

BBM444

FUNDAMENTALS OF COMPUTATIONAL PHOTOGRAPHY

Lecture #02 – Image Formation

Erkut Erdem // Hacettepe University // Spring 2023



HACETTEPE
UNIVERSITY
COMPUTER
VISION LAB

Today's Lecture

- Pinhole camera
- Basics of geometric optics and lenses
- Field of view
- Magnification and perspective
- Zooming
- Orthographic camera and telecentric lenses

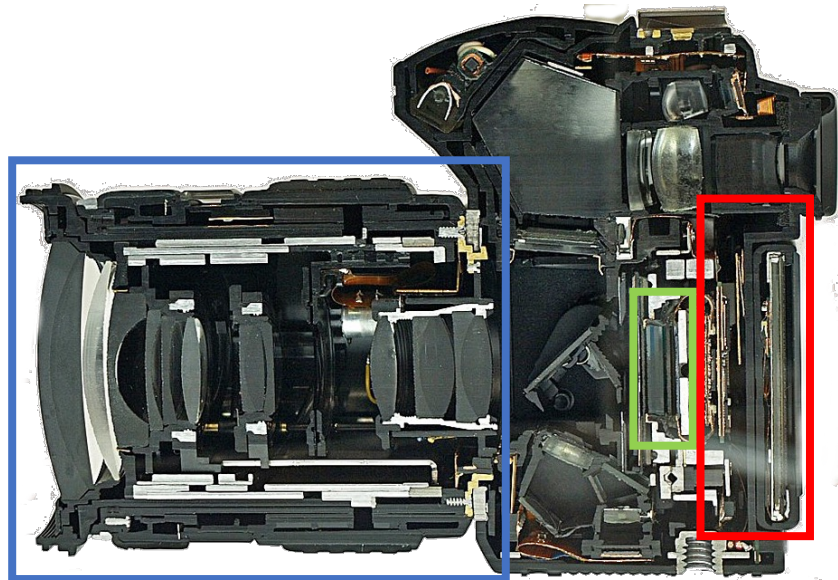
Disclaimer: The material and slides for this lecture were borrowed from

- Ioannis Gkioulekas' 15-463/15-663/15-862 "Computational Photography" class
- Steve Marschner's CS6640 "Computational Photography" class
- David Lindell's CSC2529 "Computational Imaging" class

The modern photography pipeline



post-capture processing



optics and optical controls



sensor, analog front-end, and color filter array



in-camera image processing pipeline

Some motivational imaging experiments

Let's say we have a sensor...



digital sensor
(CCD or CMOS)

... and an object we like to photograph

real-world
object



digital sensor
(CCD or CMOS)



What would an image taken like this look like?

Bare-sensor imaging

real-world
object



digital sensor
(CCD or CMOS)

