Acknowledgement: The course slides are adapted from the slides prepared by R. Sedgewick and K. Wayne of Princeton University.
Today

- Tries
- R-way tries
- Ternary search tries
- Character-based operations
### Review: summary of the performance of symbol-table implementations

**Order of growth of the frequency of operations.**

<table>
<thead>
<tr>
<th>implementation</th>
<th>typical case</th>
<th>ordered operations</th>
<th>operations on keys</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>search</td>
<td>insert</td>
<td>delete</td>
</tr>
<tr>
<td>red-black BST</td>
<td>log N</td>
<td>log N</td>
<td>log N</td>
</tr>
<tr>
<td></td>
<td>hashtable</td>
<td>1 †</td>
<td>1 †</td>
</tr>
</tbody>
</table>

- † under uniform hashing assumption

**Q.** Can we do better?  
**A.** Yes, if we can avoid examining the entire key, as with string sorting.
String symbol table. Symbol table specialized to string keys.

String symbol table basic API

public class StringST<Value>

StringST()  
create an empty symbol table

void put(String key, Value val)  
put key-value pair into the symbol table

Value get(String key)  
return value paired with given key

void delete(String key)  
delete key and corresponding value

Goal. Faster than hashing, more flexible than BSTs.
# String symbol table implementations cost summary

<table>
<thead>
<tr>
<th>implementation</th>
<th>character accesses (typical case)</th>
<th>dedup</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>search hit</td>
<td>search miss</td>
</tr>
<tr>
<td>red-black BST</td>
<td>L + c ( \log N )</td>
<td>c ( \log N )</td>
</tr>
<tr>
<td>hashing (linear probing)</td>
<td>L</td>
<td>L</td>
</tr>
</tbody>
</table>

**Parameters**

- \( N \) = number of strings
- \( L \) = length of string
- \( R \) = radix

<table>
<thead>
<tr>
<th>file</th>
<th>size</th>
<th>words</th>
<th>distinct</th>
</tr>
</thead>
<tbody>
<tr>
<td>moby.txt</td>
<td>1.2 MB</td>
<td>210 K</td>
<td>32 K</td>
</tr>
<tr>
<td>actors.txt</td>
<td>82 MB</td>
<td>11.4 M</td>
<td>900 K</td>
</tr>
</tbody>
</table>

**Challenge. Efficient performance for string keys.**
R-way tries
Ternary search tries
Character-based operations
**Tries.** [from retrieval, but pronounced "try"]

- Store characters in nodes (not keys).
- Each node has \( R \) children, one for each possible character.
- Store values in nodes corresponding to last characters in keys.

<table>
<thead>
<tr>
<th>key</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>by</td>
<td>4</td>
</tr>
<tr>
<td>sea</td>
<td>6</td>
</tr>
<tr>
<td>sells</td>
<td>1</td>
</tr>
<tr>
<td>she</td>
<td>0</td>
</tr>
<tr>
<td>shells</td>
<td>3</td>
</tr>
<tr>
<td>shore</td>
<td>7</td>
</tr>
<tr>
<td>the</td>
<td>5</td>
</tr>
</tbody>
</table>
Search in a trie

Follow links corresponding to each character in the key.

- **Search hit:** node where search ends has a non-null value.
- **Search miss:** reach a null link or node where search ends has null value.

\[\text{get("shells")}\]
Search in a trie

Follow links corresponding to each character in the key.

- **Search hit**: node where search ends has a non-null value.
- **Search miss**: reach a null link or node where search ends has null value.

get("she")

[Diagram of trie with nodes and edges labeled with characters, showing the path for the search and highlighting the node where the search ends, with an annotation indicating that the search may be terminated at an intermediate node (return 0).]
Search in a trie

Follow links corresponding to each character in the key.

- **Search hit**: node where search ends has a non-null value.
- **Search miss**: reach a null link or node where search ends has null value.

`get("shell")`

![Trie diagram showing search for "shell" with no value associated with last key character (return null).]
Search in a trie

Follow links corresponding to each character in the key.

