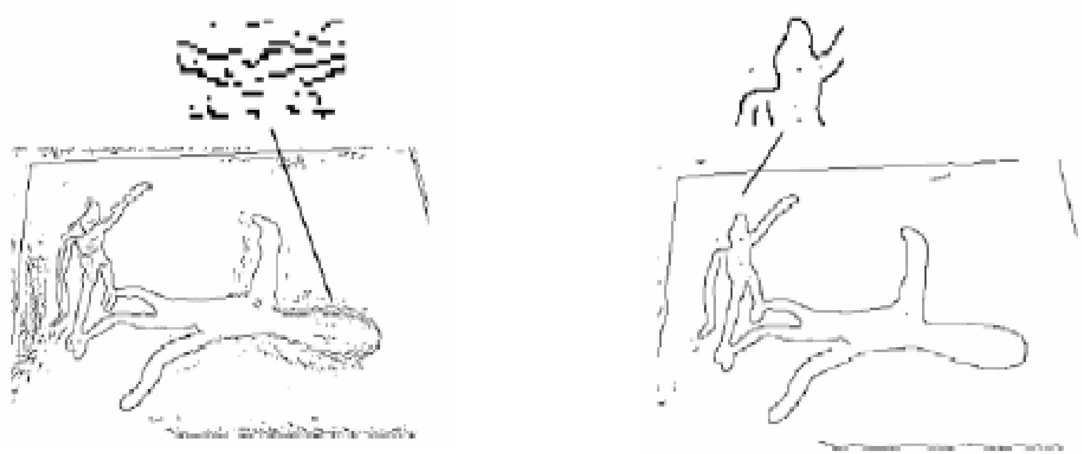
A: Depends

- SSD: faster, sensitive to overall intensity
- Normalized cross-correlation: slower, invariant to local average intensity and contrast

Source: Hays, Brown

# Q: What if we want to find larger or smaller eyes?

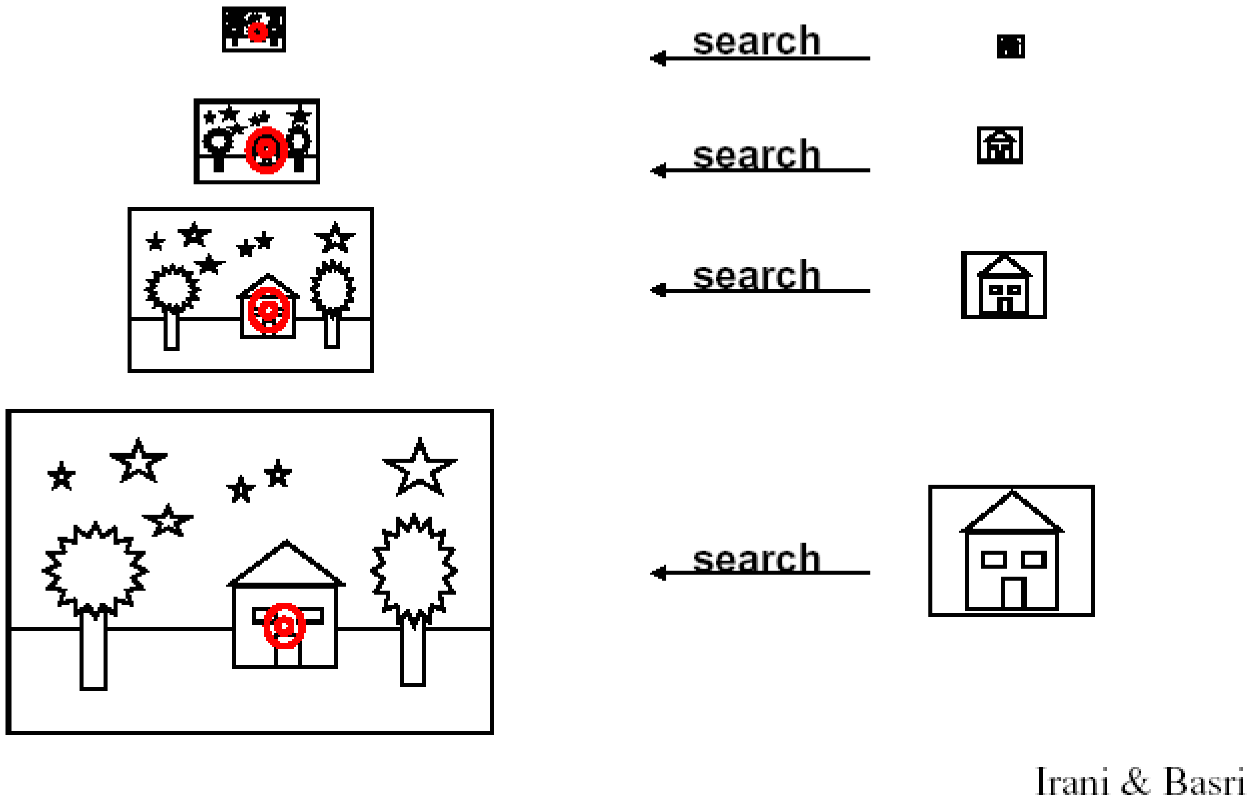
Motivation for studying scale.



