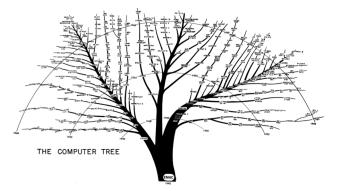


Last time... What is computation

Computer science is about logic, problem solving, and creativity

Fixed Program Computers

- Abacus
- Antikythera Mechanism
- Pascaline
- Leibniz Wheel
- Jacquard's Loom
- Babbage Difference Engine
- The Hollerith Electric Tabulating System
- Atanasoff-Berry Computer (ABC)
- Turing Bombe



Declarative knowledge

- Axioms (definitions)
- Statements of fact

Imperative knowledge

- How to do something
- A sequence of specific instructions (what computation is about)

Stored Program Computers

Problem solving



- What if input is a machine (description) itself?
- Universal Turing machines
 - An abstract general purpose computer

Lecture Overview

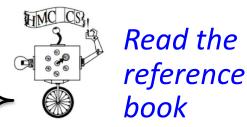
- Building a Computer
- The Harvey Mudd Miniature Machine (HMMM)

Disclaimer: Much of the material and slides for this lecture were borrowed from

- Gregory Kesden's CMU 15-110 class
- David Stotts' UNC-CH COMP 110H class
- —Swami Iyer's Umass Boston CS110 class

Lecture Overview

- Building a Computer
- The Harvey Mudd Miniature Machine (HMMM)



CS for All, by C. Alvarado, Z. Dodds, G. Kuenning & R. Libeskind-Hadas

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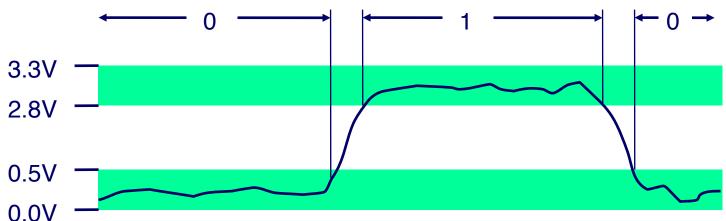
Lecture Overview

- Building a Computer
- The Harvey Mudd Miniature Machine (HMMM)

Building a Computer

- Numbers
- Letters and Strings
- Structured Information
- Boolean Algebra and Functions
- Logic Using Electrical Circuits
- Computing With Logic
- Memory
- von Neumann Architecture

- At the most fundamental level, a computer manipulates electricity according to specific rules
- To make those rules produce something useful, we need to associate the electrical signals with the numbers and symbols that we, as humans, like to use
- To represent integers, computers use combinations of numbers that are powers of 2, called the base 2 or **binary representation**
 - bit = 0 or 1
 - False or True
 - Off or On
 - Low voltage or High voltage

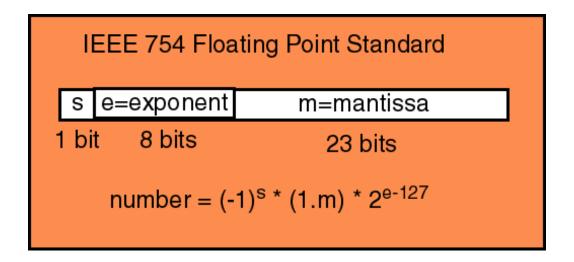


- With four consecutive powers 2⁰, 2¹, 2², 2³, we can make all of the integers from 0 to 15 using 0 or 1 of each of the four powers
- For example, $13_{10} = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 1101_2$; in other words, 1101 in base 2 means $1101_2 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 13_{10}$
- Analogously, 603 in base 10 means $603_{10} = 6.10^2 + 0.10^1 + 3.10^0$ and 207 in base 8 means $207_8 = 2.8^2 + 0.8^1 + 7.8^0 = 135_{10}$
- In general, if we choose some base $b \ge 2$, every positive integer between 0 and $b^d 1$ can be uniquely represented using d digits, with coefficients having values 0 through b-1
- A modern 64-bit computer can represent integers up to 2⁶⁴ 1

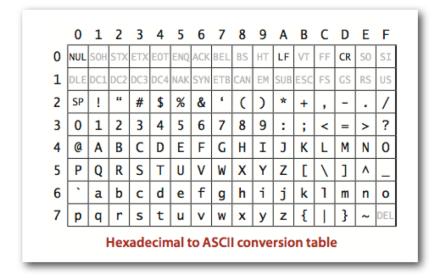
- Arithmetic in any base is analogous to arithmetic in base 10
- Examples of addition in base 10 and base 2

