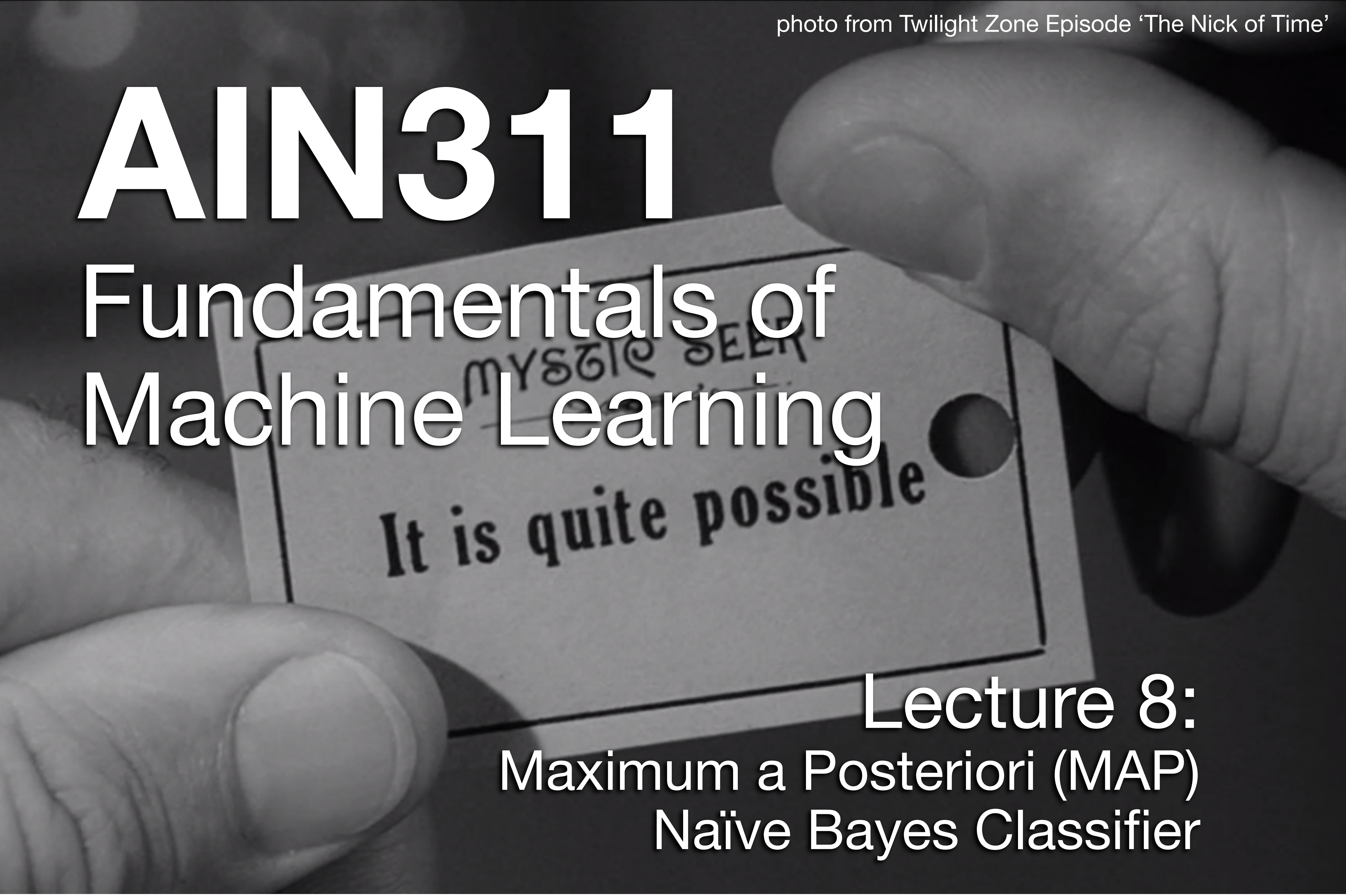
photo from Twilight Zone Episode ‘The Nick of Time’

AIN311

Fundamentals of Machine Learning



Lecture 8:

Maximum a Posteriori (MAP)
Naïve Bayes Classifier

HACETTEPE UNIVERSITY COMPUTER VISION LAB

Erkut Erdem // Hacettepe University // Fall 2023

Recap: MLE

- Maximum Likelihood estimation (MLE)

  Choose value that maximizes the probability of observed data

$$\widehat{\theta}_{MLE} = \arg\max_{\theta} P(D|\theta)$$

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Recap: MLE for Bernoulli distribution

Data, D =

$$D = \{X_i\}_{i=1}^n, \ X_i \in \{\mathrm{H}, \mathrm{T}\}$$

$P(Heads) = \theta, \ P(Tails) = 1-\theta$

Flips are **i.i.d.**:
- **Independent** events
  - **Identically distributed** according to Bernoulli distribution

MLE: Choose θ that maximizes the probability of observed data

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Recap: How many flips do I need?

I flipped the coins 5 times: 3 heads, 2 tails

$$\widehat{\theta}_{MLE} = \frac{3}{5}$$

What if I flipped 30 heads and 20 tails?

$$\widehat{\theta}_{MLE} = \frac{30}{50}$$

- **Which estimator should we trust more?**
- **The more the merrier???**

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Recap: Simple Bound

Let θ* be the true parameter.

For n = $\alpha_H + \alpha_T$, and $\widehat{\theta}_{MLE} = \frac{\alpha_H}{\alpha_H + \alpha_T}$

For any ε>0:

Hoeffding's inequality:

$$P(|\,\widehat{\theta} - \theta^*\,| \geq \epsilon) \;\leq\; 2e^{-2n\epsilon^2}$$

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Recap: MLE for Gaussian mean and variance

Choose θ= (μ,σ²) that maximizes the probability of observed data

$$\widehat{\theta}_{MLE} = \arg\max_{\theta} \; P(D \mid \theta)$$

$$= \arg\max_{\theta} \prod_{i=1}^{n} P(X_i|\theta)$$

Independent draws

$$= \arg\max_{\theta} \prod_{i=1}^{n} \frac{1}{2\sigma^2} e^{-(X_i-\mu)^2/2\sigma^2}$$

Identically distributed

$$= \arg \max_{\theta=(\mu,\sigma^2)} \underbrace{\frac{1}{2\sigma^2} e^{-\sum_{i=1}^{n}(X_i-\mu)^2/2\sigma^2}}_{J(\theta)}$$

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Recap: MLE for Gaussian mean and variance

$$\widehat{\mu}_{MLE} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\widehat{\sigma}^2_{MLE} = \frac{1}{n}\sum_{i=1}^{n} (x_i - \widehat{\mu})^2$$

Note: MLE for the variance of a Gaussian is **biased**

[Expected result of estimation is not the true parameter!]

Unbiased variance estimator: $\widehat{\sigma}^2_{unbiased} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \widehat{\mu})^2$

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Today

- Maximum a Posteriori (MAP)

- Bayes rule
  - Naïve Bayes Classifier

- Application
  - Text classification
  - “Mind reading” = fMRI data processing

What about prior knowledge? (MAP Estimation)

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What about prior knowledge?

We know the coin is "close" to 50-50. What can we do now?

