# One Time Pad, Block Ciphers, Encryption Modes

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# Basic Ciphers

- Shift Cipher
  - Brute-force attack can easily break
- Substitution Cipher
  - Frequency analysis can reduce the search space
- Vigenere Cipher
  - Kasiski test can reveal the length of key
- Enigma Machine
  - Reveal of the internals of the machine and the capture of the daily codebook

- How perfect secrecy can be satisfied?

# One Time Pad

- Basic Idea: Extend Vigenère cipher so that the key is as long as the plaintext
  - Key is a random string and is used only once
  - Encryption is similar to Vigenère
  - Cannot be broken by frequency analysis or Kasiski test

Plaintext $P = (p_1 \ p_2 \ \ldots \ p_n)$

Key $K = (k_1 \ k_2 \ \ldots \ k_n)$

Ciphertext $C = (p_1 \ p_2 \ \ldots \ p_n)$

$$E_k(X) = (p_1+k_1 \ p_2+k_2 \ \ldots \ p_n+k_n) \bmod m$$

$$D_k(Y) = (c_1-k_1 \ c_2-k_2 \ \ldots \ c_n-k_n) \bmod m$$

# The Binary Version of One-Time Pad

- Plaintext space = Ciphtertext space = Keyspace = $\{0,1\}^n$
- Key is chosen randomly
- For example:

| | |
|---|---|
| Plaintext | 11011011 |
| Key | 01101001 |
| Ciphertext | 10110010 |

# Security of One Time Pad

- How good is the security of one time pad?
  - The key is random, so ciphertext is completely random
  - Any plaintext can correspond to a ciphertext with the same length
- A scheme has perfect secrecy if ciphertext provides no "information" about plaintext
  - *C. E. Shannon, 1949*
- One-time pad has perfect secrecy
  - For example, suppose that the ciphertext is "Hello", can we say any plaintext is more likely than another plaintext?

# Importance of Key Randomness

- For perfect secrecy, key-length ≥ msg-length
- What if a One-Time Pad key is not chosen randomly, instead, texts from, e.g., a book is used.
  - this is not One-Time Pad anymore
  - this does not have perfect secrecy and can be broken
- The key in One-Time Pad should never be reused.
  - If it is reused, it is insecure!
  - How to send the key to the receiver of the ciphertext?
- These requirements make One Time Pad impractical.

# Block Ciphers

- Block Cipher = Symmetric key encryption = Conventional Encryption
- Block ciphers can be considered as substitution ciphers with large block size ($\geq$ 64 bits)
- Map n-bit plaintext blocks to n-bit ciphertext blocks (n: block size).
  - For n-bit plaintext and ciphertext blocks and a fixed key, the encryption function is a one-to-one function

# Block Ciphers

- Block size: in general larger block sizes mean greater security.
- Key size: larger key size means greater security (larger key space).
- Number of rounds: multiple rounds offer increasing security.
- Encryption modes: define how messages larger than the block size are encrypted, very important for the security of the encrypted message.

# A Simple Block Cipher: Hill Cipher

- The key k is a matrix. The message is considered as vectors. Encryption and decryption operations are matrix multiplication operations
  - Encryption: `C = k.P (mod 26)`
  - Decryption: `P = k`$^{-1}$`.C (mod 26)`
- Example: The plaintext is `CAT` converted to numeric values, namely 2, 0, 19.
- If the key is $\begin{pmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{pmatrix}$
- Encryption: $\begin{pmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 19 \end{pmatrix} \equiv \begin{pmatrix} 31 \\ 216 \\ 325 \end{pmatrix} \equiv \begin{pmatrix} 5 \\ 8 \\ 13 \end{pmatrix} \pmod{26}$
- C=`FIN`

# An Insecure Block Cipher

- Hill cipher is insecure since it uses linear matrix operations.
  - Each output bit is a linear combination of the input bits
  - An insecure block cipher uses linear equations
- Hill Cipher can easily be broken by known-plaintext attack
  - An attacker knowing a plaintext and ciphertext pair can easily figure out the key matrix.

