Lecture 5:
Stacks and Queues
Stacks

• A list on which insertion and deletion can be performed.
  • Based on Last-in-First-out (LIFO)

• Stacks are used for a number of applications:
  • Converting a decimal number into binary
  • Program execution
  • Parsing
  • Evaluating postfix expressions
  • Towers of Hanoi
  …
Stacks

A stack is an ordered list in which insertions and deletions are made at one end called the **top**.
Stacks
Object of the game is to move all the disks (animals) over to Tower 3. But you cannot place a larger disk onto a smaller disk.
Towers of Hanoi
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Towers of Hanoi
Towers of Hanoi
Stack Operations

1. Pop()
2. Push(x)
3. Top()
4. IsEmpty()

- An insertion (of, say x) is called push operation and removing the most recent element from stack is called pop operation.
- Top returns the element at the top of the stack.
- IsEmpty returns true if the stack is empty, otherwise returns false.

All of these take constant time - \( O(1) \)
Example

- Push(2)
- Push(10)
- Pop()
- Push(7)
- Push(5)
- Top(): 5
- isEmpty(): False
Array implementation of stack (pseudocode)

```plaintext
int A[10]
top ← -1  //empty stack
Push(x)
{
    top ← top + 1
    A[top] ← x
}
Pop()
{
    top ← top - 1
}
```

For an empty stack, top is set to -1. In push function, we increment top. In pop, we decrement top by 1.
Array implementation of stack (pseudocode)

```plaintext
Top()
{
    return A[top]
}
IsEmpty()
{
    if(top == -1)
        return true
    else
        return false
}
```

![Stack implementation with array and push operations](image)
#define MAX_STACK_SIZE 100

typedef struct{
    int VALUE;
} element;

element stack[MAX_STACK_SIZE];
int top=-1;
Push Stack

```c
void push (element item)
{
    if(top >= MAX_STACK_SIZE-1){
        isFull();
        return;
    }
    stack[++top]=item;
}
```
element pop()
{
    if(top==-1)
        return empty_stack();
    return stack[top--];
}
Implementation of Stacks Using Arrays
More array implementation

```c
// Stack - Array based implementation.
#include<stdio.h>
#define MAX_SIZE 101
int A[MAX_SIZE];
int top = -1;
void Push(int x) {
    if(top == MAX_SIZE -1) {
        printf("Error: stack overflow\n");
        return;
    }
    A[++top] = x;
}
void Pop() {
    if(top == -1) {
        printf("Error: No element to pop\n");
        return;
    }
    top--;
}
int Top() {
    return A[top];
}
int main() {
}
```
void Print() {
    int i;
    printf("Stack: ");
    for(i = 0; i <= top; i++)
        printf("%d ", A[i]);
    printf("\n");
}

int main() {
    Push(2); Print();
    Push(5); Print();
    Push(10); Print();
    Pop(); Print();
    Push(12); Print();
}
Check For Balanced Parentheses using Stack

<table>
<thead>
<tr>
<th>Expression</th>
<th>Balanced?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A+B)</td>
<td></td>
</tr>
<tr>
<td>{(A+B)+(C+D)}</td>
<td></td>
</tr>
<tr>
<td>{(x+y)*(z)</td>
<td></td>
</tr>
<tr>
<td>[2*3]+(A)</td>
<td></td>
</tr>
<tr>
<td>{a+z}</td>
<td></td>
</tr>
</tbody>
</table>
Check For Balanced Parantheses using Stack

<table>
<thead>
<tr>
<th>Expression</th>
<th>Balanced?</th>
</tr>
</thead>
<tbody>
<tr>
<td>()</td>
<td>Yes</td>
</tr>
<tr>
<td>{(()())}</td>
<td>Yes</td>
</tr>
<tr>
<td>{(()()}</td>
<td>No</td>
</tr>
<tr>
<td>[1()]</td>
<td>No</td>
</tr>
<tr>
<td>{}</td>
<td>No</td>
</tr>
</tbody>
</table>

The count of opening should be equal to the count of closings. AND Any parenthesis opened last should be closed first.
Idea: Create an empty list