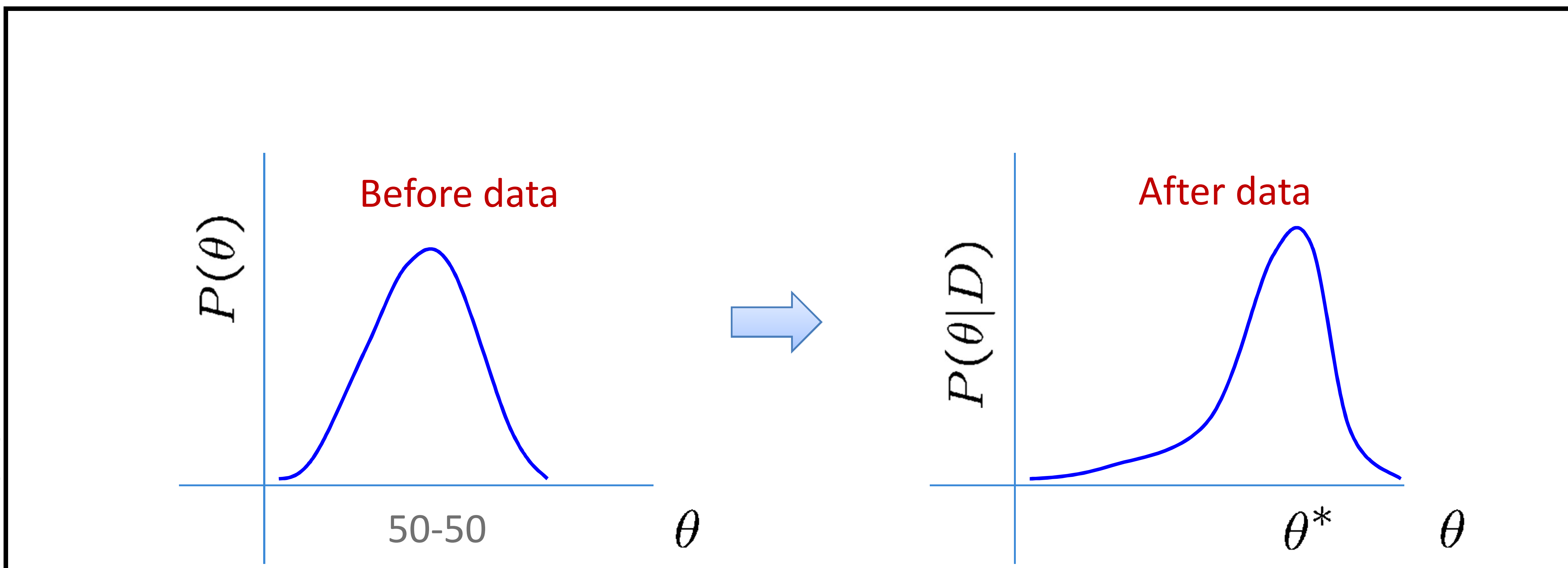
The Bayesian way…

What about prior knowledge?

We know the coin is “close” to 50-50. What can we do now?

The Bayesian way…

Rather than estimating a single θ, we obtain a distribution over possible values of θ



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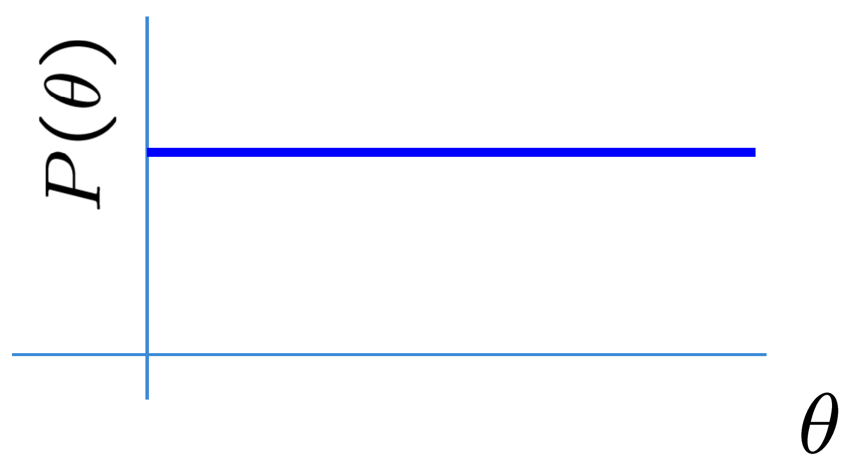
Prior distribution

- What prior? What distribution do we want for a prior?
  - Represents expert knowledge (philosophical approach)
  - Simple posterior form (engineer's approach)

- Uninformative priors:
  - Uniform distribution



- Conjugate priors:
  - Closed-form representation of posterior
  - P(θ) and P(θ|D) have the same form

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In order to proceed we will need:

Bayes Rule

Bayes, Thomas (1763) An essay towards solving a problem in the doctrine of chances. *Philosophical Transactions of the Royal Society of London,* **53:370-418**

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Chain Rule & Bayes Rule

Chain rule:

$$P(X,Y) = P(X|Y)P(Y) = P(Y|X)P(X)$$

Bayes rule:

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

Bayes rule is important for reverse conditioning.

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Bayesian Learning

- Use Bayes rule:

$$P(\theta \mid \mathcal{D}) = \frac{P(\mathcal{D} \mid \theta) P(\theta)}{P(\mathcal{D})}$$

- Or equivalently:

$$P(\theta \mid \mathcal{D}) \propto P(\mathcal{D} \mid \theta) P(\theta)$$

posterior likelihood prior

Bayes, Thomas (1763) An essay towards solving a problem in the doctrine of chances. *Philosophical Transactions of the Royal Society of London*, **53:370-418**

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MLE vs. MAP

- Maximum Likelihood estimation (MLE)

  Choose value that maximizes the probability of observed data

$$\widehat{\theta}_{MLE} = \arg\max_{\theta} P(D|\theta)$$

MLE vs. MAP

- Maximum Likelihood estimation (MLE)

  Choose value that maximizes the probability of observed data

$$\widehat{\theta}_{MLE} = \arg\max_{\theta} P(D|\theta)$$

- Maximum *a posteriori* (MAP) estimation

  Choose value that is most probable given observed data and prior belief

$$\widehat{\theta}_{MAP} = \arg\max_{\theta} P(\theta|D)$$
$$= \arg\max_{\theta} P(D|\theta)P(\theta)$$

When is MAP same as MLE?

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MAP estimation for Binomial distribution

Coin flip problem

Beta function: $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt$

Likelihood is Binomial $P(\mathcal{D} \mid \theta) = \binom{n}{\alpha_H} \theta^{\alpha_H}(1-\theta)^{\alpha_T}$

If the prior is Beta distribution,

$$P(\theta) = \frac{\theta^{\beta_H - 1}(1-\theta)^{\beta_T - 1}}{B(\beta_H, \beta_T)} \sim Beta(\beta_H, \beta_T)$$

$\Rightarrow$ posterior is Beta distribution

$$P(\theta|D) \sim Beta(\beta_H + \alpha_H, \beta_T + \alpha_T)$$

P(θ) and P(θ| D) have the same form! [Conjugate prior]

$$\widehat{\theta}_{MAP} = \arg\max_{\theta}\ P(\theta \mid D) = \arg\max_{\theta}\ P(D \mid \theta)P(\theta) = \frac{\alpha_H + \beta_H - 1}{\alpha_H + \beta_H + \alpha_T + \beta_T - 2}$$

