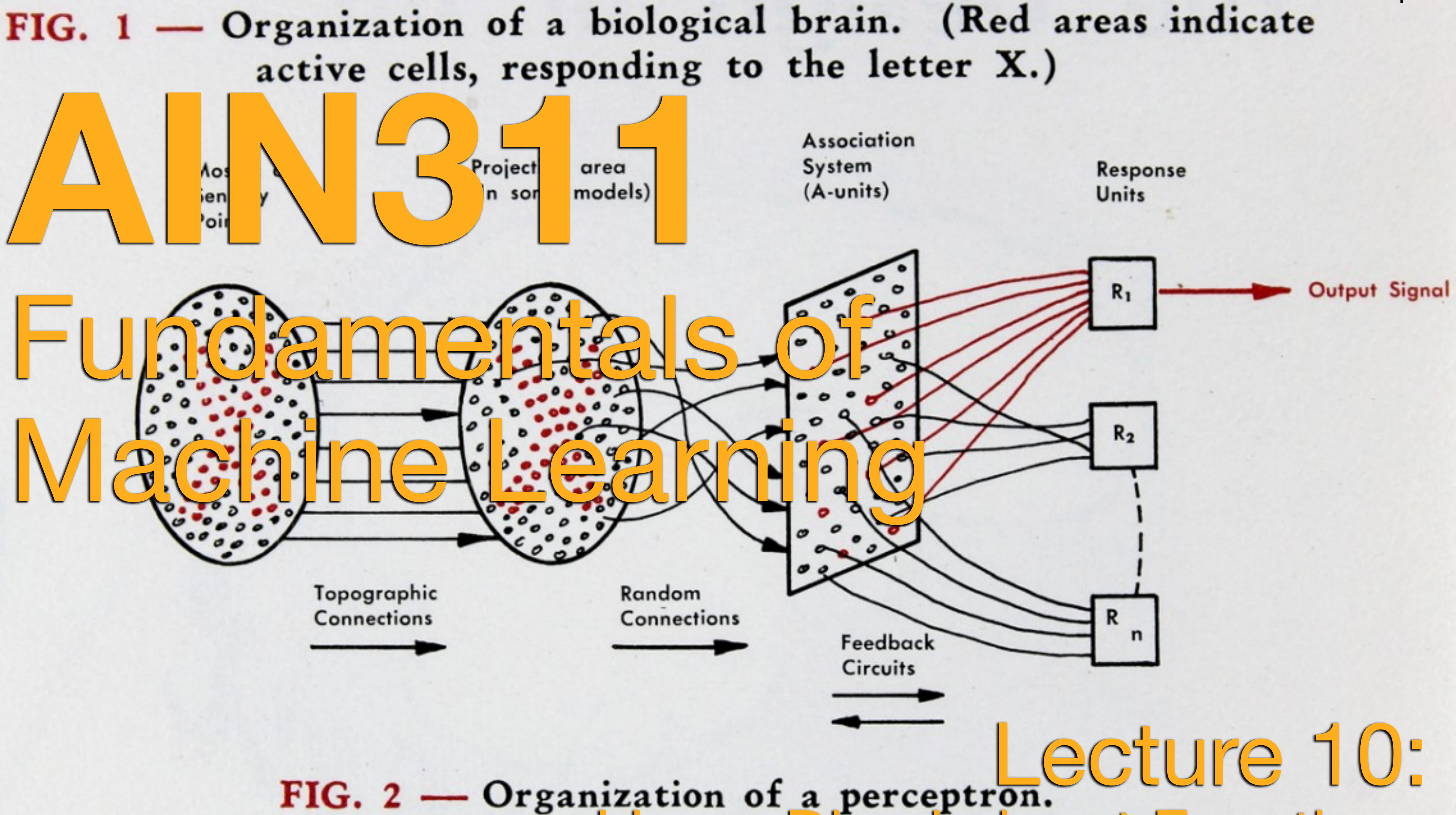
Illustration: Frank Rosenblatt's Perceptron



AIN311

Fundamentals of Machine Learning

Lecture 10: Linear Discriminant Functions Perceptron

HACETTEPE UNIVERSITY COMPUTER VISION LAB

Erkut Erdem // Hacettepe University // Fall 2024

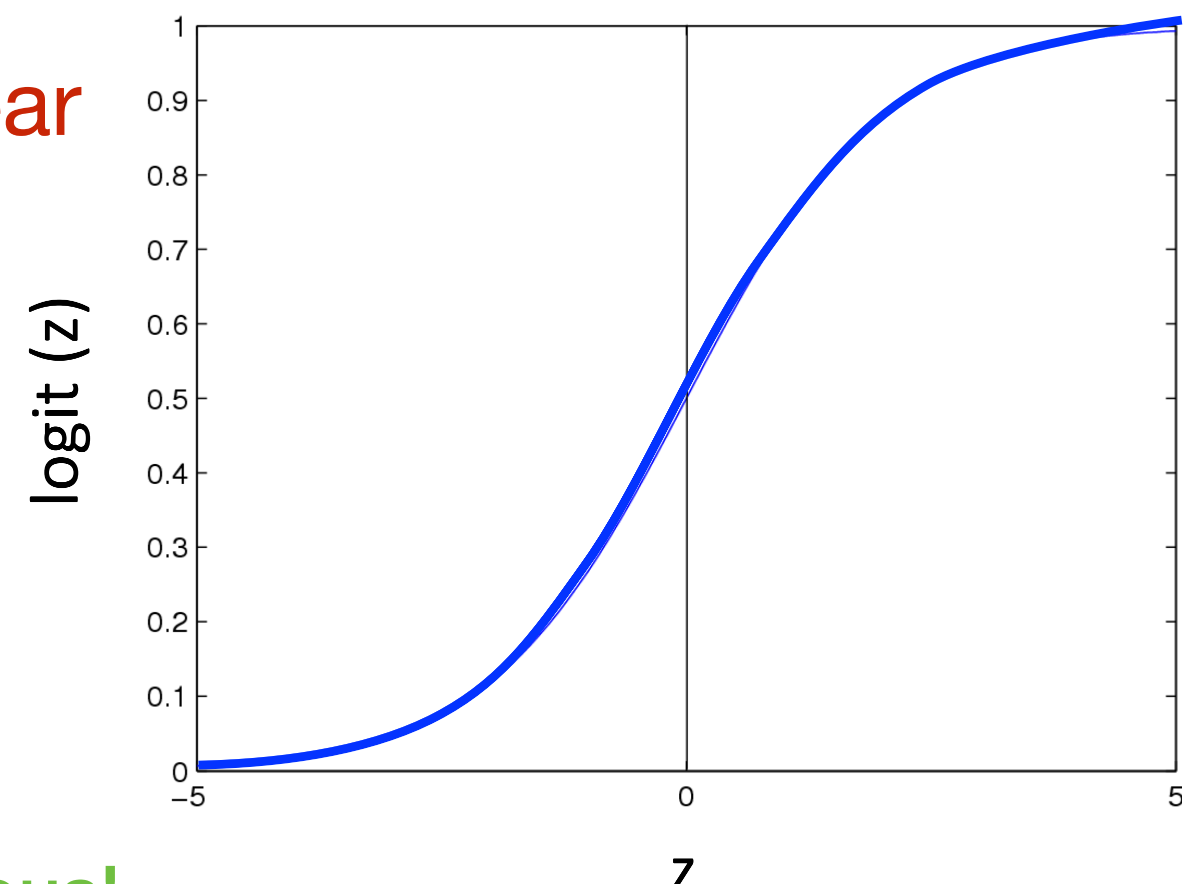
Last time… **Logistic Regression**

Assumes the following functional form for P(Y|X):

$$P(Y=1|X) = \frac{1}{1+\exp(w_0 + \sum_i w_i X_i)}$$

Logistic function applied to linear function of the data

Logistic function (or Sigmoid): $\frac{1}{1+exp(-z)}$



Features can be discrete or continuous!

slide by Aarti Singh & Barnabás Póczos

Last time.. **Logistic Regression vs. Gaussian Naïve Bayes**

- LR is a linear classifier
  - decision rule is a hyperplane
- LR optimized by maximizing conditional likelihood
  - no closed-form solution
  - concave ! global optimum with gradient ascent
- Gaussian Naïve Bayes with class-independent variances representationally equivalent to LR
  - Solution differs because of objective (loss) function
- In general, NB and LR make different assumptions
  - NB: Features independent given class! assumption on P(**X**|Y)
  - LR: Functional form of P(Y|**X**), no assumption on P(**X**|Y)
- Convergence rates
  - GNB (usually) needs less data
  - LR (usually) gets to better solutions in the limit

slide by Aarti Singh & Barnabás Póczos

Linear Discriminant Functions

Linear Discriminant Function

- Linear discriminant function for a vector $\mathbf{x}$

$$y(\mathbf{x}) = \mathbf{w}^T \mathbf{x} + w_0$$

where $\mathbf{w}$ is called weight vector, and w_0 is a bias.

- The classification function is

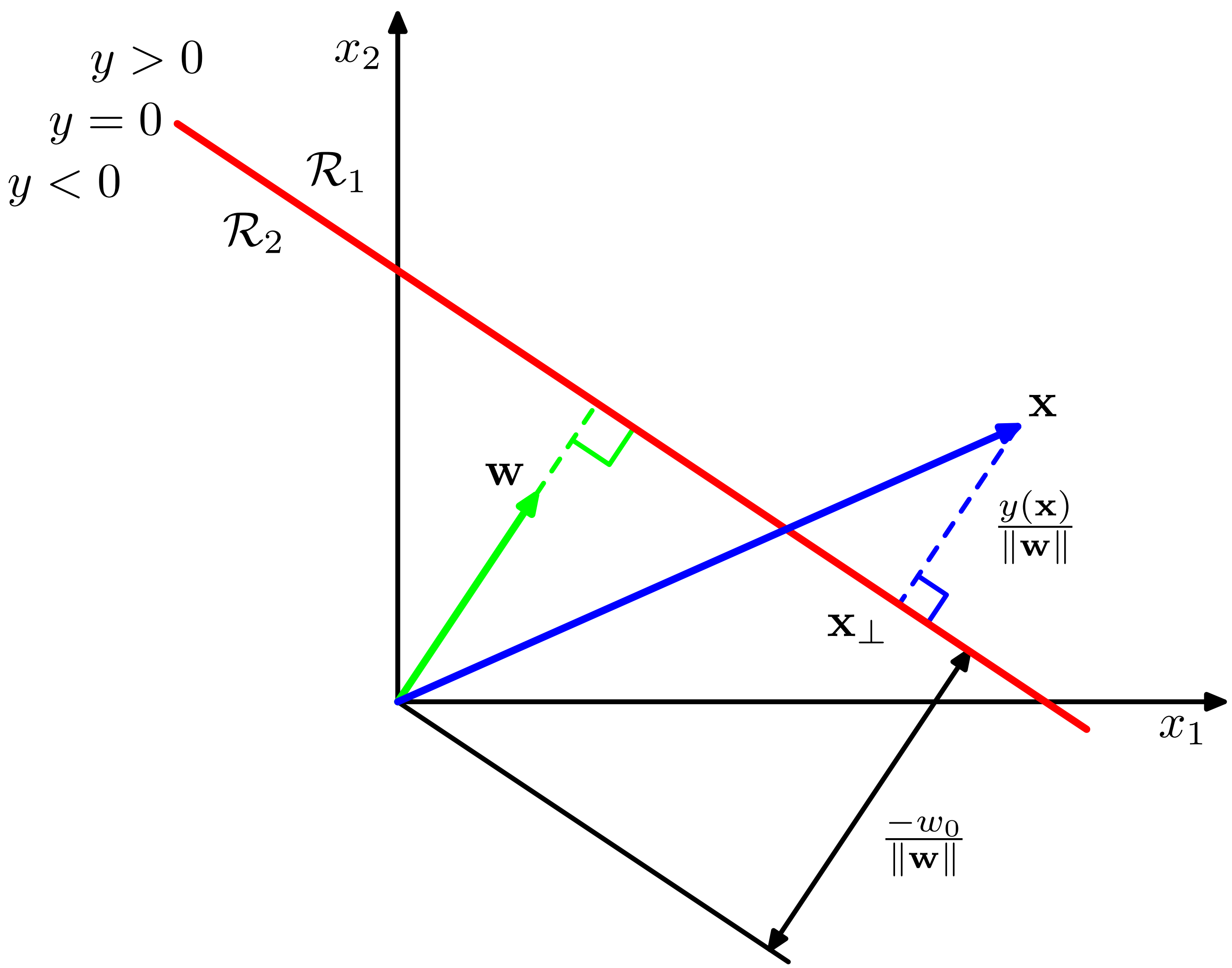
$$C(\mathbf{x}) = \mathrm{sign}(\mathbf{w}^T \mathbf{x} + w_0)$$

where step function sign(·) is defined as

$$\mathrm{sign}(a) = \begin{cases} +1, & a \geqslant 0 \\ -1, & a < 0 \end{cases}$$

slide by Ce Liu

Properties of Linear Discriminant Functions



- The decision surface, shown in red, is perpendicular to $\mathbf{w}$, and its displacement from the origin is controlled by the bias parameter w_0.
- The signed orthogonal distance of a general point $\mathbf{x}$ from the decision surface is given by $y(\mathbf{x})/\|\mathbf{w}\|$
- $y(\mathbf{x})$ gives a signed measure of the perpendicular distance r of the point $\mathbf{x}$ from the decision surface

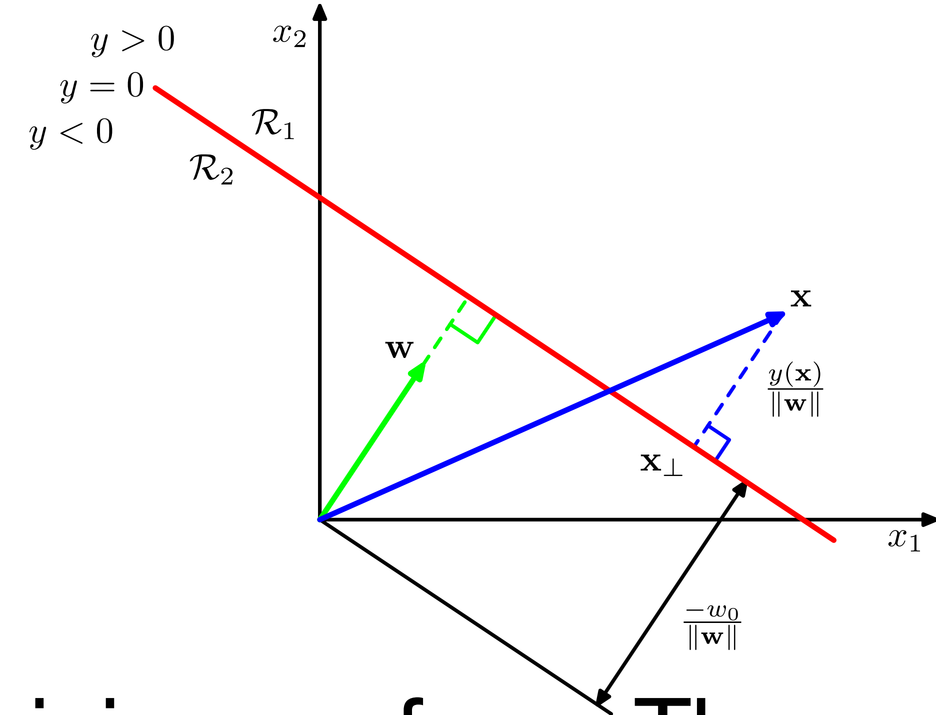
- $y(\mathbf{x}) = 0$ for $\mathbf{x}$ on the decision surface. The normal distance from the origin to the decision surface is

$$\frac{\mathbf{w}^T\mathbf{x}}{\|\mathbf{w}\|} = -\frac{w_0}{\|\mathbf{w}\|}$$

- So w_0 determines the location of the decision surface.

slide by Ce Liu

Properties of Linear Discriminant Functions



- Let

$$\mathbf{x} = \mathbf{x}_\perp + r\frac{\mathbf{w}}{\|\mathbf{w}\|}$$

where $\mathbf{x}_\perp$ is the projection $\mathbf{x}$ on the decision surface. Then

$$\mathbf{w}^T\mathbf{x} = \mathbf{w}^T\mathbf{x}_\perp + r\frac{\mathbf{w}^T\mathbf{w}}{\|\mathbf{w}\|}$$

$$\mathbf{w}^T\mathbf{x} + w_0 = \mathbf{w}^T\mathbf{x}_\perp + w_0 + r\|\mathbf{w}\|$$

$$y(\mathbf{x}) = r\|\mathbf{w}\|$$

$$r = \frac{y(\mathbf{x})}{\|\mathbf{w}\|}$$

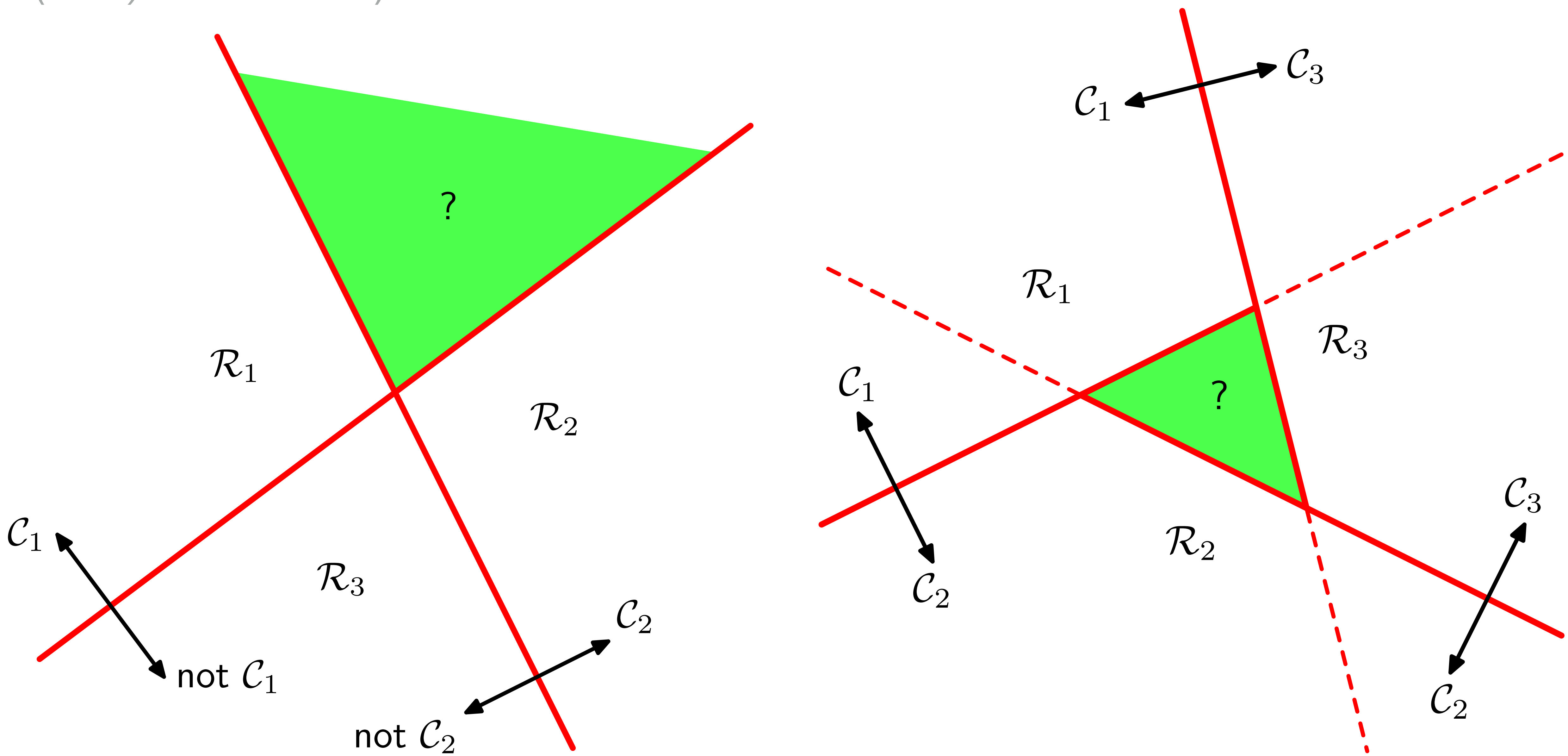
- Simpler notion: define $\widetilde{\mathbf{w}} = (w_0, \mathbf{w})$ and $\widetilde{\mathbf{x}} = (1, \mathbf{x})$ so that

$$y(\mathbf{x}) = \widetilde{\mathbf{w}}^T\widetilde{\mathbf{x}}$$

slide by Ce Liu

Multiple Classes: Simple Extension

- **One-versus-the-rest** classifier: classify C_k and samples not in C_k. *(K –1 classifiers)*
- **One-versus-one** classifier: classify every pair of classes. K*(K –1)/2 classifiers*)



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Multiple Classes: K-Class Discriminant

- A single *K*-class discriminant comprising *K* linear functions

$$y_k(\mathbf{x}) = \mathbf{w}_k^T \mathbf{x} + w_{k0}$$

- Decision function

$$\mathcal{C}(\mathbf{x}) = k, \text{ if } y_k(\mathbf{x}) > y_j(\mathbf{x}) \; \forall \, j \neq k$$

- The decision boundary between class C_k and C_j is given by $y_k(\mathbf{x}) = y_j(\mathbf{x})$

$$(\mathbf{w}_k - \mathbf{w}_j)^T \mathbf{x} + (w_{k0} - w_{j0}) = 0$$

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Property of the Decision Regions

Theorem

The decision regions of the K-class discriminant $y_k(\mathbf{x}) = \mathbf{w}_k^T\mathbf{x} + w_{k0}$ are singly connected and convex.