- **Search hit:** node where search ends has a non-null value.
- **Search miss:** reach a null link or node where search ends has null value.

```
get("shelter")
```

```
+---+---+---+---+---+
| b | s | h | e | l |
+---+---+---+---+---+
   |   |   | y | e |
+---+---+---+---+---+
      |   | a | l | e |
+---+---+---+---+---+
          |   | l | l | l |
+---+---+---+---+---+
                  |   | s | s | s |
+---+---+---+---+---+
                      |   | t | h | e |
  +---+---+---+---+---+
     |   |   | o | e | 7
  +---+---+---+---+---+

no link to 't' (return null)
```
Follow links corresponding to each character in the key.
- Encounter a null link: create new node.
- Encounter the last character of the key: set value in that node.

```
put("shore", 7)
```
Trie construction demo

put("she", 0)

key is sequence of characters from root to value
value is in node corresponding to last character
Trie construction demo

```
trie
she
```

Diagram:
```
s
h

she
```
```
e
0
```
Trie construction demo

she

trie

trie

she

e 0
she

put("sells", 1)
Trie construction demo

she
sells
trie
she
sells
trie
Trie construction demo

she
sells

`put("sea", 2)`
Trie construction demo

she
sells
sea
trie
Trie construction demo

she
sells
sea

put("shells", 3)
Trie construction demo

she
sells
sea
trie
Trie construction demo

she
sells
sea
put("by", 4)
Trie construction demo

she
sells
sea
by
trie
Trie construction demo

she
sells
sea
by
put("the", 5)
Trie construction demo

she
sells
sea
by
the
trie
put("sea", 6)

overwrite old value with new value
Trie construction demo

trie
Trie construction demo
Trie construction demo

she
sells
sea
by
the
put("shore", 7)
Trie construction demo

she
sells
sea
by
the
shore
trie

trie

trie

trie

trie

trie

trie

trie

Node. A value, plus references to $R$ nodes.

```java
private static class Node {
    private Object value;
    private Node[] next = new Node[R];
}
```

Trie representation: each node has an array of links and a value.

Neither keys nor characters are explicitly stored.

Characters are implicitly defined by link index.

Use `Object` instead of `Value` since no generic array creation in Java.
**Trie representation: Java implementation**

**Node.** A value, plus references to $R$ nodes.

```java
private static class Node
{
    private Object value;
    private Node[] next = new Node[R];
}
```

A child node for each character in Alphabet. No need to search for character, but a pointer reserved for each character in memory.

![Trie representation (R = 26)](image-url)

- Characters are implicitly defined by link index.
- Each node has an array of links and a value.
public class TrieST<Value>
{
    private static final int R = 256;
    private Node root;

    private static class Node
    {
        /* see previous slide */
    }

    public void put(String key, Value val)
    {
        root = put(root, key, val, 0);
    }

    private Node put(Node x, String key, Value val, int d)
    {
        if (x == null) x = new Node();
        if (d == key.length()) { x.val = val; return x; }
        char c = key.charAt(d);
        x.next[c] = put(x.next[c], key, val, d+1);
        return x;
    }

    :
    :
}
public boolean contains(String key)
{
    return get(key) != null;
}

delimiter

public Value get(String key)
{
    Node x = get(root, key, 0);
    if (x == null) return null;
    return (Value) x.val;  \[ cast needed \]
}

delimiter

private Node get(Node x, String key, int d)
{
    if (x == null) return null;
    if (d == key.length()) return x;
    char c = key.charAt(d);
    return get(x.next[c], key, d+1);
}

Trie performance

Search hit. Need to examine all $L$ characters for equality.

Search miss.
- Could have mismatch on first character.
- Typical case: examine only a few characters (sublinear).

Space. $R$ null links at each leaf.
(but sublinear space possible if many short strings share common prefixes)

Bottom line. Fast search hit and even faster search miss, but wastes space.
To delete a key-value pair:

- Find the node corresponding to key and set value to null.
- If that node has all null links, remove that node (and recur).

```
delete("shells")
```
To delete a key-value pair:

- Find the node corresponding to key and set value to null.
- If that node has all null links, remove that node (and recur).

```
delete("shells");
```
**String symbol table implementations cost summary**

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<td>$L + c \log^2 N$</td>
<td>$c \log^2 N$</td>
</tr>
<tr>
<td>hashing (linear probing)</td>
<td>$L$</td>
<td>$L$</td>
</tr>
<tr>
<td>R-way trie</td>
<td>$L$</td>
<td>$\log_R N$</td>
</tr>
</tbody>
</table>

**R-way trie.**

- Method of choice for small $R$.
- Too much memory for large $R$.

