

BBM 202 - ALGORITHMS



HACETTEPE UNIVERSITY

DEPT. OF COMPUTER ENGINEERING

ELEMENTARY SORTING ALGORITHMS

Acknowledgement: The course slides are adapted from the slides prepared by R. Sedgwick and K. Wayne of Princeton University.

ELEMENTARY SORTING ALGORITHMS

- ▶ **Sorting review**
- ▶ **Rules of the game**
- ▶ **Selection sort**
- ▶ **Insertion sort**
- ▶ **Shellsort**

ELEMENTARY SORTING ALGORITHMS

- ▶ **Sorting review**
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- ▶ Shellsort

Sorting problem

Ex. Student records in a university.

	Chen	3	A	991-878-4944	308 Blair
	Rohde	2	A	232-343-5555	343 Forbes
	Gazsi	4	B	766-093-9873	101 Brown
item →	Furia	1	A	766-093-9873	101 Brown
	Kanaga	3	B	898-122-9643	22 Brown
	Andrews	3	A	664-480-0023	097 Little
key →	Battle	4	C	874-088-1212	121 Whitman

Sort. Rearrange array of N items into ascending order.

Andrews	3	A	664-480-0023	097 Little
Battle	4	C	874-088-1212	121 Whitman
Chen	3	A	991-878-4944	308 Blair
Furia	1	A	766-093-9873	101 Brown
Gazsi	4	B	766-093-9873	101 Brown
Kanaga	3	B	898-122-9643	22 Brown
Rohde	2	A	232-343-5555	343 Forbes

Sample sort client

Goal. Sort **any** type of data.

Ex 1. Sort random real numbers in ascending order.

 seems artificial, but stay tuned for an application

```
public class Experiment
{
    public static void main(String[] args)
    {
        int N = Integer.parseInt(args[0]);
        Double[] a = new Double[N];
        for (int i = 0; i < N; i++)
            a[i] = StdRandom.uniform();
        Insertion.sort(a);
        for (int i = 0; i < N; i++)
            StdOut.println(a[i]);
    }
}
```

```
% java Experiment 10
0.08614716385210452
0.09054270895414829
0.10708746304898642
0.21166190071646818
0.363292849257276
0.460954145685913
0.5340026311350087
0.7216129793703496
0.9003500354411443
0.9293994908845686
```

Sample sort client

Goal. Sort **any** type of data.

Ex 2. Sort strings from file in alphabetical order.

```
public class StringSorter
{
    public static void main(String[] args)
    {
        String[] a = In.readStrings(args[0]);
        Insertion.sort(a);
        for (int i = 0; i < a.length; i++)
            StdOut.println(a[i]);
    }
}
```

```
% more words3.txt
bed bug dad yet zoo ... all bad yes

% java StringSorter words3.txt
all bad bed bug dad ... yes yet zoo
```

Sample sort client

Goal. Sort **any** type of data.

Ex 3. Sort the files in a given directory by filename.

```
import java.io.File;
public class FileSorter
{
    public static void main(String[] args)
    {
        File directory = new File(args[0]);
        File[] files = directory.listFiles();
        Insertion.sort(files);
        for (int i = 0; i < files.length; i++)
            StdOut.println(files[i].getName());
    }
}
```

```
% java FileSorter .
Insertion.class
Insertion.java
InsertionX.class
InsertionX.java
Selection.class
Selection.java
Shell.class
Shell.java
ShellX.class
ShellX.java
```

Callbacks

Goal. Sort **any** type of data.

Q. How can `sort()` know how to compare data of type `Double`, `String`, and `java.io.File` without any information about the type of an item's key?

Callback = reference to executable code.

- Client passes array of objects to `sort()` function.
- The `sort()` function calls back object's `compareTo()` method as needed.

Implementing callbacks.

- Java: **interfaces**.
- C: function pointers.
- C++: class-type functors.
- C#: delegates.
- Python, Perl, ML, Javascript: first-class functions.

Callbacks: roadmap

client

```
import java.io.File;
public class FileSorter
{
    public static void main(String[] args)
    {
        File directory = new File(args[0]);
        File[] files = directory.listFiles();
        Insertion.sort(files);
        for (int i = 0; i < files.length; i++)
            StdOut.println(files[i].getName());
    }
}
```

object implementation

```
public class File
implements Comparable<File>
{
    ...
    public int compareTo(File b)
    {
        ...
        return -1;
        ...
        return +1;
        ...
        return 0;
    }
}
```

Comparable interface (built in to Java)

```
public interface Comparable<Item>
{
    public int compareTo(Item that);
}
```

key point: no dependence
on `File` data type

sort implementation

```
public static void sort(Comparable[] a)
{
    int N = a.length;
    for (int i = 0; i < N; i++)
        for (int j = i; j > 0; j--)
            if (a[j].compareTo(a[j-1]) < 0)
                exch(a, j, j-1);
            else break;
}
```

Total order

A **total order** is a binary relation \leq that satisfies

- Antisymmetry: if $v \leq w$ and $w \leq v$, then $v = w$.
- Transitivity: if $v \leq w$ and $w \leq x$, then $v \leq x$.
- Totality: either $v \leq w$ or $w \leq v$ or both.

Ex.

- Standard order for natural and real numbers.
- Alphabetical order for strings.
- Chronological order for dates.
- ...

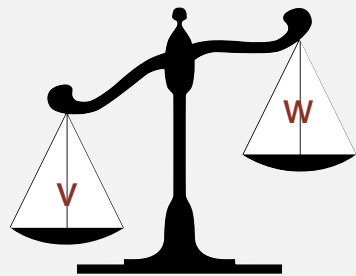


an intransitive relation

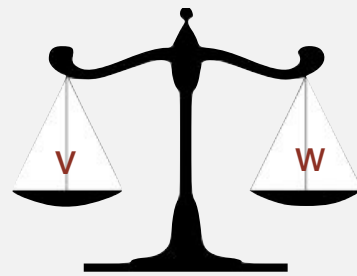
Comparable API

Implement `compareTo()` so that `v.compareTo(w)`

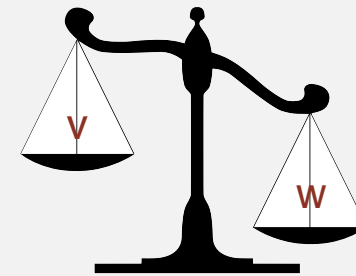
- Is a total order.
- Returns a negative integer, zero, or positive integer if v is less than, equal to, or greater than w , respectively.
- Throws an exception if incompatible types (or either is `null`).



less than (return -1)



equal to (return 0)



greater than (return +1)

Built-in comparable types. Integer, Double, String, Date, File, ...

User-defined comparable types. Implement the `Comparable` interface.

Implementing the Comparable interface

Date data type. Simplified version of `java.util.Date`.

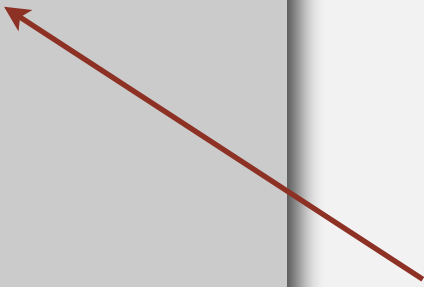
```
public class Date implements Comparable<Date>
{
    private final int month, day, year;

    public Date(int m, int d, int y)
    {
        month = m;
        day    = d;
        year   = y;
    }
}
```

```
public int compareTo(Date that)
{
    if (this.year < that.year ) return -1;
    if (this.year > that.year ) return +1;
    if (this.month < that.month) return -1;
    if (this.month > that.month) return +1;
    if (this.day < that.day ) return -1;
    if (this.day > that.day ) return +1;
    return 0;
}
```

```
}
```

only compare dates
to other dates



Two useful sorting abstractions

Helper functions. Refer to data through compares and exchanges.

Less. Is item v less than w ?

```
private static boolean less(Comparable v, Comparable w)
{   return v.compareTo(w) < 0;   }
```

Exchange. Swap item in array $a[]$ at index i with the one at index j .

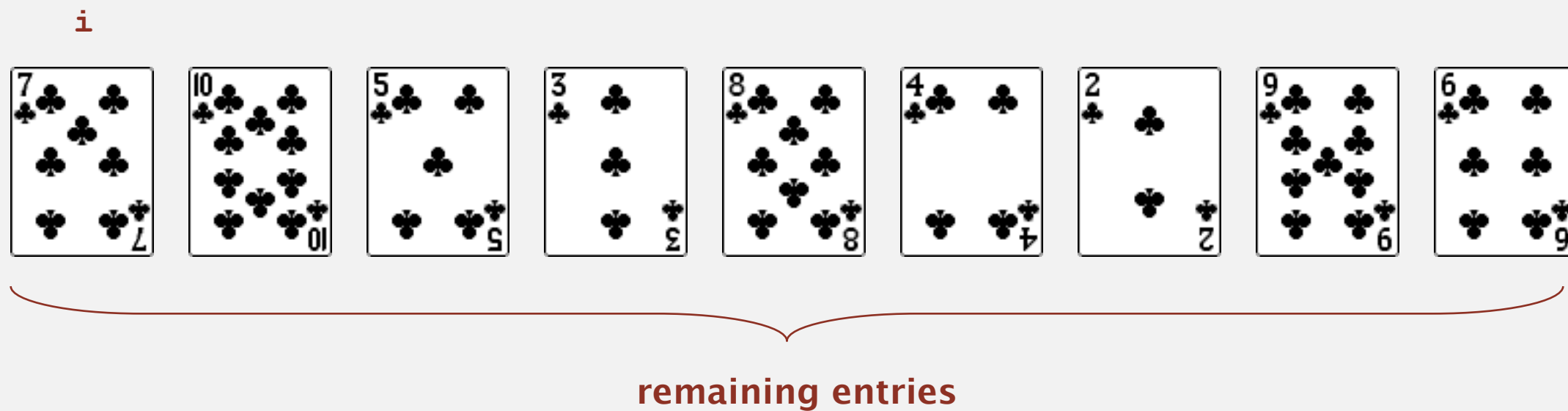
```
private static void exch(Comparable[] a, int i, int j)
{
    Comparable swap = a[i];
    a[i] = a[j];
    a[j] = swap;
}
```

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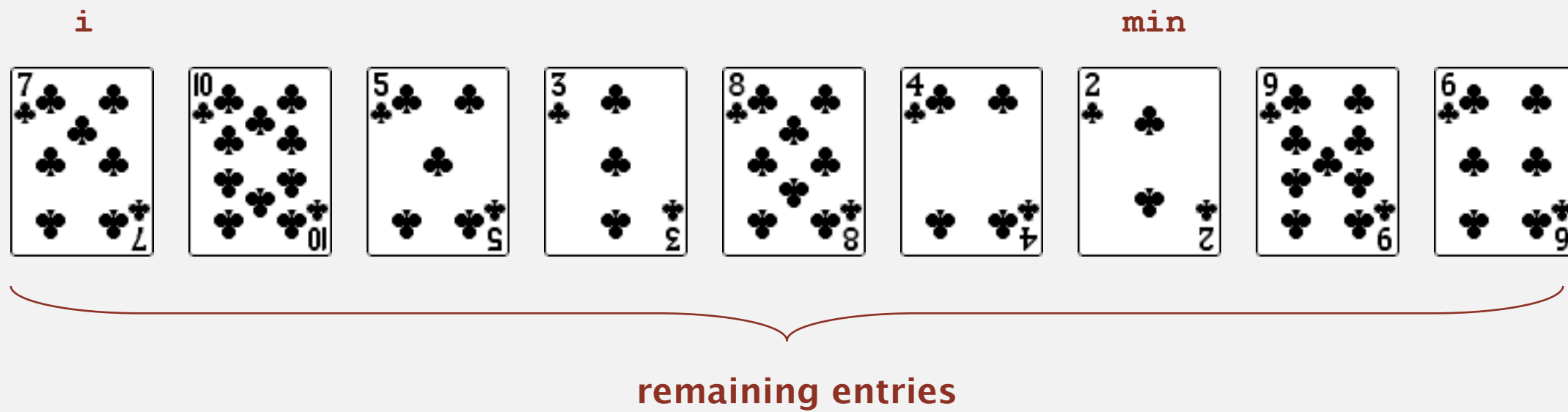
Selection sort

- In iteration i , find index \min of smallest remaining entry.
- Swap $a[i]$ and $a[\min]$.



