One Time Pad, Block Ciphers, Encryption Modes

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Basic Ciphers

- Shift Cipher
  - Brute-force attack can easily break
- Substitution Cipher
  - Frequency analysis can reduce the search space
- Vigenere Cipher
  - Kasiski test can reveal the length of key
- Enigma Machine
  - The capture of the daily codebook

- How perfect secrecy can be satisfied?

One Time Pad

- Basic Idea: Extend Vigenère cipher so that the key is as long as the plaintext
  - Key is a random string and is used only once
  - Encryption is similar to Vigenère
  - Cannot be broken by frequency analysis or Kasiski test

Plaintext $P = (x_1\ x_2\ \dots\ x_n)$
Key $K = (k_1\ k_2\ \dots\ k_n)$
Ciphertext $C = (y_1\ y_2\ \dots\ y_n)$

$E_k(X) = (x_1+k_1\ x_2+k_2\ \dots\ x_n+k_n) \bmod m$
$D_k(Y) = (y_1-k_1\ y_2-k_2\ \dots\ y_n-k_n) \bmod m$

The Binary Version of One-Time Pad

- Plaintext space = Ciphtertext space = Keyspace = $\{0,1\}^n$
- Key is chosen randomly
- For example:

Plaintext	11011011
Key	01101001
Ciphertext	10110010

Security of One Time Pad

- How good is the security of one time pad?
  - The key is random, so ciphertext is completely random
  - Any plaintext can correspond to a ciphertext with the same length
- A scheme has perfect secrecy if ciphertext provides no "information" about plaintext
  - *C. E. Shannon, 1949*
- One-time pad has perfect secrecy
  - For example, suppose that the ciphertext is "Hello", can we say any plaintext is more likely than another plaintext?

Importance of Key Randomness

- For perfect secrecy, key-length ≥ msg-length

- What if a One-Time Pad key is not chosen randomly, instead, texts from, e.g., a book is used.
  - this is not One-Time Pad anymore
  - this does not have perfect secrecy and can be broken

- The key in One-Time Pad should never be reused.
  - If it is reused, it is insecure!
  - How to send the key to the receiver of the ciphertext?

- These requirements make One Time Pad impractical.

Block Ciphers

- Block Cipher = Symmetric key encryption = Conventional Encryption
- Block ciphers can be considered as substitution ciphers with large block size (≥ 64 bits)
- Map n-bit plaintext blocks to n-bit ciphertext blocks (n: block size).
  - For n-bit plaintext and ciphertext blocks and a fixed key, the encryption function is a one-to-one function

Block Ciphers

- Block size: in general larger block sizes mean greater security.
- Key size: larger key size means greater security (larger key space).
- Number of rounds: multiple rounds offer increasing security.
- Encryption modes: define how messages larger than the block size are encrypted, very important for the security of the encrypted message.

A Simple Block Cipher: Hill Cipher

- The key k is a matrix. The message is considered as vectors. Encryption and decryption operations are matrix multiplication operations
  - Encryption: `C = k.P (mod 26)`
  - Decryption: `P = k-1.C (mod 26)` ($P = k^{-1}.C \pmod{26}$)
- Example:
  - The plaintext is `CAT` converted to numeric values (2, 0, 19).
  - If the key is $\begin{pmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{pmatrix}$
  - Encryption: $\begin{pmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 19 \end{pmatrix} \equiv \begin{pmatrix} 31 \\ 216 \\ 325 \end{pmatrix} \equiv \begin{pmatrix} 5 \\ 8 \\ 13 \end{pmatrix} \pmod{26}$
  - C=`FIN`

An Insecure Block Cipher

- Hill cipher is insecure since it uses linear matrix operations.
  - Each output bit is a linear combination of the input bits
  - An insecure block cipher uses linear equations
- Hill Cipher can easily be broken by known-plaintext attack
  - An attacker knowing a plaintext and ciphertext pair can easily figure out the key matrix.

