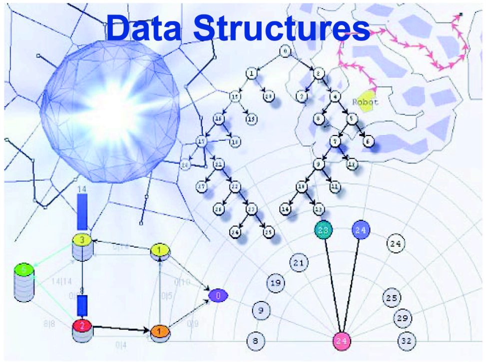
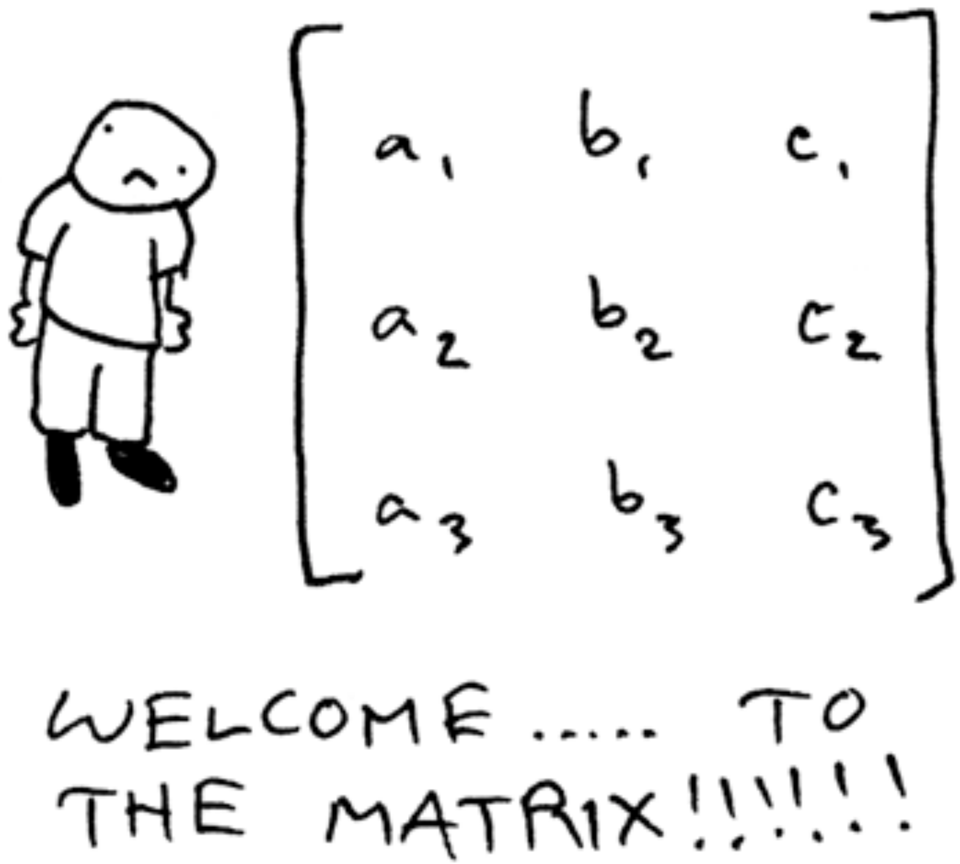
BBM 201
DATA STRUCTURES

Lecture 4:
Lower/Upper Triangular Matrix
Band Matrix
Sparse Matrix



2019-2020 Fall



$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$
WELCOME ..... TO
THE MATRIX!!!!!!