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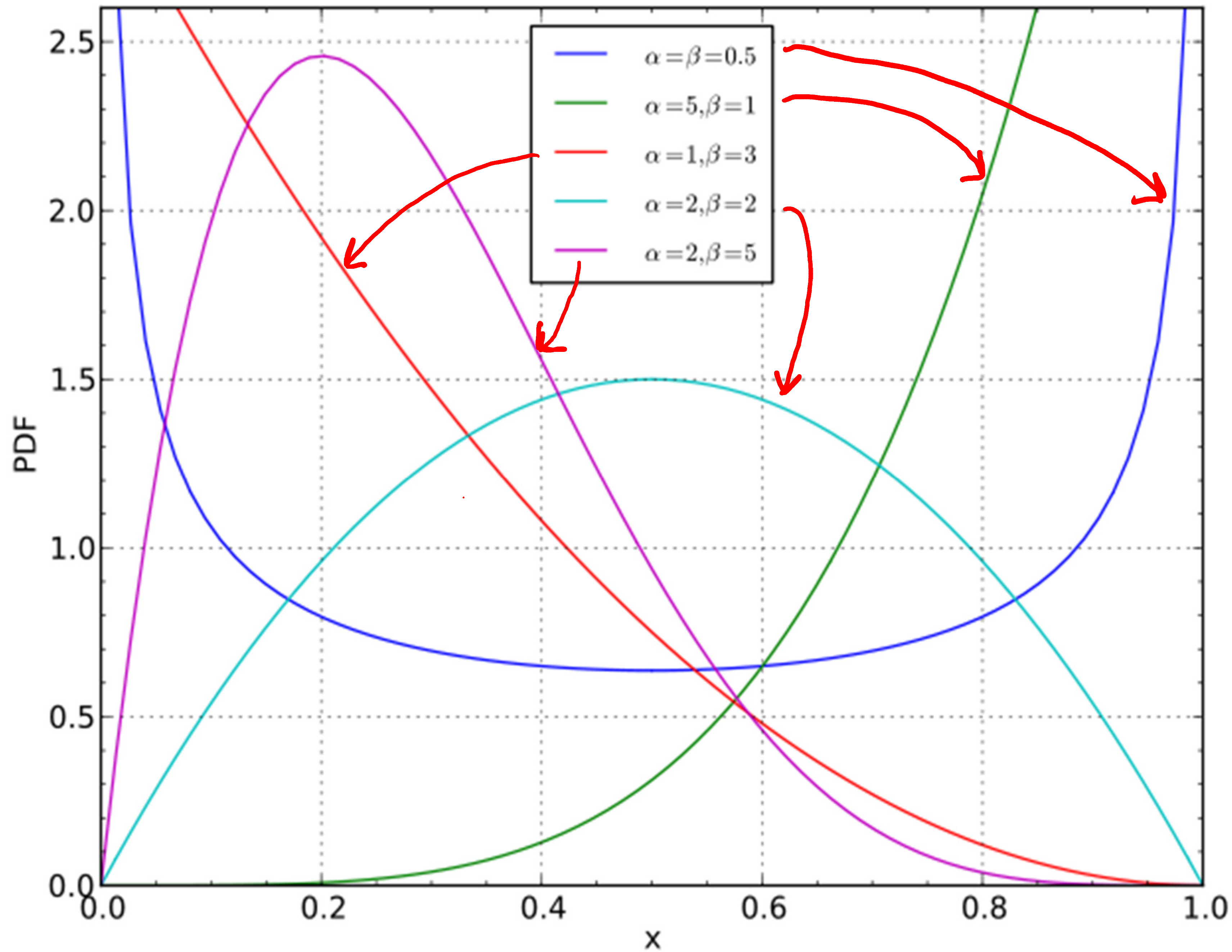
Beta distribution



More concentrated as values of α, β increase

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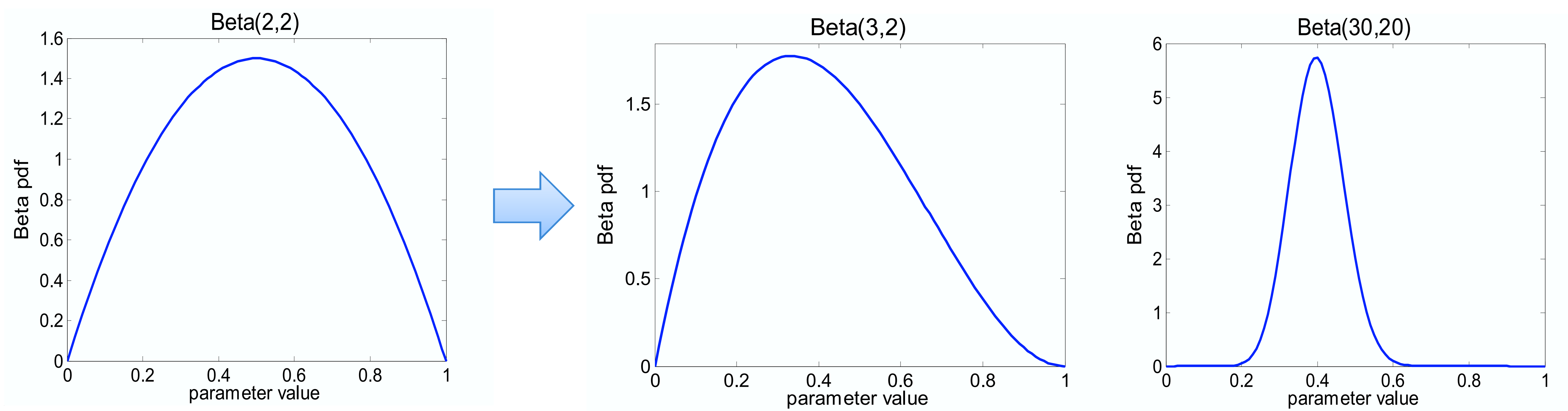
Beta conjugate prior

$$P(\theta) \sim Beta(\beta_H, \beta_T) \qquad P(\theta|D) \sim Beta(\beta_H + \alpha_H, \beta_T + \alpha_T)$$



As $n = \alpha_H + \alpha_T$ increases

As we get more samples, effect of prior is "washed out"

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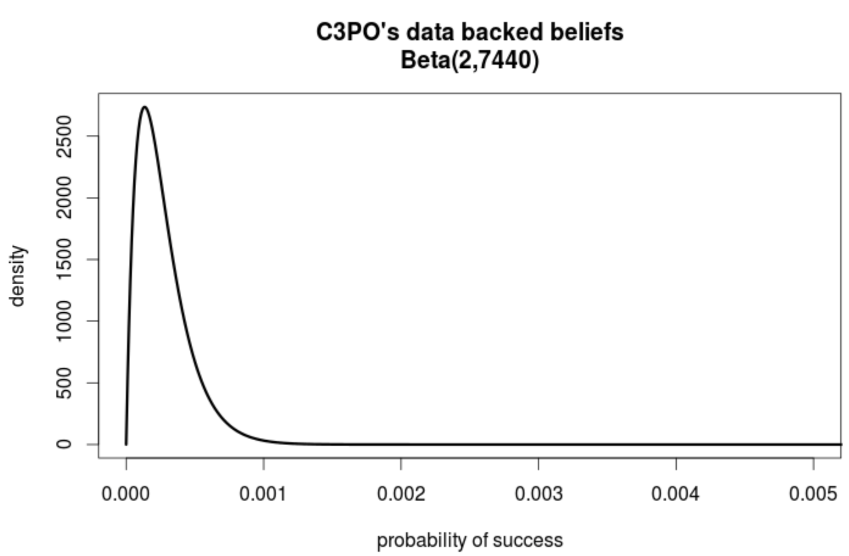
Han Solo and Bayesian Priors

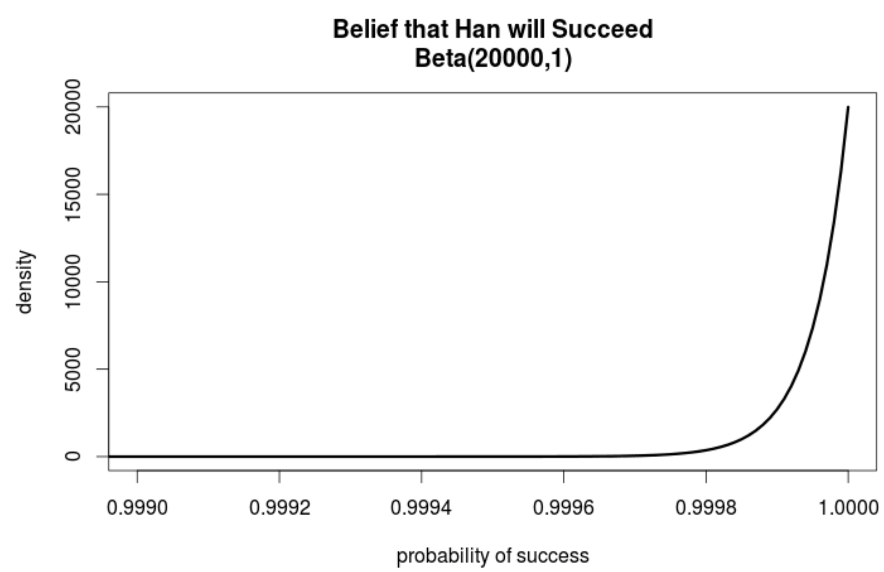
C3PO: Sir, the possibility of successfully navigating an asteroid field is approximately 3,720 to 1!