- To represent a negative integer, a computer typically uses a system called two's complement, which involves flipping the bits of the positive number and then adding 1
- For example, on an 8-bit computer, 3 = 00000011, so
 -3 = 11111101

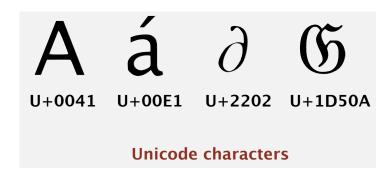
- If we are using base 10 and only have eight digits to represent our numbers, we might use the first six digits for the fractional part of a number and last two for the exponent
- For example, 31415901 would represent 0.314159 × 10¹ = 3.14159
- Computers use a similar idea to represent fractional numbers



- In order to represent letters numerically, we need a convention on the encoding
- The American National Standards Institute (ANSI) has established such a convention, called ASCII (American Standard Code for Information Interchange)
- ASCII defines encodings for the upperand lower-case letters, numbers, and a select set of special characters
- ASCII, being an 8-bit code, can only represent 256 different symbols, and doesn't provide for characters used in many languages



- The International Standards Organization's (ISO) 16-bit Unicode system can represent every character in every known language, with room for more
- Unicode being somewhat wasteful of space for English documents, ISO also defined several "Unicode Transformation Formats" (UTF), the most popular being UTF-8



• Emojis are just like characters, and they have a standard, too

face-positive															
Nº Co	ode	Browser	Appl	Googd	Twtr.	One	FB	FBM	Sams.	Wind.	GMail	SB	DCM	KDDI	CLDR Short Name
1 0+11	F600		<u></u>		U		U	÷		(iii)	*	_	_	_	grinning face
2 (0+1)	F601				6	ô	00		63	<u></u>	8	뜐	***	(a)	beaming face with smilling eyes
3 0+11	F602			&	(3)	(3)	(2)	~		6	(iii)	ૄ ૺ૾	_	(4)	face with tears of joy
4 0+11	F923	3	3	3	0	70	2	_			_	-	_	-	rolling on the floor laughing
5 0+11	F603		<u></u>	<u></u>	U	:	¥	:	3	<u>•</u>	್	<u>@</u>	22	@	grinning face with big eyes
6 (0+1)	F604			~	9	6	\$	~	(<u></u>	20	ê	_	-	grinning face with smilir eyes
7 0+11	F605			=	8	3	6	=	(2)	<u></u>	₩.	-	20	_	grinning face with swea
8 0+11	F606	&	2	3	35	3	25	3	3	\sigma	v	-	梦	_	grinning squinting face
9 0+11	F609	6	(-	<u>:</u>	53	(5)	•5	C	•	(E)	8	ૄ	ıţ	0	winking face

:

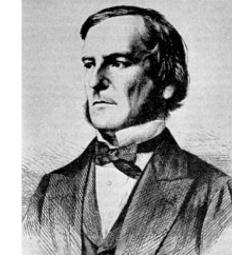
Full Emoji List, v5.0
 https://unicode.org/emoji/charts/full-emoji-list.html

- A string is represented as a sequence of numbers, with a "length field" at the very beginning that specifies the length of the string
- For example, in ASCII the sequence 99, 104, 111, 99, 111, 108, 97, 116, 101 translates to the string "chocolate", with the length field set to 9

Structured Information

- We can represent any information as a sequence of numbers
- Examples
 - A picture can be represented as a sequence of pixels, each represented as three numbers giving the amount of red, green, and blue at that pixel
 - A sound can be represented as a temporal sequence of "sound pressure levels" in the air
 - A movie can be represented as a temporal sequence of individual pictures, usually 24 or 30 per second, along with a matching sound sequence

- Boolean variables are variables that take the value True (1) or False (0)
- With Booleans 1 and 0 we could use the operations (functions) AND, OR, and NOT to build up more interesting Boolean functions



