

BBM 202 - ALGORITHMS



DEPT. OF COMPUTER ENGINEERING

ELEMENTARY SORTING ALGORITHMS

Acknowledgement: The course slides are adapted from the slides prepared by R. Sedgwick and K. Wayne of Princeton University.

ELEMENTARY SORTING ALGORITHMS

- ▶ Sorting review
- ▶ Rules of the game
- ▶ Selection sort
- ▶ Insertion sort
- ▶ Shellsort

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Sorting problem

Ex. Student records in a university.

	Chen	3	A	991-878-4944	308 Blair
	Rohde	2	A	232-343-5555	343 Forbes
	Gazsi	4	B	766-093-9873	101 Brown
item →	Furia	1	A	766-093-9873	101 Brown
	Kanaga	3	B	898-122-9643	22 Brown
	Andrews	3	A	664-480-0023	097 Little
key →	Battle	4	C	874-088-1212	121 Whitman

Sort. Rearrange array of N items into ascending order.

Andrews	3	A	664-480-0023	097 Little
Battle	4	C	874-088-1212	121 Whitman
Chen	3	A	991-878-4944	308 Blair
Furia	1	A	766-093-9873	101 Brown
Gazsi	4	B	766-093-9873	101 Brown
Kanaga	3	B	898-122-9643	22 Brown
Rohde	2	A	232-343-5555	343 Forbes

Sample sort client

Goal. Sort *any* type of data.

Ex 1. Sort random real numbers in ascending order.

seems artificial, but stay tuned for an application

```
public class Experiment
{
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);
        Double[] a = new Double[N];
        for (int i = 0; i < N; i++)
            a[i] = StdRandom.uniform();
        Insertion.sort(a);
        for (int i = 0; i < N; i++)
            StdOut.println(a[i]);
    }
}
```

```
% java Experiment 10
0.08614716385210452
0.09054270895414829
0.10708746304898642
0.21166190071646818
0.363292849257276
0.460954145685913
0.5340026311350087
0.7216129793703496
0.9003500354411443
0.9293994908845686
```

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Sample sort client

Goal. Sort *any* type of data.

Ex 2. Sort strings from file in alphabetical order.

```
public class StringSorter
{
    public static void main(String[] args)
    {
        String[] a = In.readStrings(args[0]);
        Insertion.sort(a);
        for (int i = 0; i < a.length; i++)
            StdOut.println(a[i]);
    }
}
```

```
% more words3.txt
bed bug dad yet zoo ... all bad yes

% java StringSorter words3.txt
all bad bed bug dad ... yes yet zoo
```

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Sample sort client

Goal. Sort *any* type of data.

Ex 3. Sort the files in a given directory by filename.

```
import java.io.File;
public class FileSorter
{
    public static void main(String[] args)
    {
        File directory = new File(args[0]);
        File[] files = directory.listFiles();
        Insertion.sort(files);
        for (int i = 0; i < files.length; i++)
            StdOut.println(files[i].getName());
    }
}
```

```
% java FileSorter .
Insertion.class
Insertion.java
InsertionX.class
InsertionX.java
Selection.class
Selection.java
Shell.class
Shell.java
ShellX.class
ShellX.java
```

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Callbacks

Goal. Sort *any* type of data.

Q. How can `sort()` know how to compare data of type `Double`, `String`, and `java.io.File` without any information about the type of an item's key?

Callback = reference to executable code.

- Client passes array of objects to `sort()` function.
- The `sort()` function calls back object's `compareTo()` method as needed.

Implementing callbacks.

- Java: *interfaces*.
- C: function pointers.
- C++: class-type functors.
- C#: delegates.
- Python, Perl, ML, Javascript: first-class functions.

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Callbacks: roadmap

client

```
import java.io.File;
public class FileSorter
{
    public static void main(String[] args)
    {
        File directory = new File(args[0]);
        File[] files = directory.listFiles();
        Insertion.sort(files);
        for (int i = 0; i < files.length; i++)
            StdOut.println(files[i].getName());
    }
}
```

object implementation

```
public class File
implements Comparable<File>
{
    ...
    public int compareTo(File b)
    {
        ...
        return -1;
        ...
        return +1;
        ...
        return 0;
    }
}
```

Comparable interface (built in to Java)

```
public interface Comparable<Item>
{
    public int compareTo(Item that);
}
```

sort implementation

```
public static void sort(Comparable[] a)
{
    int N = a.length;
    for (int i = 0; i < N; i++)
        for (int j = i; j > 0; j--)
            if (a[j].compareTo(a[j-1]) < 0)
                exch(a, j, j-1);
            else break;
}
```

key point: no dependence
on File data type

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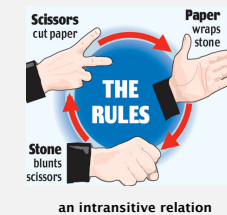
Total order

A **total order** is a binary relation \leq that satisfies

- Antisymmetry: if $v \leq w$ and $w \leq v$, then $v = w$.
- Transitivity: if $v \leq w$ and $w \leq x$, then $v \leq x$.
- Totality: either $v \leq w$ or $w \leq v$ or both.

Ex.

- Standard order for natural and real numbers.
- Alphabetical order for strings.
- Chronological order for dates.
- ...



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Comparable API

Implement `compareTo()` SO THAT `v.compareTo(w)`

- Is a total order.
- Returns a negative integer, zero, or positive integer if v is less than, equal to, or greater than w , respectively.
- Throws an exception if incompatible types (or either is `null`).



less than (return -1)



equal to (return 0)



greater than (return +1)

Built-in comparable types. Integer, Double, String, Date, File, ...

User-defined comparable types. Implement the Comparable interface.

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Implementing the Comparable interface

Date data type. Simplified version of `java.util.Date`.

```
public class Date implements Comparable<Date>
{
    private final int month, day, year;

    public Date(int m, int d, int y)
    {
        month = m;
        day = d;
        year = y;
    }

    public int compareTo(Date that)
    {
        if (this.year < that.year) return -1;
        if (this.year > that.year) return +1;
        if (this.month < that.month) return -1;
        if (this.month > that.month) return +1;
        if (this.day < that.day) return -1;
        if (this.day > that.day) return +1;
        return 0;
    }
}
```

only compare dates
to other dates

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Two useful sorting abstractions

Helper functions. Refer to data through compares and exchanges.

