Acknowledgement: The course slides are adapted from the slides prepared by R. Sedgewick and K. Wayne of Princeton University.
TODAY

- Undirected Graphs
- Graph API
- Depth-first search
- Breadth-first search
- Connected components
- Challenges
Graph. Set of vertices connected pairwise by edges.

Why study graph algorithms?

- Thousands of practical applications.
- Hundreds of graph algorithms known.
- Interesting and broadly useful abstraction.
- Challenging branch of computer science and discrete math.
Graph applications

<table>
<thead>
<tr>
<th>graph</th>
<th>vertex</th>
<th>edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>communication</td>
<td>telephone, computer</td>
<td>fiber optic cable</td>
</tr>
<tr>
<td>circuit</td>
<td>gate, register, processor</td>
<td>wire</td>
</tr>
<tr>
<td>mechanical</td>
<td>joint</td>
<td>rod, beam, spring</td>
</tr>
<tr>
<td>financial</td>
<td>stock, currency</td>
<td>transactions</td>
</tr>
<tr>
<td>transportation</td>
<td>street intersection, airport</td>
<td>highway, airway route</td>
</tr>
<tr>
<td>internet</td>
<td>class C network</td>
<td>connection</td>
</tr>
<tr>
<td>game</td>
<td>board position</td>
<td>legal move</td>
</tr>
<tr>
<td>social relationship</td>
<td>person, actor</td>
<td>friendship, movie cast</td>
</tr>
<tr>
<td>neural network</td>
<td>neuron</td>
<td>synapse</td>
</tr>
<tr>
<td>protein network</td>
<td>protein</td>
<td>protein-protein interaction</td>
</tr>
<tr>
<td>chemical compound</td>
<td>molecule</td>
<td>bond</td>
</tr>
</tbody>
</table>
**Graph terminology**

**Path.** Sequence of vertices connected by edges.

**Cycle.** Path whose first and last vertices are the same.

Two vertices are *connected* if there is a path between them.
Some graph-processing problems

Path. Is there a path between $s$ and $t$?
Shortest path. What is the shortest path between $s$ and $t$?

Cycle. Is there a cycle in the graph?
Euler tour. Is there a cycle that uses each edge exactly once?
Hamilton tour. Is there a cycle that uses each vertex exactly once?

Connectivity. Is there a way to connect all of the vertices?
MST. What is the best way to connect all of the vertices?
Biconnectivity. Is there a vertex whose removal disconnects the graph?

Planarity. Can you draw the graph in the plane with no crossing edges?
Graph isomorphism. Do two adjacency lists represent the same graph?

Challenge. Which of these problems are easy? difficult? intractable?
Undirected Graphs

- Graph API
- Depth-first search
- Breadth-first search
- Connected components
- Challenges
Graph representation

Graph drawing. Provides intuition about the structure of the graph.

two drawings of the same graph

Caveat. Intuition can be misleading.
Vertex representation.

- This lecture: use integers between 0 and $V - 1$.
- Applications: convert between names and integers with symbol table.

Anomalies.

Graph representation
# Graph API

```java
text
```

```java
In in = new In(args[0]);
Graph G = new Graph(in);
for (int v = 0; v < G.V(); v++)
    for (int w : G.adj(v))
        StdOut.println(v + "-" + w);
```

- **public class** `Graph`:
  - `Graph(int V)`: create an empty graph with `V` vertices
  - `Graph(In in)`: create a graph from input stream
  - `void addEdge(int v, int w)`: add an edge `v-w`
  - `Iterable<Integer> adj(int v)`: vertices adjacent to `v`
  - `int V()`: number of vertices
  - `int E()`: number of edges
  - `String toString()`: string representation

- `read graph from input stream`
- `print out each edge (twice)`
Graph API: sample client

Graph input format.

```java
In in = new In(args[0]);
Graph G = new Graph(in);
for (int v = 0; v < G.V(); v++)
    for (int w : G.adj(v))
        StdOut.println(v + "-" + w);
```