# Feistel Network

- A Feistel Network is fully specified given
  - the block size: $n = 2w$
  - number of rounds: $d$
  - $d$ round functions $f_1, f_2, \ldots, f_d: \{0,1\}^w \rightarrow \{0,1\}^w$
  - Each f function is a SP cipher
- Used in DES, IDEA, RC5, and many other block ciphers.
- Not used in AES

# Feistel Network

- Encryption

$L_1=R_0 \quad R_1=L_0 \oplus f_0(R_0,K_0)$

$L_2=R_1 \quad R_2=L_1 \oplus f_1(R_1,K_1)$

…

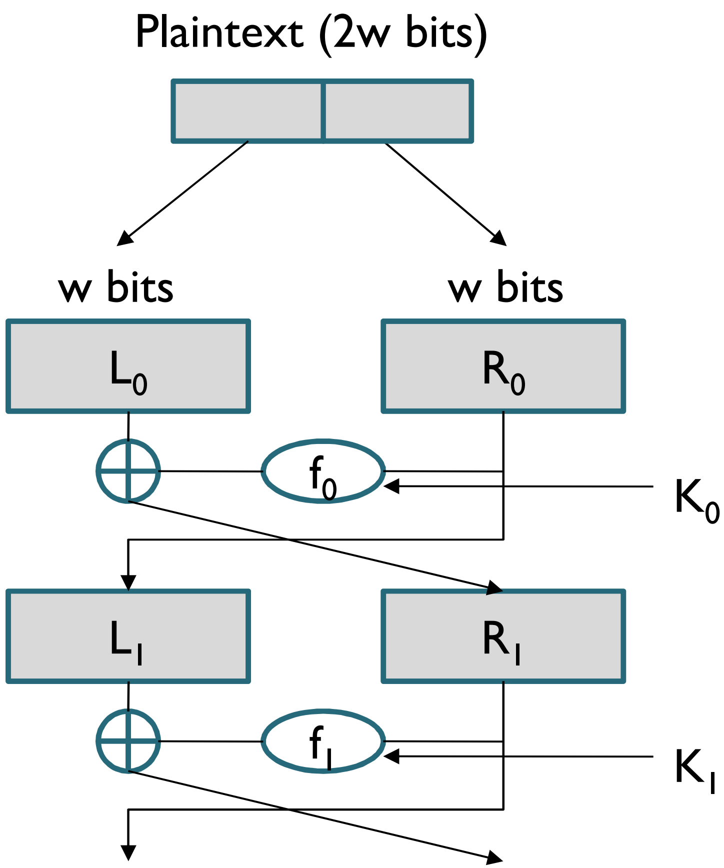
$L_d=R_{d-1} \quad R_d=L_{d-1} \oplus f_{d-1}(R_{d-1},K_{d-1})$

- Decryption

$R_{d-1}=L_d \quad L_{d-1}=R_d \oplus f_{d-1}(L_d,K_{d-1})$

…

$R_0=L_1 \quad L_0=R_1 \oplus f_0(L_1,K_0)$



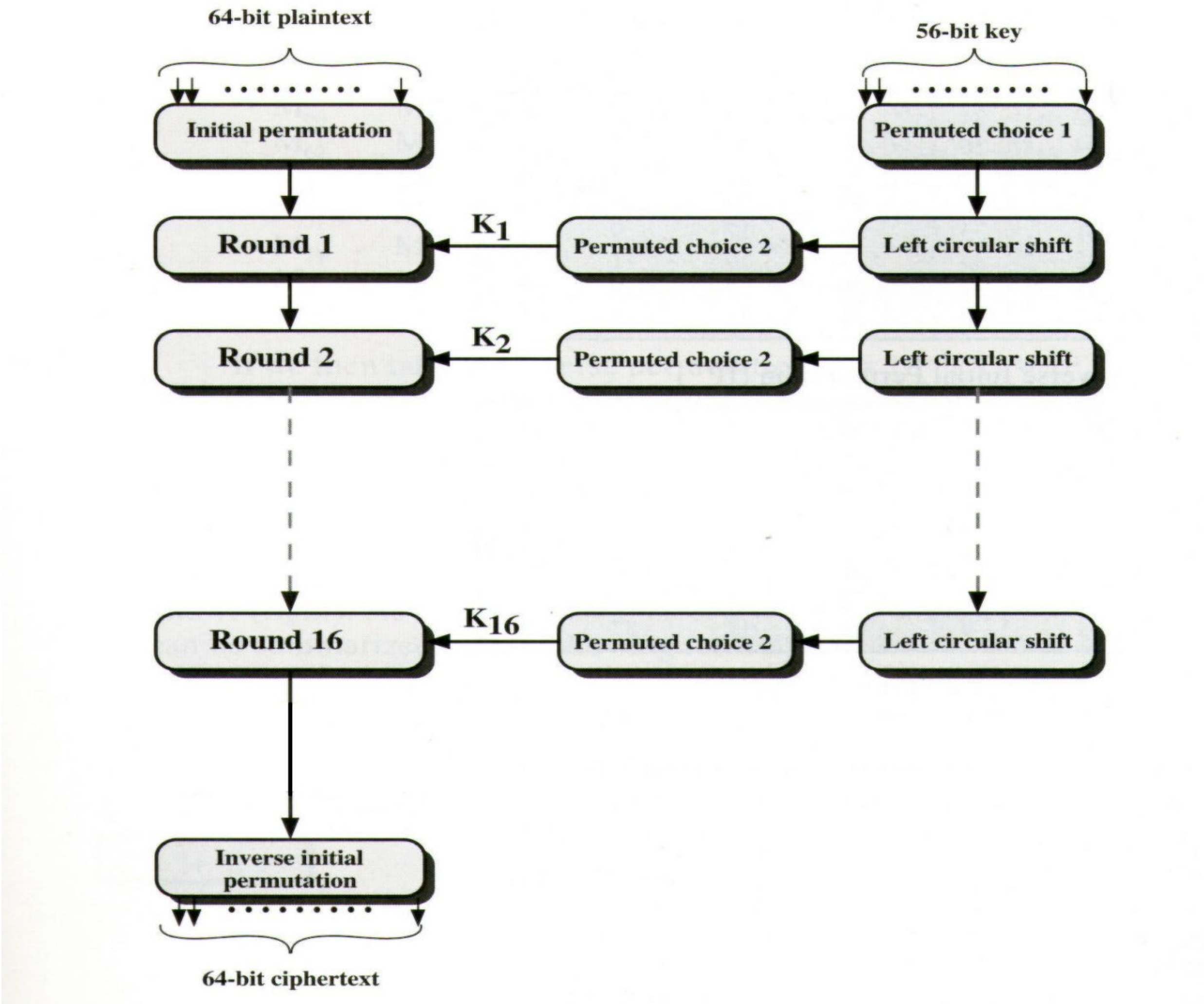
# History of Data Encryption Standard (DES)