- A truth table for a Boolean function is a listing of all possible combinations of values of the input variables, together with the result produced by the function
- Truth tables for AND, OR, and NOT functions

x	y	$\mid x$ and y
0	0	0
0	1	0
1	0	0
1	1	1

x	y	x or y
0	0	0
0	1	1
1	0	1
1	1	1

\boldsymbol{x}	NOT x
0	1
1	0

- Any function of Boolean variables, no matter how complex, can be expressed in terms of AND, OR, and NOT
- Consider the proposition "if you score over 93% in midterm and final exams, then you will get an A"
- The truth values for the above proposition is given by the "implication" function ($x \implies y$) having the following truth table

x	y	$x \implies y$
0	0	1
0	1	1
1	0	0
1	1	1

• The function can be compactly written as NOT x OR x AND y (or $\bar{x}+xy$)

- The minterm expansion algorithm, due to Claude Shannon, provides a systematic approach for building Boolean functions from truth tables
- Minterm expansion algorithm
 - Write down the truth table for the Boolean function under consideration
 - 2. Delete all rows from the truth table where the value of the function is 0
 - For each remaining row, create something called a "minterm" as follows
 - For each variable that has a 1 in that row, write the name of the variable. If the input variable is 0 in that row, write the variable with a negation symbol to ${\tt NOT}$ it
 - Now AND all of these variables together
 - 4. Combine all of the minterms for the rows using OR

For the implication function, the minterm expansion algorithm applied as follows

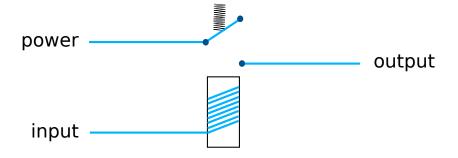
x	y	$x \implies y$	minterm
0	0	1	ar xar y
0	1	1	$ar{x}y$
		0	
1	1	1	xy

produces the Boolean function $\, \bar x \bar y + \bar x y + x y$, which is equivalent to the simpler function $\, \bar x + x y$

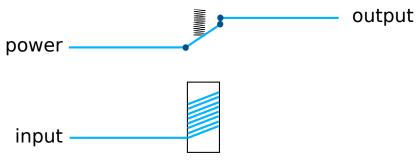
 Finding the simplest form of a Boolean function is provably as hard as some of the hardest (unsolved) problems in mathematics and computer science

Logic using Electrical Circuits

An electromechanical switch in which when the input is off, the output is "low" (0), and when the input is on, the output is "high" (1)



The NOT gate constructed using a switch that conducts only when the input is off

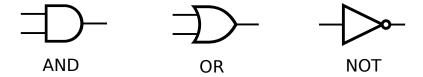


• The AND and OR gates for computing x and y and x or y, constructed using electromechanical switches

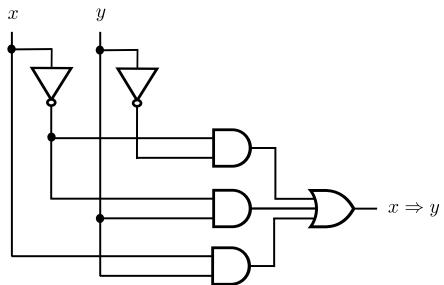


Logic using Electrical Circuits

- Computers today are built with much smaller, much faster, more reliable, and more efficient transistorized switches
- Since the details of the switches aren't terribly important at this level of abstraction, we represent, or "abstract", the gates using the following symbols



• A logical circuit for the implication function ar xar y + ar xy + xy



Computing with Logic

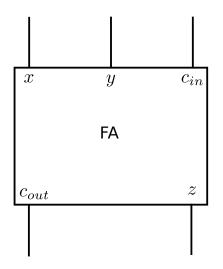
 A truth table describing the addition of two two-bit numbers to get a three-bit result

x	y	x + y
00	00	000
00	01	001
00	10	010
:		:
01	10	011
01	11	100
	:	
11	11	110

 Building a corresponding circuit using the minterm expansion algorithm is infeasible — adding two 16-bit numbers, for example, will result in a circuit with several billion gates

Computing with Logic

• We build a relatively simple circuit called a full adder (FA) that does just one column of addition