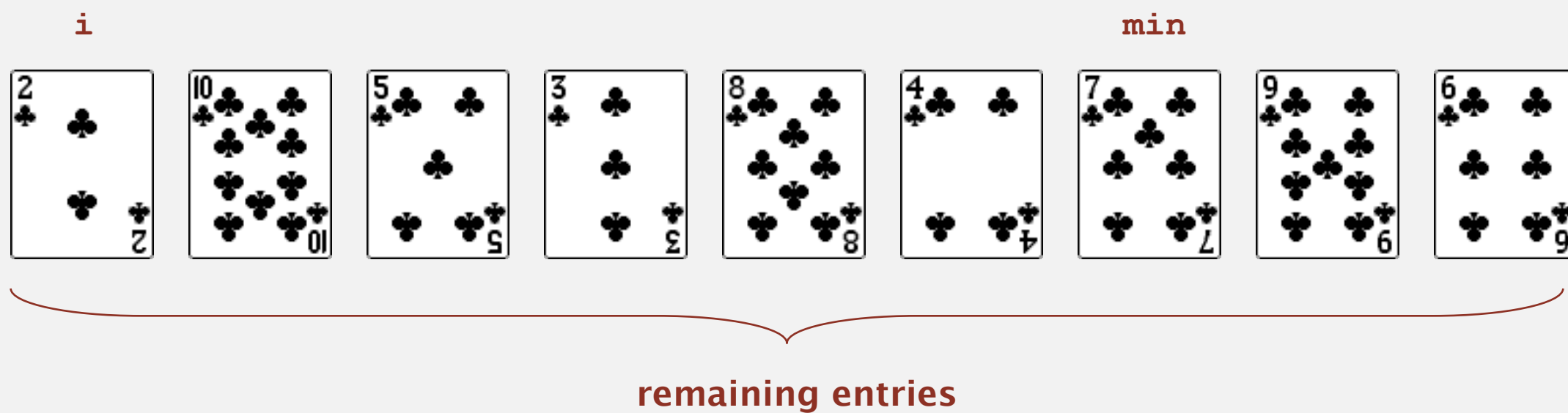
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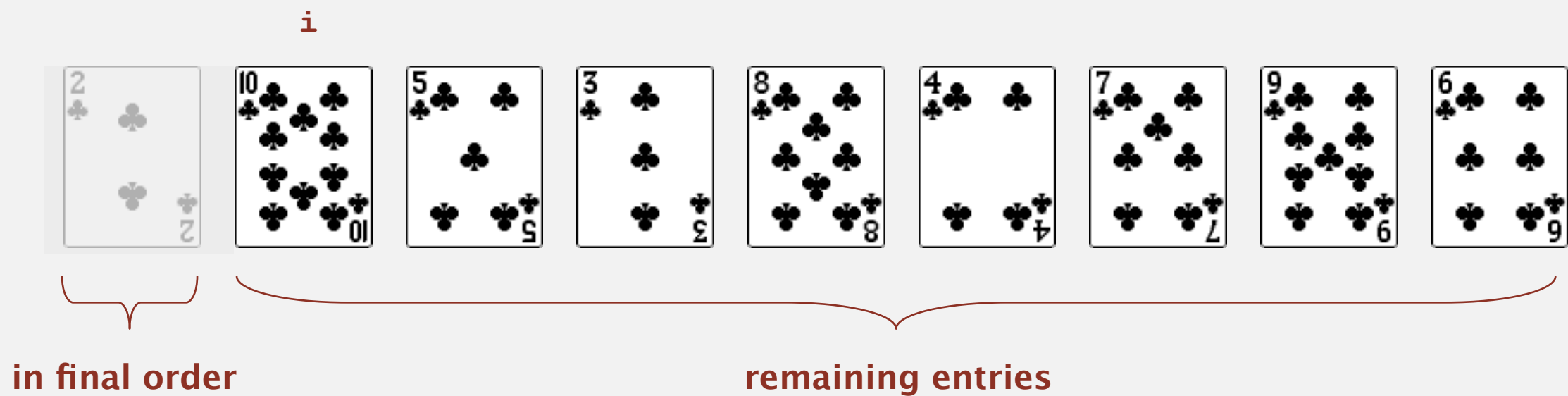
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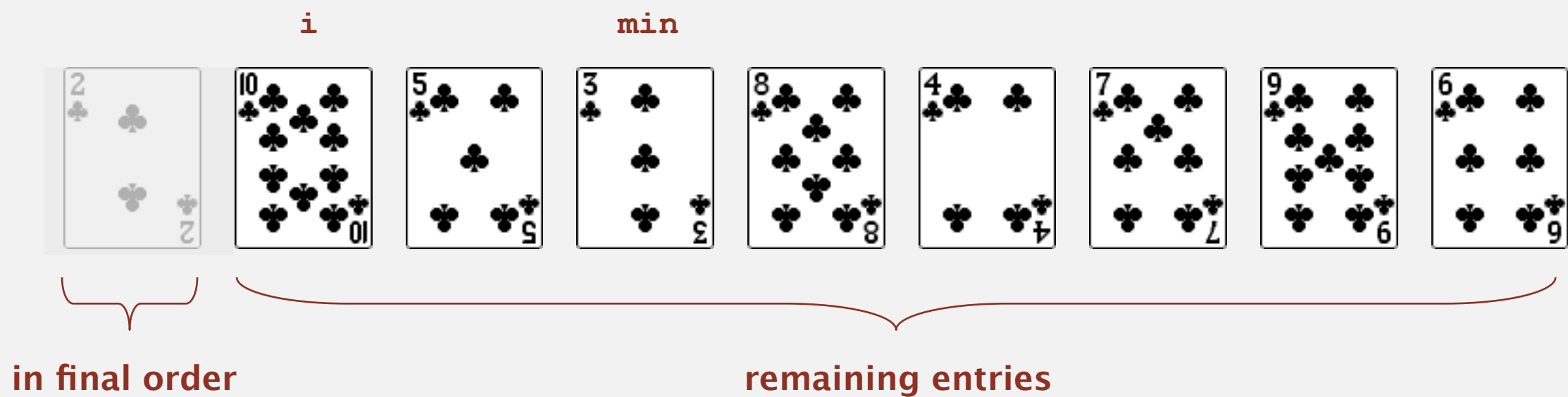
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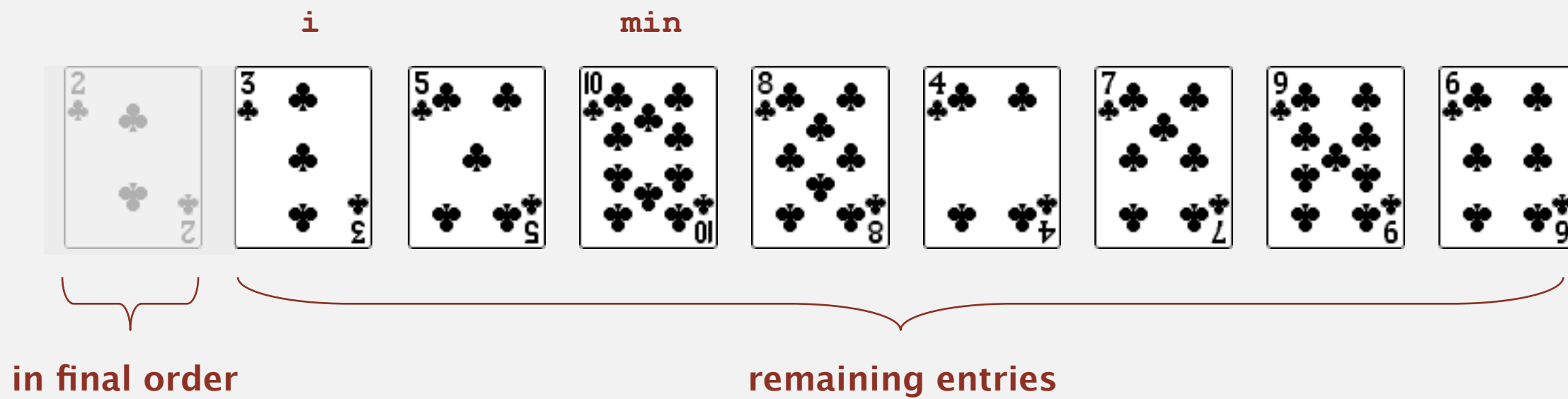
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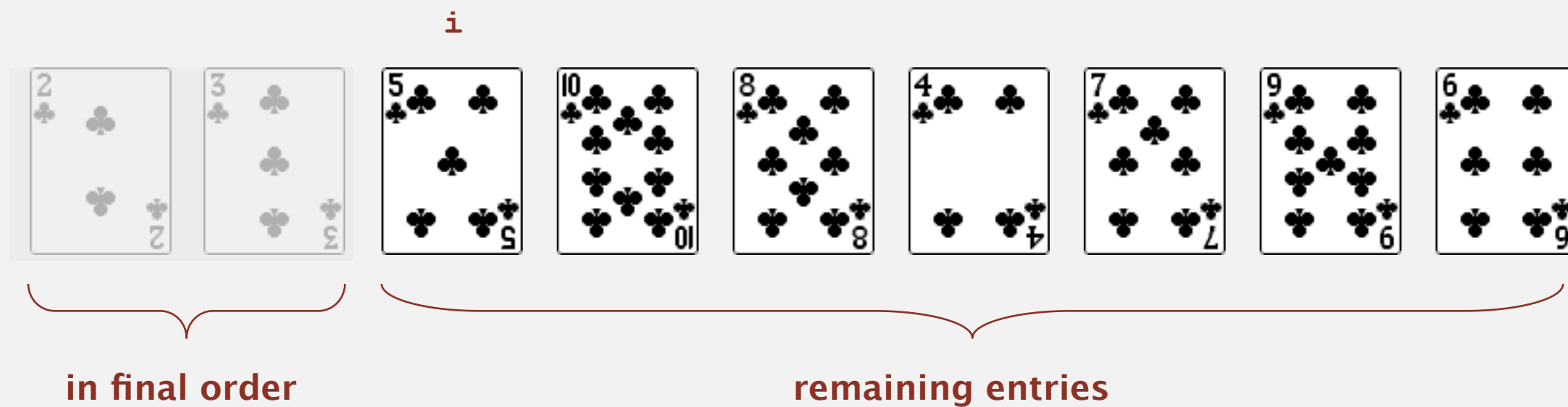
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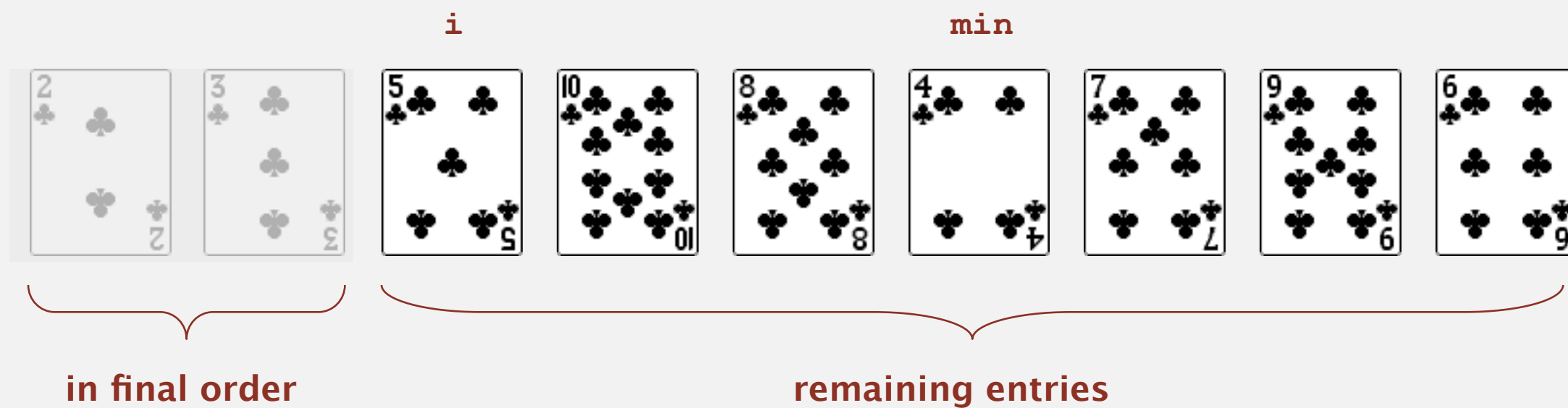
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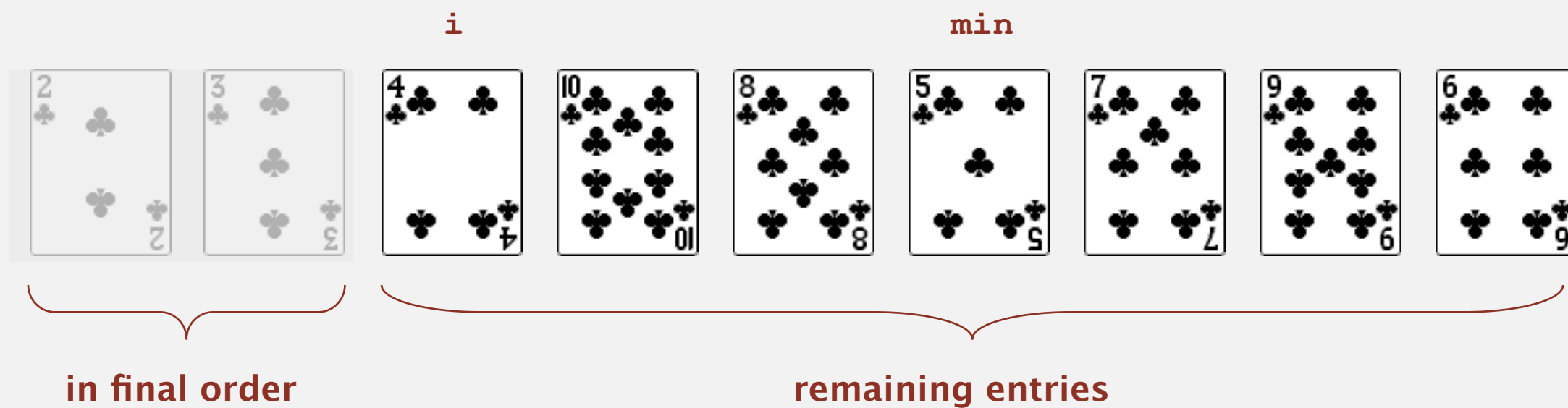
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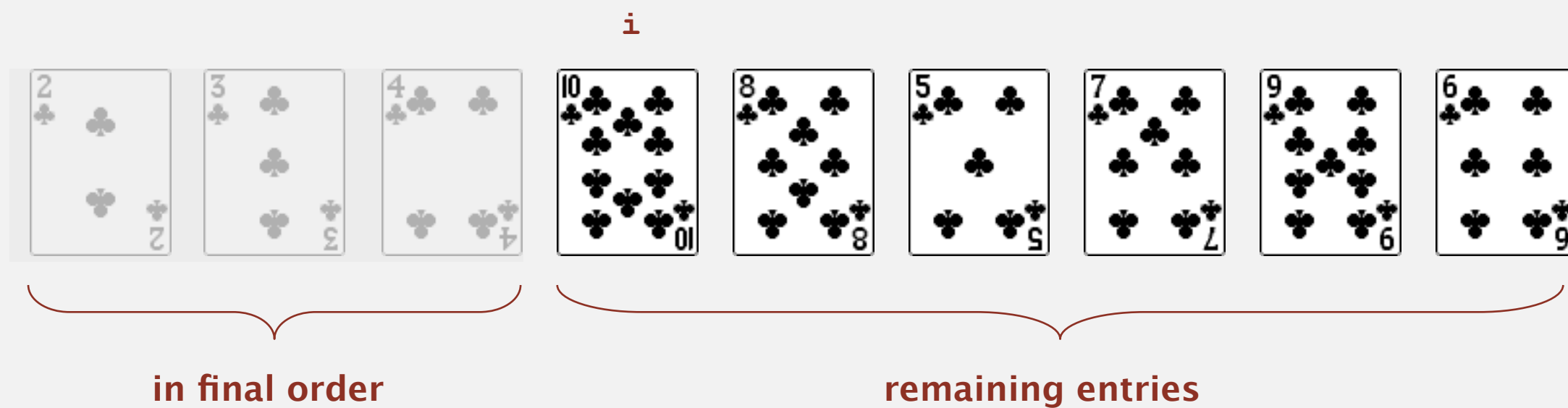
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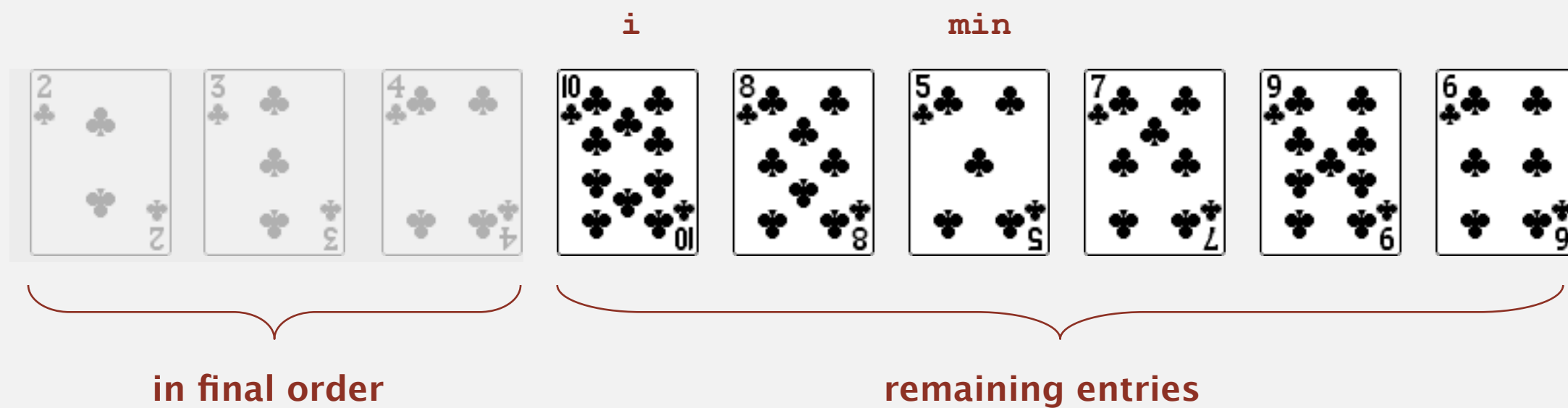
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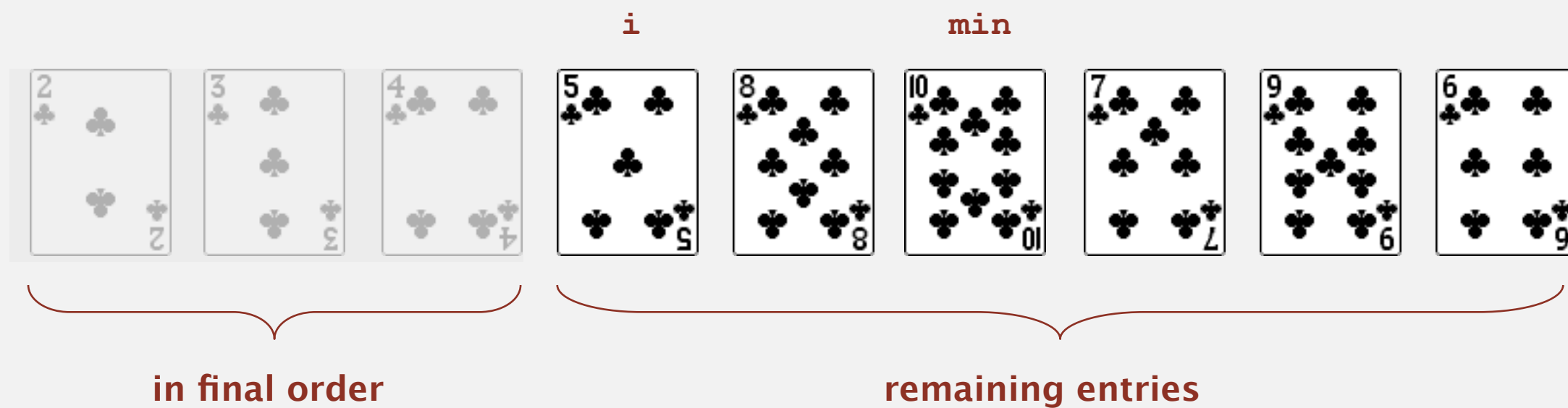
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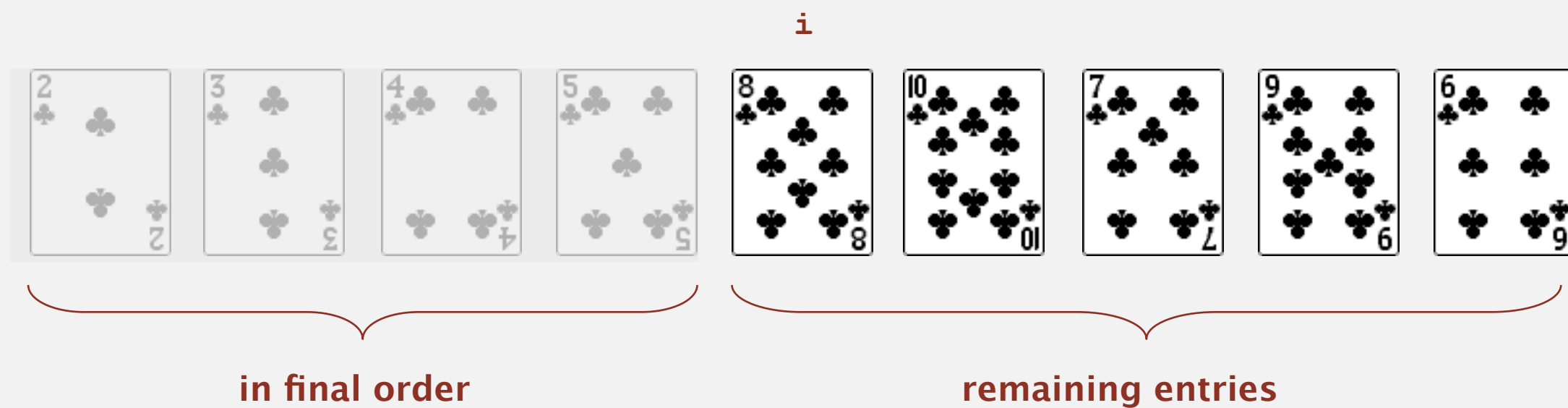
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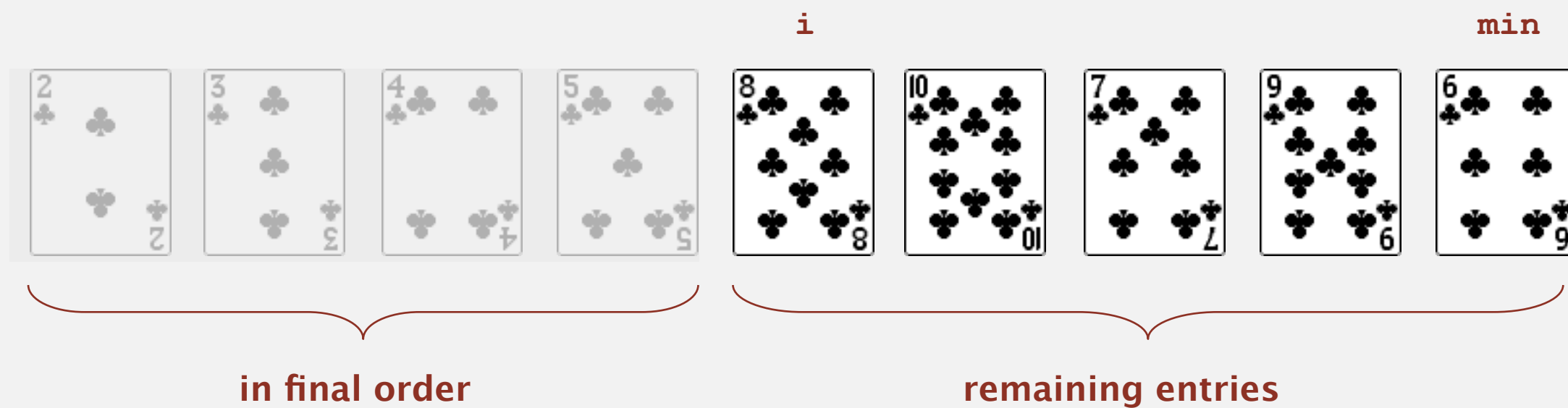
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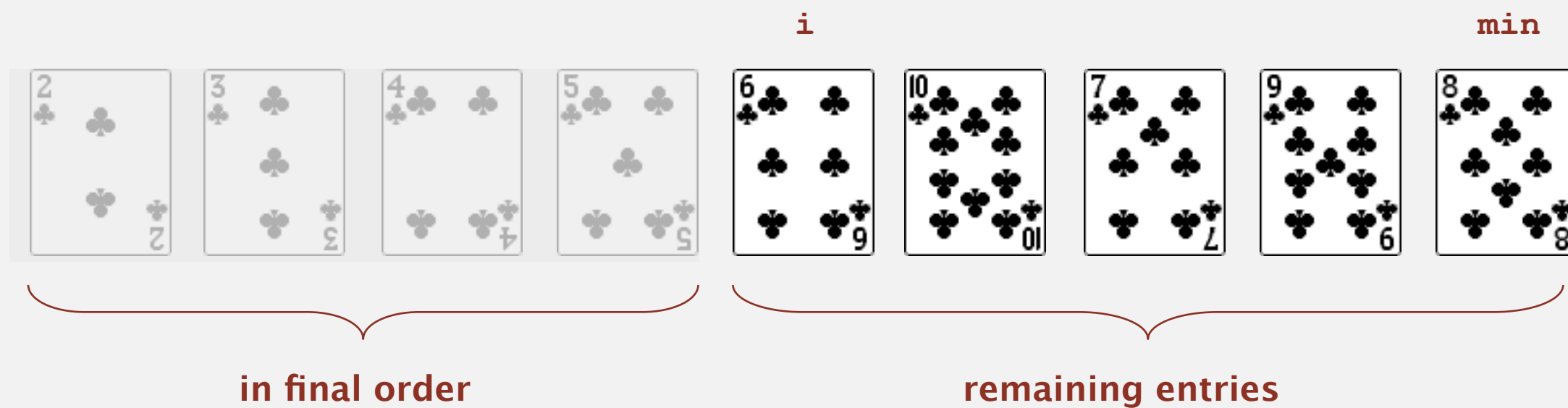
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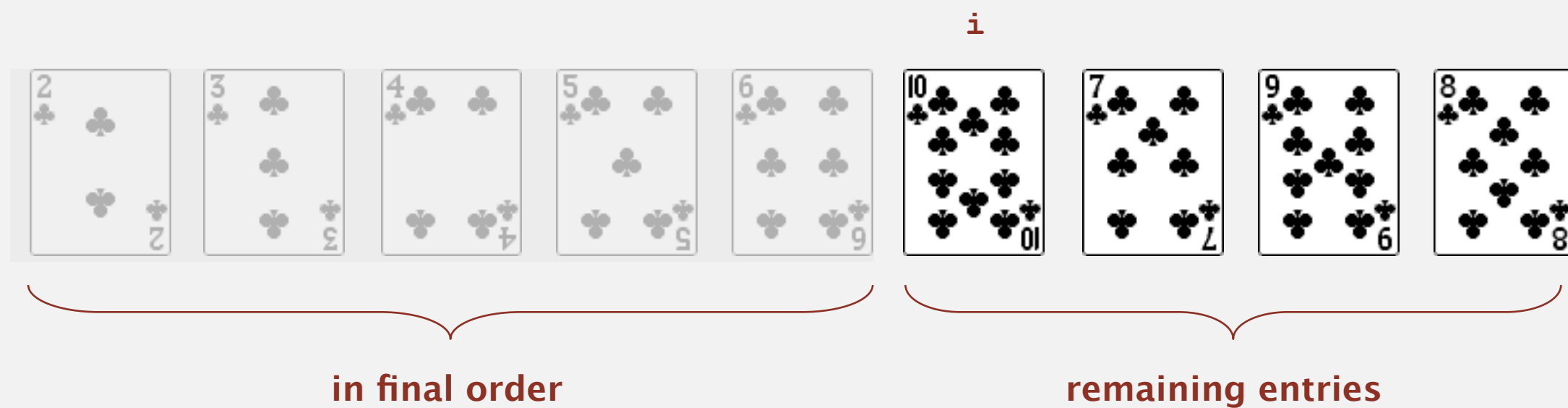
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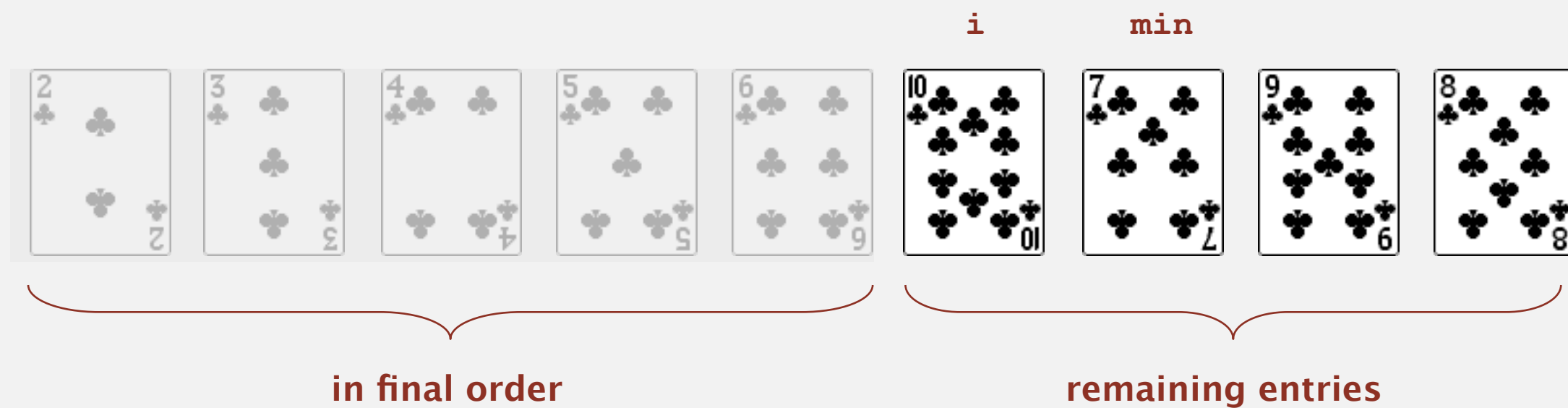
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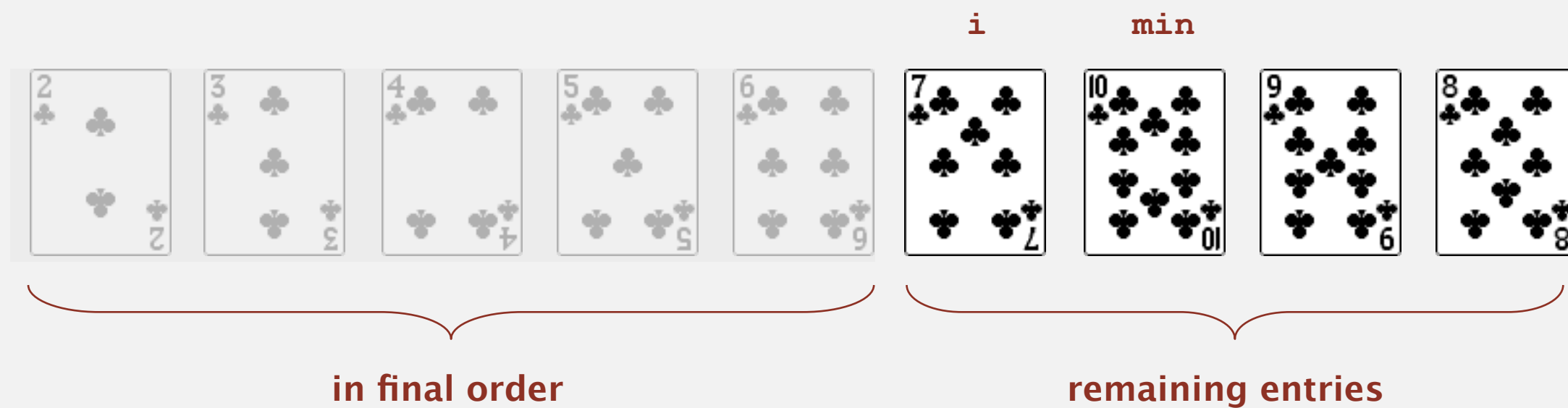
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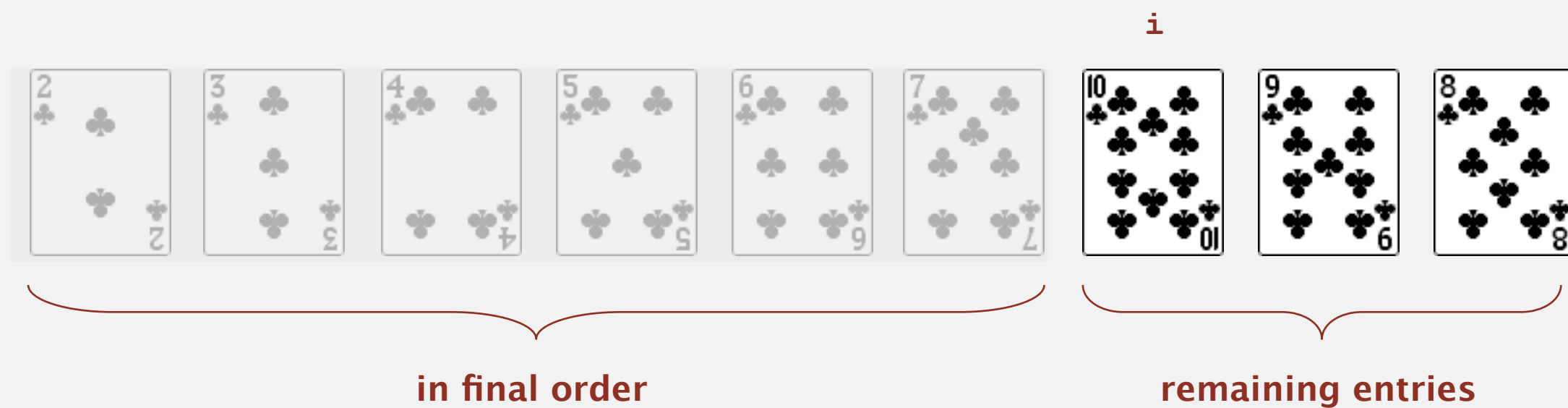
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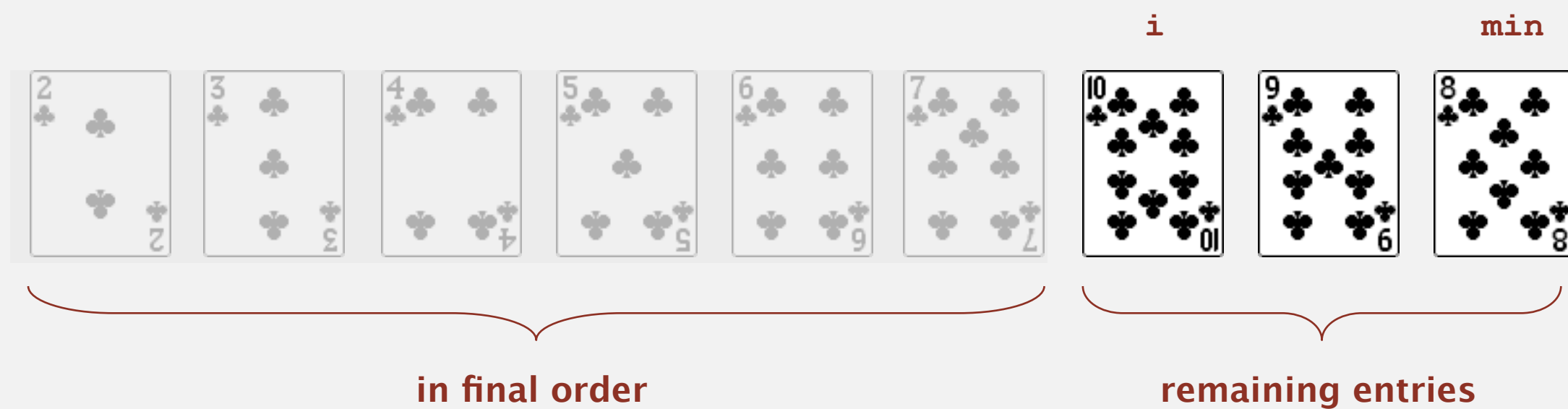
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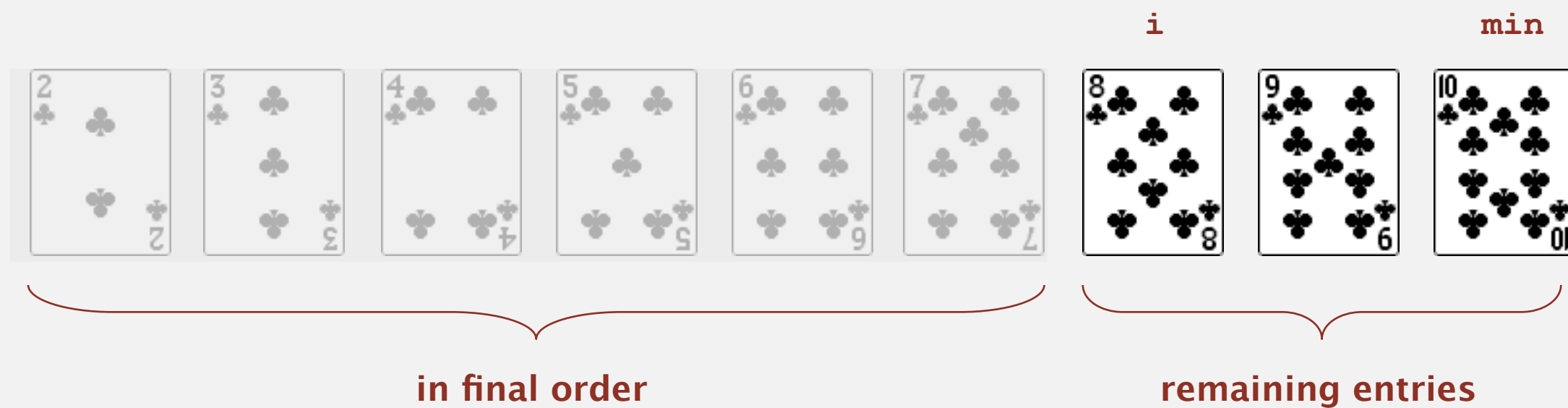
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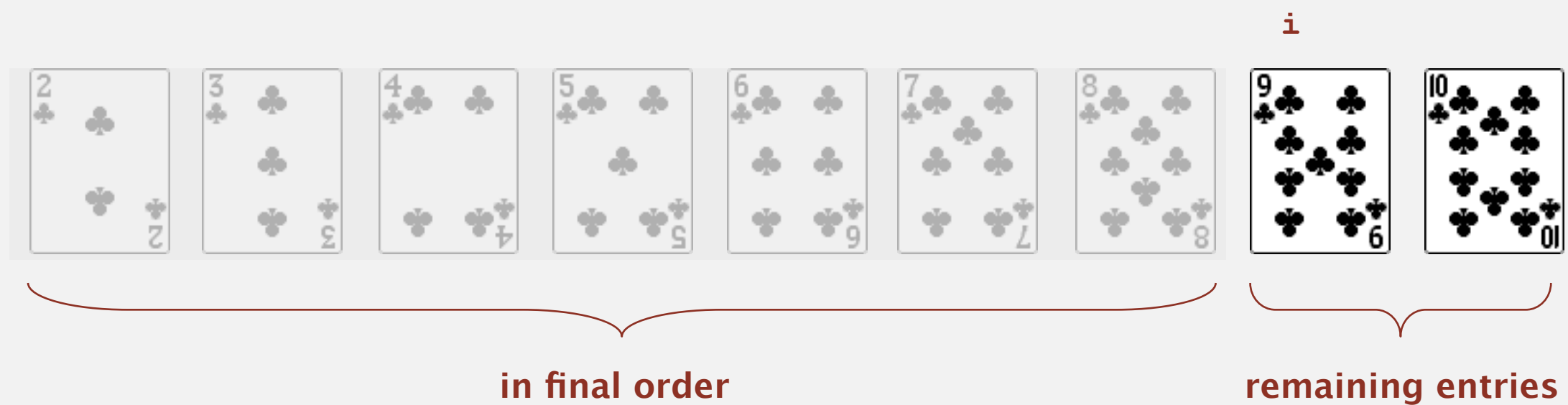
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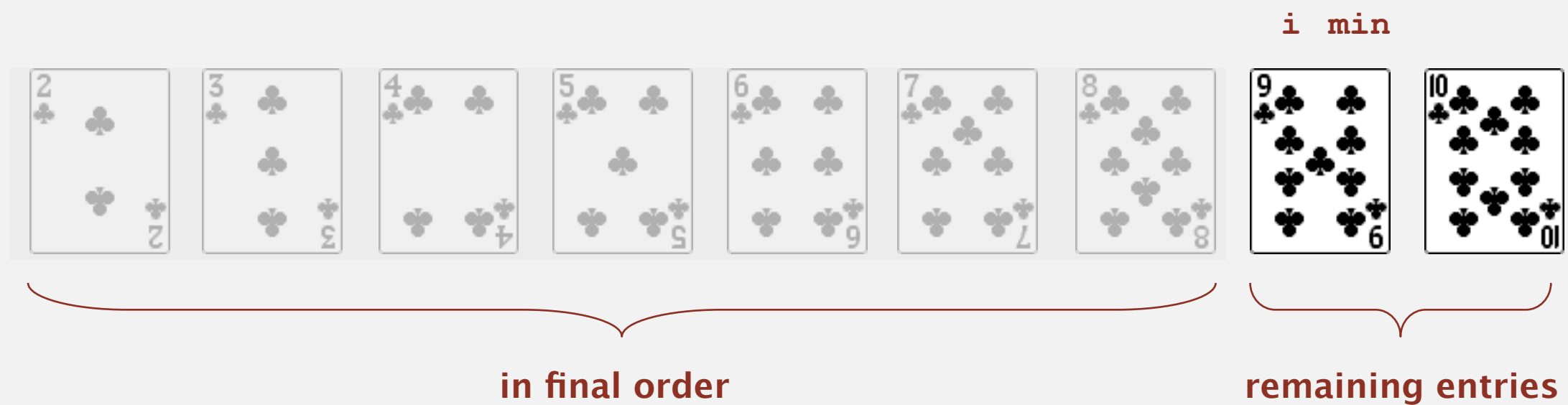
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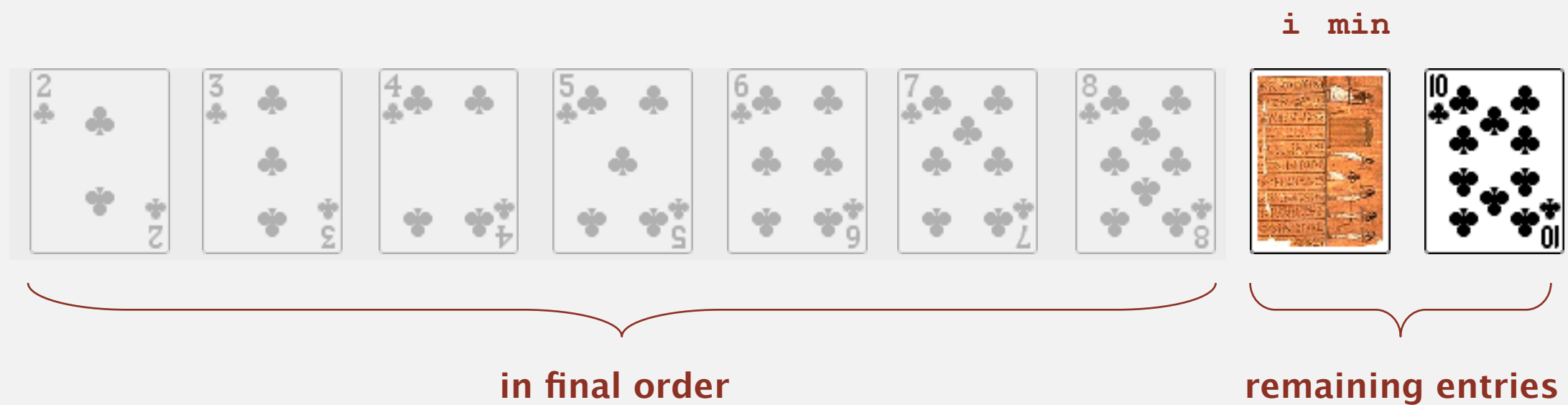
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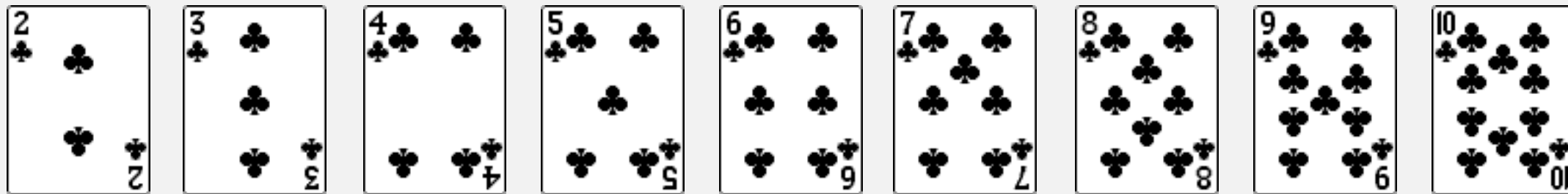
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Selection sort