- 1967: Feistel at IBM
  - Lucifer: block size 128; key size 128 bit
- 1972: NBS asks for an encryption standard
- 1975: IBM developed DES (modification of Lucifer)
  - block size 64 bits; key size 56 bits
- 1975: NSA suggests modification
- 1977: NBS adopts DES as encryption standard in (FIPS 46-1, 46-2).
- 2001: NIST adopts Rijndael (AES) as replacement to DES.

# DES Features

- Features:
  - Block size = 64 bits
  - Key size = 56 bits
  - Number of rounds = 16
  - 16 intermediary keys, each 48 bits

# DES Structure



# Details of DES Rounds

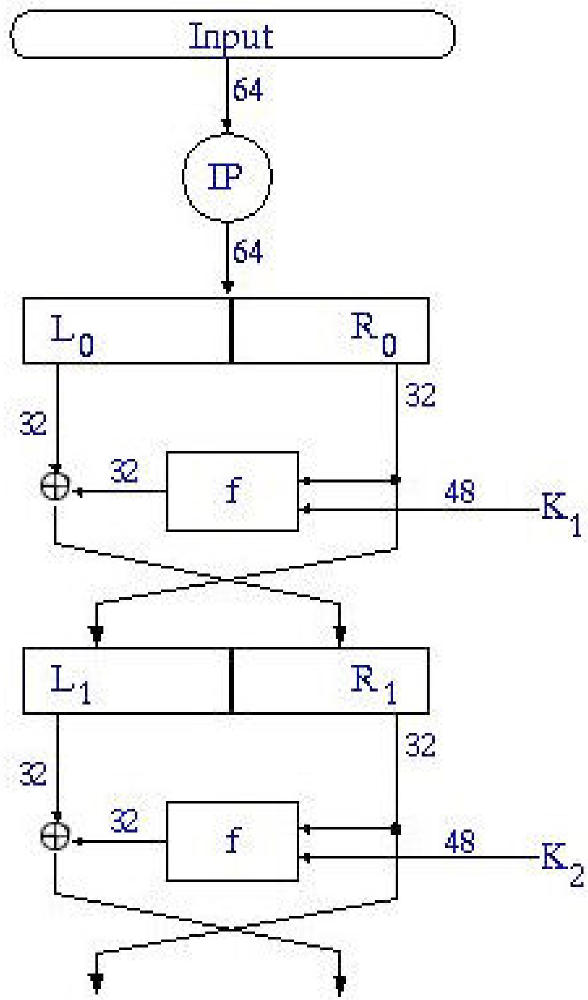
- An initial permutation is applied on the plaintext

  $IP(x) = L_0 R_0$

- In each round:

  $L_i = R_{i-1}$

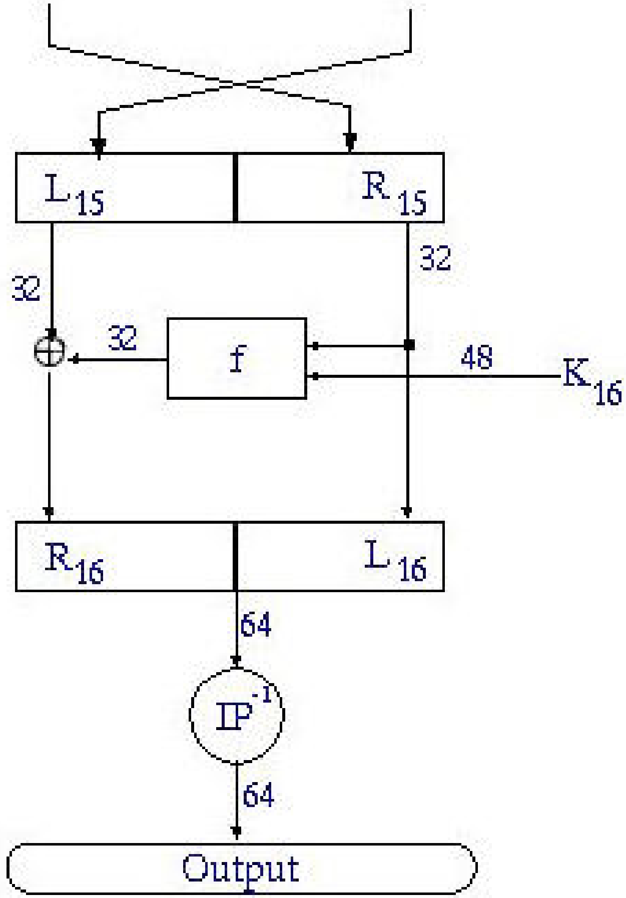
  $R_i = L_{i-1} \oplus f(R_{i-1}, K_i)$



