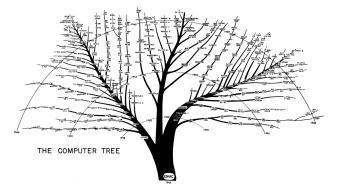


Last time... What is computation

Computer science is about logic, problem solving, and creativity

Fixed Program Computers

- Abacus
- Antikythera Mechanism
- Pascaline
- Leibniz Wheel
- Jacquard's Loom
- Babbage Difference Engine
- The Hollerith Electric Tabulating System
- Atanasoff-Berry Computer (ABC)
- Turing Bombe



Declarative knowledge

- Axioms (definitions)
- Statements of fact

Imperative knowledge

- How to do something
- A sequence of specific instructions (what computation is about)

Stored Program Computers

Problem solving



- What if input is a machine (description) itself?
- Universal Turing machines
 - An abstract general purpose computer

Lecture Overview

- Building a Computer
- The Harvey Mudd Miniature Machine (HMMM)

Disclaimer: Much of the material and slides for this lecture were borrowed from

- Gregory Kesden's CMU 15-110 class
- —David Stotts' UNC-CH COMP 110H class
- —Swami lyer's Umass Boston CS110 class

Lecture Overview

- Building a Computer
- The Harvey Mudd Miniature Machine (HMMM)



CS for All, by C. Alvarado, Z. Dodds, G. Kuenning & R. Libeskind-Hadas

Programming languages (PLs)

Disclaimer: Much of the material and slides for this lecture were borrowed from

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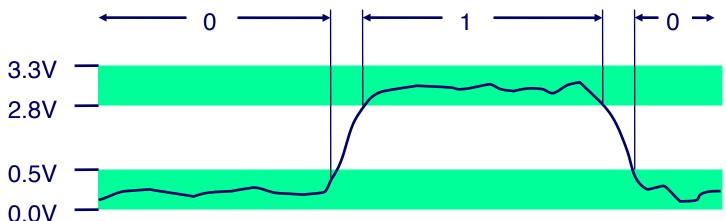
Lecture Overview

- Building a Computer
- The Harvey Mudd Miniature Machine (HMMM)

Building a Computer

- Numbers
- Letters and Strings
- Structured Information
- Boolean Algebra and Functions
- Logic Using Electrical Circuits
- Computing With Logic
- Memory
- von Neumann Architecture

- At the most fundamental level, a computer manipulates electricity according to specific rules
- To make those rules produce something useful, we need to associate the electrical signals with the numbers and symbols that we, as humans, like to use
- To represent integers, computers use combinations of numbers that are powers of 2, called the base 2 or **binary representation**
 - **bit** = **0** or **1**
 - False or True
 - Off or On
 - Low voltage or High voltage

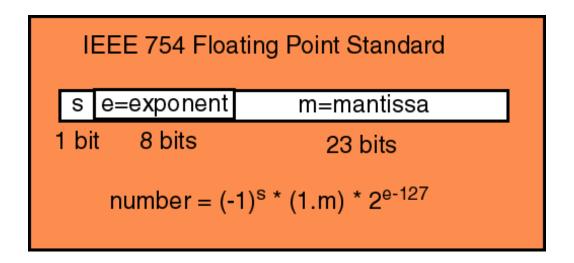


- With four consecutive powers 2⁰, 2¹, 2², 2³, we can make all of the integers from 0 to 15 using 0 or 1 of each of the four powers
- For example, $13_{10} = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 1101_2$; in other words, 1101 in base 2 means $1101_2 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 13_{10}$
- Analogously, 603 in base 10 means $603_{10} = 6.10^2 + 0.10^1 + 3.10^0$ and 207 in base 8 means $207_8 = 2.8^2 + 0.8^1 + 7.8^0 = 135_{10}$
- In general, if we choose some base $b \ge 2$, every positive integer between 0 and $b^d 1$ can be uniquely represented using d digits, with coefficients having values 0 through b-1
- A modern 64-bit computer can represent integers up to 2⁶⁴ 1

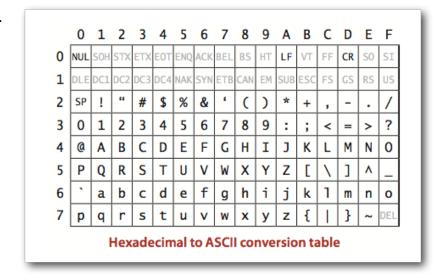
- Arithmetic in any base is analogous to arithmetic in base 10
- Examples of addition in base 10 and base 2