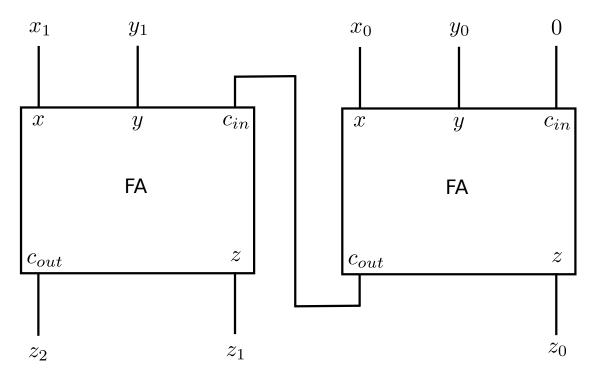
\boldsymbol{x}	y	c_{in}	z	c_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

The minterm expansion principle applied to the truth table for the FA circut yields the following Boolean functions

$$z = \bar{x}\bar{y}c_{in} + \bar{x}y\bar{c}_{in} + x\bar{y}\bar{c}_{in} + xyc_{in}$$
$$c_{out} = \bar{x}yc_{in} + x\bar{y}c_{in} + xy\bar{c}_{in} + xyc_{in}$$

Computing with Logic

- We can "chain" n full adders together to add two n-bit numbers, and the resulting circuit is called a ripple-carry adder
- A 2-bit ripple-carry adder

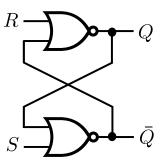


Memory

Truth table for a NOR gate (OR followed by NOT)

x	y	x nor y
0	0	1
0	1	0
1	0	0
1	1	0

- A latch is a device that allows us to "lock" a bit and retrieve it later
- By aggregating millions of latches we have the Random Access Memory (RAM)
- A latch can be constructed from two NOR gates as shown below



where the input S is known as "set" while the input R is known as "reset"

Recall: Stored Program Concept

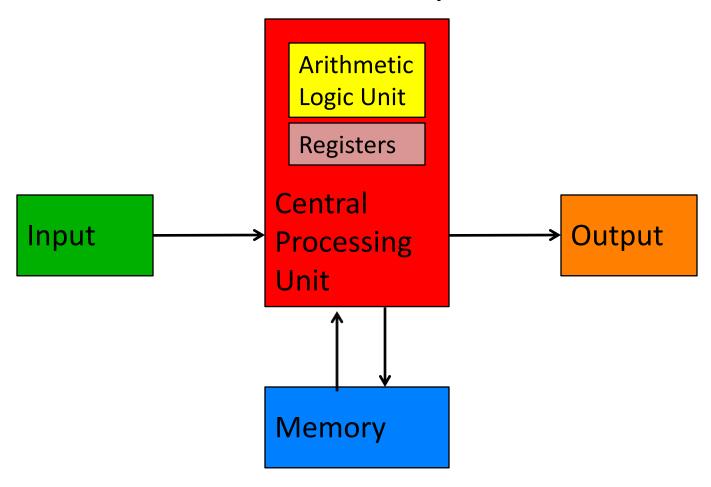
- Stored-program concept is the fundamental principle of the ENIAC's successor, the EDVAC (Electronic Discrete Variable Automatic Computer)
- Instructions were stored in memory sequentially with their data
- Instructions were executed sequentially except where a conditional instruction would cause a jump to an instruction someplace other than the next instruction

- Mauchly and Eckert are generally credited with the idea of the stored-program
- BUT: John von Neumann publishes a draft report that describes the concept and earns the recognition as the inventor of the concept
 - "von Neumann architecture"
 - A First Draft of a Report of the EDVAC published in 1945
 - http://www.worldpowersystems.com/J/EDVAC/

