BBM 202 - ALGORITHMS

HACETTEPE UNIVERSITY

DEPT. OF COMPUTER ENGINEERING

ELEMENTARY SEARCH ALGORITHMS

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R. Sedgewick and K. Wayne of Princeton University.

Today

- Symbol Tables
- API
- Elementary implementations
- Ordered operations

Symbol Tables

- **API**
- Elementary implementations
- Ordered operations

Symbol tables

Key-value pair abstraction.

- Insert a value with specified key.
- Given a key, search for the corresponding value.

Ex. DNS lookup.

- Insert URL with specified IP address.
- Given URL, find corresponding IP address.

URL	IP address
`www.cs.princeton.edu`	`128.112.136.11`
`www.princeton.edu`	`128.112.128.15`
`www.yale.edu`	`130.132.143.21`
`www.harvard.edu`	`128.103.060.55`
`www.simpsons.com`	`209.052.165.60`

↑ key ↑ value

Symbol table applications

application	purpose of search	key	value
dictionary	find definition	word	definition
book index	find relevant pages	term	list of page numbers
file share	find song to download	name of song	computer ID
financial account	process transactions	account number	transaction details
web search	find relevant web pages	keyword	list of page names
compiler	find properties of variables	variable name	type and value
routing table	route Internet packets	destination	best route
DNS	find IP address given URL	URL	IP address
reverse DNS	find URL given IP address	IP address	URL
genomics	find markers	DNA string	known positions
file system	find file on disk	filename	location on disk

Basic symbol table API

Associative array abstraction. Associate one value with each key.

`public class ST<Key, Value>`

	`ST()`	*create a symbol table*	
`void`	`put(Key key, Value val)`	*put key-value pair into the table (remove* `key` *from table if value is* `null`*)*	`a[key] = val;`
`Value`	`get(Key key)`	*value paired with* `key` *(*`null` *if* `key` *is absent)*	`a[key]`
`void`	`delete(Key key)`	*remove* `key` *(and its value) from table*	
`boolean`	`contains(Key key)`	*is there a value paired with* `key`*?*	
`boolean`	`isEmpty()`	*is the table empty?*	
`int`	`size()`	*number of key-value pairs in the table*	
`Iterable<Key>`	`keys()`	*all the keys in the table*	

Conventions

- Values are not `null`.
- Method `get()` returns `null` if key not present.
- Method `put()` overwrites old value with new value.

Intended consequences.

- Easy to implement `contains()`.

```
public boolean contains(Key key)
{  return get(key) != null;  }
```

- Can implement lazy version of `delete()`.

```
public void delete(Key key)
{  put(key, null);  }
```

Keys and values

Value type. Any generic type.

specify `Comparable` in API.

Key type: several natural assumptions.

- Assume keys are `Comparable`, use `compareTo()`.
- Assume keys are any generic type, use `equals()` to test equality.
- Assume keys are any generic type, use `equals()` to test equality; use `hashCode()` to scramble key.

built-in to Java
(stay tuned)

Best practices. Use immutable types for symbol table keys.

- Immutable in Java: `String`, `Integer`, `Double`, `java.io.File`, …
- Mutable in Java: `StringBuilder`, `java.net.URL`, arrays, ...

Equality test

All Java classes inherit a method `equals()`.

Java requirements. For any references `x`, `y` and `z`:

- Reflexive: `x.equals(x)` is `true`.
- Symmetric: `x.equals(y)` iff `y.equals(x)`.
- Transitive: if `x.equals(y)` and `y.equals(z)`, then `x.equals(z)`.
- Non-null: `x.equals(null)` is `false`.

equivalence relation

do `x` and `y` refer to the same object?

Default implementation. `(x == y)`

Customized implementations. `Integer`, `Double`, `String`, `File`, `URL`, ...

User-defined implementations. Some care needed.

Implementing equals for user-defined types

Seems easy.

```
public       class Date implements Comparable<Date>
{
   private final int month;
   private final int day;
   private final int year;
   ...

   public boolean equals(Date that)
   {









      if (this.day   != that.day  ) return false;
      if (this.month != that.month) return false;
      if (this.year  != that.year ) return false;
      return true;
   }
}
```

check that all significant fields are the same

Implementing equals for user-defined types

Seems easy, but requires some care.