Feistel Network

- A Feistel Network is fully specified given
  - the block size: n = 2w
  - number of rounds: d
  - d round functions $f_1, f_2 \ldots, f_d: \{0,1\}^w \to \{0,1\}^w$
  - Each f function is a SP cipher
- Feistel Network are used in DES, IDEA, RC5, and many other block ciphers.
- Not used in AES

Feistel Network

- Encryption

  $L_1=R_0 \quad R_1=L_0 \oplus f_0(R_0)$

  $L_2=R_1 \quad R_2=L_1 \oplus f_1(R_1)$

  …

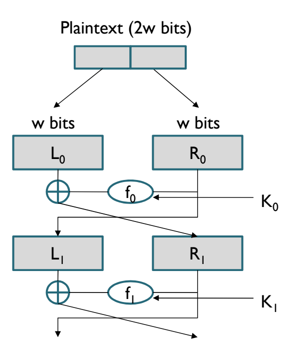
  $L_d=R_{d-1} \quad R_d=L_{d-1} \oplus f_{d-1}(R_{d-1})$

- Decryption

  $R_{d-1}=L_d \quad L_{d-1}=R_d \oplus f_{d-1}(L_d)$

  …

  $R_0=L_1 \quad L_0=R_1 \oplus f_0(L_1)$



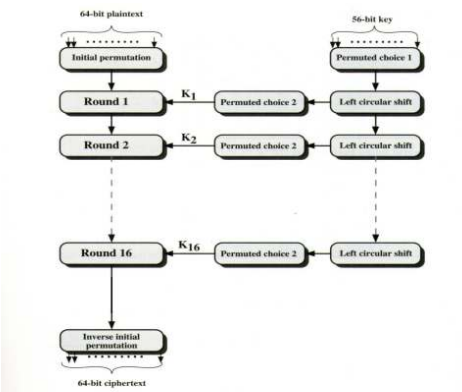
History of Data Encryption Standard (DES)

- 1967: Feistel at IBM
  - Lucifer: block size 128; key size 128 bit
- 1972: NBS asks for an encryption standard
- 1975: IBM developed DES (modification of Lucifer)
  - block size 64 bits; key size 56 bits
- 1975: NSA suggests modification
- 1977: NBS adopts DES as encryption standard in (FIPS 46-1, 46-2).
- 2001: NIST adopts Rijndael (AES) as replacement to DES.

DES Features

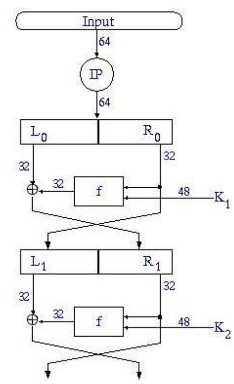
- Features:
  - Block size = 64 bits
  - Key size = 56 bits
  - Number of rounds = 16
  - 16 intermediary keys, each 48 bits

DES Structure



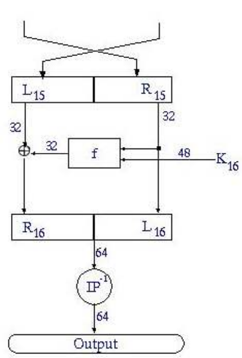
Details of DES Rounds

- An initial permutation is applied on the plaintext
  $IP(x) = L_0 R_0$
- In each round:
  $L_i = R_{i-1}$
  $R_i = L_{i-1} \oplus f(R_{i-1}, K_i)$

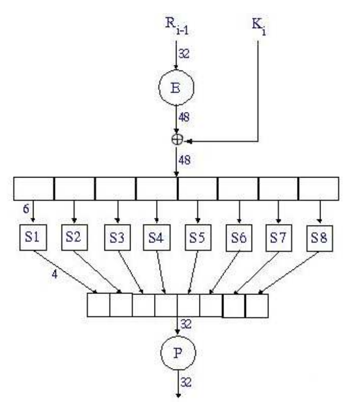


Details of DES Rounds

- After the last round
  $y = IP^{-1}(R_{16} L_{16})$

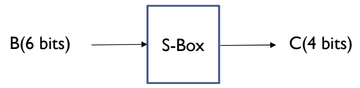


DES f Function



DES S-boxes

- S-boxes are the only non-linear elements in DES design



- $B = b_1b_2b_3b_4b_5b_6$ -> row=b_1b_6 column=$b_2b_3b_4b_5$
- Example:
  - B = 011011 row= 01 column=1101