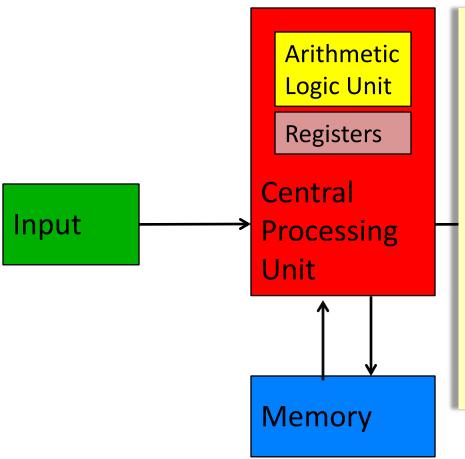


von Neumann, Member of the Navy Bureau of Ordinance 1941-1955

"Fetch-Decode-Execute" cycle



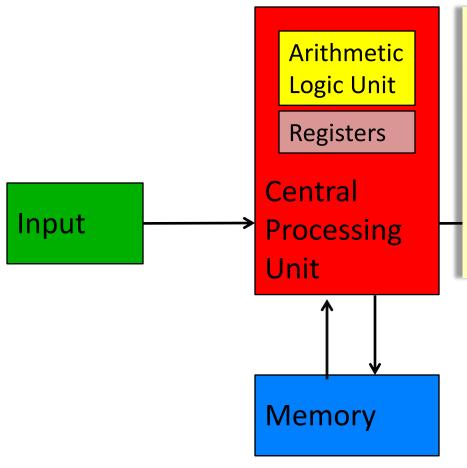
"Fetch-Decode-Execute" cycle



Central Processing Unit (CPU)

- In a modern computer, the CPU is where all the computation takes place
- The CPU has devices such as ripplecarry adders, multipliers, etc. for doing arithmetic. In addition, it has a small amount of (scratch) memory called registers
- The computer's main memory, which allows storing large amounts of data, is separate from the CPU and is connected to it by wires on the computer's circuit board

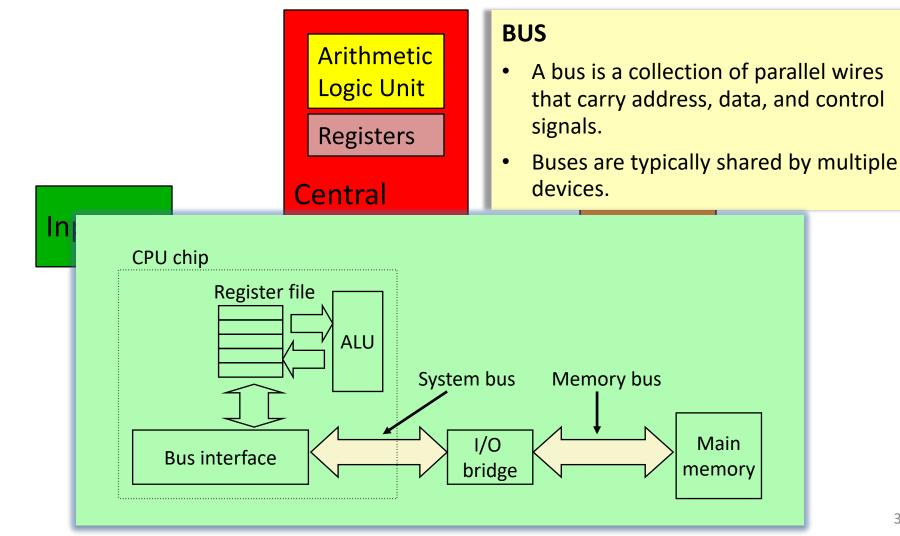
"Fetch-Decode-Execute" cycle



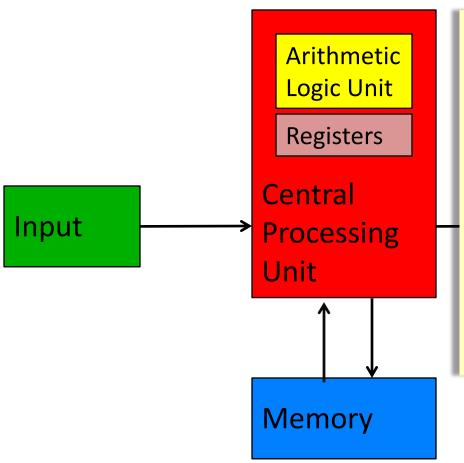
Central Processing Unit (CPU)

- ALU + Control = Processor
- Registers. Storage cells that holds heavily used program data
- Without address, specific purpose
- e.g. the operands of an arithmetic operation, the result of an operation, etc.