- To represent a negative integer, a computer typically uses a system called two's complement, which involves flipping the bits of the positive number and then adding 1
- For example, on an 8-bit computer, 3 = 00000011, so
 -3 = 11111101

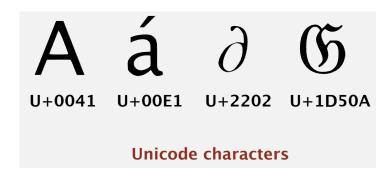
- If we are using base 10 and only have eight digits to represent our numbers, we might use the first six digits for the fractional part of a number and last two for the exponent
- For example, 31415901 would represent 0.314159 × 10¹ = 3.14159
- Computers use a similar idea to represent fractional numbers



- In order to represent letters numerically, we need a convention on the encoding
- The American National Standards Institute (ANSI) has established such a convention, called ASCII (American Standard Code for Information Interchange)
- ASCII defines encodings for the upperand lower-case letters, numbers, and a select set of special characters
- ASCII, being an 8-bit code, can only represent 256 different symbols, and doesn't provide for characters used in many languages



- The International Standards Organization's (ISO) 16-bit Unicode system can represent every character in every known language, with room for more
- Unicode being somewhat wasteful of space for English documents, ISO also defined several "Unicode Transformation Formats" (UTF), the most popular being UTF-8



• Emojis are just like characters, and they have a standard, too

Sm	ileys & Peop	ole													
ace	e-positive														
Nº	Code	Browser	Appl	Googd	Twtr.	One	FB	FBM	Sams.	Wind.	GMail	SB	DCM	KDDI	CLDR Short Name
1	U+1F600	\odot	<u></u>		:		U			(iii)	~	_	_	_	grinning face
2	U+1F601				6	8	00		63	(8	步	222	(a)	beaming face with smilin eyes
3	U+1F602			\(\text{\cond}\)	(2)		(2)	~	(2)	6	(i)	਼ੌ	_	@	face with tears of joy
4	U+1F923	3	€	②	2	10	½	_		Ø	_	_	_	-	rolling on the floor laughing
5	U+1F603		<u></u>	<u></u>	U		e e	¥	()	<u>•</u>	**	<u>a</u>	**	<u>@</u>	grinning face with big eyes
6	U+1F604			~	9	\(\text{\ti}\xitit}\\ \text{\texi\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\te}\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\tetx{\texi{\text{\texi}\text{\text{\text{\text{\text{\text{\text{\texi}\text{\text{\text{\texi}\text{\text{\texi}\text{\text{\text{\texi}\text{\text{\text{\texi}\text{\text{\text{\text{\text{\tetx{\texi}\text{\texi}\text{\texi}\text{\texi}\text{\text{\texi}\text{\texi}\text{\texi}\text{\texi}\text{\texi}\text{\text{\texit}\tint{\texi}\text{\texi}\text{\texitit}\\\ \ti}\text{\tex	\(\text{\te}\text{\texi}\text{\text{\text{\text{\text{\texi}\text{\text{\text{\text{\text{\texi}\text{\text{\texit{\tex{\text{\text{\text{\text{\text{\texi}\text{\texi}\text{\tex	~	(<u></u>	**	ê	_	-	grinning face with smilin eyes
7	U+1F605			=	8	3	6	3	(2)	<u></u>	₩.	_	200	_	grinning face with sweat
8	U+1F606	&	23	\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\	25	**	24	3	3	8	••	_	₩	-	grinning squinting face
9	U+1F609	<u> </u>	6	<u></u>	13	•5	•5	0	•	<u> </u>	2	્	U	0	winking face

:

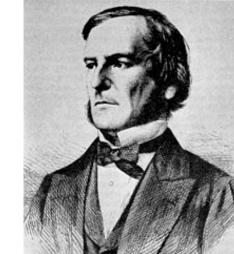
 Full Emoji List, v5.0 https://unicode.org/emoji/charts/full-emoji-list.html

- A string is represented as a sequence of numbers, with a "length field" at the very beginning that specifies the length of the string
- For example, in ASCII the sequence 99, 104, 111, 99, 111, 108, 97, 116, 101 translates to the string "chocolate", with the length field set to 9