**Challenge.** Use less memory, e.g., 65,536-way trie for Unicode!
Digression: out of memory?

“640 K ought to be enough for anybody.”
— (mis)attributed to Bill Gates, 1981
(commenting on the amount of RAM in personal computers)

“64 MB of RAM may limit performance of some Windows XP features; therefore, 128 MB or higher is recommended for best performance.”
— Windows XP manual, 2002

“64 bit is coming to desktops, there is no doubt about that. But apart from Photoshop, I can't think of desktop applications where you would need more than 4GB of physical memory, which is what you have to have in order to benefit from this technology. Right now, it is costly.”
— Bill Gates, 2003
## Digression: out of memory?

A short (approximate) history.

<table>
<thead>
<tr>
<th>machine</th>
<th>year</th>
<th>address bits</th>
<th>addressable memory</th>
<th>typical actual memory</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDP-8</td>
<td>1960s</td>
<td>12</td>
<td>6 KB</td>
<td>6 KB</td>
<td>$16K</td>
</tr>
<tr>
<td>PDP-10</td>
<td>1970s</td>
<td>18</td>
<td>256 KB</td>
<td>256 KB</td>
<td>$1M</td>
</tr>
<tr>
<td>IBM S/360</td>
<td>1970s</td>
<td>24</td>
<td>4 MB</td>
<td>512 KB</td>
<td>$1M</td>
</tr>
<tr>
<td>VAX</td>
<td>1980s</td>
<td>32</td>
<td>4 GB</td>
<td>1 MB</td>
<td>$1M</td>
</tr>
<tr>
<td>Pentium</td>
<td>1990s</td>
<td>32</td>
<td>4 GB</td>
<td>1 GB</td>
<td>$1K</td>
</tr>
<tr>
<td>Xeon</td>
<td>2000s</td>
<td>64</td>
<td>enough</td>
<td>4 GB</td>
<td>100 $</td>
</tr>
<tr>
<td>??</td>
<td>future</td>
<td>128+</td>
<td>enough</td>
<td>enough</td>
<td>1 $</td>
</tr>
</tbody>
</table>

“512-bit words ought to be enough for anybody.”
— Kevin Wayne, 1995
TRIES

- R-way tries
- Ternary search tries
- Character-based operations
• Store characters and values in nodes (not keys).
• Each node has three children: smaller (left), equal (middle), larger (right).

Fast Algorithms for Sorting and Searching Strings

Jon L. Bentley*  Robert Sedgewick#

Abstract

We present theoretical algorithms for sorting and searching multikey data, and derive from them practical C implementations for applications in which keys are character strings. The sorting algorithm blends Quicksort and radix sort, it is competitive with the best known C sort codes. The searching algorithm blends tries and binary search trees, it is faster than hashing and other commonly used search methods. The basic ideas behind the algorithm that is competitive with the most efficient string sorting programs known. The second program is a symbol table implementation that is faster than hashing, which is commonly regarded as the fastest symbol table implementation. The symbol table implementation is much more space-efficient than multiway trees, and supports more advanced searches.

In many application programs, sorts use a Quicksort implementation based on an abstract compare operation,
• Store characters and values in nodes (not keys).
• Each node has three children: smaller (left), equal (middle), larger (right).

Ternary search tries

TST representation of a trie
Search in a TST

Follow links corresponding to each character in the key.
- If less, take left link; if greater, take right link.
- If equal, take the middle link and move to the next key character.