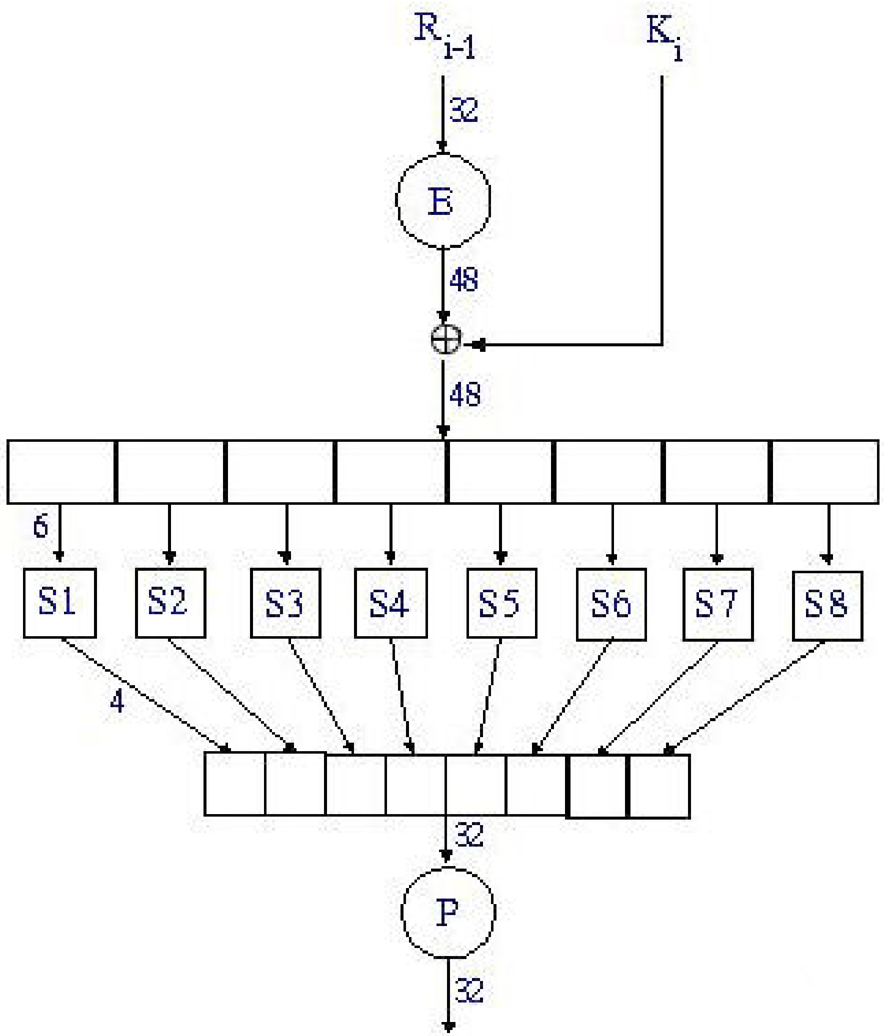
# Details of DES Rounds

- After the last round

  $y = IP^{-1}(R_{16}L_{16})$

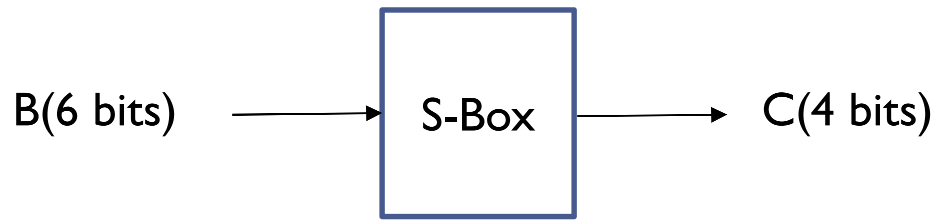


# DES f Function



# DES S-boxes

- S-boxes are the only non-linear elements in DES design




- $B = b_1b_2b_3b_4b_5b_6$ row=$b_1b_6$ column=$b_2b_3b_4b_5$
- Example: B = 011011 row= 01 column=1101

Middle 4 bits of input

| $S_5$ (Outer bits) | 0000 | 0001 | 0010 | 0011 | 0100 | 0101 | 0110 | 0111 | 1000 | 1001 | 1010 | 1011 | 1100 | 1101 | 1110 | 1111 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | 0010 | 1100 | 0100 | 0001 | 0111 | 1010 | 1011 | 0110 | 1000 | 0101 | 0011 | 1111 | 1101 | 0000 | 1110 | 1001 |
| 01 | 1110 | 1011 | 0010 | 1100 | 0100 | 0111 | 1101 | 0001 | 0101 | 0000 | 1111 | 1010 | 0011 | 1001 | 1000 | 0110 |
| 10 | 0100 | 0010 | 0001 | 1011 | 1010 | 1101 | 0111 | 1000 | 1111 | 1001 | 1100 | 0101 | 0110 | 0011 | 0000 | 1110 |
| 11 | 1011 | 1000 | 1100 | 0111 | 0001 | 1110 | 0010 | 1101 | 0110 | 1111 | 0000 | 1001 | 1010 | 0100 | 0101 | 0011 |

C = 1001

# DES Weak Keys

- Weak keys: keys make the same sub-key to be generated in more than one round.
  - Result: reduce cipher complexity
  - Weak keys can be avoided at key generation. DES has 4 weak keys:

    0000000 0000000
    0000000 FFFFFFF
    FFFFFFF 0000000
    FFFFFFF FFFFFFF
- Semi-weak keys: A pair of DES semi-weak keys is a pair $(K_1,K_2)$ with $E_{K_1}(E_{K_2}(x))=x$
- There are six pairs of DES semi-weak keys

# Dictionary Attack to DES

- Even without having weak/semi-weak keys DES is vulnerable to dictionary attacks:
- Each plaintext may result in $2^{64}$ different ciphertexts, but there are only $2^{56}$ possible different key values.
- Given a PT/CT pair (*M,C*)
  - Encrypt the known plaintext M with all possible keys.
  - Keep a look up table of size $2^{56}$.
  - Look up C in the table

# Double DES

- DES uses a 56-bit key, this raised concerns about brute force attacks.
- One proposed solution: double DES.
- Apply DES twice using two keys, K1 and K2.
  - $C = E_{K2} [ E_{K1} [ P ] ]$
  - $P = D_{K1} [ D_{K2} [ C ] ]$
- This leads to a 2x56=112 bit key, so it is more secure than DES. Is it?

# Meet-in-the-middle Attack

- Goal: given the pair (P, C) find keys $K_1$ and $K_2$.
- Based on the observation:

$$C = E_{K2} [ E_{K1} [ P ] ]$$

$$D_{K2}[ C ] = E_{K1}[ P ]$$

1. Encrypt P with all $2^{56}$ possible keys $K_1$
   - Store all pairs ( $K_1$, $E_{K1}[P]$ ), sorted by $E_{K1}[P]$.
2. Decrypt C using all $2^{56}$ possible keys $K_2$
   - For each decrypted result, check to see if there is a match $D_{K2}(C) = E_{K1}(P)$. If a match is found, ($K_1$ ,$K_2$) is a possible match
3. The attack has a higher chance of succeeding if another pair (P', C') is available to the cryptanalysis.