Middle 4 bits of input

Outer bits	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
00	0010	1100	0100	0001	0111	1010	1011	0110	1000	0101	0011	1111	1101	0000	1110	1001
01	1110	1011	0010	1100	0100	0111	1101	0001	0101	0000	1111	1010	0011	1001	1000	0110
10	0100	0010	0001	1011	1010	1101	0111	1000	1111	1001	1100	0101	0110	0011	0000	1110
11	1011	1000	1100	0111	0001	1110	0010	1101	0110	1111	0000	1001	1010	0100	0101	0011

C = 1001

DES Weak Keys

- Weak keys: keys make the same sub-key to be generated in more than one round.
  - Result: reduce cipher complexity
  - Weak keys can be avoided at key generation. DES has 4 weak keys:
    - 0000000 0000000
    - 0000000 FFFFFFF
    - FFFFFFF 0000000
    - FFFFFFF FFFFFFF
- Semi-weak keys: A pair of DES semi-weak keys is a pair (K_1,K_2) with $E_{K_1}(E_{K_2}(x))=x$
  - There are six pairs of DES semi-weak keys

Dictionary Attack to DES

- Even without having weak/semi-weak keys DES is vulnerable to dictionary attacks:
- Each plaintext may result in 2^{64} different ciphertexts, but there are only 2^{56} possible different key values.
- Given a PT/CT pair *(M,C)*
  - Encrypt the known plaintext M with all possible keys.
  - Keep a look up table of size 2^{56}.
  - Look up C in the table

Double DES

- DES uses a 56-bit key, this raised concerns about brute force attacks.
- One proposed solution: Double DES.
- Apply DES twice using two keys, K_1 and K_2.
  - $C = E_{K2} [E_{K1} [P]]$
  - $P = D_{K1} [D_{K2} [C]]$
- This leads to a 2x56=112 bit key, so it is more secure than DES. Is it?

Meet-in-the-middle Attack

- Goal: given the pair (P, C) find keys K_1 and K_2.
- Based on the observation:

$$C = E_{K2} [E_{K1} [P]]$$
$$D_{K2}[C] = E_{K1}[P]$$

- Encrypt P with all 2^{56} possible keys K_1
  - Store all pairs (K_1, $E_{K1}[P]$), sorted by $E_{K1}[P]$.
- Decrypt C using all 2^{56} possible keys K_2
  - For each decrypted result, check to see if there is a match $D_{K2}(C) = E_{K1}(P)$. If a match is found, (K_1, K_2) is a possible match
- The attack has a higher chance of succeeding if another pair (P', C') is available to the cryptanalysis.

Triple DES

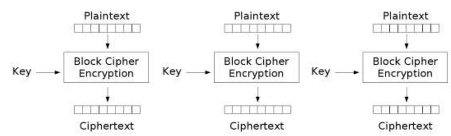
- Two key version is widely used and standard
  - Key space is 56 x 2 = 112 bits
    - Encrypt: $C = E_{K1} [D_{K2} [E_{K1} [P]]]$
    - Decrypt: $P = D_{K1} [E_{K2} [D_{K1} [C]]]$
- Three key version is possible but not standard
  - Key space is 56 x 3 = 168 bits
    - Encrypt: $C = E_{K3} [D_{K2} [E_{K1} [P]]]$
    - Decrypt: $P = D_{K1} [E_{K2} [D_{K3} [C]]]$
- No known practical attack against it.
- Some protocols/applications use 3DES (such as PGP)