Less. Is item v less than w ?

```
private static boolean less(Comparable v, Comparable w)
{ return v.compareTo(w) < 0; }
```

Exchange. Swap item in array $a[]$ at index i with the one at index j .

```
private static void exch(Comparable[] a, int i, int j)
{
    Comparable swap = a[i];
    a[i] = a[j];
    a[j] = swap;
}
```

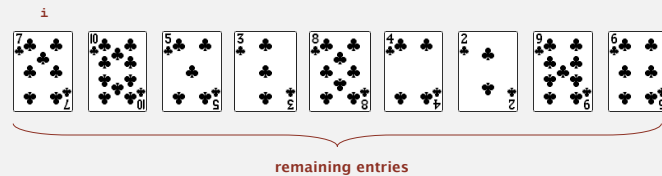
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ELEMENTARY SORTING ALGORITHMS

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- ▶ Rules of the game
- ▶ **Selection sort**
- ▶ Insertion sort
- ▶ Shellsort

Selection sort

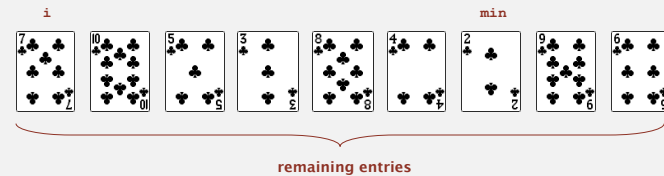
- In iteration i , find index min of smallest remaining entry.
- Swap $a[i]$ and $a[\text{min}]$.



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Selection sort

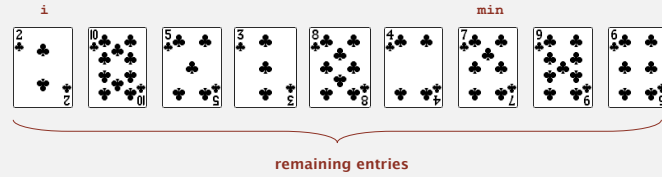
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Selection sort

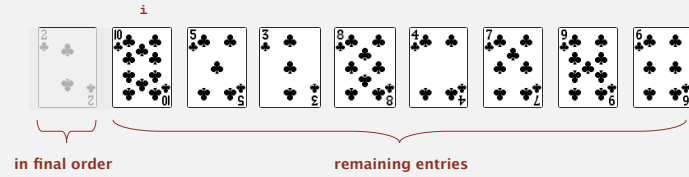
- In iteration i , find index \min of smallest remaining entry.
- Swap $a[i]$ and $a[\min]$.



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Selection sort

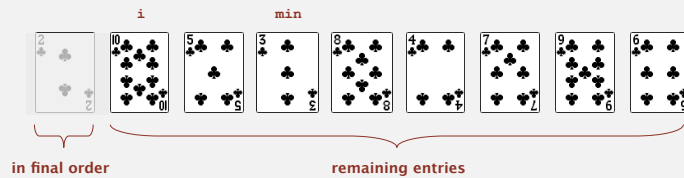
- In iteration i , find index \min of smallest remaining entry.
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18

Selection sort

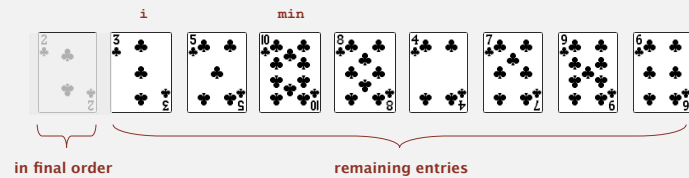
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- Swap $a[i]$ and $a[\min]$.



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Selection sort

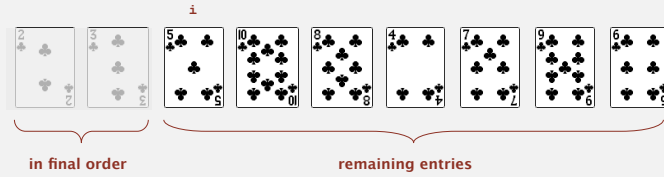
- In iteration i , find index \min of smallest remaining entry.
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20

Selection sort

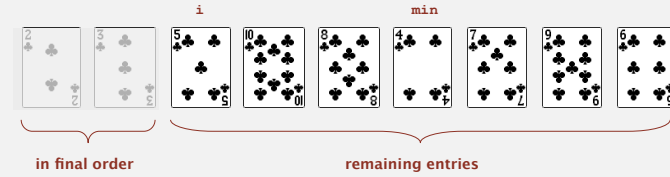
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- Swap $a[i]$ and $a[\min]$.



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Selection sort

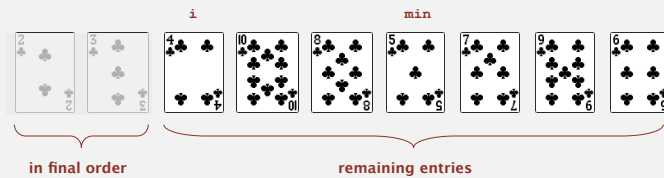
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Selection sort

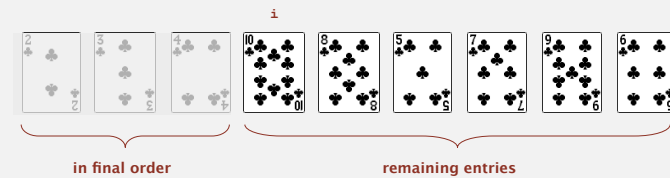
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23

Selection sort

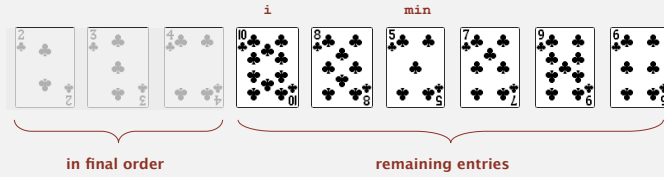
- In iteration i , find index \min of smallest remaining entry.
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Selection sort

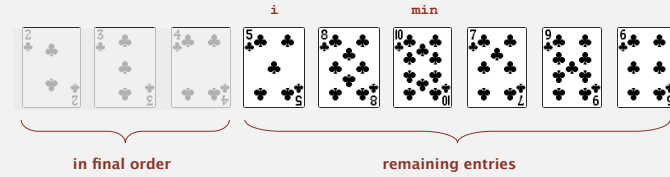
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25

Selection sort

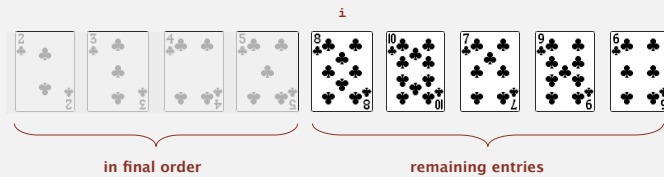
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Selection sort

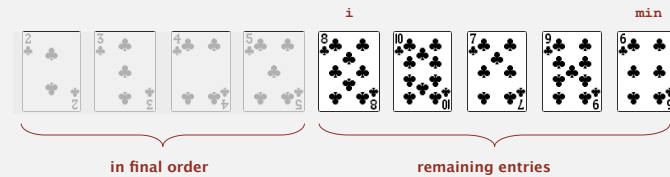
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27