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sorted

Selection sort: Java implementation

```
public class Selection
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
        {
            int min = i;
            for (int j = i+1; j < N; j++)
                if (less(a[j], a[min]))
                    min = j;
            exch(a, i, min);
        }
    }

    private static boolean less(Comparable v, Comparable w)
    { /* as before */ }

    private static void exch(Comparable[] a, int i, int j)
    { /* as before */ }
}
```

Selection sort: mathematical analysis

Proposition. Selection sort uses $(N-1) + (N-2) + \dots + 1 + 0 \sim N^2/2$ compares and N exchanges.

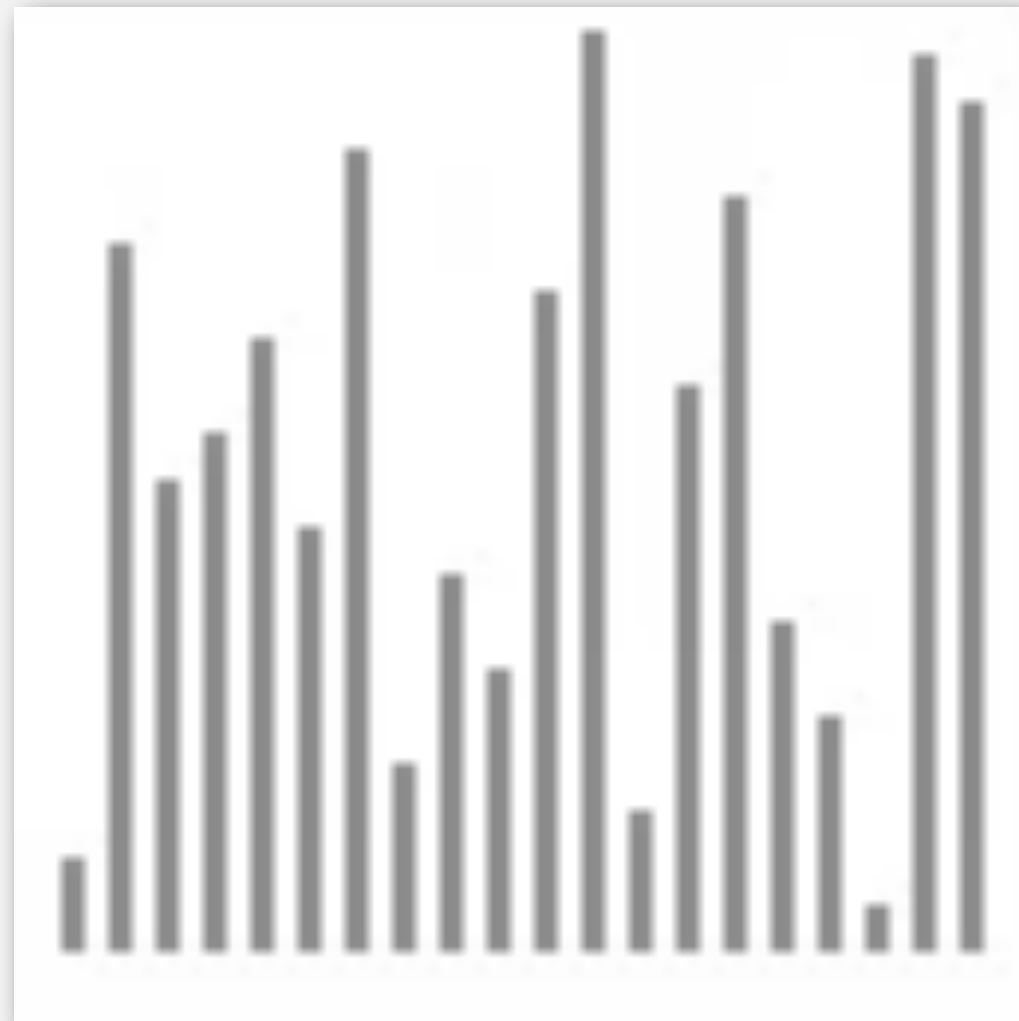
		a[]												
	i	min	0	1	2	3	4	5	6	7	8	9	10	
			S	O	R	T	E	X	A	M	P	L	E	
	0	6	S	O	R	T	E	X	A	M	P	L	E	<i>entries in black are examined to find the minimum</i>
	1	4	A	O	R	T	E	X	S	M	P	L	E	<i>entries in red are a[min]</i>
	2	10	A	E	R	T	O	X	S	M	P	L	E	
	3	9	A	E	E	T	O	X	S	M	P	L	R	
	4	7	A	E	E	L	O	X	S	M	P	T	R	
	5	7	A	E	E	L	M	X	S	O	P	T	R	
	6	8	A	E	E	L	M	O	S	X	P	T	R	
	7	10	A	E	E	L	M	O	P	X	S	T	R	
	8	8	A	E	E	L	M	O	P	R	S	T	X	
	9	9	A	E	E	L	M	O	P	R	S	T	X	
	10	10	A	E	E	L	M	O	P	R	S	T	X	<i>entries in gray are in final position</i>
			A	E	E	L	M	O	P	R	S	T	X	

Trace of selection sort (array contents just after each exchange)

Running time insensitive to input. Quadratic time, even if input array is sorted.
Data movement is minimal. Linear number of exchanges.