Lower Triangular Matrix

Triangular matrix

Upper triangular matrix

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & . & . & . & u_{1n} \\ 0 & u_{22} & u_{23} & & & & . \\ 0 & 0 & u_{33} & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & . \\ 0 & . & . & . & . & 0 & u_{nn} \end{bmatrix}$$

Lower triangular matrix

$$L = \begin{bmatrix} l_{11} & 0 & 0 & . & . & . & 0 \\ l_{21} & l_{22} & 0 & & & & . \\ l_{31} & l_{32} & l_{33} & & & & . \\ . & & & . & & & . \\ . & & & & . & & . \\ . & & & & & . & 0 \\ l_{n1} & . & . & . & . & . & l_{nn} \end{bmatrix}$$

Lower Triangular Matrix

- Does the definition of a special data structure for triangular matrix provide any benefits over a typical matrix in terms of memory and processing time?
- We can insert the items in a single dimensional array:
- ***ALT***

a_{00}	a_{10}	a_{11}	a_{20}	a_{21}	a_{22}	a_{30}	a_{31}	a_{32}	a_{33}
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]

- Number of items in the array becomes:

$$1+2+\ldots+(n-1)+n=\frac{n(n+1)}{2}$$

$$a \begin{bmatrix} a_{00} & 0 & 0 & 0 \\ a_{10} & a_{11} & 0 & 0 \\ a_{20} & a_{21} & a_{22} & 0 \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Lower Triangular Matrix

- How can we find the position of u[i][j] in the array?
  - Answer: i=1, there is one item in the 0^{th} row, 2 items in the 1^{st} row.
  - i=2, there is one item in the 0^{th} row, 2 items in the 1^{st} row, 3 items in the 2^{nd} row.
  - Therefore the address of u[i][j] in the array is calculated as below:

$$k = \sum_{t=1}^{i}(t) + (j) = (1 + 2 + \cdots + i) + (j)$$

$$= \frac{i(i+1)}{2} + (j)$$

Lower Triangular Matrix

```
void main(void){
    int alt[MAX_SIZE];
    int i, n;
    scanf("%d", &n); //matrix size
    readtriangularmatrix(alt,n);
    for(i=0; i<=n*(n+1)/2-1; i++)
       printf("%d", alt[i]);

    i = gettriangularmatrix(3,0,n);
    if(i == -2)
       printf("\n invalid index\n");
    else if(i == -1)
       printf("\n access to the upper triangular\n");
    else
       printf("\n  the position in 'alt' matrix: %d value: %d \n", i, alt[i]);
}
```

Lower Triangular Matrix

```
void readtriangularmatrix(int alt[], int n)
{
   int i, j, k;
   if(n*(n+1)/2 > MAX_SIZE){
      printf("\n invalid array size \n");
      exit(-1);
   }
   else
      for(i=0; i<=n-1; i++){
         k=(i+1)*i/2;
         for(j=0; j<=i; j++)
            scanf("%d", &alt[k+j]);
      }
}
```

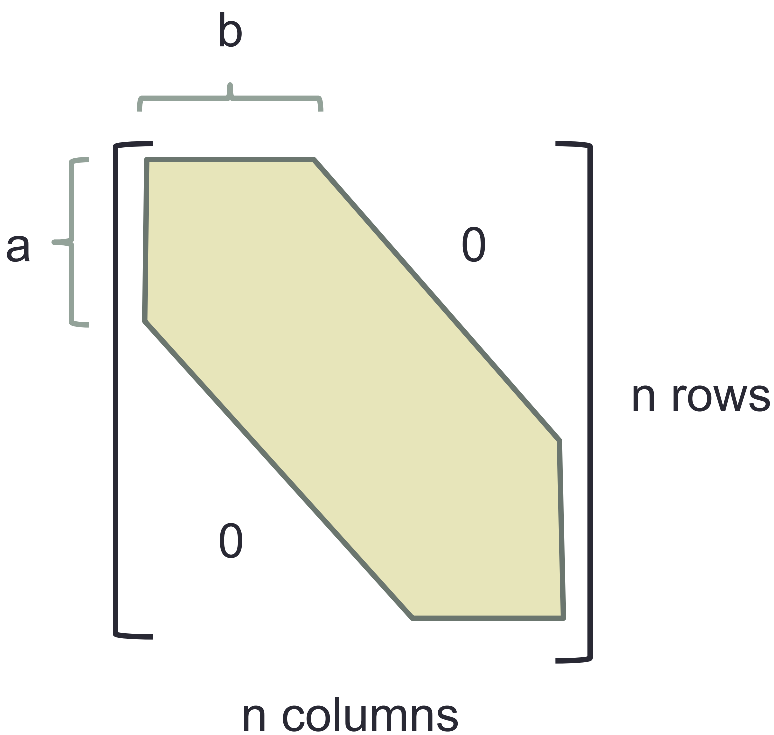
```
int gettriangularmatrix(int i, int j, int n){

   if(i<0 || i>=n || j<0 || j>=n){
      printf("\n invalid index\n");
      exit(-2);
   }
   else if(i>=j) // valid index
      return (i+1)*i/2 + j;
   else return -1; // outside of the triangle
                   // value is zero
}
```