Selection sort

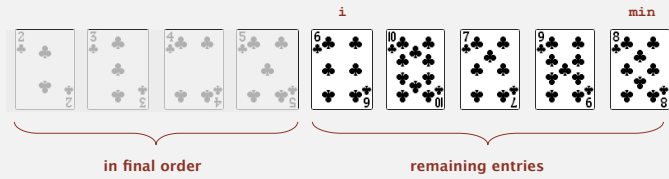
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28

Selection sort

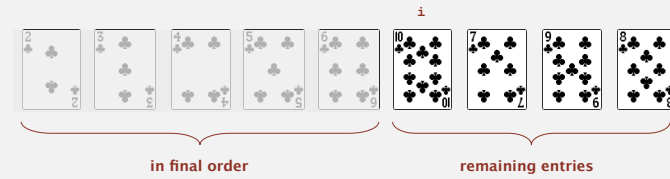
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29

Selection sort

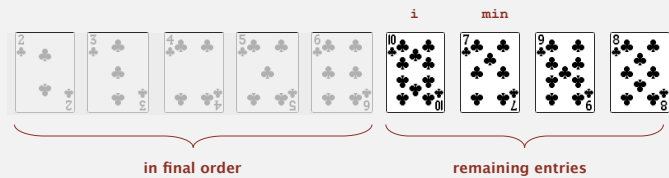
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- Swap $a[i]$ and $a[\text{min}]$.



30

Selection sort

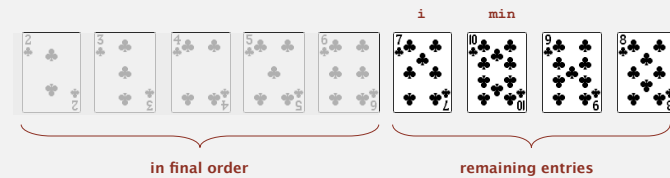
- In iteration i , find index min of smallest remaining entry.
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Selection sort

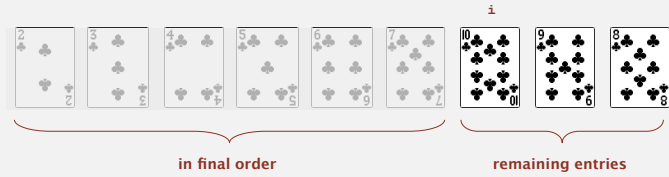
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32

Selection sort

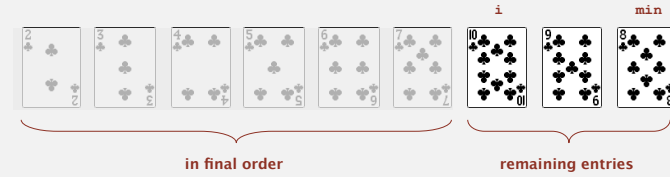
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33

Selection sort

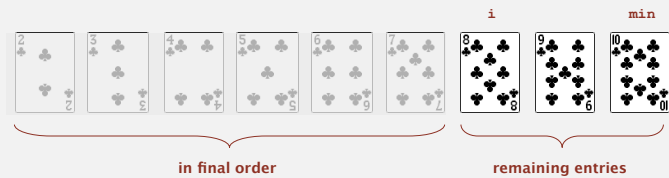
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34

Selection sort

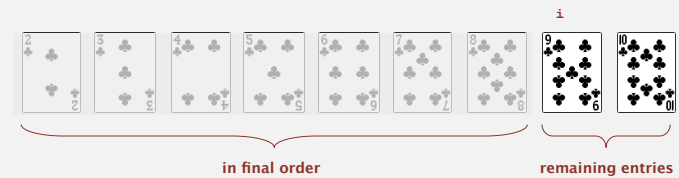
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35

Selection sort

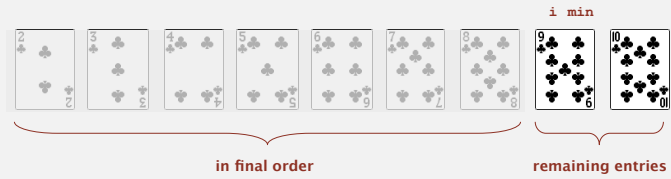
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Selection sort

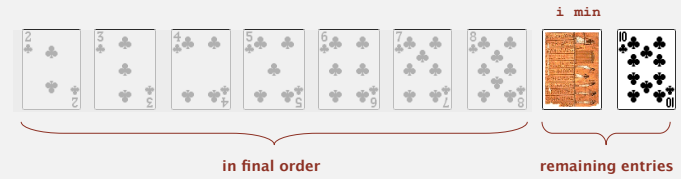
- In iteration i , find index \min of smallest remaining entry.
- Swap $a[i]$ and $a[\min]$.



37

Selection sort

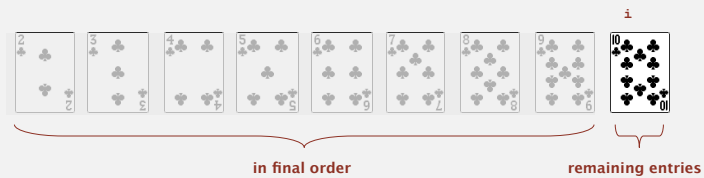
- In iteration i , find index \min of smallest remaining entry.
- Swap $a[i]$ and $a[\min]$.



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Selection sort

- In iteration i , find index \min of smallest remaining entry.
- Swap $a[i]$ and $a[\min]$.



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Selection sort

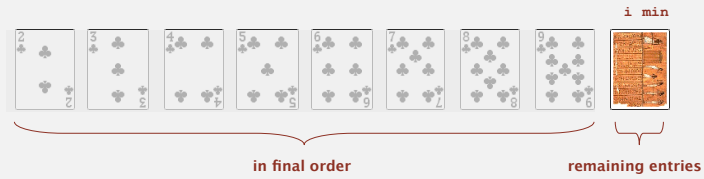
- In iteration i , find index \min of smallest remaining entry.
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40

Selection sort

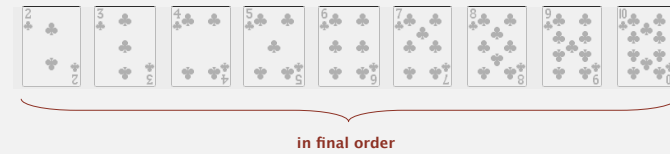
- In iteration i , find index \min of smallest remaining entry.
- Swap $a[i]$ and $a[\min]$.



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Selection sort

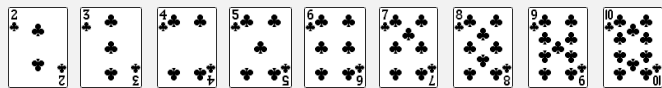
- In iteration i , find index \min of smallest remaining entry.
- Swap $a[i]$ and $a[\min]$.