Structured Information

- We can represent any information as a sequence of numbers
- Examples
 - A picture can be represented as a sequence of pixels, each represented as three numbers giving the amount of red, green, and blue at that pixel
 - A sound can be represented as a temporal sequence of "sound pressure levels" in the air
 - A movie can be represented as a temporal sequence of individual pictures, usually 24 or 30 per second, along with a matching sound sequence

- Boolean variables are variables that take the value True (1) or False (0)
- With Booleans 1 and 0 we could use the operations (functions) AND, OR, and NOT to build up more interesting Boolean functions



- A truth table for a Boolean function is a listing of all possible combinations of values of the input variables, together with the result produced by the function
- Truth tables for AND, OR, and NOT functions

x	y	x and y
0	0	0
0	1	0
1	0	0
1	1	1

x	y	x or y
0	0	0
0	1	1
1	0	1
1	1	1

\boldsymbol{x}	NOT x
0	1
1	0

- Any function of Boolean variables, no matter how complex, can be expressed in terms of AND, OR, and NOT
- Consider the proposition "if you score over 93% in this course, then you will get an A"
- The truth values for the above proposition is given by the "implication" function ($x \implies y$) having the following truth table

x	y	$x \implies y$
0	0	1
0	1	1
1	0	0
1	1	1

• The function can be compactly written as NOT x OR x AND y (or $\bar{x} + xy$)

- The minterm expansion algorithm, due to Claude Shannon, provides a systematic approach for building Boolean functions from truth tables
- Minterm expansion algorithm
 - Write down the truth table for the Boolean function under consideration
 - 2. Delete all rows from the truth table where the value of the function is 0
 - For each remaining row, create something called a "minterm" as follows
 - For each variable that has a 1 in that row, write the name of the variable. If the input variable is 0 in that row, write the variable with a negation symbol to ${\tt NOT}$ it
 - Now AND all of these variables together
 - 4. Combine all of the minterms for the rows using OR

For the implication function, the minterm expansion algorithm applied as follows

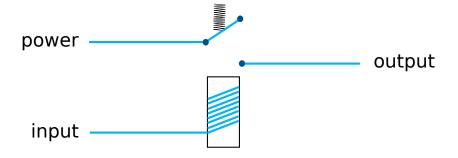
x	y	$x \implies y$	minterm
0	0	1	ar x ar y
0	1	1	$ar{x}y$
		0	
1	1	1	xy

produces the Boolean function $\bar x \bar y + \bar x y + x y$, which is equivalent to the simpler function $\bar x + x y$

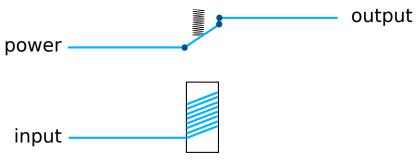
 Finding the simplest form of a Boolean function is provably as hard as some of the hardest (unsolved) problems in mathematics and computer science

Logic using Electrical Circuits

• An electromechanical switch in which when the input is off, the output is "low" (0), and when the input is on, the output is "high" (1)



The NOT gate constructed using a switch that conducts only when the input is off

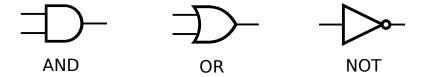


• The AND and OR gates for computing x and y and x or y, constructed using electromechanical switches

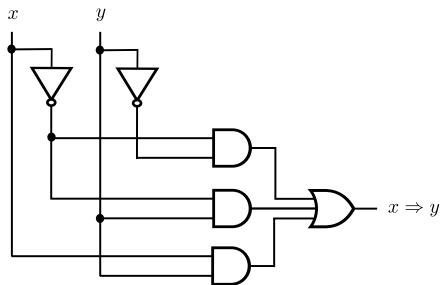


Logic using Electrical Circuits

- Computers today are built with much smaller, much faster, more reliable, and more efficient transistorized switches
- Since the details of the switches aren't terribly important at this level of abstraction, we represent, or "abstract", the gates using the following symbols



ullet A logical circuit for the implication function ar xar y + ar xy + xy



Computing with Logic

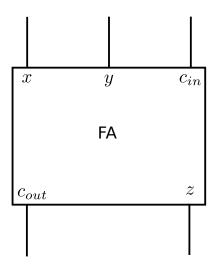
 A truth table describing the addition of two two-bit numbers to get a three-bit result

x	y	x+y
00	00	000
00	01	001
00	10	010
:	:	:
01	10	011
01	11	100
	:	
11	11	110

 Building a corresponding circuit using the minterm expansion algorithm is infeasible — adding two 16-bit numbers, for example, will result in a circuit with several billion gates