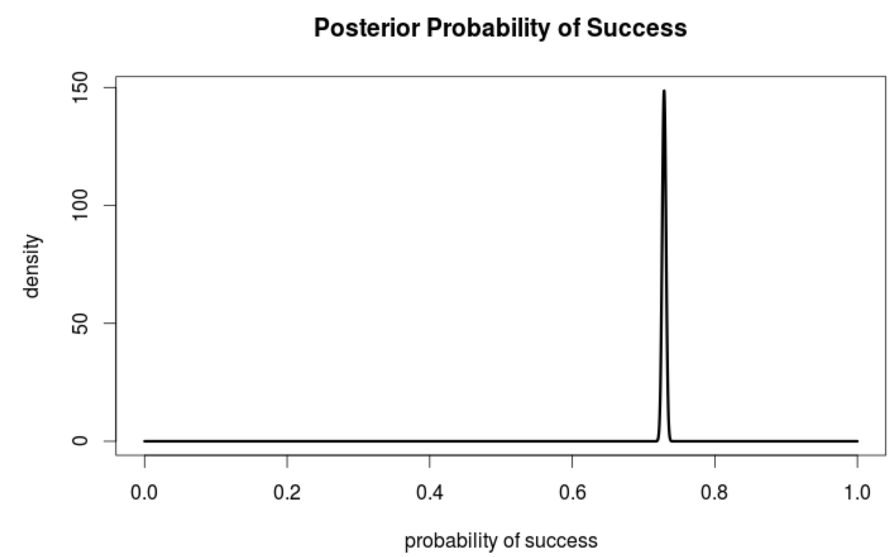
Han: Never tell me the odds!

$P(\theta) \sim Beta(\beta_H, \beta_T)$

$P(\theta|D) \sim Beta(\beta_H + \alpha_H, \beta_T + \alpha_T)$







https://www.countbayesie.com/blog/2015/2/18/hans-solo-and-bayesian-priors

From Binomial to Multinomial

Example: Dice roll problem (6 outcomes instead of 2)

Likelihood is ~ Multinomial($\theta = \{\theta_1, \theta_2, \ldots, \theta_k\}$)

$$P(\mathcal{D} \mid \theta) = \theta_1^{\alpha_1}\theta_2^{\alpha_2} \ldots \theta_k^{\alpha_k}$$

If prior is Dirichlet distribution,

$$P(\theta) = \frac{\prod_{i=1}^{k} \theta_i^{\beta_i - 1}}{B(\beta_1, \ldots, \beta_k)} \sim \text{Dirichlet}(\beta_1, \ldots, \beta_k)$$

Then posterior is Dirichlet distribution

$$P(\theta|D) \sim \text{Dirichlet}(\beta_1 + \alpha_1, \ldots, \beta_k + \alpha_k)$$

For Multinomial, conjugate prior is Dirichlet distribution.

http://en.wikipedia.org/wiki/Dirichlet_distribution

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Bayesians vs. Frequentists

You are no good when sample is small

You give a different answer for different priors

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Application of Bayes Rule

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AIDS test (Bayes rule)

Data

- **Approximately 0.1% are infected**
- **Test detects all infections**
- **Test reports positive for 1% healthy people**

Probability of having AIDS if test is positive

$$P(a=1|t=1) = \frac{P(t=1|a=1)P(a=1)}{P(t=1)}$$

$$= \frac{P(t=1|a=1)P(a=1)}{P(t=1|a=1)P(a=1) + P(t=1|a=0)P(a=0)}$$

$$= \frac{1 \cdot 0.001}{1 \cdot 0.001 + 0.01 \cdot 0.999} = 0.091$$

Only 9%!...

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Improving the diagnosis

Use a weaker follow-up test!

- Approximately 0.1% are infected
- Test 2 reports positive for 90% infections
- Test 2 reports positive for 5% healthy people

$$P(a=0|t_1=1,t_2=1) = \frac{P(t_1=1,t_2=1|a=0)P(a=0)}{P(t_1=1,t_2=1|a=1)P(a=1)+P(t_1=1,t_2=1|a=0)P(a=0)}$$

$$= \frac{0.01 \cdot 0.05 \cdot 0.999}{1 \cdot 0.9 \cdot 0.001 + 0.01 \cdot 0.05 \cdot 0.999} = 0.357$$

$$P(a=1|t_1=1,t_2=1) = 0.643$$

64%!...

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AIDS test (Bayes rule)

Why can't we use Test 1 twice?

- Outcomes are **not** independent,
- but tests 1 and 2 **conditionally independent (by assumption)**:

$$p(t_1, t_2|a) = p(t_1|a) \cdot p(t_2|a)$$

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The Naïve Bayes Classifier

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Data for spam filtering

- date
- time
- recipient path
- IP number
- sender
- encoding
- many more features

```
Delivered-To: alex.smola@gmail.com
Received: by 10.216.47.73 with SMTP id s51cs361171web;
        Tue, 3 Jan 2012 14:17:53 -0800 (PST)
Received: by 10.213.17.145 with SMTP id s17mr2519891eba.147.1325629071725;
        Tue, 03 Jan 2012 14:17:51 -0800 (PST)
Return-Path: <alex+caf_=alex.smola=gmail.com@smola.org>
Received: from mail-ey0-f175.google.com (mail-ey0-f175.google.com [209.85.215.175])
        by mx.google.com with ESMTPS id n4si29264232eef.57.2012.01.03.14.17.51
        (version=TLSv1/SSLv3 cipher=OTHER);
        Tue, 03 Jan 2012 14:17:51 -0800 (PST)
Received-SPF: neutral (google.com: 209.85.215.175 is neither permitted nor denied by best
guess record for domain of alex+caf_=alex.smola=gmail.com@smola.org) client-
ip=209.85.215.175;
Authentication-Results: mx.google.com; spf=neutral (google.com: 209.85.215.175 is neither
permitted nor denied by best guess record for domain of
alex+caf_=alex.smola=gmail.com@smola.org)
smtp.mail=alex+caf_=alex.smola=gmail.com@smola.org; dkim=pass (test mode)
header.i=@googlemail.com
Received: by eaal1 with SMTP id l1so15092746eaa.6
        for <alex.smola@gmail.com>; Tue, 03 Jan 2012 14:17:51 -0800 (PST)
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        Tue, 03 Jan 2012 14:17:51 -0800 (PST)
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        (version=TLSv1/SSLv3 cipher=OTHER);
        Tue, 03 Jan 2012 14:17:48 -0800 (PST)
Received-SPF: pass (google.com: domain of althoff.tim@googlemail.com designates
209.85.220.179 as permitted sender) client-ip=209.85.220.179;
Received: by vcbf13 with SMTP id f13so11295098vcb.10
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        h=mime-version:sender:date:x-google-sender-auth:message-id:subject
         :from:to:content-type;
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Subject: CS 281B. Advanced Topics in Learning and Decision Making
```

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Naïve Bayes Assumption

Naïve Bayes assumption: Features X_1 and X_2 are conditionally independent given the class label Y:

$$P(X_1, X_2|Y) = P(X_1|Y)P(X_2|Y)$$

More generally:

$$P(X_1...X_d|Y) = \prod_{i=1}^{d} P(X_i|Y)$$

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Naïve Bayes Assumption, Example

Task: Predict whether or not a picnic spot is enjoyable

Training Data: $X = (X_1 \quad X_2 \quad X_3 \quad \dots \quad \dots \quad X_d) \quad Y$

n rows

Sky	Temp	Humid	Wind	Water	Forecst	EnjoySpt
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

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Naïve Bayes assumption: $$P(X_1 \ldots X_d | Y) = \prod_{i=1}^{d} P(X_i | Y)$$

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Naïve Bayes assumption: $$P(X_1 ... X_d|Y) = \prod_{i=1}^{d} P(X_i|Y)$$

How many parameters to estimate?