ELDER AND ZUCKER: LOCAL SCALE CONTROL FOR EDGE DETECTION AND BLUR ESTIMATION

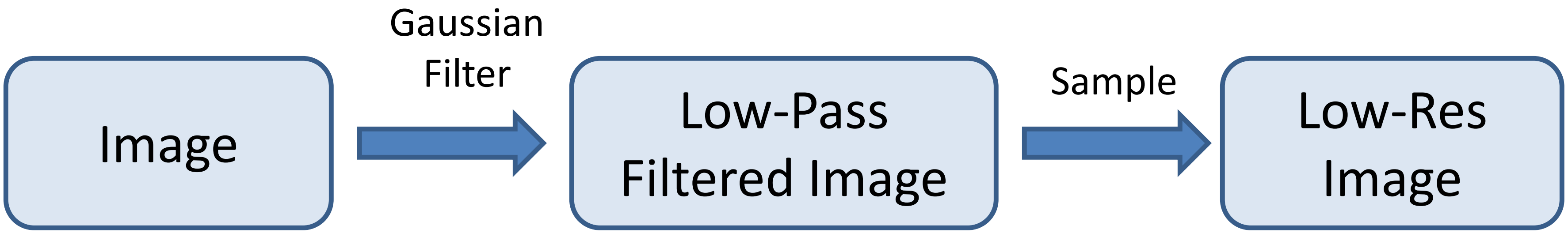
IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 20, NO. 7, JULY 1998