Safer to use **equals()** with inheritance if fields in extending class contribute to **equals()** the symmetry violated

```
public final class Date implements Comparable<Date>
{
   private final int month;
   private final int day;
   private final int year;
   ...

   public boolean equals(Object y)
   {
      if (y == this) return true;

      if (y == null) return false;

      if (y.getClass() != this.getClass())
         return false;

      Date that = (Date) y;
      if (this.day   != that.day  ) return false;
      if (this.month != that.month) return false;
      if (this.year  != that.year ) return false;
      return true;
   }
}
```

must be **Object**.

optimize for true object equality

check for **null**

objects must be in the same class

cast is guaranteed to succeed

check that all significant fields are the same

Equals design

"Standard" recipe for user-defined types.

- Optimization for reference equality.
- Check against `null`.
- Check that two objects are of the same type and cast.
- Compare each significant field:
  - if field is a primitive type, use `==`
  - if field is an object, use `equals()` ← apply rule recursively
  - if field is an array, apply to each entry ← alternatively, use `Arrays.equals(a, b)` or `Arrays.deepEquals(a, b)`, but not `a.equals(b)`

Best practices.

- No need to use calculated fields that depend on other fields.
- Compare fields mostly likely to differ first.
- Only use necessary fields, e.g. a webpage is best defined by URL, not number of views.
- Make `compareTo()` consistent with `equals()`.

`x.equals(y)` if and only if `(x.compareTo(y) == 0)`

ST test client for traces

Build ST by associating value i with i^{th} string from standard input.

```
public static void main(String[] args)
{
   ST<String, Integer> st = new ST<String, Integer>();
   for (int i = 0; !StdIn.isEmpty(); i++)
   {
     String key = StdIn.readString();
     st.put(key, i);
   }
   for (String s : st.keys())
      StdOut.println(s + " " + st.get(s));
}
```

The order of output depends on the underlying data structure!

keys	S	E	A	R	C	H	E	X	A	M	P	L	E
values	0	1	2	3	4	5	6	7	8	9	10	11	12

output

A	8
C	4
E	12
H	5
L	11
M	9
P	10
R	3
S	0
X	7

ST test client for analysis

Frequency counter. Read a sequence of strings from standard input and print out one that occurs with highest frequency.

```
% more tinyTale.txt
it was the best of times
it was the worst of times
it was the age of wisdom
it was the age of foolishness
it was the epoch of belief
it was the epoch of incredulity
it was the season of light
it was the season of darkness
it was the spring of hope
it was the winter of despair

% java FrequencyCounter 1 < tinyTale.txt
it 10

% java FrequencyCounter 8 < tale.txt
business 122

% java FrequencyCounter 10 < leipzig1M.txt
government 24763
```

tiny example (60 words, 20 distinct)

real example (135,635 words, 10,769 distinct)

real example (21,191,455 words, 534,580 distinct)

Frequency counter implementation

```
public class FrequencyCounter
{
   public static void main(String[] args)
   {
      int minlen = Integer.parseInt(args[0]);
      ST<String, Integer> st = new ST<String, Integer>();
      while (!StdIn.isEmpty())
      {
         String word = StdIn.readString();
         if (word.length() < minlen) continue;
         if (!st.contains(word)) st.put(word, 1);
         else                    st.put(word, st.get(word) + 1);
      }
      String max = "";
      st.put(max, 0);
      for (String word : st.keys())
         if (st.get(word) > st.get(max))
            max = word;
      StdOut.println(max + " " + st.get(max));
   }
}
```