Computing with Logic

• We build a relatively simple circuit called a full adder (FA) that does just one column of addition



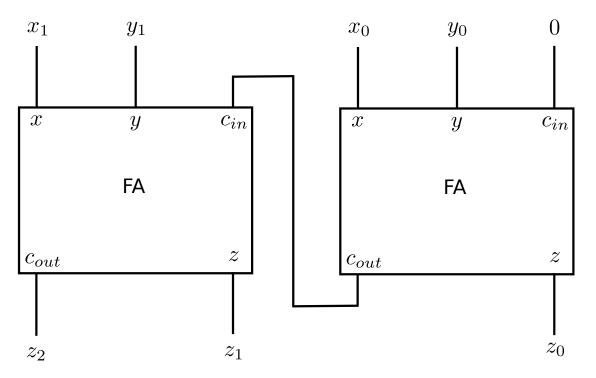
\boldsymbol{x}	y	c_{in}	z	c_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

The minterm expansion principle applied to the truth table for the FA circut yields the following Boolean functions

$$z = \bar{x}\bar{y}c_{in} + \bar{x}y\bar{c}_{in} + x\bar{y}\bar{c}_{in} + xyc_{in}$$
$$c_{out} = \bar{x}yc_{in} + x\bar{y}c_{in} + xy\bar{c}_{in} + xyc_{in}$$

Computing with Logic

- We can "chain" n full adders together to add two n-bit numbers, and the resulting circuit is called a ripple-carry adder
- A 2-bit ripple-carry adder

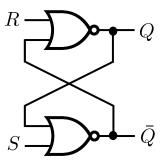


Memory

Truth table for a NOR gate (OR followed by NOT)

x	y	x nor y
0	0	1
0	1	0
1	0	0
1	1	0

- A latch is a device that allows us to "lock" a bit and retrieve it later
- By aggregating millions of latches we have the Random Access Memory (RAM)
- A latch can be constructed from two NOR gates as shown below



where the input S is known as "set" while the input R is known as "reset"

Recall: Stored Program Concept

- Stored-program concept is the fundamental principle of the ENIAC's successor, the EDVAC (Electronic Discrete Variable Automatic Computer)
- Instructions were stored in memory sequentially with their data

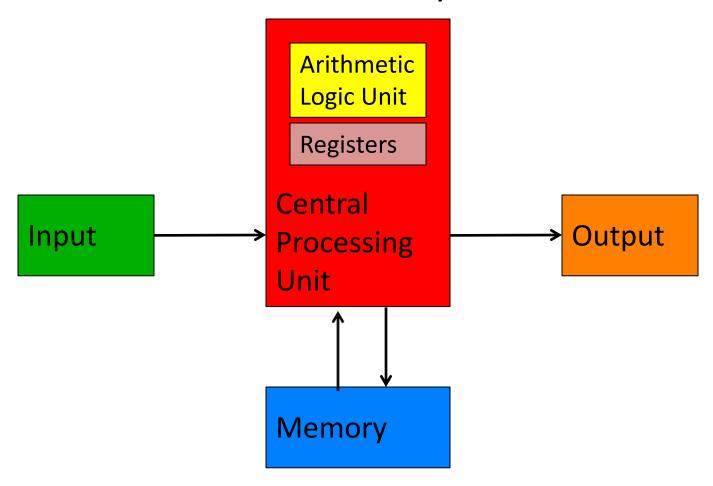
 Instructions were executed sequentially except where a conditional instruction would cause a jump to an instruction someplace other than the next instruction

- Mauchly and Eckert are generally credited with the idea of the stored-program
- BUT: John von Neumann publishes a draft report that describes the concept and earns the recognition as the inventor of the concept
 - "von Neumann architecture"
 - A First Draft of a Report of the EDVAC published in 1945
 - http://www.worldpowersystems.com/J/EDVAC/