Band Matrix

$$\begin{bmatrix} B_{11} & B_{12} & 0 & \cdots & \cdots & 0 \\ B_{21} & B_{22} & B_{23} & \ddots & \ddots & \vdots \\ 0 & B_{32} & B_{33} & B_{34} & \ddots & \vdots \\ \vdots & \ddots & B_{43} & B_{44} & B_{45} & 0 \\ \vdots & \ddots & \ddots & B_{54} & B_{55} & B_{56} \\ 0 & \cdots & \cdots & 0 & B_{65} & B_{66} \end{bmatrix}$$

Matrix (n, a) : n by n matrix, non-zero entries are confined to a diagonal band, comprising the main diagonal and zero or more diagonals (a-1) on either side.

Band Matrix



$$\begin{bmatrix} a_{00} & a_{01} & 0 & 0 \\ a_{10} & a_{11} & a_{12} & 0 \\ a_{20} & a_{21} & a_{22} & a_{23} \\ 0 & a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$b = 2$$
$$a = 3$$
$$n = 4$$

Band Matrix

- What kind of a data structure can we use?
- We can insert the items in a single dimensional array:
- ***ALT***

a_{20}	a_{31}	a_{10}	a_{21}	a_{32}	a_{00}	a_{11}	a_{22}	a_{33}	a_{01}	a_{12}	a_{23}
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

$$\begin{bmatrix} a_{00} & a_{01} & 0 & 0 \\ a_{10} & a_{11} & a_{12} & 0 \\ a_{20} & a_{21} & a_{22} & a_{23} \\ 0 & a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \begin{aligned} a &= 3 \\ b &= 2 \\ n &= 4 \end{aligned}$$

- What is the number of items in the array?

Band Matrix

- What is the number of items in the array?
  - Number of items **on** and **below** the diagonal:

$$n + (n-1) + (n-2) + \ldots + n - (a-1)$$

  - Number of items **above** the diagonal:

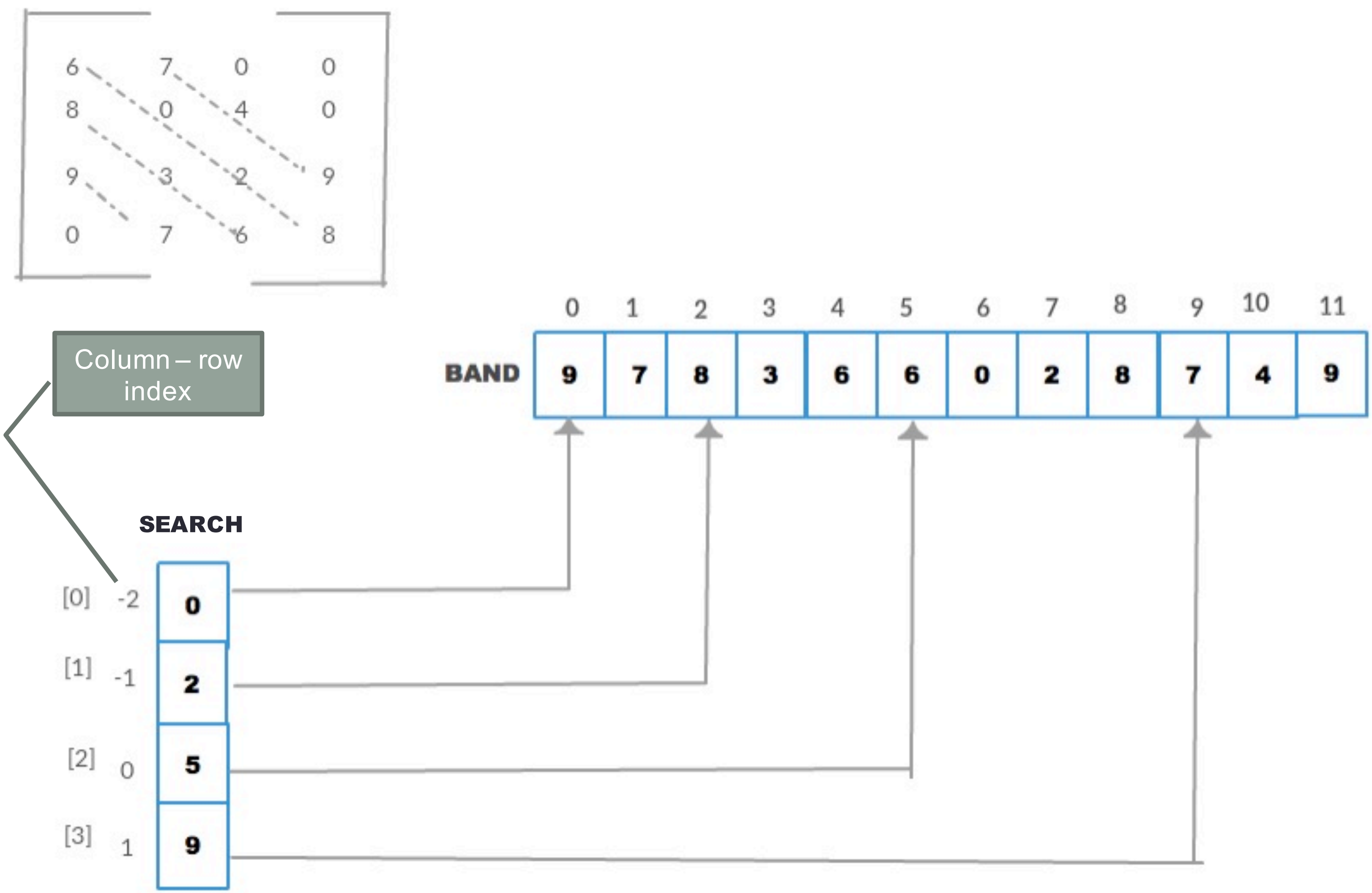
$$(n-1) + (n-2) + \ldots + n - (b-1)$$

  - Sum of these becomes:

$$Sum = n + (n-1) + (n-2) + \ldots + n - (a-1) + (n-1) + (n-2) + \ldots + n - (b-1)$$

$$= n(a+b-1) - \frac{(a-1)a}{2} - \frac{(b-1)b}{2}$$

Band Matrix


6 7 0 0
8 0 4 0
9 3 2 9
0 7 6 8
Column – row index
BAND
0 1 2 3 4 5 6 7 8 9 10 11
9 7 8 3 6 6 0 2 8 7 4 9
SEARCH
[0] -2 0
[1] -1 2
[2] 0 5
[3] 1 9

Band Matrix

```
void main(void){
   int band[MAX_SIZE];
   int search[MAX_SIZE];

   int i, n, a, b;
   printf(" n:", &n); scanf("%d", &n);
   printf(" a:", &a); scanf("%d", &a);
   printf(" b:", &b); scanf("%d", &b);

   buildbandmatrix(band,search,a,b);

   for(i=0; i <= n*(a+b-1) - a*(a-1)/2 - b*(b-1)/2 - 1; i++)
      printf(" %d ",band[i];

   printf("\n");
   for(i=0; i <= a+b-2; i++)
      printf(" %d", search[i]);

   i = getbandmatrix(3, 3, n, a, b, search);

   if(i == -2)
      printf("\n invalid index");
   else if(i == -1)
      printf("\n item to be searched: 0");
   else
      printf("\n item to be searched: %d -> %d", i, band[i]);
}
```