- Scan from left to right
  - If opening symbol, add it to the list
    - Push it into the stack
  - If closing symbol, remove last opening symbol of the same type using Pop from the stack

Should end with an empty list
Check For Balanced Parantheses: Pseudocode

```
CheckBalancedParanthesis(exp) {
    n ← length(exp)
    Create a stack: S
    for i ← 0 to n-1{
        if exp[i] is '(' or '{' or '['
            Push(exp[i])
        else if exp[i] is ')' or '}' or ']'{
            if (S is empty or
top does not pair with exp[i])
                return false
        else
            pop()
    }
    return S is empty?
}
```

Create a stack of characters and scan this string by using push if the character is an opening parenthesis and by using pop if the character is a closing parenthesis. (See next slide)
Examples

The pseudo code will return false.

The pseudo code will return true.
Queues

- A queue is an ordered list on which
  - all insertions take place at one end called the **rear/back** and
  - all deletions take place at the opposite end called the **front**.
- Based on **First-in-First-out (FIFO)**
Comparison of Queue and Stack

Queue ADT

Queue - First-In-First-Out (FIFO)

Stack - Last-In-First-Out (LIFO)
Queue is a list with the restriction that insertion can be made at one end (rear) and deletion can be made at other end (front).
Built-in Operations for Queue

Enqueue(x) or Push(x)
Dequeue() or Pop()
Front(): Returns the element in the front without removing it.
IsEmpty(): Returns true or false as an answer.
IsFull()

Each operation takes constant time, therefore has $O(1)$ time complexity.
Example

Enqueue (2)
Enqueue (5)
Enqueue (3)
Dequeue () → 2
Front () → 5
IsEmpty () → False

Applications:
• Printer queue
• Process scheduling
Array implementation of queue (Pseudocode)

```plaintext
int A[10]
front ← -1
rear ← -1
IsEmpty(){
    if (front == -1 && rear == -1)
        return true
    else
        return false
}
Enqueue(x){
    if IsFull()
        return
    else if IsEmpty()
        front ← rear ← 0
    else
        rear ← rear+1
    A[rear]← x
}
Array implementation of queue (Pseudocode)

Dequeue() {
    if IsEmpty() {
        return
    } else if (front == rear) {
        front ← rear ← -1
    } else {
        front ← front + 1
    }
}

At this stage, we cannot Enqueue an element anymore.
#define MAX_QUEUE_SIZE 100

typedef struct{
    int value;
} element;

element queue[MAX_QUEUE_SIZE];
int front= -1;
int rear= -1;
Add Queue

```c
void addq( element item) {
    if(rear == MAX_QUEUE_SIZE-1) {
        isFull();
        return;
    }
    queue[++rear]=item;
}
```
Delete Queue

element deleteq(element item)
{
    if(front == rear)
        return isEmpty();
    return queue[++front];
}
Circular Queue

- When the queue is full (the rear index equals to MAX_QUEUE_SIZE)
  - We should move the entire queue to the left
  - Recalculate the rear

Shifting an array is time-consuming!
- O(MAX_QUEUE_SIZE)
Circular Queue

- More efficient queue representation
Full Circular Queue

![Diagram of full circular queue with front = 0 and rear = 5 on the left, and front = 4 and rear = 3 on the right.](image-url)
Enqueue for circular array (Pseudocode)

Current position = i
Next position = (i+1) % N
previous position = (i+N-1) % N

Enqueue(x) {
    if (rear+1) % N == front
        return
    else if IsEmpty()
        front <- rear <- 0
    else
        rear <- (rear+1) % N
        A[rear] <- x
}
Dequeue for circular array (Pseudocode)

Dequeue(x)
{
    if IsEmpty()
        return
    else if(front == rear)
        front ← rear ← -1
    else
        front ← (front+1)\%N
}
Add Circular Queue

```c
void addcircularq( element item)
{
    rear=(rear+1)% MAX_QUEUE_SIZE;
    if(front == rear)
    {
        isFull(rear);
        return;
    }
    queue[rear]=item;
}
```
Delete Circular Queue

```c
void deletecircularq()
{
    if(front == rear)
        return isEmpty();
    front=(front+1)% MAX_QUEUE_SIZE;
    return queue[front];
}
```