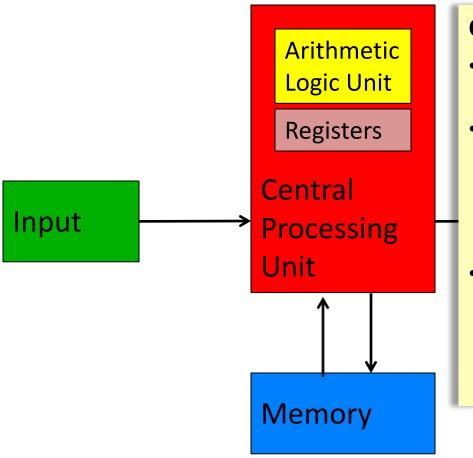


von Neumann, Member of the Navy Bureau of Ordinance 1941-1955

"Fetch-Decode-Execute" cycle



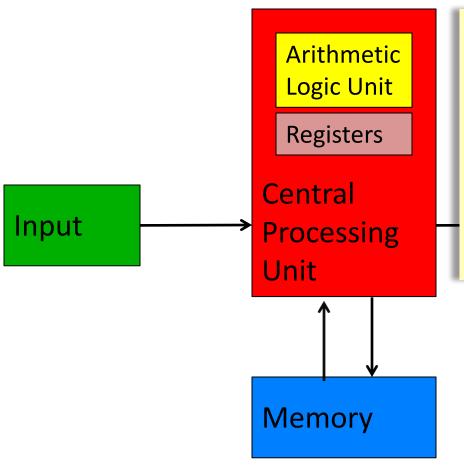
"Fetch-Decode-Execute" cycle



Central Processing Unit (CPU)

- In a modern computer, the CPU is where all the computation takes place
- The CPU has devices such as ripplecarry adders, multipliers, etc. for doing arithmetic. In addition, it has a small amount of (scratch) memory called registers
- The computer's main memory, which allows storing large amounts of data, is separate from the CPU and is connected to it by wires on the computer's circuit board

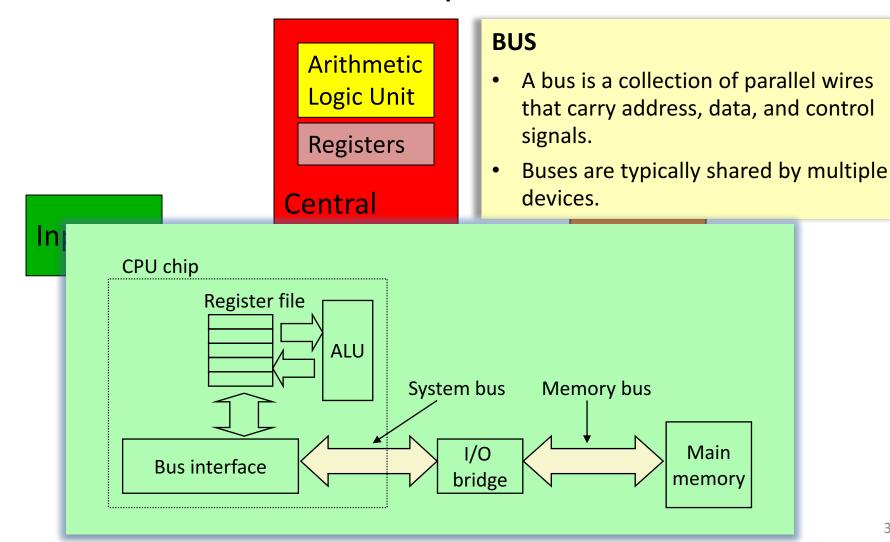
"Fetch-Decode-Execute" cycle



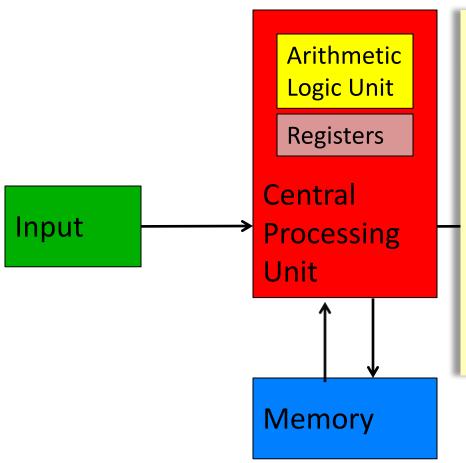
Central Processing Unit (CPU)

- ALU + Control = Processor
- Registers. Storage cells that holds heavily used program data
- Without address, specific purpose
- e.g. the operands of an arithmetic operation, the result of an operation, etc.