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Selection sort

- In iteration i , find index \min of smallest remaining entry.
- Swap $a[i]$ and $a[\min]$.



sorted

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Selection sort: Java implementation

```
public class Selection
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
        {
            int min = i;
            for (int j = i+1; j < N; j++)
                if (less(a[j], a[min]))
                    min = j;
            exch(a, i, min);
        }
    }

    private static boolean less(Comparable v, Comparable w)
    { /* as before */ }

    private static void exch(Comparable[] a, int i, int j)
    { /* as before */ }
}
```

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Selection sort: mathematical analysis

Proposition. Selection sort uses $(N-1) + (N-2) + \dots + 1 + 0 \sim N^2/2$ compares and N exchanges.

i	min	0	1	2	3	4	5	6	7	8	9	10
		S	O	R	T	E	X	A	M	P	L	E
0	6	S	O	R	T	E	X	A	M	P	L	E
1	4	A	O	R	T	E	X	S	M	P	L	E
2	10	A	E	R	T	O	X	S	M	P	L	E
3	9	A	E	E	T	O	X	S	M	P	L	R
4	7	A	E	E	L	O	X	S	M	P	T	R
5	7	A	E	E	L	M	X	S	O	P	T	R
6	8	A	E	E	L	M	O	S	X	P	T	R
7	10	A	E	E	L	M	O	P	X	S	T	R
8	8	A	E	E	L	M	O	P	R	S	T	X
9	9	A	E	E	L	M	O	P	R	S	T	X
10	10	A	E	E	L	M	O	P	R	S	T	X

Trace of selection sort (array contents just after each exchange)

entries in black are examined to find the minimum

entries in red are a[min]

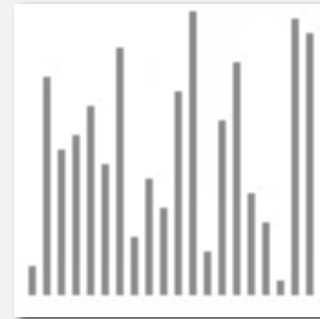
entries in gray are in final position

Running time insensitive to input. Quadratic time, even if input array is sorted.
Data movement is minimal. Linear number of exchanges.

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Selection sort: animations

20 random items



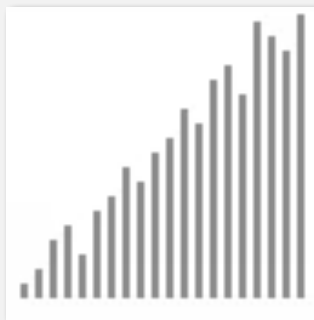
▲ algorithm position
 ■ in final order
 ■ not in final order

<http://www.sorting-algorithms.com/selection-sort>

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Selection sort: animations

20 partially-sorted items



▲ algorithm position
 ■ in final order
 ■ not in final order

<http://www.sorting-algorithms.com/selection-sort>

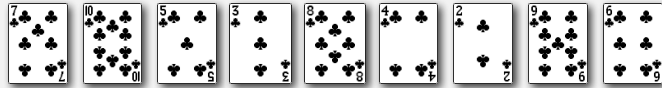
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Insertion sort

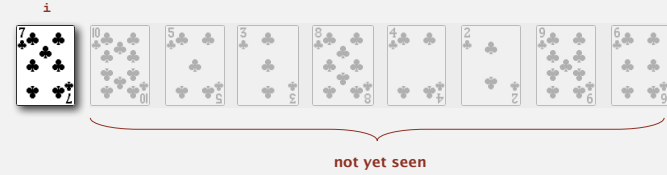
- In iteration i , swap $a[i]$ with each larger entry to its left.



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Insertion sort

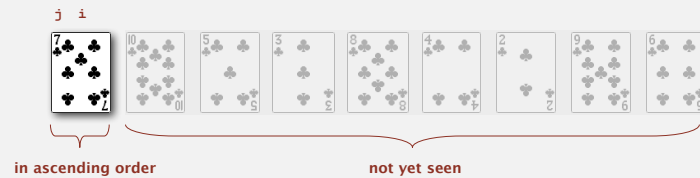
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50

Selection sort

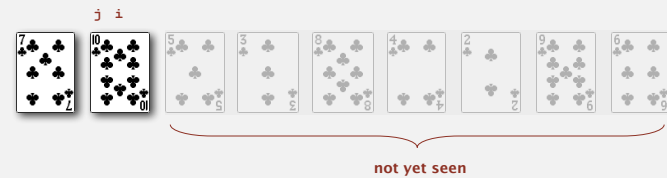
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Insertion sort

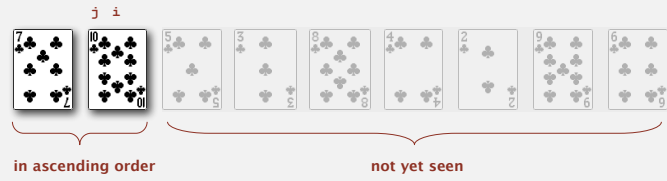
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52

Insertion sort

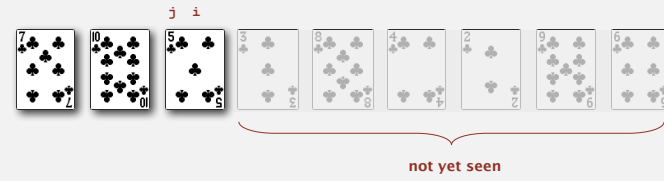
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53

Insertion sort

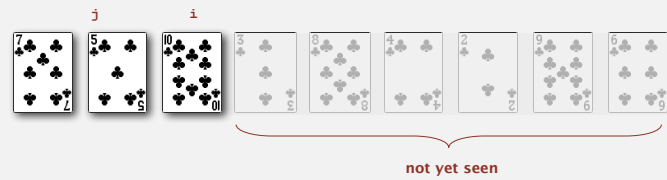
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Insertion sort

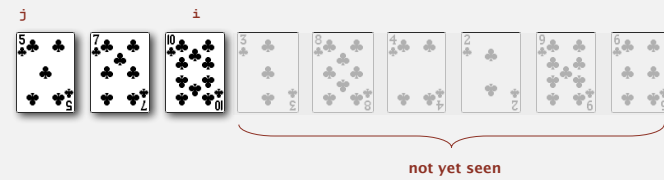
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55

Insertion sort

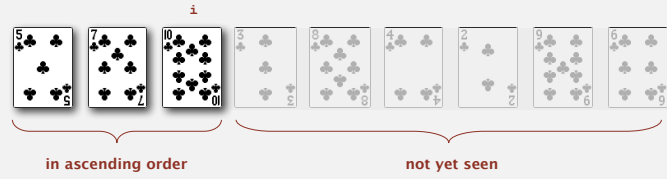
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Insertion sort

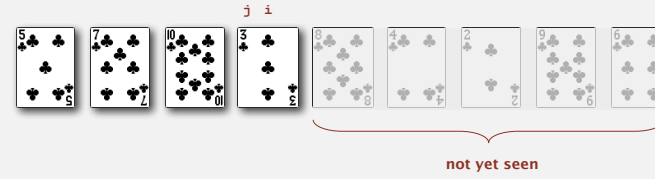
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57

Insertion sort

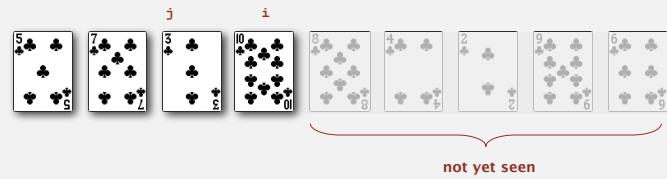
- In iteration i , swap $a[i]$ with each larger entry to its left.