Search hit.  Node where search ends has a non-null value.
Search miss.  Reach a null link or node where search ends has null value.
Search in a TST

get("sea")

return value associated with last key character
Search in a TST

get("shelter")

no link to 't'
(return null)
Ternary search trie insertion demo

ternary search trie
Ternary search trie insertion demo

`put("she", 0)`

- Key is sequence of characters from root to value using middle links.
- Value is in node corresponding to last character.
Ternary search trie insertion demo

```plaintext
put("she", 0)
```
put("sells", 1)
Ternary search trie insertion demo

ternary search trie

`nodes`
put("sea", 2)
Ternary search trie insertion demo

ternary search trie
put("shells", 3)
Ternary search trie insertion demo

ternary search trie
put("by", 4)
Ternary search trie insertion demo

ternary search trie
Ternary search trie insertion demo

put("the", 5)
Ternary search trie insertion demo
put("sea", 6)

overwrite old value with new value
Ternary search trie insertion demo

ternary search trie

```
        s
       /|
      b y
     /|
    4 h
   /|
  e l
 /|
6 a
```

```
   0 e l
   |
   1
```

```
   3 s
  /|
  s
```
put("shore", 7)
Ternary search trie insertion demo

ternary search trie
Ternary search trie insertion demo

ternary search trie

```
  b
  /\  
 /   \ 
 y 4  e
  /\  /\  
 /   /   
 h  e  l
  /\  /\  /\  
 l  e  l  r
 /\  /\  /\  /\  
 s  s  e  e  e
```

66
26-way trie vs. TST

26-way trie. 26 null links in each leaf.

TST. 3 null links in each leaf.
A TST node is five fields:

- A value.
- A character $c$. 
- A reference to a left TST.
- A reference to a middle TST.
- A reference to a right TST.

```java
private class Node {
    private Value val;
    private char c;
    private Node left, mid, right;
}
```
public class TST<Value> {
    private Node root;

    private class Node {
        /* see previous slide */
    }

    public void put(String key, Value val) {
        root = put(root, key, val, 0);
    }

    private Node put(Node x, String key, Value val, int d) {
        char c = key.charAt(d);
        if (x == null) {
            x = new Node();
            x.c = c;
        }
        if (c < x.c) x.left = put(x.left, key, val, d);
        else if (c > x.c) x.right = put(x.right, key, val, d);
        else if (d < key.length() - 1) x.mid = put(x.mid, key, val, d+1);
        else x.val = val;
        return x;
    }

    ...
public boolean contains(String key)
{
    return get(key) != null;
}

public Value get(String key)
{
    Node x = get(root, key, 0);
    if (x == null) return null;
    return x.val;
}

private Node get(Node x, String key, int d)
{
    if (x == null) return null;
    char c = key.charAt(d);
    if   (c < x.c) return get(x.left, key, d);
    else if (c > x.c) return get(x.right, key, d);
    else if (d < key.length() - 1) return get(x.mid, key, d+1);
    else return x;
}
## String symbol table implementation cost summary

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<td>L + c (\lg^2 N)</td>
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</tr>
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<td>hashing (linear probing)</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>R-way trie</td>
<td>L</td>
<td>(\log R N)</td>
</tr>
<tr>
<td>TST</td>
<td>L + (\ln N)</td>
<td>(\ln N)</td>
</tr>
</tbody>
</table>

**Remark.** Can build balanced TSTs via rotations to achieve \(L + \log N\) worst-case guarantees.

**Bottom line.** TST is as fast as hashing (for string keys), space efficient.
TST vs. hashing

Hashing.
- Need to examine entire key.
- Search hits and misses cost about the same.
- Performance relies on hash function.
- Does not support ordered symbol table operations.

TSTs.
- Works only for strings (or digital keys).
- Only examines just enough key characters.
- Search miss may involve only a few characters.
- Supports ordered symbol table operations (plus others!).

Bottom line. TSTs are:
- Faster than hashing (especially for search misses).
  More flexible than red-black BSTs.  
  [stay tuned]
Tries.
- Store characters in nodes (not keys).
- Each node has $R$ children, one for each possible character.
- Store values in nodes corresponding to last characters in keys.

Ternary Search Trees (TSTs)
- Store characters and values in nodes (not keys).
- Each node has three children: smaller (left), equal (middle), larger (right).
Tries

- R-way tries
- Ternary search tries
- Character-based operations
Character-based operations. The string symbol table API supports several useful character-based operations.