% java Test tinyG.txt
0-6
0-2
0-1
0-5
1-0
2-0
3-5
3-4
...
12-11
12-9
Typical graph-processing code

```java
public static int degree(Graph G, int v)
{
    int degree = 0;
    for (int w : G.adj(v)) degree++;
    return degree;
}

compute the degree of v

public static int maxDegree(Graph G)
{
    int max = 0;
    for (int v = 0; v < G.V(); v++)
        if (degree(G, v) > max)
            max = degree(G, v);
    return max;
}

compute maximum degree

public static double averageDegree(Graph G)
{
    return 2.0 * G.E() / G.V();
}

compute average degree

public static int numberOfSelfLoops(Graph G)
{
    int count = 0;
    for (int v = 0; v < G.V(); v++)
        for (int w : G.adj(v))
            if (v == w) count++;
    return count/2; // each edge counted twice
}

count self-loops
```
Maintain a list of the edges (linked list or array).
Maintain a two-dimensional $V$-by-$V$ boolean array; for each edge $v \rightarrow w$ in graph: $\text{adj}[v][w] = \text{adj}[w][v] = \text{true}$.
Adjacency-list graph representation

Maintain vertex-indexed array of lists.

Adjacency-lists representation (undirected graph)

Bag objects

representations of the same edge
Adjacency-list graph representation: Java implementation

```java
public class Graph {
    private final int V;
    private Bag<Integer>[] adj;

    public Graph(int V) {
        this.V = V;
        adj = (Bag<Integer>[]) new Bag[V];
        for (int v = 0; v < V; v++)
            adj[v] = new Bag<Integer>();
    }

    public void addEdge(int v, int w) {
        adj[v].add(w);
        adj[w].add(v);
    }

    public Iterable<Integer> adj(int v) {
        return adj[v];
    }
}
```
Graph representations

In practice. Use adjacency-lists representation.

• Algorithms based on iterating over vertices adjacent to \( v \).
• Real-world graphs tend to be sparse.
In practice. Use adjacency-lists representation.

- Algorithms based on iterating over vertices adjacent to \( v \).
- Real-world graphs tend to be sparse.

<table>
<thead>
<tr>
<th>representation</th>
<th>space</th>
<th>add edge</th>
<th>edge between ( v ) and ( w )?</th>
<th>iterate over vertices adjacent to ( v )?</th>
</tr>
</thead>
<tbody>
<tr>
<td>list of edges</td>
<td>( E )</td>
<td>1</td>
<td>( E )</td>
<td>( E )</td>
</tr>
<tr>
<td>adjacency matrix</td>
<td>( V^2 )</td>
<td>1 *</td>
<td>1</td>
<td>( V )</td>
</tr>
<tr>
<td>adjacency lists</td>
<td>( E + V )</td>
<td>1</td>
<td>degree(( v ))</td>
<td>degree(( v ))</td>
</tr>
</tbody>
</table>

* disallows parallel edges

huge number of vertices, small average vertex degree
Undirected Graphs

- Graph API
- Depth-first search
- Breadth-first search
- Connected components
- Challenges
Maze exploration

Maze graphs.
• Vertex = intersection.
• Edge = passage.

Goal. Explore every intersection in the maze.
Trémaux maze exploration

**Algorithm.**

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options.
Depth-first search

**Goal.** Systematically search through a graph.

**Idea.** Mimic maze exploration.

**DFS (to visit a vertex v)**

- Mark v as visited.
- Recursively visit all unmarked vertices w adjacent to v.

**Typical applications.**
- Find all vertices connected to a given source vertex.
- Find a path between two vertices.

**Design challenge.** How to implement?
Design pattern for graph processing

Design pattern. Decouple graph data type from graph processing.

- Create a `Graph` object.
- Pass the `Graph` to a graph-processing routine, e.g., `Paths`.
- Query the graph-processing routine for information.

```java
public class Paths {
    Paths(Graph G, int s) {
        // find paths in G from source s
    }

    boolean hasPathTo(int v) {
        // is there a path from s to v?
    }

    Iterable<Integer> pathTo(int v) {
        // path from s to v; null if no such path
    }
}

Paths paths = new Paths(G, s);
for (int v = 0; v < G.V(); v++)
    if (paths.hasPathTo(v))
        StdOut.println(v);
```

print all vertices connected to s
Depth-first search

To visit a vertex \( v \):

- Mark vertex \( v \) as visited.
- Recursively visit all unmarked vertices adjacent to \( v \).