(X is composed of d binary features,
Y has K possible class labels)

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Naïve Bayes Assumption, Example

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How many parameters to estimate?

(X is composed of d binary features,
Y has K possible class labels)

$\boxed{(2^d-1)K \text{ vs } (2-1)dK}$

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Naïve Bayes Classifier

Given:

- Class prior $P(Y)$
- d conditionally independent features $X_1,\ldots X_d$ given the class label Y
- For each X_i feature, we have the conditional likelihood $P(X_i|Y)$

Naïve Bayes Decision rule:

$$
\begin{aligned}
f_{NB}(\mathbf{x}) &= \arg\max_y \; P(x_1,\ldots,x_d \mid y)P(y) \\
&= \arg\max_y \prod_{i=1}^{d} P(x_i|y)P(y)
\end{aligned}
$$

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Naïve Bayes Algorithm for discrete features

Training data: $\{(X^{(j)}, Y^{(j)})\}_{j=1}^{n}$

$$X^{(j)} = (X_1^{(j)}, \ldots, X_d^{(j)})$$

n d-dimensional discrete features + K class labels

$$f_{NB}(\mathbf{x}) = \arg\max_y \prod_{i=1}^{d} P(x_i|y)P(y)$$

We need to estimate these probabilities!

Estimate them with MLE (Relative Frequencies)!

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Naïve Bayes Algorithm for discrete features

$$f_{NB}(\mathbf{x}) = \arg\max_y \prod_{i=1}^{d} P(x_i|y)P(y)$$

We need to estimate these probabilities!

Estimators

For Class Prior

$$\widehat{P}(y) = \frac{\{\#j : Y^{(j)} = y\}}{n}$$

For Likelihood

$$\frac{\widehat{P}(x_i, y)}{\widehat{P}(y)} = \frac{\{\#j : X_i^{(j)} = x_i, Y^{(j)} = y\}/n}{\{\#j : Y^{(j)} = y\}/n}$$

NB Prediction for test data: $X = (x_1, \ldots, x_d)$

$$Y = \arg\max_y \widehat{P}(y) \prod_{i=1}^{d} \frac{\widehat{P}(x_i, y)}{\widehat{P}(y)}$$

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Subtlety: Insufficient training data

What if you never see a training instance where $X_1 = a$ when $Y = b$?

For example,

there is no X_1='Earn' when Y='SpamEmail' in our dataset.

$$\Rightarrow P(X_1 = a, Y = b) = 0 \Rightarrow P(X_1 = a | Y = b) = 0$$

$$\boxed{\Rightarrow P(X_1 = a, X_2 ... X_n | Y) = P(X_1 = a | Y) \prod_{i=2}^{d} P(X_i | Y) = 0}$$

Thus, no matter what the values $X_2, \ldots, X_d$ take:

$$P(Y = b \mid X_1 = a, X_2, \ldots, X_d) = 0$$

What now???

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Naïve Bayes Alg — Discrete features

Training data: $\{(X^{(j)}, Y^{(j)})\}_{j=1}^{n}$ $X^{(j)} = (X_1^{(j)}, \ldots, X_d^{(j)})$

Use your expert knowledge & apply prior distributions:

- Add m "virtual" examples
- Same as assuming conjugate priors

Assume priors: $Q(Y = b)$ $Q(X_i = a, Y = b)$

MAP Estimate:

$$\widehat{P}(X_i = a|Y = b) = \frac{\{\#j : X_i^{(j)} = a, Y^{(j)} = b\} + mQ(X_i = a, Y = b)}{\{\#j : Y^{(j)} = b\} + \underbrace{mQ(Y = b)}_{\text{\# virtual examples with Y = b}}}$$

called Laplace smoothing

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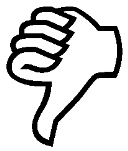
Case Study:
Text Classification

Positive or negative movie review?



- unbelievably disappointing

- Full of zany characters and richly applied satire, and some great plot twists

- this is the greatest screwball comedy ever filmed

- It was pathetic. The worst part about it was the boxing scenes.

slide by Dan Jurafsky

What is the subject of this article?

MEDLINE Article


Syntactic frame and verb bias in aphasia: Plausibility judgments of undergoer-subject sentences

?

MeSH Subject Category Hierarchy

- Antogonists and Inhibitors
- Blood Supply
- Chemistry
- Drug Therapy
- Embryology
- Epidemiology
- …

slide by Dan Jurafsky

Text Classification

- Assigning subject categories, topics, or genres
- Spam detection
- Authorship identification
- Age/gender identification
- Language Identification
- Sentiment analysis
- …

slide by Dan Jurafsky

Text Classification: definition

- Input:
  - a document d
  - a fixed set of classes $C = \{c_1, c_2, \ldots, c_J\}$
- Output: a predicted class $c \in C$

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Hand-coded rules

- Rules based on combinations of words or other features
    - spam: black-list-address OR (“dollars” AND“have been selected”)
- Accuracy can be high
    - If rules carefully refined by expert
- But building and maintaining these rules is expensive

slide by Dan Jurafsky

Text Classification and Naive Bayes

- Classify emails
  - Y = {Spam, NotSpam}
- Classify news articles
  - Y = {what is the topic of the article?}

What are the features X?

The text!

Let X_i represent i^{th} word in the document

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X_i represents i^{th} word in document

Article from rec.sport.hockey

```
Path: cantaloupe.srv.cs.cmu.edu!das-news.harvard.e
From: xxx@yyy.zzz.edu (John Doe)
Subject: Re: This year's biggest and worst (opinic
Date: 5 Apr 93 09:53:39 GMT

I can only comment on the Kings, but the most
obvious candidate for pleasant surprise is Alex
Zhitnik. He came highly touted as a defensive
defenseman, but he's clearly much more than that.
Great skater and hard shot (though wish he were
more accurate). In fact, he pretty much allowed
the Kings to trade away that huge defensive
liability Paul Coffey. Kelly Hrudey is only the
biggest disappointment if you thought he was any
good to begin with. But, at best, he's only a
mediocre goaltender. A better choice would be
Tomas Sandstrom, though not through any fault of
his own, but because some thugs in Toronto decided
```

NB for Text Classification

A problem: The support of $P(\mathbf{X}|Y)$ is huge!

– Article at least 1000 words, $\mathbf{X}=\{X_1,\ldots,X_{1000}\}$

– X_i represents i^{th} word in document, i.e., the domain of X_i is the entire vocabulary, e.g., Webster Dictionary (or more).

$X_i \in \{1,\ldots,50000\} \Rightarrow K(1000^{50000} - 1)$

parameters to estimate without the NB assumption….

$$h_{MAP}(\mathbf{x}) = \arg\max_{1\le k\le K} P(Y=k)P(X_1=x_1,\ldots,X_{1000}=x_{1000}|Y=k)$$

slide by Barnabás Póczos & Aarti Singh

NB for Text Classification

$X_i \in \{1,\ldots,50000\} \Rightarrow K(1000^{50000} - 1)$ parameters to estimate….

NB assumption helps a lot!!!

If $P(X_i=x_i|Y=y)$ is the probability of observing word x_i at the i^{th} position in a document on topic y

$\Rightarrow 1000K(50000-1)$ parameters to estimate with NB assumption

NB assumption helps, but still lots of parameters to estimate.

$$h_{NB}(\mathbf{x}) = \arg\max_y P(y) \prod_{i=1}^{LengthDoc} P(X_i = x_i|y)$$

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Bag of words model

Typical additional assumption:

Position in document doesn't matter:

$P(X_i=x_i|Y=y) = P(X_k=x_i|Y=y)$

- "Bag of words" model – order of words on the page ignored
  The document is just a bag of words: i.i.d. words
- Sounds really silly, but often works very well!

