

BBM 201

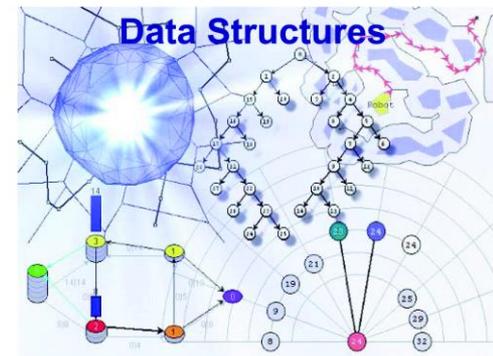
DATA STRUCTURES

Lecture 10:

Implementation of Linked Lists
(Stacks, Queue, Hashtable)



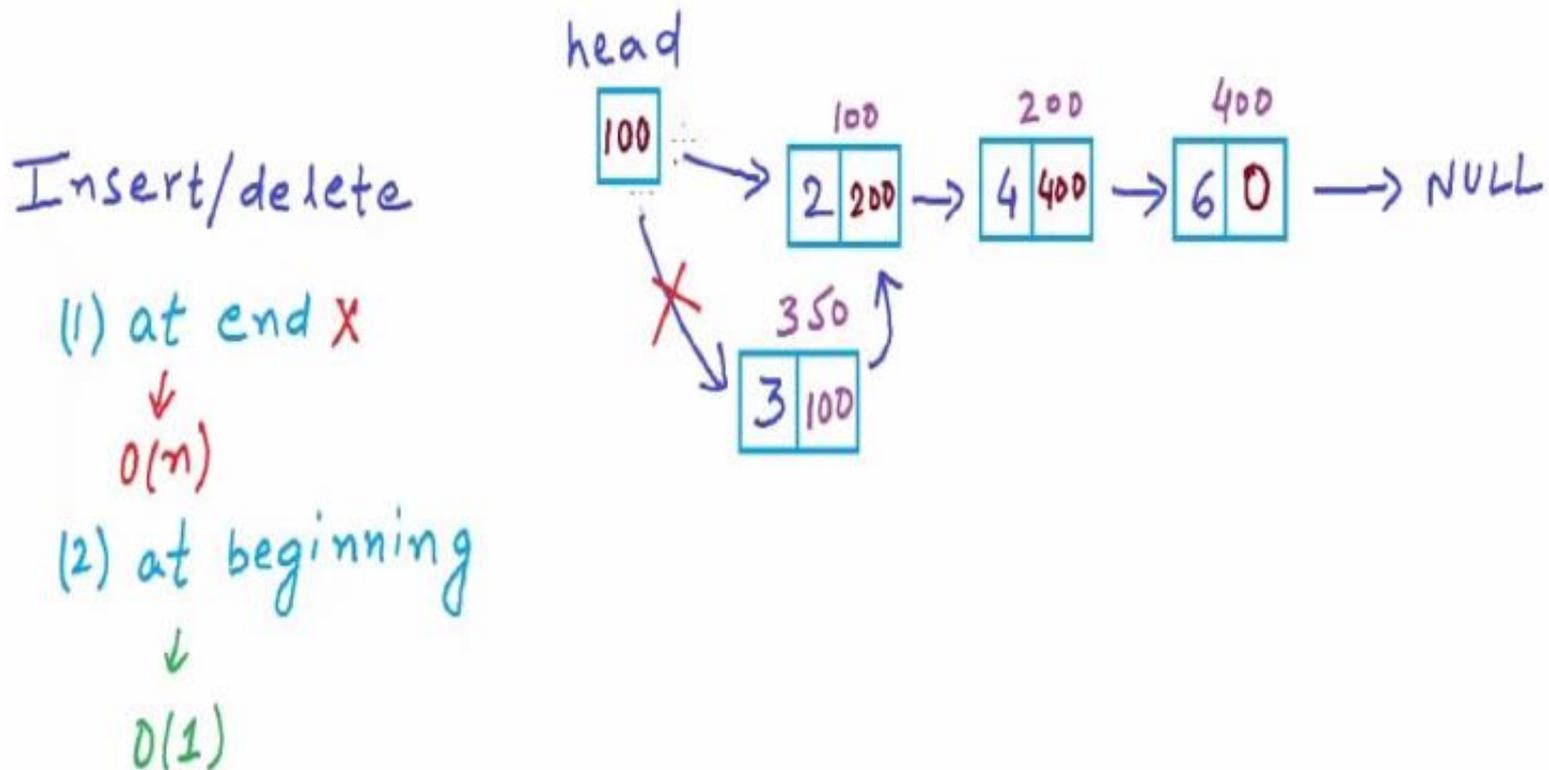
2017-2018 Fall



Linked list implementation of stacks

The **cost of insert and delete at the beginning** of a linked list is constant, that is $O(1)$.

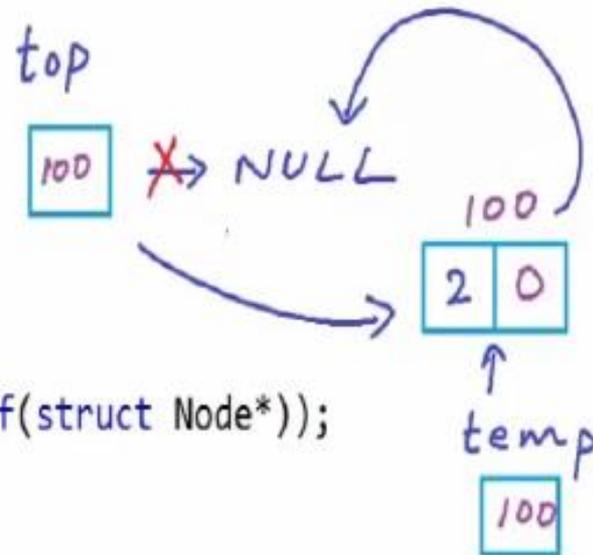
So, in the linked list implementation of stack, we insert and delete a node at the beginning so that time complexity of Push and Pop is $O(1)$.



Instead of head, we use variable name **top**. When top is NULL, the stack is empty.

With dereferencing **temp->data=x** and **temp->link = top**, we fill the fields of the new node, called **temp**. Finally, with **top=temp**, top points to the new node.

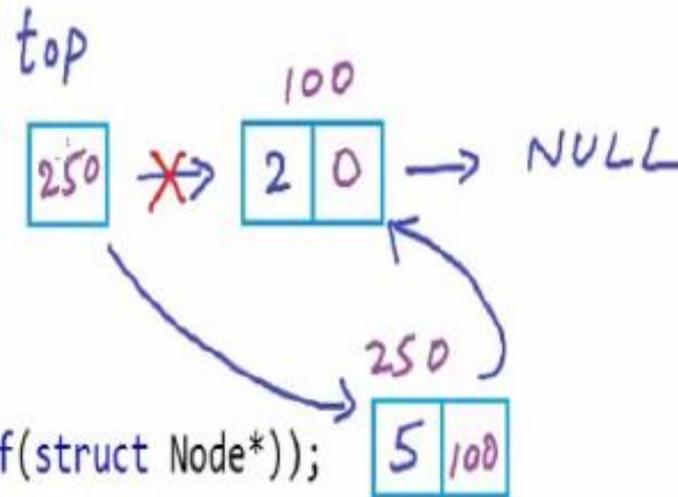
```
struct Node {
    int data;
    struct Node* link;
};
struct Node* top = NULL;
void Push(int x) {
    struct Node* temp =
        (struct Node*)malloc(sizeof(struct Node*));
    temp->data = x;
    temp->link = top;
    top = temp;
}
```



Push(2)

Stack data structure uses the memory efficiently by using push when something is needed and pop when not needed in an array or linked list.

```
struct Node {  
    int data;  
    struct Node* link;  
};  
struct Node* top = NULL;  
void Push(int x) {  
    struct Node* temp =  
        (struct Node*)malloc(sizeof(struct Node*));  
    temp->data = x;  
    temp->link = top;  
    top = temp;  
}
```