Graph \( G \)
To visit a vertex \( v \):

- Mark vertex \( v \) as visited.
- Recursively visit all unmarked vertices adjacent to \( v \).
Depth-first search

Goal. Find all vertices connected to \( s \) (and a path).


Algorithm.

- Use recursion (ball of string).
- Mark each visited vertex (and keep track of edge taken to visit it).
- Return (retrace steps) when no unvisited options.

Data structures.

- `boolean[] marked` to mark visited vertices.
- `int[] edgeTo` to keep tree of paths.
  
  \((\text{edgeTo}[w] == v)\) means that edge \( v-w \) taken to visit \( w \) for first time
public class DepthFirstPaths
{
    private boolean[] marked;
    private int[] edgeTo;
    private int s;

    public DepthFirstSearch(Graph G, int s)
    {
        ...
        dfs(G, s);
    }

    private void dfs(Graph G, int v)
    {
        marked[v] = true;
        for (int w : G.adj(v))
        {
            if (!marked[w])
            {
                dfs(G, w);
                edgeTo[w] = v;
            }
        }
    }
}
Depth-first search properties

**Proposition.** DFS marks all vertices connected to \( s \) in time proportional to the sum of their degrees.

**Pf.**

- **Correctness:**
  - if \( w \) marked, then \( w \) connected to \( s \) (why?)
  - if \( w \) connected to \( s \), then \( w \) marked
    (if \( w \) unmarked, then consider last edge on a path from \( s \) to \( w \) that goes from a marked vertex to an unmarked one)

- **Running time:**
  Each vertex connected to \( s \) is visited once.

![Diagram of DFS properties](image)
Proposition. After DFS, can find vertices connected to \( s \) in constant time and can find a path to \( s \) (if one exists) in time proportional to its length.

\[ \text{Pf.} \quad \text{edgeTo[]} \] is a parent-link representation of a tree rooted at \( s \).

```java
public boolean hasPathTo(int v)
{  return marked[v];  }

public Iterable<Integer> pathTo(int v)
{  if (!hasPathTo(v)) return null;
    Stack<Integer> path = new Stack<Integer>();
    for (int x = v; x != s; x = edgeTo[x])
    { path.push(x);
      path.push(s);
    return path;
  }
```
UNDIRECTED GRAPHS

- Graph API
- Depth-first search
- **Breadth-first search**
- Connected components
- Challenges
Breadth-first search

Repeat until queue is empty:

- Remove vertex \( v \) from queue.
- Add to queue all unmarked vertices adjacent to \( v \) and mark them.
Breadth-first search

Repeat until queue is empty:

- Remove vertex $v$ from queue.
- Add to queue all unmarked vertices adjacent to $v$ and mark them.

```
<table>
<thead>
<tr>
<th>queue</th>
<th>v</th>
<th>edgeTo[v]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
```

add 0 to queue
Breadth-first search

Repeat until queue is empty:

• Remove vertex $v$ from queue.
• Add to queue all unmarked vertices adjacent to $v$ and mark them.

```
queue   v   edgeTo[v]
0       -
1       -
2       -
3       -
4       -
5       -
0
```

dequeue 0
Breadth-first search

Repeat until queue is empty:
• Remove vertex $v$ from queue.
• Add to queue all unmarked vertices adjacent to $v$ and mark them.

 dequeue 0
Repeat until queue is empty:

- Remove vertex $v$ from queue.
- Add to queue all unmarked vertices adjacent to $v$ and mark them.