$\Rightarrow$ K(50000-1) parameters to estimate

The probability of a document with words $x_1, x_2, \ldots$

$$\prod_{i=1}^{LengthDoc} P(x_i|y) = \prod_{w=1}^{W} P(w|y)^{count_w}$$

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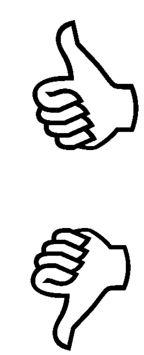
The bag of words representation

$\gamma($

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun… It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet.

$)=c$



slide by Dan Jurafsky

The bag of words representation

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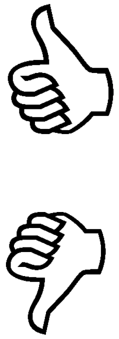

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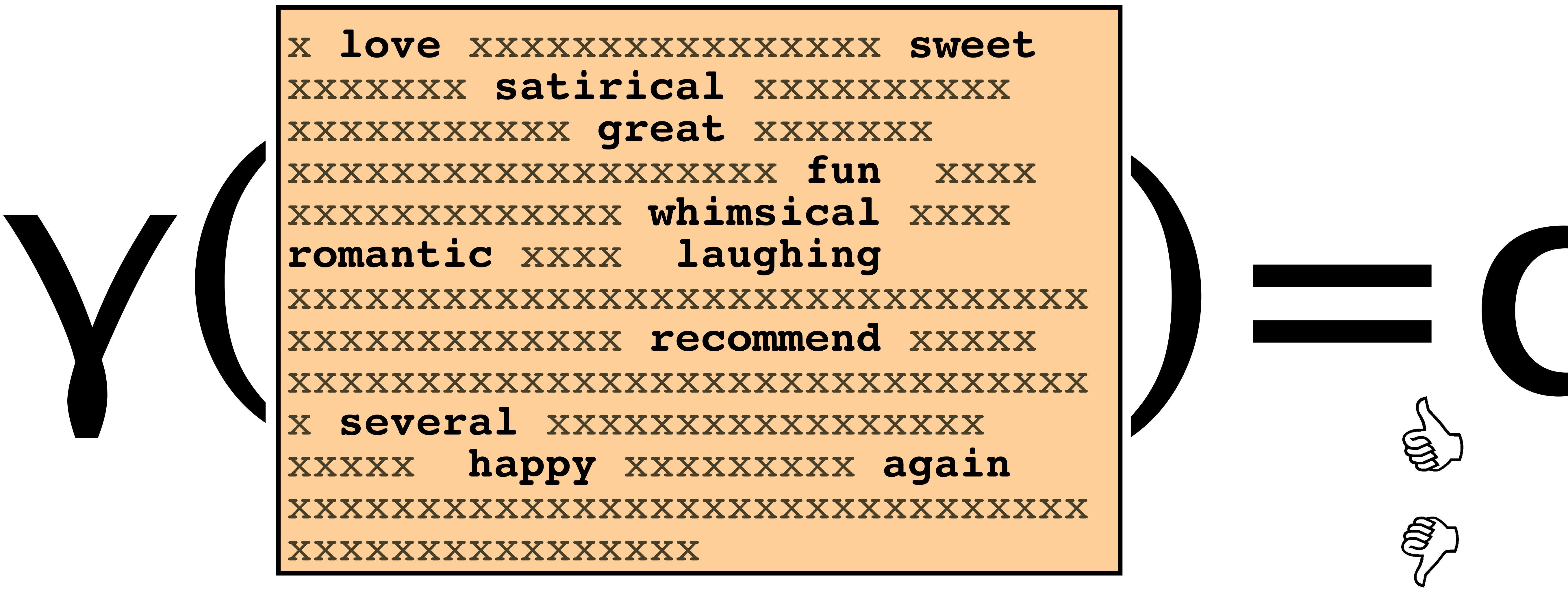
The bag of words representation: using a subset of words



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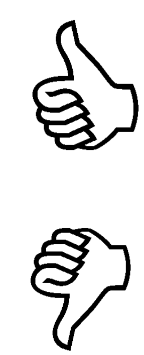
The bag of words representation

$\gamma($

great	2
love	2
recommend	1
laugh	1
happy	1
...	...

$)=c$



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$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(w \mid c) = \frac{count(w,c)+1}{count(c)+|V|}$$

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

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Priors:

$$P(c) = \frac{3}{4}$$

$$P(j) = \frac{1}{4}$$

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$$P(c) = \frac{3}{4}$$

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Conditional Probabilities:

P(Chinese|*c*) = (5+1) / (8+6) = 6/14 = 3/7

P(Tokyo|*c*) = (0+1) / (8+6) = 1/14

P(Japan|*c*) = (0+1) / (8+6) = 1/14

P(Chinese|*j*) = (1+1) / (3+6) = 2/9

P(Tokyo|*j*) = (1+1) / (3+6) = 2/9

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P(Japan|*j*) = (1+1) / (3+6) = 2/9

Choosing a class:

$$P(c|d_5) \propto 3/4 * (3/7)^3 * 1/14 * 1/14 \approx 0.0003$$

$$P(j|d_5) \propto 1/4 * (2/9)^3 * 2/9 * 2/9 \approx 0.0001$$

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Twenty news groups results

Given 1000 training documents from each group
Learn to classify new documents according to
which newsgroup it came from

comp.graphics	misc.forsale
comp.os.ms-windows.misc	rec.autos
comp.sys.ibm.pc.hardware	rec.motorcycles
comp.sys.mac.hardware	rec.sport.baseball
comp.windows.x	rec.sport.hockey

alt.atheism	sci.space
soc.religion.christian	sci.crypt
talk.religion.misc	sci.electronics
talk.politics.mideast	sci.med
talk.politics.misc	
talk.politics.guns	

Naïve Bayes: 89% accuracy

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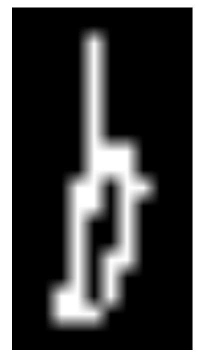
What if features are continuous?

e.g., character recognition: X_i is intensity at i^{th} pixel




Gaussian Naïve Bayes (GNB):

$$P(X_i = x \mid Y = y_k) = \frac{1}{\sigma_{ik}\sqrt{2\pi}} \; e^{\frac{-(x-\mu_{ik})^2}{2\sigma_{ik}^2}}$$

Different mean and variance for each class k and each pixel i.