create ST

ignore short strings

read string and update frequency

print a string with max freq

Symbol Tables

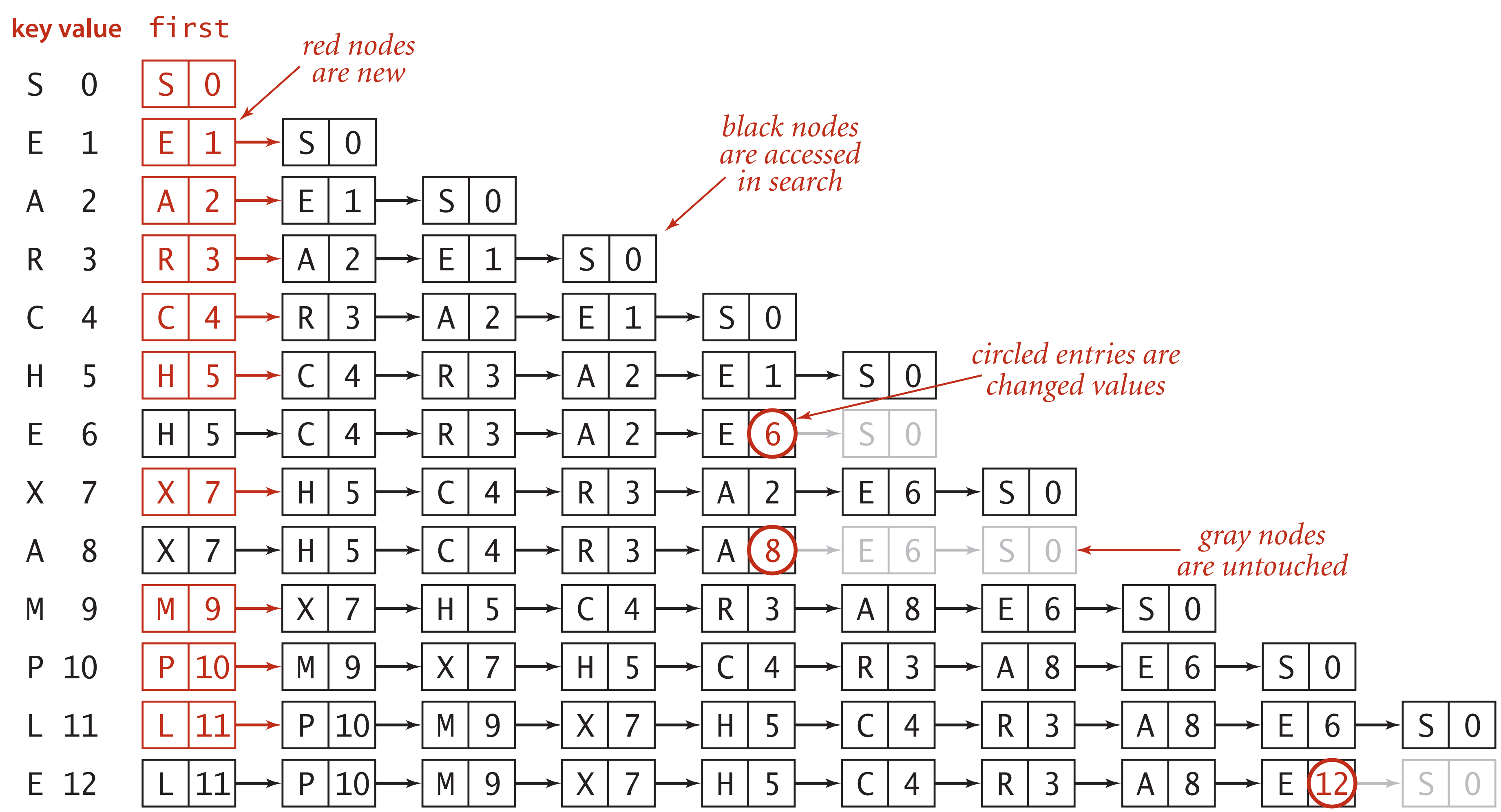
- API
- **Elementary implementations**
- Ordered operations

Sequential search in a linked list

Data structure. Maintain an (unordered) linked list of key-value pairs.

Search. Scan through all keys until find a match.

Insert. Scan through all keys until find a match; if no match add to front.