"Fetch-Decode-Execute" cycle



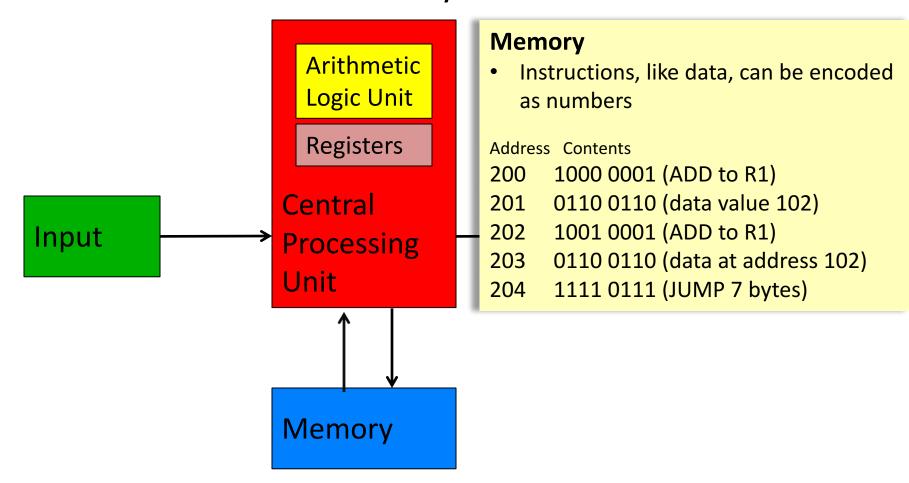
"Fetch-Decode-Execute" cycle



Memory

- A program, which is usually a long list of instructions, is stored in the main memory, and is copied, one instruction at a time, into a register in the CPU for execution
- The CPU has two special registers: a
 program counter that keeps track of
 the location in memory where it will
 find the next instruction and an
 instruction register that stores the
 next instruction to execute

"Fetch-Decode-Execute" cycle



von Neumann Architecture

• Let's assume an 8-bit computer with only four instructions:

_	add,	subtract,	multiply,	and	divide
---	------	-----------	-----------	-----	--------

Opcode	Meaning
00	Add
01	Subtract
10	Multiply
11	Divide

- Each of the instructions will need a number,
 which is called an operation code (or opcode), to represent it
- Next, let's assume that our computer has four registers, numbered 0 through 3, and 256 8-bit memory cells
- An instruction will be encoded as: the first two bits represent the instruction, the next two bits encode the "destination register", the next four bits encode the registers containing two operands
- For example, the instruction add 3 0 2 (meaning add the contents of register 2 with the contents of register 0 and store the result in register 3) will be encoded as 00110010

von Neumann Architecture

- Our computer operates by repeatedly performing the following procedure
 - 1. Send the address in the program counter (commonly called the PC) to the memory, asking it to read that location
 - 2. Load the value from memory into the instruction register
 - 3. Decode the instruction register to determine what instruction to execute and which registers to use
 - 4. Execute the requested instruction, which involves reading operands from registers, performing arithmetic, and sending the results back to the destination register
 - 5. Increment PC so that it contains the address of the next instruction in memory

Program Counter	00000000
Instruction Register	00100001
-	
Register 0	00000101
Register 1	0000010
Register 2	00000111
Register 3	00000000

Loca (Binary)	tion (Base 10)	Contents
00000000	0	00100001
0000001	1	00000000
0000010	2	00001010
00000011	3	
11111111	255	

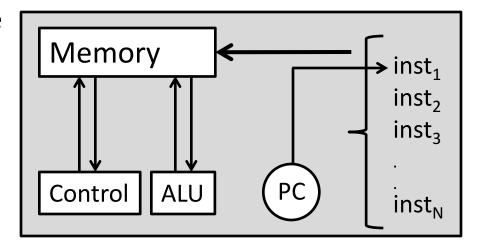
Assembly Language

- A low-level programming language for computers
- More readable, English-like abbreviations for instructions
- Architecture-specific
- Example:

```
MOV AL, 61h
MOV AX, BX
ADD EAX, 10
XOR EAX, EAX
```

Summary: Components of a Computer

- Sequential execution of machine instructions
 - The sequence of instructions are stored in the memory.
 - One instruction at a time is fetched from the memory to the control unit.
 - They are read in and treated just like data.



- PC (program counter) is responsible from the flow of control.
- PC points a memory location containing an instruction on the sequence.
- Early programmers (coders) write programs via machine instructions.