Encryption Modes

- Electronic Code Book (ECB)
- Cipher Block Chaining (CBC)
- Output Feedback Mode (OFB)
- Cipher Feedback Mode (CFB)
- Counter Mode (CTR)

Electronic Code Book (ECB)

- Message is broken into independent blocks of block_size bits.
- Electronic Code Book (ECB): each block encrypted separately.
  - Encryption: $C_i = E_k(P_i)$
  - Decrytion: $P_i = D_k(C_i)$



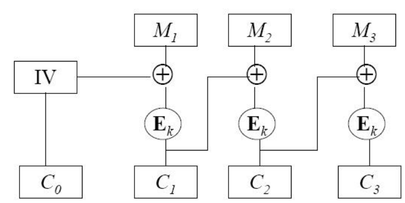
Electronic Codebook (ECB) mode encryption

Properties of ECB

- Deterministic: the same data block gets encrypted the same way.
  - This reveals patterns of data when a data block repeats.
- Malleable: reordering ciphertext results in reordered plaintext.
- Errors in one ciphertext block do not propagate.
- Usage: not recommended to encrypt more than one block of data.

Cipher Block Chaining (CBC)

- Cipher Block Chaining (CBC): next input depends upon previous output
  - Encryption: $C_i = E_k(M_i \oplus C_{i-1})$, with C_0=IV
  - Decryption: $M_i = C_{i-1} \oplus D_k(C_i)$, with C_0=IV



Properties of CBC

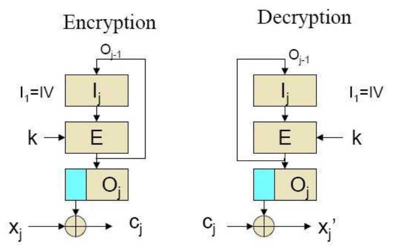
- Randomized encryption: repeated text gets mapped to different encrypted data.
  - can be proven to be "secure" assuming that the block cipher has desirable properties and that random IV's are used
- A ciphertext block depends on all preceding plaintext blocks
  - Sequential encryption, cannot use parallel hardware
- Errors in one block of ciphertext propagate to two blocks
  - one bit error in C_j affects all bits in M_j and one bit in M_{j+1}

Block Ciphers vs. Stream Ciphers

- A block cipher operates on blocks of fixed length.
- A stream cipher is a symmetric key cipher where plaintext bits are combined with a pseudorandom cipher bit stream (keystream), typically by an exclusive-or (xor) operation.

Output Feedback (OFB)

- Output feedback (OFB): construct a pseudorandom number generator (PRNG) to obtain a one time pad and XOR the message with the pad
  - y_0=IV $y_i = E_k[y_{i-1}]$

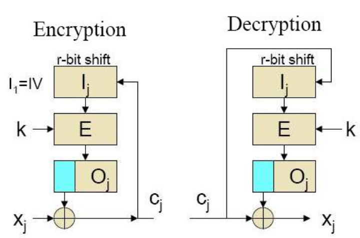


Properties of OFB

- Randomized encryption
- Sequential encryption, but preprocessing possible
  - Generate the key before the message comes
- Error propagation limited
  - Only the changed bits are lost
- It can only be used as a stream cipher

Cipher Feedback (CFB)

- Cipher Feedback (CFB): the message is XORed with the feedback of encrypting the previous block



Counter Mode (CTR)

- Counter Mode (CTR): Another way to construct pseudo random number generator using DES
  - $Y_i = E_k[\text{counter}+i]$
  - $C_i = Y_i \oplus P_i$
  - Sender and receiver share a counter value (does not need to be secret) and the secret key

Properties of CTR

- Software and hardware efficiency: different blocks can be encrypted in parallel.
- Preprocessing: the encryption part can be done offline and when the message is known, just do the XOR.
- Random Access: decryption of a block can be done in random order, very useful for hard-disk encryption.
- Messages of Arbitrary Length: ciphertext is the same length with the plaintext (i.e., no IV).