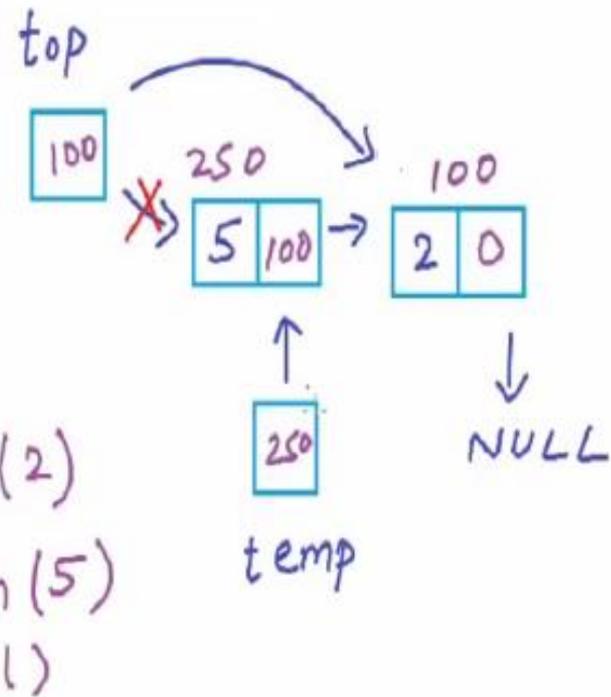


Push(2)
Push(5)

```

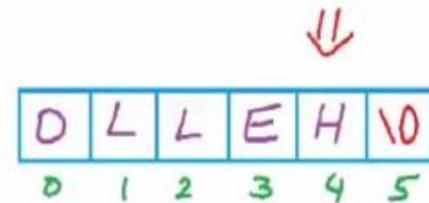
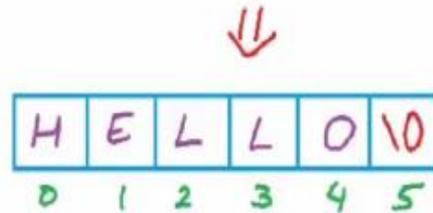
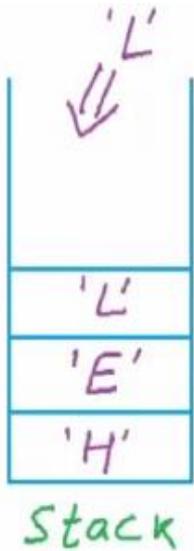
struct Node* top = NULL;
void Push(int x) {
    struct Node* temp =
        (struct Node*)malloc(sizeof(struct Node*));
    temp->data = x;
    temp->link = top;
    top = temp;
}
void Pop() {
    struct Node *temp;
    if(top == NULL) return;
    temp = top;
    ⇒ top = top->link;
    free(temp);
}

```



Reverse a string using stack

We can create a stack of characters by traversing the string from left to right and using the push operation. Then, we can pop the elements from the stack and overwrite all the positions in the string.



We pass the string and the length of the string to the [Reverse function](#).

```
// string reversal using stack
#include<iostream>
using namespace std;

void Reverse(char C[],int n)
{

}

int main() {
    char C[51];
    printf("Enter a string: ");
    gets(C);
    Reverse(C,strlen(C));
    printf("Output = %s",C);
}
```

In C++, we can use these operations available from the standard temporary library called stack.

```
// string reversal using stack
#include<iostream>
using namespace std;
class Stack
{
private:
    char A[101];
    int top;| I
public:
    void Push(int x);
    void Pop();
    int Top();
    bool IsEmpty();
};
```

As we traverse the string from left to right, we use the push function. In the second loop, we use the top and pop functions for each character in the stack to define the string C. See next slide for output.

```
#include<iostream>
#include<stack> // stack from standard template library (STL)
using namespace std;
void Reverse(char *C,int n)
{
    stack<char> S;
    //loop for push
    for(int i=0;i<n;i++){
        S.push(C[i]);
    }
    //loop for pop
    for(int i =0;i<n;i++){
        C[i] = S.top(); //overwrite the character at index i.
        S.pop(); // perform pop.
    }
}
```

```

#include<stack> // stack from standard template library (STL)
using namespace std;
void Reverse(char *C,int n)
{
    stack<char> S;
    //loop for push
    for(int i=0;i<n;i++){
        S.push(C[i]);
    }
    //loop for pop
    for(int i =0;i<n;i++){
        C[i] = S.top(); //overwrite the character at index i.
        S.pop(); // perform pop.
    }
}
int main() {
    char C[51];
    printf("Enter a string: ");
    gets(C);
    Reverse(C,strlen(C));
    printf("Output = %s",C);
}

```

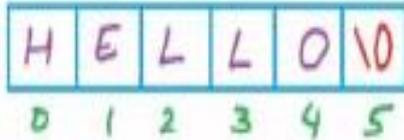
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```

Enter a string: HELLO
Output = OLLEH_

```

Problem: Reverse a string

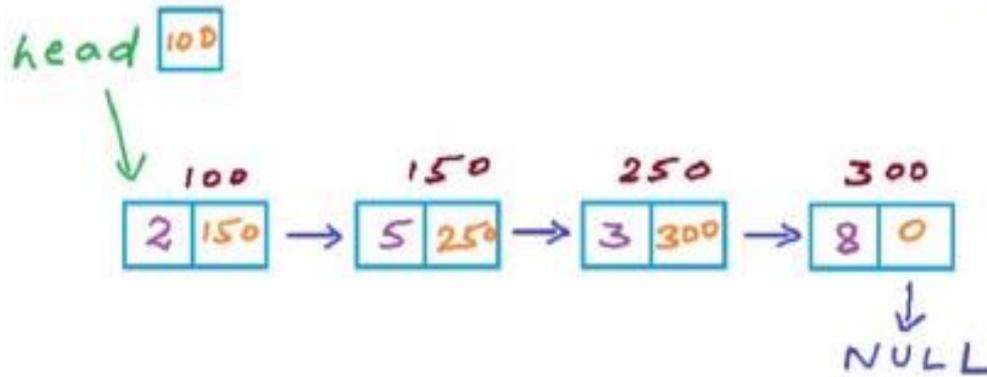


```
void Reverse(char *C, int n)
{
    stack<char> S;
    //loop for push
    O(n) { for(int i=0; i<n; i++){
           S.push(C[i]);
         }
    //loop for pop
    O(n) { for(int i =0; i<n; i++){
           C[i] = S.top();
           S.pop();
         }
    }
```

- All operations on stack take constant time, $O(1)$.
- Thus, for each loop, time cost is $O(n)$.
- Space complexity is also $O(n)$.

Reverse a linked list using stack

Time complexity of reversing a linked list



Iterative Solution

Time - $O(n)$

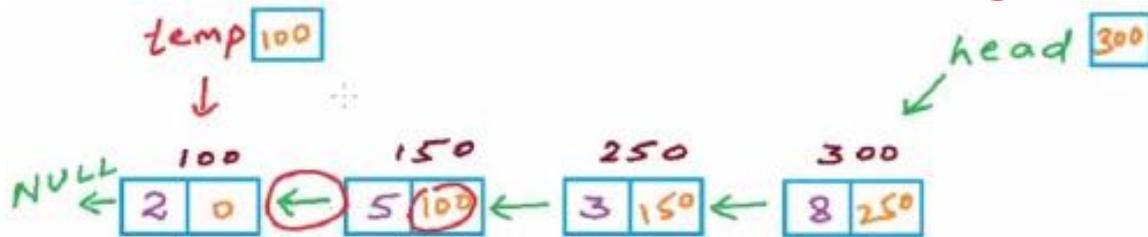
Space - $O(1)$

Recursive Solution
(Implicit Stack)

Time - $O(n)$

Space - $O(n)$

Reverse a linked list using stack



```
Node *temp = S.top();  
head = temp;  
S.pop();  
while(!S.empty()){  
    temp->next = S.top();  
    S.pop();  
    temp = temp->next;  
}  
=> temp->next = NULL;
```

```
stack<struct Node*> S;  
  