Lecture Overview

Building a Computer

The Harvey Mudd Miniature Machine (HMMM)

The Harvey Mudd Miniature Machine (HMMM)

- HMMM
- A Simple HMMM Program
- Looping
- Functions
- HMMM Instruction Set

HMMM

- A real computer must be able to
 - Move information between registers and memory
 - Get data from the outside world
 - Print results
 - Make decisions
- The Harvey Mudd Miniature Machine (HMMM) is organized as follows
 - Both instructions and data are 16 bit wide
 - In addition to the program counter and instruction register, there are 16 registers named ${\tt r0}$ through ${\tt r15}$
 - There are 256 memory locations
- Instead of programming in binary (0's and 1's), we'll use assembly language, a
 programming language where each instruction has a symbolic representation
- For example, to compute r3 = r1+r2, we'll write add r3 r1 r2
- We'll use a program to convert the assembly language into 0's and 1's the machine language that the computer can execute

A Simple HMMM Program

triangle1.hmmm: Calculate the approximate area of a triangle.

```
oread r1 # Get base b
read r2 # Get height h
mul r1 r1 r2 # b times h into r1
setn r2 2
div r1 r1 r2 # Divide by 2
write r1
halt
```

Assemble! →

```
ASSEMBLY SUCCESSFUL
 : 0000 0001 0000 0001
                                         r1 # Get base b
                                  read
 : 0000 0010 0000 0001
                                 read
                                            # Get height h
 : 1000 0001 0001 0010
                                         r1 r1 r2 # b times h into r1
                                  mul
 : 0001 0010 0000 0010
                                  setn r2 2
 : 1001 0001 0001 0010
                                  div
                                         r1 r1 r2 # Divide by 2
 : 0000 0001 0000 0010
                                  write
                                         r1
6 : 0000 0000 0000 0000
                                  halt
```

```
4
5
10
```

Looping

• Unconditional jump (jumpn N): set program counter to address N

triangle2.hmmm: Calculate the approximate areas of many triangles.

```
4
5
10
5
5
5
12
<ctrl-d>
End of input, halting program execution...
```

Looping

Conditional jump (jeqzn rX N): if rX == 0, then jump to line N

triangle3.hmmm: Calculate the approximate areas of many triangles. Stop when a base or height of zero is given.

```
read
            r1
                   # Get base b
     jeqzn r1 9  # Jump to halt if base is zero
read r2  # Get height h
     jeqzn r2 9 # Jump to halt if height is zero
3
     mul r1 r1 r2 # b times h into r1
5
           r2 2
     setn
6
     div
            r1 r1 r2 # Divide by 2
7
     write
            r1
8
     jumpn
             0
9
     halt
```

```
4
5
10
5
5
5
12
```

Looping

is_it_a_prime_number.hmmm: Calculate whether a given positive number is prime or not

```
# read the number. Please enter positive integers.
0
     read r1
      setn r2 2
                      # use this register for arithmetic operations with 2.
      setn r9 1
                      # use this register for arithmetic operations with 1.
      sub r15 r1 r9
3
      jegz r15 17
                      # check if the number is 1
      div r3 r1 r2
                      # Divide to 2. The biggest divider (denominator) should (may) be this number.
      nop
                      # there is no reason. Deleted a line, but too lazy to change all the line numbers.
      # the number is 2 or 3. So it is prime.
      sub r15 r3 r9
7
      jegz r15
      # The number is not 1, 2 or 3. The main loop starts here-----
      mod r15 r1 r3
                      # mod to check if the number is aliquot.
10
      jegz r15
              17
                      # it is not a prime number. Jump to line 17.
      sub r3 r3 r9 # subtract one from the divider
11
12
      sub r5 r3 r9 # subtract one, but on a different register to check the divider is 1 or not.
13
      jegz r5 15
                     # we successfully reduced the divider to 1. This is a prime number. Jump to line 15.
                      # jump to the start of the main loop.
14
      jumpn 9
      #----- Write 1 for prime numbers.
      write r9
                 # r9 is already 1.
15
16
      Halt
      #----- Write 0 for non-prime numbers.
17
      setn r8 0
      write r8
18
19
      halt
```

Functions

- Call a function (calln rX N): copy the next address (aka return address) into rX and then jump to address N
- Return from a function (jumpr rX): set program counter to the return address in rX
- By convention, we use register r14 to store the return address

```
square.hmmm: Calculate the square of a number \,N_{\,\cdot}\,
```

```
read
                 # Get. N
           r1
   calln
          r14 5 # Calculate N^2
   write
           r2
                 # Write answer
  halt
   nop
                 # Waste some space
Square function. N is in r1. Result (N^2) is in r2. Return address is in r14.
           r2 r1 r1 # Calculate and store N^2 in r2
   mul
   jumpr
           r14
                    # Done; return to caller
```

```
Simulate! →
```