# Triple DES

- Two key version is widely used and standard
  - Key space is 56 x 2 = 112 bits

    Encrypt: $C = E_{K1} [ D_{K2} [ E_{K1} [P] ] ]$

    Decrypt: $P = D_{K1} [ E_{K2} [ D_{K1} [C] ] ]$
- Three key version is possible but not standard
  - Key space is 56 x 3 = 168 bits

    Encrypt: $C = E_{K3} [ D_{K2} [ E_{K1} [P] ] ]$

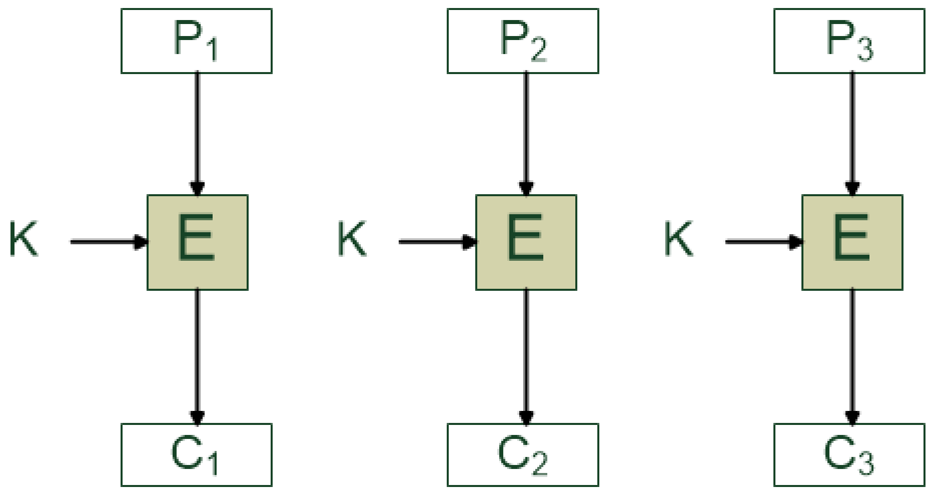
    Decrypt: $P = D_{K1} [ E_{K2} [ D_{K3} [C] ] ]$
- No known practical attack against it.
- Some protocols/applications use 3DES (such as PGP)

# Encryption Modes

- Electronic Code Book (ECB)
- Cipher Block Chaining (CBC)
- Output Feedback Mode (OFB)
- Cipher Feedback Mode (CFB)
- Counter Mode (CTR)

# Electronic Code Book (ECB)

- Message is broken into independent blocks of block_size bits.
- Electronic Code Book (ECB): each block encrypted separately.
  - Encryption: $C_i = E_k[P_i]$
  - Decrytion: $P_i = D_k[C_i]$

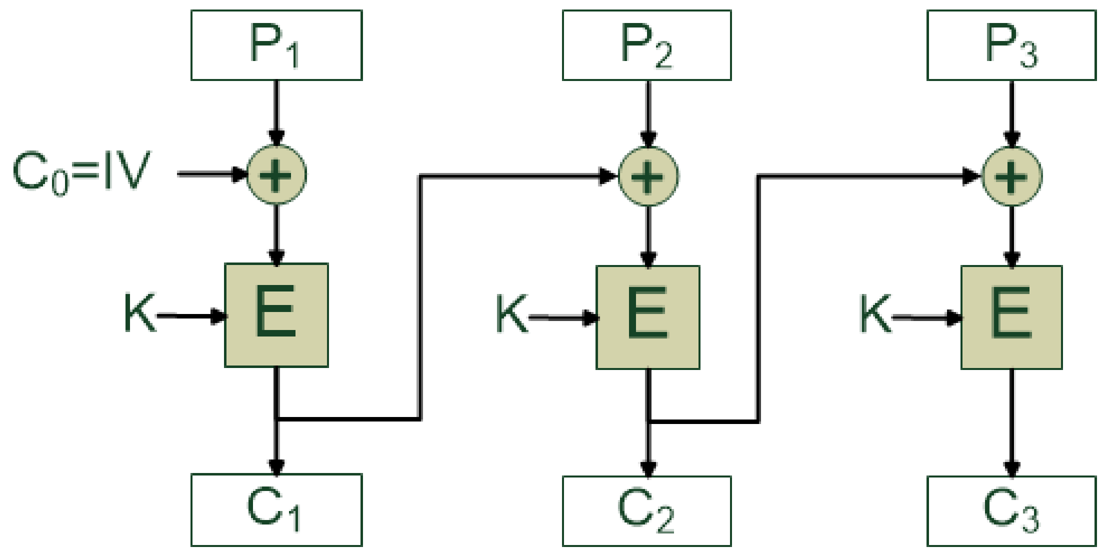


# Properties of ECB

- Deterministic: the same data block gets encrypted the same way.
  - This reveals patterns of data when a data block repeats.
- Malleable: reordering ciphertext results in reordered plaintext.
- Errors in one ciphertext block do not propagate.
- Usage: not recommended to encrypt more than one block of data.