Property of the Decision Regions

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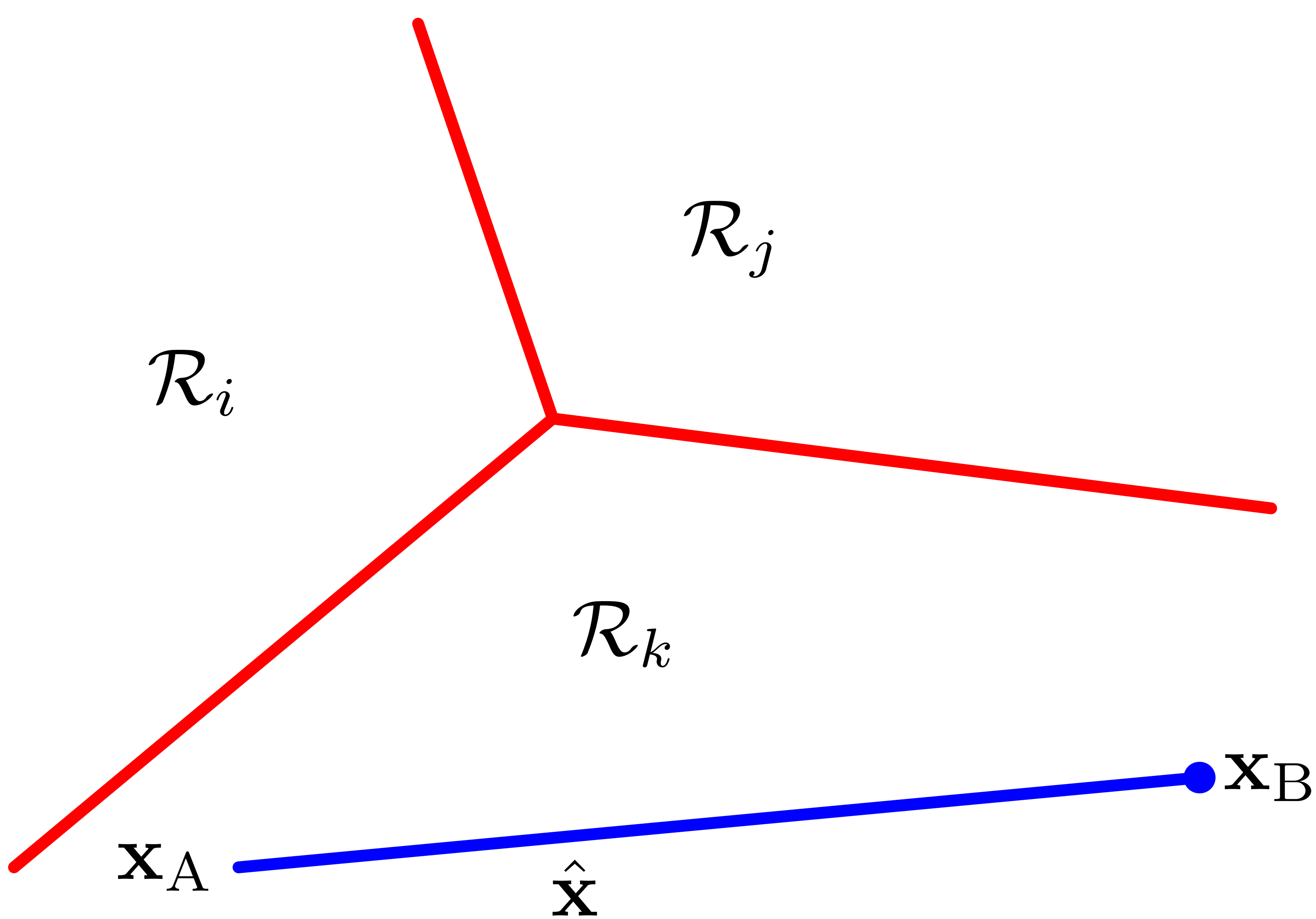

If two points $\mathbf{x}_\mathrm{A}$ and $\mathbf{x}_\mathrm{B}$ both lie inside the same decision region R_k, then any point $\mathbf{x}$ that lies on the line connecting these two points must also lie in R_k, and hence the decision region must be singly connected and convex.

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Fisher's Linear Discriminant

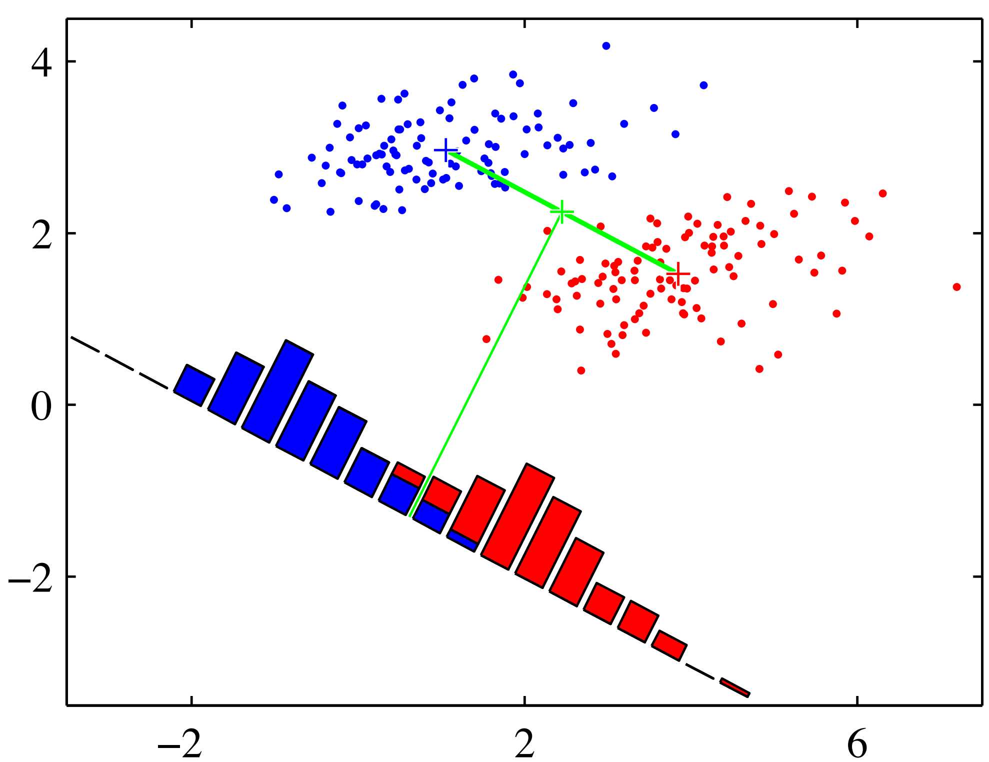
- Pursue the optimal linear projection on which the two classes can be maximally separated

$$y = \mathbf{w}^T \mathbf{x}$$

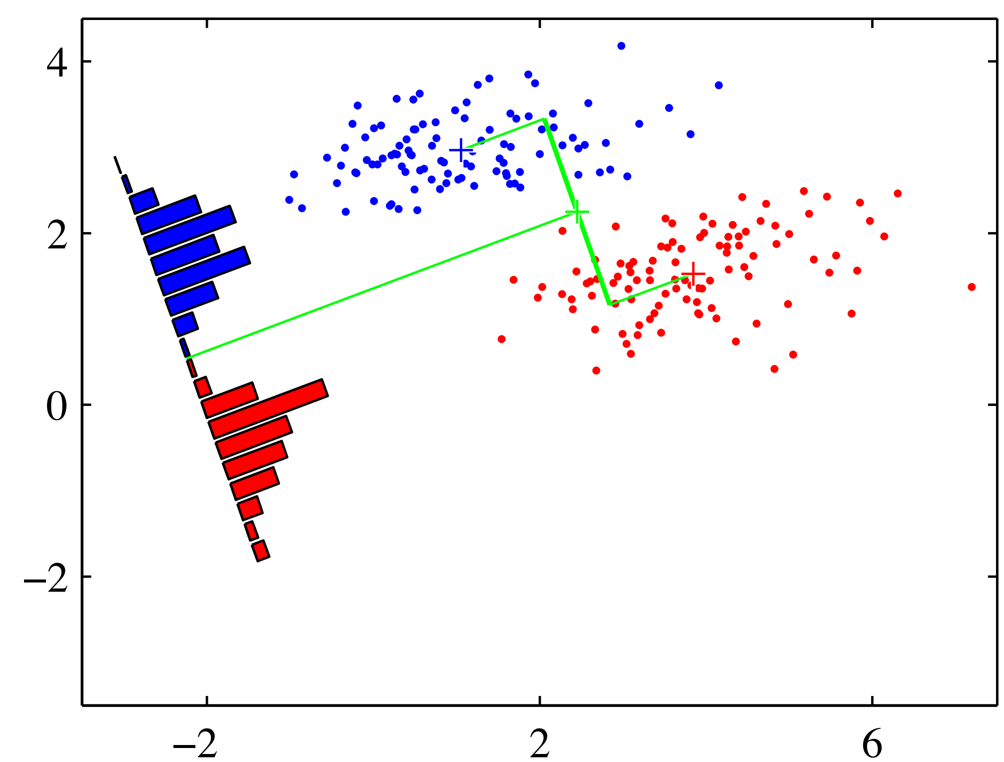
- The mean vectors of the two classes

$$\mathbf{m}_1 = \frac{1}{N_1} \sum_{n \in \mathcal{C}_1} \mathbf{x}_n, \quad \mathbf{m}_2 = \frac{1}{N_2} \sum_{n \in \mathcal{C}_2} \mathbf{x}_n$$

A way to view a linear classification model is in terms of dimensionality reduction.



Difference of means



Fisher's Linear Discriminant

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What's a Good Projection?

- After projection, the two classes are separated as much as possible. Measured by the distance between projected center

$$\left(\mathbf{w}^T(\mathbf{m}_1 - \mathbf{m}_2)\right)^2 = \mathbf{w}^T(\mathbf{m}_1 - \mathbf{m}_2)(\mathbf{m}_1 - \mathbf{m}_2)^T\mathbf{w}$$
$$= \mathbf{w}^T\mathbf{S}_B\mathbf{w}$$

where $\mathbf{S}_B = (\mathbf{m}_1 - \mathbf{m}_2)(\mathbf{m}_1 - \mathbf{m}_2)^T$ is called **between-class** covariance matrix.

- After projection, the variances of the two classes are as small as possible. Measured by the within-class covariance

$$\mathbf{w}^T\mathbf{S}_W\mathbf{w}$$

where

$$\mathbf{S}_W = \sum_{n\in\mathcal{C}_1}(\mathbf{x}_n - \mathbf{m}_1)(\mathbf{x}_n - \mathbf{m}_1)^T + \sum_{n\in\mathcal{C}_2}(\mathbf{x}_n - \mathbf{m}_2)(\mathbf{x}_n - \mathbf{m}_2)^T$$

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Fisher's Linear Discriminant

- Fisher criterion: maximize the ratio w.r.t. $\mathbf{w}$

$$J(\mathbf{w}) = \frac{\texttt{Between-class variance}}{\texttt{Within-class variance}} = \frac{\mathbf{w}^T \mathbf{S}_B \mathbf{w}}{\mathbf{w}^T \mathbf{S}_W \mathbf{w}}$$

- Recall the quotient rule: for $f(x) = \frac{g(x)}{h(x)}$

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{h^2(x)}$$

- Setting $\nabla J(\boldsymbol{w}) = 0$, we obtain

$$(\mathbf{w}^T \mathbf{S}_B \mathbf{w}) \mathbf{S}_W \mathbf{w} = (\mathbf{w}^T \mathbf{S}_W \mathbf{w}) \mathbf{S}_B \mathbf{w}$$

$$(\mathbf{w}^T \mathbf{S}_B \mathbf{w}) \mathbf{S}_W \mathbf{w} = (\mathbf{w}^T \mathbf{S}_W \mathbf{w}) (\mathbf{m}_2 - \mathbf{m}_1) \Big((\mathbf{m}_2 - \mathbf{m}_1)^T \mathbf{w} \Big)$$

- Terms $\mathbf{w}^T\mathbf{S}_B\mathbf{w}$, $\mathbf{w}^T\mathbf{S}_W\mathbf{w}$ and $(\mathbf{m}_2 - \mathbf{m}_1)^T\mathbf{w}$ are scalars, and we only care about directions. So the scalars are dropped. Therefore

$$\mathbf{w} \propto \mathbf{S}_W^{-1} (\mathbf{m}_2 - \mathbf{m}_1)$$

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From Fisher's Linear Discriminant to Classifiers

- Fisher's Linear Discriminant is not a classifier; it only decides on an optimal projection to convert high-dimensional classification problem to 1D.

- A bias (threshold) is needed to form a linear classifier (multiple thresholds lead to nonlinear classifiers). The final classifier has the form

$$y(\mathbf{x}) = \mathrm{sign}(\mathbf{w}^T\mathbf{x} + w_0)$$

  where the nonlinear activation function sign(·) is a step function

$$\mathrm{sign}(a) = \begin{cases} +1, & a \geqslant 0 \\ -1, & a < 0 \end{cases}$$

- How to decide the bias w_0?

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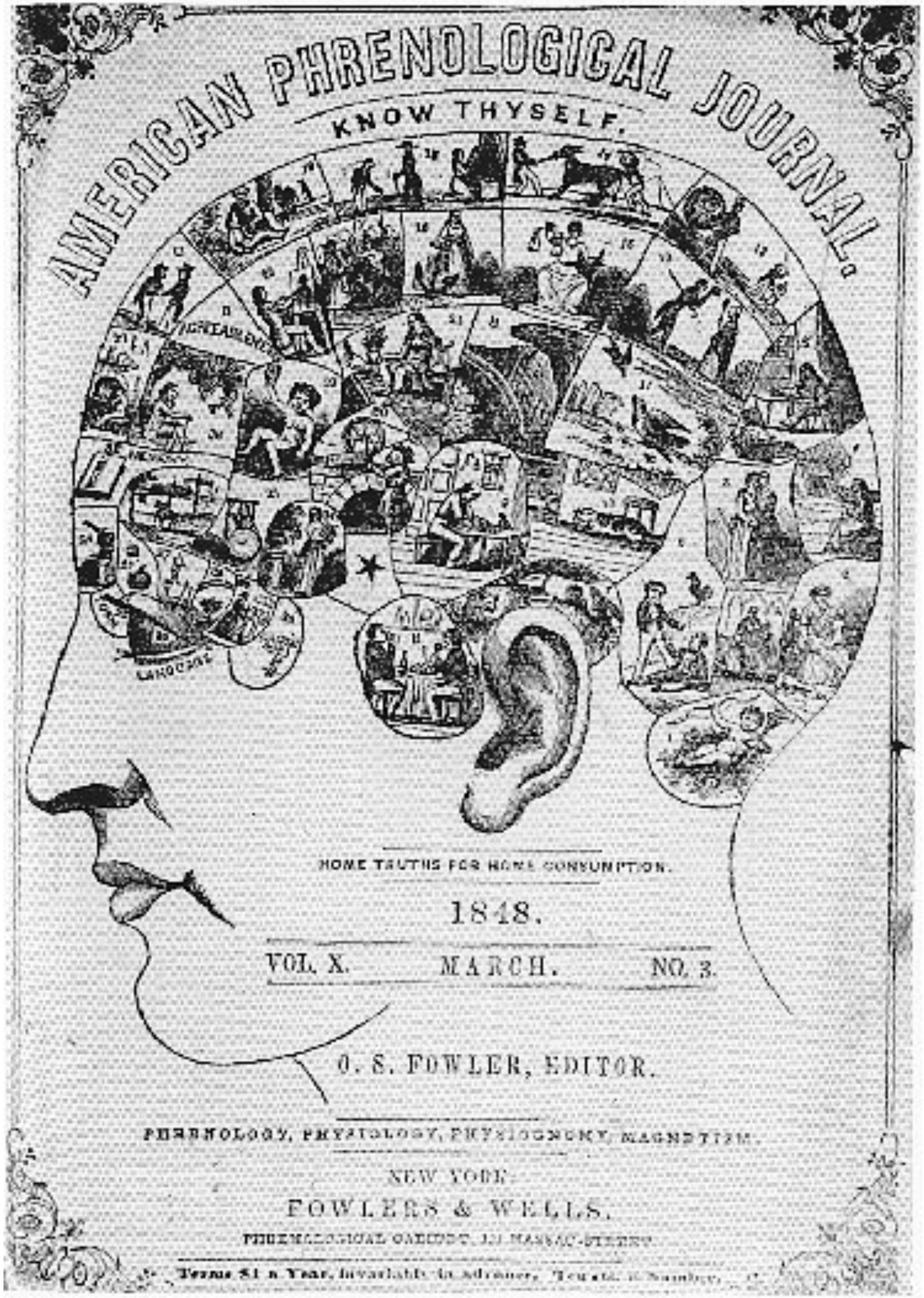
Perceptron



early theories
of the brain

slide by Alex Smola

Biology and Learning

- Basic Idea
  - Good behavior should be rewarded, bad behavior punished (or not rewarded). This improves system fitness.
  - Killing a sabertooth tiger should be rewarded ...
  - Correlated events should be combined.
  - Pavlov's salivating dog.

- Training mechanisms
  - Behavioral modification of individuals (learning)
    Successful behavior is rewarded (e.g. food).
  - Hard-coded behavior in the genes (instinct)
    The wrongly coded animal does not reproduce.

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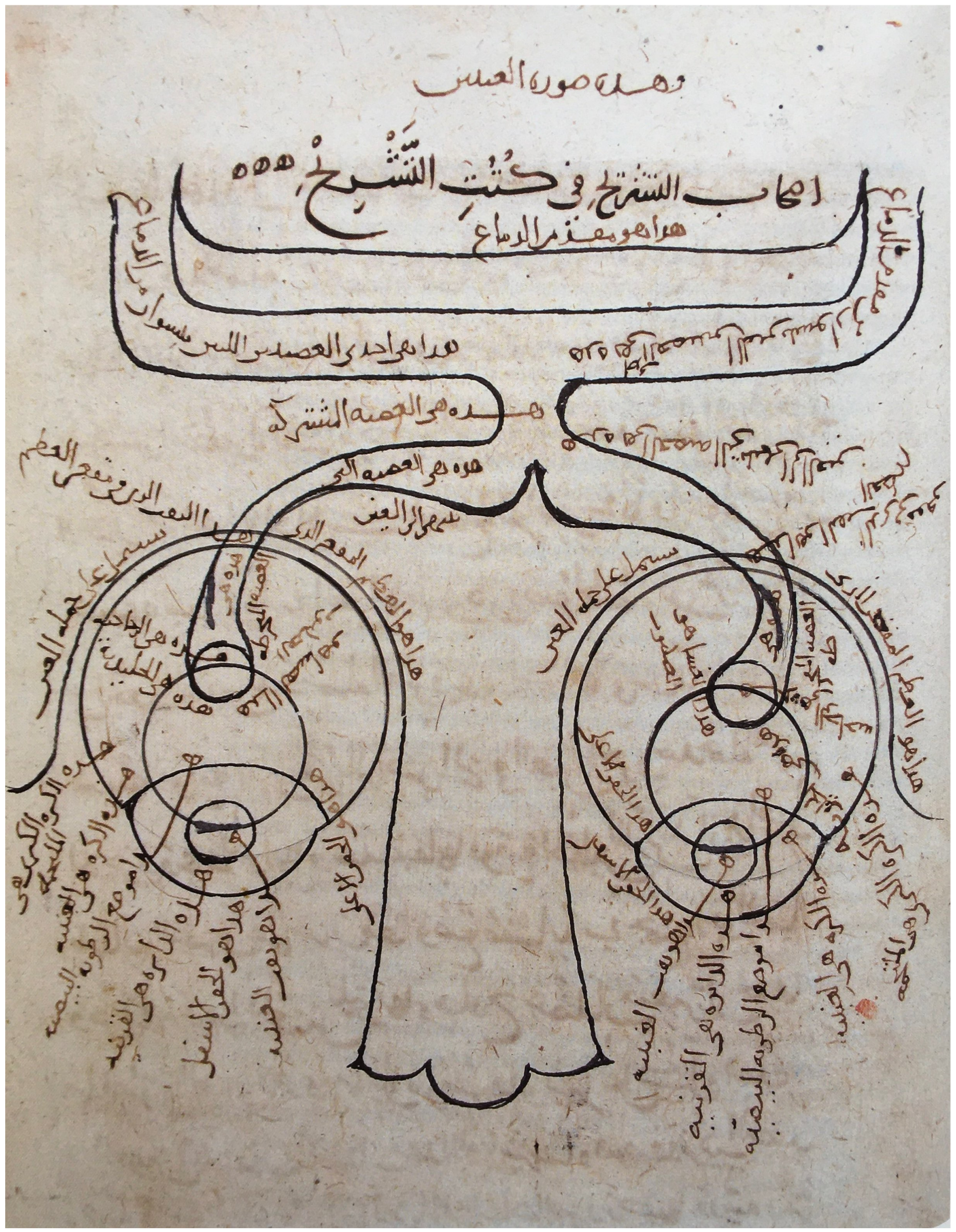
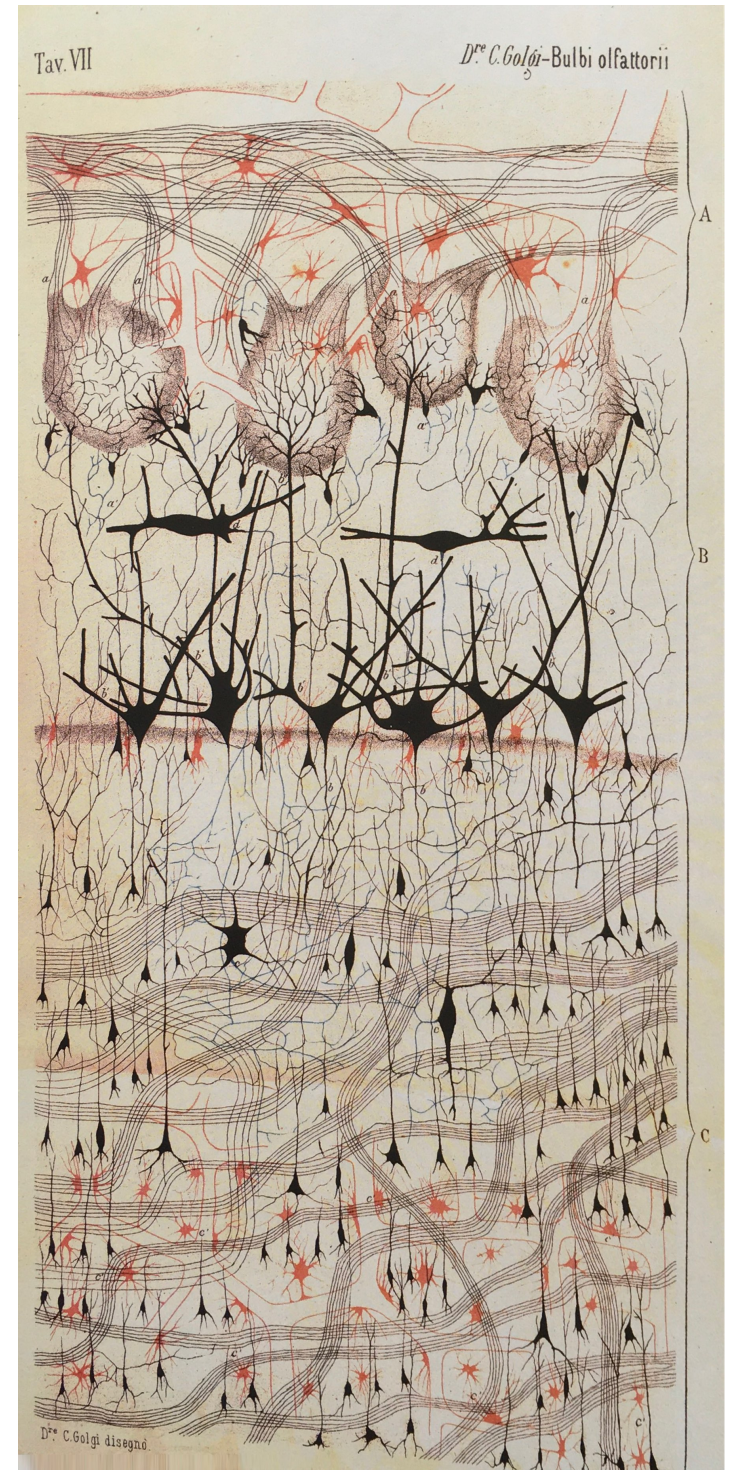
Nervous System