Selection sort: animations

20 random items

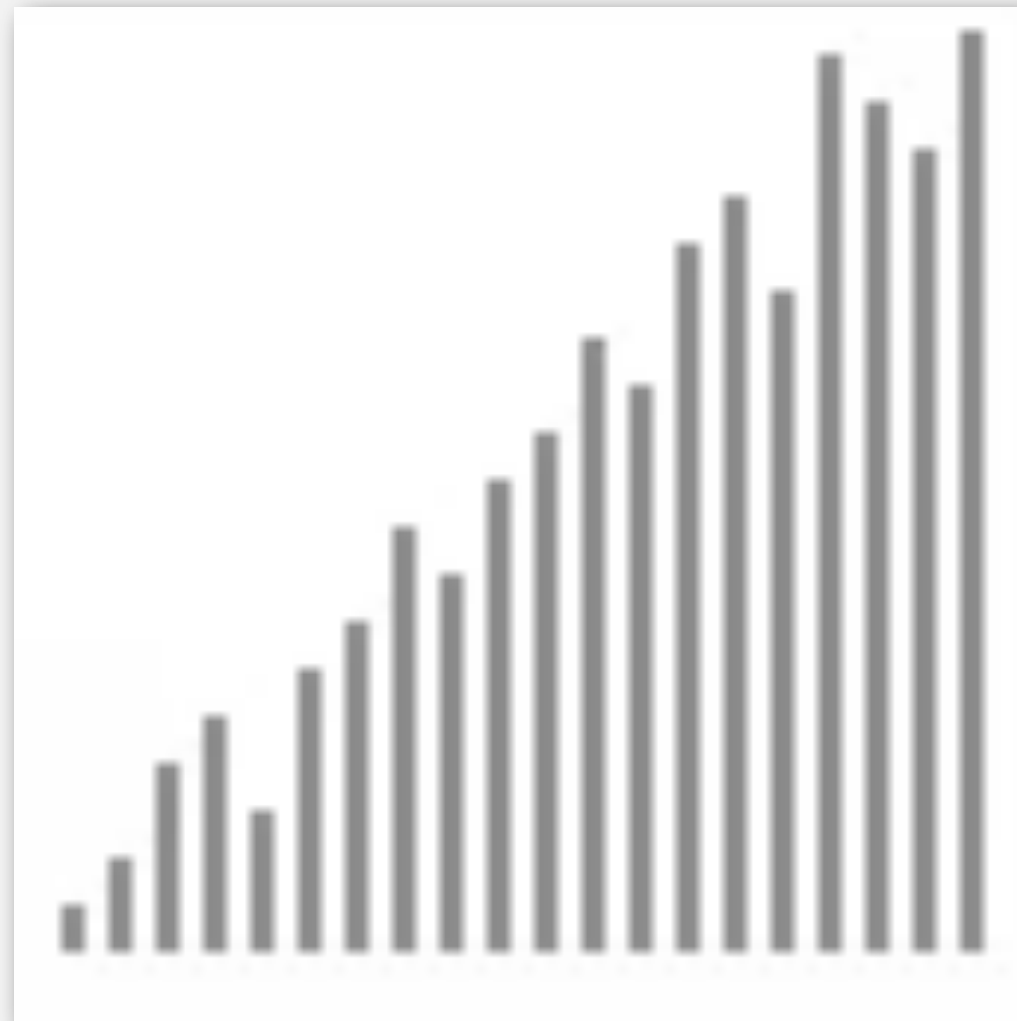


- ▲ algorithm position
- █ in final order
- ▒ not in final order

<http://www.sorting-algorithms.com/selection-sort>

Selection sort: animations

20 partially-sorted items



- ▲ algorithm position
- █ in final order
- ▒ not in final order

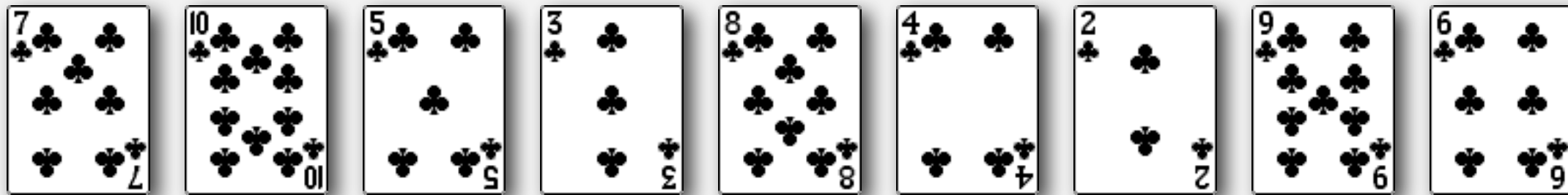
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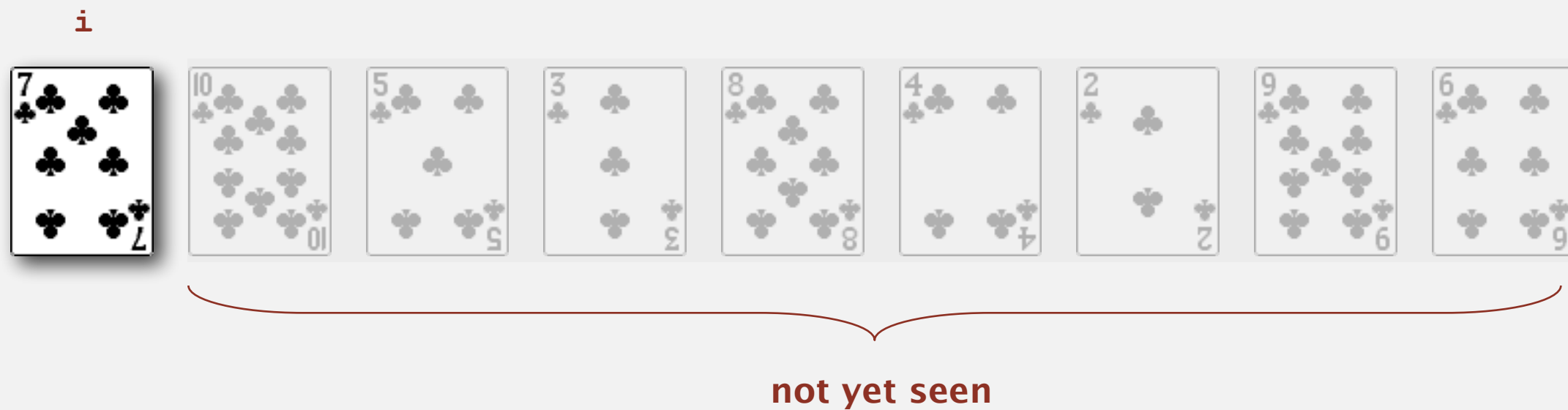
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



Selection sort

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j i

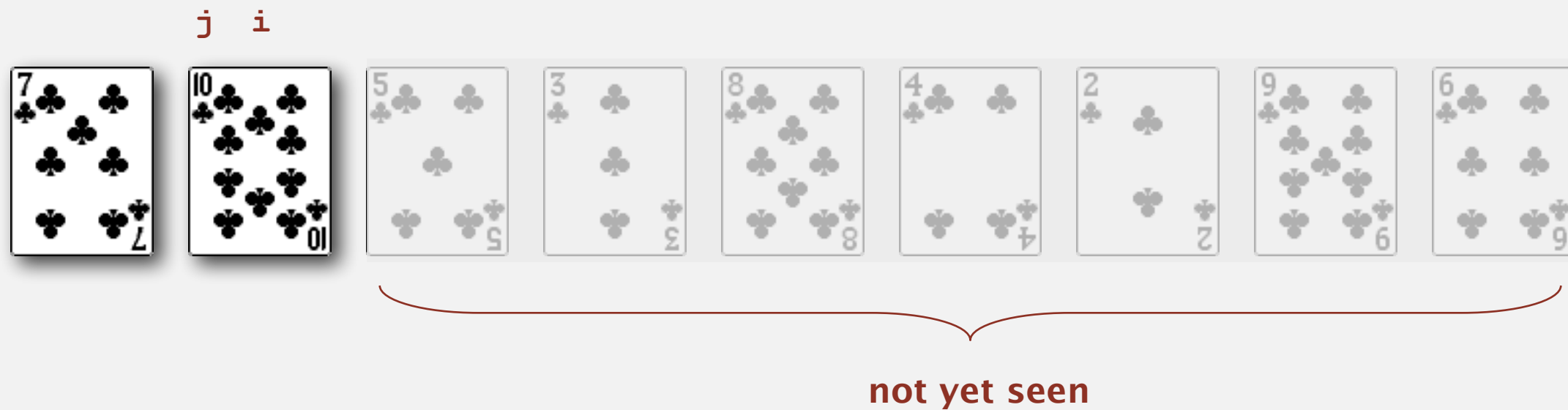


in ascending order

not yet seen

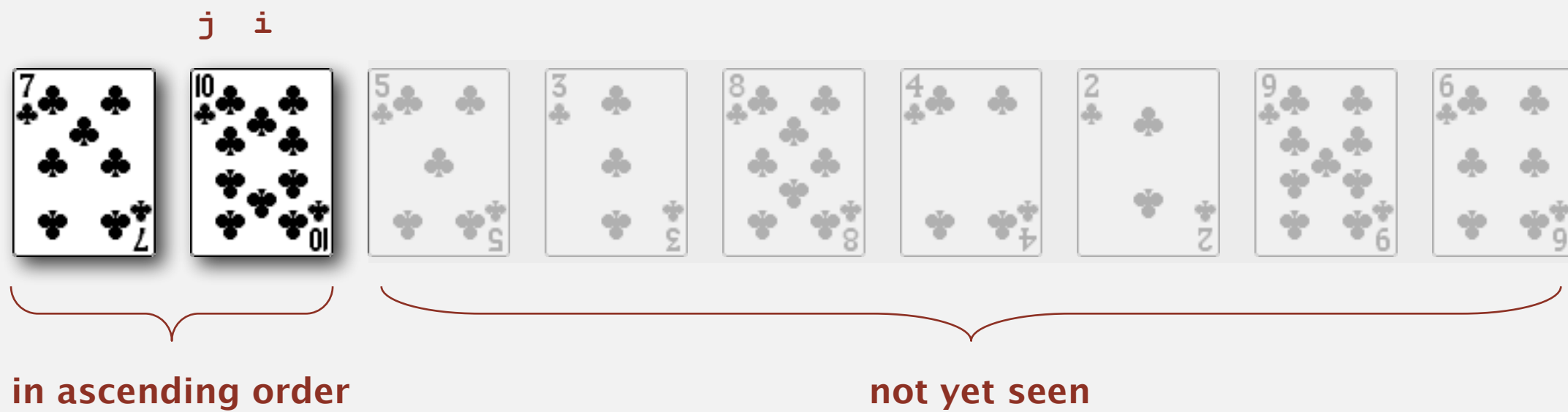
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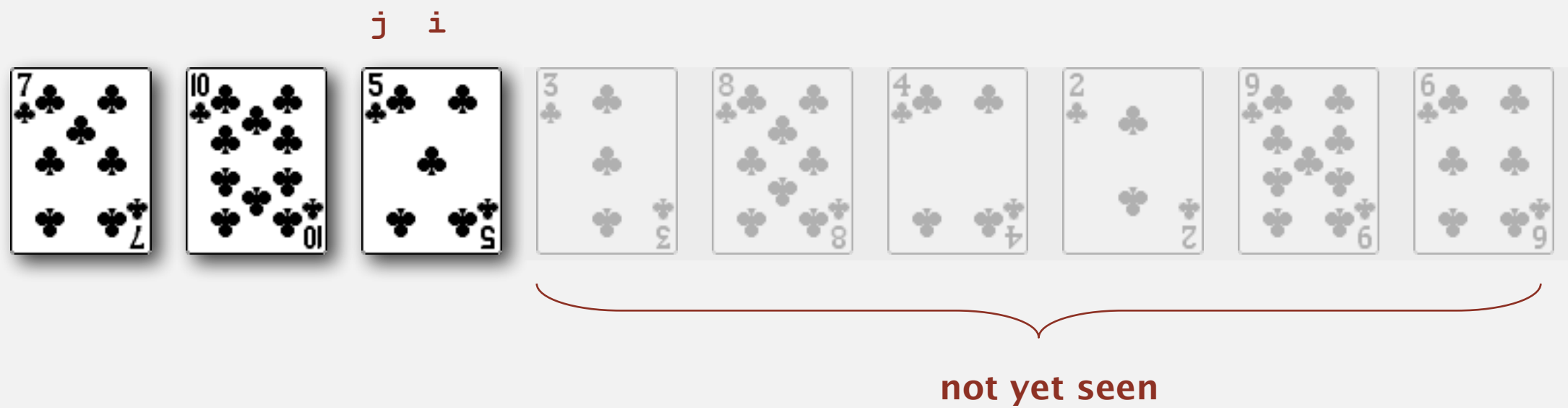
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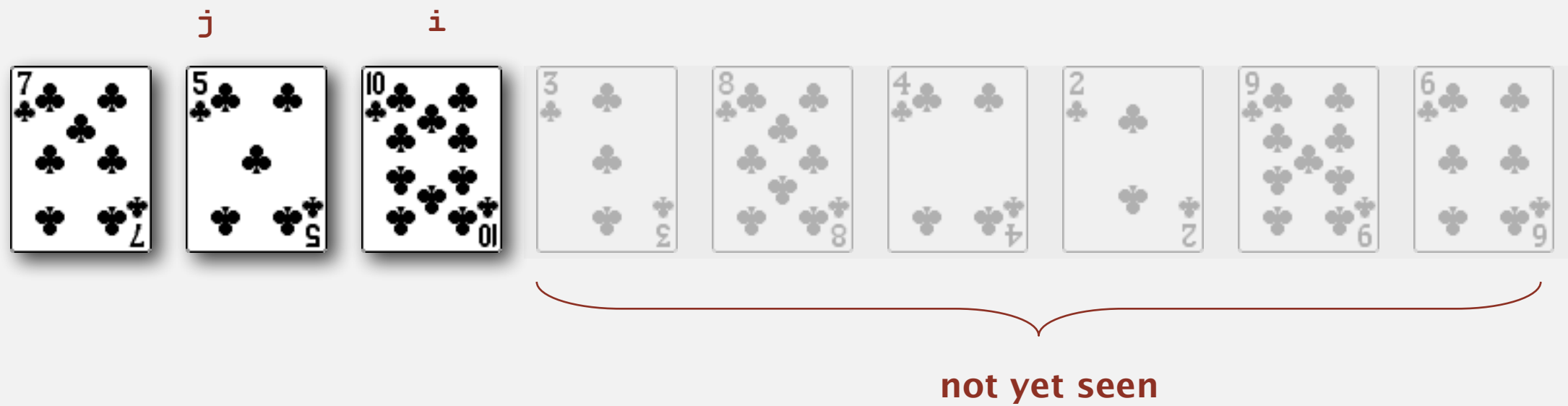
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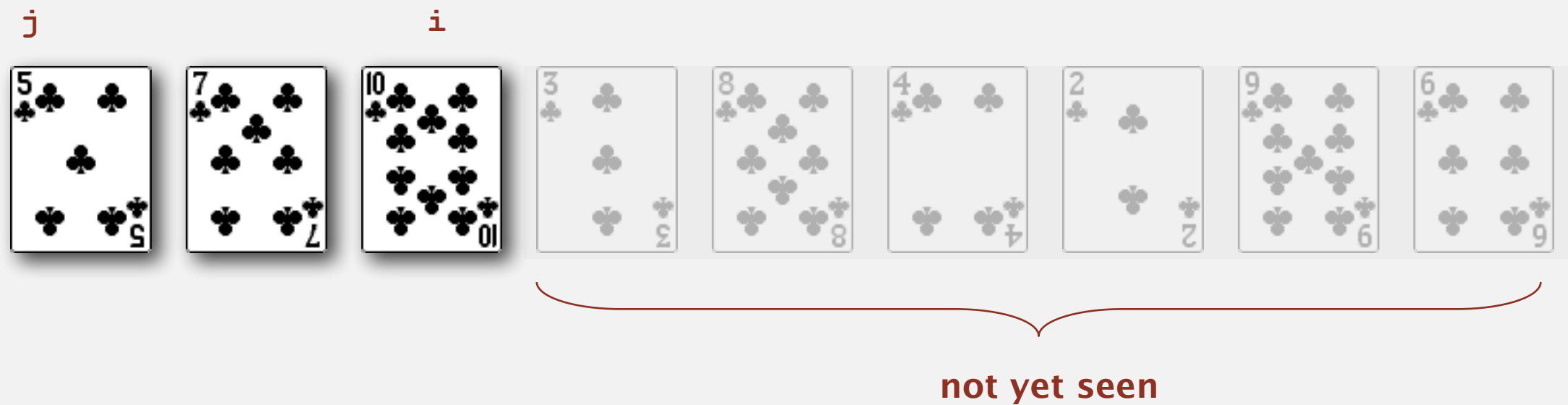
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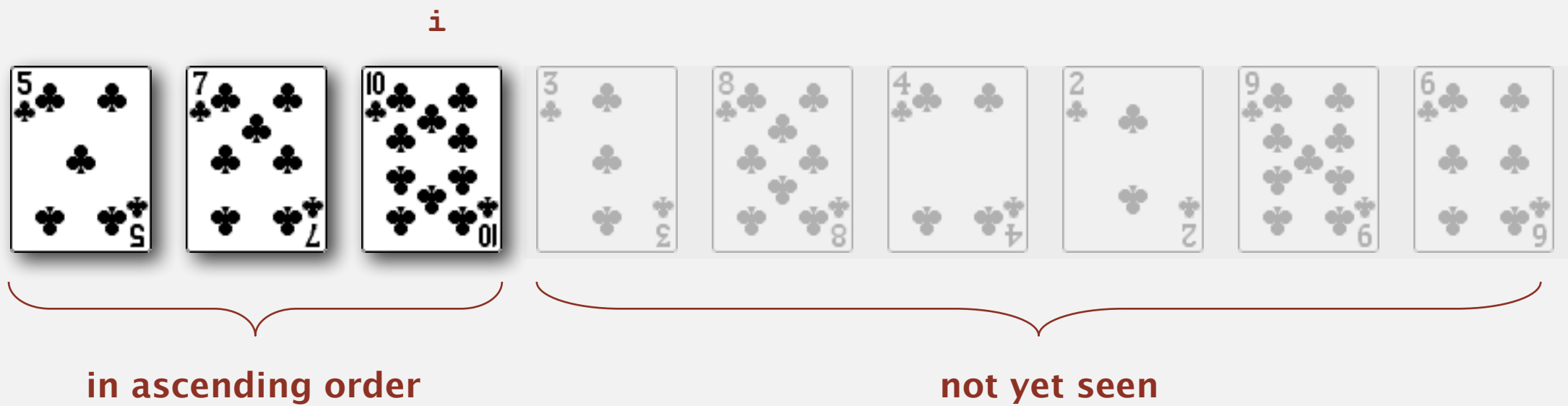
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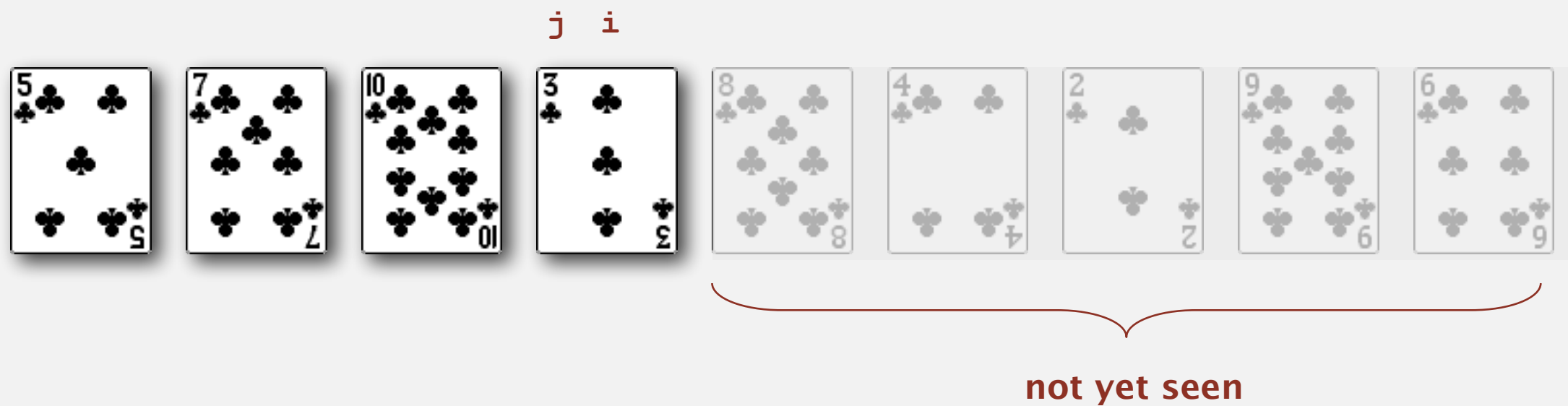
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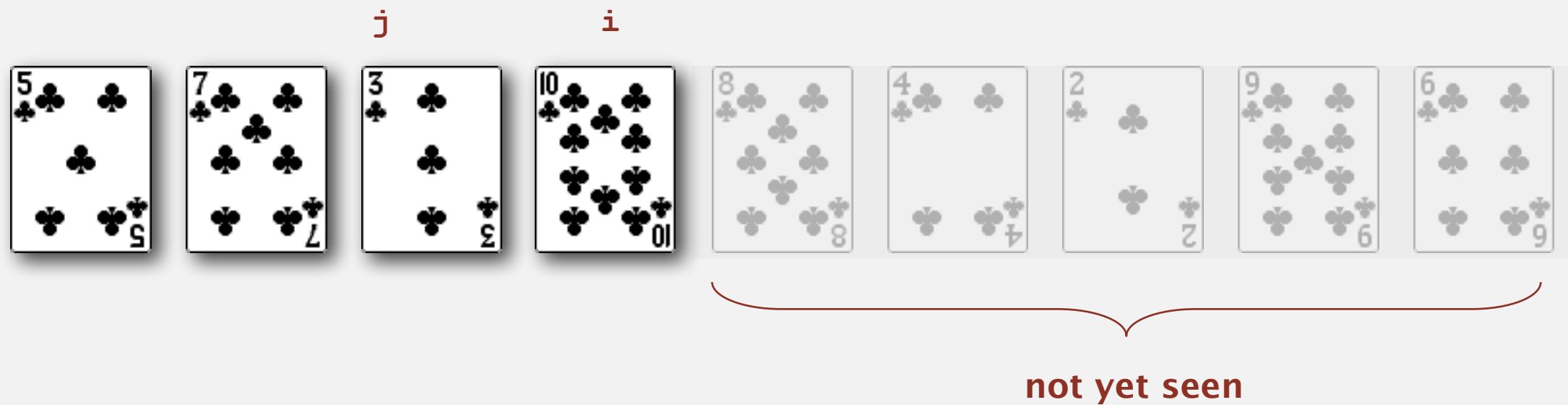
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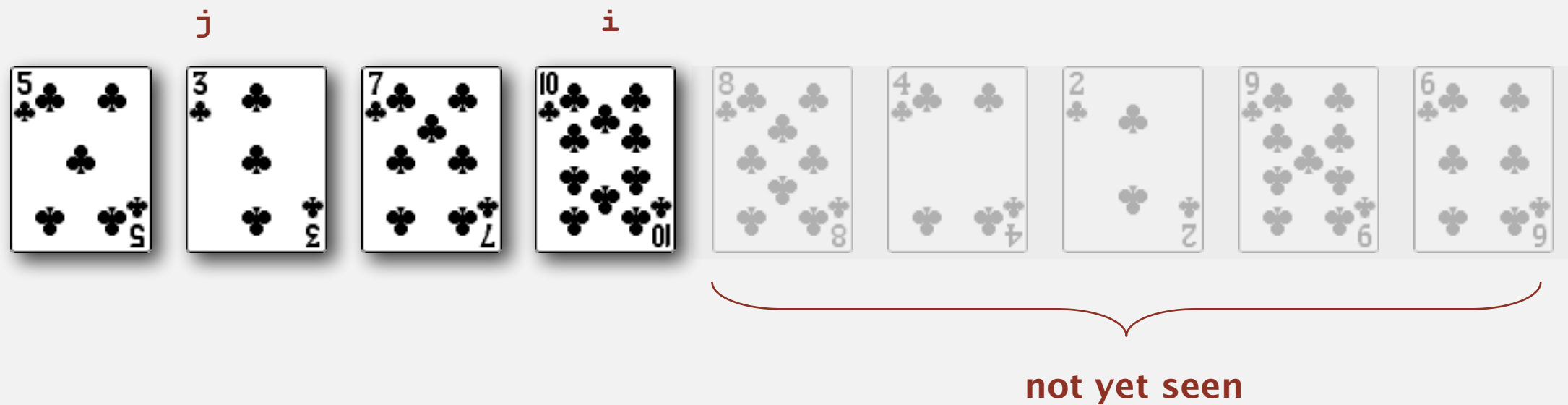
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