Sometimes assume variance

- is independent of Y (i.e., σ_i),
- or independent of X_i (i.e., σ_k)
- or both (i.e., σ)

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Estimating parameters: Y discrete, X_i continuous

$$h_{NB}(\mathbf{x}) = \arg\max_y P(y) \prod_i P(X_i = x_i | y)$$
$$\approx \arg\max_k \widehat{P}(Y = k) \prod_i \mathcal{N}(\widehat{\mu}_{ik}, \widehat{\sigma}_{ik})$$

$$\boxed{\widehat{\mu}_{MLE} = \frac{1}{N} \sum_{j=1}^{N} x_j}$$

$$\boxed{\widehat{\sigma}^2_{unbiased} = \frac{1}{N-1} \sum_{j=1}^{N} (x_j - \widehat{\mu})^2}$$

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Estimating parameters: Y discrete, X_i continuous

Maximum likelihood estimates:

$$\widehat{\mu}_{MLE} = \frac{1}{N}\sum_{j=1}^{N} x_j$$

$$\widehat{\mu}_{ik} = \frac{1}{\sum_j \delta(Y^j = y_k)} \sum_j X_i^j \delta(Y^j = y_k)$$

k^{th} class

j^{th} training image

i^{th} pixel in j^{th} training image

$$\widehat{\sigma}^2_{unbiased} = \frac{1}{N-1}\sum_{j=1}^{N}(x_j - \widehat{\mu})^2$$

$$\widehat{\sigma}^2_{ik} = \frac{1}{\sum_j \delta(Y^j = y_k) - 1} \sum_j (X_i^j - \widehat{\mu}_{ik})^2 \delta(Y^j = y_k)$$

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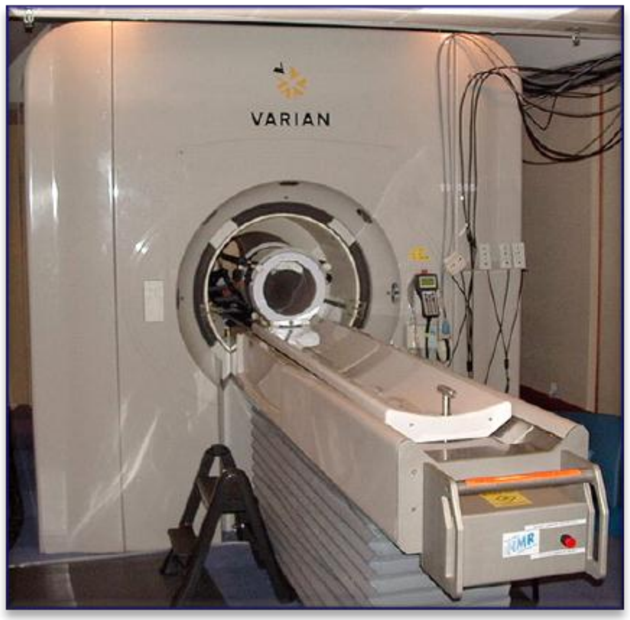
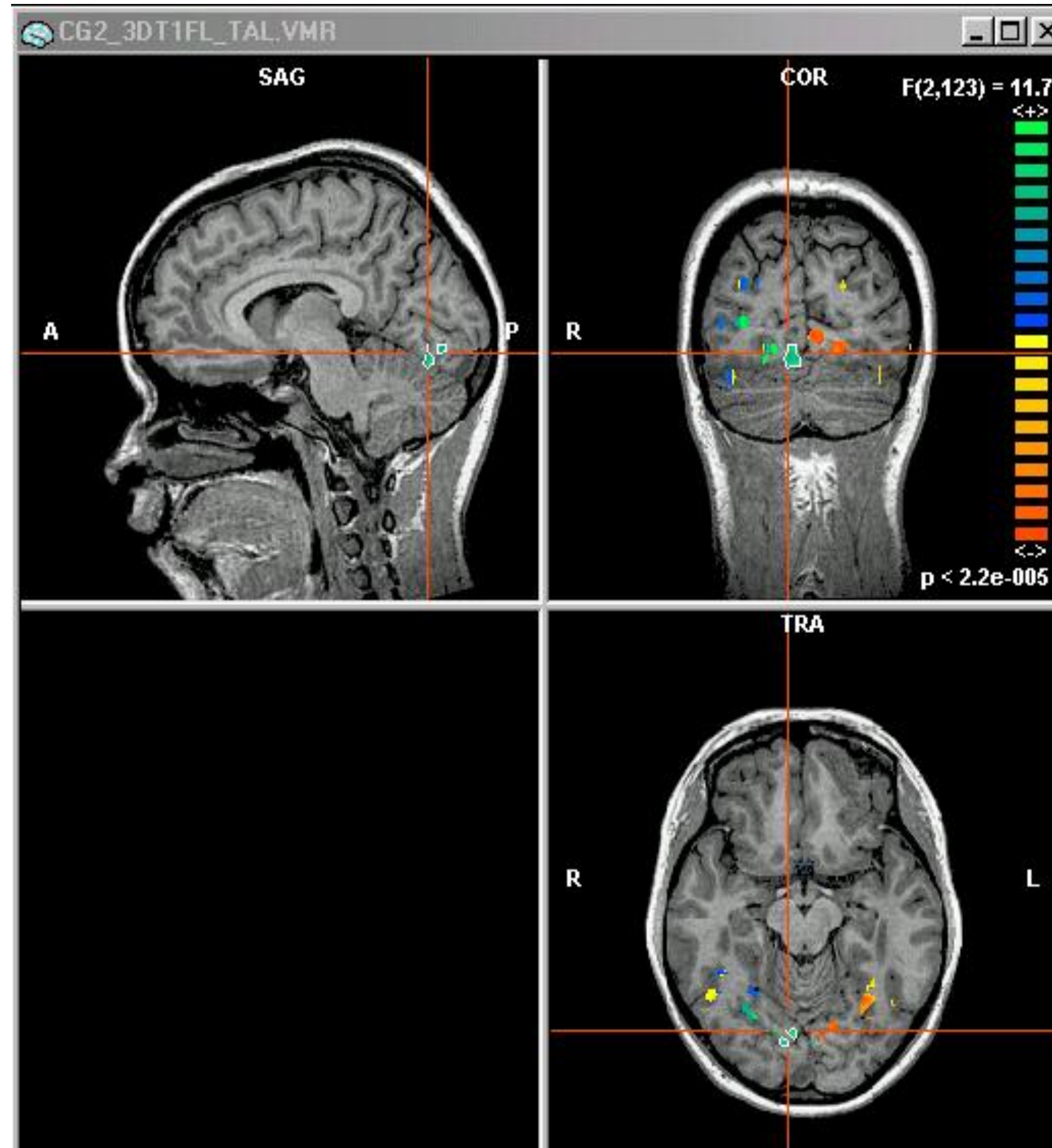
Case Study:
Classifying Mental States

Example: GNB for classifying mental states





[Mitchell et al.]

~1 mm resolution

~2 images per sec.