The oldest known drawing of the nervous system by Ibn al-Haytham (published in 1083)



Olfactory bulb, Camillo Golgi, 1875

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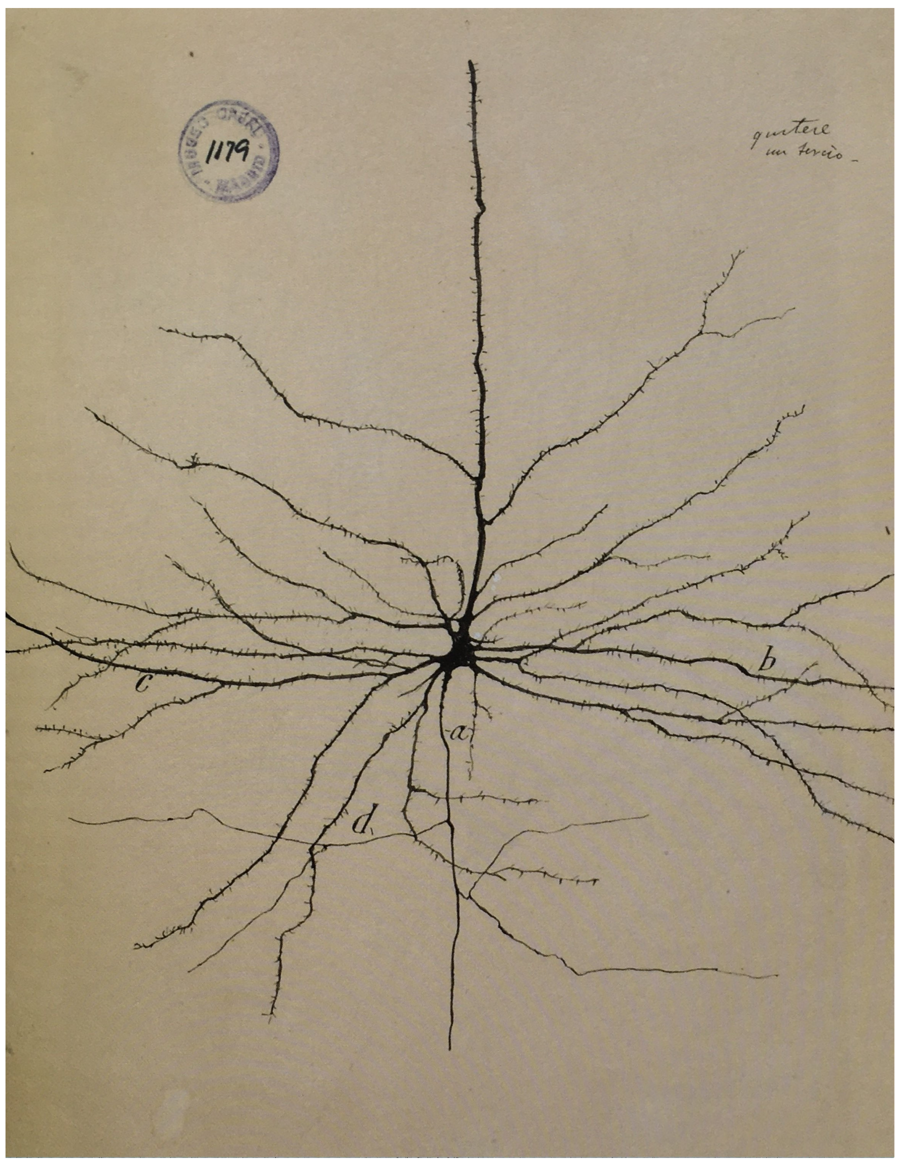
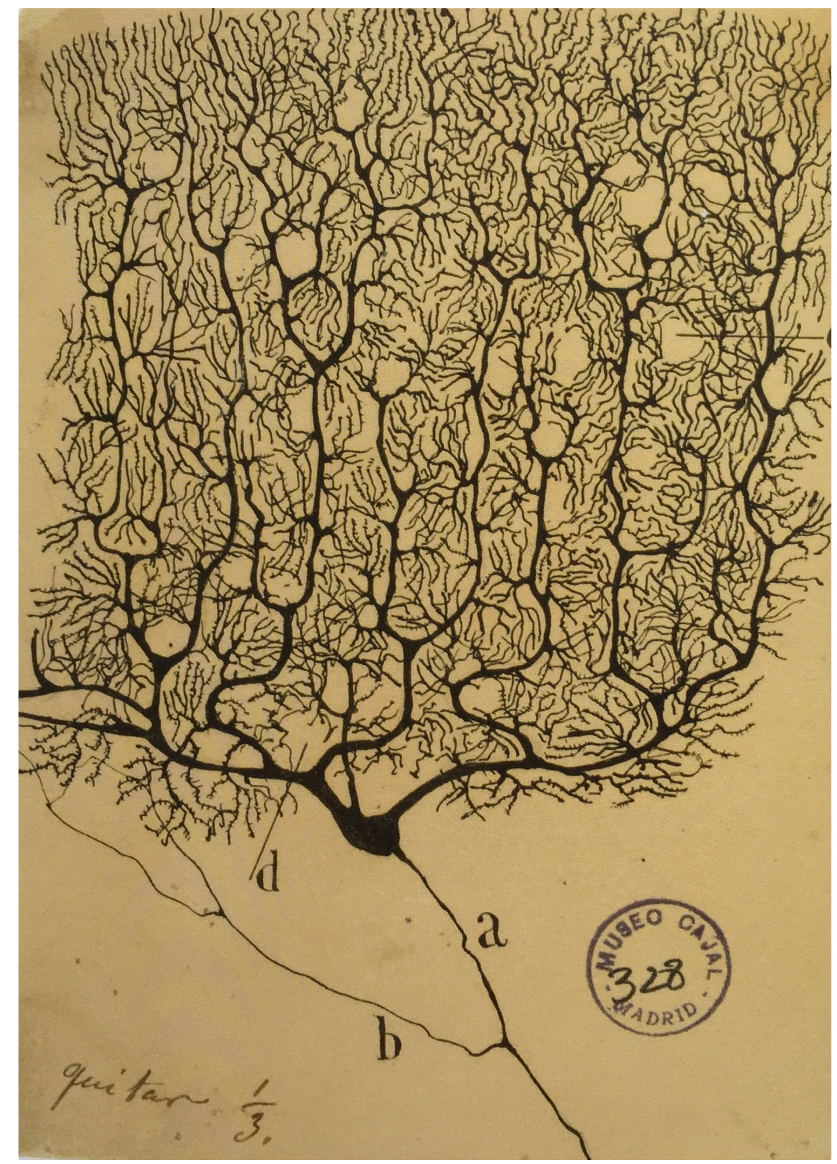
Santiago Ramón Y Cajal & The Neuron Doctrine

- Neuron as the discrete distinct entities in the brain as opposed to a continuous network.



pyramidal neuron. Cajal, 1899



purkinje neuron. Cajal, 1899

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Santiago Ramón Y Cajal & The Neuron Doctrine

The Nobel Prize in Physiology or Medicine 1906

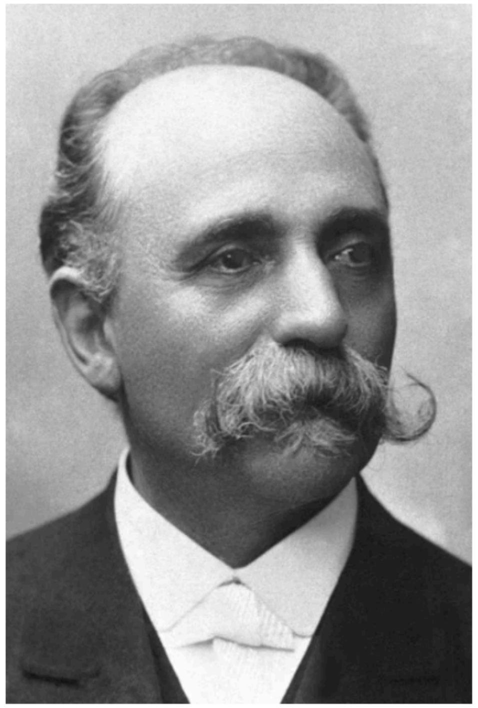


Photo from the Nobel Foundation archive.

Camillo Golgi

Prize share: 1/2

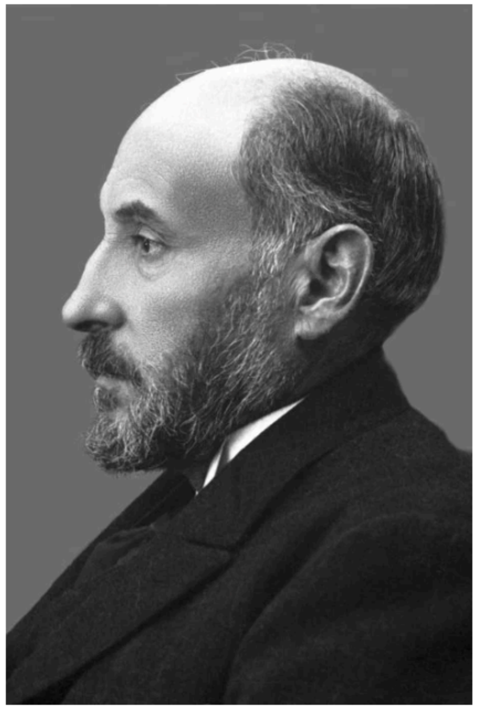


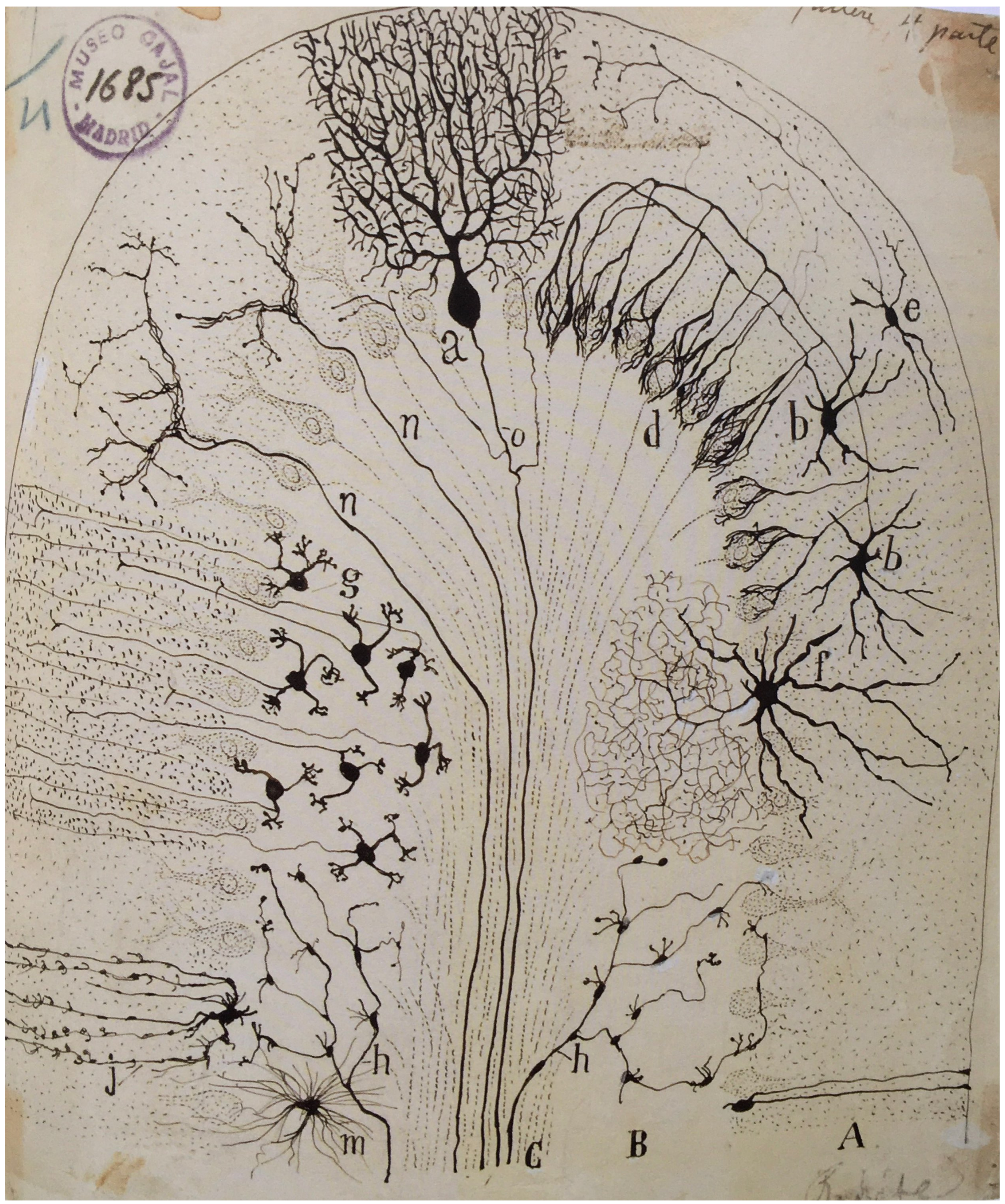
Photo from the Nobel Foundation archive.

Santiago Ramón y Cajal

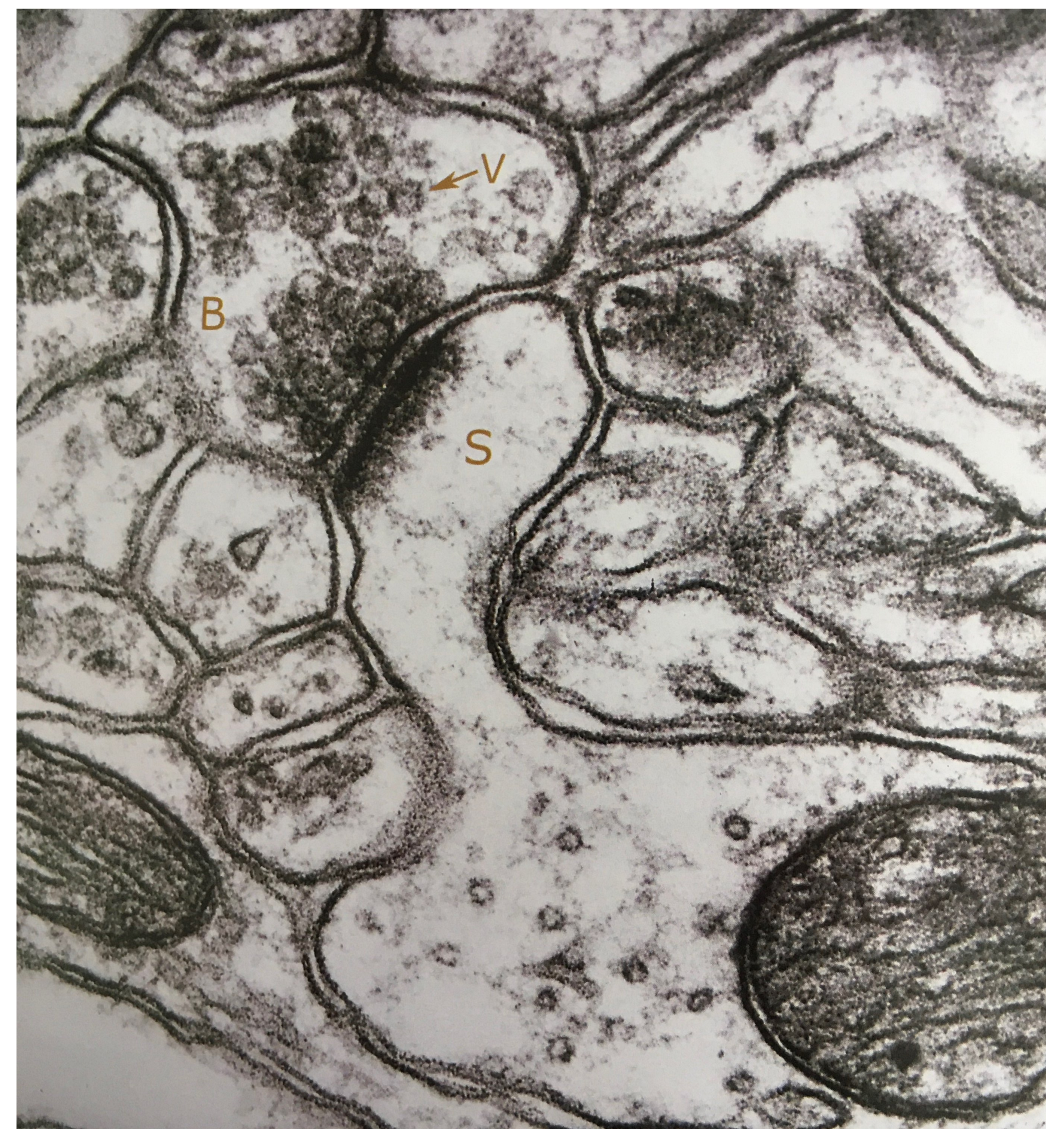
Prize share: 1/2

The Nobel Prize in Physiology or Medicine 1906 was awarded jointly to Camillo Golgi and Santiago Ramón y Cajal "in recognition of their work on the structure of the nervous system"