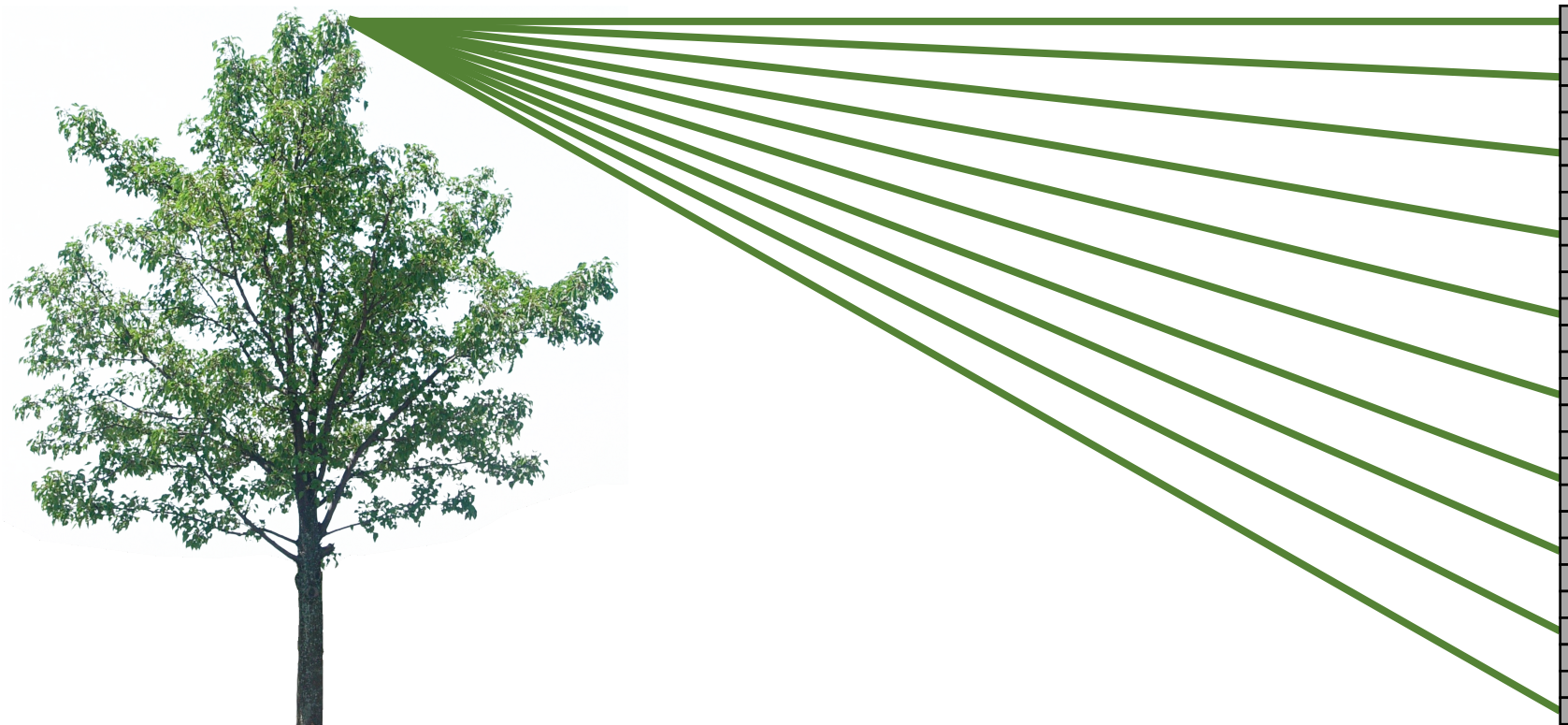


Bare-sensor imaging

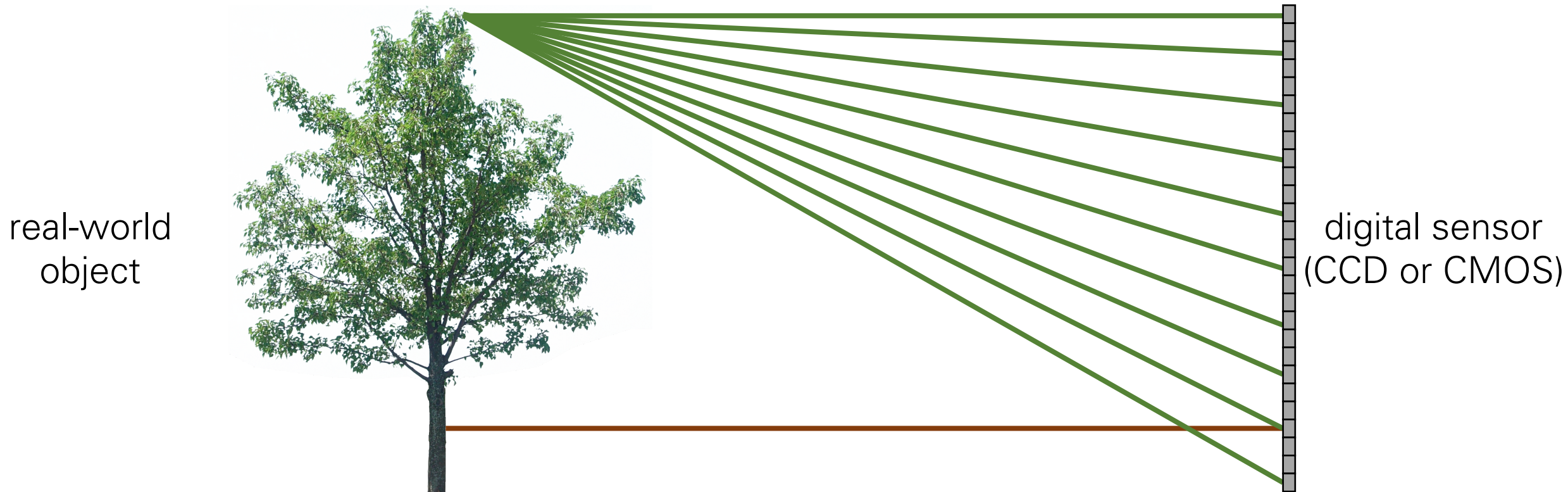
real-world
object



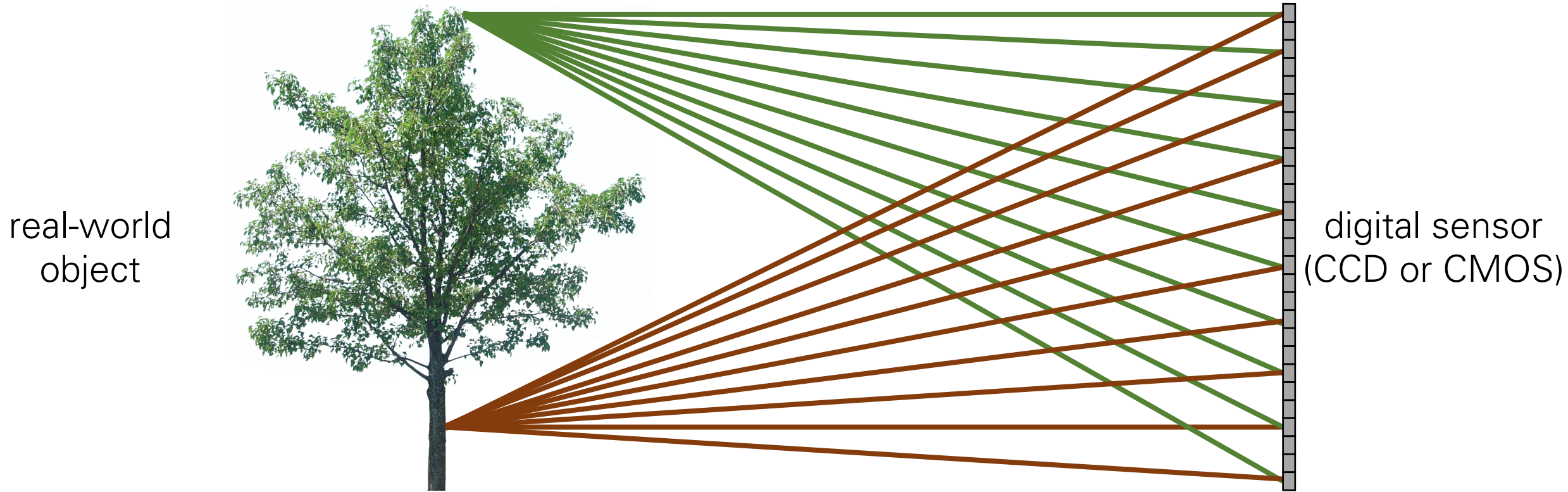
digital sensor
(CCD or CMOS)



Bare-sensor imaging



Bare-sensor imaging



All scene points contribute to all sensor pixels

What does the image on the sensor look like?

Bare-sensor imaging



All scene points contribute to all sensor pixels

What can we do to make our image look better?

real-world
object



digital sensor
(CCD or CMOS)



Let's add something to this scene

real-world
object



barrier (diaphragm)



pinhole
(aperture)



digital sensor
(CCD or CMOS)

What would an image taken like this look like?