Trace of linked-list ST implementation for standard indexing client

Elementary ST implementations: summary

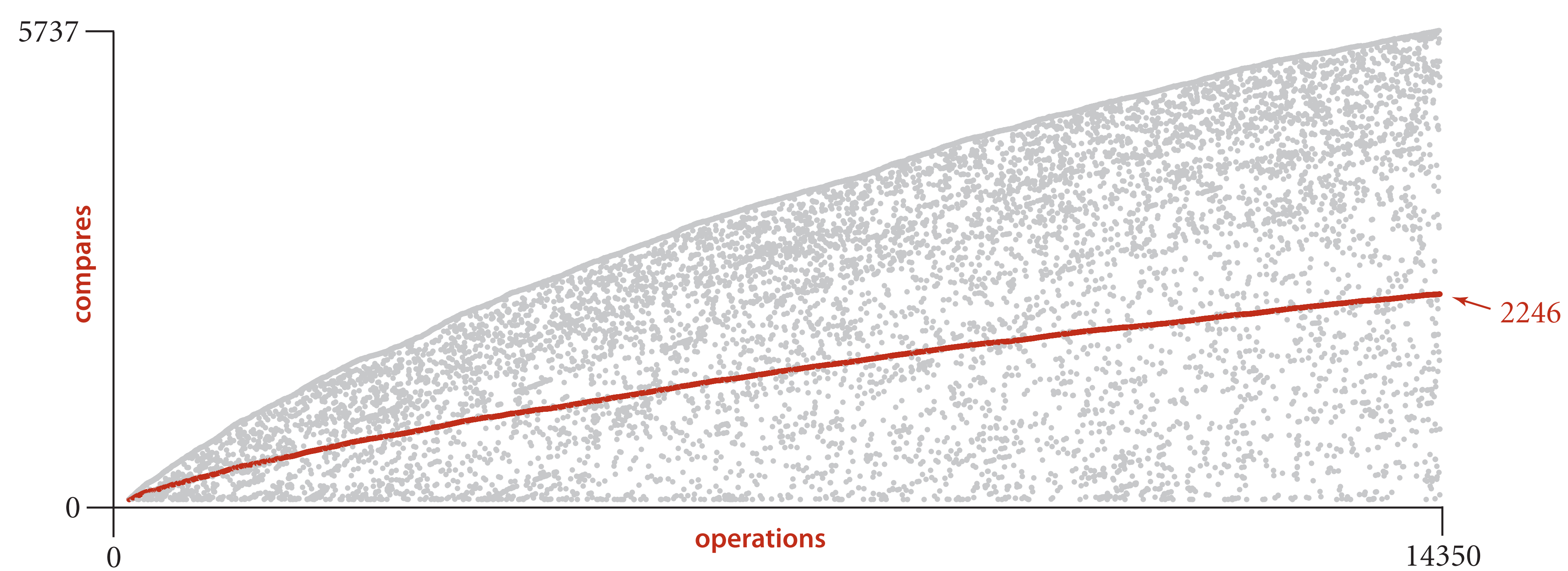
ST implementation	worst-case cost (after N inserts)		average case (after N random inserts)		ordered iteration?	key interface
	search	insert	search hit	insert		
sequential search (unordered list)	N	N	N / 2	N	no	`equals()`

Must search first to avoid duplicates

Challenge. Efficient implementations of both search and insert.

Elementary ST implementations: summary

ST implementation	worst case		average case		ordered iteration?	operations on keys
	search	insert	search hit	insert		
sequential search (unordered list)	N	N	N / 2	N	no	`equals()`



Costs for `java FrequencyCounter 8 < tale.txt` **using** `SequentialSearchST`

Grey data points are observed costs for i^{th} operation, reds are their averages

Challenge. Efficient implementations of both search and insert.

Binary search

Data structure. Maintain an ordered array of key-value pairs.