**Breadth-first search**

<table>
<thead>
<tr>
<th>queue</th>
<th>$v$</th>
<th>edgeTo[$v$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

** dequeue 0**
Breadth-first search

Repeat until queue is empty:
• Remove vertex $v$ from queue.
• Add to queue all unmarked vertices adjacent to $v$ and mark them.
Breadth-first search

Repeat until queue is empty:

• Remove vertex $v$ from queue.
• Add to queue all unmarked vertices adjacent to $v$ and mark them.
Breadth-first search

Repeat until queue is empty:

- Remove vertex \( v \) from queue.
- Add to queue all unmarked vertices adjacent to \( v \) and mark them.
Repeat until queue is empty:

- Remove vertex \( v \) from queue.
- Add to queue all unmarked vertices adjacent to \( v \) and mark them.

**Breadth-first search**

dequeue 2
Breadth-first search

Repeat until queue is empty:

- Remove vertex $v$ from queue.
- Add to queue all unmarked vertices adjacent to $v$ and mark them.

**Queue:**

```
<table>
<thead>
<tr>
<th>queue</th>
<th>v</th>
<th>edgeTo[v]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

dequeue 2
Breadth-first search

Repeat until queue is empty:

• Remove vertex $v$ from queue.
• Add to queue all unmarked vertices adjacent to $v$ and mark them.

Breadth-first search

dequeue 2
Breadth-first search

Repeat until queue is empty:
• Remove vertex $v$ from queue.
• Add to queue all unmarked vertices adjacent to $v$ and mark them.

<table>
<thead>
<tr>
<th>queue</th>
<th>$v$</th>
<th>edgeTo[$v$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>
Repeat until queue is empty:
• Remove vertex $v$ from queue.
• Add to queue all unmarked vertices adjacent to $v$ and mark them.
Breadth-first search

Repeat until queue is empty:
• Remove vertex $v$ from queue.
• Add to queue all unmarked vertices adjacent to $v$ and mark them.

\[
\begin{array}{c|c|c}
\text{queue} & v & \text{edgeTo}[v] \\
\hline
0 & - \\
1 & 0 \\
2 & 0 \\
3 & 2 \\
4 & 2 \\
5 & 0 \\
\end{array}
\]
Breadth-first search

Repeat until queue is empty:
• Remove vertex \( v \) from queue.
• Add to queue all unmarked vertices adjacent to \( v \) and mark them.

dequeue 1
Breadth-first search

Repeat until queue is empty:

• Remove vertex $v$ from queue.
• Add to queue all unmarked vertices adjacent to $v$ and mark them.

dequeue 1
Repeat until queue is empty:

- Remove vertex $v$ from queue.
- Add to queue all unmarked vertices adjacent to $v$ and mark them.
Breadth-first search

Repeat until queue is empty:
- Remove vertex \( v \) from queue.
- Add to queue all unmarked vertices adjacent to \( v \) and mark them.

![Graph with Breadth-first search](image)

<table>
<thead>
<tr>
<th>queue</th>
<th>( v )</th>
<th>edgeTo[( v )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

decqueue 5
Breadth-first search

Repeat until queue is empty:

• Remove vertex $v$ from queue.
• Add to queue all unmarked vertices adjacent to $v$ and mark them.
Repeat until queue is empty:

- Remove vertex $v$ from queue.
- Add to queue all unmarked vertices adjacent to $v$ and mark them.

**queue**

<table>
<thead>
<tr>
<th>queue</th>
<th>v</th>
<th>edgeTo[v]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**dequeue 5**
Breadth-first search

Repeat until queue is empty:

- Remove vertex \( v \) from queue.
- Add to queue all unmarked vertices adjacent to \( v \) and mark them.
Repeat until queue is empty:
- Remove vertex $v$ from queue.
- Add to queue all unmarked vertices adjacent to $v$ and mark them.

**Queue and edgeTo array**
- **Queue**: 3, 4, 5, 1, 2
- **edgeTo**:
  - 0: -
  - 1: 0
  - 2: 0
  - 3: 2
  - 4: 2
  - 5: 0

**Dequeue 3**
Repeat until queue is empty:

- Remove vertex $v$ from queue.
- Add to queue all unmarked vertices adjacent to $v$ and mark them.

**Breadth-first search**

dequeue 3
Repeat until queue is empty:

- Remove vertex $v$ from queue.
- Add to queue all unmarked vertices adjacent to $v$ and mark them.
Repeat until queue is empty:
• Remove vertex $v$ from queue.
• Add to queue all unmarked vertices adjacent to $v$ and mark them.

Breadth-first search

dequeue 3
Repeat until queue is empty:

- Remove vertex $v$ from queue.
- Add to queue all unmarked vertices adjacent to $v$ and mark them.
Breadth-first search

Repeat until queue is empty:

- Remove vertex \( v \) from queue.
- Add to queue all unmarked vertices adjacent to \( v \) and mark them.

dequeue 4
Repeat until queue is empty:
- Remove vertex \( v \) from queue.
- Add to queue all unmarked vertices adjacent to \( v \) and mark them.

**Breadth-first search**

**queue**

\[
\begin{array}{l|l}
\text{v} & \text{edgeTo[v]} \\
\hline
0 & - \\
1 & 0 \\
2 & 0 \\
3 & 2 \\
4 & 2 \\
5 & 0 \\
\end{array}
\]

**dequeue 4**
Breadth-first search

Repeat until queue is empty:
• Remove vertex \( v \) from queue.
• Add to queue all unmarked vertices adjacent to \( v \) and mark them.
Breadth-first search

Repeat until queue is empty:

- Remove vertex \( v \) from queue.
- Add to queue all unmarked vertices adjacent to \( v \) and mark them.
Repeat until queue is empty:
- Remove vertex $v$ from queue.
- Add to queue all unmarked vertices adjacent to $v$ and mark them.

**Breadth-first search**