58

Insertion sort

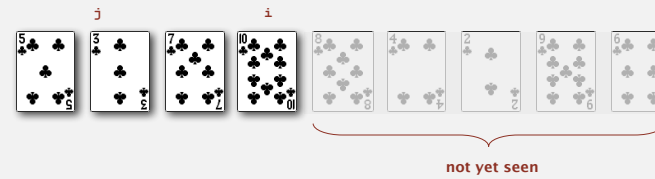
- In iteration i , swap $a[i]$ with each larger entry to its left.



59

Insertion sort

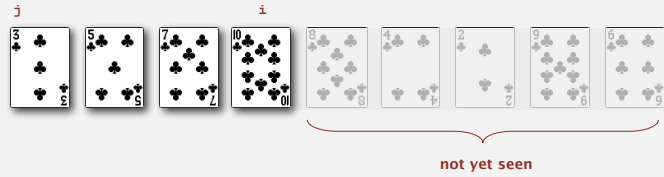
- In iteration i , swap $a[i]$ with each larger entry to its left.



60

Insertion sort

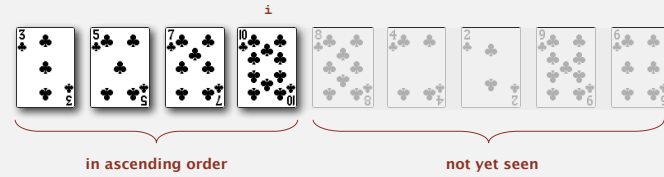
- In iteration i , swap $a[i]$ with each larger entry to its left.



61

Insertion sort

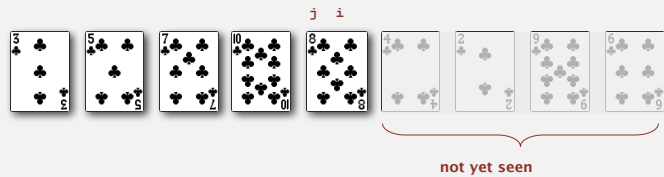
- In iteration i , swap $a[i]$ with each larger entry to its left.



62

Insertion sort

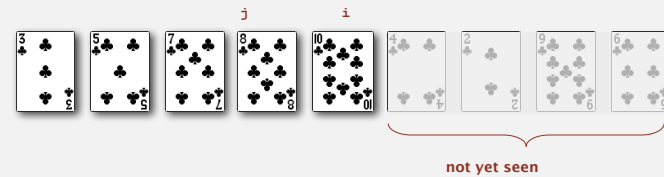
- In iteration i , swap $a[i]$ with each larger entry to its left.



63

Insertion sort

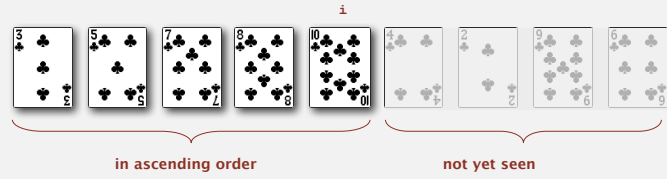
- In iteration i , swap $a[i]$ with each larger entry to its left.



64

Insertion sort

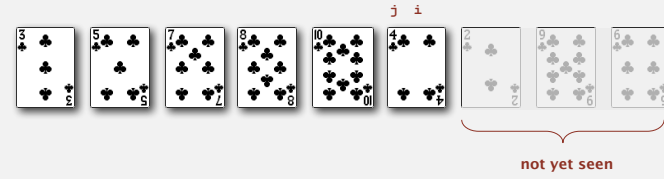
- In iteration i , swap $a[i]$ with each larger entry to its left.



65

Insertion sort

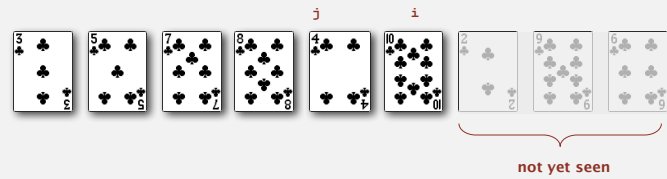
- In iteration i , swap $a[i]$ with each larger entry to its left.



66

Insertion sort

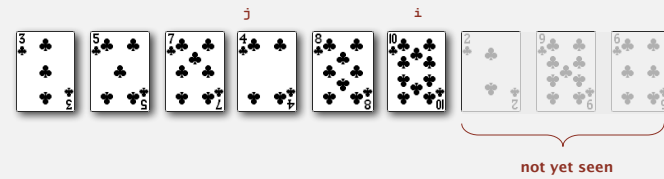
- In iteration i , swap $a[i]$ with each larger entry to its left.



67

Insertion sort

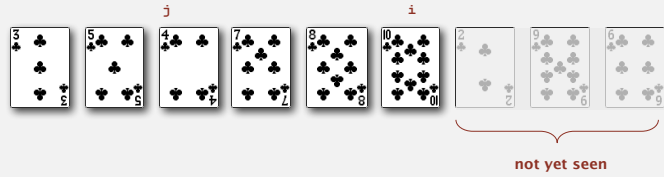
- In iteration i , swap $a[i]$ with each larger entry to its left.



68

Insertion sort

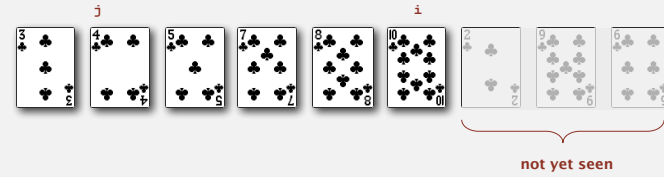
- In iteration i , swap $a[i]$ with each larger entry to its left.



69

Insertion sort

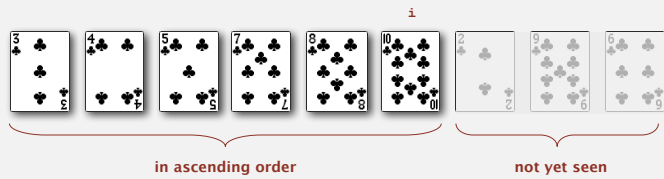
- In iteration i , swap $a[i]$ with each larger entry to its left.