# A: Image Pyramid




adapted from Michael Black, Brown University

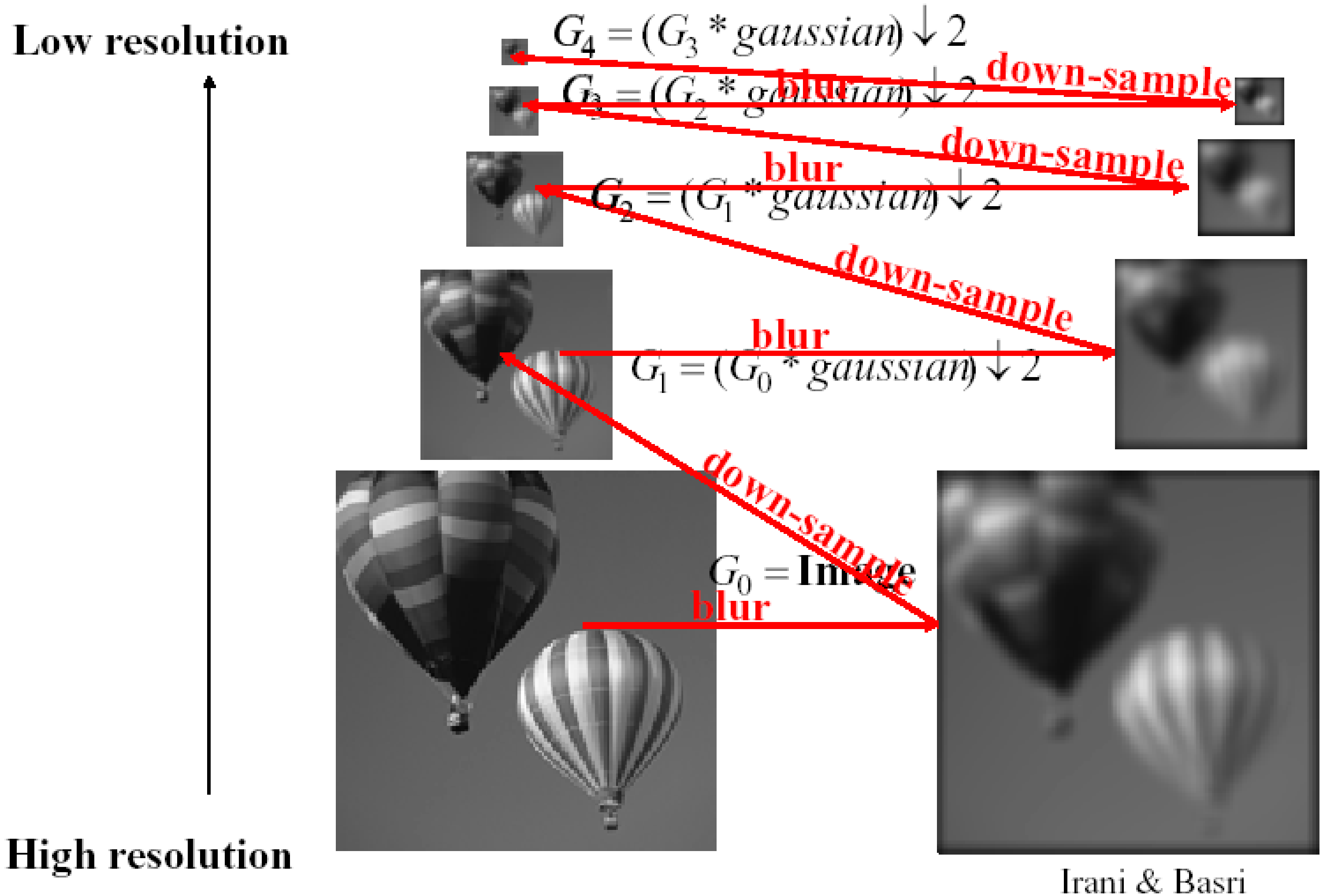
# Review of Sampling




Source: Hays, Brown

Slide: Hoiem

# Gaussian Pyramid




adapted from Michael Black, Brown University

# Gaussian pyramid


Source: Hays, Brown

Source: Forsyth

# Template Matching with Image Pyramids

Input: Image, Template

1. Match template at current scale

2. Downsample image

3. Repeat 1-2 until image is very small

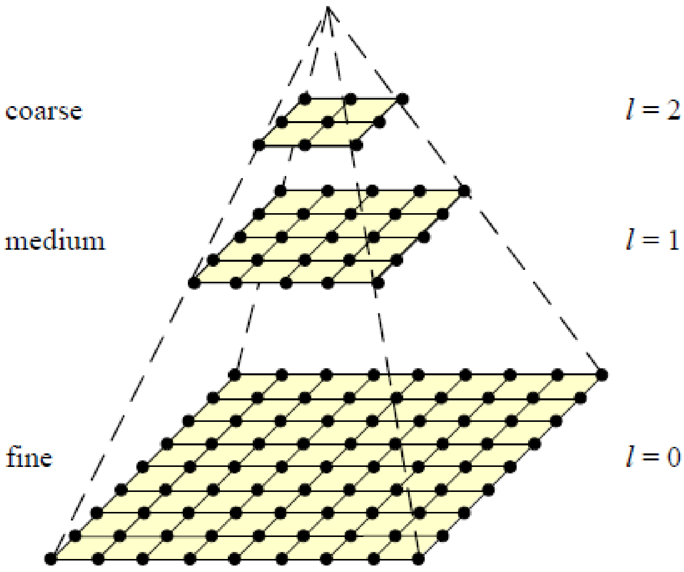
4. Take responses above some threshold, perhaps with non-maxima suppression