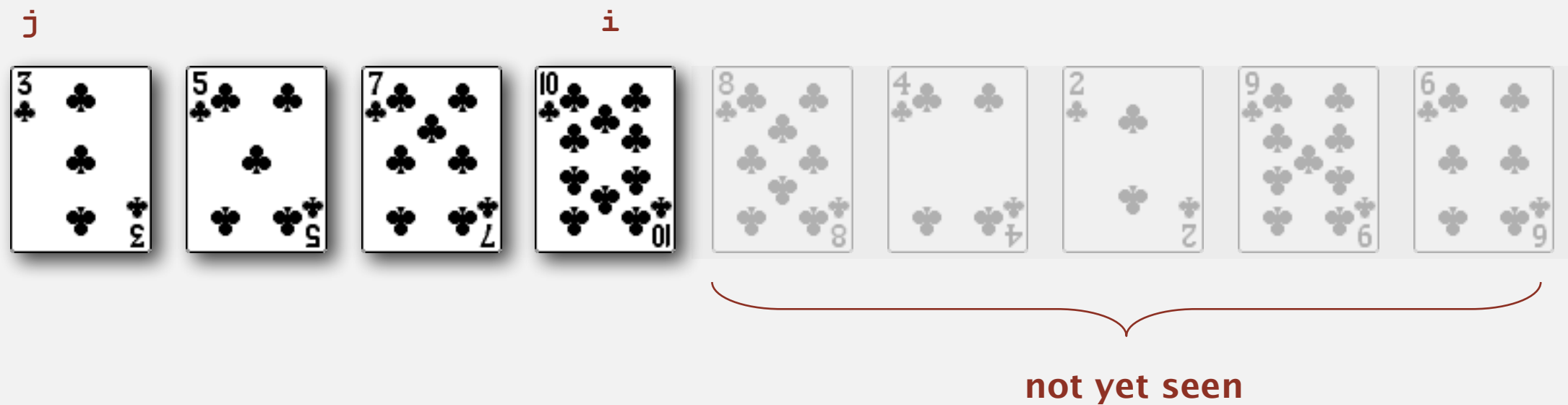
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



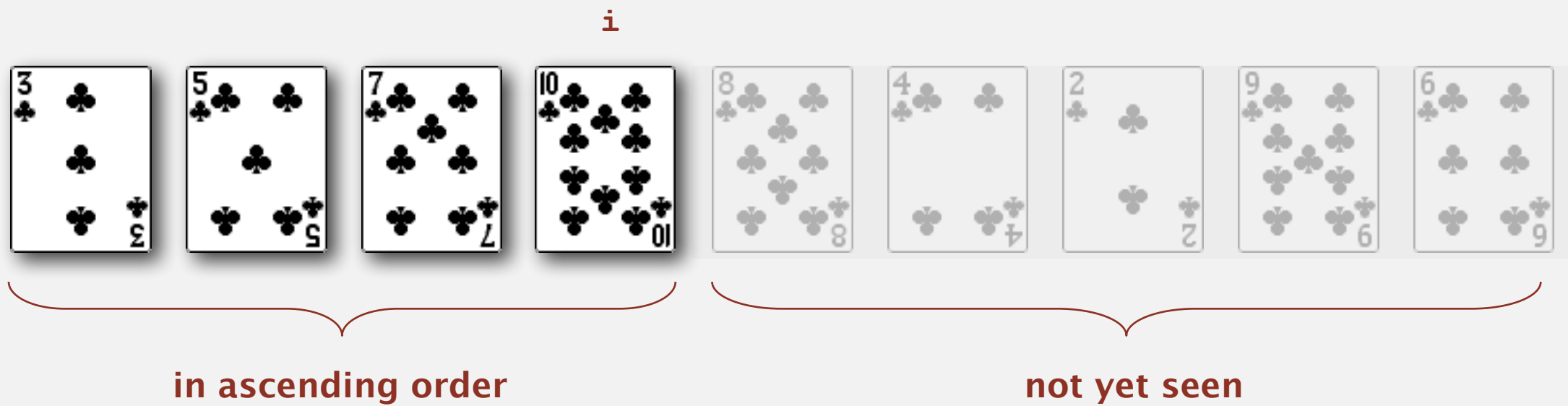
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



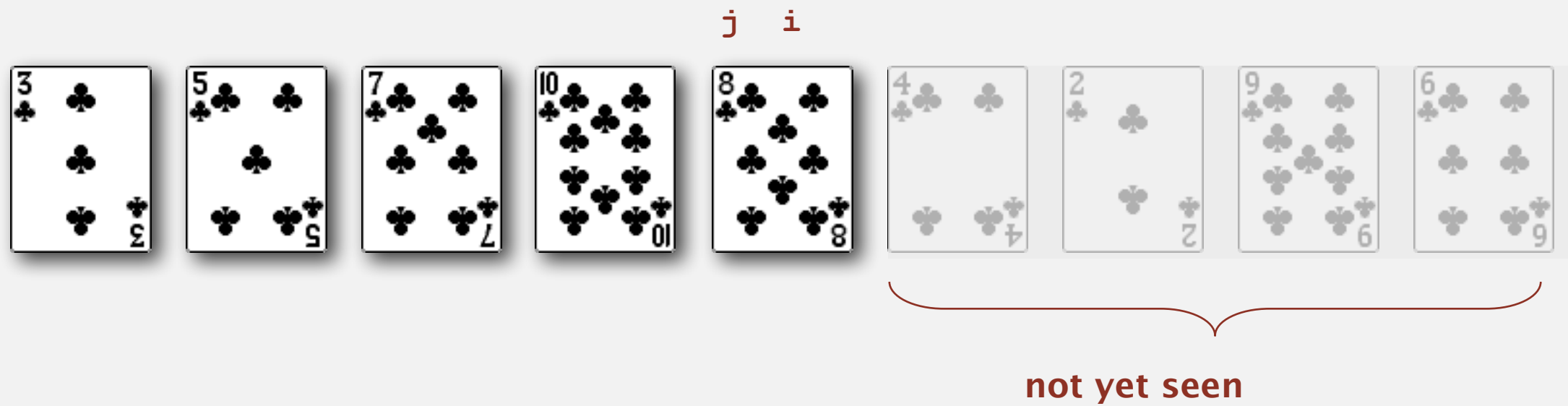
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



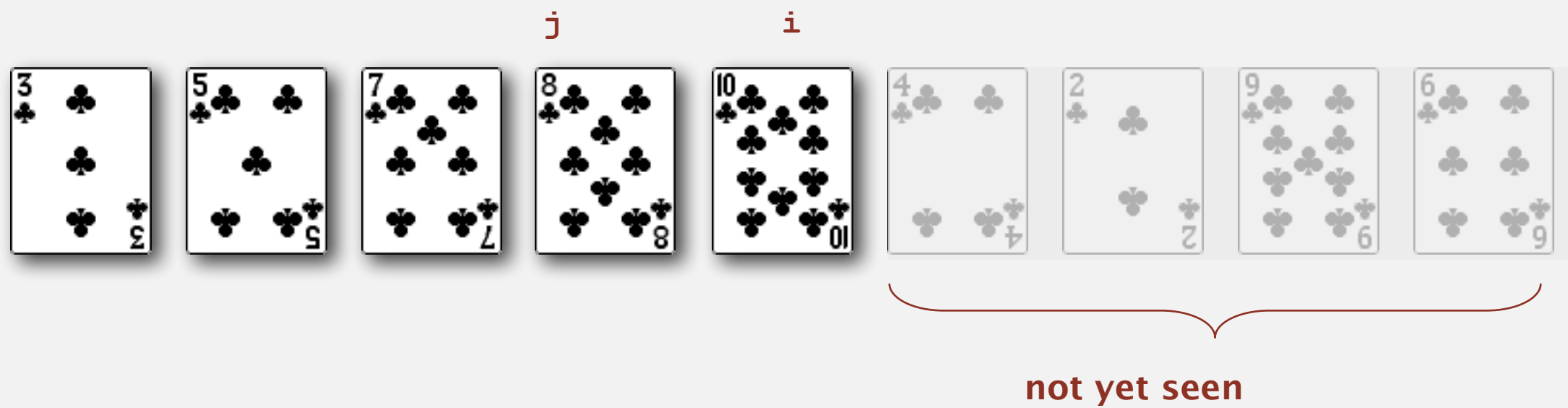
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



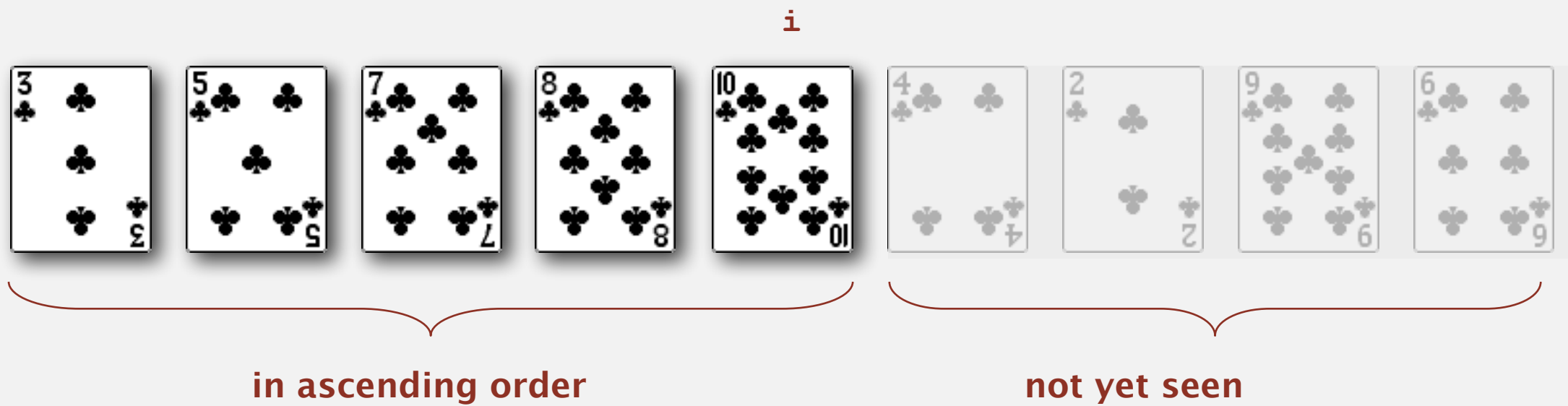
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



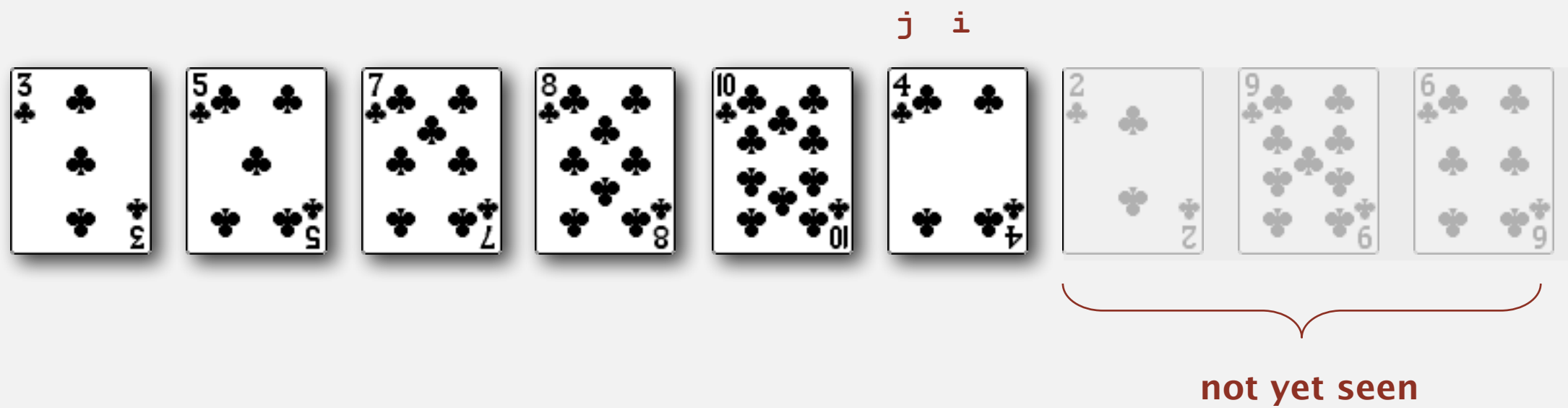
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



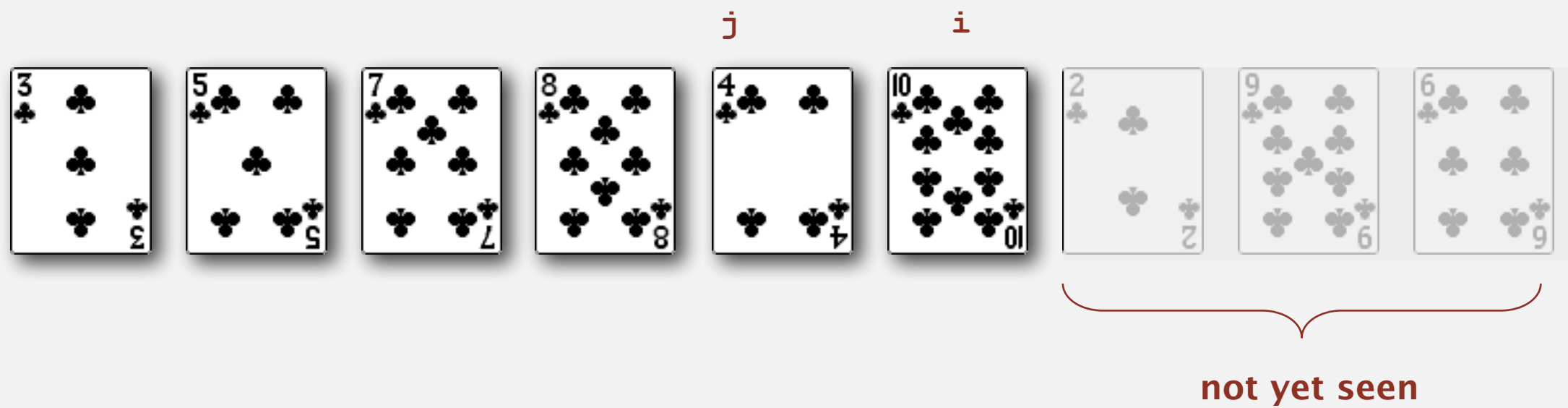
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



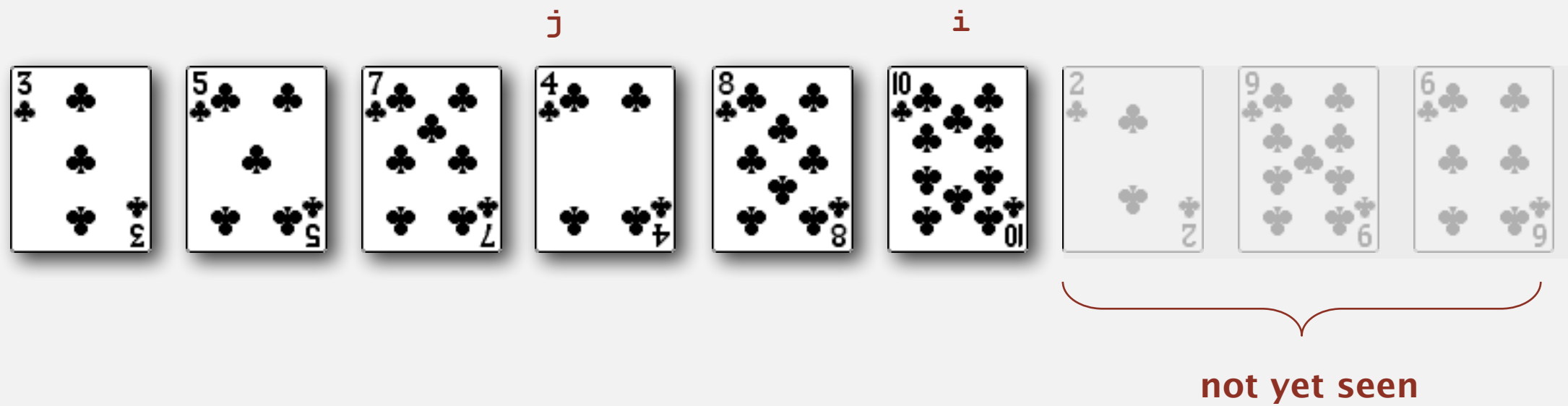
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



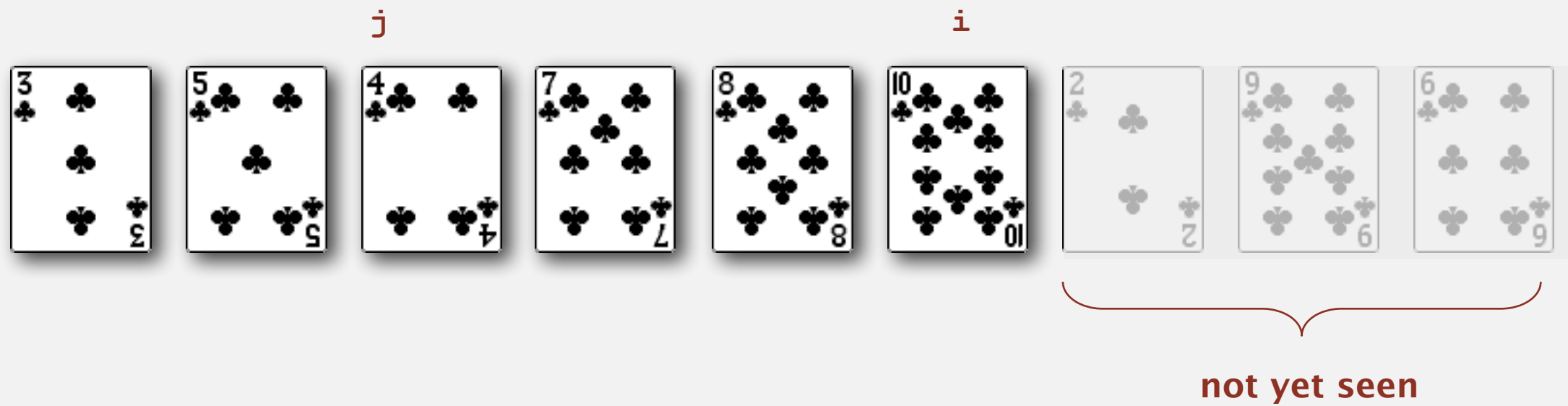
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



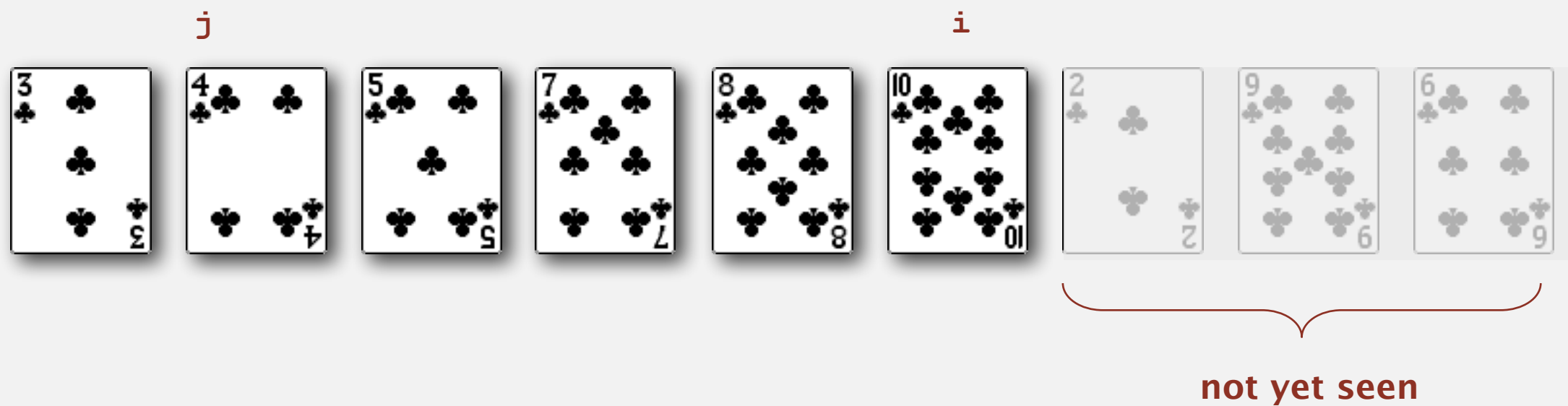
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



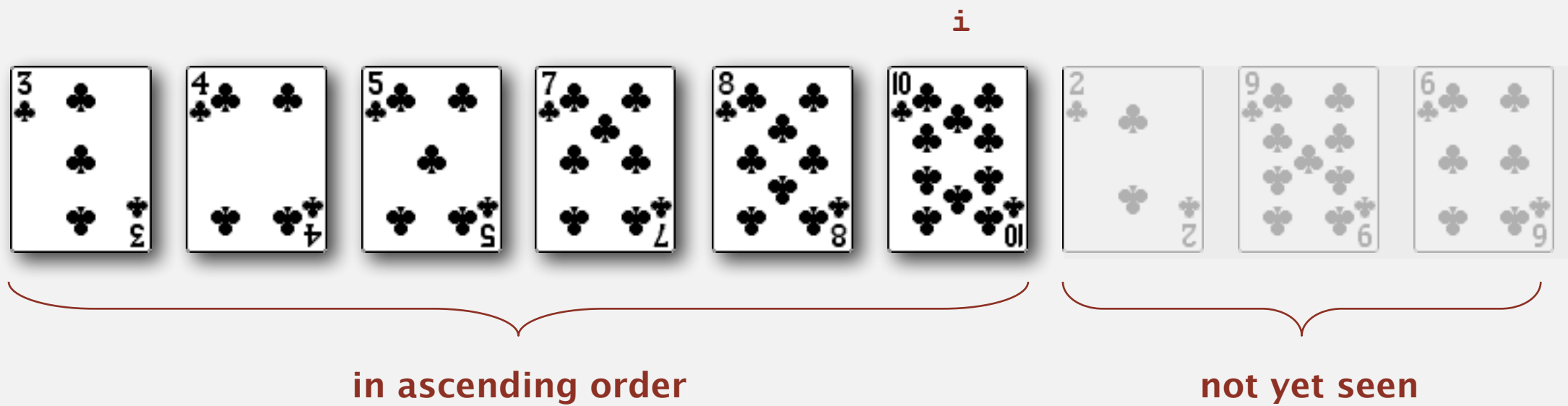
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