"Fetch-Decode-Execute" cycle



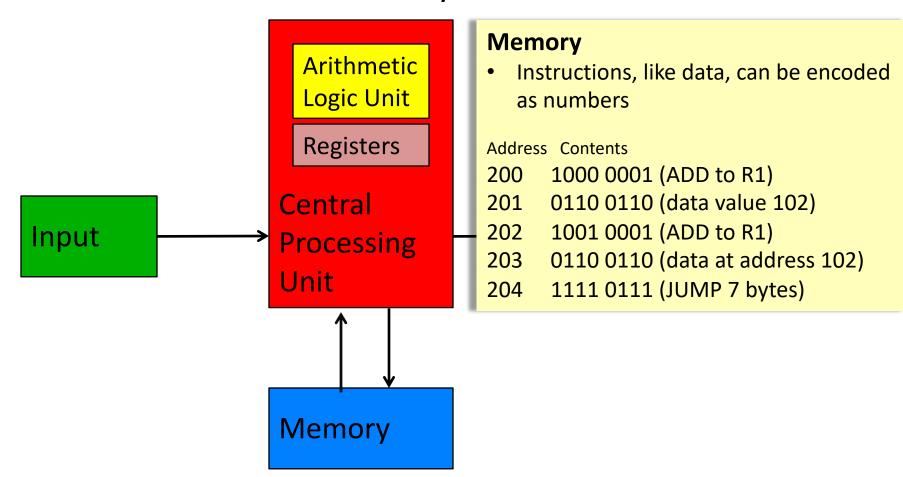
"Fetch-Decode-Execute" cycle



Memory

- A program, which is usually a long list of instructions, is stored in the main memory, and is copied, one instruction at a time, into a register in the CPU for execution
- The CPU has two special registers: a program counter that keeps track of the location in memory where it will find the next instruction and an instruction register that stores the next instruction to execute

"Fetch-Decode-Execute" cycle



von Neumann Architecture

• Let's assume an 8-bit computer with only four instructions:

_	add, subtract,	multiply,	and divide
---	----------------	-----------	------------

Opcode	Meaning
00	Add
01	Subtract
10	Multiply
11	Divide

- Each of the instructions will need a number,
 which is called an operation code (or opcode), to represent it
- Next, let's assume that our computer has four registers, numbered 0 through 3, and 256 8-bit memory cells
- An instruction will be encoded as: the first two bits represent the instruction, the next two bits encode the "destination register", the next four bits encode the registers containing two operands
- For example, the instruction add 3 0 2 (meaning add the contents of register 2 with the contents of register 0 and store the result in register 3) will be encoded as 00110010

von Neumann Architecture

- Our computer operates by repeatedly performing the following procedure
 - 1. Send the address in the program counter (commonly called the PC) to the memory, asking it to read that location
 - 2. Load the value from memory into the instruction register
 - 3. Decode the instruction register to determine what instruction to execute and which registers to use
 - 4. Execute the requested instruction, which involves reading operands from registers, performing arithmetic, and sending the results back to the destination register
 - 5. Increment PC so that it contains the address of the next instruction in memory

Program Counter	00000000
Instruction Register	00100001
Register 0	00000101
Register 1	00000010
Register 2	00000111
Register 3	00000000
Register 3	00000000

Location		Contents
(Binary)	(Base 10)	
00000000	0	00100001
0000001	1	00000000
00000010	2	00001010
00000011	3	
	111	
11111111	255	

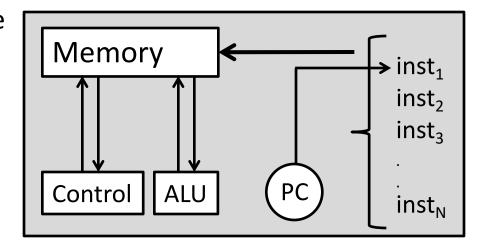
Assembly Language

- A low-level programming language for computers
- More readable, English-like abbreviations for instructions
- Architecture-specific
- Example:

```
MOV AL, 61h
MOV AX, BX
ADD EAX, 10
XOR EAX, EAX
```

Summary: Components of a Computer

- Sequential execution of machine instructions
 - The sequence of instructions are stored in the memory.
 - One instruction at a time is fetched from the memory to the control unit.
 - They are read in and treated just like data.



- PC (program counter) is responsible from the flow of control.
- PC points a memory location containing an instruction on the sequence.
- Early programmers (coders) write programs via machine instructions.