Pinhole imaging

real-world
object

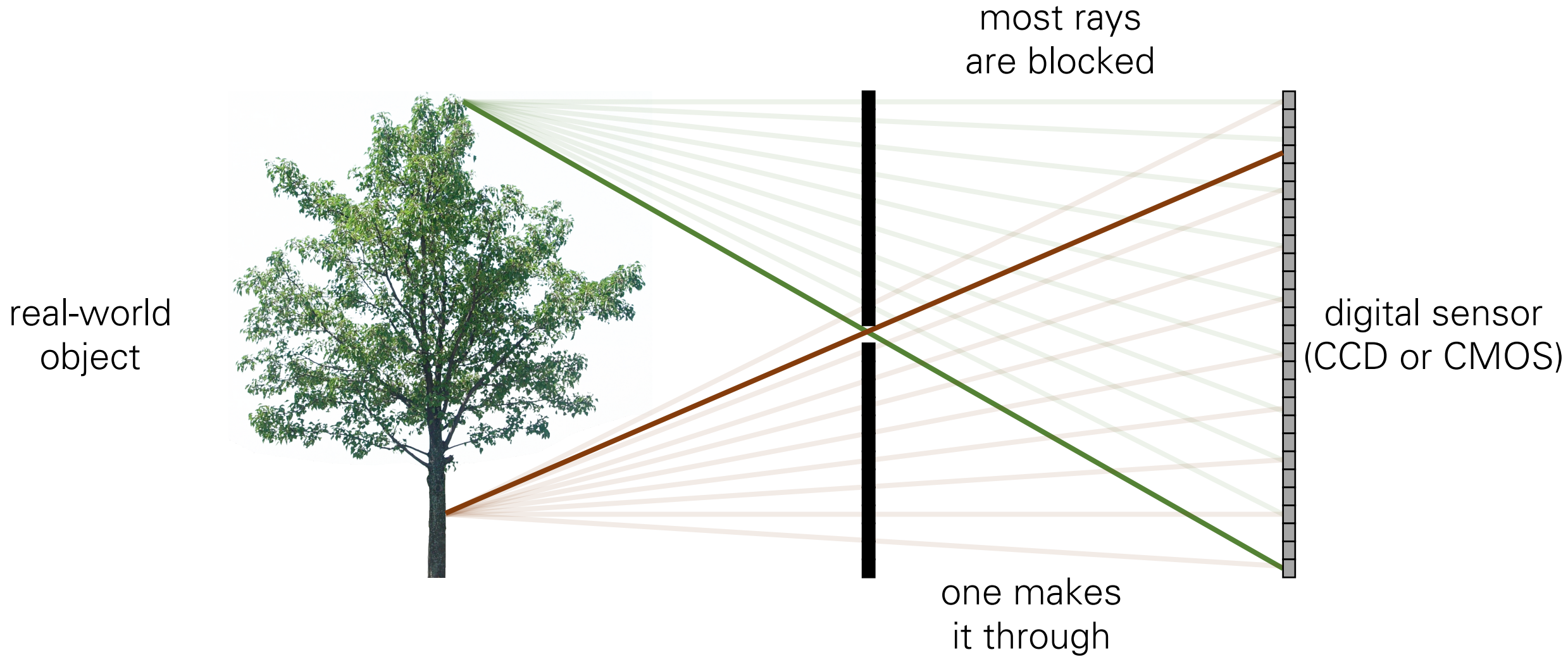


most rays
are blocked

one makes
it through

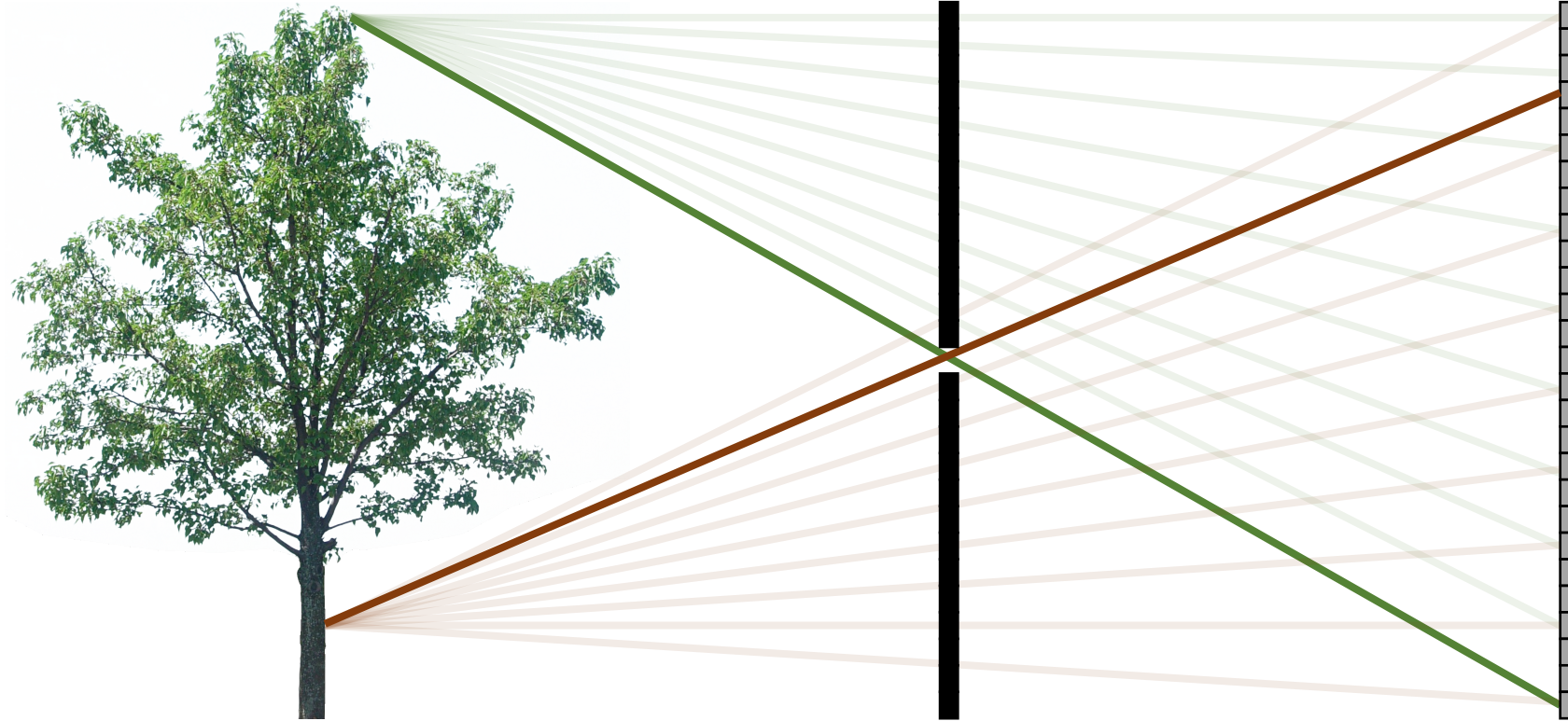
digital sensor
(CCD or CMOS)