Network of Neurons:
Axons, Dendrites, and Synapses



network of neurons. Cajal, 1899



Synapse. Spacek and Harris, 2000

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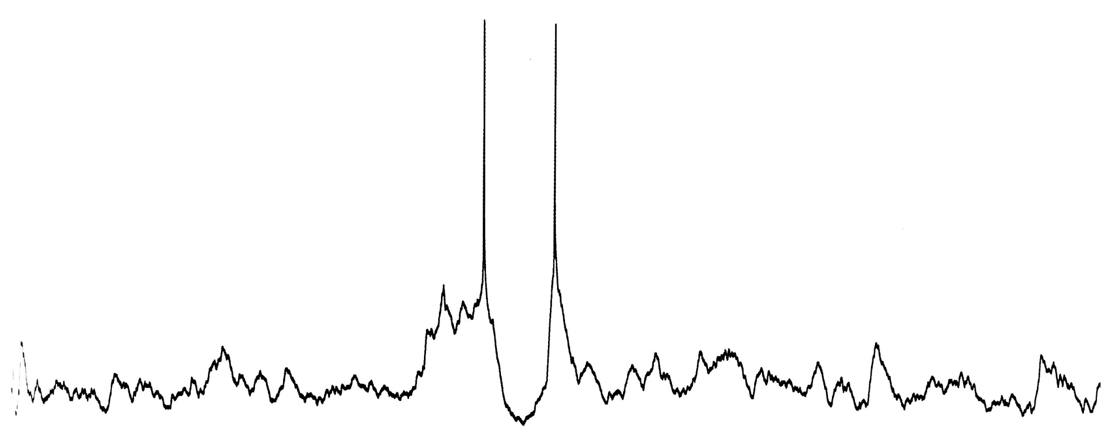
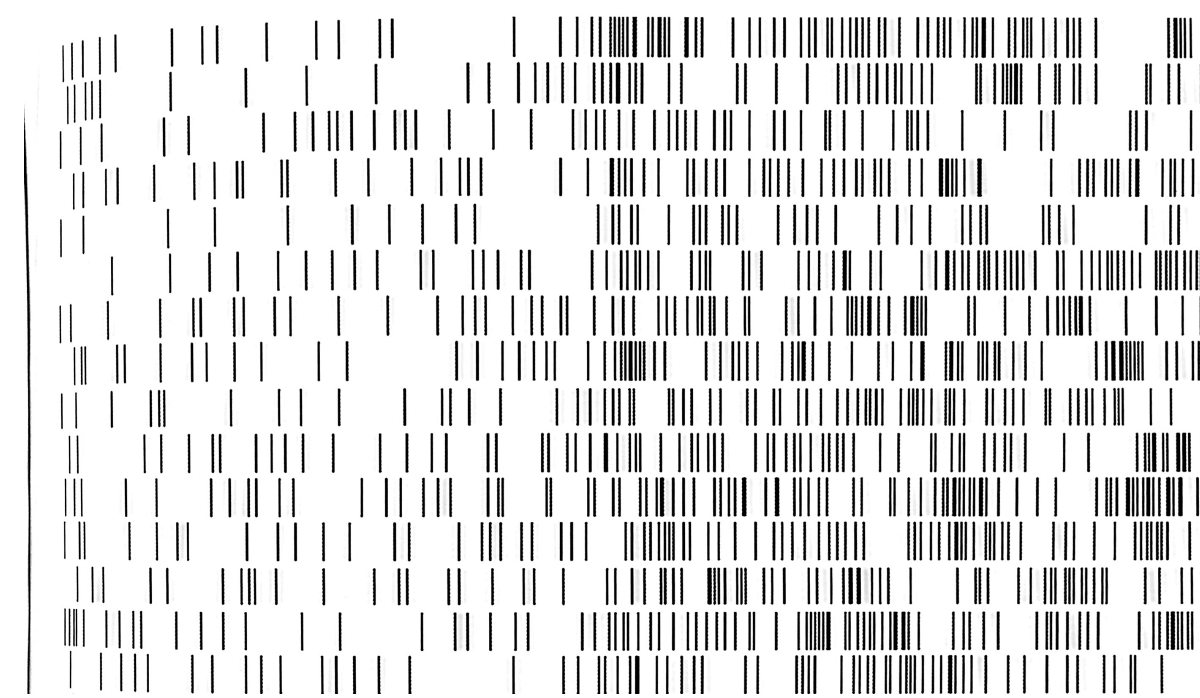
Electricity in the brain



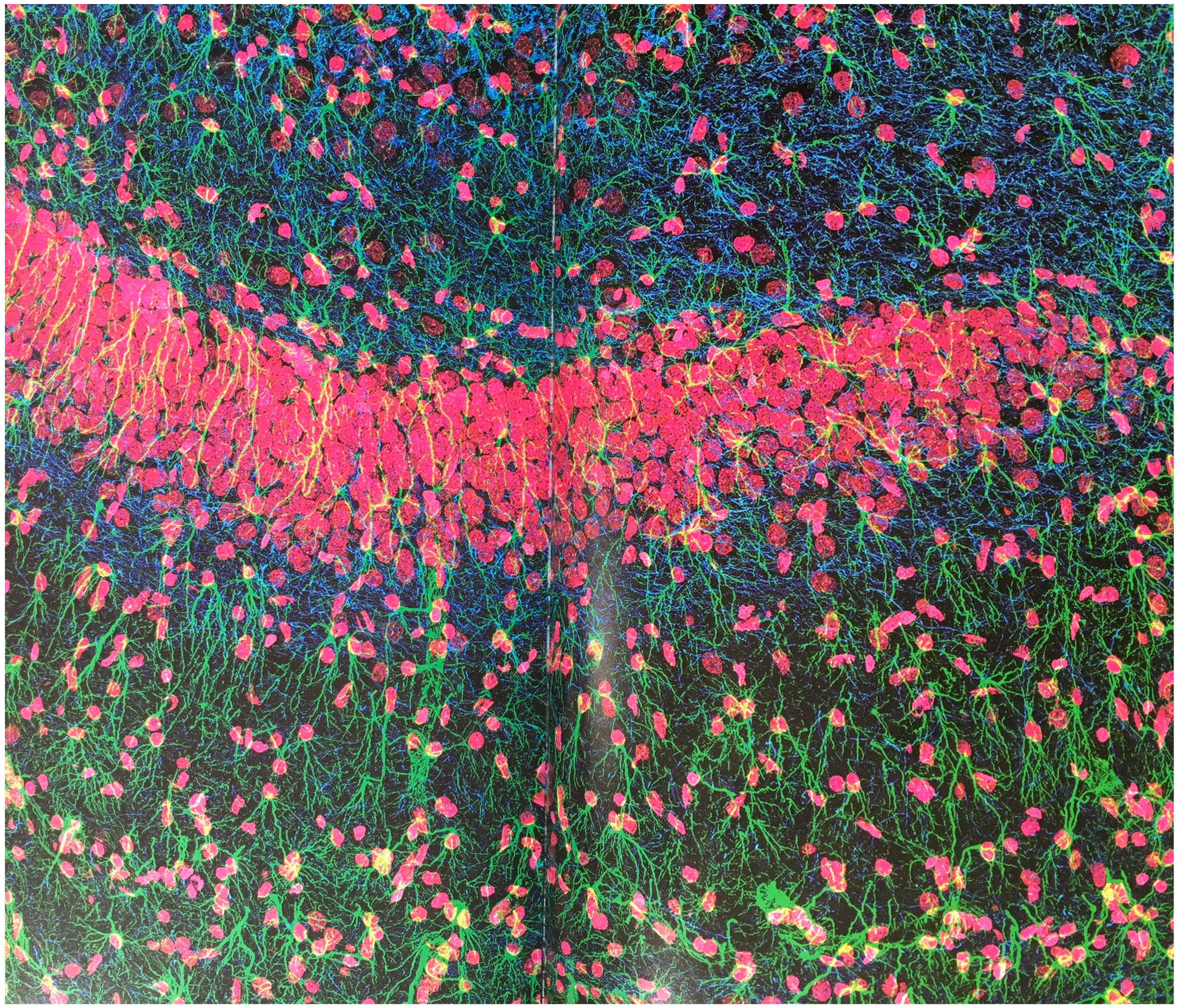
Whole-cell recording in an awake rat. Contantinople and Bruno, 2009



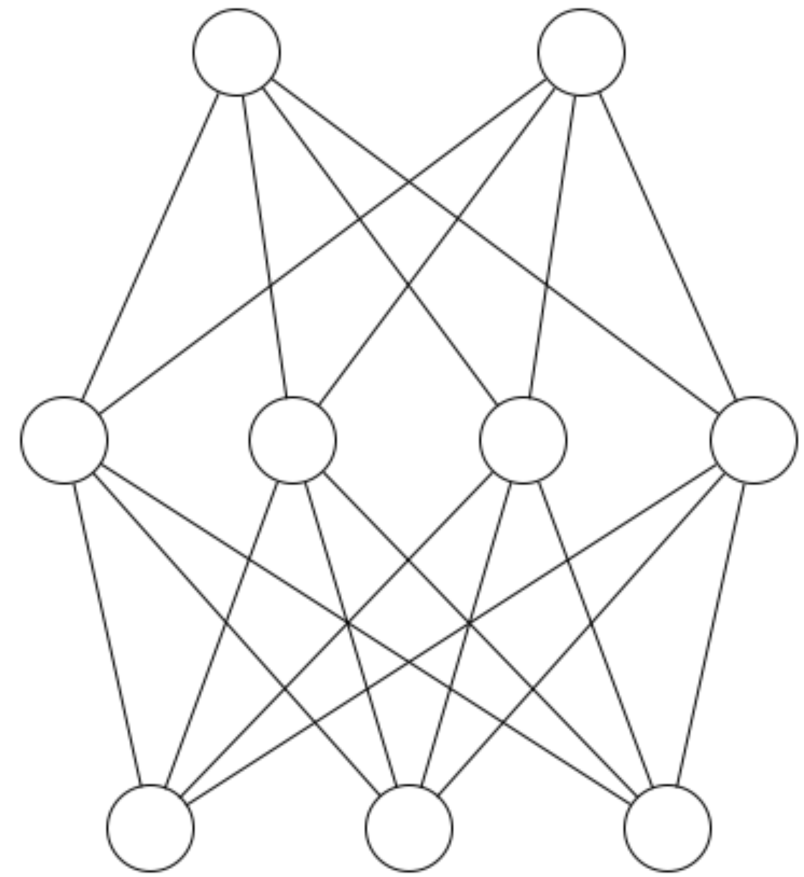
Action potentials in a live monkey brain. Saez and Salzman, 2009. (Each row, 4 seconds.)

slide by Saeed Saremi

Electricity in the brain



human brain: 10^{11} neurons, 10^{15} synapses



GPT-4: 10^{6} neurons, 10^{11} parameters (weights), 100 layers

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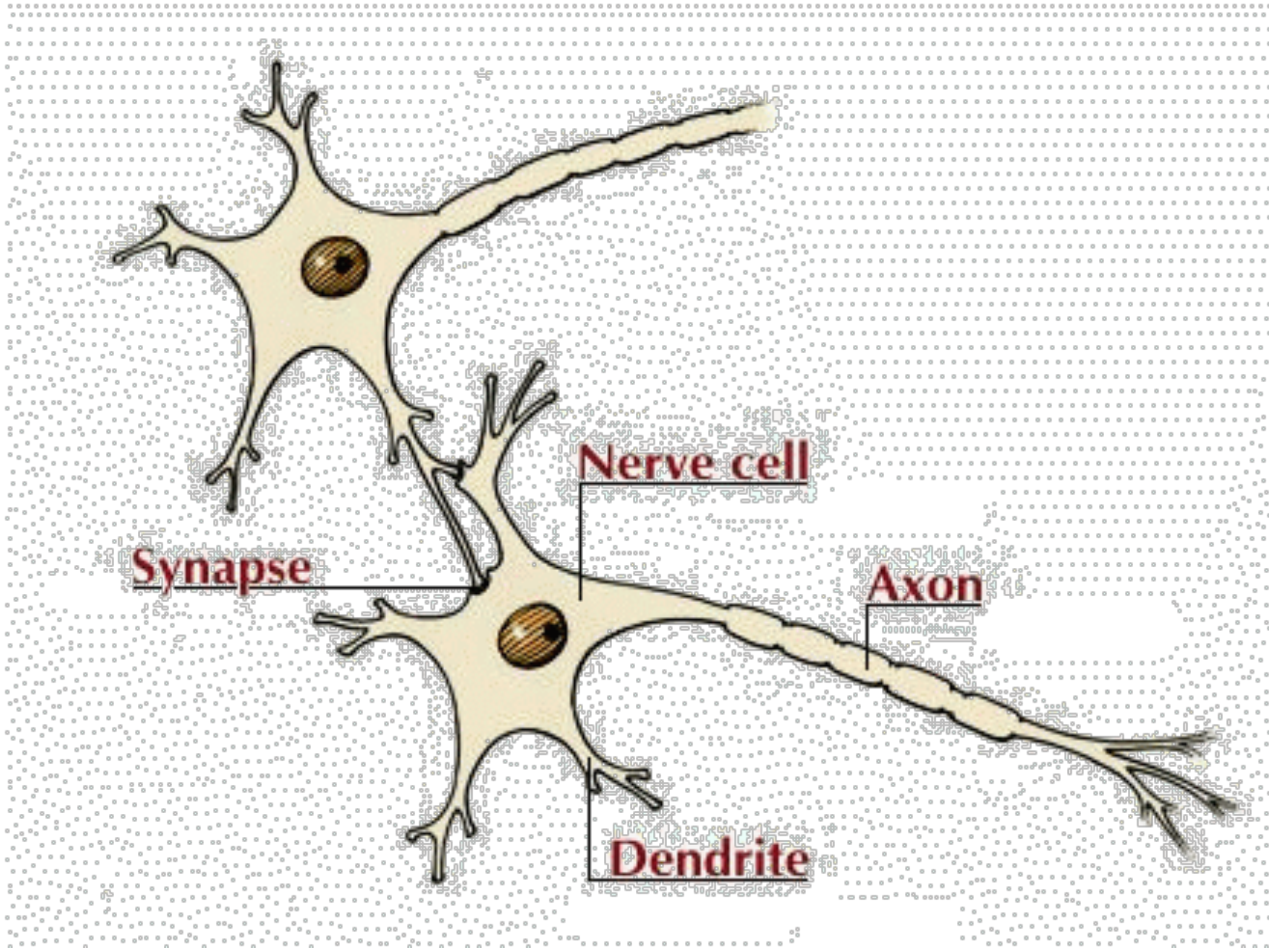
Neurons



- Soma (CPU)
  Cell body - combines signals

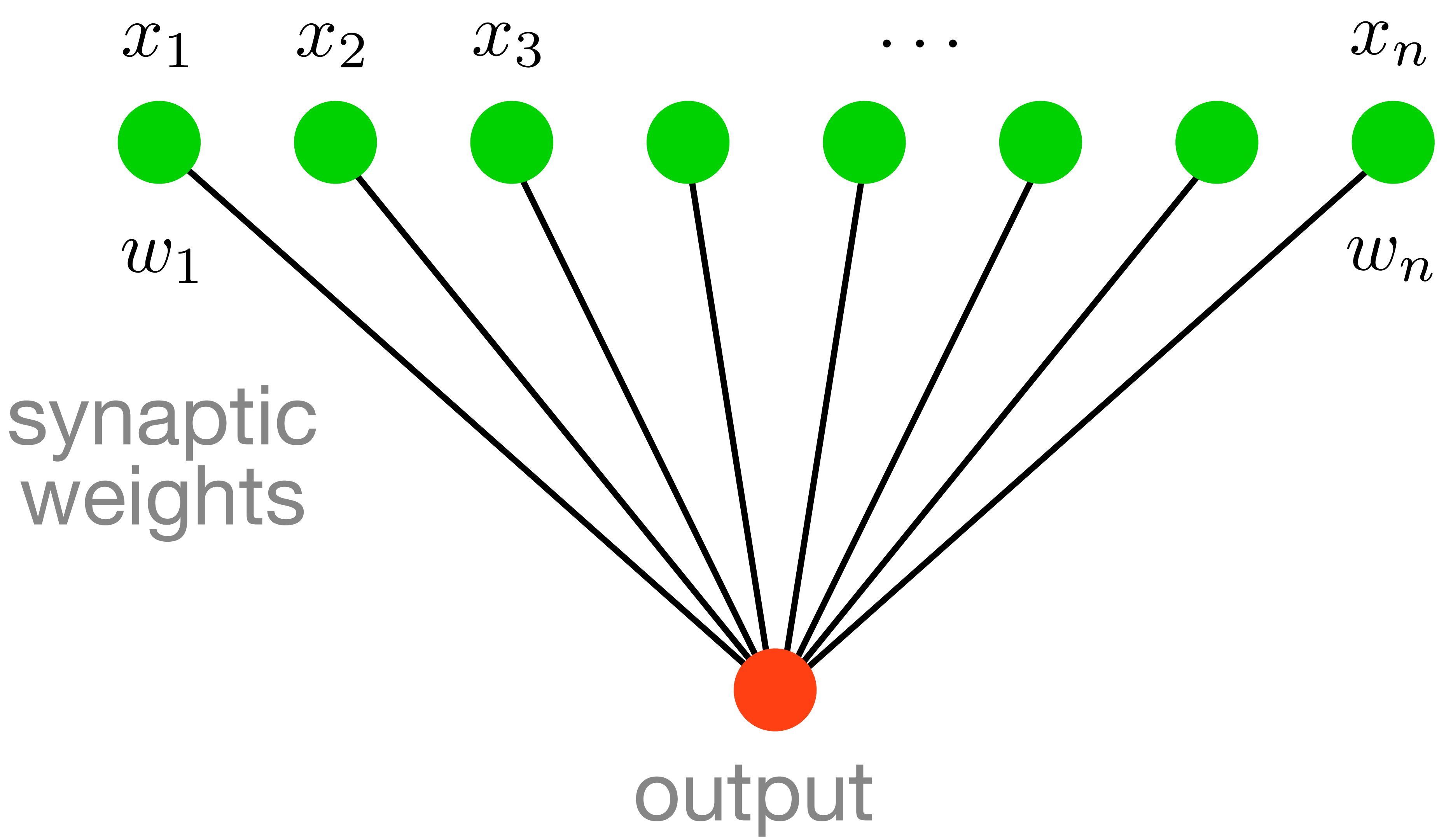
- Dendrite (input bus)
  Combines the inputs from several other nerve cells

- Synapse (interface)
  Interface and parameter store between neurons

- Axon (cable)
  May be up to 1m long and will transport the activation signal to neurons at different locations

slide by Alex Smola

Neurons



$$f(x) = \sum_i w_i x_i = \langle w, x \rangle$$

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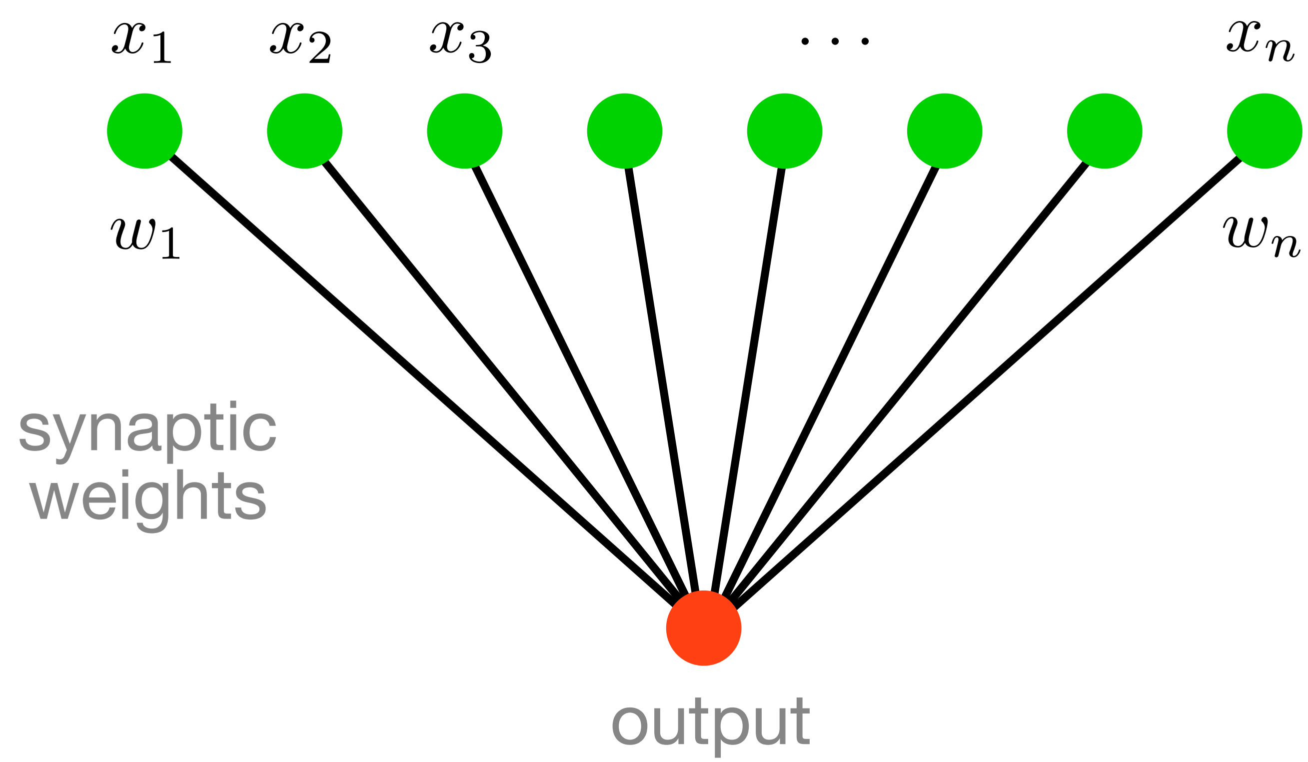
Perceptron

- Weighted linear combination
- Nonlinear decision function
- Linear offset (bias)



$$f(x) = \sigma\left(\langle w, x\rangle + b\right)$$

- Linear separating hyperplanes (spam/ham, novel/typical, click/no click)
- Learning
  Estimating the parameters w and b