15,000 voxels/image

non-invasive, safe

measures Blood Oxygen Level Dependent (BOLD) response

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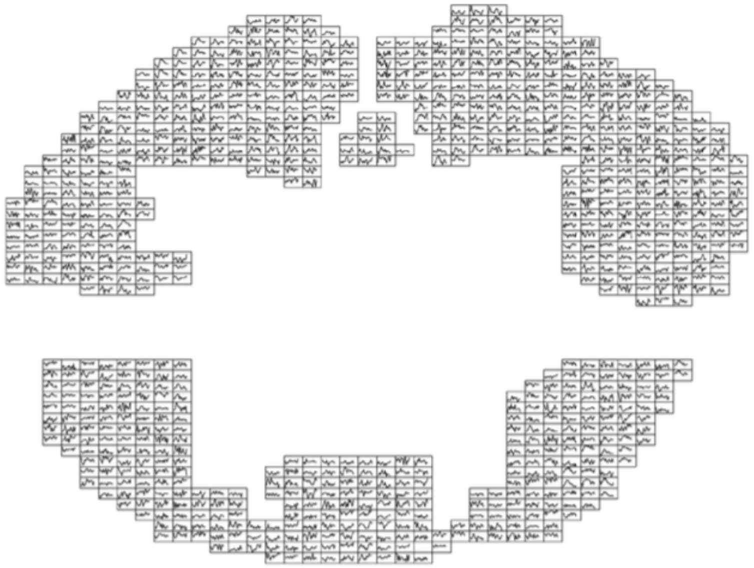
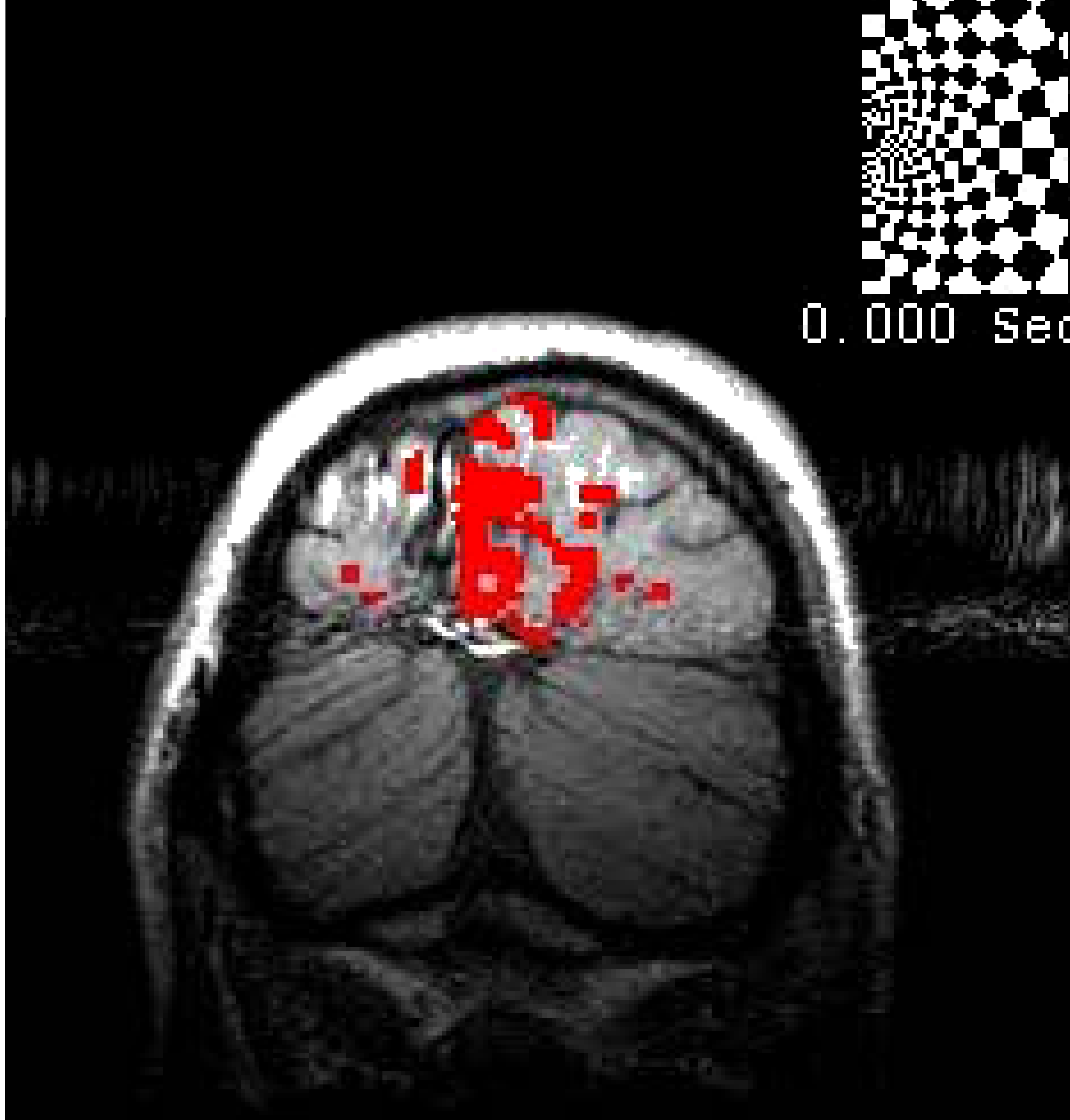


Brain scans can track activation with precision and sensitivity



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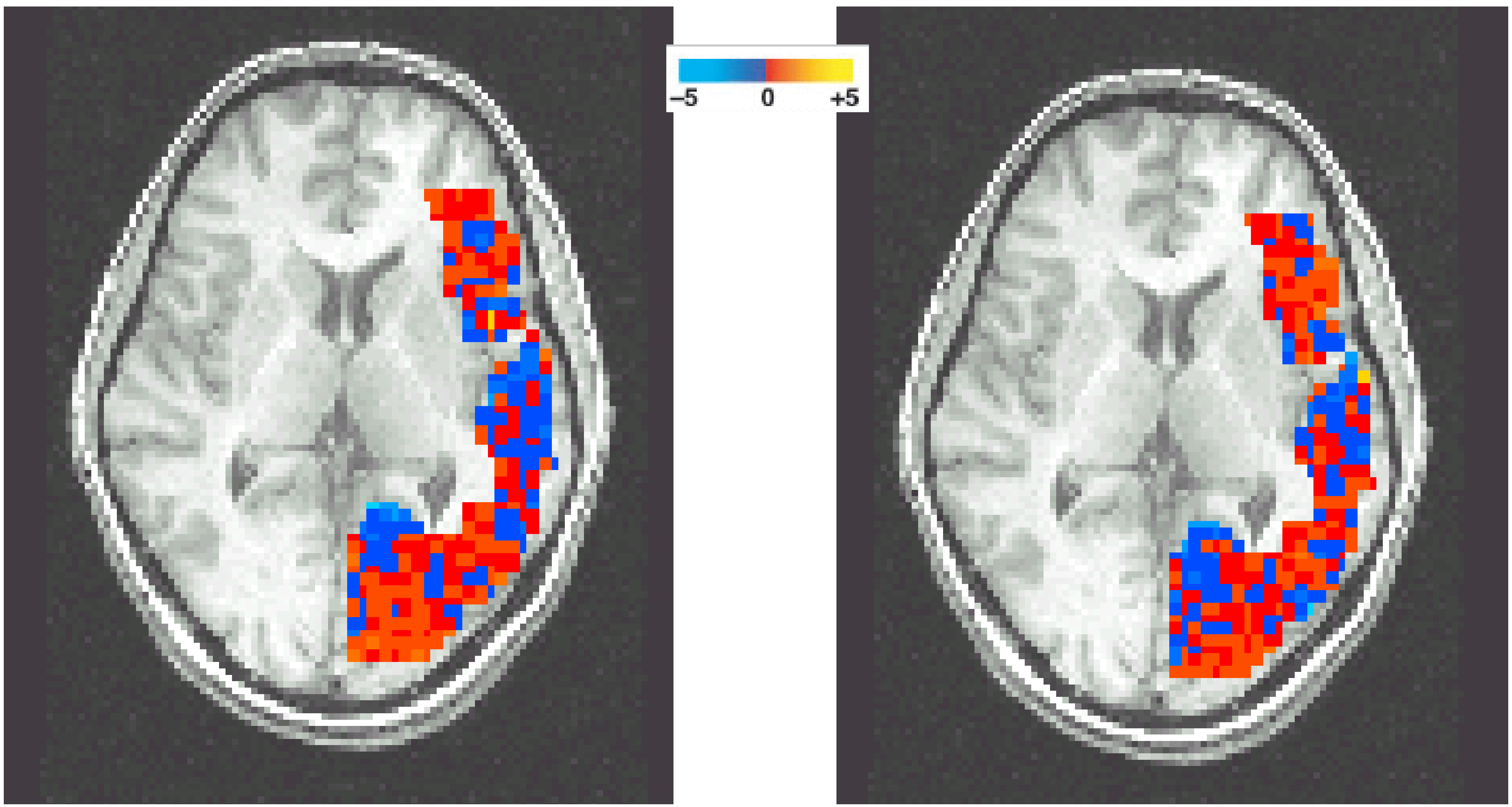
Learned Naïve Bayes Models – Means for P(BrainActivity | WordCategory)

Pairwise classification accuracy: 78-99%, 12 participants [Mitchell et al.]

Tool words | Building



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What you should know…

Naïve Bayes classifier
- What's the assumption
- Why we use it
- How do we learn it
- Why is Bayesian (MAP) estimation important

Text classification
- Bag of words model

Gaussian NB
- Features are still conditionally independent
- Each feature has a Gaussian distribution given class

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Next Class:

Logistic Regression,
Discriminant vs. Generative
Classification