struct Node {  
    int data;  
    Node* next;  
};
```

The values we are pushing into the stack are the pointers to the node. Initially, `S.top()` returns 300. We store head as this address, so `head` stores 300.

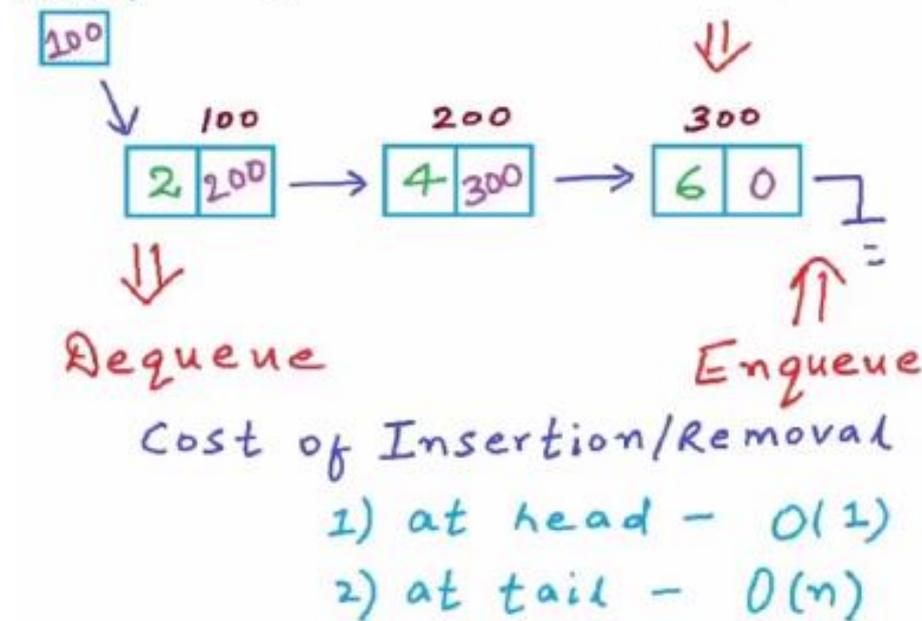
In the loop, while stack is not empty, set `temp->next` to the address at the top of the stack, which is 250.

And finally, `temp` points to the node at address 250.

The last line is needed so that the end of the reversed linked list points to NULL and not to 150.

Linked list implementation of queue

Queue - Linked List implementation



Operations

- (1) Enqueue(x)
 - (2) Dequeue()
 - (3) front()
 - (4) IsEmpty()
- } Constant time or $O(1)$

In this case above, Dequeue will take constant time, but Enqueue will take $O(n)$ time.

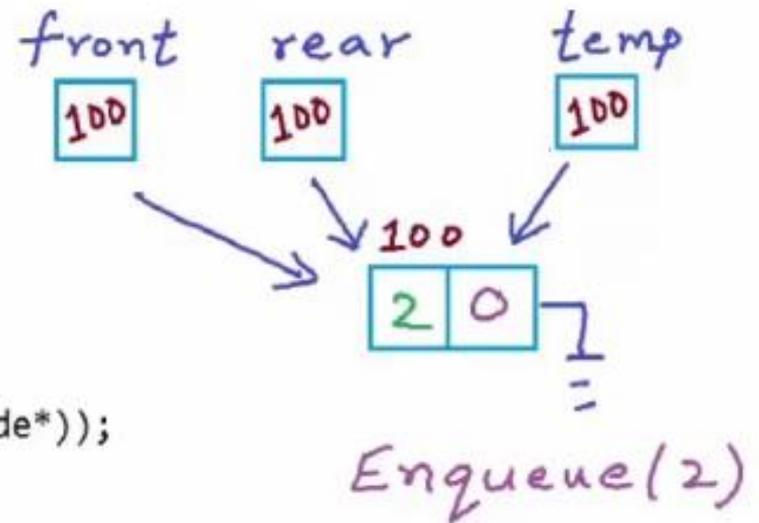
As before, the traversal of the linked list to learn the address of the last node takes $O(n)$ time.

Solution: Keep a pointer called rear that stores the address of the node at the rear position.

```

struct Node {
    int data;
    struct Node* next;
};
struct Node* front = NULL;
struct Node* rear = NULL;
void Enqueue(int x) {
    struct Node* temp =
    {
        (struct Node*)malloc(sizeof(struct Node*));
        temp->data = x;
        temp->next = NULL;
        if(front == NULL && rear == NULL){
            front = rear = temp;
            return;
        }
        rear->next = temp;
        rear = temp;
    }
}

```



Instead of declaring a pointer variable pointing to head, we **declare two pointer variables** one pointing to the front node and another pointing to the rear node.

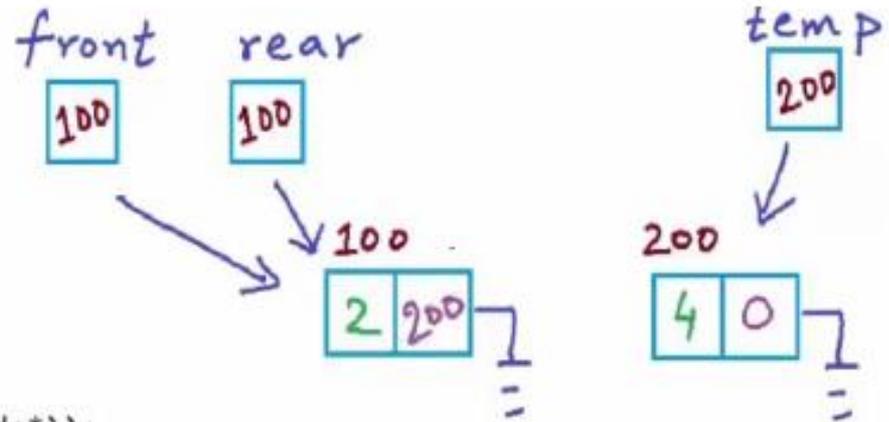
For the **Enqueue**, we again use a pointer called `temp` to point to the new node created dynamically.

If the list is empty, both `front` and `rear` point to the new node.

```

struct Node {
    int data;
    struct Node* next;
};
struct Node* front = NULL;
struct Node* rear = NULL;
void Enqueue(int x) {
    struct Node* temp =
        (struct Node*)malloc(sizeof(struct Node*));
    temp->data = x;
    temp->next = NULL;
    if(front == NULL && rear == NULL){
        front = rear = temp;
        return;
    }
    ⇒ rear->next = temp;
    rear = temp;
}

```



Enqueue(2)

Enqueue(4)

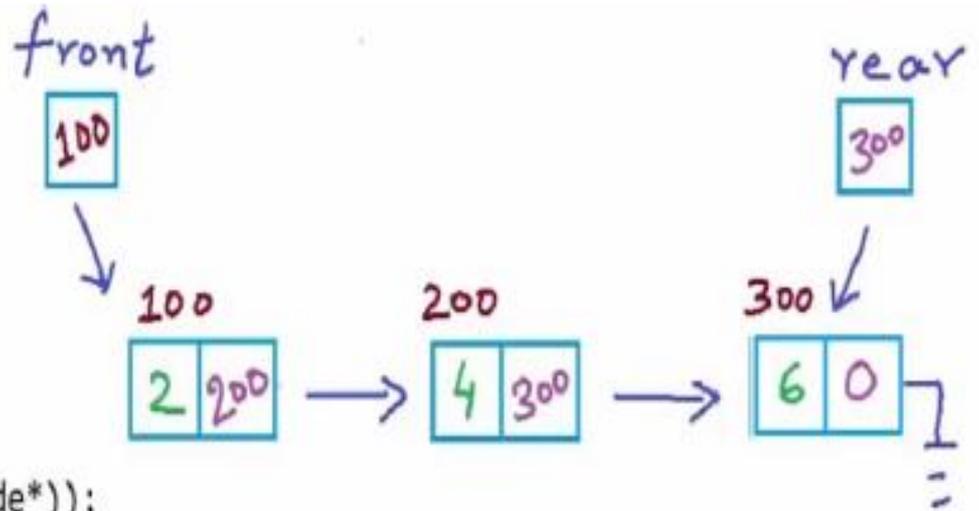
If the list is not empty, first, set the address part of the current **rear node** to the address of the new node.

Then let the rear pointer point to the new node.

```

struct Node {
    int data;
    struct Node* next;
};
struct Node* front = NULL;
struct Node* rear = NULL;
void Enqueue(int x) {
    struct Node* temp =
        (struct Node*)malloc(sizeof(struct Node*));
    temp->data = x;
    temp->next = NULL;
    if(front == NULL && rear == NULL){
        front = rear = temp;
        return;
    }
    rear->next = temp;
    rear = temp;
}