Rank helper function. How many keys < k ?

keys[]

successful search for P

lo	hi	m	0	1	2	3	4	5	6	7	8	9
0	9	4	A	C	E	H	L	M	P	R	S	X
5	9	7	A	C	E	H	L	M	P	R	S	X
5	6	5	A	C	E	H	L	M	P	R	S	X
6	6	6	A	C	E	H	L	M	P	R	S	X

entries in black are `a[lo..hi]`

entry in red is `a[m]`

loop exits with `keys[m] = P:` *return* 6

unsuccessful search for Q

lo	hi	m	0	1	2	3	4	5	6	7	8	9
0	9	4	A	C	E	H	L	M	P	R	S	X
5	9	7	A	C	E	H	L	M	P	R	S	X
5	6	5	A	C	E	H	L	M	P	R	S	X
7	6	6	A	C	E	H	L	M	P	R	S	X

loop exits with `lo > hi:` *return* 7

Trace of binary search for rank in an ordered array

Binary search: Java implementation

```
public Value get(Key key)
{
   if (isEmpty()) return null;
   int i = rank(key);
   if (i < N && keys[i].compareTo(key) == 0) return vals[i];
   else  return null;
}
```

```
private int rank(Key key)
{
   int lo = 0, hi = N-1;
   while (lo <= hi)
   {
       int mid = lo + (hi - lo) / 2;
       int cmp = key.compareTo(keys[mid]);
       if      (cmp  < 0) hi = mid - 1;
       else if (cmp  > 0) lo = mid + 1;
       else if (cmp == 0) return mid;
   }
   return lo;
}
```

number of keys < `key`

Binary search: mathematical analysis

Proposition. Binary search uses $\sim \lg N$ compares to search any array of size N.

Pf. $\mathrm{T}(N) \equiv$ number of compares to binary search in a sorted array of size N.

$$\leq \underbrace{\mathrm{T}(\lfloor N/2 \rfloor)}_{\text{left or right half}} + 1$$

Recall lecture 2.

Binary search: trace of standard indexing client

Problem. To insert, need to shift all greater keys over.

		keys[]											vals[]									
key	value	0	1	2	3	4	5	6	7	8	9	N	0	1	2	3	4	5	6	7	8	9
S	0	S										1	0									
E	1	E	S									2	1	0								
A	2	A	E	S								3	2	1	0							
R	3	A	E	R	S							4	2	1	3	0						
C	4	A	C	E	R	S						5	2	4	1	3	0					
H	5	A	C	E	H	R	S					6	2	4	1	5	3	0				
E	6	A	C	E	H	R	S					6	2	4	6	5	3	0				
X	7	A	C	E	H	R	S	X				7	2	4	6	5	3	0	7			
A	8	A	C	E	H	R	S	X				7	8	4	6	5	3	0	7			
M	9	A	C	E	H	M	R	S	X			8	8	4	6	5	9	3	0	7		
P	10	A	C	E	H	M	P	R	S	X		9	8	4	6	5	9	10	3	0	7	
L	11	A	C	E	H	L	M	P	R	S	X	10	8	4	6	5	11	9	10	3	0	7
E	12	A	C	E	H	L	M	P	R	S	X	10	8	4	12	5	11	9	10	3	0	7
		A	C	E	H	L	M	P	R	S	X		8	4	12	5	11	9	10	3	0	7