Lecture Overview

Building a Computer

The Harvey Mudd Miniature Machine (HMMM)

The Harvey Mudd Miniature Machine (HMMM)

- HMMM
- A Simple HMMM Program
- Looping
- Functions
- HMMM Instruction Set

HMMM

- A real computer must be able to
 - Move information between registers and memory
 - Get data from the outside world
 - Print results
 - Make decisions
- The Harvey Mudd Miniature Machine (HMMM) is organized as follows
 - Both instructions and data are 16 bit wide
 - In addition to the program counter and instruction register, there are 16 registers named r0 through r15
 - There are 256 memory locations
- Instead of programming in binary (0's and 1's), we'll use assembly language, a programming language where each instruction has a symbolic representation
- For example, to compute r3 = r1+r2, we'll write add r3 r1 r2
- We'll use a program to convert the assembly language into 0's and 1's the machine language that the computer can execute

A Simple HMMM Program

triangle1.hmmm: Calculate the approximate area of a triangle.

```
oread r1 # Get base b
read r2 # Get height h
mul r1 r1 r2 # b times h into r1
setn r2 2
div r1 r1 r2 # Divide by 2
write r1
halt
```

Assemble! →

```
ASSEMBLY SUCCESSFUL
 : 0000 0001 0000 0001
                                         r1 # Get base b
                                 read
 : 0000 0010 0000 0001
                                 read
                                            # Get height h
 : 1000 0001 0001 0010
                                 mul
                                         r1 r1 r2 # b times h into r1
 : 0001 0010 0000 0010
                                 setn r2 2
 : 1001 0001 0001 0010
                                 div
                                         r1 r1 r2 # Divide by 2
 : 0000 0001 0000 0010
                                 write
                                         r1
6: 0000 0000 0000 0000
                                 halt
```

```
4
5
10
```

Looping

• Unconditional jump (jumpn N): set program counter to address N

triangle2.hmmm: Calculate the approximate areas of many triangles.

```
4
5
10
5
5
5
12
<ctrl-d>
End of input, halting program execution...
```

Looping

• Conditional jump (jeqzn rx N): if rx == 0, then jump to line N

triangle3.hmmm: Calculate the approximate areas of many triangles. Stop when a base or height of zero is given.

```
read
            r1
                   # Get base b
     jeqzn r1 9  # Jump to halt if base is zero
read r2  # Get height h
     jeqzn r2 9 # Jump to halt if height is zero
3
     mul r1 r1 r2 # b times h into r1
5
           r2 2
     setn
6
     div
            r1 r1 r2 # Divide by 2
7
     write
            r1
8
     jumpn
             0
9
     halt
```

```
4
5
10
5
5
5
12
```

Looping

is_it_a_prime_number.hmmm: Calculate whether a given positive number is prime or not

```
0
      read r1
                      # read the number. Please enter positive integers.
      setn r2 2
                      # use this register for arithmetic operations with 2.
1
                      # use this register for arithmetic operations with 1.
      setn r9 1
      sub
          r15 r1 r9
      jegzn r15 17
                      # check if the number is 1
      div
            r3 r1 r2
                      # Divide to 2. The biggest divider (denominator) should (may) be this number.
                      # there is no reason. Deleted a line, but too lazy to change all the line numbers.
      gon
      # the number is 2 or 3. So it is prime.
7
      sub
            r15 r3 r9
      jegz r15
      # The number is not 1, 2 or 3. The main loop starts here----------
            r15 r1 r3
                       # mod to check if the number is aliquot.
      jegzn r15
10
                17
                       # it is not a prime number. Jump to line 17.
11
      sub
            r3 r3 r9
                       # subtract one from the divider
12
          r5 r3 r9
                       # subtract one, but on a different register to check the divider is 1 or not.
      sub
                       # we successfully reduced the divider to 1. This is a prime number. Jump to line 15.
13
      jegz r5 15
14
                      # jump to the start of the main loop.
      jumpn 9
      #----- Write 1 for prime numbers.
15
      write r9 # r9 is already 1.
16
      halt
                  ----- Write 0 for non-prime numbers.
      #-----
17
      setn
           r8 0
18
      write r8
19
      halt
```

Functions

- Call a function (calln rX N): copy the next address (aka return address)
 into rX and then jump to address N
- Return from a function (jumpr rX): set program counter to the return address in rX
- By convention, we use register r14 to store the return address

```
square.hmmm: Calculate the square of a number \,N_{\,\cdot}\,
```

```
read
                 # Get. N
          r1
   calln
          r14 5 # Calculate N^2
  write
           r2
                 # Write answer
  halt
   nop
                 # Waste some space
Square function. N is in r1. Result (N^2) is in r2. Return address is in r14.
           r2 r1 r1 # Calculate and store N^2 in r2
   mul
   jumpr
           r14
                    # Done; return to caller
```