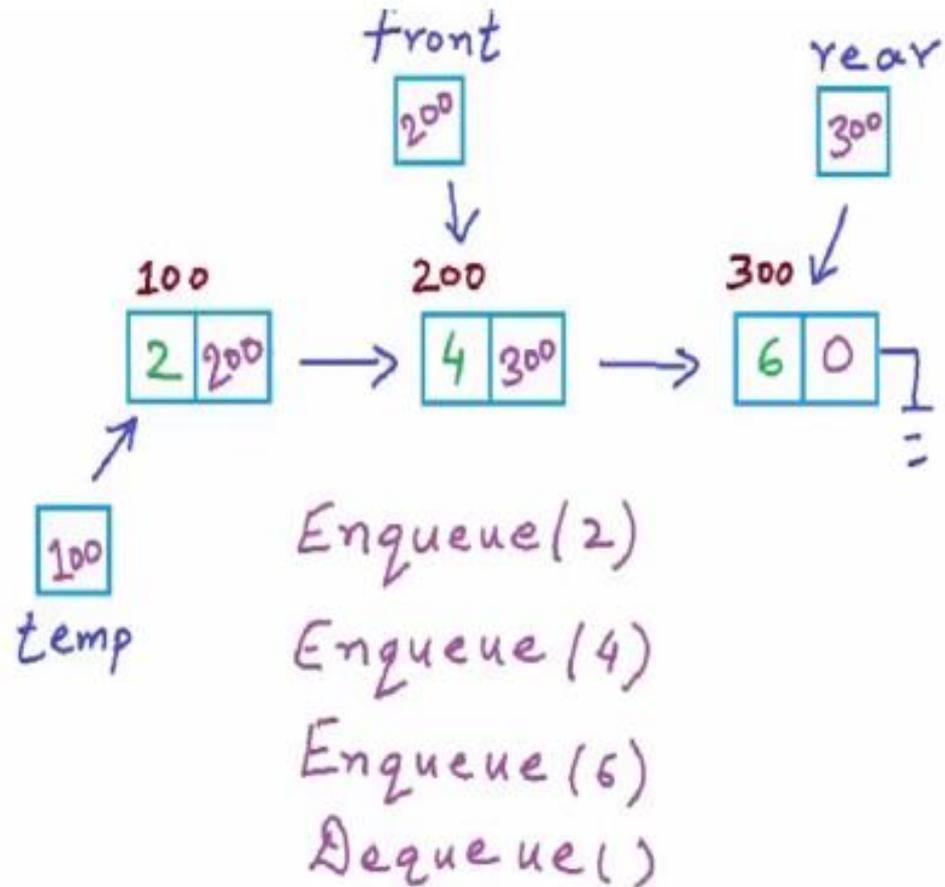
```



Enqueue(2)
 Enqueue(4)
 Enqueue(6)

Implementation of Dequeue

```
void Dequeue() {  
    struct Node* temp = front;  
    if(front == NULL) return;  
    if(front == rear) {  
        front = rear = NULL;  
    }  
    else {  
        front = front->next;  
    }  
    ⇒ free(temp);  
}
```



Multiple Stack Implementation Using Linked List

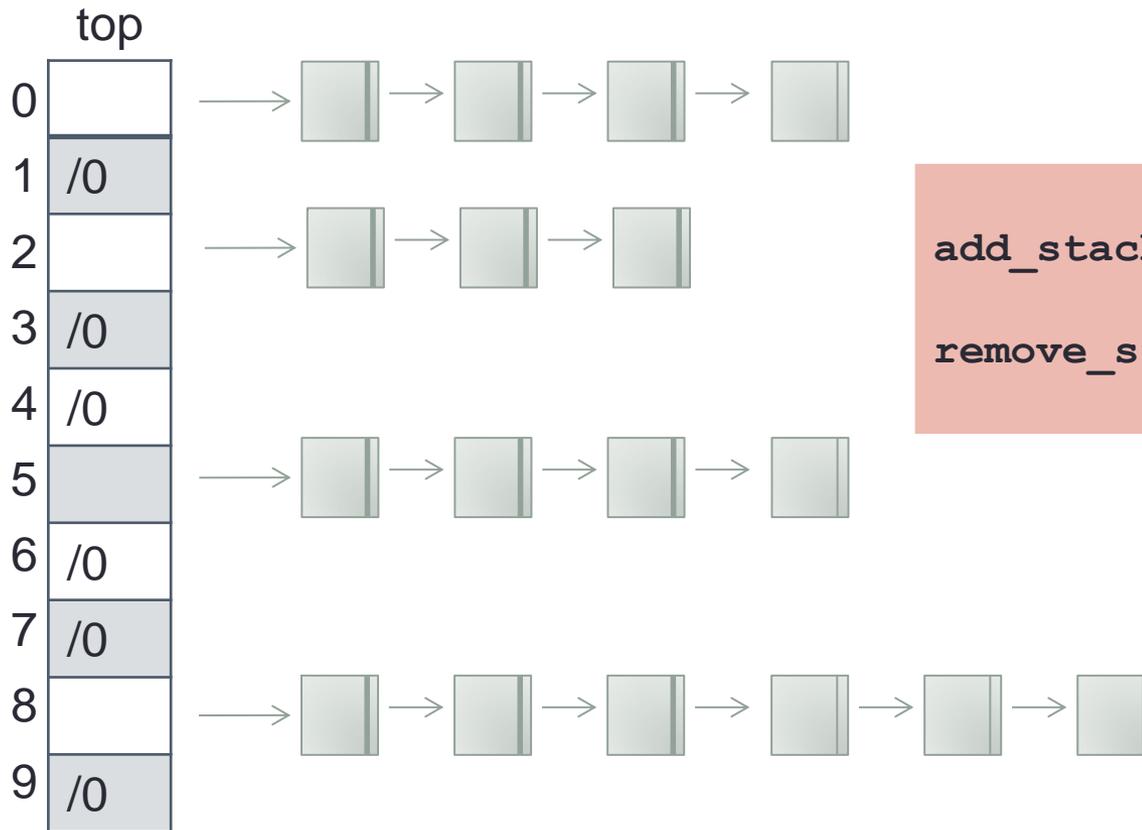
Multiple Stack Implementation Using Linked List

A single stack:



Multiple Stack Implementation Using Linked List

Multiple stacks:



```
add_stack(&top[stack_i], x);  
remove_stack(&top[stack_i]);
```

Linked List

```
#define MAX_STACK 10 // number of max stacks

typedef struct{
    int value;
    //other fields
}item;
typedef struct stack *stack_pointer;
typedef struct stack{
    item x;
    stack * next;
};

stack_pointer top [MAX_STACK];
```

Linked List

```
#define ISFULL (p) (!p)
#define IEMPTY (p) (!p)

for(int i=0;i<MAX_STACK;i++)
    top[i] = NULL;
```

Linked List

```
void add_stack(stack_pointer *top, item x)
{
    stack_pointer p = (stack *)malloc (sizeof(stack));
    if(ISFULL(p))
    {
        printf("\n Memory allocation failed");
        exit(1);
    }
    p->x = x;
    p->next = *top;
    *top = p;
}
```

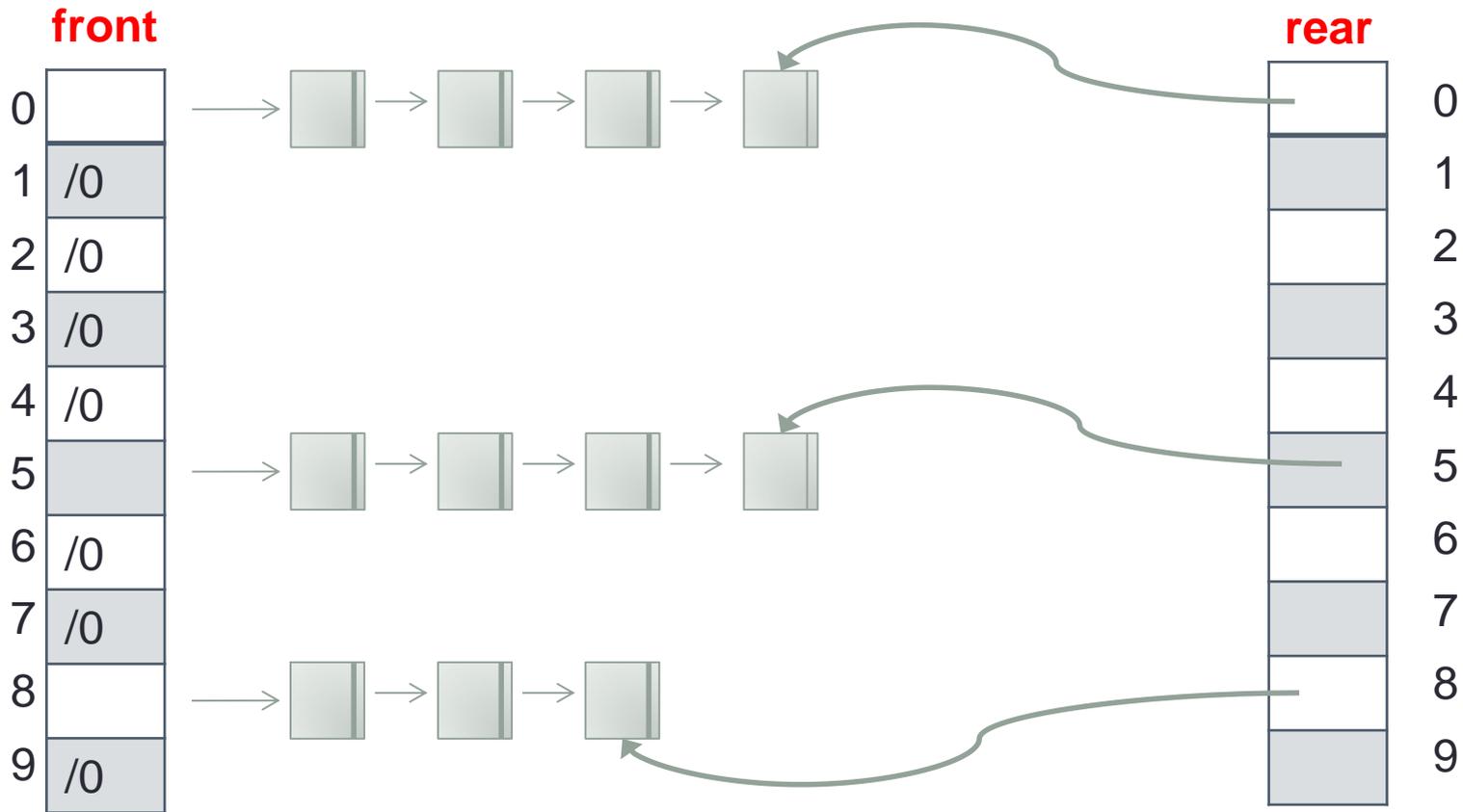
Linked List