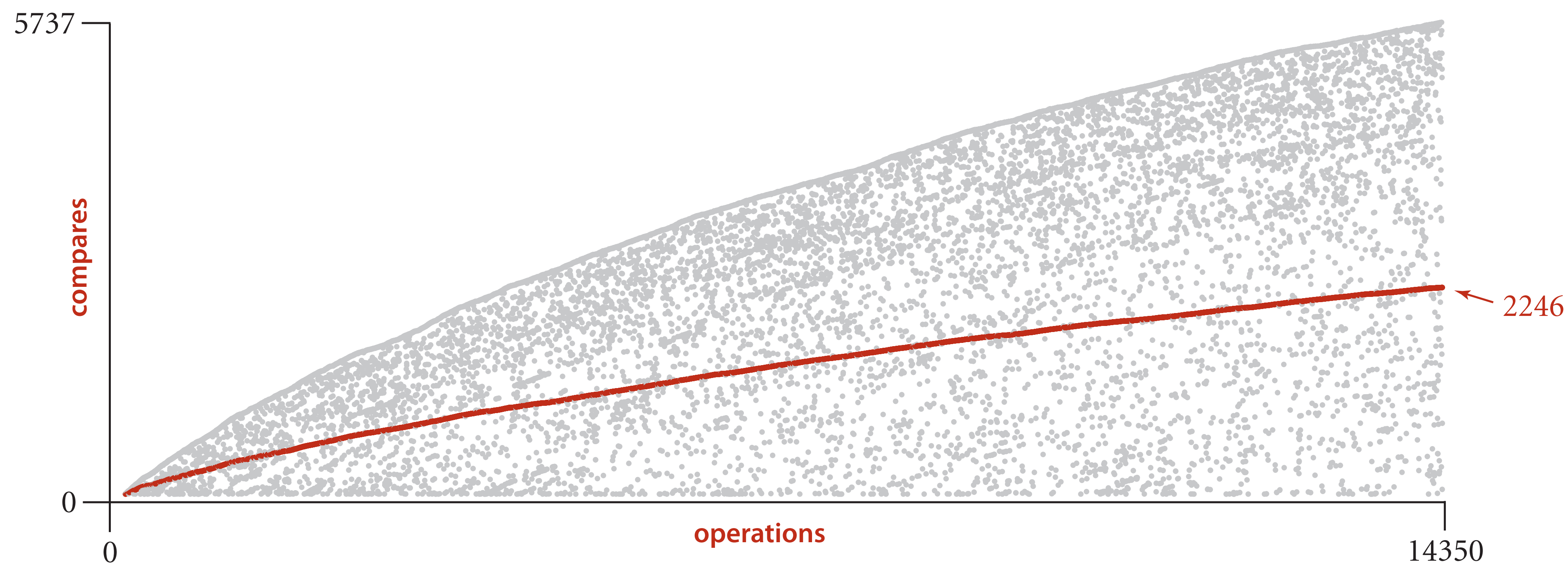
entries in red were inserted

entries in gray did not move

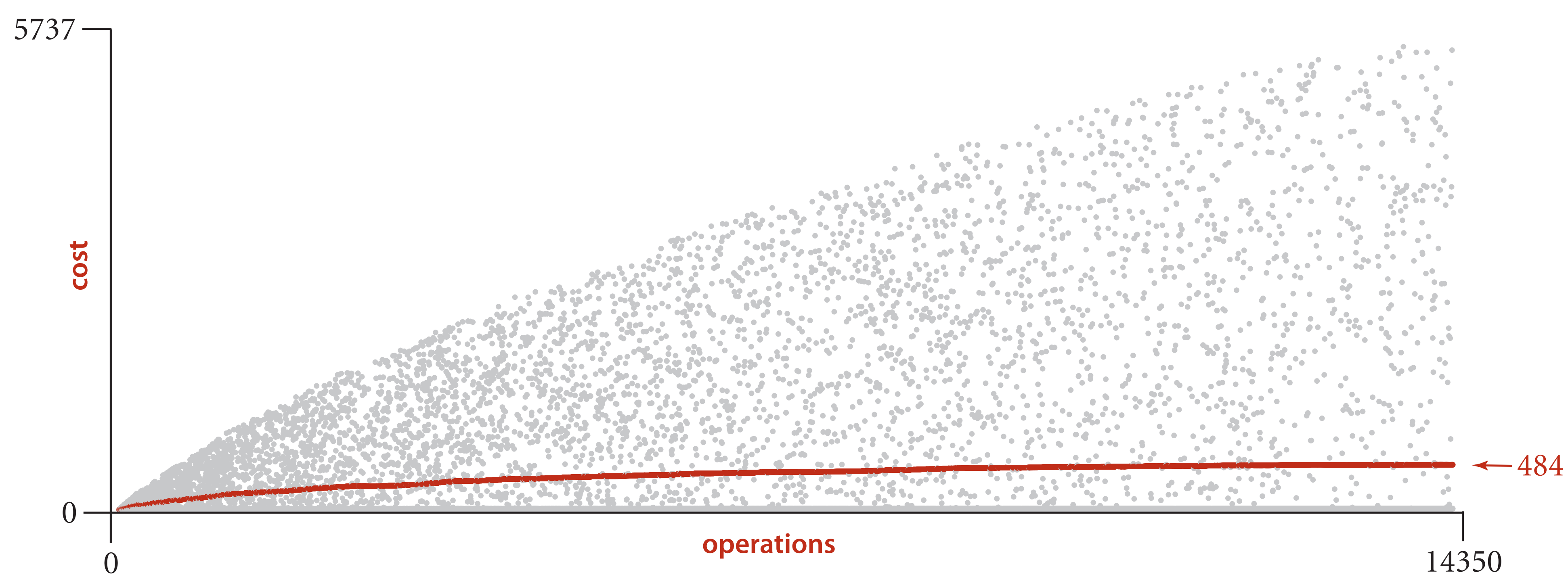
entries in black moved to the right

circled entries are changed values

Elementary ST implementations: frequency counter



Costs for `java FrequencyCounter 8 < tale.txt` using `SequentialSearchST`



Costs for `java FrequencyCounter 8 < tale.txt` using `BinarySearchST`

Elementary ST implementations: summary

ST implementation	worst-case cost (after N inserts)		average case (after N random inserts)		ordered iteration?	key interface
	search	insert	search hit	insert		
sequential search (unordered list)	N	N	N / 2	N	no	`equals()`
binary search (ordered array)	log N	N	log N	N / 2	yes	`compareTo()`

Challenge. Efficient implementations of both search and insert.

Symbol Tables

- API
- Elementary implementations
- Ordered operations

Ordered symbol table API (Example Operations)

	keys	*values*
min() →	09:00:00	Chicago
	09:00:03	Phoenix
get(09:00:13) →	09:00:13	Houston
	09:00:59	Chicago
	09:01:10	Houston
floor(09:05:00) →	09:03:13	Chicago
	09:10:11	Seattle
select(7) →	09:10:25	Seattle
	09:14:25	Phoenix
	09:19:32	Chicago
	09:19:46	Chicago
keys(09:15:00, 09:25:00) →	09:21:05	Chicago
	09:22:43	Seattle
	09:22:54	Seattle
	09:25:52	Chicago
ceiling(09:30:00) →	09:35:21	Chicago
	09:36:14	Seattle
max() →	09:37:44	Phoenix

size(09:15:00, 09:25:00) *is* 5

rank(09:10:25) *is* 7

Examples of ordered symbol-table operations