70

Insertion sort

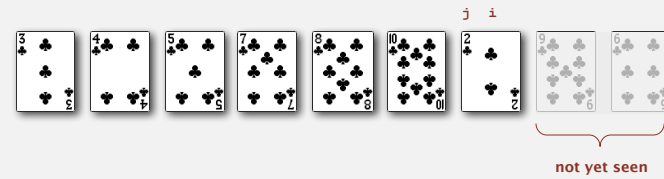
- In iteration i , swap $a[i]$ with each larger entry to its left.



71

Insertion sort

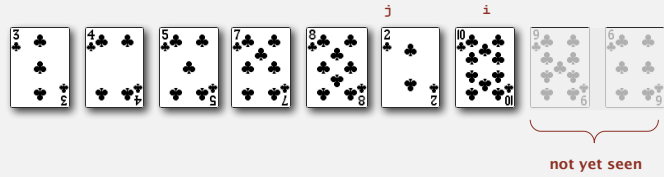
- In iteration i , swap $a[i]$ with each larger entry to its left.



72

Insertion sort

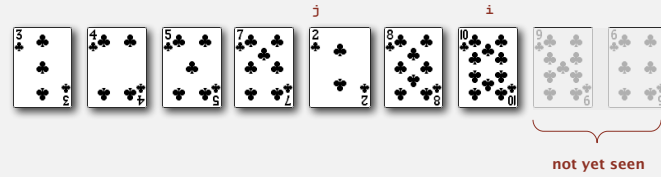
- In iteration i , swap $a[i]$ with each larger entry to its left.



73

Insertion sort

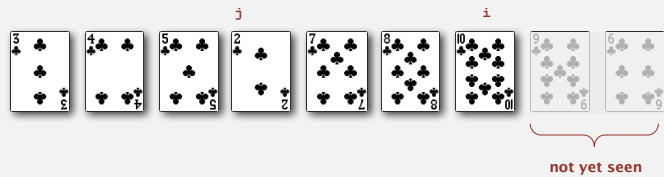
- In iteration i , swap $a[i]$ with each larger entry to its left.



74

Insertion sort

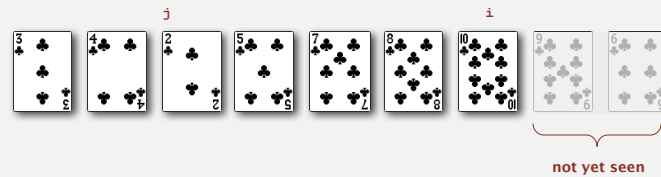
- In iteration i , swap $a[i]$ with each larger entry to its left.



75

Insertion sort

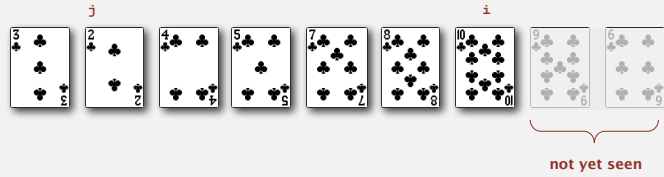
- In iteration i , swap $a[i]$ with each larger entry to its left.



76

Insertion sort

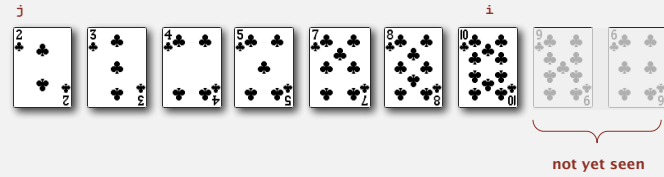
- In iteration i , swap $a[i]$ with each larger entry to its left.



77

Insertion sort

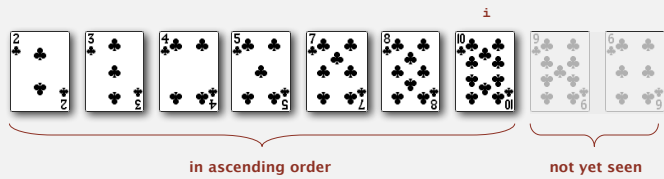
- In iteration i , swap $a[i]$ with each larger entry to its left.



78

Insertion sort

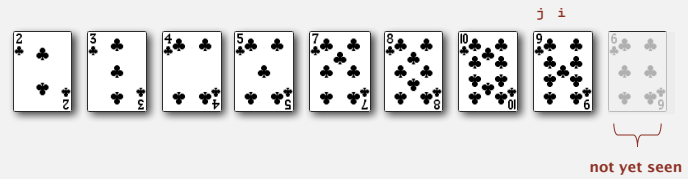
- In iteration i , swap $a[i]$ with each larger entry to its left.



79

Insertion sort

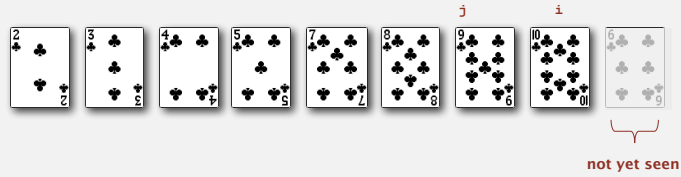
- In iteration i , swap $a[i]$ with each larger entry to its left.



80

Insertion sort

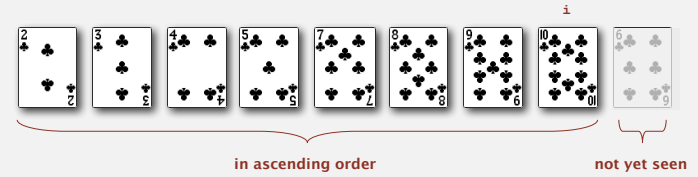
- In iteration i , swap $a[i]$ with each larger entry to its left.



81

Insertion sort

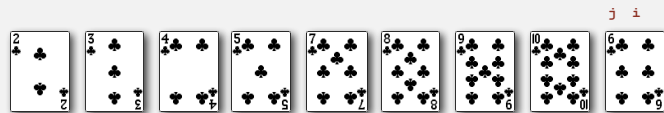
- In iteration i , swap $a[i]$ with each larger entry to its left.



82

Insertion sort

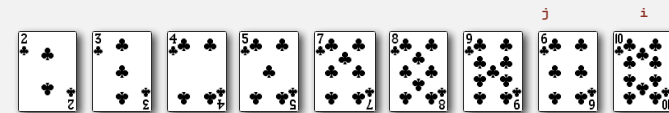
- In iteration i , swap $a[i]$ with each larger entry to its left.



83

Insertion sort

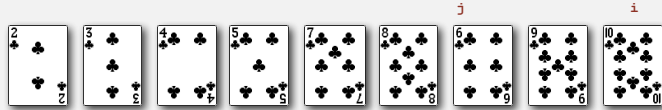
- In iteration i , swap $a[i]$ with each larger entry to its left.