```
item remove_stack(stack_pointer *top)
{
    stack_pointer p = *top;
    item x;

    if(ISEMPTY(p))
    {
        printf("\n Stack is empty!");
        exit(1);
    }
    x = p->x;
    *top = p->next;
    free(p);
    return(x);
}
```

Multiple Queue Implementation Using Linked List

Multiple Queue Implementation Using Linked List



Linked List

```
#define MAX_QUEUE 10 // number of max queues

typedef struct queue *queue_pointer;
typedef struct queue{
    item value;
    queue_pointer link;
};

queue_pointer front[MAX_QUEUE], rear[MAX_QUEUE];
for(i=0; i<MAX_QUEUE; i++)
    front[i]=NULL;
```

Linked List

```
void add_queue(queue_pointer *front, queue_pointer* rear, item x)
{
    queue_pointer p=(queue_pointer)malloc(sizeof(queue));
    if(ISFULL(p)){
        printf("\n Memory allocation failed!!!\n");
        exit(1);
    }
    p->value=x;
    p->link=NULL;
    if(*front)
        (*rear)->link=p;
    else
        *front=p;
    *rear=p;
}
```

Linked List

```
item remove_queue(queue_pointer *front)
{
    queue_pointer p=*front;
    item x;
    if(ISEMPTY(*front)){
        printf("\nQueue is empty!!!");
        exit(1);
    }

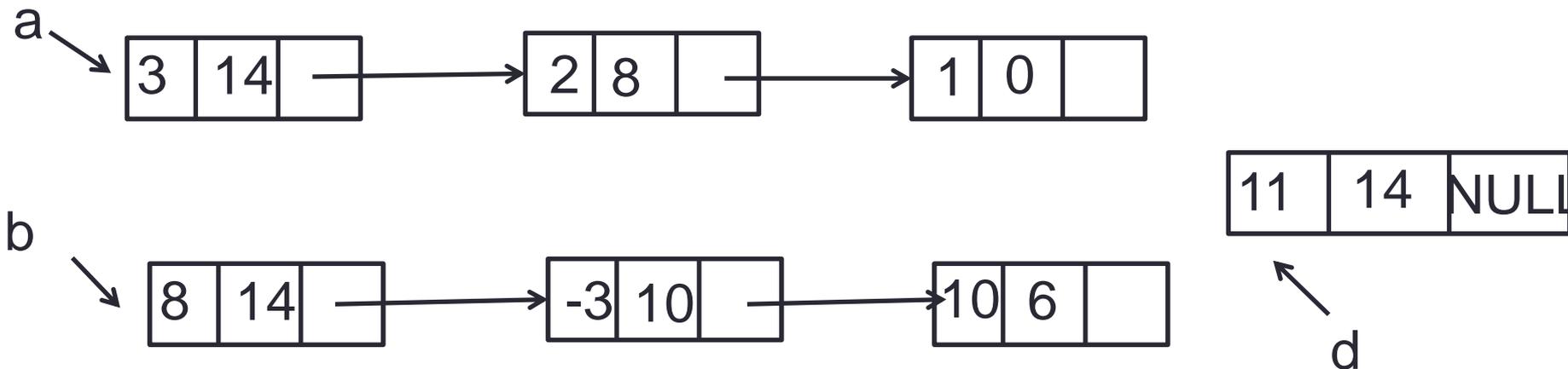
    x=p->value;
    *front=p->link;
    free(p);
    return(x);
}
```

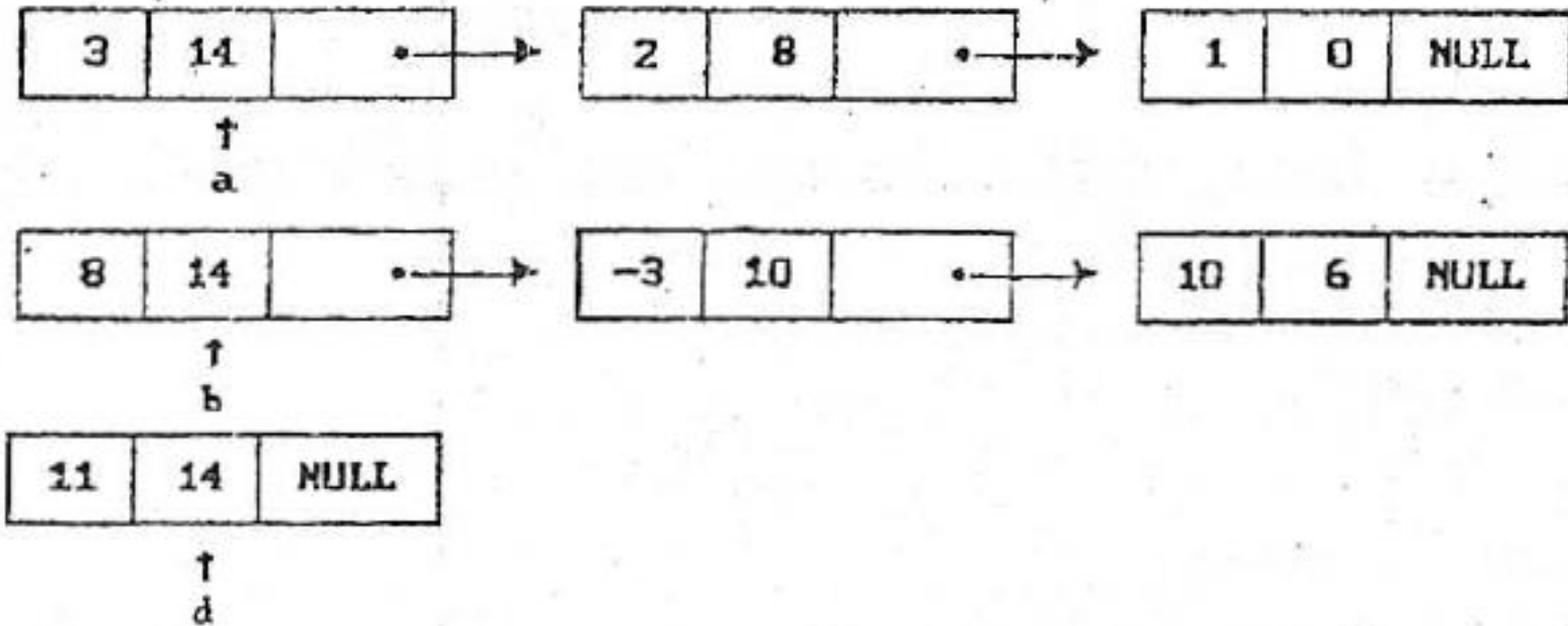
Addition of polynomials by using singly linked lists

Adding Polynomials using linked list