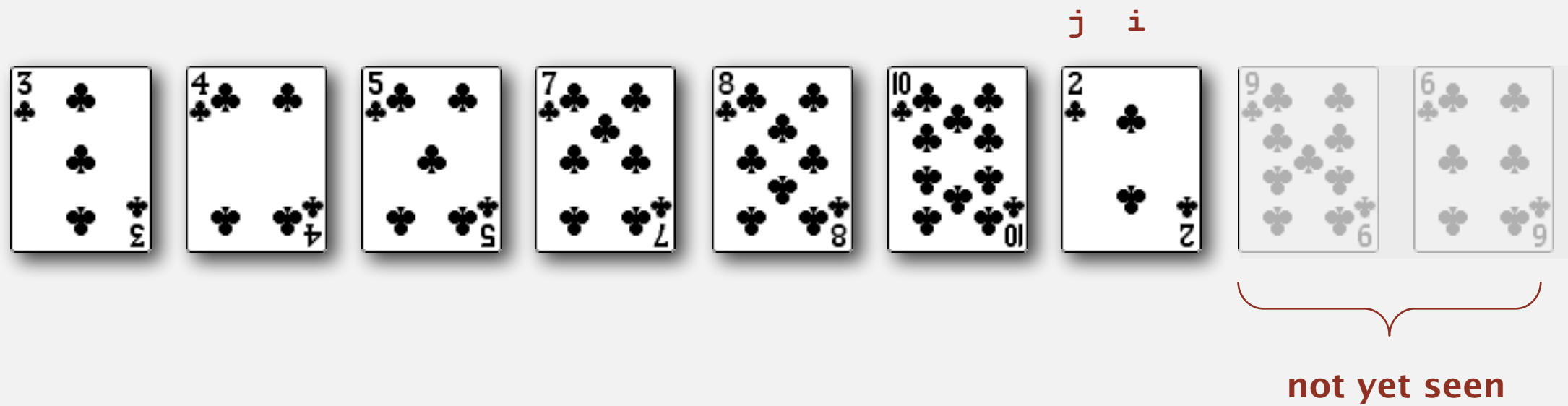
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



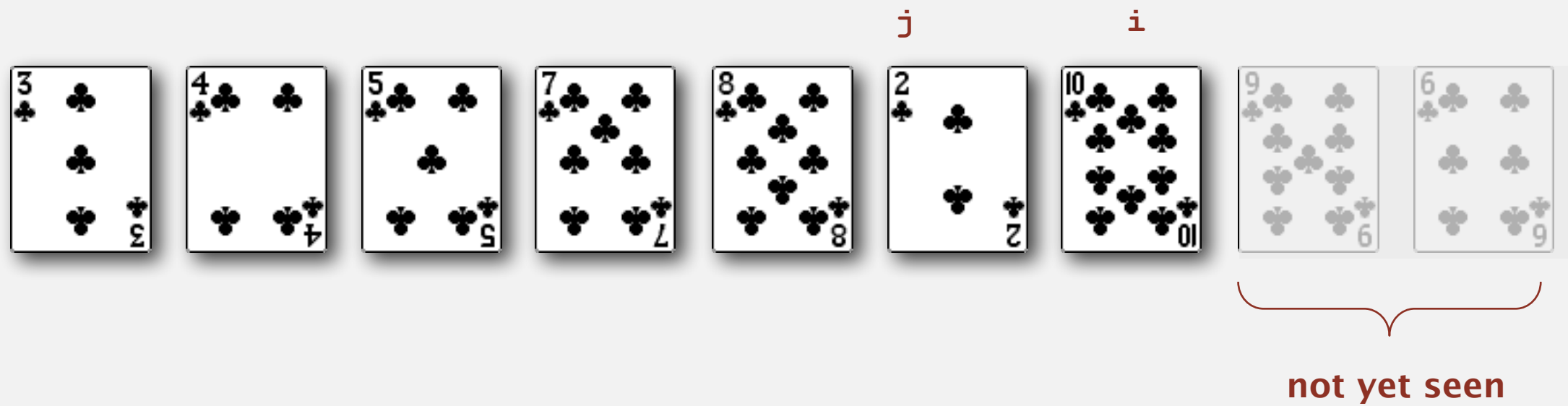
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



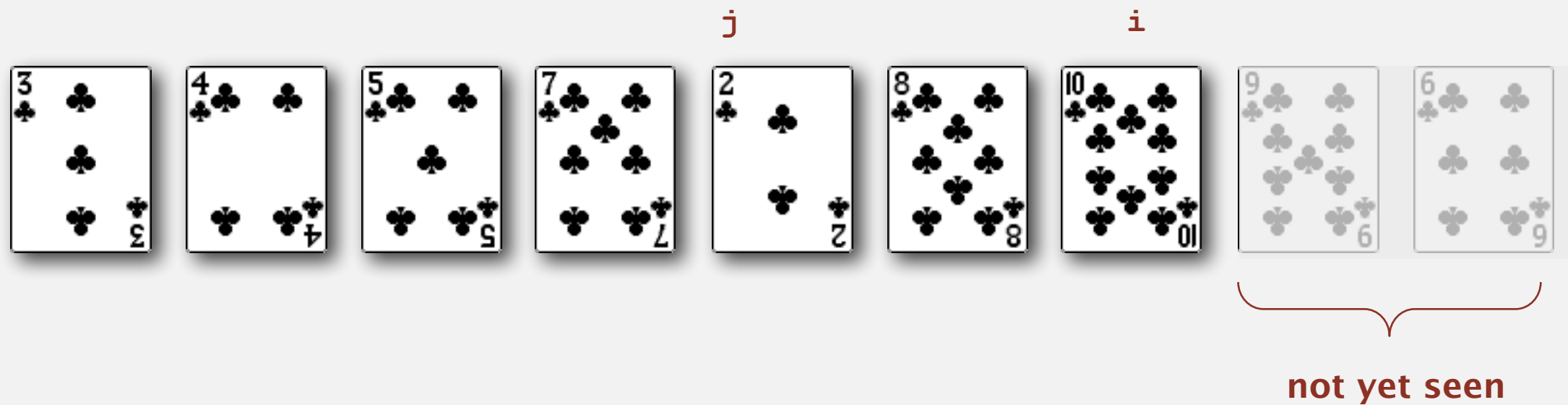
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



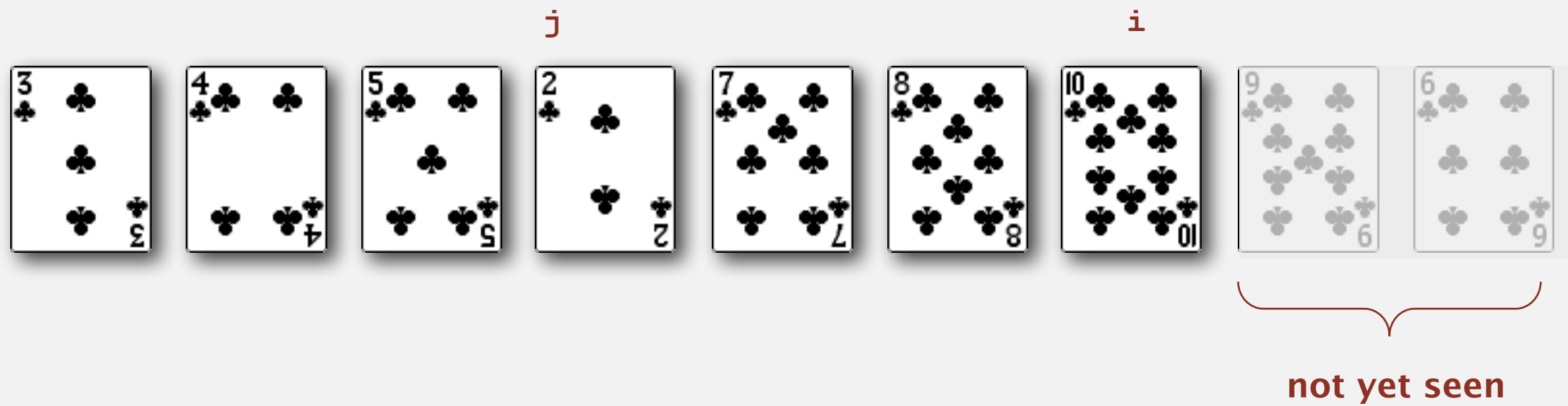
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



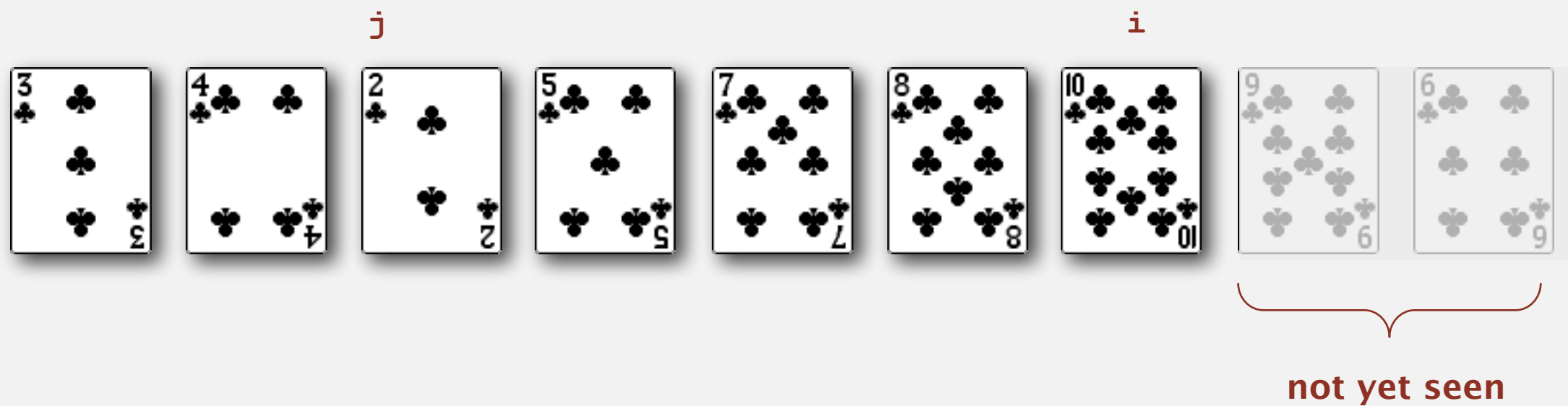
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



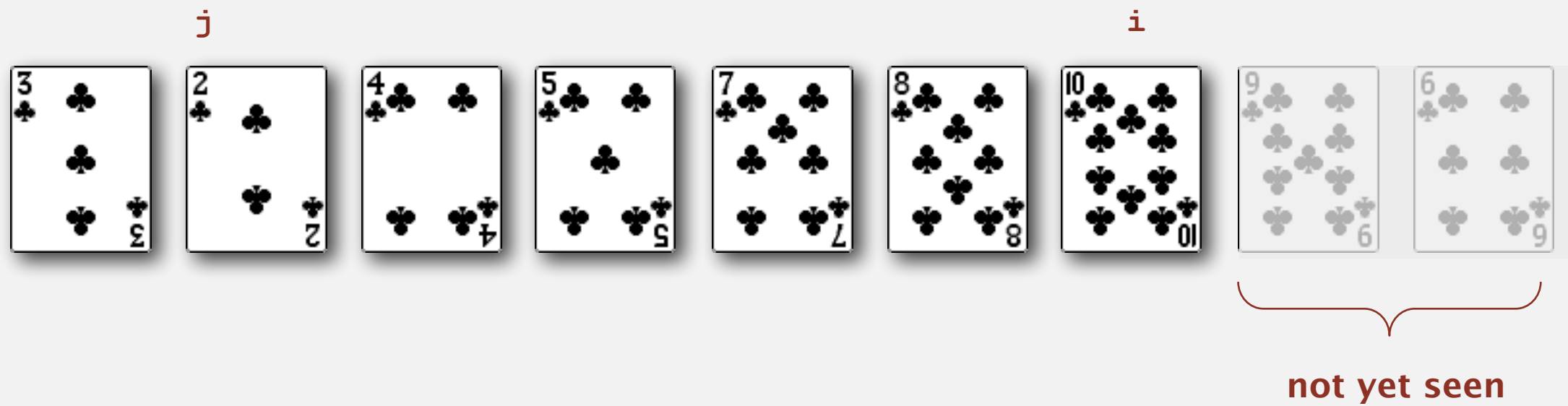
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



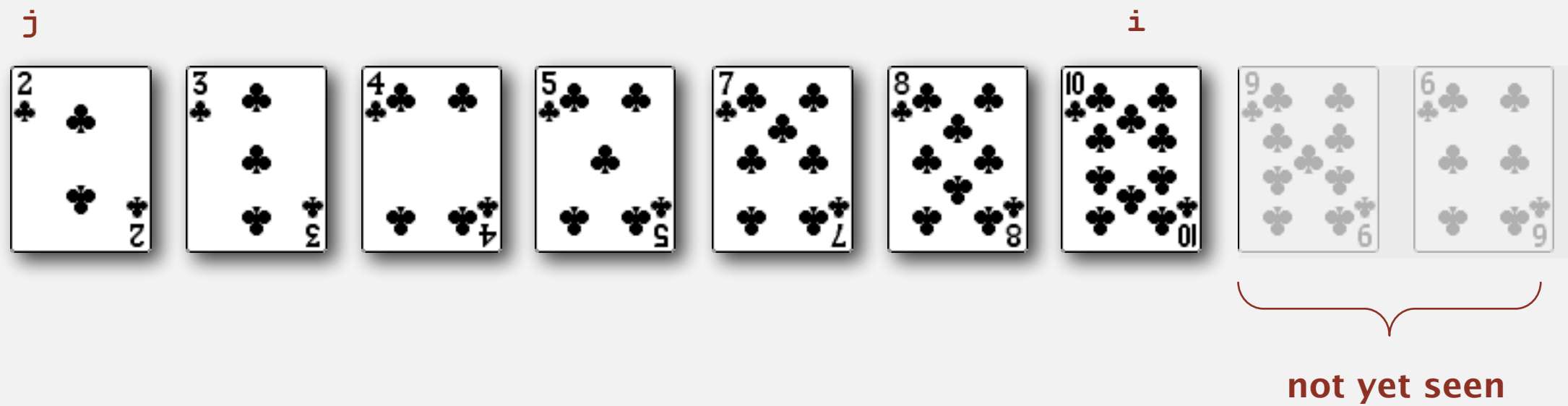
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



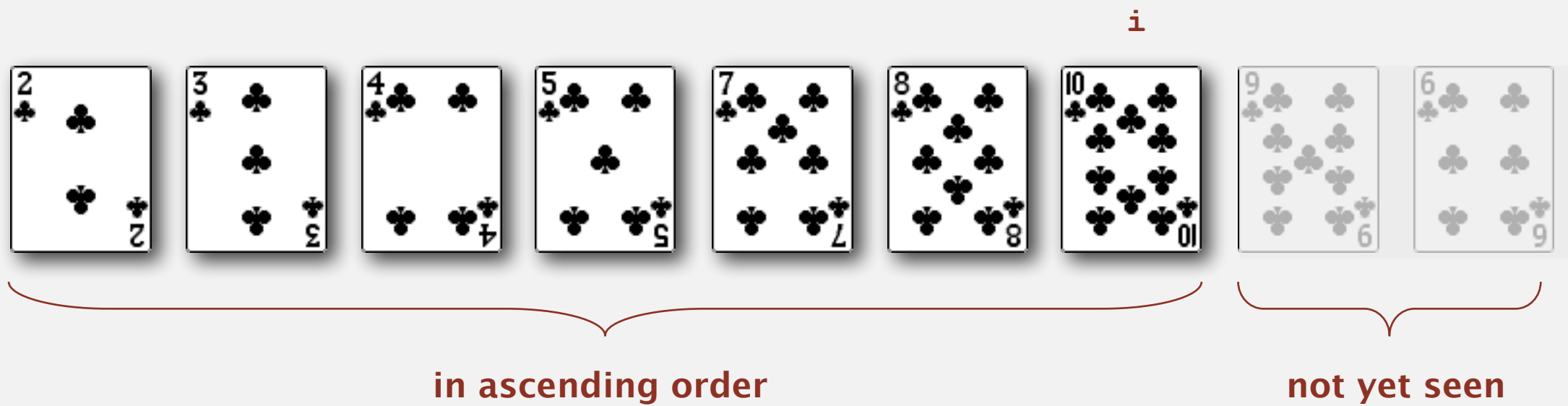
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



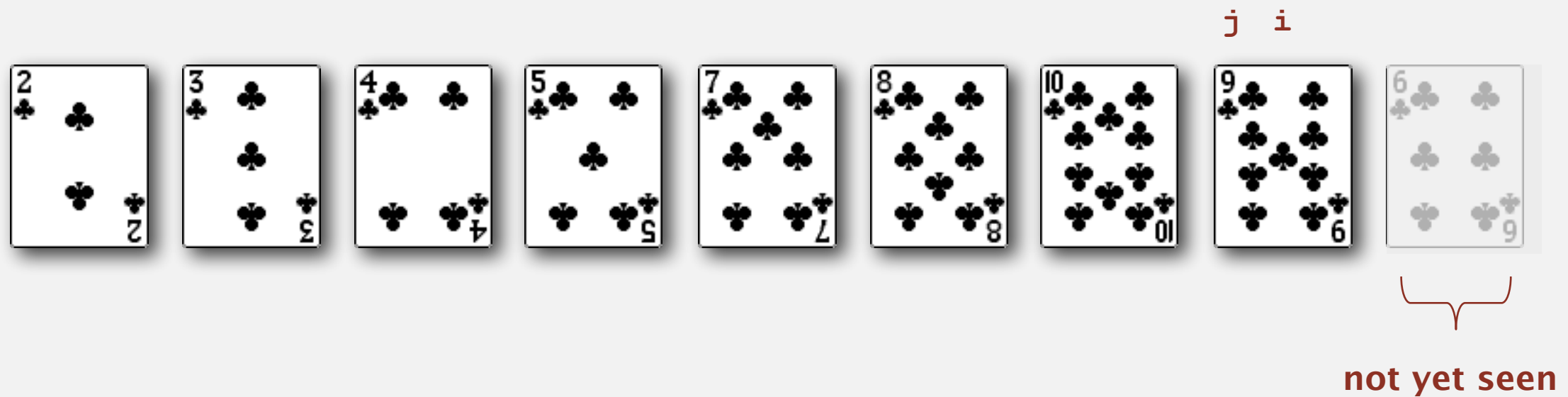
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



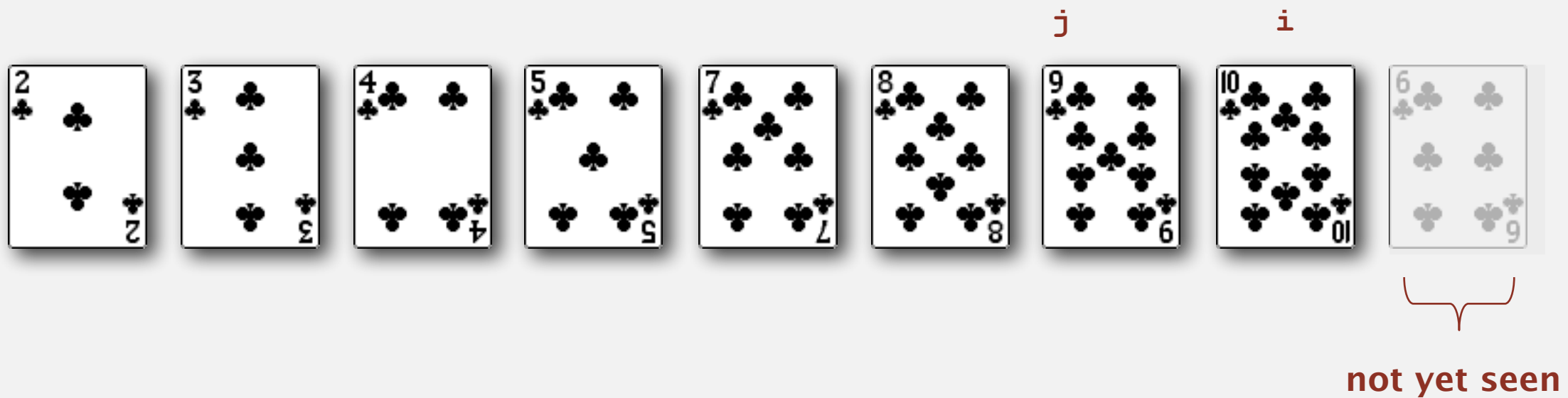
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