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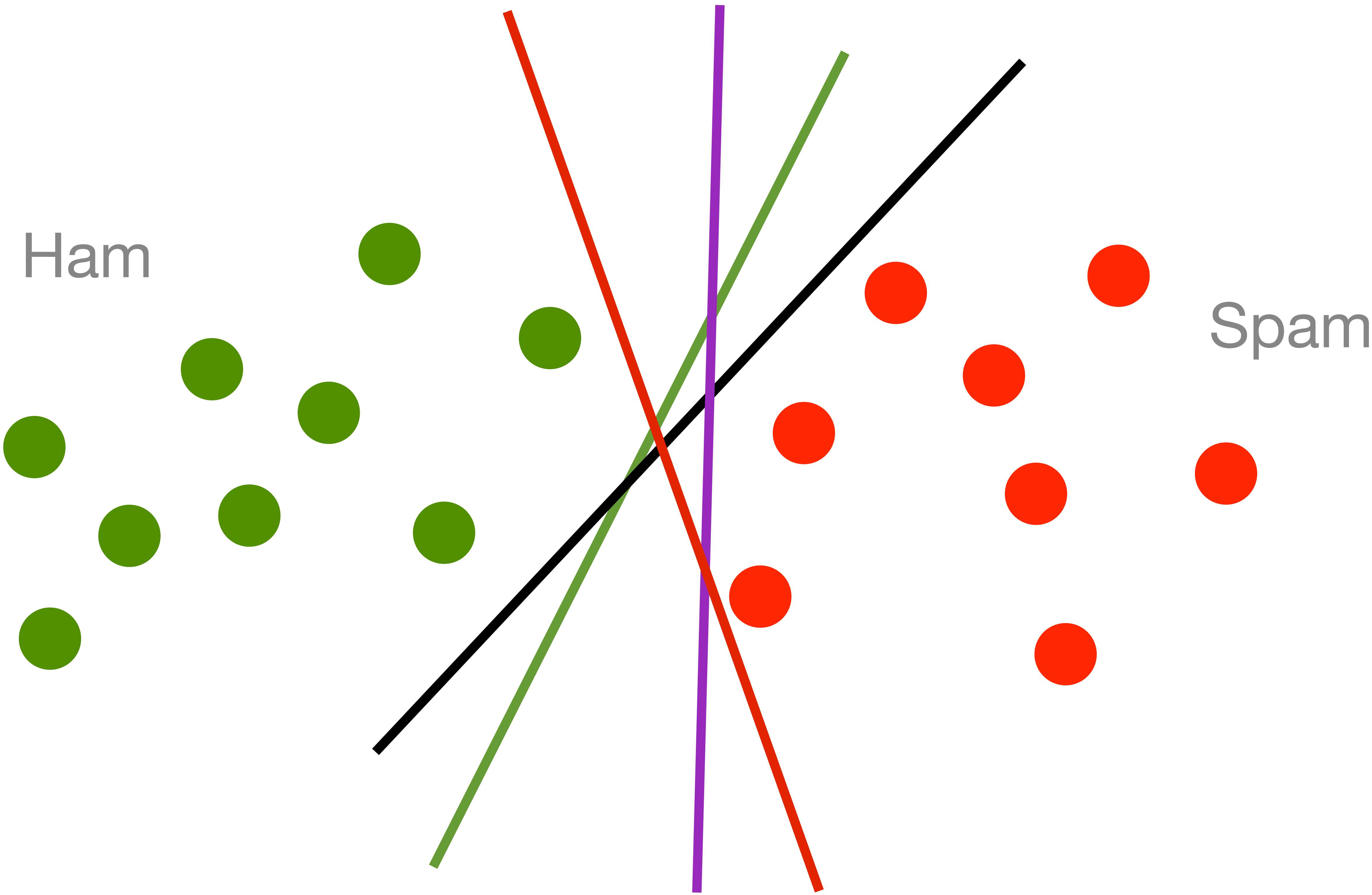
Perceptron



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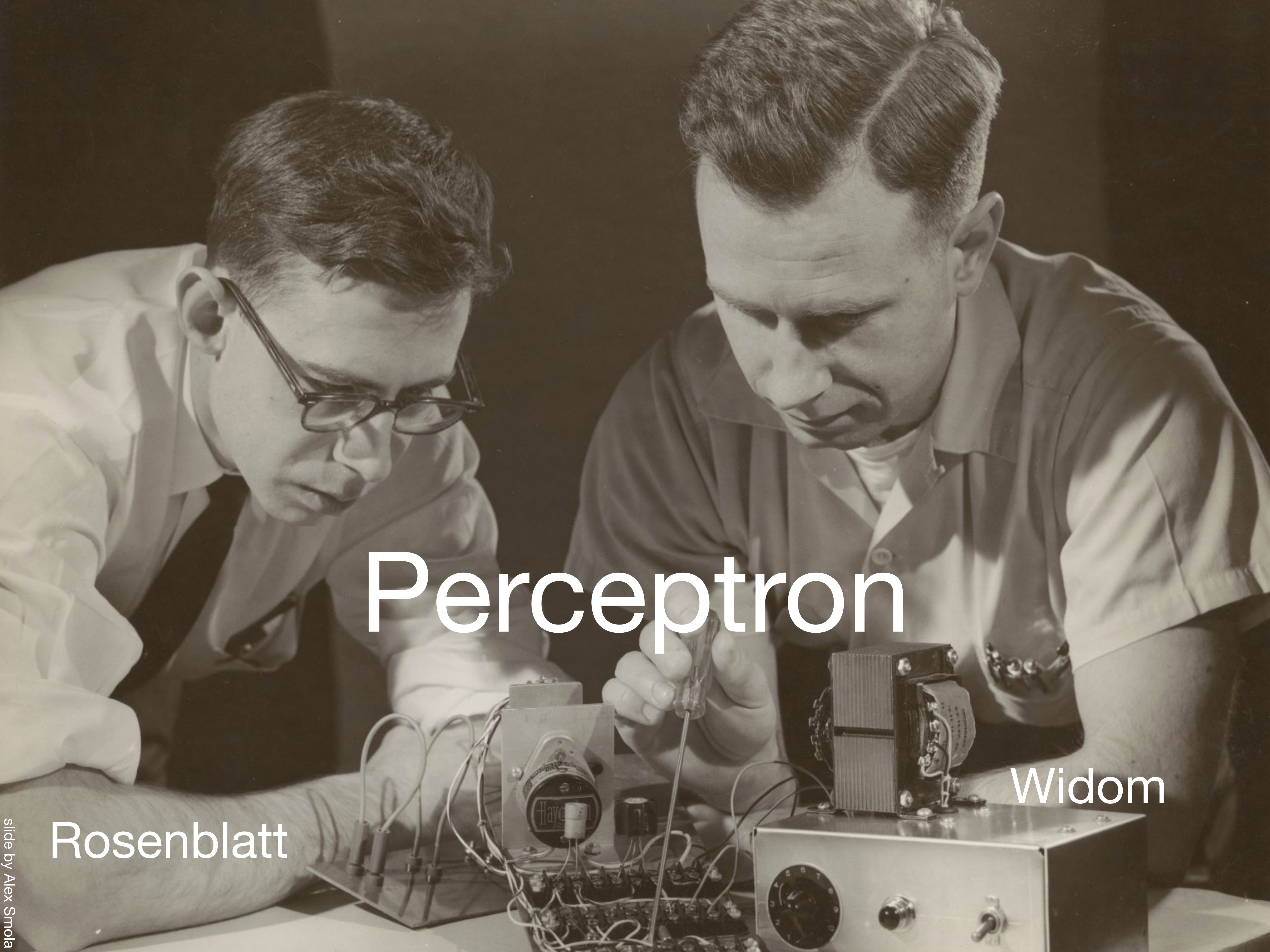
Perceptron
Widom
Rosenblatt
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Perceptron Inductive Bias

1. Decision boundary should be linear
2. Most recent mistakes are most important (and should be corrected)

Background: Hyperplanes

Notation Trick: fold the bias *b* and the weights ***w*** into a single vector **θ** by prepending a constant to ***x*** and increasing dimensionality by one to get **x'**!

Hyperplane (Definition 1):

$$\mathcal{H} = \{\mathbf{x} : \mathbf{w}^T\mathbf{x} = b\}$$

Hyperplane (Definition 2):

$$\mathcal{H} = \{\mathbf{x'} : \boldsymbol{\theta}^T\mathbf{x'} = 0 \text{ and } x'_1 = 1\}$$

$$\boldsymbol{\theta} = [b, w_1, \ldots, w_M]^T$$

Half-spaces:

$$\mathcal{H}^+ = \{\mathbf{x} : \boldsymbol{\theta}^T\mathbf{x} > 0 \text{ and } x_1 = 1\}$$

$$\mathcal{H}^- = \{\mathbf{x} : \boldsymbol{\theta}^T\mathbf{x} < 0 \text{ and } x_1 = 1\}$$

(Online) Perceptron Algorithm

Data: Inputs are continuous vectors of length *M*. Outputs are discrete.

$$(\mathbf{x}^{(1)}, y^{(1)}), (\mathbf{x}^{(2)}, y^{(2)}), \ldots$$

where $\mathbf{x} \in \mathbb{R}^M$ and $y \in \{+1, -1\}$

Prediction: Output determined by hyperplane.

$$\hat{y} = h_{\boldsymbol{\theta}}(\mathbf{x}) = \text{sign}(\boldsymbol{\theta}^T \mathbf{x})$$

$$\text{sign}(a) = \begin{cases} 1, & \text{if } a \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

Assume $\boldsymbol{\theta} = [b, w_1, \ldots, w_M]^T$ and $x_1 = 1$

Learning: Iterative procedure:

- **initialize** parameters to vector of all zeroes
- **while** not converged
  - **receive** next example ($\mathbf{x}^{(i)}$, $y^{(i)}$)
  - **predict** $y' = h(\mathbf{x}^{(i)})$
  - **if** positive mistake: **add** $\mathbf{x}^{(i)}$ to parameters
  - **if** negative mistake: **subtract** $\mathbf{x}^{(i)}$ from parameters

slide by Matt Gormley & Henry Chai

(Online) Perceptron Algorithm

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Assume $\boldsymbol{\theta} = [b, w_1, \ldots, w_M]^T$ and $x_1 = 1$

Learning:

Algorithm 1 Perceptron Learning Algorithm (Online)

```
procedure PERCEPTRON(𝒟 = {(x^(1), y^(1)), (x^(2), y^(2)), ...})
    θ ← 0                                 ▷ Initialize parameters
    for i ∈ {1, 2, ...} do                ▷ For each example
        ŷ ← sign(θ^T x^(i))               ▷ Predict
        if ŷ ≠ y^(i) then                 ▷ If mistake
            θ ← θ + y^(i) x^(i)           ▷ Update parameters
    return θ
```

(Online) Perceptron Algorithm

Data: Inputs are continuous vectors of length *M*. Outputs are discrete.

$$(\mathbf{x}^{(1)}, y^{(1)}), (\mathbf{x}^{(2)}, y^{(2)}), \ldots$$

where $\mathbf{x} \in \mathbb{R}^M$ and $y \in \{+1, -1\}$

Prediction: Output determine

$$\hat{y} = h_{\boldsymbol{\theta}}(\mathbf{x}) = \text{sign}(\boldsymbol{\theta}^T \mathbf{x})$$

Assume $\boldsymbol{\theta} = [b, w_1, \ldots, w_M]$

Learning:

Algorithm 1 Perceptron Learning Alg

procedure PERCEPTRON($\mathcal{D} = \{(\mathbf{x}$

$\boldsymbol{\theta} \leftarrow \mathbf{0}$ ▷ Initialize parameters

for $i \in \{1, 2, \ldots\}$ **do** ▷ For each example

$\hat{y} \leftarrow$ sign$(\boldsymbol{\theta}^T \mathbf{x}^{(i)})$ ▷ Predict

if $\hat{y} \neq y^{(i)}$ **then** ▷ If mistake

$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + y^{(i)} \mathbf{x}^{(i)}$ ▷ Update parameters

return $\boldsymbol{\theta}$

Implementation Trick: same behavior as our "*add on positive mistake and subtract on negative mistake*" version, because $y^{(i)}$ takes care of the sign

slide by Matt Gormley & Henry Chai

(Batch) Perceptron Algorithm

Learning for Perceptron also works if we have a fixed training dataset, D. We call this the "batch" setting in contrast to the "online" setting that we've discussed so far.

Algorithm 1 Perceptron Learning Algorithm (Batch)

```
procedure PERCEPTRON(𝒟 = {(x^(1), y^(1)), ..., (x^(N), y^(N))})
    θ ← 0                                   ▷ Initialize parameters
    while not converged do
        for i ∈ {1, 2, ..., N} do            ▷ For each example
            ŷ ← sign(θ^T x^(i))              ▷ Predict
            if ŷ ≠ y^(i) then                ▷ If mistake
                θ ← θ + y^(i) x^(i)          ▷ Update parameters
    return θ
```

$$\textbf{procedure } \textsc{Perceptron}(\mathcal{D} = \{(\mathbf{x}^{(1)}, y^{(1)}), \ldots, (\mathbf{x}^{(N)}, y^{(N)})\})$$
$$\boldsymbol{\theta} \leftarrow \mathbf{0} \quad \triangleright \text{ Initialize parameters}$$
$$\textbf{while } \text{not converged } \textbf{do}$$
$$\textbf{for } i \in \{1, 2, \ldots, N\} \textbf{ do} \quad \triangleright \text{ For each example}$$
$$\hat{y} \leftarrow \text{sign}(\boldsymbol{\theta}^T \mathbf{x}^{(i)}) \quad \triangleright \text{ Predict}$$
$$\textbf{if } \hat{y} \neq y^{(i)} \textbf{ then} \quad \triangleright \text{ If mistake}$$
$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + y^{(i)} \mathbf{x}^{(i)} \quad \triangleright \text{ Update parameters}$$
$$\textbf{return } \boldsymbol{\theta}$$

(Batch) Perceptron Algorithm

Learning for Perceptron also works if we have a fixed training dataset, D. We call this the "batch" setting in contrast to the "online" setting that we've discussed so far.

Discussion:

The Batch Perceptron Algorithm can be derived in two ways.

1. By extending the online Perceptron algorithm to the batch setting (as mentioned above)
2. By applying **Stochastic Gradient Descent (SGD)** to minimize a so-called **Hinge Loss** on a linear separator

The Perceptron

initialize $w = 0$ and $b = 0$
repeat
  if $y_i \left[\langle w, x_i \rangle + b\right] \leq 0$ **then**
    $w \leftarrow w + y_i x_i$ and $b \leftarrow b + y_i$
  end if
until all classified correctly

- Nothing happens if classified correctly
- Weight vector is linear combination $w = \sum_{i \in I} y_i x_i$
- Classifier is linear combination of inner products $f(x) = \sum_{i \in I} y_i \langle x_i, x \rangle + b$

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Convergence Theorem

- If there exists some (w^*, b^*) with unit length and
$$y_i \left[\langle x_i, w^* \rangle + b^*\right] \geq \rho \text{ for all } i$$
then the perceptron converges to a linear separator after a number of steps bounded by
$$\left(b^{*2} + 1\right)\left(r^2 + 1\right)\rho^{-2} \text{ where } \|x_i\| \leq r$$

- Dimensionality independent
- Order independent (i.e. also worst case)
- Scales with 'difficulty' of problem

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Consequences

- Only need to store errors.
  This gives a compression bound for perceptron.
- Stochastic gradient descent on hinge loss
  $$l(x_i, y_i, w, b) = \max\left(0, 1 - y_i\left[\langle w, x_i\rangle + b\right]\right)$$
- Fails with noisy data

do NOT train your
avatar with perceptrons

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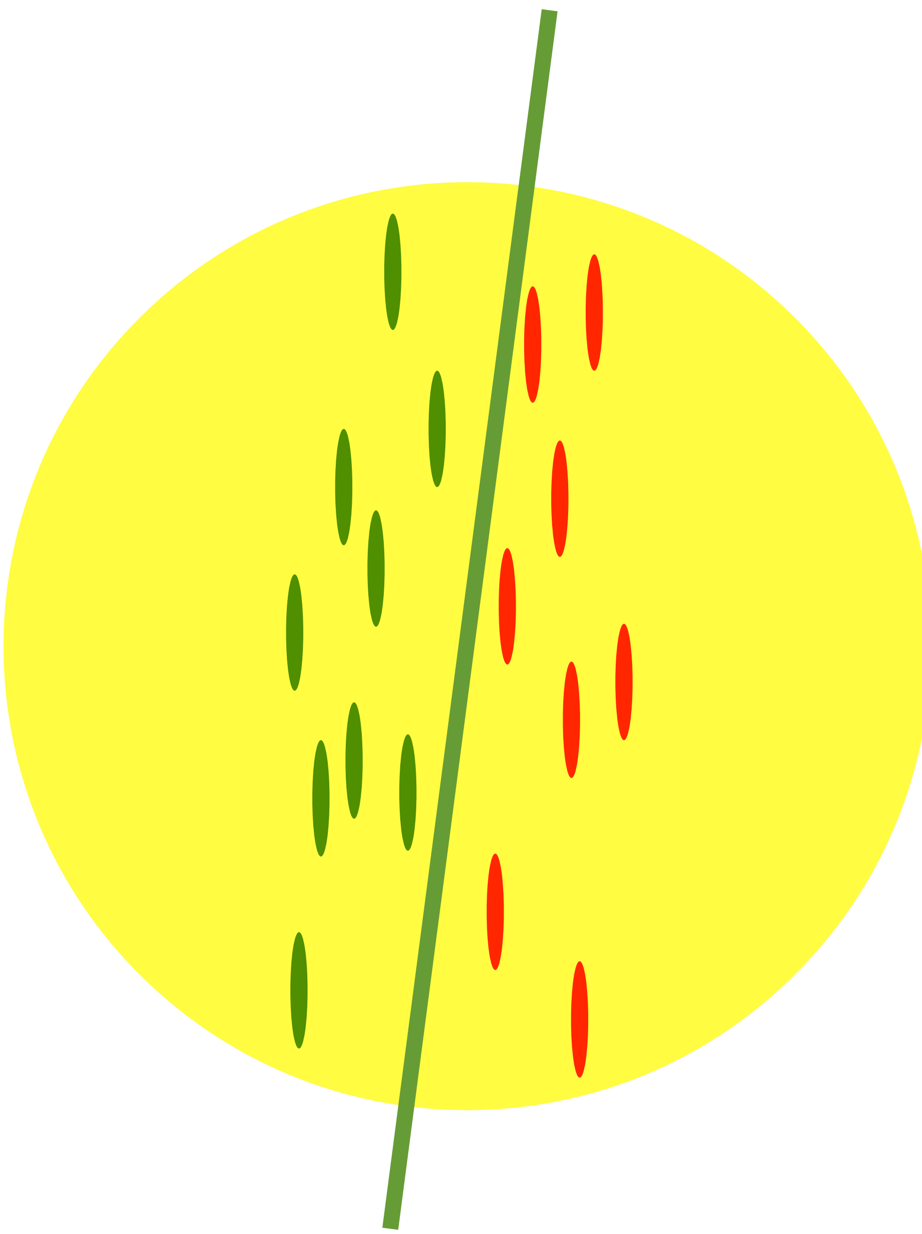
Hardness: margin vs. size



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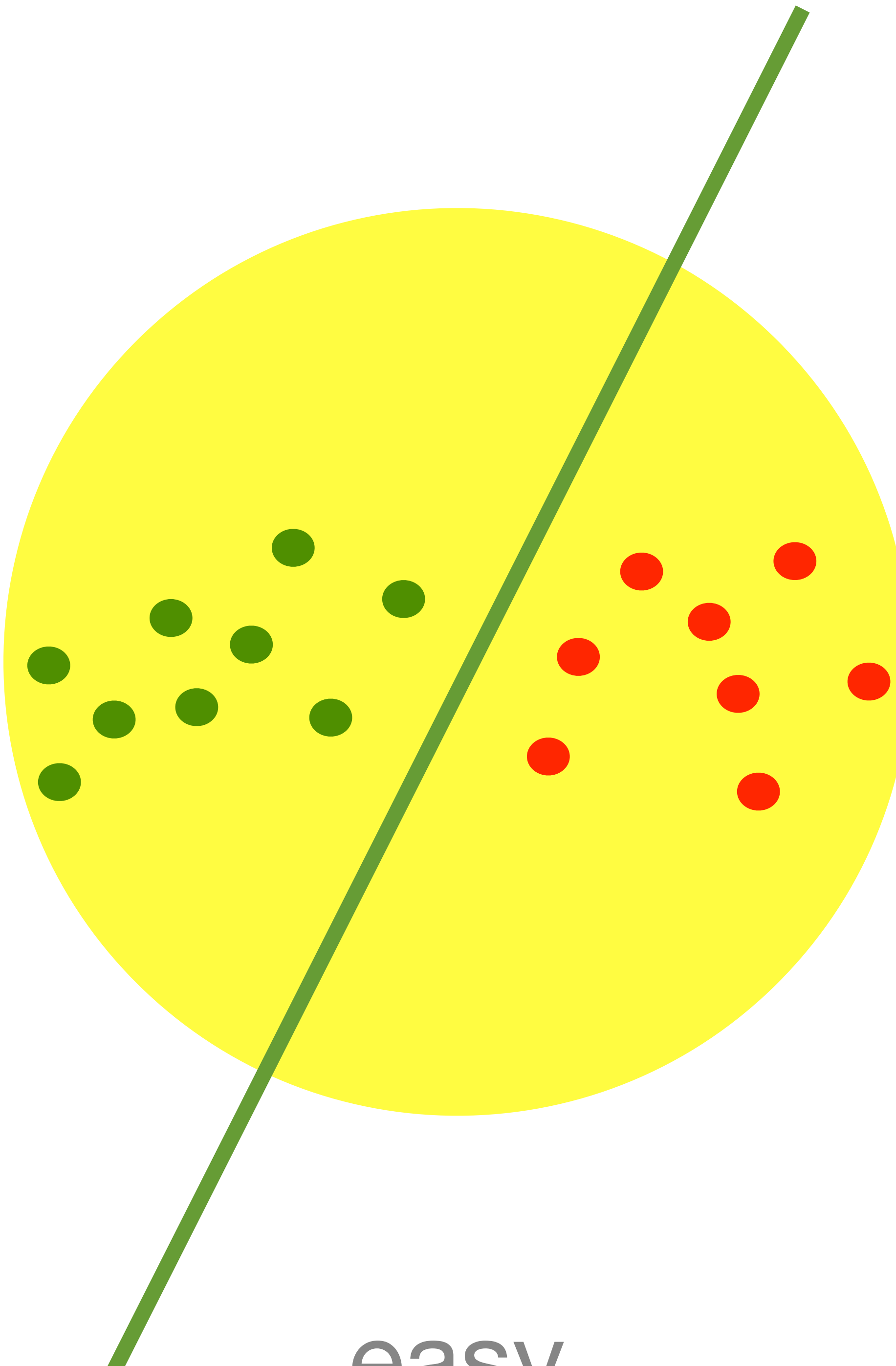