Source: Hays, Brown

Slide: Hoiem

# Coarse-to-fine Image Registration

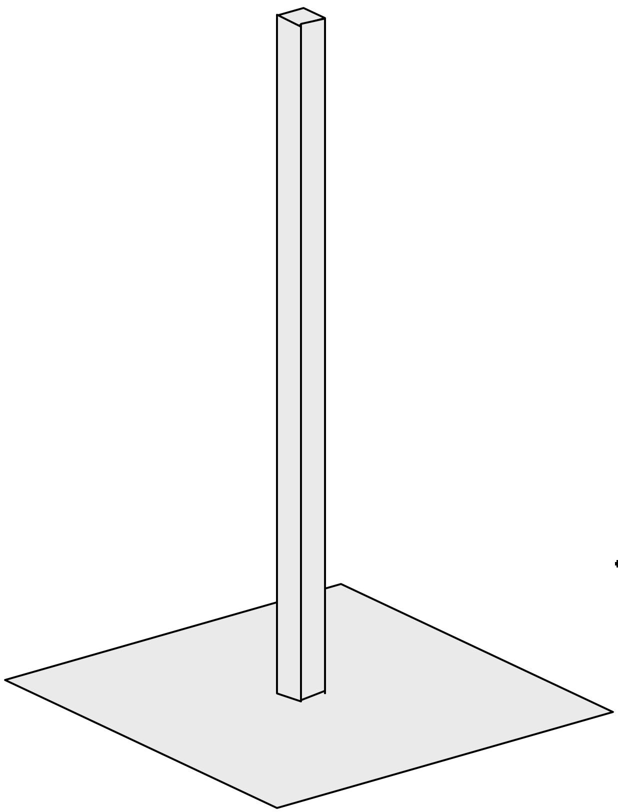
1. Compute Gaussian pyramid
2. Align with coarse pyramid
3. Successively align with finer pyramids
   – Search smaller range




Why is this faster?

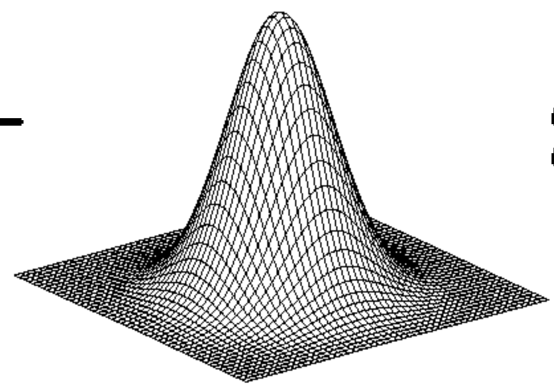
Are we guaranteed to get the same result?