```
typedef struct poly_node* poly_pointer;  
typedef struct poly_node{  
    int coefficient;  
    int expon;  
    poly_pointer link;  
};  
poly_pointer a,b,d;
```

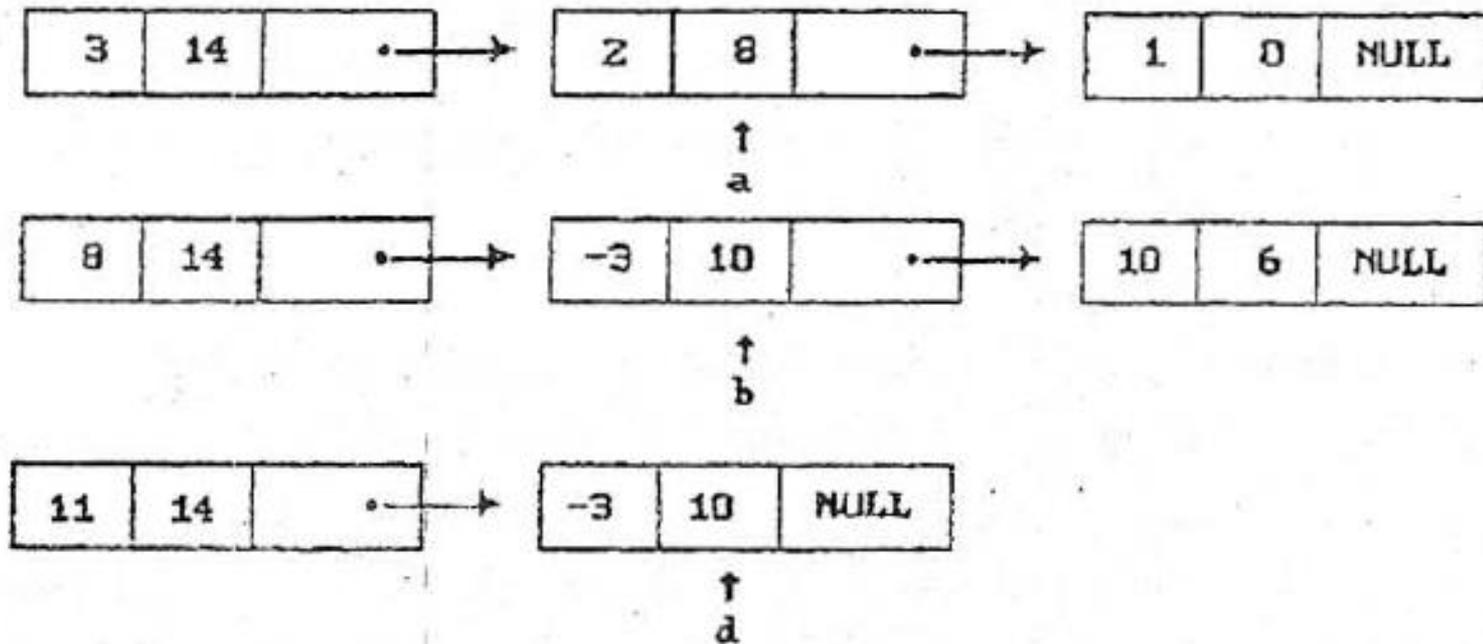
$$a(x) = 3x^{14} + 2x^8 + 1$$
$$b(x) = 8x^{14} - 3x^{10} + 10x^6$$





(a) $a \rightarrow \text{expon} == b \rightarrow \text{expon}$

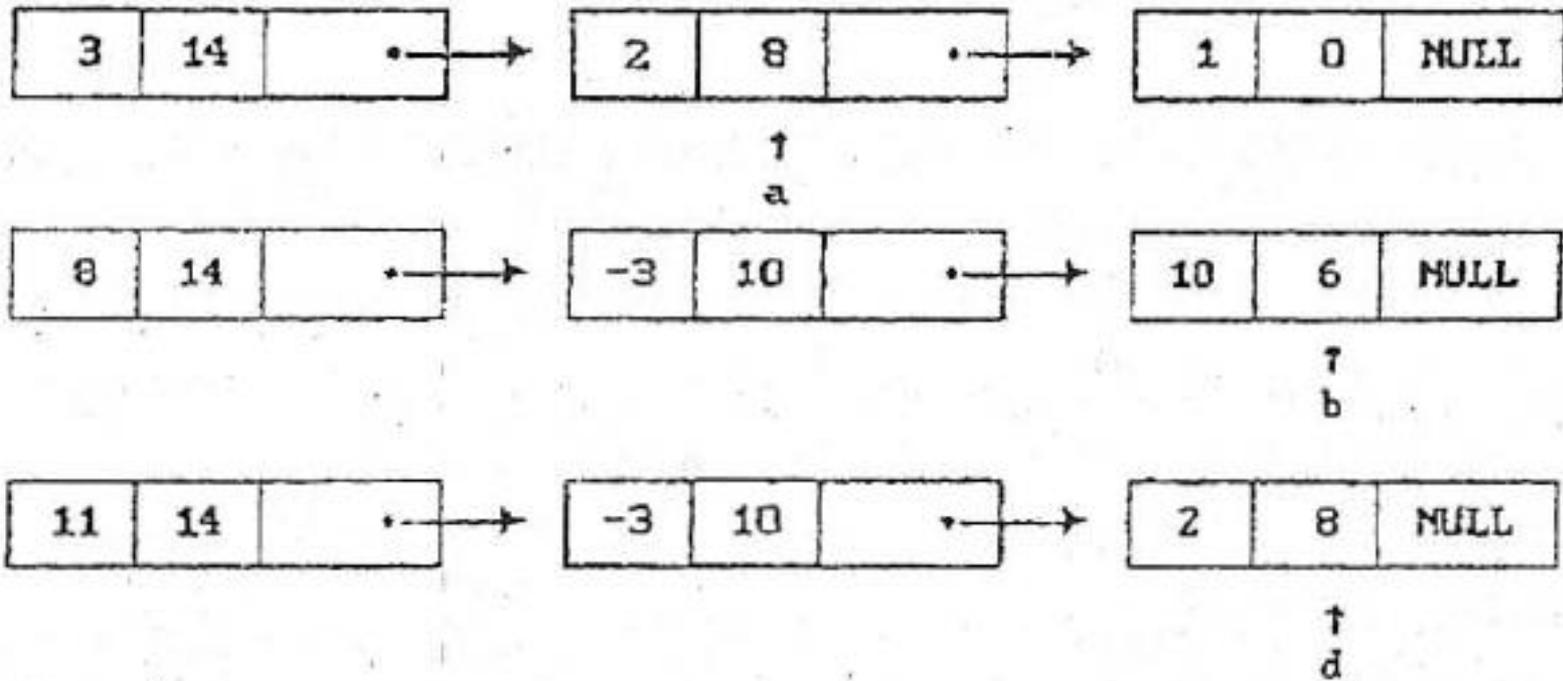
If the exponents of two terms are equal, we add the two coefficients and create a new term, also move the pointers to the next nodes in a and b.



(b) $a \rightarrow \text{expon} < b \rightarrow \text{expon}$

If the exponent of the current term in a is less than the exponent of the current term in b, then we create a duplicate term of b, Attach this term to the result, called d. Then move the pointer to the next term in b.

(Similarly, if the exponent of the current term in a is larger than the exponent of the current term in b.)



(c) $a \rightarrow \text{expon} > b \rightarrow \text{expon}$

To avoid having to search for the last node in d, each time we add a new node, we keep a pointer, "rear", which points to the current last node in d.

Complexity of padd: $O(m+n)$.,
 m and n, the number of terms in the
 polynomials a and b. Why?

```
poly_pointer padd(poly_pointer a, poly_pointer b) {
/* return a polynomial which is the sum of a and b */
    poly_pointer front, rear, temp;
    int sum;
    rear = (poly_pointer)malloc(sizeof(poly_node));
    if (IS_FULL(rear)) {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    front = rear;
    while (a && b)
        switch (COMPARE(a->expon, b->expon)) {
            case -1: /* a->expon < b->expon */
                attach(b->coef, b->expon, &rear);
                b = b->link; break;
            case 0: /* a->expon = b->expon */
                sum = a->coef + b->coef;
                if(sum) attach(sum, a->expon, &rear);
                a = a->link; b = b->link; break;
            case 1: /* a->expon > b->expon */
                attach(a->coef, a->expon, &rear);
                a = a->link;
        }
}
```