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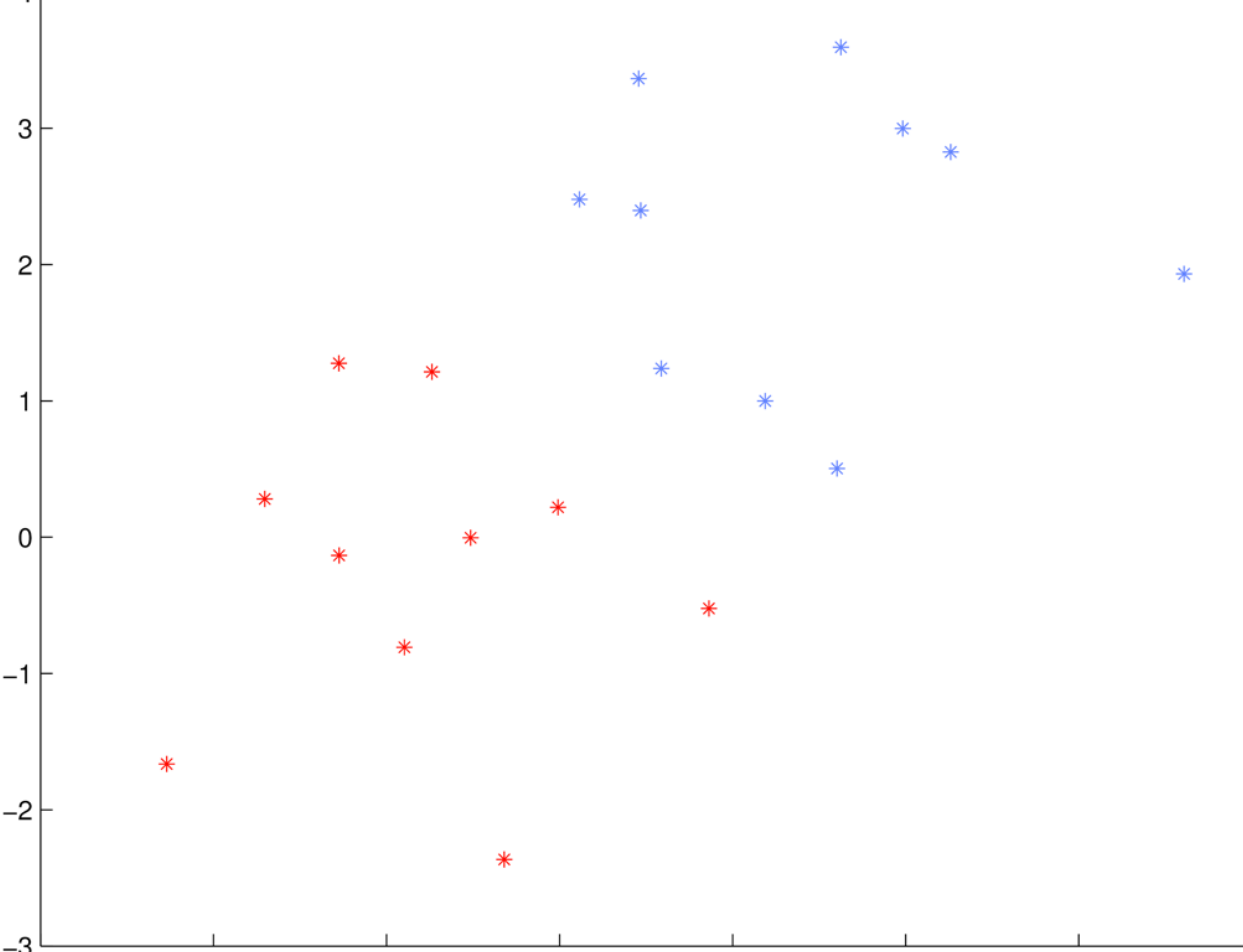

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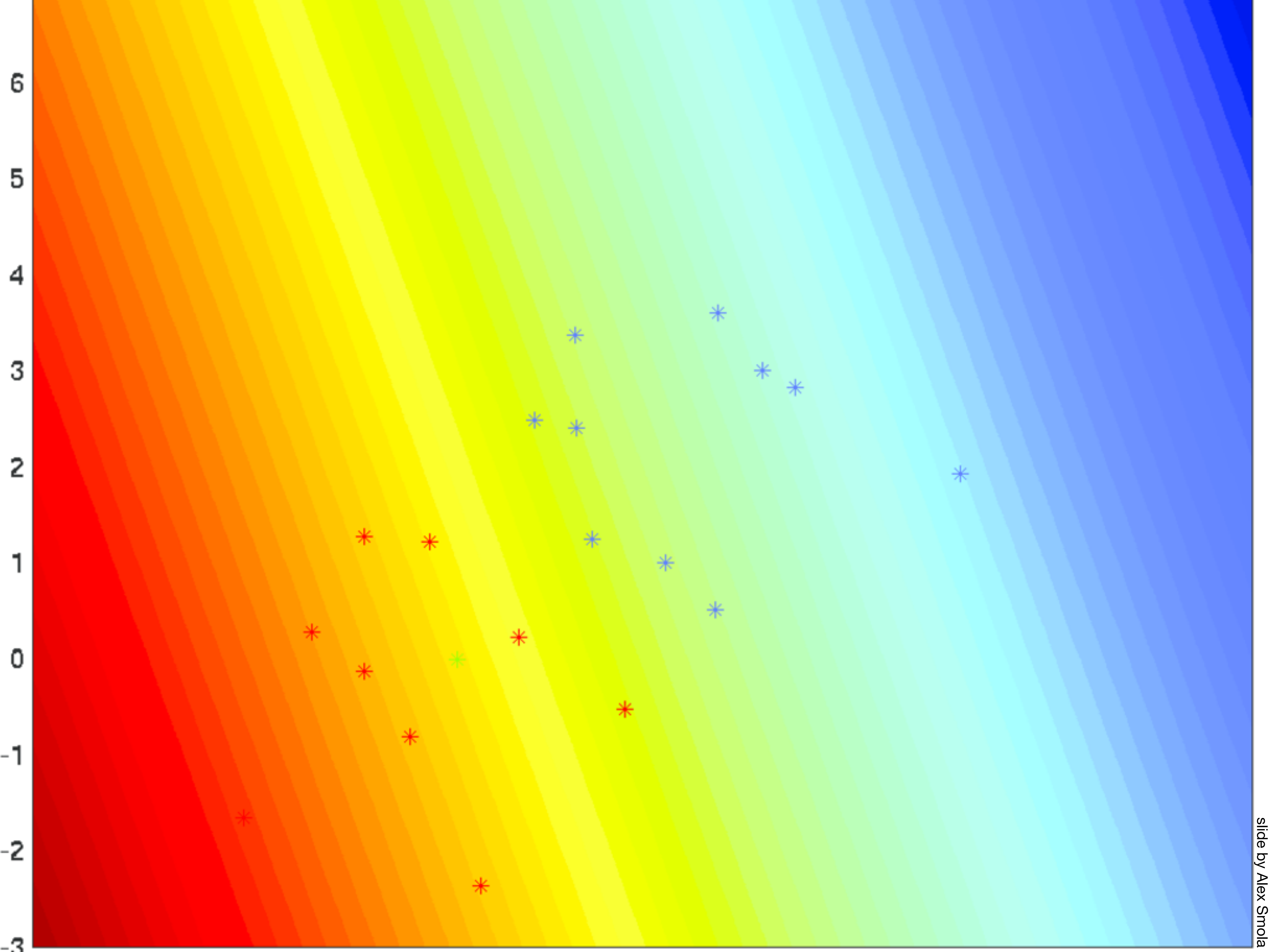
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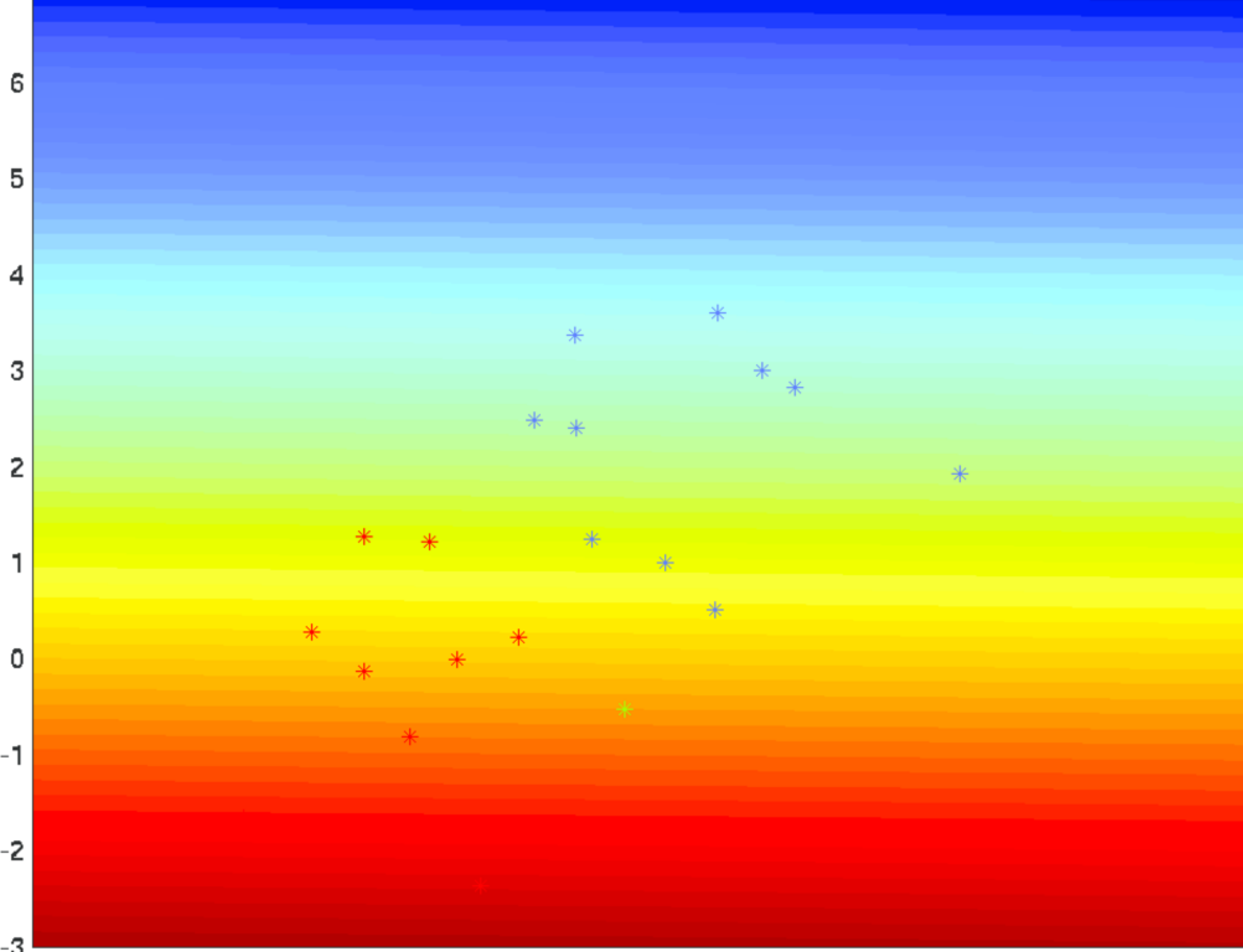