<table>
<thead>
<tr>
<th>key</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>by</td>
<td>4</td>
</tr>
<tr>
<td>sea</td>
<td>6</td>
</tr>
<tr>
<td>sells</td>
<td>1</td>
</tr>
<tr>
<td>she</td>
<td>0</td>
</tr>
<tr>
<td>shells</td>
<td>3</td>
</tr>
<tr>
<td>shore</td>
<td>7</td>
</tr>
<tr>
<td>the</td>
<td>5</td>
</tr>
</tbody>
</table>

Prefix match. Keys with prefix "sh": "she", "shells", and "shore".

Wildcard match. Keys that match ".he": "she" and "the".

Longest prefix. Key that is the longest prefix of "shellsort": "shells".
## String symbol table API

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>public class <code>StringST&lt;Value&gt;</code></td>
<td></td>
</tr>
<tr>
<td><code>StringST()</code></td>
<td>create a symbol table with string keys</td>
</tr>
<tr>
<td><code>void put(String key, Value val)</code></td>
<td>put key-value pair into the symbol table</td>
</tr>
<tr>
<td><code>Value get(String key)</code></td>
<td>value paired with key</td>
</tr>
<tr>
<td><code>void delete(String key)</code></td>
<td>delete key and corresponding value</td>
</tr>
<tr>
<td><code>Iterable&lt;String&gt; keys()</code></td>
<td>all keys</td>
</tr>
<tr>
<td><code>Iterable&lt;String&gt; keysWithPrefix(String s)</code></td>
<td>keys having s as a prefix</td>
</tr>
<tr>
<td><code>Iterable&lt;String&gt; keysThatMatch(String s)</code></td>
<td>keys that match s (where . is a wildcard)</td>
</tr>
<tr>
<td><code>String longestPrefixOf(String s)</code></td>
<td>longest key that is a prefix of s</td>
</tr>
</tbody>
</table>

**Remark.** Can also add other ordered ST methods, e.g., `floor()` and `rank()`. 
Warmup: ordered iteration

To iterate through all keys in sorted order:
- Do inorder traversal of trie; add keys encountered to a queue.
- Maintain sequence of characters on path from root to node.
To iterate through all keys in sorted order:

- Do inorder traversal of trie; add keys encountered to a queue.
- Maintain sequence of characters on path from root to node.

```java
public Iterable<String> keys()
{
    Queue<String> queue = new Queue<String>();
collect(root, "", queue);
return queue;
}

private void collect(Node x, String prefix, Queue<String> q)
{
    if (x == null) return;
    if (x.val != null) q.enqueue(prefix);
for (char c = 0; c < R; c++)
    collect(x.next[c], prefix + c, q);
}
```
Prefix matches

Find all keys in symbol table starting with a given prefix.

Ex. Autocomplete in a cell phone, search bar, text editor, or shell.

• User types characters one at a time.
• System reports all matching strings.
Find all keys in symbol table starting with a given prefix.

public Iterable<String> keysWithPrefix(String prefix) {
    Queue<String> queue = new Queue<String>();
    Node x = get(root, prefix, 0);
    collect(x, prefix, queue);
    return queue;
}
Use wildcard . to match any character in alphabet.

<table>
<thead>
<tr>
<th>co.....er</th>
<th>.c.....c.</th>
</tr>
</thead>
<tbody>
<tr>
<td>coalizer</td>
<td>acresce</td>
</tr>
<tr>
<td>coberger</td>
<td>acroach</td>
</tr>
<tr>
<td>codifier</td>
<td>acuracy</td>
</tr>
<tr>
<td>cofaster</td>
<td>octarch</td>
</tr>
<tr>
<td>cofather</td>
<td>science</td>
</tr>
<tr>
<td>cognizer</td>
<td>scranch</td>
</tr>
<tr>
<td>cohelper</td>
<td>scratch</td>
</tr>
<tr>
<td>colander</td>
<td>scrauch</td>
</tr>
<tr>
<td>coleader</td>
<td>screich</td>
</tr>
<tr>
<td>...</td>
<td>scrincher</td>
</tr>
<tr>
<td>compiler</td>
<td>scritch</td>
</tr>
<tr>
<td>...</td>
<td>scrunch</td>
</tr>
<tr>
<td>composer</td>
<td>scudick</td>
</tr>
<tr>
<td>computer</td>
<td>scutock</td>
</tr>
<tr>
<td>cowkeeper</td>
<td></td>
</tr>
</tbody>
</table>
Wildcard matches