```
/* copy rest of list a and then list b */
for (; a; a=a->link) attach(a->coef,a->expon,&rear);
for (; b; b=b->link) attach(b->coef,b->expon,&rear);
rear->link = NULL;
/* delete extra initial node */
temp = front; front = front->link; free(temp);
return front;}
```

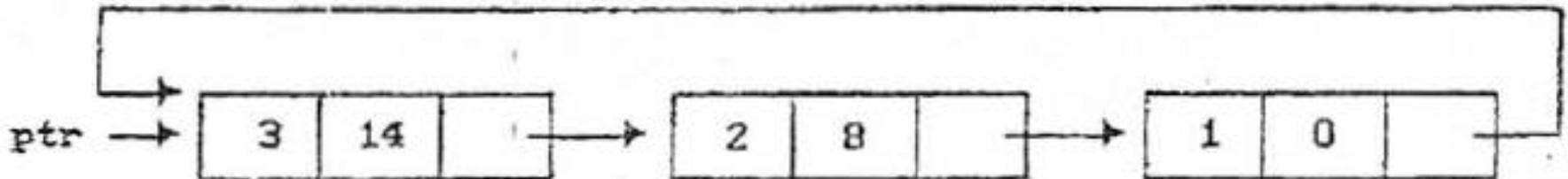
```
void attach(float coefficient,int exponent,poly_pointer *ptr){
/* create a new node with coef = coefficient and expon =
exponent,attach it to the node pointed to by ptr. ptr is
updated to point to this new node */
poly_pointer temp;
temp = (poly_pointer)malloc(sizeof(poly_node));
if (IS_FULL(temp)){
    fprintf(stderr, "The memory is full\n");
    exit(1);}
temp->coef = coefficient;
temp->expon = exponent;
(*ptr)->link = temp;
*ptr = temp;}
```

Singly-linked circular lists

Representing polynomials as circularly linked List

We can free the nodes of a polynomial more efficiently if we have a circular list:

In a circular list, the last node is linked to the first node.



$$\text{ptr}(x) = 3x^{14} + 2x^8 + 1$$

Methodology:

We free nodes that are no longer in use so that we may reuse these nodes later by:

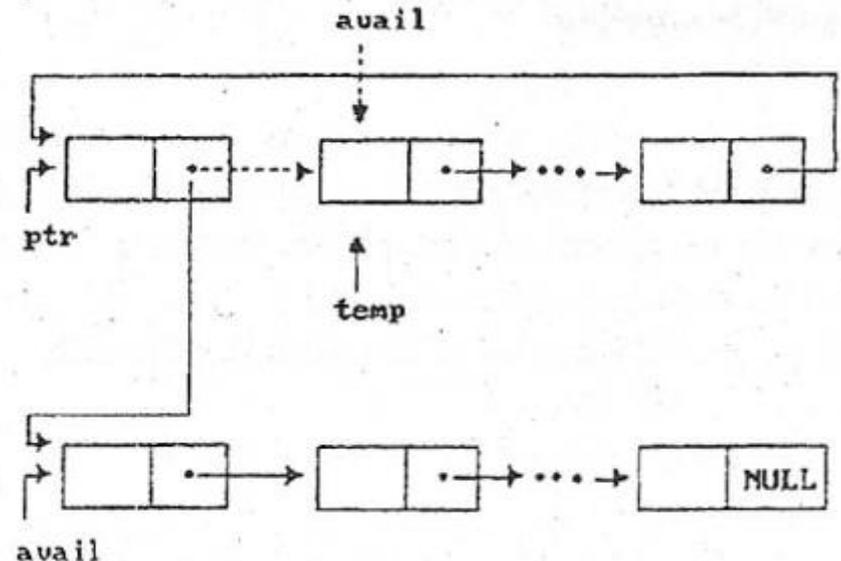
- Obtaining an efficient erase algorithm for circular lists
- Maintaining our own list (as a chain) of nodes that have been “freed”
- Examining this list, when we need a new node and using one of the nodes if this list is not empty
- And using “malloc” only if this list is empty

- Let `avail` be a variable of type `poly_pointer` that points to the first node in our list of free nodes.

- Initially we set `avail` to NULL.

- Instead of using “malloc” and “free”, we use `get_node` and `ret_node`.

(See next slide)



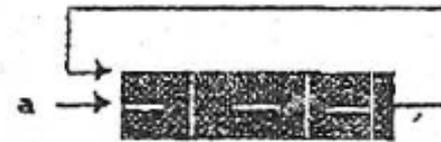
```
poly_pointer get_node() {
    /*provide a node for use*/
    poly_pointer node;
    if (avail){
        node = avail;
        avail = avail->link;
    }
    else{
        node = (poly_pointer) malloc(sizeof(poly_node));
        if(IS_FULL(node)){
            fprintf(stderr, "The memory is full\n");
            exit(1);
        }
    }
    return node;
}
```

```
void ret_node(poly_pointer ptr) {
    /*return a node to the available list*/
    ptr->link = avail;
    avail = ptr;
}
```

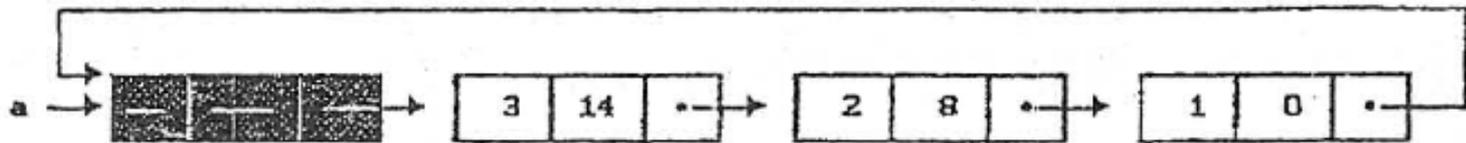
Erasing in a circular list:

- We may erase a circular list in a fixed amount of time independent of the number of nodes in the list using *cerase*.
- To avoid the special case for **zero polynomial**, a head node is introduced for each polynomial. Therefore, also the zero polynomial contains a node.

```
void cerase(poly_pointer *ptr) {  
  /*erase the circular list ptr*/  
  poly_pointer temp;  
  if(*ptr) {  
    temp = (*ptr)->link;  
    (*ptr)->link = avail;  
    avail = temp;  
    *ptr = NULL;  
  }  
}
```



(a) Zero polynomial



(b) $3x^{14} + 2x^8 + 1$

To change padd for circularly lists

- Add two variables, **starta = a** and **startb = b**.
- Prior to the while loop, assign `a = a->link` and `b = b->link`.
- Change the while loop to `while(a!=starta && b!=startb)`
- Change the first for loop to `for (;a!=starta; a=a->link)`
- Change the second for loop to `for (;b!=startb; b=b->link)`
- Delete the lines: `rear->link = NULL;`
`/*delete extra initial node */`
- Change the lines: `temp = front;`
`front = front->link;`
`free(temp);`
to **`rear->link = front;`**