84

Insertion sort

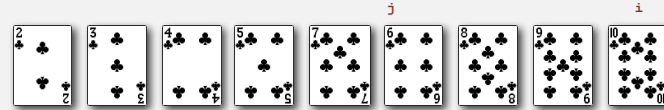
- In iteration i , swap $a[i]$ with each larger entry to its left.



85

Insertion sort

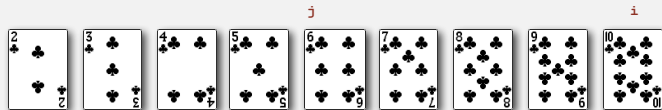
- In iteration i , swap $a[i]$ with each larger entry to its left.



86

Insertion sort

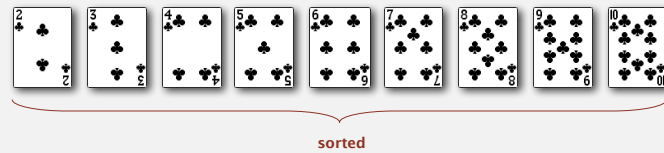
- In iteration i , swap $a[i]$ with each larger entry to its left.



87

Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



88

Insertion sort: Java implementation

```
public class Insertion
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
            for (int j = i; j > 0; j--)
                if (less(a[j], a[j-1]))
                    exch(a, j, j-1);
                else break;
    }

    private static boolean less(Comparable v, Comparable w)
    { /* as before */ }

    private static void exch(Comparable[] a, int i, int j)
    { /* as before */ }
}
```

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Insertion sort: mathematical analysis

Proposition. To sort a randomly-ordered array with distinct keys, insertion sort uses $\sim \frac{1}{4} N^2$ compares and $\sim \frac{1}{4} N^2$ exchanges on average.

Pf. Expect each entry to move halfway back.

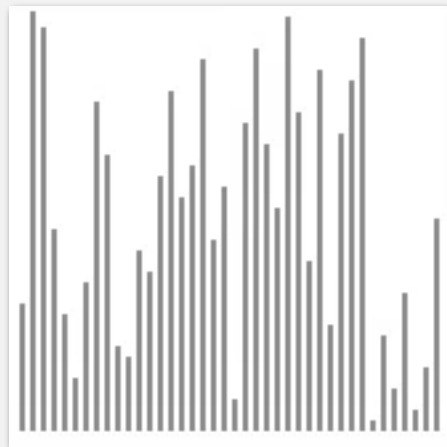
		a[j]											
i	j	0	1	2	3	4	5	6	7	8	9	10	
		S	O	R	T	E	X	A	M	P	L	E	<i>entries in gray do not move</i>
1	0	O	S	R	T	E	X	A	M	P	L	E	
2	1	O	R	S	T	E	X	A	M	P	L	E	
3	3	O	R	S	T	E	X	A	M	P	L	E	
4	0	E	O	R	S	T	X	A	M	P	L	E	<i>entry in red is a[j]</i>
5	5	E	O	R	S	T	X	A	M	P	L	E	
6	0	A	E	O	R	S	T	X	M	P	L	E	
7	2	A	E	M	O	R	S	T	X	P	L	E	
8	4	A	E	M	O	P	R	S	T	X	L	E	<i>entries in black moved one position right for insertion</i>
9	2	A	E	L	M	O	P	R	S	T	X	E	
10	2	A	E	E	L	M	O	P	R	S	T	X	
		A	E	E	L	M	O	P	R	S	T	X	

Trace of insertion sort (array contents just after each insertion)

90

Insertion sort: animation

40 random items



<http://www.sorting-algorithms.com/insertion-sort>

▲ algorithm position
 ■ in order
 ■ not yet seen

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Insertion sort: best and worst case

Best case. If the array is in ascending order, insertion sort makes $N - 1$ compares and 0 exchanges.

A E E L M O P R S T X

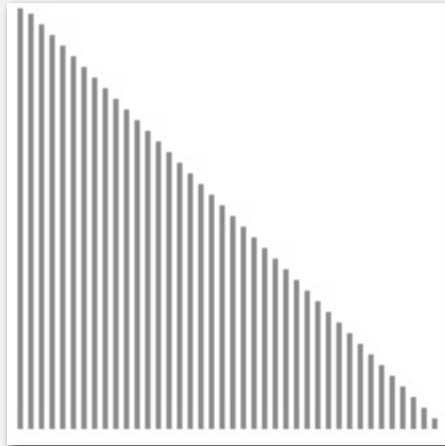
Worst case. If the array is in descending order (and no duplicates), insertion sort makes $\sim \frac{1}{2} N^2$ compares and $\sim \frac{1}{2} N^2$ exchanges.

X T S R P O M L E E A

92

Insertion sort: animation

40 reverse-sorted items



<http://www.sorting-algorithms.com/insertion-sort>

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Insertion sort: partially-sorted arrays

Def. An **inversion** is a pair of keys that are out of order.

A E E L M O T R X P S

T-R T-P T-S R-P X-P X-S

(6 inversions)

Def. An array is **partially sorted** if the number of inversions is $\leq cN$.

- Ex 1. A subarray of size 10 appended to a sorted subarray of size N .
- Ex 2. An array of size N with only 10 entries out of place.

Proposition. For partially-sorted arrays, insertion sort runs in linear time.

Pf. Number of exchanges equals the number of inversions.

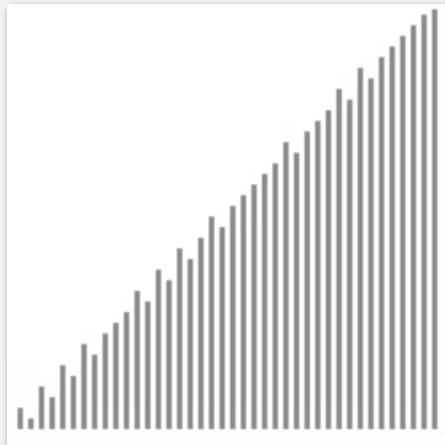
$$\uparrow$$

$$\text{number of compares} = \text{exchanges} + (N - 1)$$

94

Insertion sort: animation

40 partially-sorted items



<http://www.sorting-algorithms.com/insertion-sort>

95

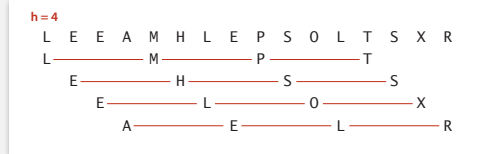
ELEMENTARY SORTING ALGORITHMS

- ▶ **Sorting review**
- ▶ Rules of the game
- ▶ Selection sort
- ▶ Insertion sort
- ▶ **Shellsort**

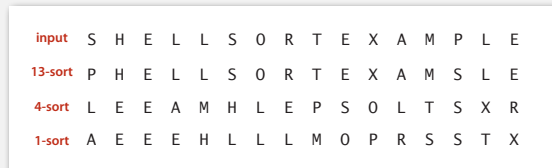
Shellsort overview

Idea. Move entries more than one position at a time by *h*-sorting the array.

an *h*-sorted array is *h* interleaved sorted subsequences



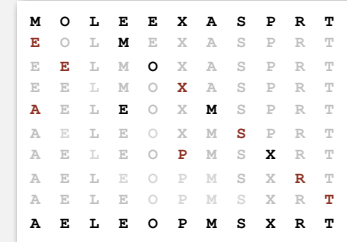
Shellsort. [Shell 1959] *h*-sort the array for decreasing seq. of values of *h*.



h-sorting

How to *h*-sort an array? Insertion sort, with stride length *h*.