# Laplacian filter



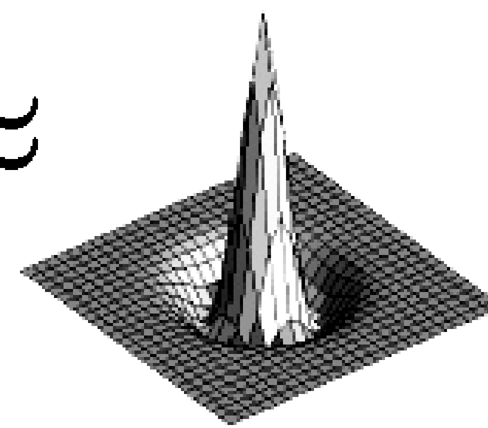
unit impulse

$-$



Gaussian

$\approx$

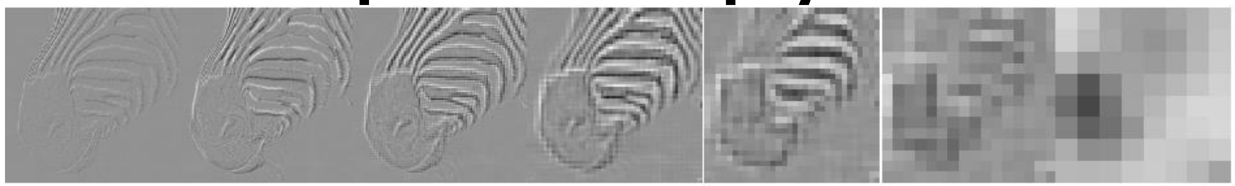


Laplacian of Gaussian

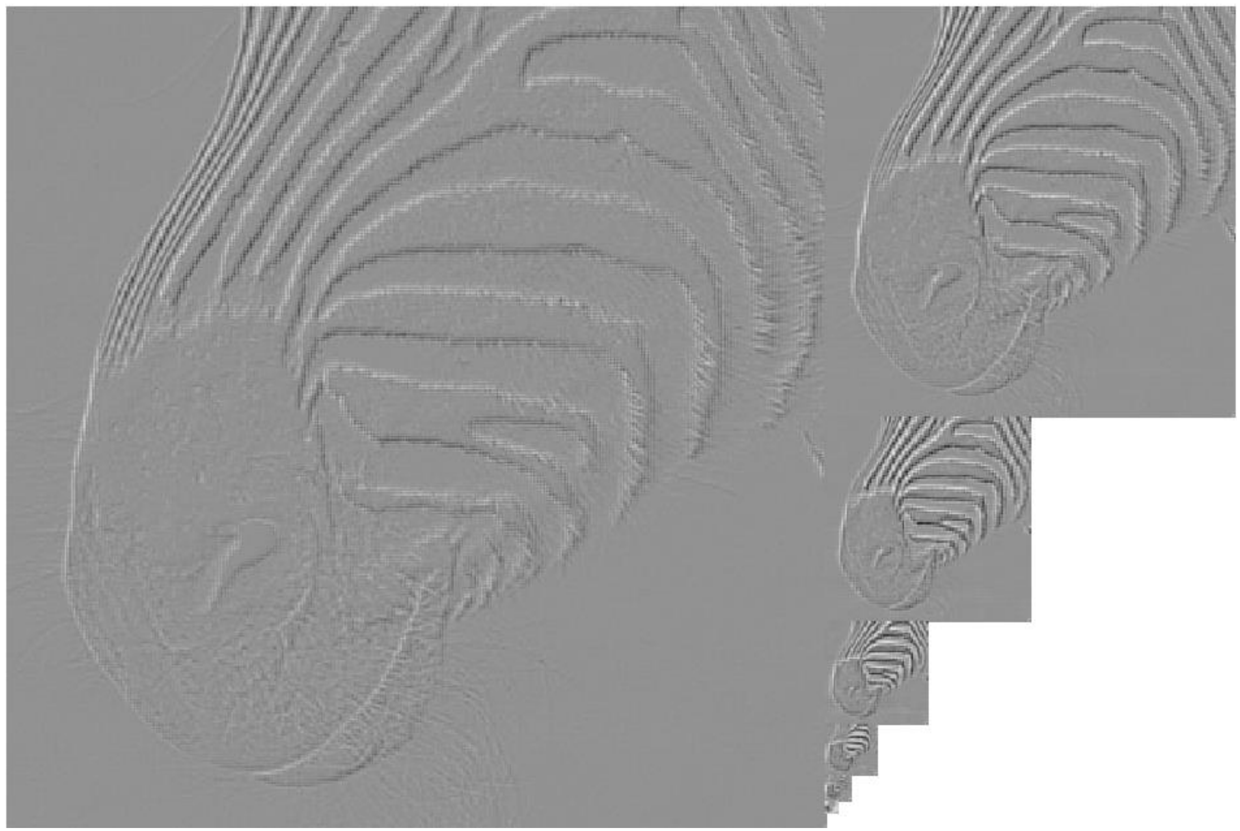
Source: Hays, Brown

Source: Lazebnik

# Laplacian pyramid



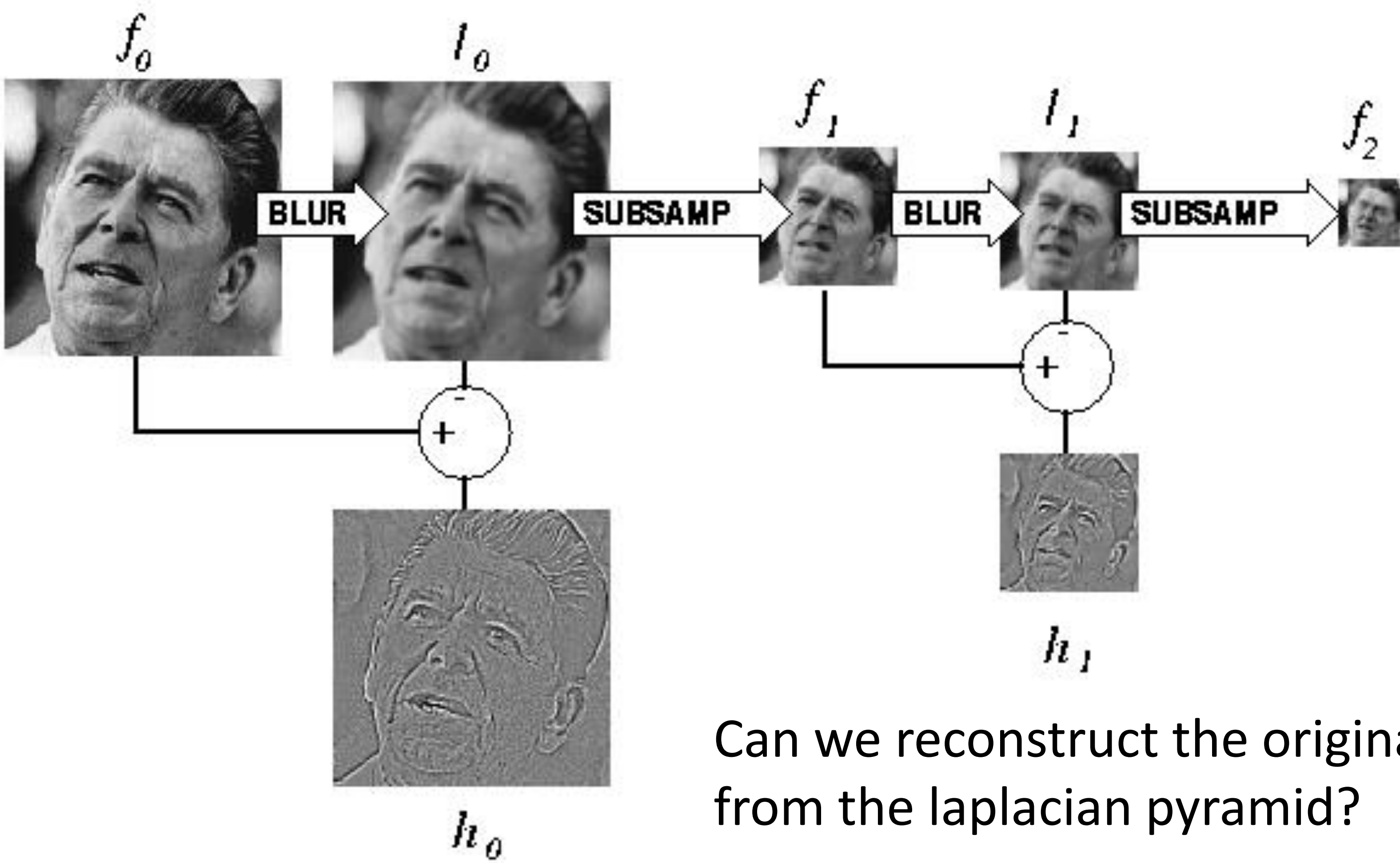
512 256 128 64 32 16 8



Source: Hays, Brown

Source: Forsyth