Functions

combinations.hmmm: Calculate C(N,K) (aka N choose K) defined as C(N,K) = N!/(K!(N-K)!), where N! (N factorial) is defined as $N! = N \times (N-1) \times (N-2) \times \cdots \times 2 \times 1$, with 0! = 1.

```
r3
                     # Get N
0
     read
            r4
                    # Get K
     read
2
           r1 r3
                    # Calculate N!
     сору
3
     calln r14 15 # ...
            r5 r2 # Save N! as C(N, K)
4
     copy
5
           r1 r4 # Calculate K!
     сору
6
     calln r14 15 # ...
7
            r5 r5 r2 # N!/K!
     div
8
            r1 r3 r4 # Calculate (N - K)!
     sub
9
                     # ...
     calln
            r14 15
            r5 r5 r2 # C(N, K)
10
     div
11
    write
            r5
                     # Write answer
12
    halt
13
    nop
                     # Waste some space
14
    nop
# Factorial function. N is in r1. Result is r2. Return address is in r14.
15
             r2 1
                      # Initial product
      setn
     jegzn r1 20
16
                      # Quit if N has reached zero
17
           r2 r1 r2 # Update product
     mul
18
      addn
             r1 -1
                      # Decrement N
19
      jumpn
             16
                     # Back for more
20
             r14
                     # Done; return to caller
      jumpr
```

Simulate! →

5 2 10

Functions

Trace of the factorial function (N=4)

	instruction				r1	r2
					4	
15	setn	r2	1		4	1
16	jeqzn	r1	20		4	1
17	mul	r2	r1	r2	4	4
18	addn	r1	-1		3	4
19	jumpn	16			3 3 3 2	4
16	jeqzn	r1	20		3	4
17	mul	r2	r1	r2	3	12
18	addn	r1	- 1		2	12
19	jumpn	16			2	12
16	jeqzn	r1	20		2	12
17	mul	r2	r1	r2	2	24
18	addn	r1	- 1		1	24
19	jumpn	16			1	24
16	jeqzn	r1	20		1	24
17	mul	r2	r1	r2	1	24
18	addn	r1	-1		0	24
19	jumpn	16			0	24
16	jeqzn	r1	20		0	24
20	jumpr	r14	1		0	24

Trace of the program (N=5, K=2)

	instru	ction	r1	r2	r3	r4	r5	r14
0	read	r3			5			
1	read	r4			5	2		
2	сору	r1 r3	5		5	2		
3	calln	r14 15	5	120	5	2		4
4	сору	r5 r2	5	120	5	2	120	4
5	сору	r1 r4	2	120	5	2	120	4
6	calln	r14 15	2	2	5	2	120	7
7	div	r5 r5 r2	2	2	5	2	60	7
8	sub	r1 r3 r4	3	2	5	2	60	7
9	calln	r14 15	3	6	5	2	60	10
10	div	r5 r5 r2	3	6	5	2	10	10
11	write	r5	3	6	5	2	10	10
12	halt		3	6	5	2	10	10

HMMM Instruction Set

System instructions

halt	stop
read rX	place user input in register rx
write rX	print contents of register rX
nop	do nothing

Setting register data

setn rX N	set register rx equal to the integer x (-128 to 127)
addn rX N	add integer N (-128 to 127) to register $\texttt{r} \texttt{X}$
copy rX rY	set rX=rY

Arithmetic

```
add rX rY rZ set rX=rY+rZ
sub rX rY rZ set rX=rY-rZ
neg rX rY set rX=-rY
mul rX rY rZ set rX=rY*rZ
div rX rY rZ set rX=rY*rZ (integer division; no remainder)
mod rX rY rZ set rX=rY%rZ (returns the remainder of integer division)
```

HMMM Instruction Set

Jumps

```
jumpn N set program counter to address N
jumpr rX set program counter to address in rX
jeqzn rX N if rX==0, then jump to line N
jnezn rX N if rX!=0, then jump to line N
jgtzn rX N if rX>0, then jump to line N
jltzn rX N if rX<0, then jump to line N
calln rX N copy the next address into rX and then jump to address N</pre>
```

Interacting with memory

```
loadn rX N load register rX with the contents of address N
storen rX N store contents of register rX into address N
loadr rX rY load register rX with data from the address location held in register rY
storer rX rY store contents of register rX into address held in register rY
```