# Cipher Block Chaining (CBC)

- Cipher Block Chaining (CBC): next input depends upon previous output
  - Encryption: $C_i = E_k [P_i \oplus C_{i-1}]$, with $C_0$=IV
  - Decryption: $P_i = C_{i-1} \oplus D_k[C_i]$, with $C_0$=IV



# Properties of CBC

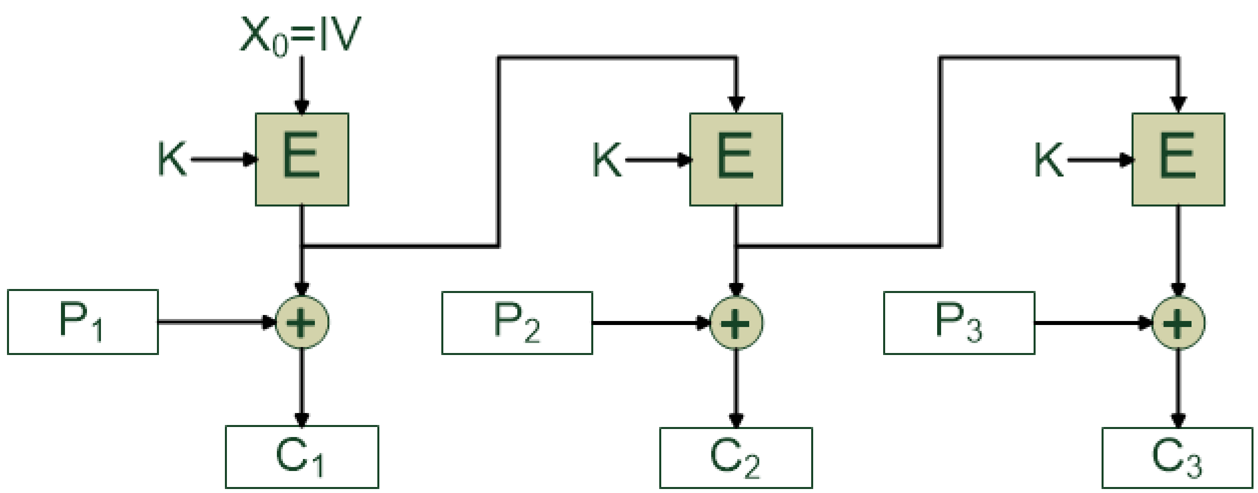
- Randomized encryption: repeated text gets mapped to different encrypted data.
  - can be proven to be "secure" assuming that the block cipher has desirable properties and that random IV's are used
- A ciphertext block depends on all preceding plaintext blocks
  - Sequential encryption, cannot use parallel hardware
- Errors in one block of ciphertext propagate to two blocks
  - one bit error in $C_j$ affects all bits in $M_j$ and one bit in $M_{j+1}$

# Block Ciphers vs. Stream Ciphers

- A block cipher operates on blocks of fixed length.
- A stream cipher is a symmetric key cipher where plaintext bits are combined with a pseudorandom cipher bit stream (keystream), typically by an exclusive-or (xor) operation.

# Output Feedback (OFB)

- Output feedback (OFB): construct a pseudorandom number generator (PRNG) to obtain a one time pad and XOR the message with the pad
  - Encryption: $X_0$=IV, $X_i$= $E_k[X_{i-1}]$ , $C_i = P_i + X_i$
  - Decryption: $X_0$=IV, $X_i$= $E_k[X_{i-1}]$ , $P_i = C_i + X_i$