Band Matrix

```
void buildbandmatrix(int band[], int search[], int n, int a, int b){

   int i, k, itemnum;

   if(n*(a+b-1) - a*(a-1)/2 - b*(b-1)/2 > MAX_SIZE){
      printf("\n not enough memory");
      exit(-1);
   }
   else{
      itemnum = 0;
      for(i=-a+1; i<=b-1; i++){ //for each diagonal
         search[i+a-1] = itemnum;

         for(k=0; k <= n-abs(i)-1; k++)   //for the current diagonal
            scanf("%d", &band[search[i+a-1]+k]);
         itemnum = itemnum+(n-abs(i));
      }
   }
}
```

Band Matrix

```
void getbandmatrix(int i, int j, int n, int a, int b, int search[]){

   if(i>=n || i<0 || j>=n || j<0)  //index overflow
   {
      printf("\n invalid index\n");
      return -2;
   }
   else
   {
      if(j>i)   //above the diagonal
         if(j-i<b)  //above the upper band
            return(search[a-1+j-i]+i);  //yes
         else  //no
            return -1;
      else if(i-j<a) //below or on the diagonal
         return(search[j-i+a-1]+j);
      else //not on the band
         return -1;
   }
}
```

Sparse Matrix

- Most of the elements are zero.
- It wastes space.

Sparsity: the fraction of zero elements.

<u>Basic matrix operations:</u>

1. Creation
2. Addition
3. Multiplication
4. Transpose

$$\begin{array}{cc}
A & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
15 & 0 & 0 & 22 & 0 & -15 \\
0 & 11 & 3 & 0 & 0 & 0 \\
0 & 0 & 0 & -6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
91 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 28 & 0 & 0 & 0
\end{bmatrix}
\end{array}$$

Sparse Matrix

Data Structure

```
#define MAX_TERMS 101
typedef struct{
            int col;
            int row;
            int value;
           }term;
term a[MAX_TERMS];
```

- a[0].row: row index
- a[0].col: column index
- a[0].value: number of items in the sparse matrix

Rows and columns are in ascending order!

Sparse Matrix

$$\begin{array}{c|c} A & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 15 & 0 & 0 & 22 & 0 & -15 \\ 0 & 11 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 91 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 28 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

	Row	Column	Value
A[0]	6	6	8
A[1]	0	0	15
A[2]	0	3	22
A[3]	0	5	-15
A[4]	1	1	11
A[5]	1	2	3
...			
A[8]	5	2	28

Matrix Transpose

- Replacement of rows and columns in a matrix is called the transpose of the matrix:

$$A = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$$

- The item a[i][j] becomes a[j][i].

Matrix Transpose

```
void transpose(term  a[],term b[])
{
  int n,i,j,currentb;
  n=a[0].value;   //number of items
  b[0].row=a[0].col;    //number of rows
  b[0].col=a[0].row;    //number of columns
  b[0].value=n;

  if(n>0){
    currentb=1;
    for(i=0; i<a[0].col; i++)
      for(j=1; j<=n; j++) //find the ones with col i in a
        if(a[j].col==i){
          b[currentb].row  = a[j].col;
          b[currentb].col  = a[j].row;
          b[currentb].value  = a[j].value;
          currentb++;
        }
  }
}
```

Question: What is the complexity of this method?

Fast Transpose

```
#define MAX_TERM 101
typedef struct{
            int row;
            int col;
            int value;
              } term;
term a[MAX_TERM];

void fastTranspose(term  a[], term b[])
{
  int ItemNum[MAX_COL],  StartPos[MAX_COL];
  int i, j, ColNum=a[0].col,  TermNum=a[0].value;
  b[0].value=TermNum;
  if(TermNum>0){  //does  the  item  exist?
    for(i  = 0;  i  < ColNum;  i++)
      ItemNum[i]  = 0;
    for(i  = 1;  i  <= TermNum;  i++)
      ItemNum[a[i].col]++;
    StartPos[0]  = 1;
    for(i  = 1;  i  < ColNum;  i++)
      StartPos[i]  = StartPos[i-1]  + ItemNum[i-1];
    for(i  = 1;  i  <= TermNum;  i++){
      j = StartPos[a[i].col]++;
      b[j].row  = a[i].col;
      b[j].col  = a[i].row;
      b[j].value  = a[i].value;
    }
  }
}
```

Fast Transpose

- Execute the fastTranspose method.
- **Question:** What is the complexity of the method?
- Compare its complexity with the previous transpose method.