Pinhole imaging



Pinhole imaging

real-world
object



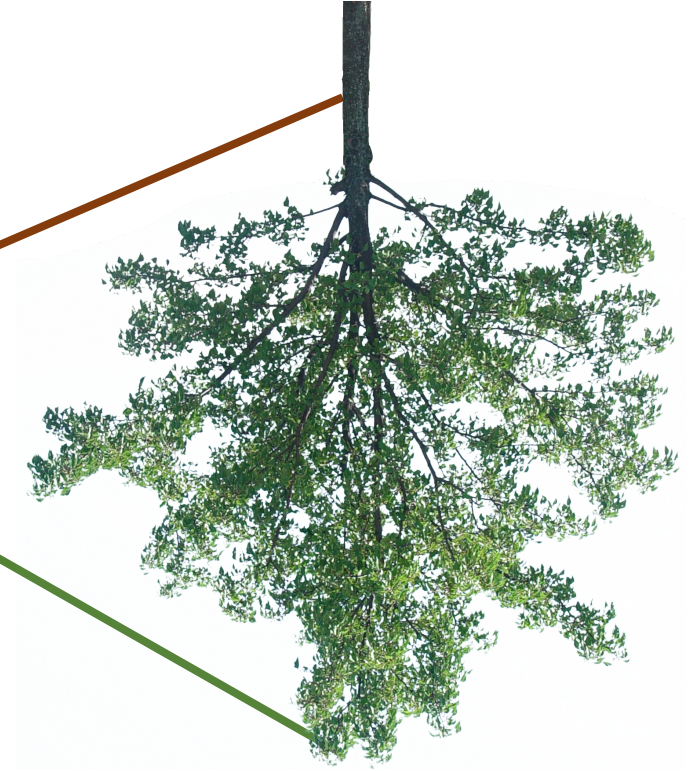
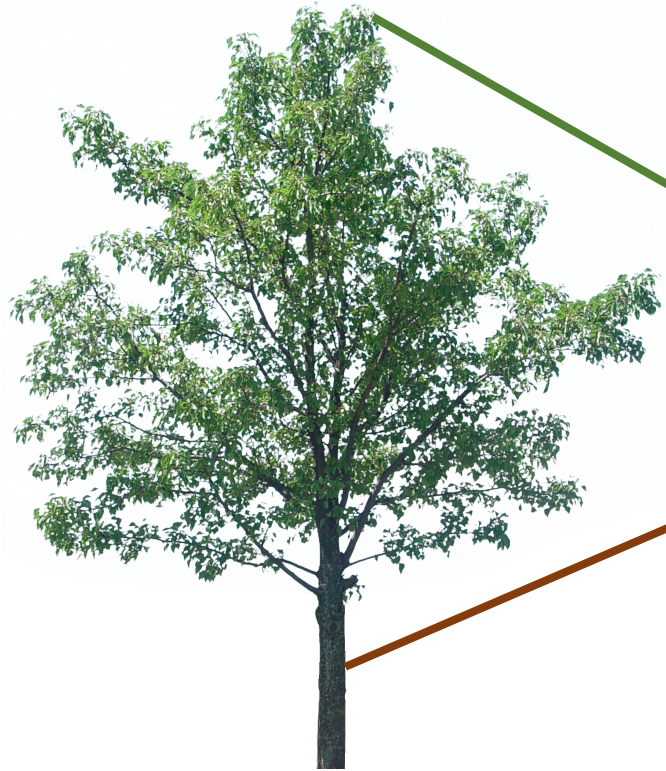
digital sensor
(CCD or CMOS)

Each scene point contributes to only one sensor pixel

What does the image
on the sensor look like?