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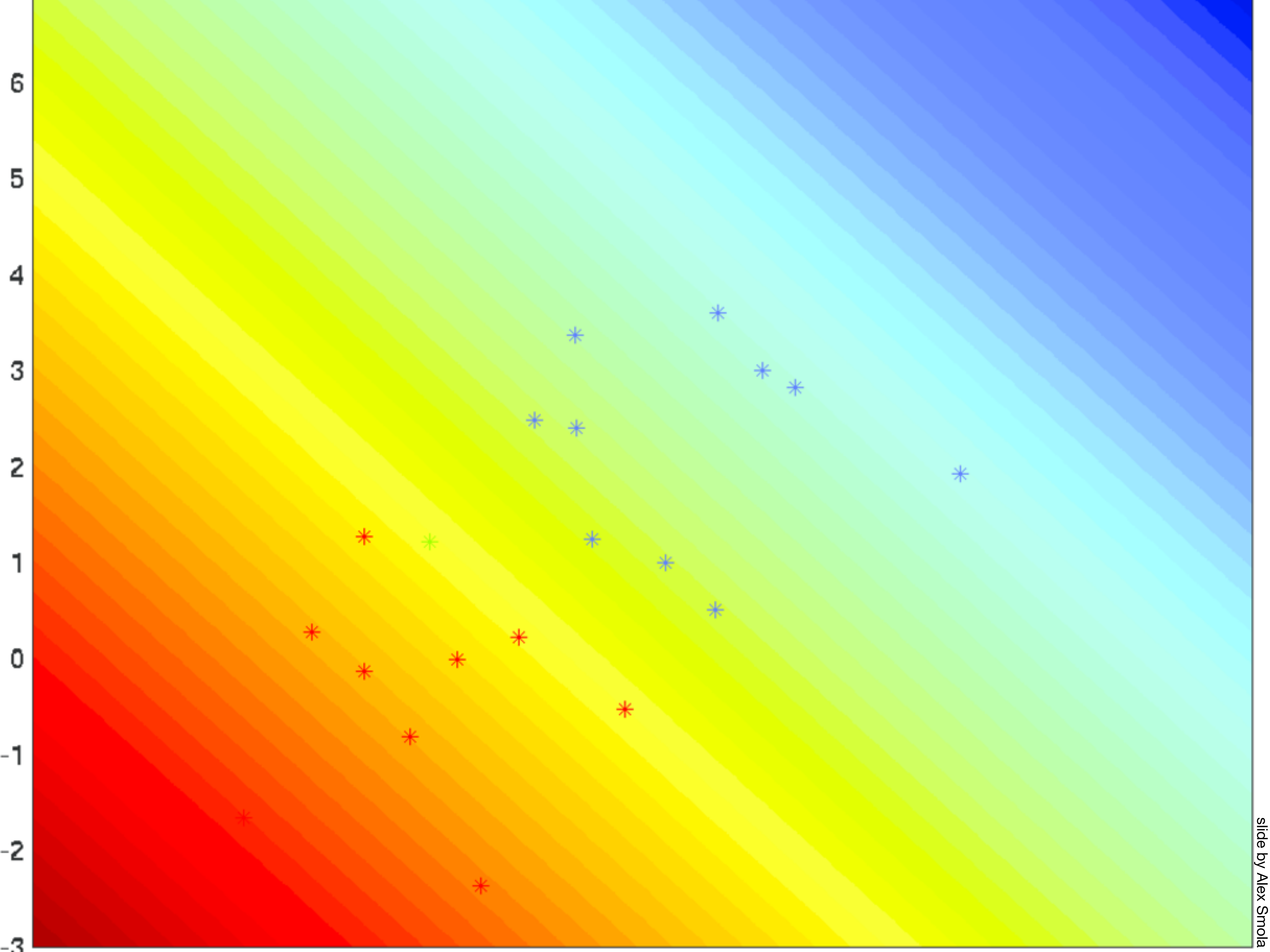
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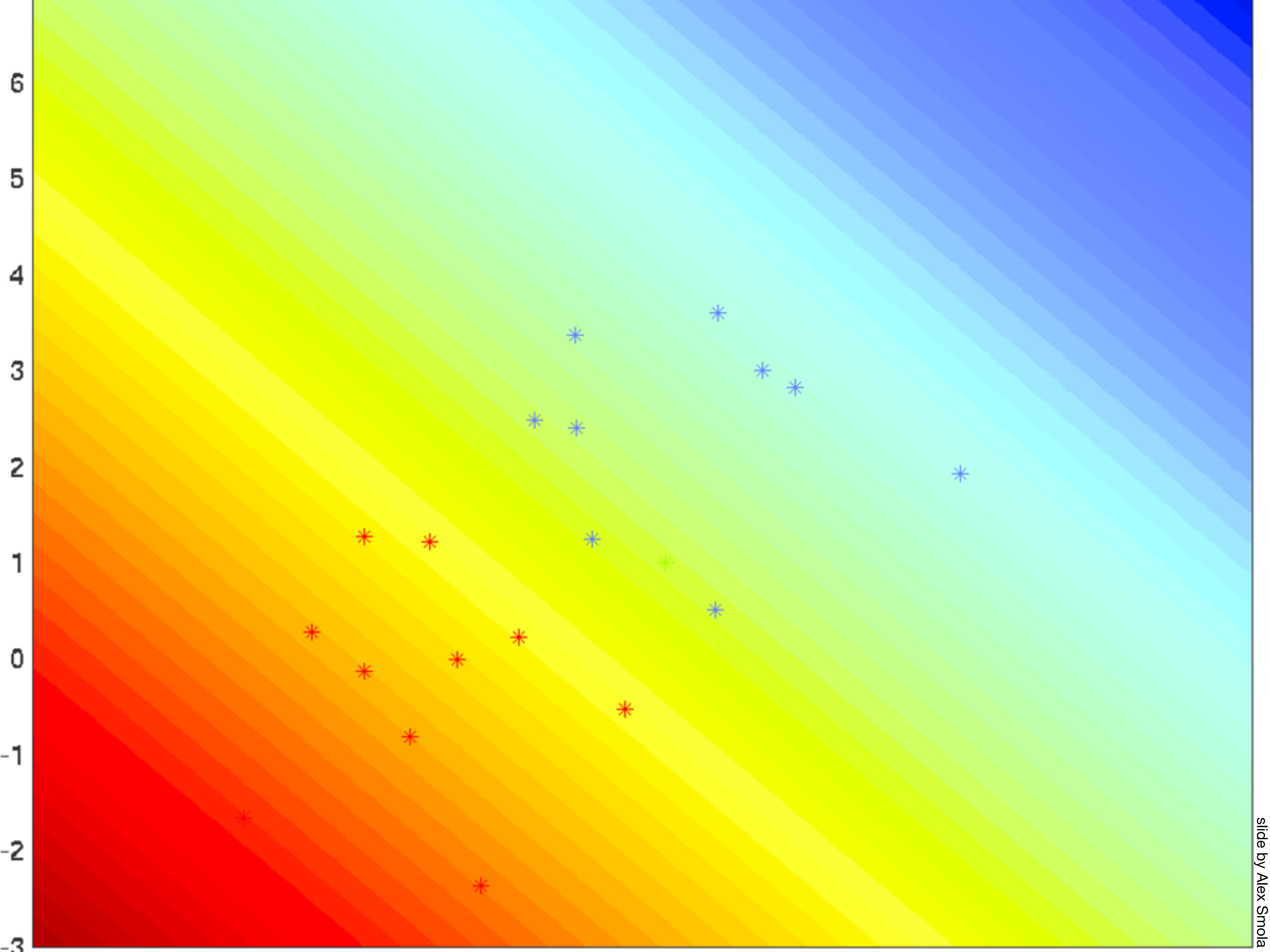
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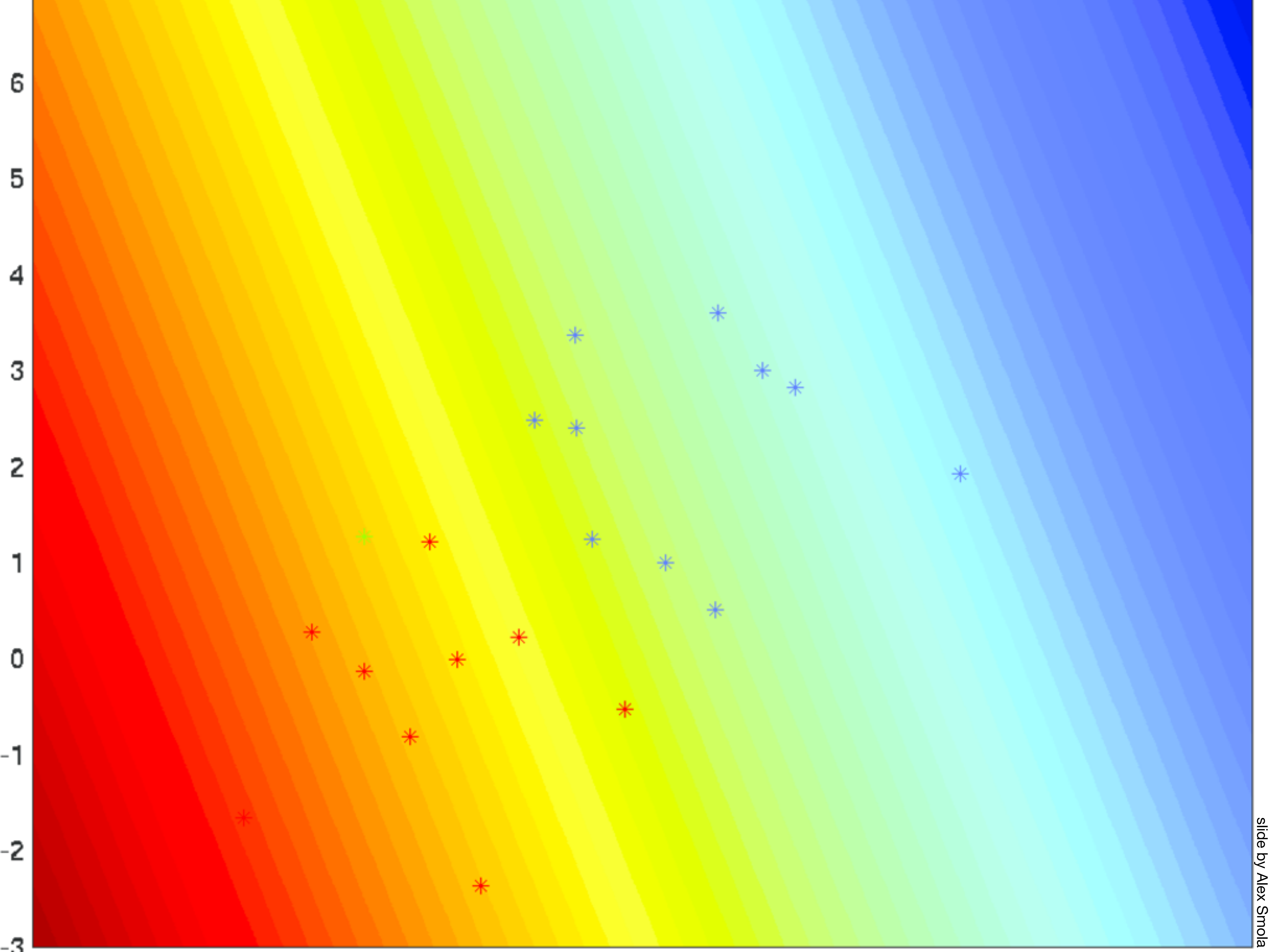
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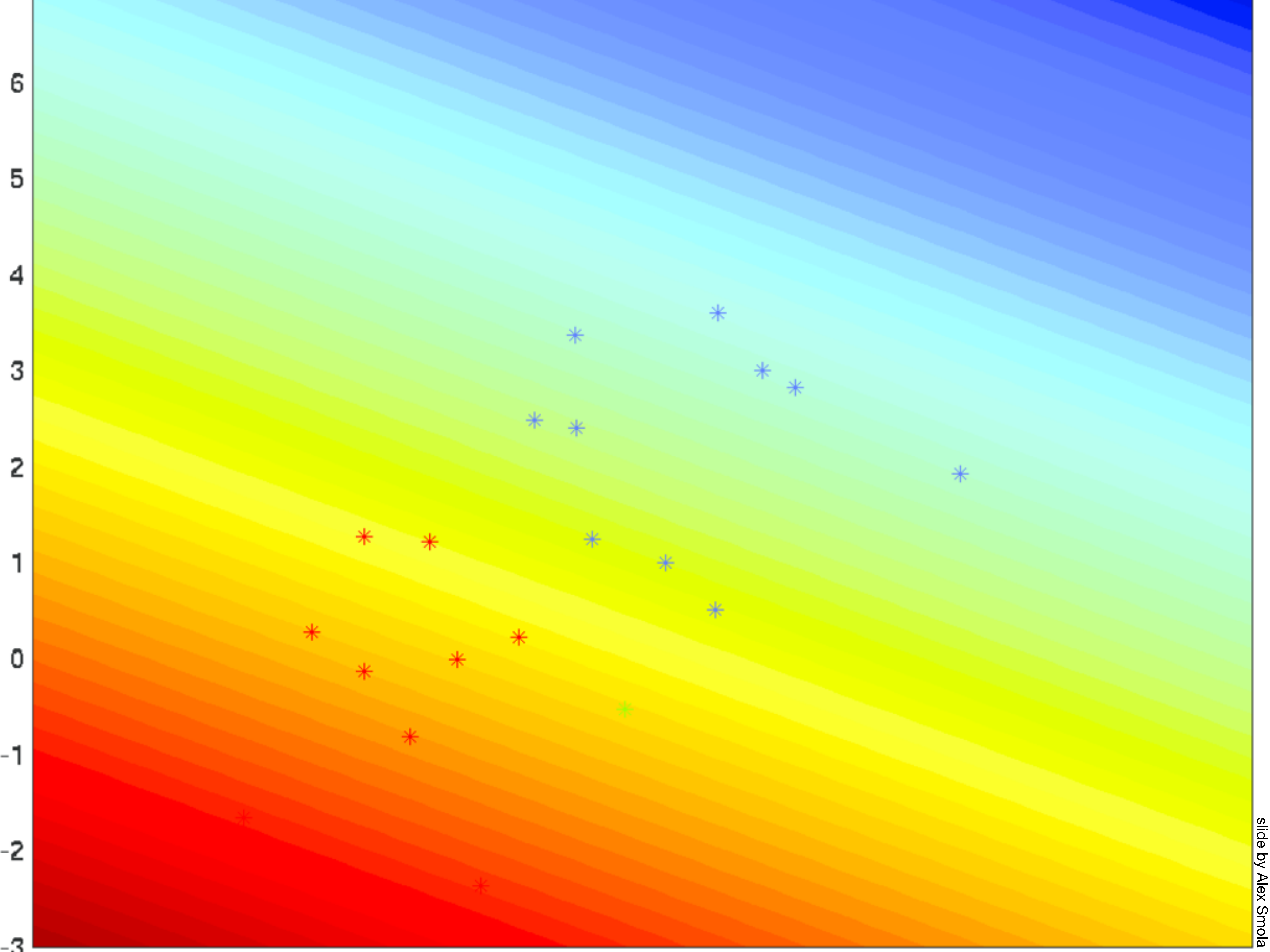
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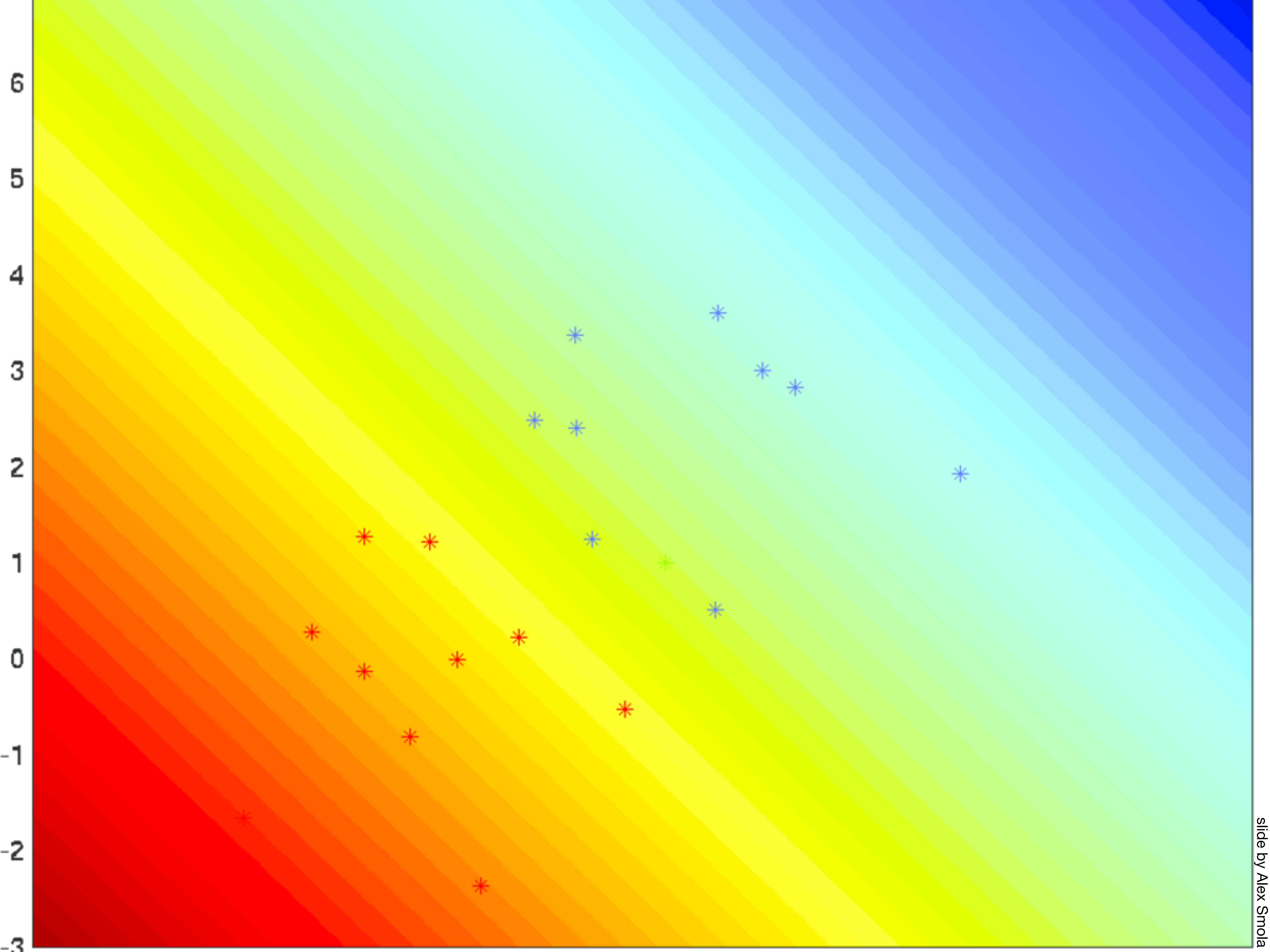
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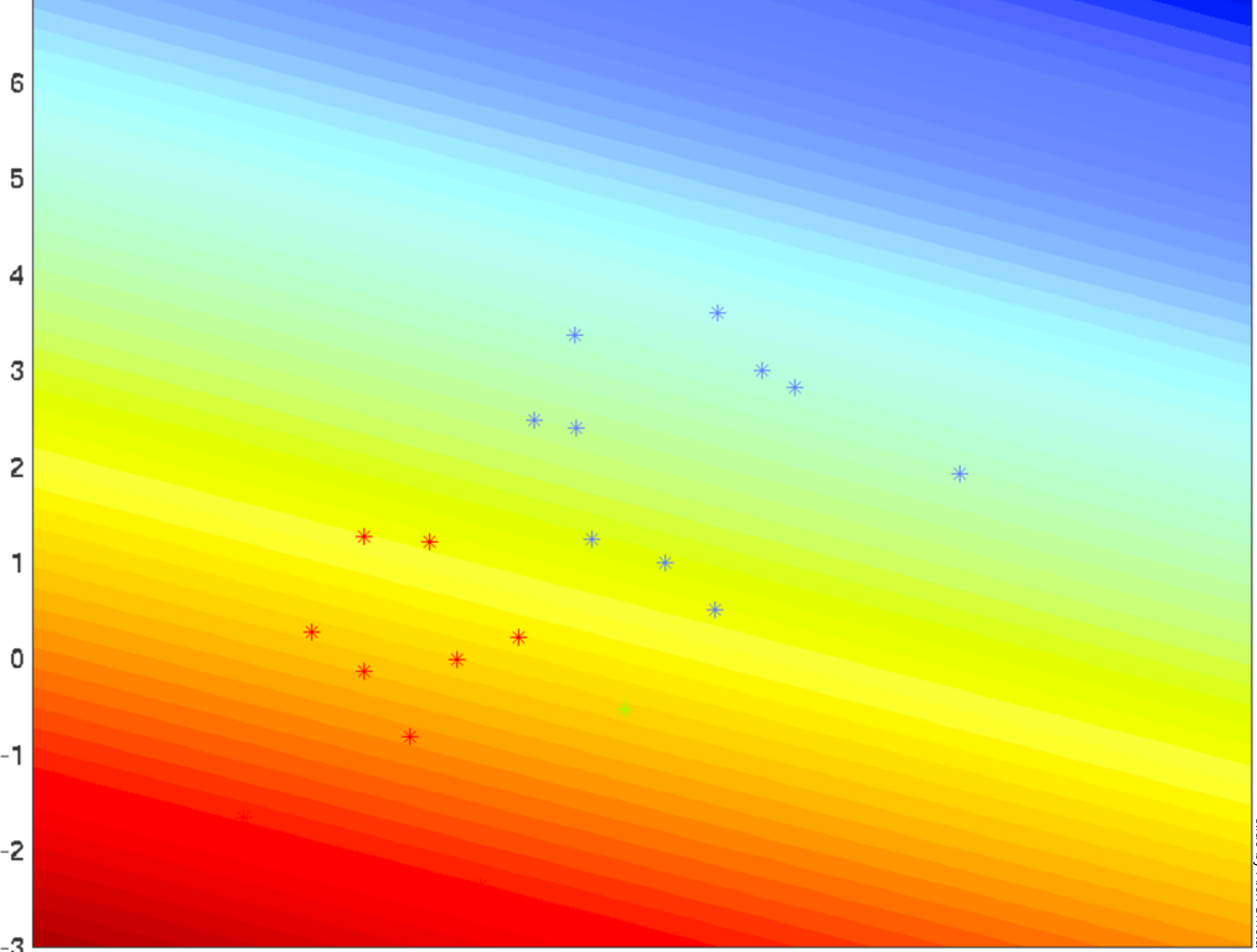
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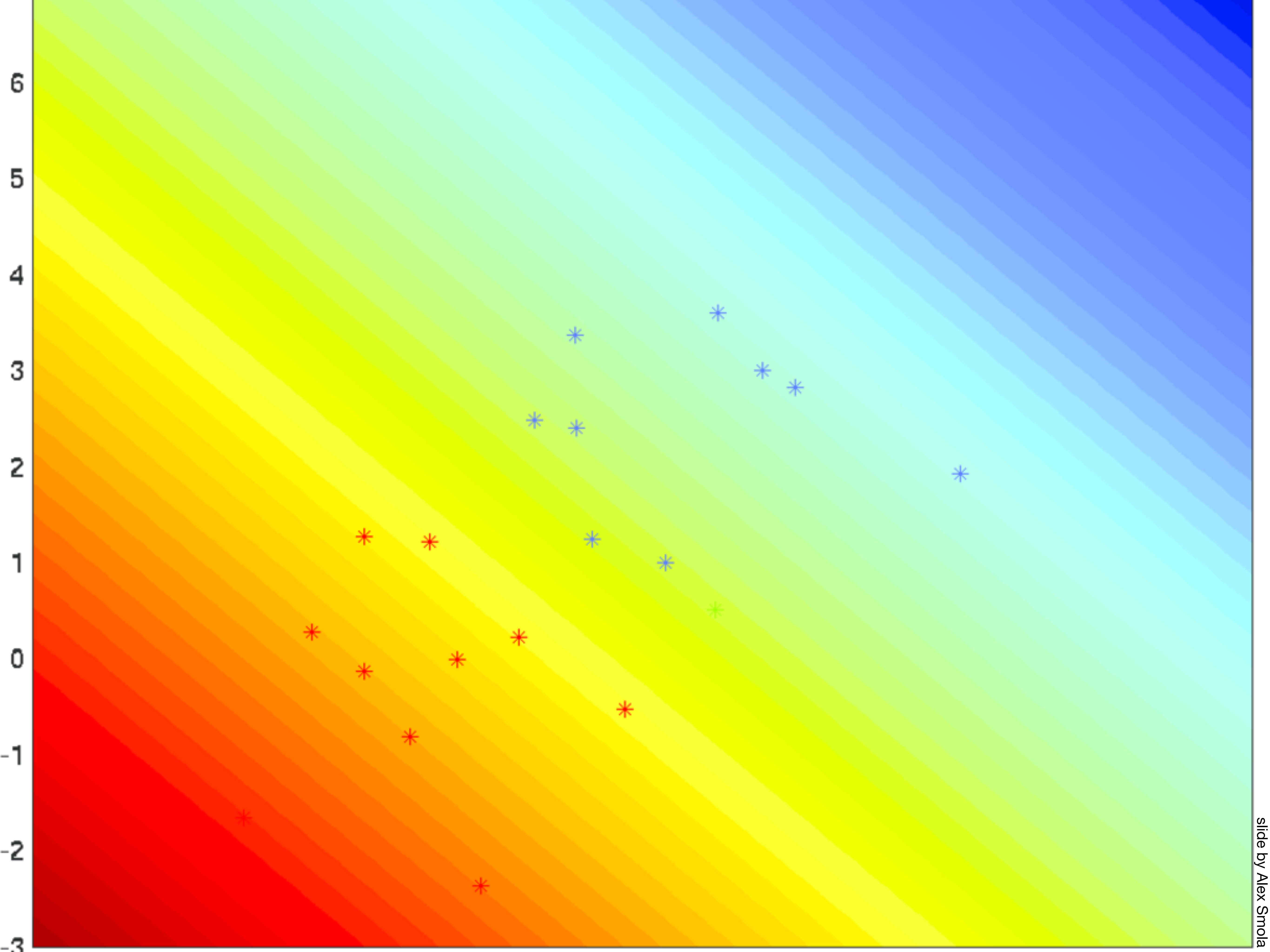
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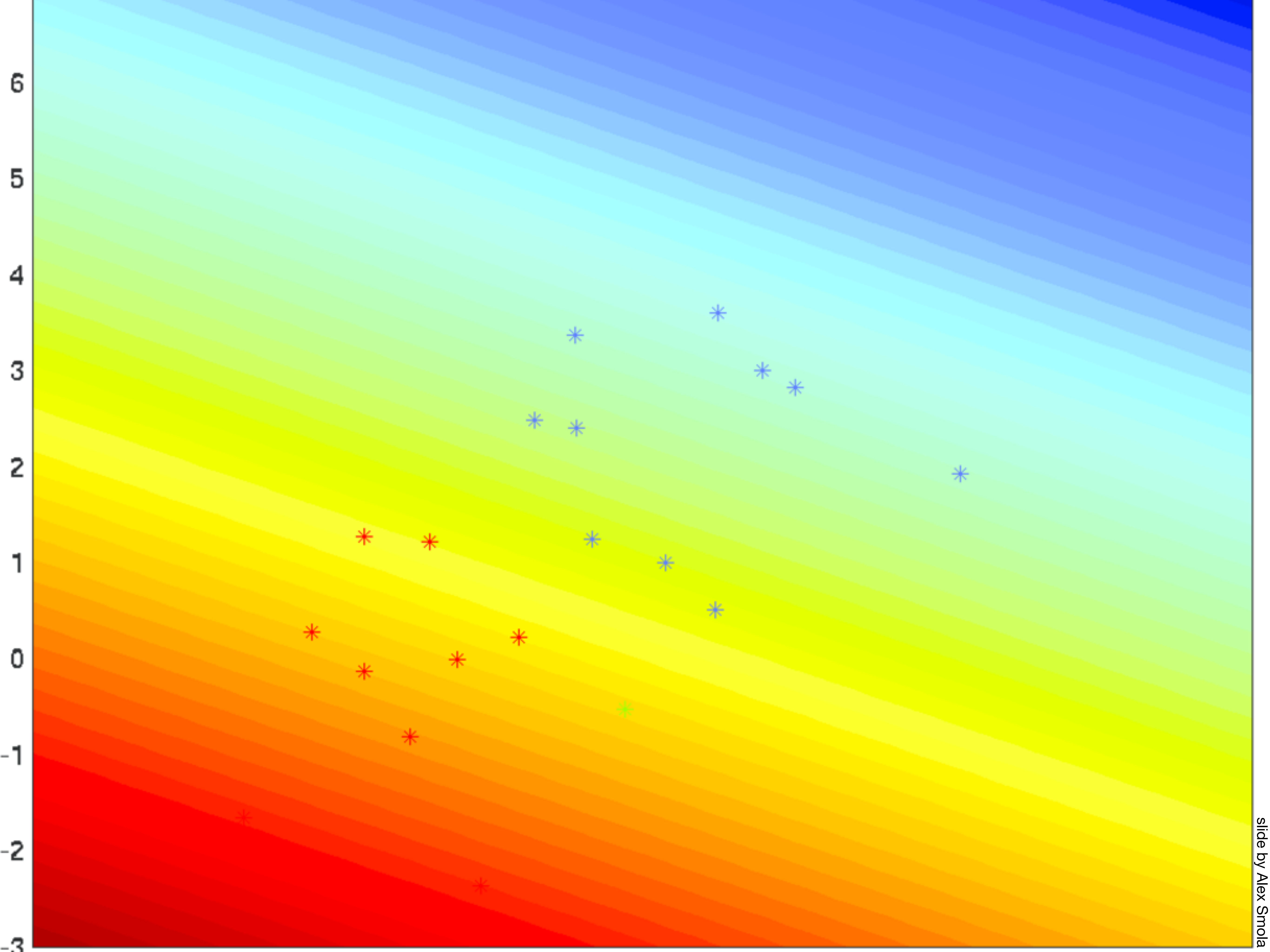
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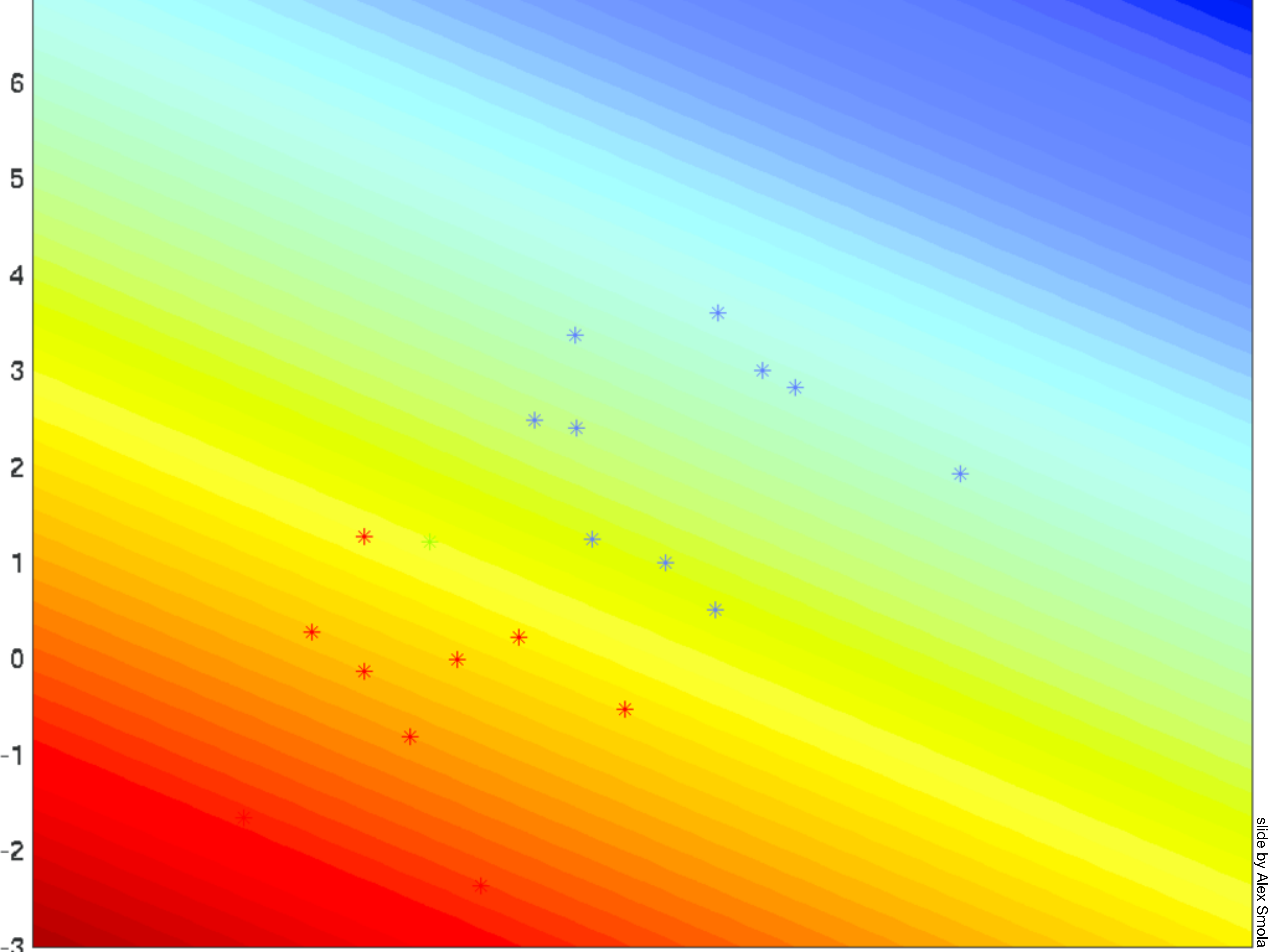
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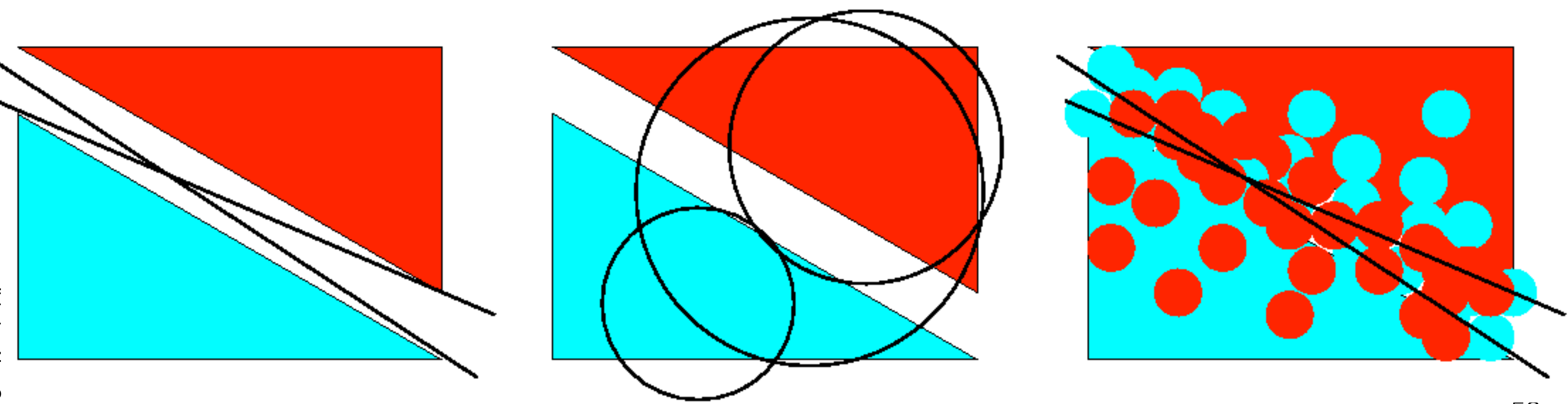
Concepts & version space