# Computing Gaussian/Laplacian Pyramid

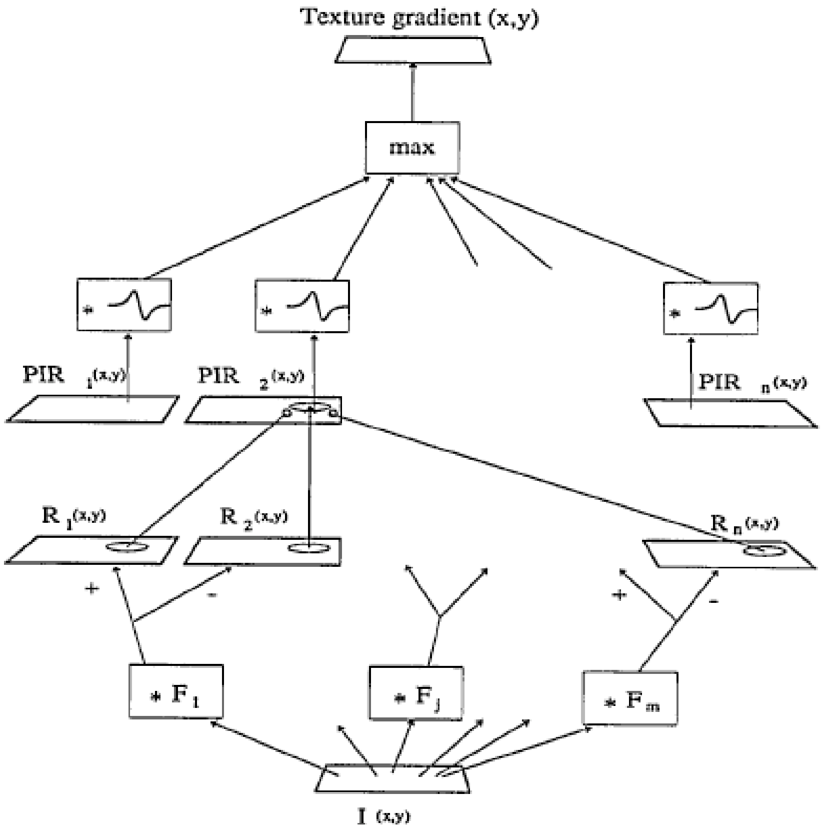


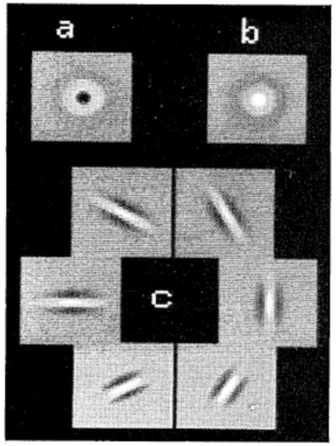

Can we reconstruct the original from the laplacian pyramid?

Source: Hays, Brown

http://sepwww.stanford.edu/~morgan/texturematch/paper_html/node3.html

# Texture segmentation

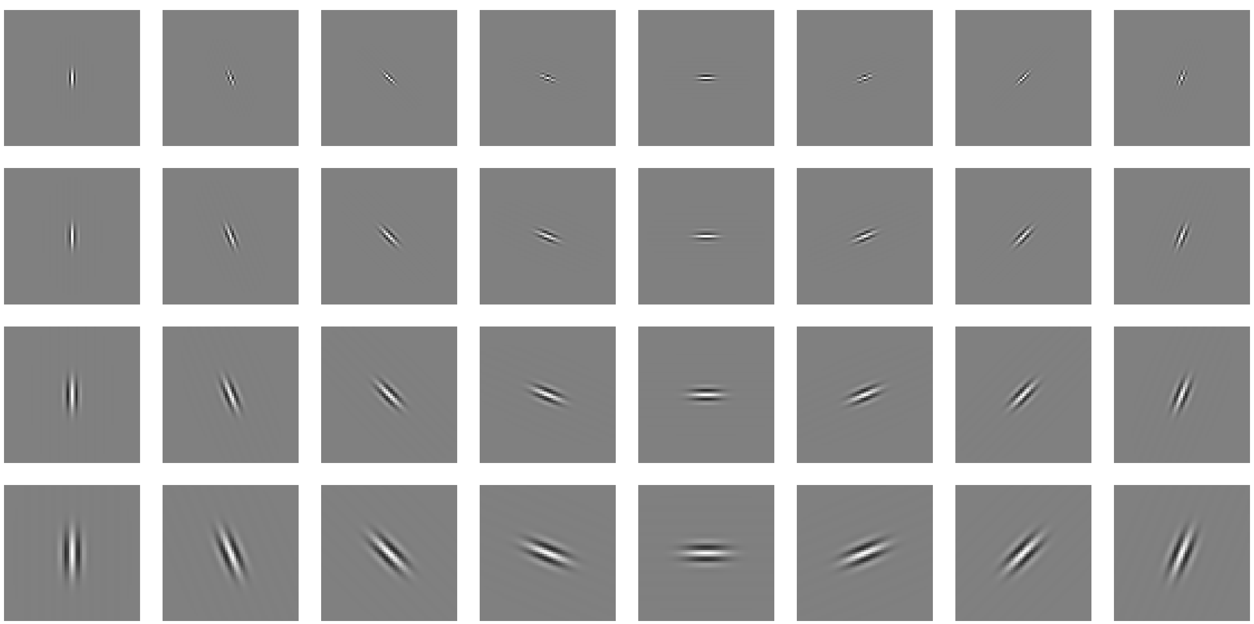



Malik & Perona, 1990. Preattentive texture discrimination with early vision mechanisms.

# Clues from Human Perception

- Early processing in humans filters for various orientations and scales of frequency
- Perceptual cues in the mid-high frequencies dominate perception
- When we see an image from far away, we are effectively subsampling it



Early Visual Processing: Multi-scale edge and blob filters

Source: Hays, Brown