Pinhole imaging

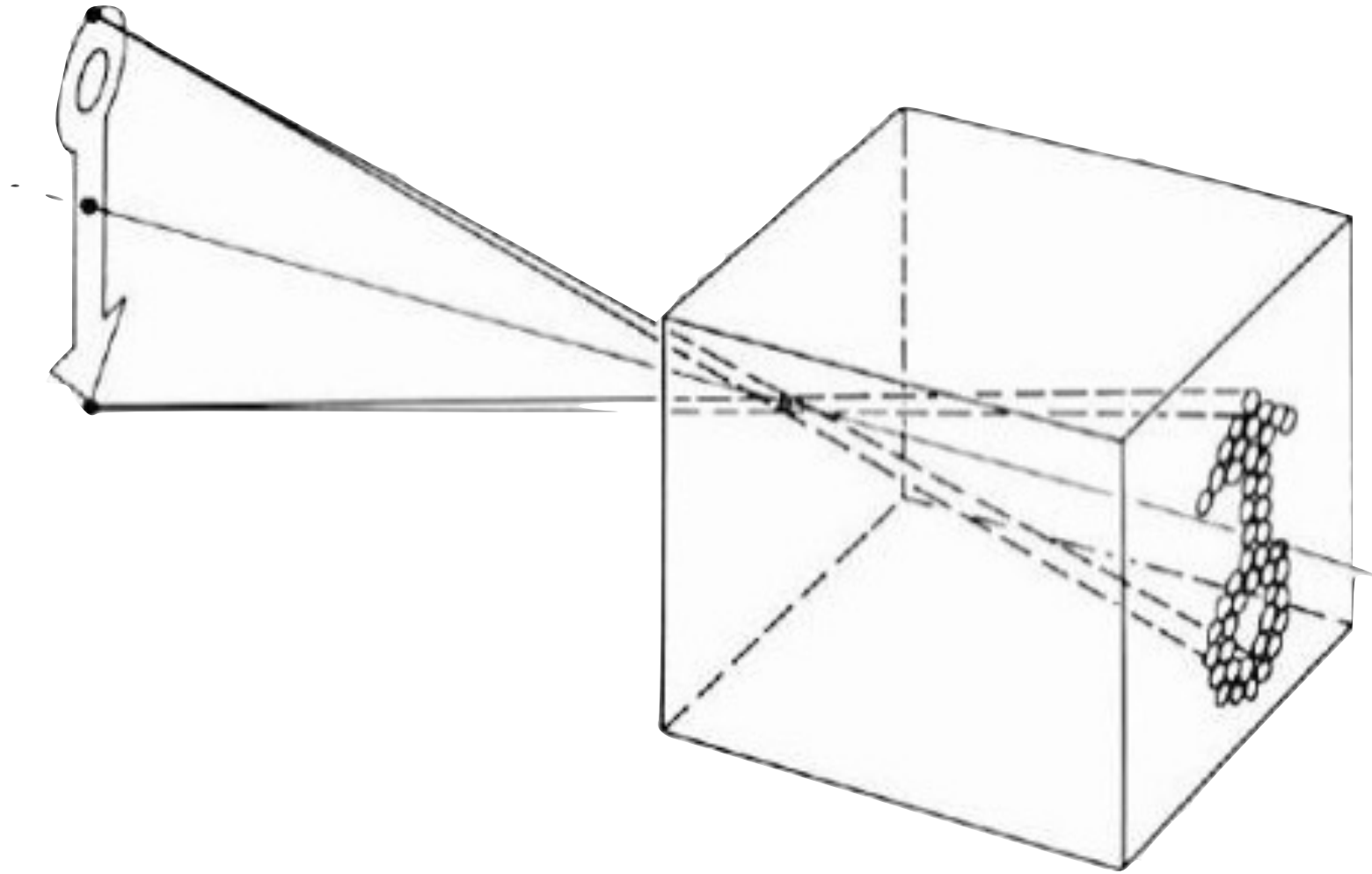
real-world
object



copy of real-world object
(inverted and scaled)

Pinhole camera

Pinhole camera a.k.a. camera obscura



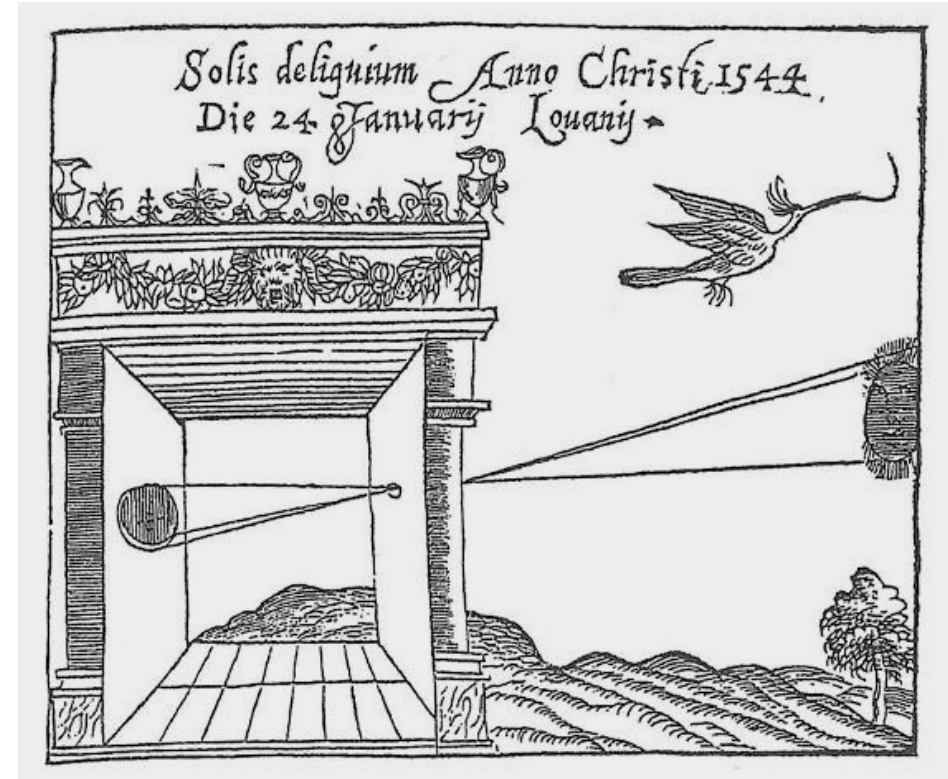
Pinhole camera a.k.a. camera obscura

First mention ...