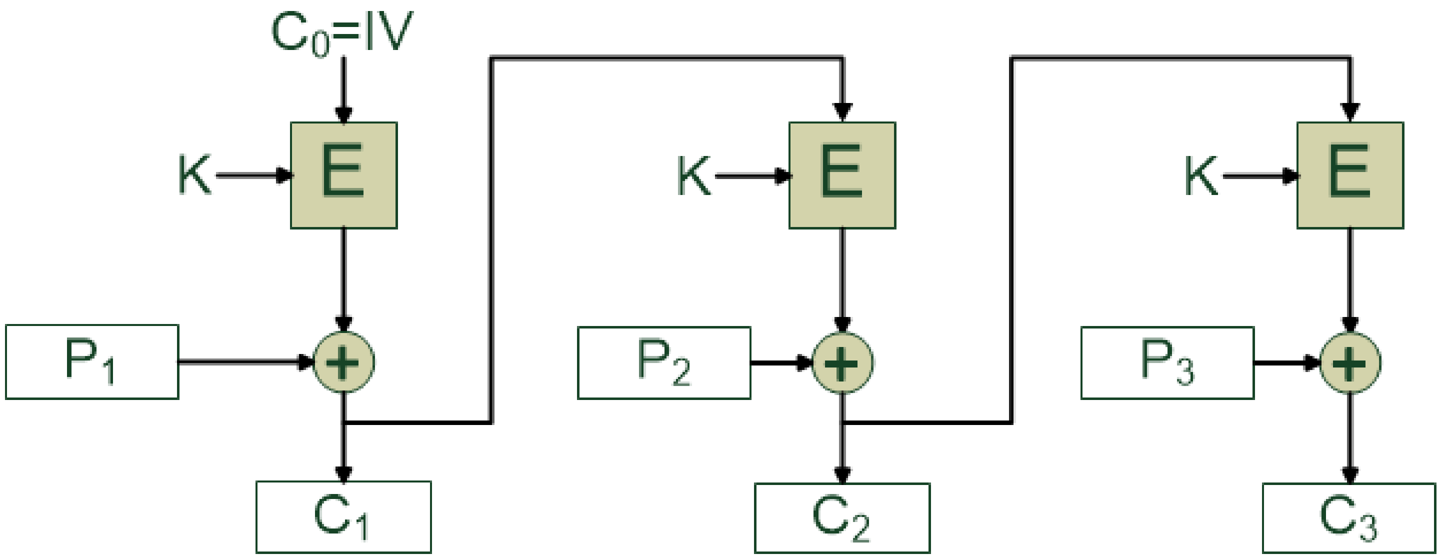


# Properties of OFB

- Randomized encryption
- Sequential encryption, but preprocessing possible
  - Generate the key before the message comes
- Error propagation limited
  - Only the changed bits are lost
- It can only be used as a stream cipher

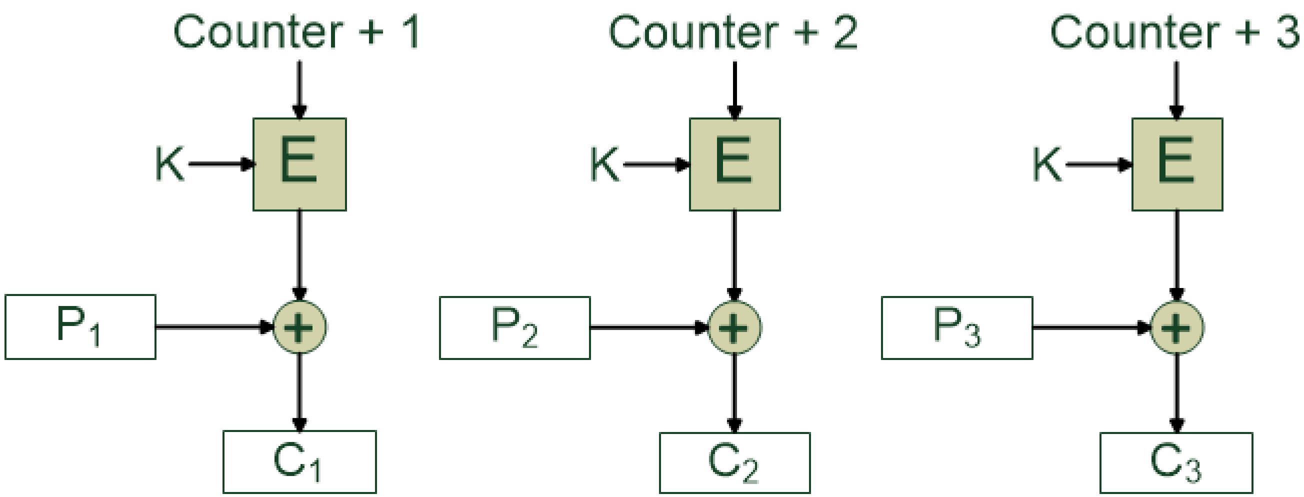
# Cipher Feedback (CFB)

- Cipher Feedback (CFB): the message is XORed with the feedback of encrypting the previous block
  - Encryption: $C_0$=IV, $C_i = E_k[C_{i-1}] + P_i$
  - Decryption: $C_0$=IV, $P_i = E_k[C_{i-1}] + C_i$



# Counter Mode (CTR)

- Counter Mode (CTR): Another way to construct pseudo random number generator using DES
  - $X_i = E_k[\text{Counter}+i]$
  - $C_i = P_i \oplus X_i$
  - Sender and receiver share a counter value (does not need to be secret) and the secret key



# Properties of CTR

- Software and hardware efficiency: different blocks can be encrypted in parallel.
- Preprocessing: the encryption part can be done offline and when the message is known, just do the XOR.
- Random Access: decryption of a block can be done in random order, very useful for hard-disk encryption.
- Messages of Arbitrary Length: ciphertext is the same length with the plaintext (i.e., no IV).