```
<table>
<thead>
<tr>
<th></th>
<th>edgeTo[v]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>
```

**done**
**Breadth-first search**

**Depth-first search.** Put unvisited vertices on a **stack**.

**Breadth-first search.** Put unvisited vertices on a **queue**.

**Shortest path.** Find path from $s$ to $t$ that uses **fewest number of edges**.

---

**BFS** *(from source vertex $s$)*

---

Put $s$ onto a FIFO queue, and mark $s$ as visited.

Repeat until the queue is empty:
- remove the least recently added vertex $v$
- add each of $v$'s unvisited neighbors to the queue, and mark them as visited.

---

**Intuition.** BFS examines vertices in increasing distance from $s$.  

**Proposition.** BFS computes shortest path (number of edges) from $s$ in a connected graph in time proportional to $E + V$.

**Pf. [correctness]** Queue always consists of zero or more vertices of distance $k$ from $s$, followed by zero or more vertices of distance $k + 1$.

**Pf. [running time]** Each vertex connected to $s$ is visited once.
Breadth-first search

```java
public class BreadthFirstPaths {
    private boolean[] marked;
    private boolean[] edgeTo[];
    private final int s;
    ...

    private void bfs(Graph G, int s) {
        Queue<Integer> q = new Queue<Integer>();
        q.enqueue(s);
        marked[s] = true;
        while (!q.isEmpty()) {
            int v = q.dequeue();
            for (int w : G.adj(v)) {
                if (!marked[w]) {
                    q.enqueue(w);
                    marked[w] = true;
                    edgeTo[w] = v;
                }
            }
        }
    }
}
```
UNDIRECTED GRAPHS

- Graph API
- Depth-first search
- Breadth-first search
- Connected components
- Challenges
Connectivity queries

**Def.** Vertices $v$ and $w$ are **connected** if there is a path between them.

**Goal.** Preprocess graph to answer queries: is $v$ connected to $w$? in **constant** time.