```

poly_pointer cpadd(poly_pointer a, poly_pointer b) {
/* polynomials a and b are singly linked circular lists with a head node. Return
a polynomial which is the sum of a and b */
    poly_pointer starta, d, lastd;
    int sum, done = FALSE;
    starta = a;          /*record start of a*/
    a = a->link;         /*skip headnode for a and b*/
    b = b->link;
    d = get_node();      /*get a head node for sum*/
    d->expon = -1; lastd = d;
    do{
        switch (COMPARE(a->expon,b->expon)) {
            case -1: /* a->expon < b->expon */
                attach(b->coef, b->expon, &lastd);
                b = b->link; break;
            case 0: /* a->expon = b->expon */
                if(starta == a) done = TRUE;
                else{
                    sum = a->coef + b->coef;
                    if(sum) attach(sum,a->expon,&lastd);
                    a = a->link; b = b->link; }
                break;
            case 1: /* a->expon > b->expon */
                attach(a->coef,a->expon,&lastd);
                a = a->link;
        }
    }while(!done);
    lastd->link = d;
    return d;}

```

Hashtable

Hashing

- Another important and widely useful technique for implementing dictionaries
- Constant time per operation (on the average)
- Worst case time proportional to the size of the set for each operation (like linked list and arrays).

Basic Idea

- Use *hash function* to map keys into positions in a *hash table*.

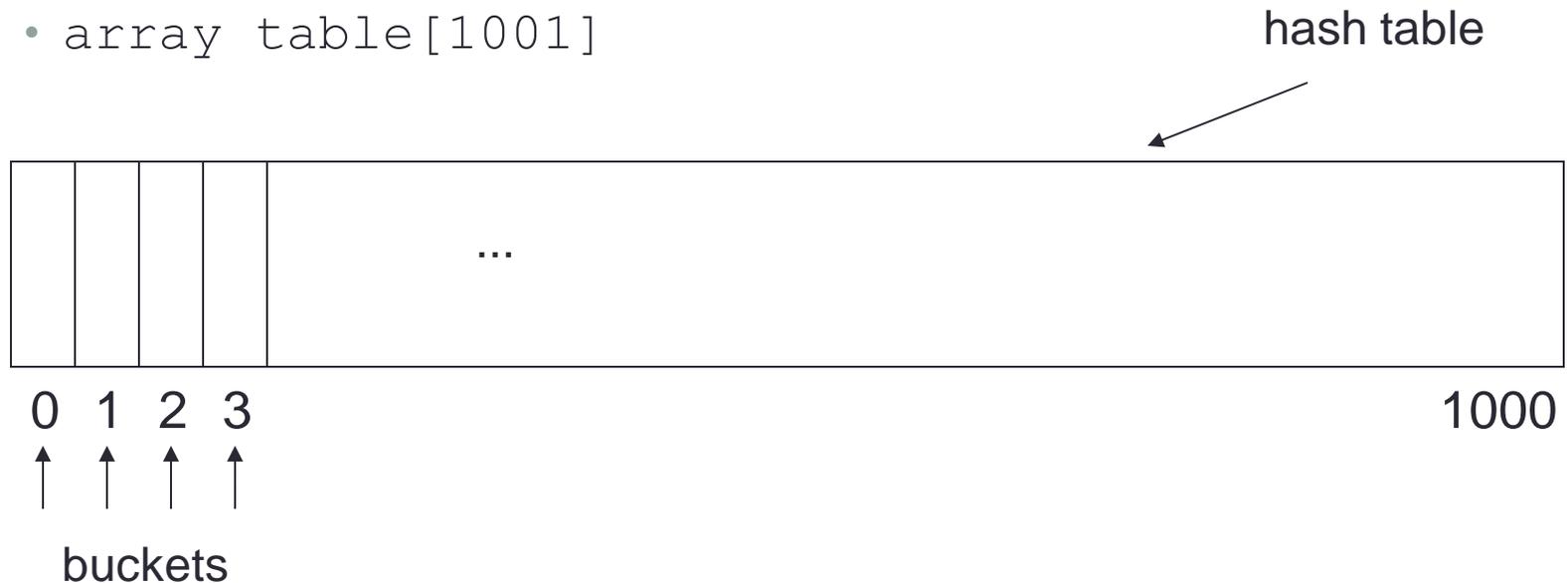
Ideally

- If element e has key k and h is hash function, then e is stored in position $h(k)$ of table
- To search for e , compute $h(k)$ to locate position. If no element, dictionary does not contain e .

Example

- Dictionary Student Records

- Keys are ID numbers (951000 - 952000), no more than 100 students
- Hash function: $h(k) = k - 951000$ maps ID into distinct table positions 0-1000
- array `table[1001]`



Analysis (Ideal Case)

- $O(b)$ time to initialize hash table (b number of positions or buckets in hash table)
- $O(1)$ time to perform *insert*, *remove*, *search*

Ideal Case is Unrealistic

- Works for implementing dictionaries, but many applications have key ranges that are too large to have 1-1 mapping between buckets and keys!

Example:

- Suppose key can take on values from 0 .. 65,535 (2 byte unsigned int)
- Expect $\approx 1,000$ records at any given time
- Impractical to use hash table with 65,536 slots!

Hash Functions

- If key range is too large, use hash table with fewer buckets and a hash function which maps multiple keys to same bucket:

$h(k_1) = \beta = h(k_2)$: k_1 and k_2 have **collision** at slot β

- Popular hash functions: hashing by division
 $h(k) = k \% D$, where D is number of buckets in hash table
- Example: hash table with 11 buckets
 $h(k) = k \% 11$
 $80 \rightarrow 3$ ($80 \% 11 = 3$), $40 \rightarrow 7$, $65 \rightarrow 10$
 $58 \rightarrow 3$ collision!

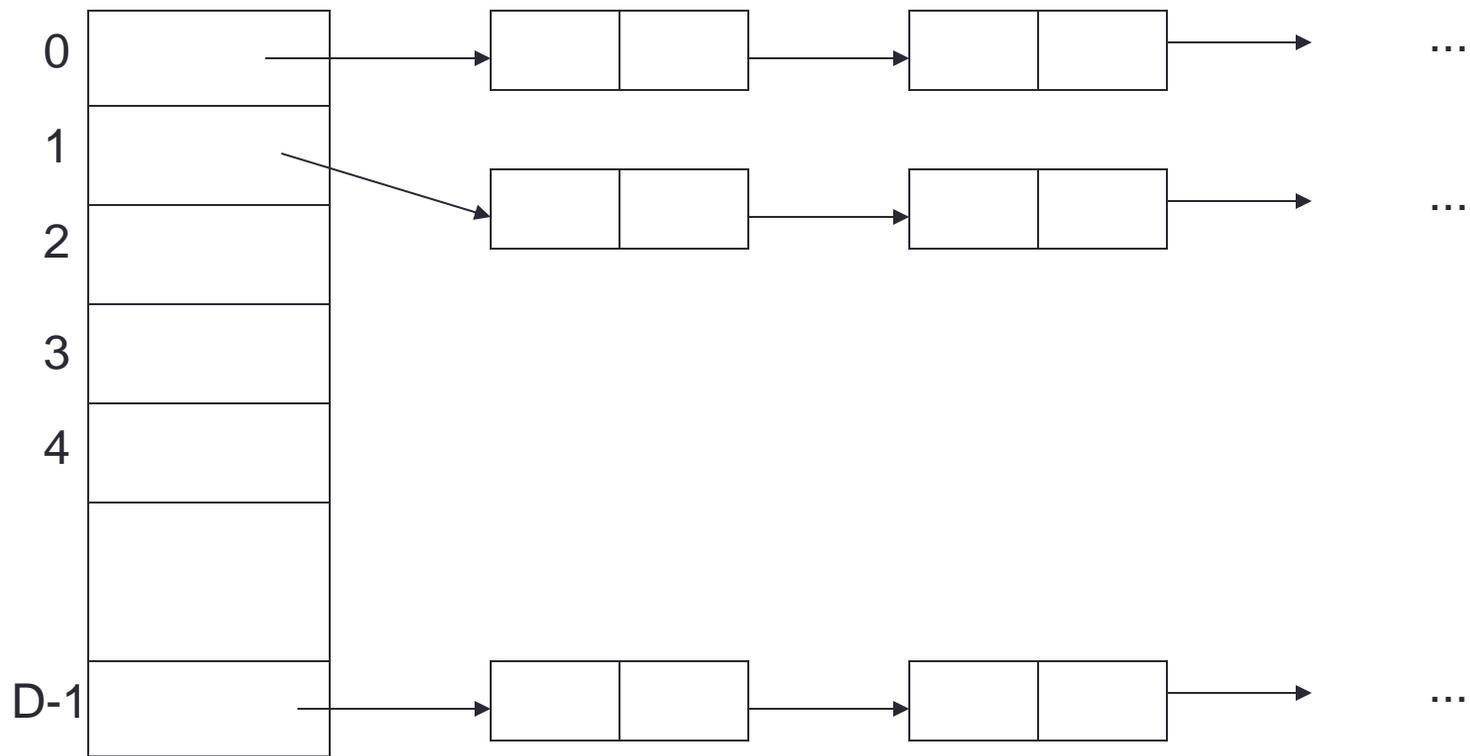
Collision Resolution Policies

- Two classes:
 - (1) Open hashing, a.k.a. separate chaining
 - (2) Closed hashing, a.k.a. open addressing
- Difference has to do with whether collisions are stored *outside the table* (open hashing) or whether collisions result in storing one of the records at *another slot in the table* (closed hashing)

Open Hashing

- Each bucket in the hash table is the head of a linked list
- All elements that hash to a particular bucket are placed on that bucket's linked list
- Records within a bucket can be ordered in several ways
 - by order of insertion, by key value order, or by frequency of access order

Open Hashing Data Organization



Analysis

- Open hashing is most appropriate when the hash table is kept in main memory, implemented with a standard in-memory linked list
- We hope that number of elements per bucket roughly equal in size, so that the lists will be short
- If there are n elements in set, then each bucket will have roughly n/D
- If we can estimate n and choose D to be roughly as large, then the average bucket will have only one or two members

Analysis Cont'd

Average time per dictionary operation:

- D buckets, n elements in dictionary \Rightarrow average n/D elements per bucket
- *insert*, *search*, *remove* operation take $O(1+n/D)$ time each
- If we can choose D to be about n , constant time
- Assuming each element is likely to be hashed to any bucket, running time constant, independent of n

Data Structure for Chaining

```
#define MAX CHAR 10
#define TABLE SIZE 13
#define IS_FULL(ptr) (! (ptr))

typedef struct {
    char key[MAX CHAR];
    /* other fields */
} element;

typedef struct list *list_pointer;
typedef struct list {
    element item;
    list_pointer link;
};

list_pointer hash_table[TABLE_SIZE];
```

Chain Insert

```
void chain_insert(element item, list_pointer ht[])
{
    int hash_value = hash(item.key);
    list_pointer ptr, trail=NULL, lead=ht[hash_value];
    for (; lead; trail=lead, lead=lead->link)
        if (!strcmp(lead->item.key, item.key)) {
            fprintf(stderr, "The key is in the table\n");
            exit(1);
        }
    ptr = (list_pointer) malloc(sizeof(list));
    if (IS_FULL(ptr)) {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    ptr->item = item;
    ptr->link = NULL;
    if (trail) trail->link = ptr;
    else ht[hash_value] = ptr;
}
```

Results of Hash Chaining

acos, atoi, char, define, exp, ceil, cos, float, atol, floor, ctime

$f(x)$ =first character of x