Ordered symbol table API

```
public class ST<Key extends Comparable<Key>, Value>
```

	`ST()`	*create an ordered symbol table*
`void`	`put(Key key, Value val)`	*put key-value pair into the table (remove* `key` *from table if value is* `null`*)*
`Value`	`get(Key key)`	*value paired with* `key` *(*`null` *if* `key` *is absent)*
`void`	`delete(Key key)`	*remove* `key` *(and its value) from table*
`boolean`	`contains(Key key)`	*is there a value paired with* `key`*?*
`boolean`	`isEmpty()`	*is the table empty?*
`int`	`size()`	*number of key-value pairs*
`Key`	`min()`	*smallest key*
`Key`	`max()`	*largest key*
`Key`	`floor(Key key)`	*largest key less than or equal to* `key`
`Key`	`ceiling(Key key)`	*smallest key greater than or equal to* `key`
`int`	`rank(Key key)`	*number of keys less than* `key`
`Key`	`select(int k)`	*key of rank* `k`
`void`	`deleteMin()`	*delete smallest key*
`void`	`deleteMax()`	*delete largest key*
`int`	`size(Key lo, Key hi)`	*number of keys in* `[lo..hi]`
`Iterable<Key>`	`keys(Key lo, Key hi)`	*keys in* `[lo..hi]`*, in sorted order*
`Iterable<Key>`	`keys()`	*all keys in the table, in sorted order*

Binary search: ordered symbol table operations summary

	sequential search	binary search
search	N	lg N
insert	N	N
min / max	N	1
floor / ceiling	N	lg N
rank	N	lg N
select	N	1
ordered iteration	N log N	N

The Problem: Insert Operation

order of growth of the running time for ordered symbol table operations