```java
public class CC{
    CC(Graph G) {
        find connected components in $G$
    }
    boolean connected(int v, int w) {
        are $v$ and $w$ connected?
    }
    int count() {
        number of connected components
    }
    int id(int v) {
        component identifier for $v$
    }
}
```

**Depth-first search.** [next few slides]
The relation "is connected to" is an equivalence relation:

- Reflexive: $v$ is connected to $v$.
- Symmetric: if $v$ is connected to $w$, then $w$ is connected to $v$.
- Transitive: if $v$ connected to $w$ and $w$ connected to $x$, then $v$ connected to $x$.

**Def.** A **connected component** is a maximal set of connected vertices.

**Remark.** Given connected components, can answer queries in constant time.
**Connected components**

**Def.** A connected component is a maximal set of connected vertices.

![Diagram showing 63 connected components](image)
**Goal.** Partition vertices into connected components.

- Initialize all vertices $v$ as unmarked.
- For each unmarked vertex $v$, run DFS to identify all vertices discovered as part of the same component.
To visit a vertex $v$:

- Mark vertex $v$ as visited.
- Recursively visit all unmarked vertices adjacent to $v$.

Connected components

graph $G$
To visit a vertex $v$:

- Mark vertex $v$ as visited.
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---

**Connected components**

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Visit 6
To visit a vertex \( v \):

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3 & T & 0 \\
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\end{array}\]
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Connected components

To visit a vertex \( \nu \):
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visit 4
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4 done
Connected components

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visit 2
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Connected components

2 done
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Connected components

To visit a vertex \( v \):

- Mark vertex \( v \) as visited.
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1 done
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![Graph diagram]

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visit 7
To visit a vertex $v$:

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Connected components

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To visit a vertex $v$:

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### Connected components

7 done

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Connected components

connected component: 7 8
Connected components

To visit a vertex $v$:

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**Connected components**

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### Connected Components Table

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Visit 11
**Connected components**

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Connected components

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visit 12
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12 done
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11 done
Connected components

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- Recursively visit all unmarked vertices adjacent to $v$.

Visit 9
To visit a vertex \( v \):  
- Mark vertex \( v \) as visited.  
- Recursively visit all unmarked vertices adjacent to \( v \).
To visit a vertex $v$:

- Mark vertex $v$ as visited.
- Recursively visit all unmarked vertices adjacent to $v$. 

### Connected components

<table>
<thead>
<tr>
<th>$v$</th>
<th>marked[]</th>
<th>cc[]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>T</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>T</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>4</td>
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<td>5</td>
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<tr>
<td>7</td>
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<tr>
<td>8</td>
<td>T</td>
<td>1</td>
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<tr>
<td>10</td>
<td>T</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>T</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>T</td>
<td>2</td>
</tr>
</tbody>
</table>

10 done
Connected components

To visit a vertex $v$:

- Mark vertex $v$ as visited.
- Recursively visit all unmarked vertices adjacent to $v$.

<table>
<thead>
<tr>
<th></th>
<th>marked[]</th>
<th>cc[]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>T</td>
<td>0</td>
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<tr>
<td>1</td>
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<tr>
<td>12</td>
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</tr>
</tbody>
</table>

9 done
To visit a vertex $v$:

- Mark vertex $v$ as visited.
- Recursively visit all unmarked vertices adjacent to $v$. 

Connected components

connected component: 9 10 11 12
Connected components

To visit a vertex $v$:
- Mark vertex $v$ as visited.
- Recursively visit all unmarked vertices adjacent to $v$.

check 10 11 12
To visit a vertex $v$:

- Mark vertex $v$ as visited.
- Recursively visit all unmarked vertices adjacent to $v$. 

---

**Connected components**

---

### Table: Connected Components

<table>
<thead>
<tr>
<th>Vertex</th>
<th>marked[]</th>
<th>cc[]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>T</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
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<td>0</td>
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<td>2</td>
</tr>
<tr>
<td>12</td>
<td>T</td>
<td>2</td>
</tr>
</tbody>
</table>
Finding connected components with DFS

```java
public class CC {
    private boolean marked[];
    private int[] id;
    private int count;

    public CC(Graph G) {
        marked = new boolean[G.V()];
        id = new int[G.V()];
        for (int v = 0; v < G.V(); v++) {
            if (!marked[v]) {
                dfs(G, v);
                count++;
            }
        }
    }

    public int count() {
        return count;
    }

    public int id(int v) {
        return id[v];
    }

    private void dfs(Graph G, int v) {
        // DFS implementation...
    }
}
```

- `id[v] = id` of component containing `v`
- Number of components
- Run DFS from one vertex in each component
- See next slide
Finding connected components with DFS (continued)