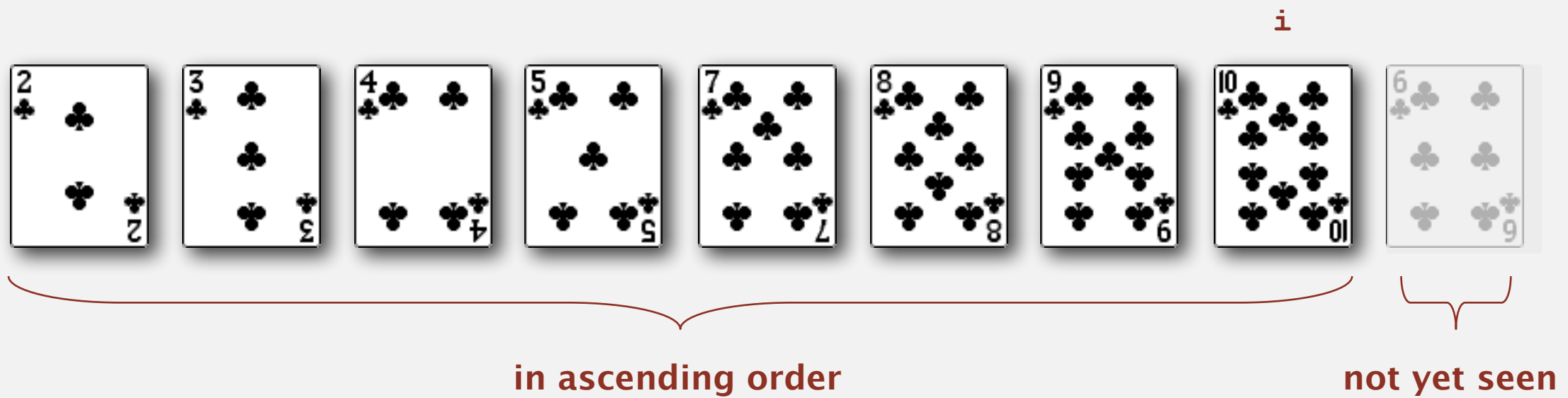
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



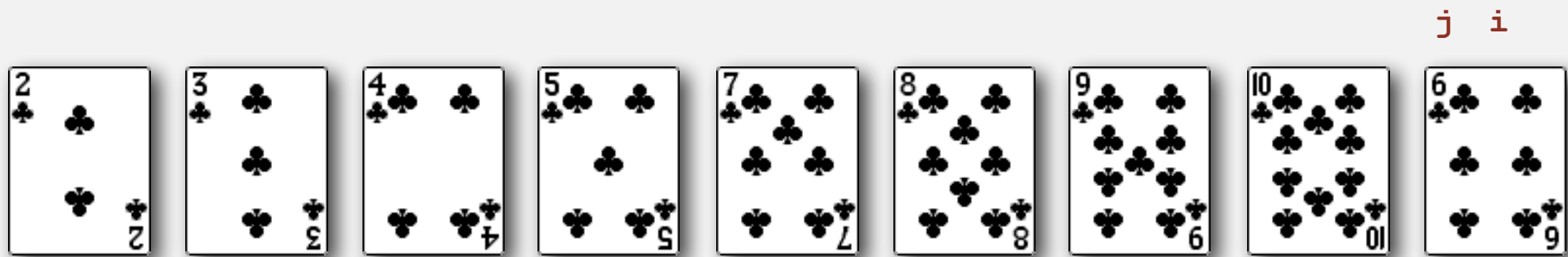
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



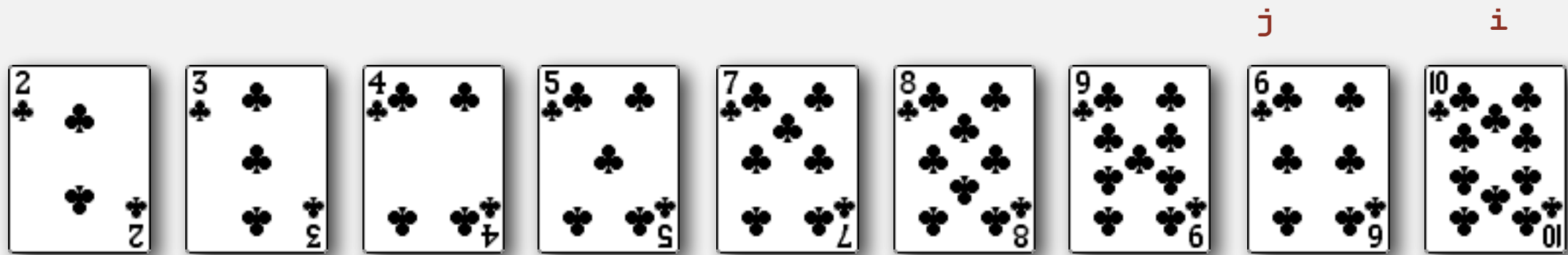
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



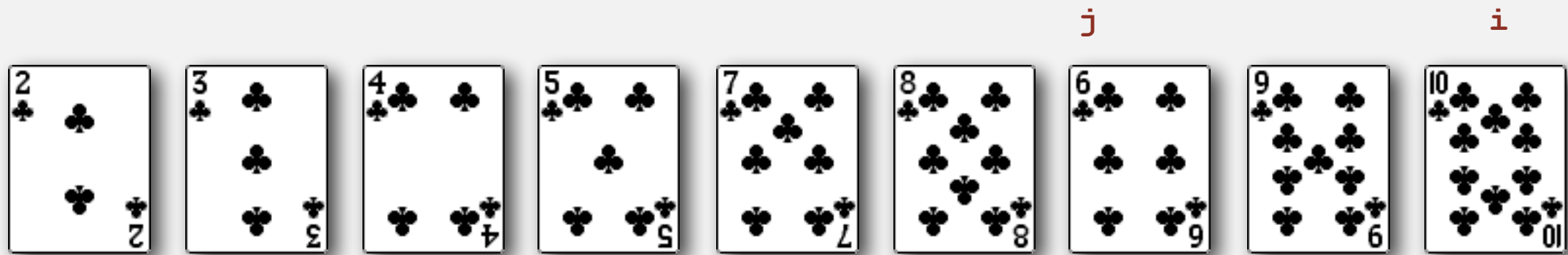
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



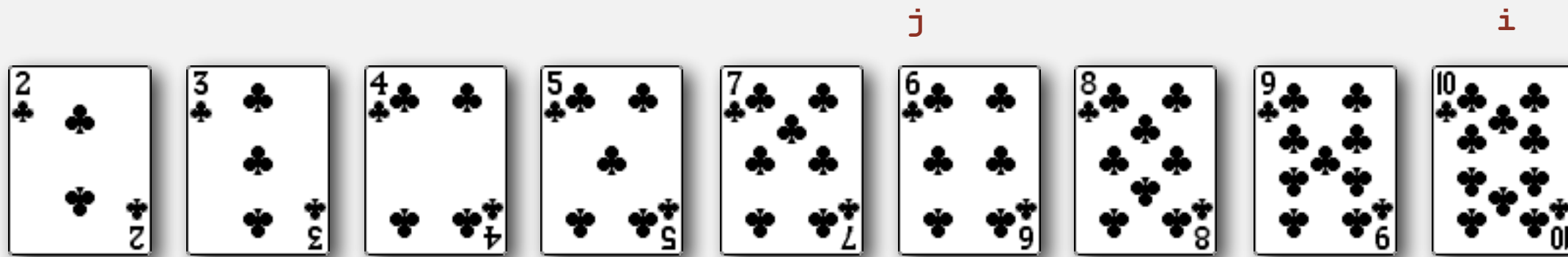
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



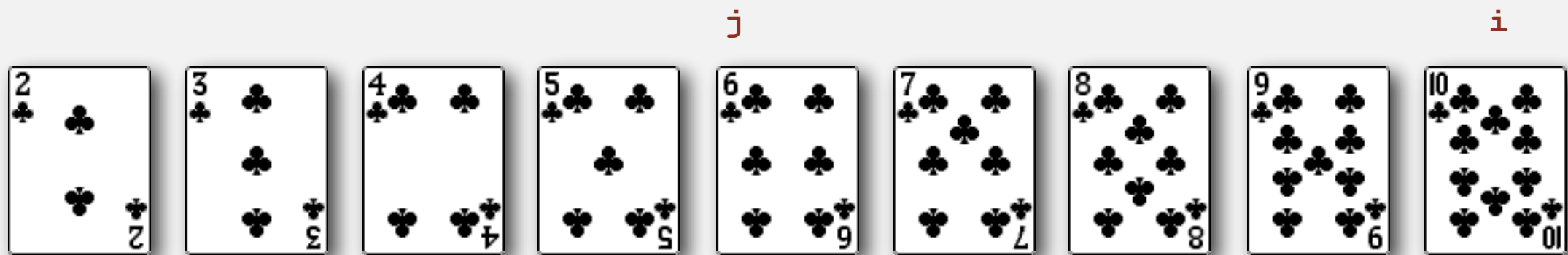
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



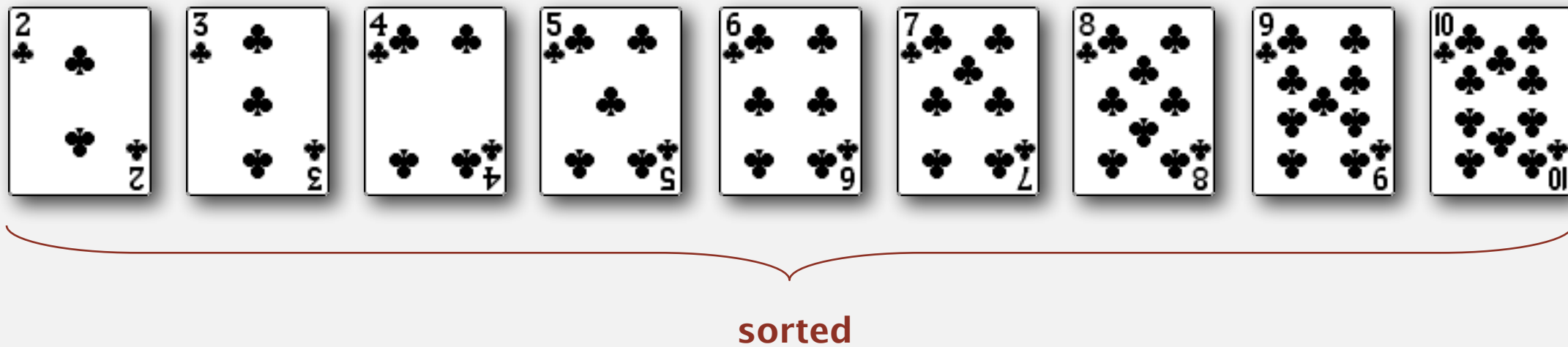
Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



Insertion sort

- In iteration i , swap $a[i]$ with each larger entry to its left.



Insertion sort: Java implementation

```
public class Insertion
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;
        for (int i = 0; i < N; i++)
            for (int j = i; j > 0; j--)
                if (less(a[j], a[j-1]))
                    exch(a, j, j-1);
                else break;
    }

    private static boolean less(Comparable v, Comparable w)
    { /* as before */ }

    private static void exch(Comparable[] a, int i, int j)
    { /* as before */ }
}
```

Insertion sort: mathematical analysis

Proposition. To sort a randomly-ordered array with distinct keys, insertion sort uses $\sim \frac{1}{4} N^2$ compares and $\sim \frac{1}{4} N^2$ exchanges on average.

Pf. Expect each entry to move halfway back.

		a[]										
i	j	0	1	2	3	4	5	6	7	8	9	10
		S	O	R	T	E	X	A	M	P	L	E
1	0	O	S	R	T	E	X	A	M	P	L	E
2	1	O	R	S	T	E	X	A	M	P	L	E
3	3	O	R	S	T	E	X	A	M	P	L	E
4	0	E	O	R	S	T	X	A	M	P	L	E
5	5	E	O	R	S	T	X	A	M	P	L	E
6	0	A	E	O	R	S	T	X	M	P	L	E
7	2	A	E	M	O	R	S	T	X	P	L	E
8	4	A	E	M	O	P	R	S	T	X	L	E
9	2	A	E	L	M	O	P	R	S	T	X	E
10	2	A	E	E	L	M	O	P	R	S	T	X
		A	E	E	L	M	O	P	R	S	T	X

Trace of insertion sort (array contents just after each insertion)

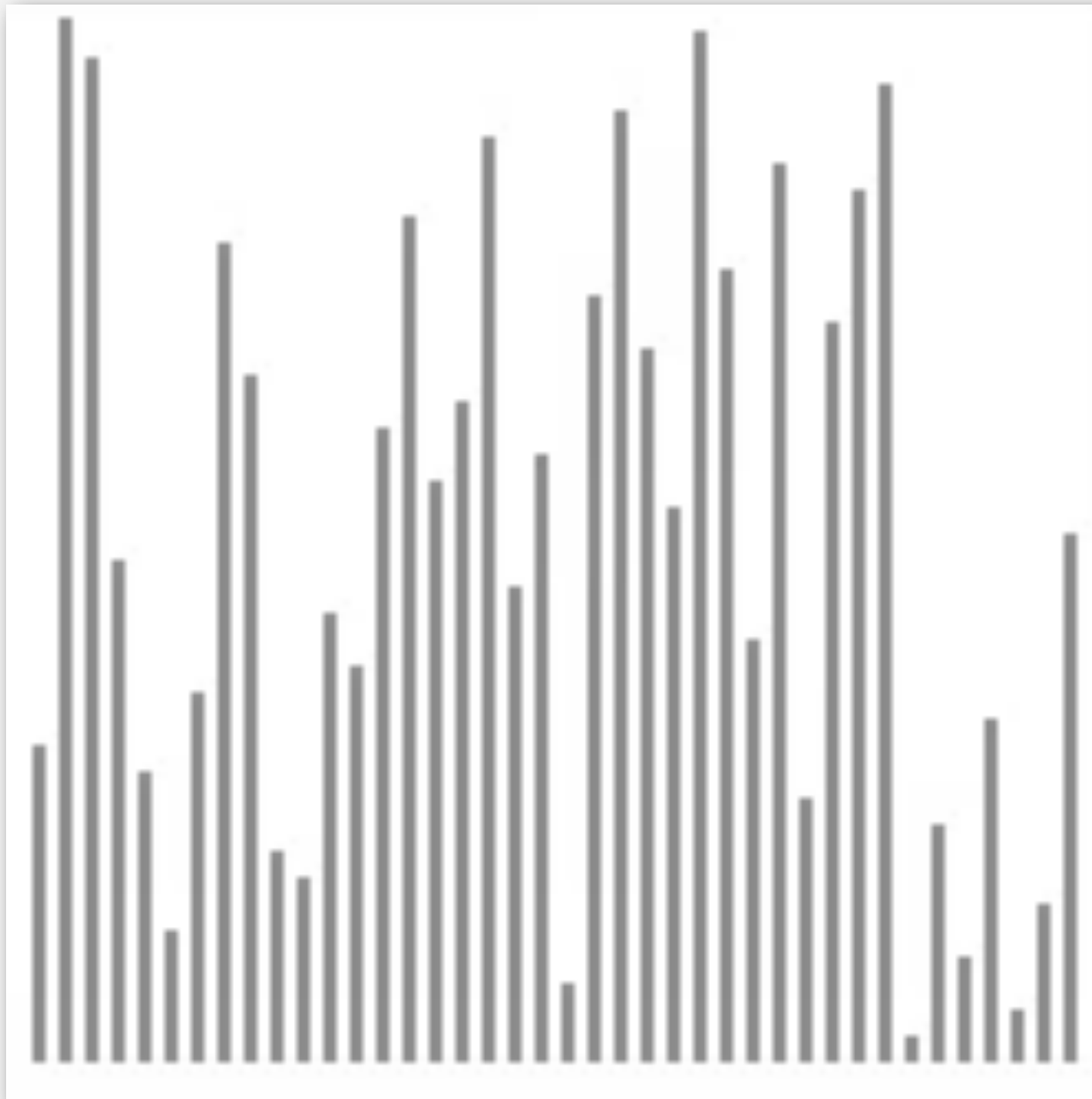
entries in gray do not move

entry in red is a[j]

entries in black moved one position right for insertion

Insertion sort: animation

40 random items



▲ algorithm position
█ in order
█ not yet seen

<http://www.sorting-algorithms.com/insertion-sort>

Insertion sort: best and worst case

Best case. If the array is in ascending order, insertion sort makes $N-1$ compares and 0 exchanges.

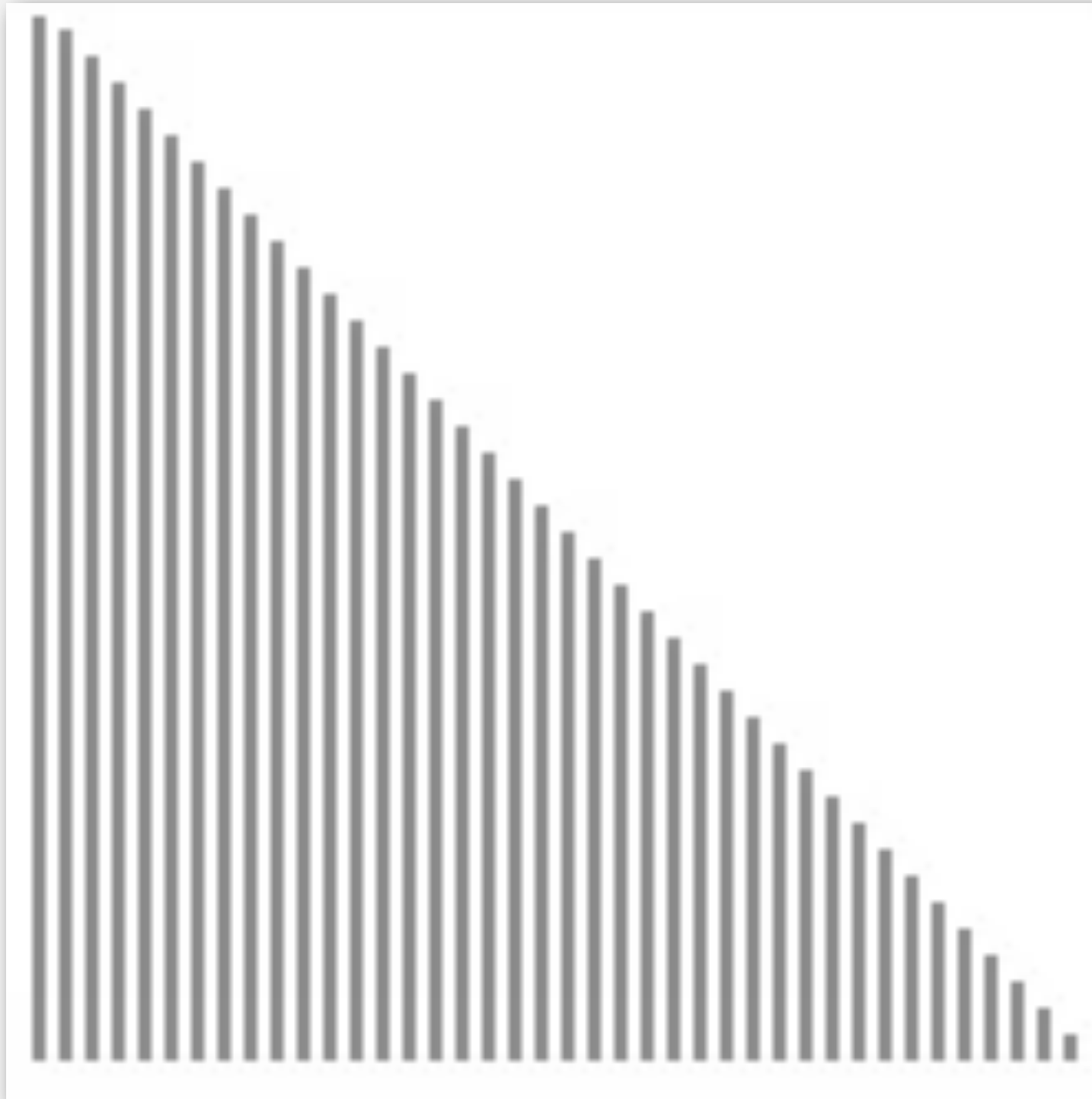
A E E L M O P R S T X

Worst case. If the array is in descending order (and no duplicates), insertion sort makes $\sim \frac{1}{2} N^2$ compares and $\sim \frac{1}{2} N^2$ exchanges.

X T S R P O M L E E A

Insertion sort: animation

40 reverse-sorted items



▲ algorithm position
■ in order
■ not yet seen

<http://www.sorting-algorithms.com/insertion-sort>

Insertion sort: partially-sorted arrays

Def. An **inversion** is a pair of keys that are out of order.

A E E L M O T R X P S

T-R T-P T-S R-P X-P X-S

(6 inversions)

Def. An array is **partially sorted** if the number of inversions is $\leq cN$.

- Ex 1. A subarray of size 10 appended to a sorted subarray of size N .
- Ex 2. An array of size N with only 10 entries out of place.

Proposition. For partially-sorted arrays, insertion sort runs in linear time.