Search as usual if character is not a period; go down all $R$ branches if query character is a period.

```java
public Iterable<String> keysThatMatch(String pat) {
    Queue<String> queue = new Queue<String>();
    collect(root, "", 0, pat, queue);
    return queue;
}

private void collect(Node x, String prefix, String pat, Queue<String> q) {
    if (x == null) return;
    int d = prefix.length();
    if (d == pat.length() && x.val != null) q.enqueue(prefix);
    if (d == pat.length()) return;
    char next = pat.charAt(d);
    for (char c = 0; c < R; c++)
        if (next == '.' || next == c)
            collect(x.next[c], prefix + c, pat, q);
}
```
Longest prefix

Find longest key in symbol table that is a prefix of query string.

Ex. To send packet toward destination IP address, router chooses IP address in routing table that is longest prefix match.

- "128"
- "128.112"
- "128.112.055"
- "128.112.055.15"
- "128.112.136"
- "128.112.155.11"
- "128.112.155.13"
- "128.222"
- "128.222.136"

represented as 32-bit binary number for IPv4 (instead of string)

- longestPrefixOf("128.112.136.11") = "128.112.136"
- longestPrefixOf("128.112.100.16") = "128.112"
- longestPrefixOf("128.166.123.45") = "128"

Note. Not the same as floor:  \( \text{floor}("128.112.100.16") = "128.112.055.15" \)
**Longest prefix**

Find longest key in symbol table that is a prefix of query string.

- Search for query string.
- Keep track of longest key encountered.

**Possibilities for `longestPrefixOf()`**

- For "she":
  - Search ends at end of string value is not null return she

- For "shell":
  - Search ends at end of string value is null return she (last key on path)

- For "shellsort":
  - Search ends at null link return shells (last key on path)
Find longest key in symbol table that is a prefix of query string.

• Search for query string.
• Keep track of longest key encountered.

```java
public String longestPrefixOf(String query) {
    int length = search(root, query, 0, 0);
    return query.substring(0, length);
}

private int search(Node x, String query, int d, int length) {
    if (x == null) return length;
    if (x.val != null) length = d;
    if (d == query.length()) return length;
    char c = query.charAt(d);
    return search(x.next[c], query, d+1, length);
}
```
**T9 texting**

**Goal.** Type text messages on a phone keypad.

**Multi-tap input.** Enter a letter by repeatedly pressing a key until the desired letter appears.

"a much faster and more fun way to enter text"

**T9 text input.**
- Find all words that correspond to given sequence of numbers.
- Press 0 to see all completion options.

**Ex.** hello
- Multi-tap: 4 4 3 3 5 5 5 5 5 5 5 6 6 6
- T9: 4 3 5 5 6

[Image: www.t9.com]
Patricia trie

Patricia trie. [Practical Algorithm to Retrieve Information Coded in Alphanumeric]

• Remove one-way branching.
• Each node represents a sequence of characters.
• Implementation: one step beyond this course.

Applications.
• Database search.
• P2P network search.
• IP routing tables: find longest prefix match.
• Compressed quad-tree for N-body simulation.
• Efficiently storing and querying XML documents.

Also known as: crit-bit tree, radix tree.
Suffix tree

- Patricia trie of suffixes of a string.
- Linear-time construction: beyond this course.

Applications.
- Linear-time: longest repeated substring, longest common substring, longest palindromic substring, substring search, tandem repeats, ....
- Computational biology databases (BLAST, FASTA).
String symbol tables summary

A success story in algorithm design and analysis.

Red-black BST.
- Performance guarantee: $\log N$ key compares.
- Supports ordered symbol table API.

Hash tables.
- Performance guarantee: constant number of probes.
- Requires good hash function for key type.

Tries. R-way, TST.
- Performance guarantee: $\log N$ characters accessed.
- Supports character-based operations.

Bottom line. You can get at anything by examining 50-100 bits (!!!)