```java
public int count()
{  return count;  }

public int id(int v)
{  return id[v];  }

private void dfs(Graph G, int v)
{
    marked[v] = true;
    id[v] = count;
    for (int w : G.adj(v))
        if (!marked[w])
            dfs(G, w);
}
```

- number of components
- id of component containing v
- all vertices discovered in same call of dfs have same id
Undirected Graphs

- Graph API
- Depth-first search
- Breadth-first search
- Connected components
- Challenges
Problem. Is a graph bipartite?

How difficult?
- Any programmer could do it.
- Typical diligent algorithms student could do it.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.
Graph-processing challenge 1

Problem. Is a graph bipartite?

How difficult?

- Any programmer could do it.
- Typical diligent algorithms student could do it.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.

✓ simple DFS-based solution (see textbook)
Problem. Find a cycle.

How difficult?
• Any programmer could do it.
• Typical diligent algorithms student could do it.
• Hire an expert.
• Intractable.
• No one knows.
• Impossible.
Graph-processing challenge 2

Problem. Find a cycle.

How difficult?
- Any programmer could do it.
- Typical diligent algorithms student could do it.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.

simple DFS-based solution (see textbook)
The Seven Bridges of Königsberg. [Leonhard Euler 1736]

“… in Königsberg in Prussia, there is an island A, called the Kneiphof; the river which surrounds it is divided into two branches ... and these branches are crossed by seven bridges. Concerning these bridges, it was asked whether anyone could arrange a route in such a way that he could cross each bridge once and only once.”

Euler tour. Is there a (general) cycle that uses each edge exactly once? Answer. Yes iff connected and all vertices have even degree. To find path. DFS-based algorithm (see textbook).
**Problem.** Find a cycle that uses every edge.

**Assumption.** Need to use each edge exactly once.

**How difficult?**
- Any programmer could do it.
- Typical diligent algorithms student could do it.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.
Graph-processing challenge 3

Problem. Find a cycle that uses every edge.
Assumption. Need to use each edge exactly once.

How difficult?
- Any programmer could do it.
- Typical diligent algorithms student could do it.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.

Eulerian tour
(classic graph-processing problem)
Graph-processing challenge 4

Problem. Find a cycle that visits every vertex exactly once.

How difficult?
• Any programmer could do it.
• Typical diligent algorithms student could do it.
• Hire an expert.
• Intractable.
• No one knows.
• Impossible.
Problem. Find a cycle that visits every vertex.
Assumption. Need to visit each vertex exactly once.

How difficult?
• Any programmer could do it.
• Typical diligent algorithms student could do it.
• Hire an expert.
✓ • Intractable.
• No one knows.
• Impossible.

Hamiltonian cycle (classical NP-complete problem)
**Graph-processing challenge 5**

**Problem.** Are two graphs identical except for vertex names?

**How difficult?**
- Any programmer could do it.
- Typical diligent algorithms student could do it.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.
Problem. Are two graphs identical except for vertex names?

How difficult?
- Any programmer could do it.
- Typical diligent algorithms student could do it.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.

\[0\leftrightarrow4, \ 1\leftrightarrow3, \ 2\leftrightarrow2, \ 3\leftrightarrow6, \ 4\leftrightarrow5, \ 5\leftrightarrow0, \ 6\leftrightarrow1\]

Graph isomorphism is longstanding open problem.
**Problem.** Lay out a graph in the plane without crossing edges?

**How difficult?**
- Any programmer could do it.
- Typical diligent algorithms student could do it.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.
Graph-processing challenge 6

Problem. Lay out a graph in the plane without crossing edges?

How difficult?

- Any programmer could do it.
- Typical diligent algorithms student could do it.
- Hire an expert.
- Intractable.
- No one knows.
- Impossible.

✓

linear-time DFS-based planarity algorithm discovered by Tarjan in 1970s (too complicated for practitioners)