Functions

combinations.hmmm: Calculate C(N,K) (aka N choose K) defined as C(N,K) = N!/(K!(N-K)!), where N! (N factorial) is defined as $N! = N \times (N-1) \times (N-2) \times \cdots \times 2 \times 1$, with 0! = 1.

```
r3
                     # Get N
0
     read
            r4
                     # Get K
     read
2
           r1 r3
                    # Calculate N!
     сору
3
     calln r14 15 # ...
            r5 r2 # Save N! as C(N, K)
4
     copy
5
           r1 r4 # Calculate K!
     сору
6
     calln r14 15 # ...
7
            r5 r5 r2 # N!/K!
     div
            r1 r3 r4 # Calculate (N - K)!
8
     sub
9
                     # ...
     calln
            r14 15
            r5 r5 r2 # C(N, K)
10
     div
11
    write
            r5
                     # Write answer
12
    halt
13
    nop
                     # Waste some space
14
    nop
# Factorial function. N is in r1. Result is r2. Return address is in r14.
15
             r2 1
                      # Initial product
      setn
     jegzn r1 20
16
                       # Quit if N has reached zero
17
             r2 r1 r2 # Update product
     mul
18
      addn
             r1 -1
                      # Decrement N
19
      jumpn
             16
                      # Back for more
20
             r14
                      # Done; return to caller
      jumpr
```

Simulate! →

5 2 10

Functions

Trace of the factorial function (N=4)

	instruc	tior	r1	r2		
					4	
15	setn	r2	1		4	1
16	jeqzn	r1	20		4	1
17	mul	r2	r1	r2	4	4
18	addn	r1	-1		3	4
19	jumpn	16			3 3 3 2 2	4
16	jeqzn	r1	20		3	4
17	mul	r2	r1	r2	3	12
18	addn	r1	-1		2	12
19	jumpn	16			2	12
16	jeqzn	r1	20		2	12
17	mul	r2	r1	r2	2	24
18	addn	r1	-1		1	24
19	jumpn	16			1	24
16	jeqzn	r1	20		1	24
17	mu1	r2	r1	r2	1	24
18	addn	r1	-1		0	24
19	jumpn	16			0	24
16	jeqzn	r1	20		0	24
20	jumpr				0	24

Trace of the program (N=5, K=2)

	instru	ction	r1	r2	r3	r4	r5	r14
0	read	r3			5			
1	read	r4			5	2		
2	сору	r1 r3	5		5	2		
3	calln	r14 15	5	120	5	2		4
4	сору	r5 r2	5	120	5	2	120	4
5	сору	r1 r4	2	120	5	2	120	4
6	calln	r14 15	2	2	5	2	120	7
7	div	r5 r5 r2	2	2	5	2	60	7
8	sub	r1 r3 r4	3	2	5	2	60	7
9	calln	r14 15	3	6	5	2	60	10
10	div	r5 r5 r2	3	6	5	2	10	10
11	write	r5	3	6	5	2	10	10
12	halt		3	6	5	2	10	10

HMMM Instruction Set

System instructions

halt	stop
read rX	place user input in register rx
write rX	print contents of register rX
nop	do nothing

Setting register data

setn rX N	set register rx equal to the integer x (-128 to 127)
addn rX N	add integer N (-128 to 127) to register $\texttt{r} \texttt{X}$
copy rX rY	set rx=ry

Arithmetic

```
add rX rY rZ set rX=rY+rZ
sub rX rY rZ set rX=rY-rZ
neg rX rY set rX=-rY
mul rX rY rZ set rX=rY*rZ
div rX rY rZ set rX=rY*rZ (integer division; no remainder)
mod rX rY rZ set rX=rY%rZ (returns the remainder of integer division)
```

HMMM Instruction Set

Jumps

```
jumpn N set program counter to address N
jumpr rX set program counter to address in rX
jeqzn rX N if rX==0, then jump to line N
jnezn rX N if rX!=0, then jump to line N
jgtzn rX N if rX>0, then jump to line N
jltzn rX N if rX<0, then jump to line N
calln rX N copy the next address into rX and then jump to address N</pre>
```

Interacting with memory

```
loadn rX N load register rX with the contents of address N
storen rX N store contents of register rX into address N
loadr rX rY load register rX with data from the address location held in register rY
storer rX rY store contents of register rX into address held in register rY
```