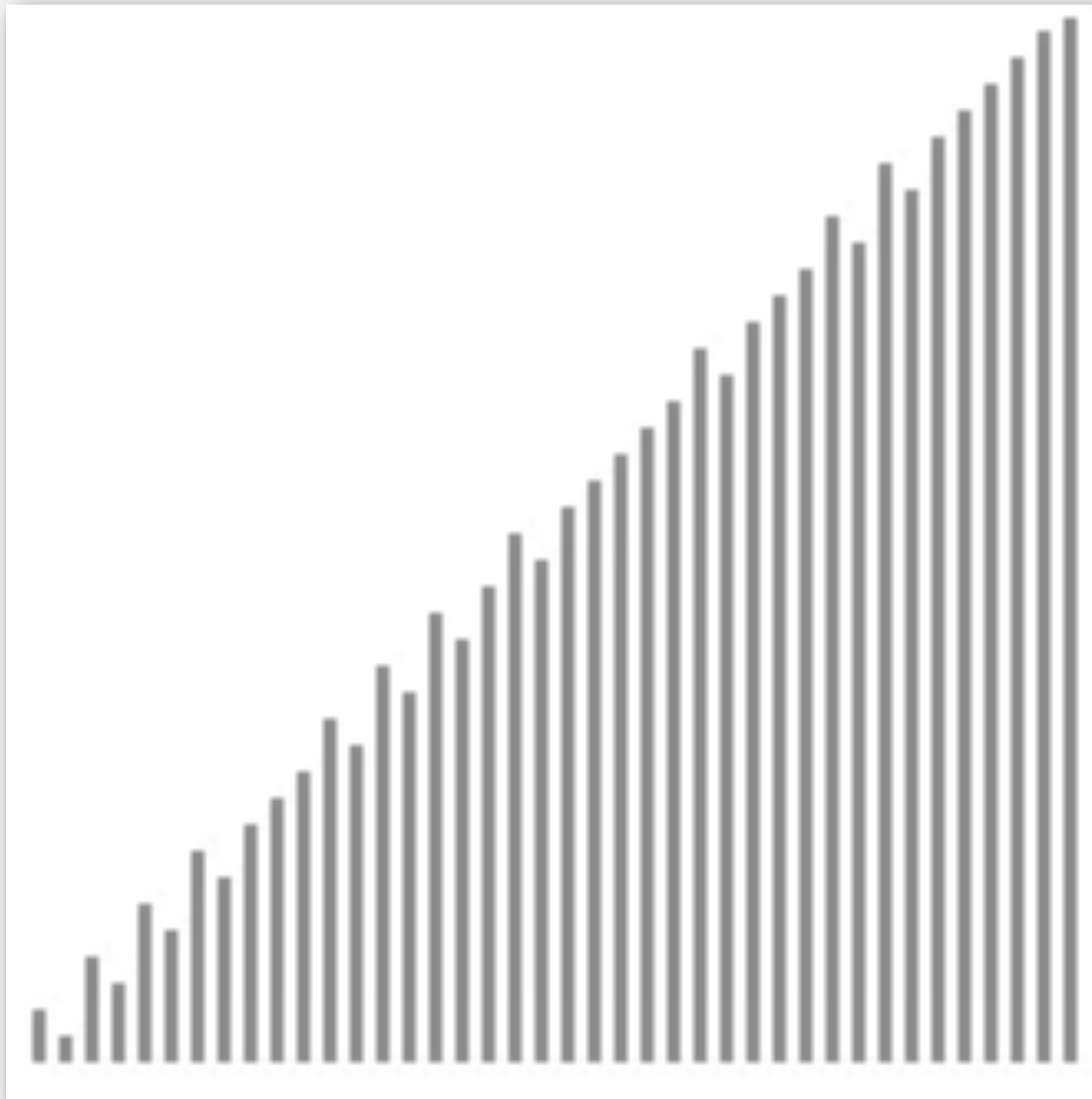
Pf. Number of exchanges equals the number of inversions.



number of compares = exchanges + $(N - 1)$

Insertion sort: animation

40 partially-sorted items



<http://www.sorting-algorithms.com/insertion-sort>

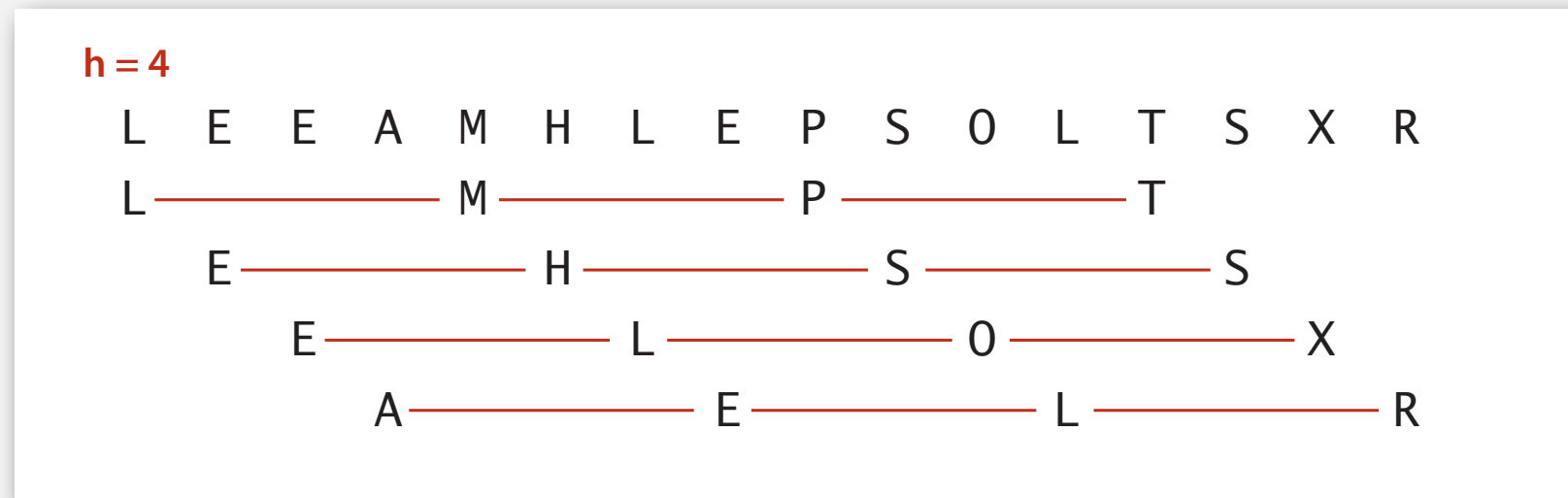
ELEMENTARY SORTING ALGORITHMS

- ▶ **Sorting review**
- ▶ Rules of the game
- ▶ Selection sort
- ▶ Insertion sort
- ▶ **Shellsort**

Shellsort overview

Idea. Move entries more than one position at a time by *h*-sorting the array.

an *h*-sorted array is *h* interleaved sorted subsequences



Shellsort. [Shell 1959] *h*-sort the array for decreasing seq. of values of *h*.

input	S	H	E	L	L	S	O	R	T	E	X	A	M	P	L	E
13-sort	P	H	E	L	L	S	O	R	T	E	X	A	M	S	L	E
4-sort	L	E	E	A	M	H	L	E	P	S	O	L	T	S	X	R
1-sort	A	E	E	E	H	L	L	L	M	O	P	R	S	S	T	X

h-sorting

How to h -sort an array? Insertion sort, with stride length h .

3-sorting an array

M	O	L	E	E	X	A	S	P	R	T
E	O	L	M	E	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T

Why insertion sort?

- Big increments \Rightarrow small subarray.
- Small increments \Rightarrow nearly in order. [stay tuned]

Shellsort example: increments 7, 3, 1

input

S O R T E X A M P L E

7-sort

S	O	R	T	E	X	A	M	P	L	E
M	O	R	T	E	X	A	S	P	L	E
M	O	R	T	E	X	A	S	P	L	E
M	O	L	T	E	X	A	S	P	R	E
M	O	L	E	E	X	A	S	P	R	T

3-sort

M	O	L	E	E	X	A	S	P	R	T
E	O	L	M	E	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T

1-sort

A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	E	L	O	P	M	S	X	R	T
A	E	E	L	O	P	M	S	X	R	T
A	E	E	L	M	O	P	S	X	R	T
A	E	E	L	M	O	P	S	X	R	T
A	E	E	L	M	O	P	R	S	X	T
A	E	E	L	M	O	P	R	S	T	X

result

A E E L M O P R S T X

Shellsort: intuition

Proposition. A g -sorted array remains g -sorted after h -sorting it.

7-sort

M	O	R	T	E	X	A	S	P	L	E
M	O	R	T	E	X	A	S	P	L	E
M	O	L	T	E	X	A	S	P	R	E
M	O	L	E	E	X	A	S	P	R	T
M	O	L	E	E	X	A	S	P	R	T

3-sort

M	O	L	E	E	X	A	S	P	R	T
E	O	L	M	E	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T

still 7-sorted

Challenge. Prove this fact—it's more subtle than you'd think!

Shellsort: which increment sequence to use?

Powers of two. 1, 2, 4, 8, 16, 32, ...

No.

Powers of two minus one. 1, 3, 7, 15, 31, 63, ...

Maybe.

→ $3x + 1$. 1, 4, 13, 40, 121, 364, ...

OK. Easy to compute.

merging of $(9 \times 4^i) - (9 \times 2^i) + 1$ and $4^i - (3 \times 2^i) + 1$



Sedgewick. 1, 5, 19, 41, 109, 209, 505, 929, 2161, 3905, ...

Good. Tough to beat in empirical studies.

=

Interested in learning more?

- See Section 6.8 of Algs, 3rd edition or Volume 3 of Knuth for details.
- Do a JP on the topic.

Shellsort: Java implementation

```
public class Shell
{
    public static void sort(Comparable[] a)
    {
        int N = a.length;

        int h = 1;
        while (h < N/3) h = 3*h + 1; // 1, 4, 13, 40, 121, 364, 1093, ...

        while (h >= 1)
        { // h-sort the array.
            for (int i = h; i < N; i++)
            {
                for (int j = i; j >= h && less(a[j], a[j-h]); j -= h)
                    exch(a, j, j-h);
            }

            h = h/3;
        }
    }

    private static boolean less(Comparable v, Comparable w)
    { /* as before */ }
    private static boolean void(Comparable[] a, int i, int j)
    { /* as before */ }
}
```

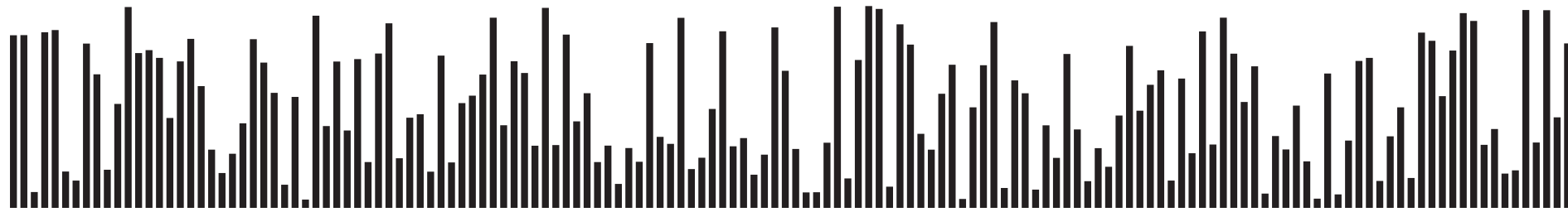
3x+1 increment
sequence

insertion sort

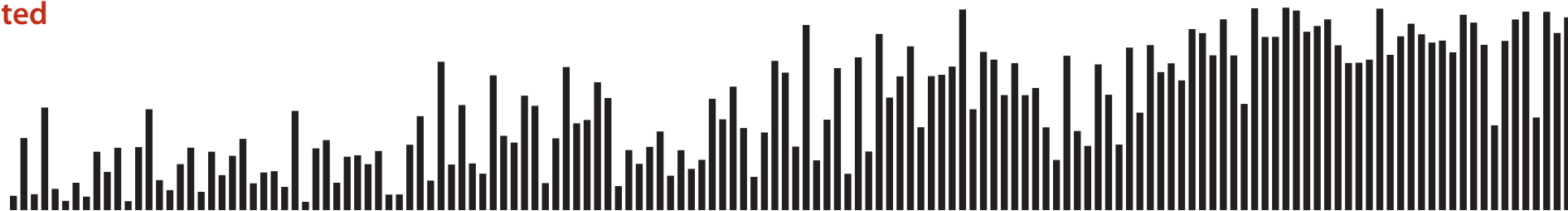
move to next
increment

Shellsort: visual trace

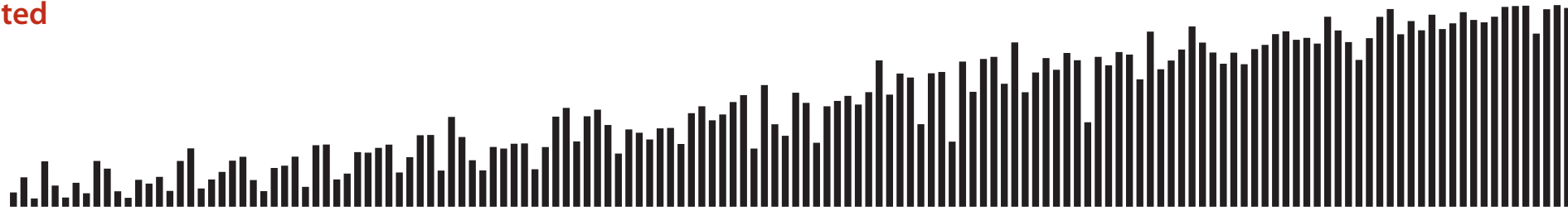
input



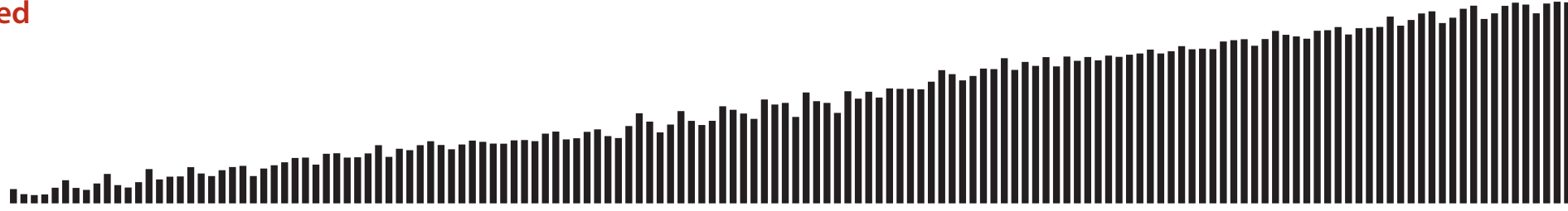
40-sorted



13-sorted



4-sorted

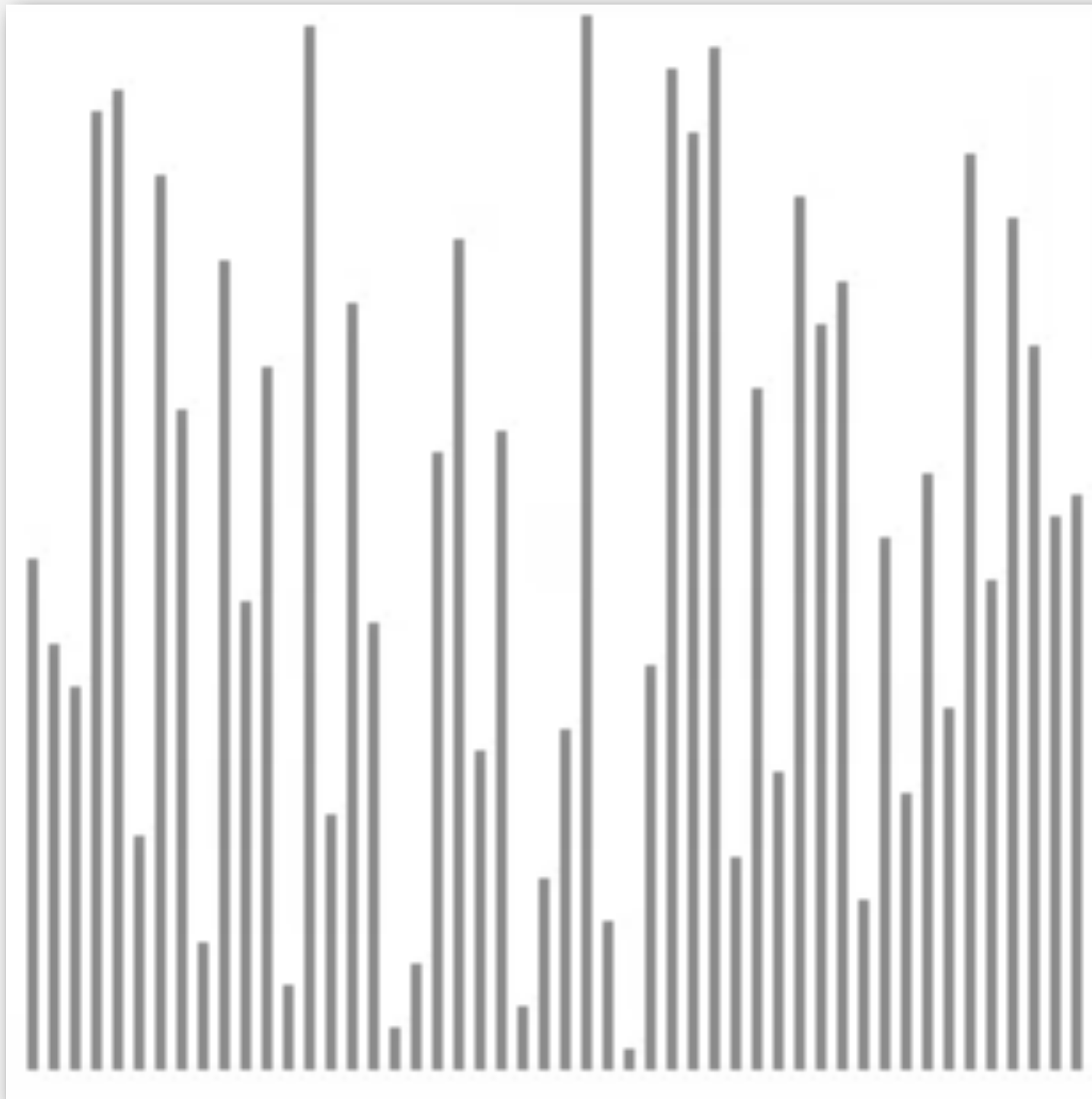


result


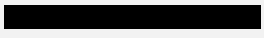
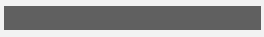
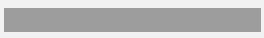


Shellsort: animation

50 random items

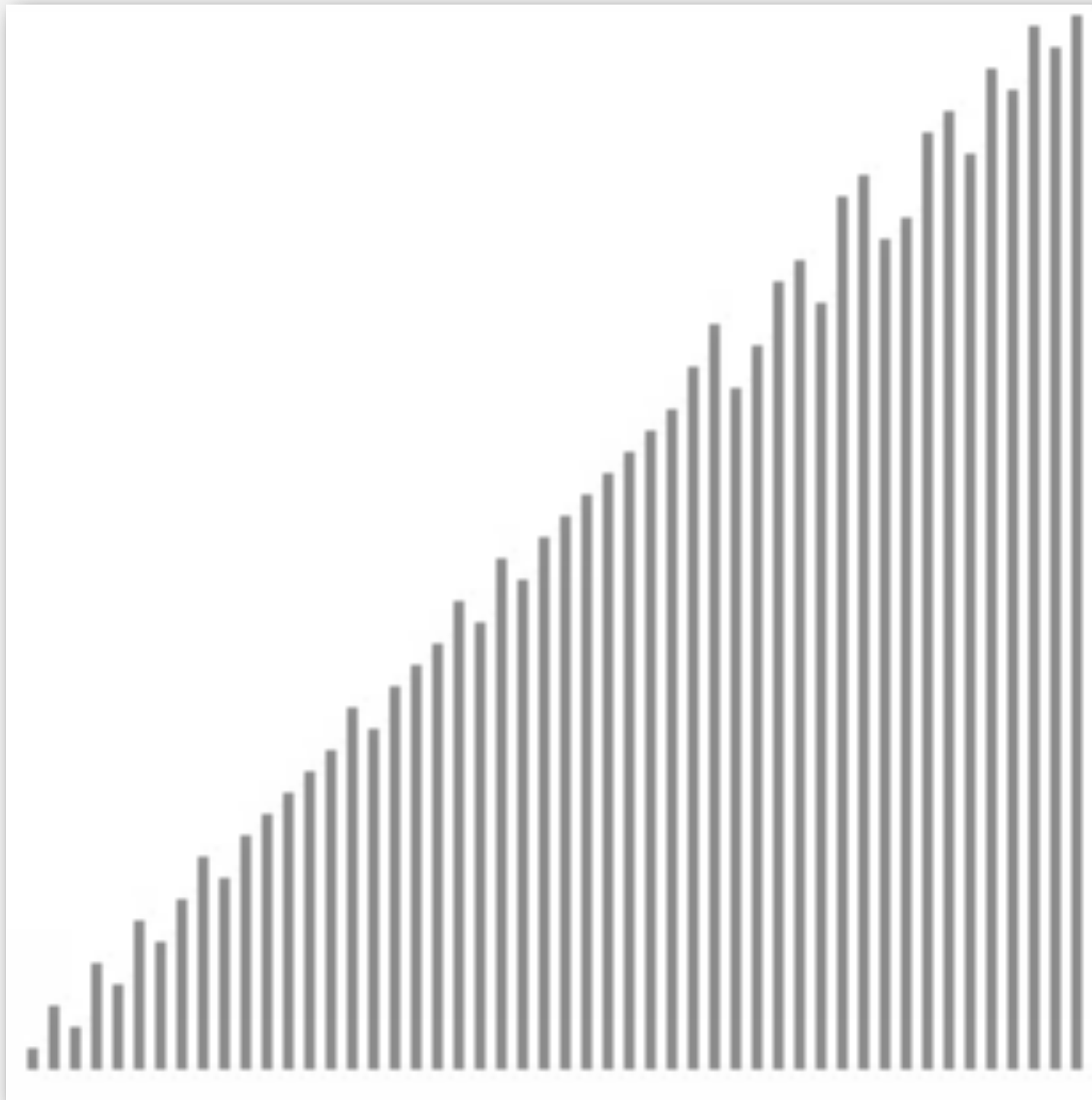


<http://www.sorting-algorithms.com/shell-sort>

-  algorithm position
-  h-sorted
-  current subsequence
-  other elements

Shellsort: animation

50 partially-sorted items



<http://www.sorting-algorithms.com/shell-sort>

- ▲ algorithm position
- h-sorted
- current subsequence
- other elements

Shellsort: analysis

Proposition. The worst-case number of compares used by shellsort with the $3x+1$ increments is $O(N^{3/2})$.

Property. The number of compares used by shellsort with the $3x+1$ increments is at most by a small multiple of N times the # of increments used.

N	compares	$N^{1.289}$	$2.5 N \lg N$
5.000	93	58	106
10.000	209	143	230
20.000	467	349	495
40.000	1022	855	1059
80.000	2266	2089	2257

measured in thousands

Remark. Accurate model has not yet been discovered (!)

Why are we interested in shellsort?

Example of simple idea leading to substantial performance gains.

Useful in practice.

- Fast unless array size is huge.
- Tiny, fixed footprint for code (used in embedded systems).
- Hardware sort prototype.

Simple algorithm, nontrivial performance, interesting questions.

- Asymptotic growth rate?
- Best sequence of increments? ← open problem: find a better increment sequence
- Average-case performance?

Lesson. Some good algorithms are still waiting discovery.