- Realizable concepts
  - Some function exists that can separate data and is included in the concept space
  - For perceptron - data is linearly separable
- Unrealizable concept
  - Data not separable
  - We don't have a suitable function class (often hard to distinguish)



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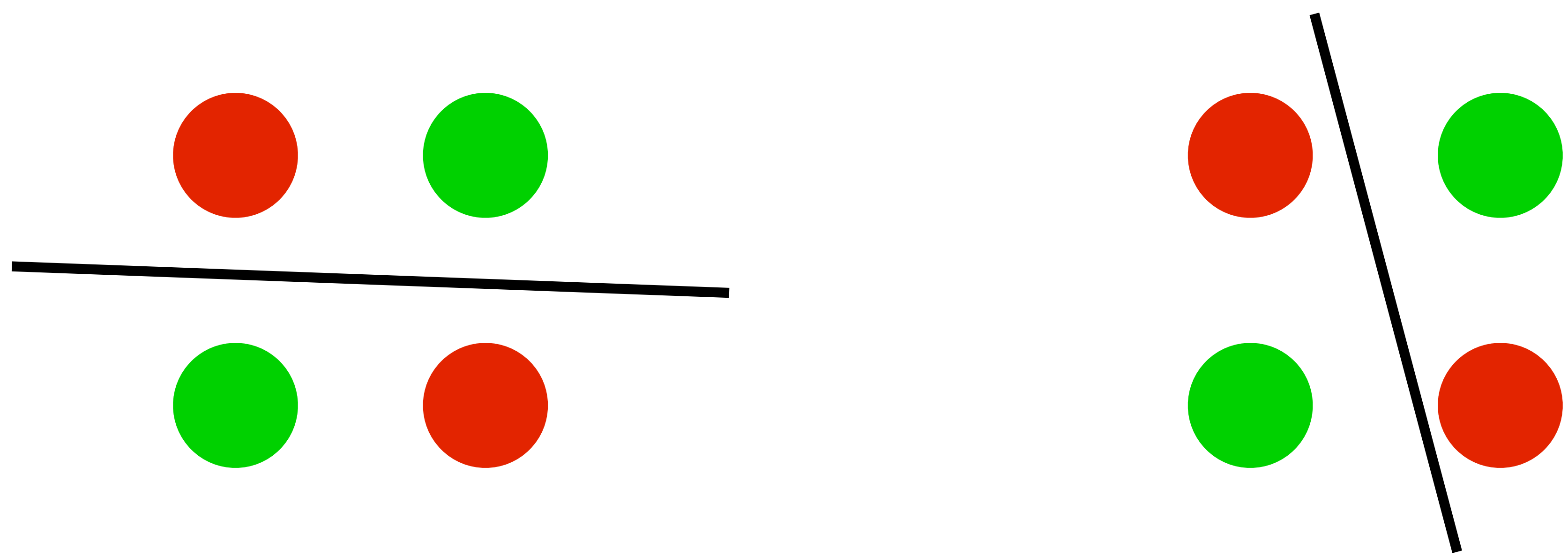
Minimum error separation



- XOR - not linearly separable
- Nonlinear separation is trivial
- Caveat (Minsky & Papert)
  Finding the minimum error linear separator is NP hard (this killed Neural Networks in the 70s).

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Nonlinear Features

- Regression
  We got nonlinear functions by preprocessing
- Perceptron
  - Map data into feature space $x \to \phi(x)$
  - Solve problem in this space
  - Query replace $\langle x, x' \rangle$ by $\langle \phi(x), \phi(x') \rangle$ for code
- Feature Perceptron
  - Solution in span of $\phi(x_i)$

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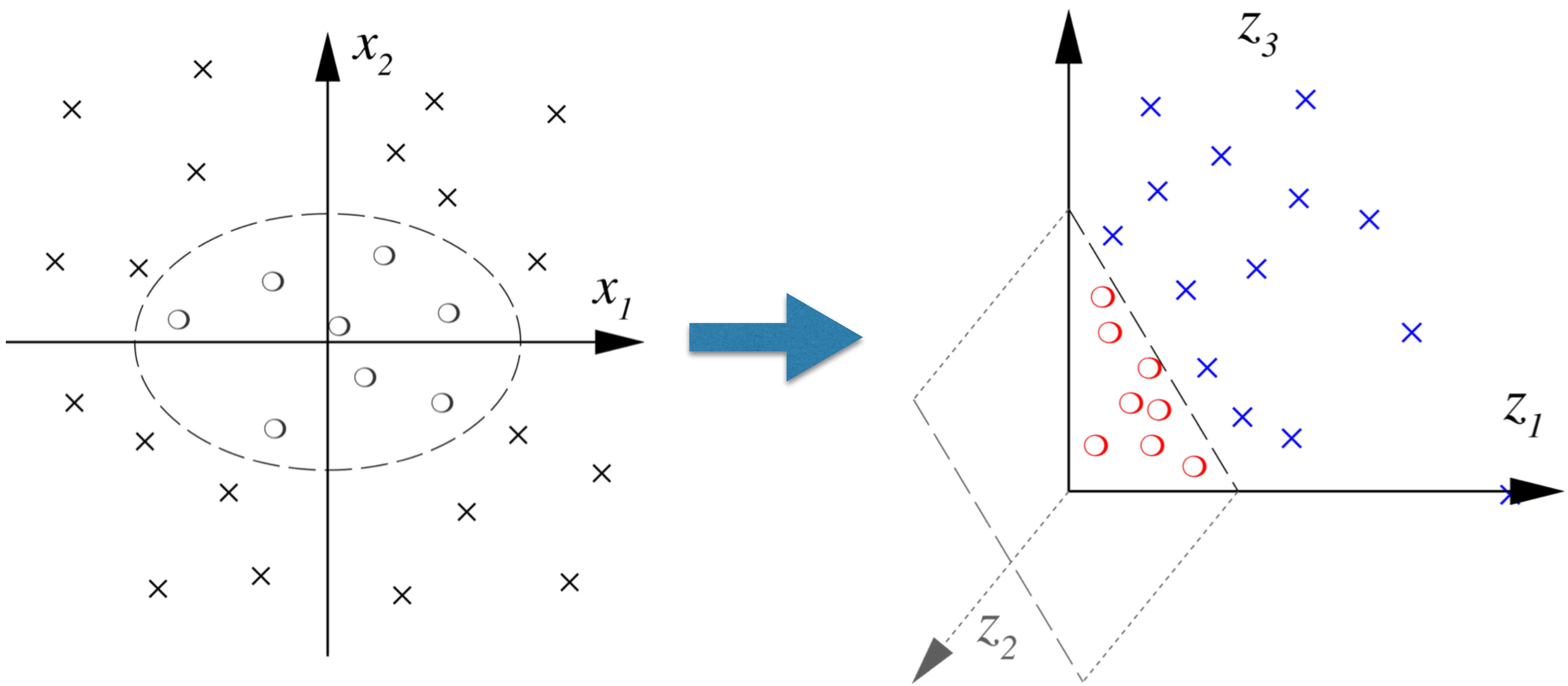
Quadratic Features



- Separating surfaces are
Circles, hyperbolae, parabolae

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Constructing Features (very naive OCR system)

	1	2	3	4	5	6	7	8	9	0
Loops	0	0	0	1	0	1	0	2	1	1
3 Joints	0	0	0	0	0	1	0	0	1	0
4 Joints	0	0	0	1	0	0	0	1	0	0
Angles	0	1	1	1	1	0	1	0	0	0
Ink	1	2	2	2	2	2	1	3	2	2

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Feature Engineering for Spam Filtering

- bag of words
- pairs of words
- date & time
- recipient path
- IP number
- sender
- encoding
- links
- ... secret sauce ...

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More feature engineering

- Two Interlocking Spirals
  Transform the data into a radial and angular part
  $$(x_1, x_2) = (r \sin \phi, r \cos \phi)$$
- Handwritten Japanese Character Recognition
  - Break down the images into strokes and recognize it
  - Lookup based on stroke order
- Medical Diagnosis
  - Physician's comments
  - Blood status / ECG / height / weight / temperature ...
  - Medical knowledge
- Preprocessing
  - Zero mean, unit variance to fix scale issue (e.g. weight vs. income)
  - Probability integral transform (inverse CDF) as alternative

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The Perceptron on features

initialize $w, b = 0$
repeat
  Pick (x_i, y_i) from data
  if $y_i(w \cdot \Phi(x_i) + b) \leq 0$ then

$$\begin{aligned} w' &= w + y_i \Phi(x_i) \\ b' &= b + y_i \end{aligned}$$

until $y_i(w \cdot \Phi(x_i) + b) > 0$ for all i

- Nothing happens if classified correctly
- Weight vector is linear combination $w = \sum_{i \in I} y_i \phi(x_i)$
- Classifier is linear combination of inner products $f(x) = \sum_{i \in I} y_i \langle \phi(x_i), \phi(x) \rangle + b$

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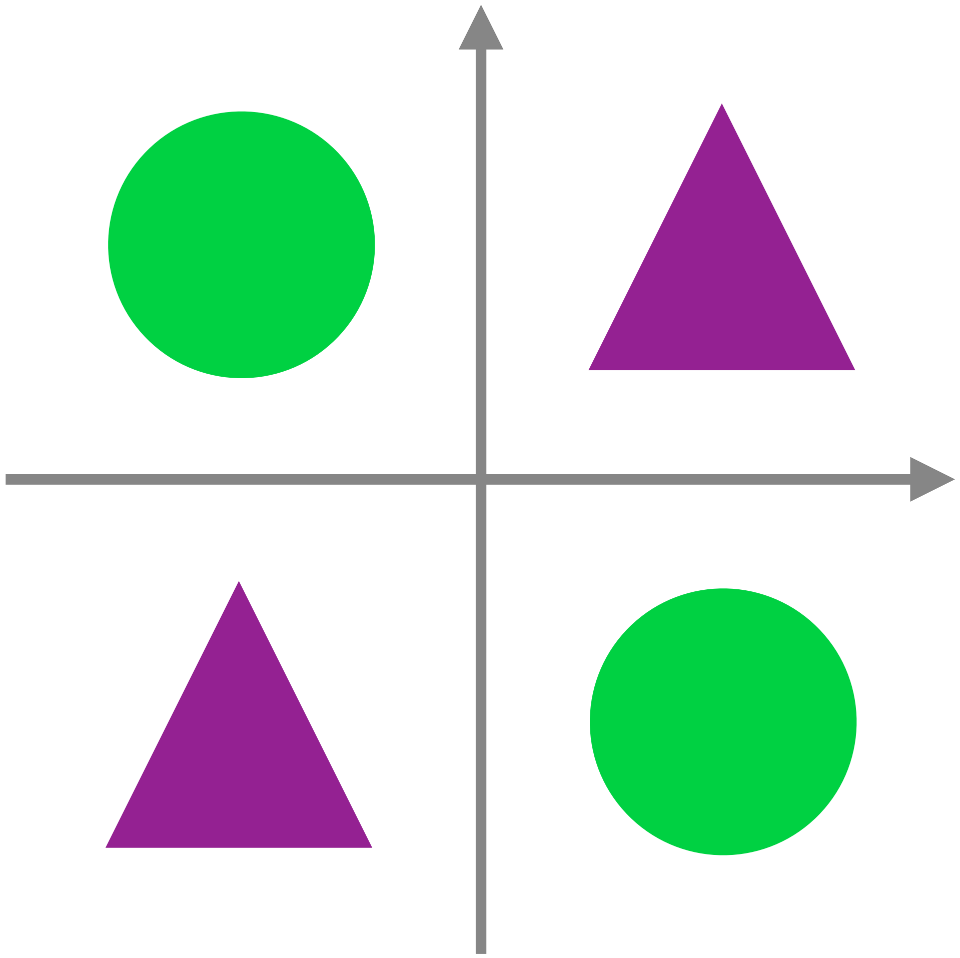
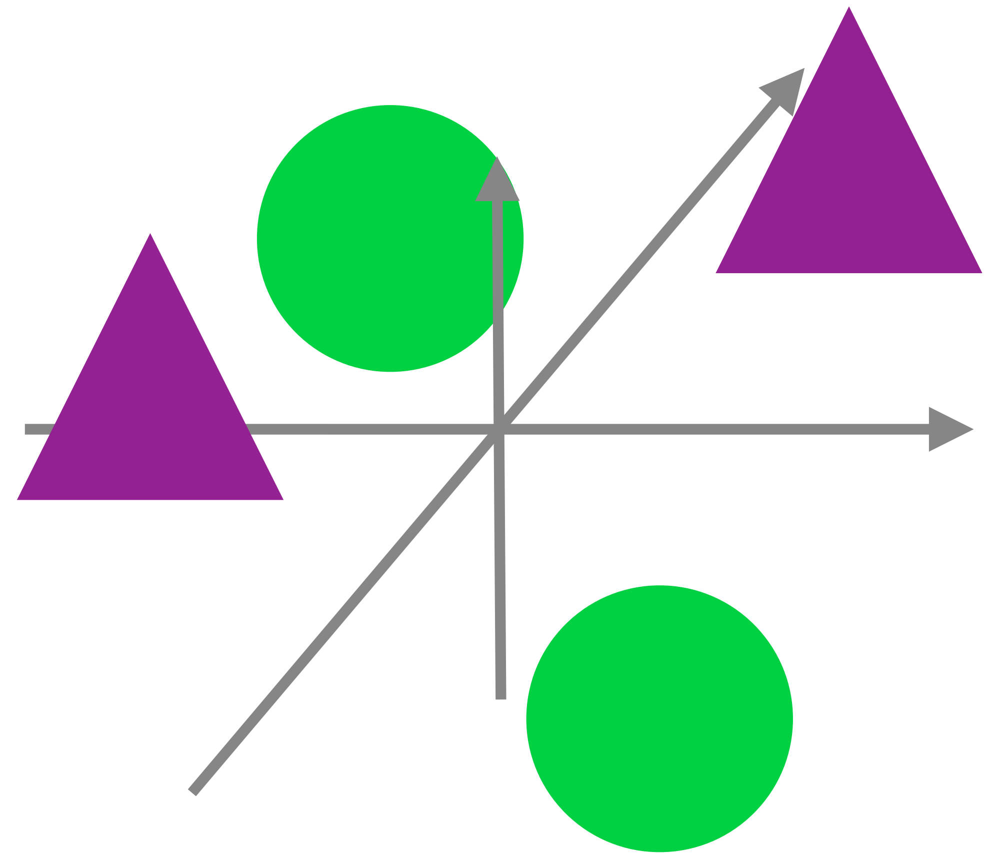
Solving XOR



(x_1, x_2)



$(x_1, x_2, x_1 x_2)$

- XOR not linearly separable
- Mapping into 3 dimensions makes it easily solvable

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Extensions of Perceptron

- **Voted Perceptron**
  - generalizes better than (standard) perceptron
  - memory intensive (keeps around every weight vector seen during training, so each one can vote)
- **Averaged Perceptron**
  - empirically similar performance to voted perceptron
  - can be implemented in a memory efficient way (running averages are efficient)

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Extensions of Perceptron

- **Kernel Perceptron**
  - Choose a kernel K(x’, x)
  - Apply the **kernel trick** to Perceptron
  - Resulting algorithm is still **very simple**
- **Structured Perceptron**
  - Basic idea can also be applied when **y** ranges over an exponentially large set
  - Mistake bound **does not** depend on the size of that set

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Summary: Perceptron

- Perceptron is a **linear classifier**
- **Simple learning algorithm:** when a mistake is made, add / subtract the features
- Perceptron will converge if the data are **linearly separable**, it will not converge if the data are **linearly inseparable**
- For linearly separable and inseparable data, we can **bound the number of mistakes** (geometric argument)
- **Extensions** support nonlinear separators and structured prediction

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Next Lecture:

Multi-layer Perceptron