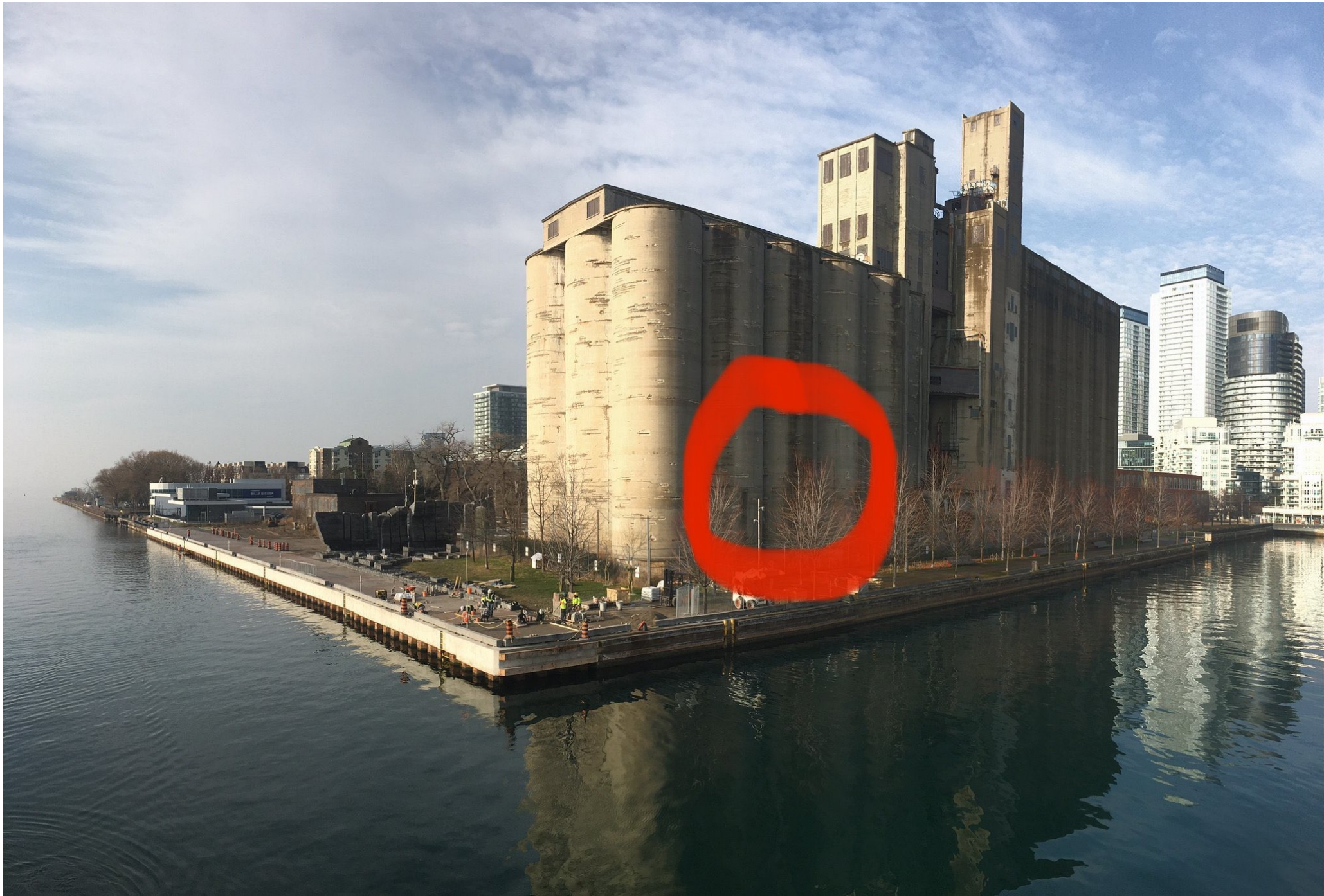


Chinese philosopher Mozi
(470 to 390 BC)

First camera ...