3-sorting an array



Why insertion sort?

- Big increments \Rightarrow small subarray.
- Small increments \Rightarrow nearly in order. [stay tuned]

Shellsort example: increments 7, 3, 1

input

S O R T E X A M P L E

7-sort

```

S O R T E X A M P L E
M O R T E X A S P L E
M O R T E X A S P L E
M O L T E X A S P R E
M O L E E X A S P R T
    
```

3-sort

```

M O L E E X A S P R T
E O L M E X A S P R T
E E L M O X A S P R T
A E L E O X M S P R T
A E L E O X M S P R T
A E L E O P M S X R T
A E L E O P M S X R T
A E L E O P M S X R T
A E L E O P M S X R T
    
```

1-sort

```

A E L E O P M S X R T
A E L E O P M S X R T
A E L E O P M S X R T
A E E L O P M S X R T
A E E L O P M S X R T
A E E L M O P S X R T
A E E L M O P S X R T
A E E L M O P R S X T
A E E L M O P R S T X
    
```

result

A E E L M O P R S T X

Shellsort: intuition

Proposition. A *g*-sorted array remains *g*-sorted after *h*-sorting it.

7-sort

```

M O R T E X A S P L E
M O R T E X A S P L E
M O L T E X A S P R E
M O L E E X A S P R T
M O L E E X A S P R T
    
```

3-sort

```

M O L E E X A S P R T
E O L M E X A S P R T
E E L M O X A S P R T
E E L M O X A S P R T
A E L E O X M S P R T
A E L E O X M S P R T
A E L E O P M S X R T
A E L E O P M S X R T
A E L E O P M S X R T
A E L E O P M S X R T
    
```

still 7-sorted

Shellsort: which increment sequence to use?

Powers of two. 1, 2, 4, 8, 16, 32, ...
No.

Powers of two minus one. 1, 3, 7, 15, 31, 63, ...
Maybe.

→ $3x + 1$. 1, 4, 13, 40, 121, 364, ...

OK. Easy to compute.

merging of $(9 \times 4) - (9 \times 2) + 1$ and $4^2 - (3 \times 2) + 1$

Sedgewick. 1, 5, 19, 41, 109, 209, 505, 929, 2161, 3905, ...

Good. Tough to beat in empirical studies.

=

Interested in learning more?

- See Section 6.8 of Algs, 3rd edition or Volume 3 of Knuth for details.
- Do a JP on the topic.

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Shellsort: Java implementation

```
public class Shell
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;

        int h = 1;
        while (h < N/3) h = 3*h + 1; // 1, 4, 13, 40, 121, 364, 1093, ...

        while (h >= 1)
        { // h-sort the array.
            for (int i = h; i < N; i++)
            {
                for (int j = i; j >= h && less(a[j], a[j-h]); j -= h)
                    exch(a, j, j-h);
            }

            h = h/3;
        }
    }

    private static boolean less(Comparable v, Comparable w)
    { /* as before */ }
    private static boolean void(Comparable[] a, int i, int j)
    { /* as before */ }
}
```

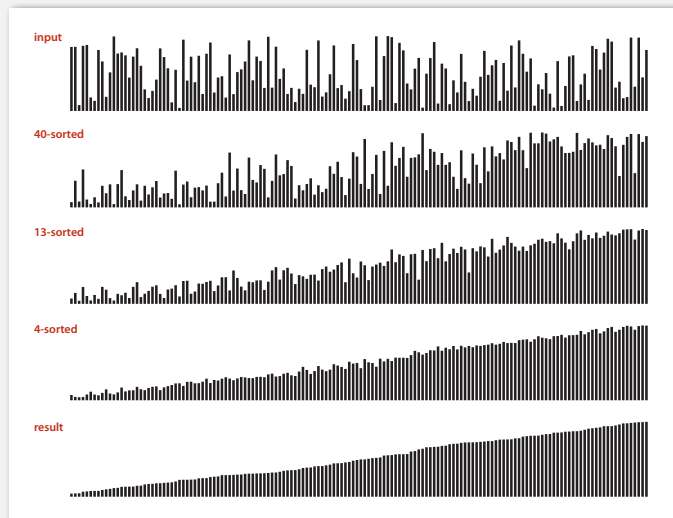
3x+1 increment sequence

insertion sort

move to next increment

102

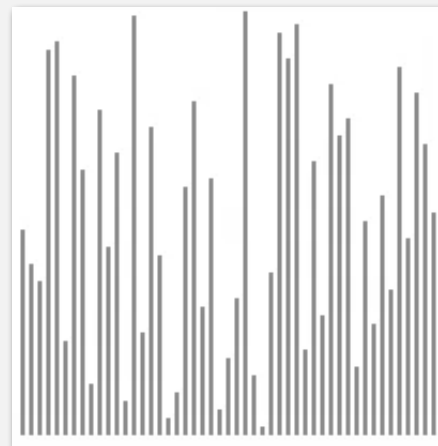
Shellsort: visual trace



103

Shellsort: animation

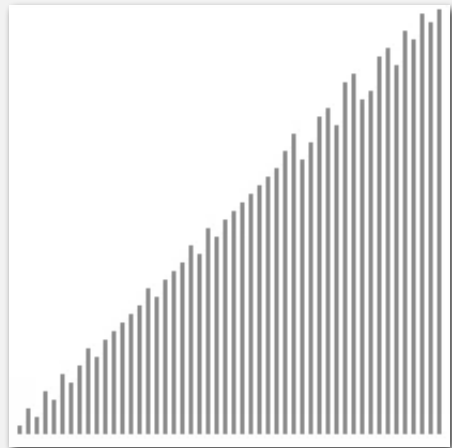
50 random items



104

Shellsort: animation

50 partially-sorted items



<http://www.sorting-algorithms.com/shell-sort>

105

Why are we interested in shellsort?

Example of simple idea leading to substantial performance gains.

Useful in practice.

- Fast unless array size is huge.
- Tiny, fixed footprint for code (used in embedded systems).
- Hardware sort prototype.

Simple algorithm, nontrivial performance, interesting questions.

- Asymptotic growth rate?
- Best sequence of increments? ← open problem: find a better increment sequence
- Average-case performance?

Lesson. Some good algorithms are still waiting discovery.

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