Greek philosopher Aristotle
(384 to 322 BC)

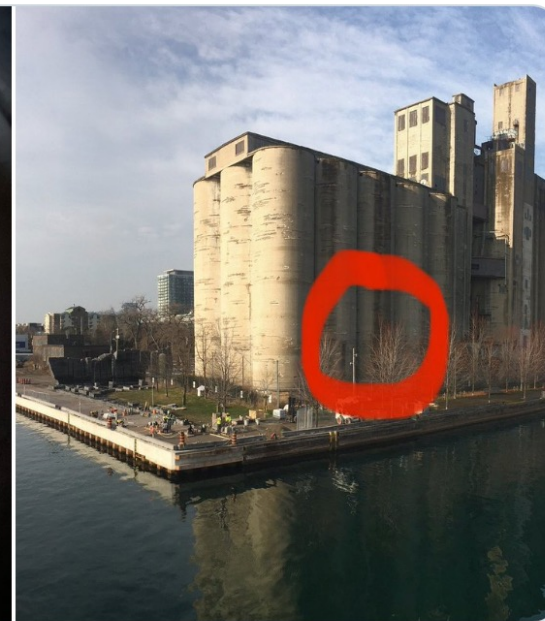
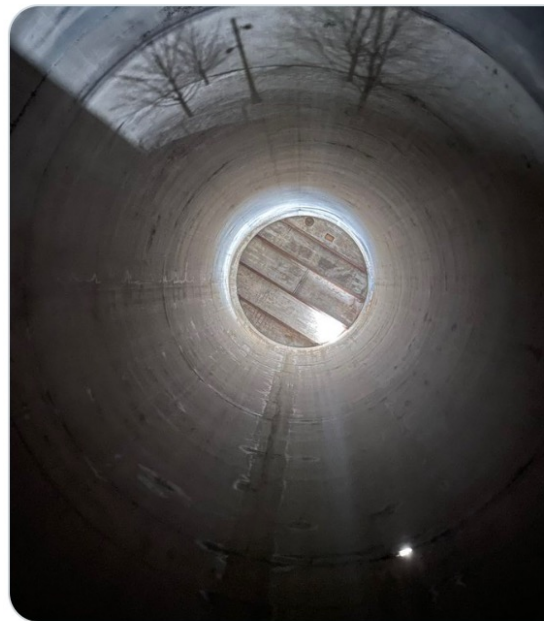




Bryan Bowen
@bryanmbowen



Fun discovery - a small crack in the eastern facade of the Canada Malting Co silos has created a perfect pinhole camera. The result: real time projection of Toronto's waterfront on the silo's interior curved surfaces. An unplugged projection show!



9:37 AM · Jan 27, 2022 · Twitter for iPhone

656 Retweets 70 Quote Tweets 2,836 Likes









Abelardo Morell



Vermeer and The Camera Obscura

[http://www.essentialvermeer.com/camera_obscura/
co_one.html](http://www.essentialvermeer.com/camera_obscura/co_one.html)



Officer and Laughing Girl, 1657

Vermeer and The Camera Obscura

The screenshot shows the IMDb page for the documentary 'Tim's Vermeer'. The page features a large video player with a play button and the text 'Play trailer 2:05'. To the right of the video player are buttons for '9 VIDEOS' and '23 PHOTOS'. Below the video player, there are tabs for 'Documentary' and 'History'. The main description reads: 'Inventor Tim Jenison seeks to understand the painting techniques used by Dutch Master Johannes Vermeer.' The page also lists the director as Teller, writers as Penn Jillette and Teller, and stars as Tim Jenison, Penn Jillette, and Martin Mull. At the bottom, there is a section for 'Nominated for 1 BAFTA Award' and a 'More to explore' section.

IMDb Menu All Search IMDb IMDbPro Watchlist Sign In EN

Cast & crew · User reviews · Trivia IMDbPro All topics

Tim's Vermeer

2013 · PG-13 · 1h 20m

IMDb RATING **7.8**/10
8.3K

YOUR RATING **Rate**

NEW YORK · TORONTO · TELLURIDE

"AWESOME!" PENN AND TELLER'S STERLING DOCUMENTARY."
"SO ENTERTAINING AUDIENCES HARDLY EVEN REALIZE HOW INCREDIBLE IT IS!"
"THRILLING TO WATCH!"

Tim's Vermeer
A Penn & Teller Film

Play trailer 2:05

9 VIDEOS

23 PHOTOS

Documentary History

Inventor Tim Jenison seeks to understand the painting techniques used by Dutch Master Johannes Vermeer.

Director [Teller](#)

Writers [Penn Jillette](#) · [Teller](#)

Stars [Tim Jenison](#) · [Penn Jillette](#) · [Martin Mull](#)

IMDbPro See production, box office & company info

+ Add to Watchlist
Added by 20.2K users

52 User reviews 84 Critic reviews 76 Metascore

Nominated for 1 BAFTA Award 1 win & 6 nominations total

More to explore

Pinhole camera terms

real-world
object



barrier (diaphragm)



pinhole
(aperture)



digital sensor
(CCD or CMOS)

Pinhole camera terms

real-world
object



barrier (diaphragm)



pinhole
(aperture)



camera center
(center of projection)

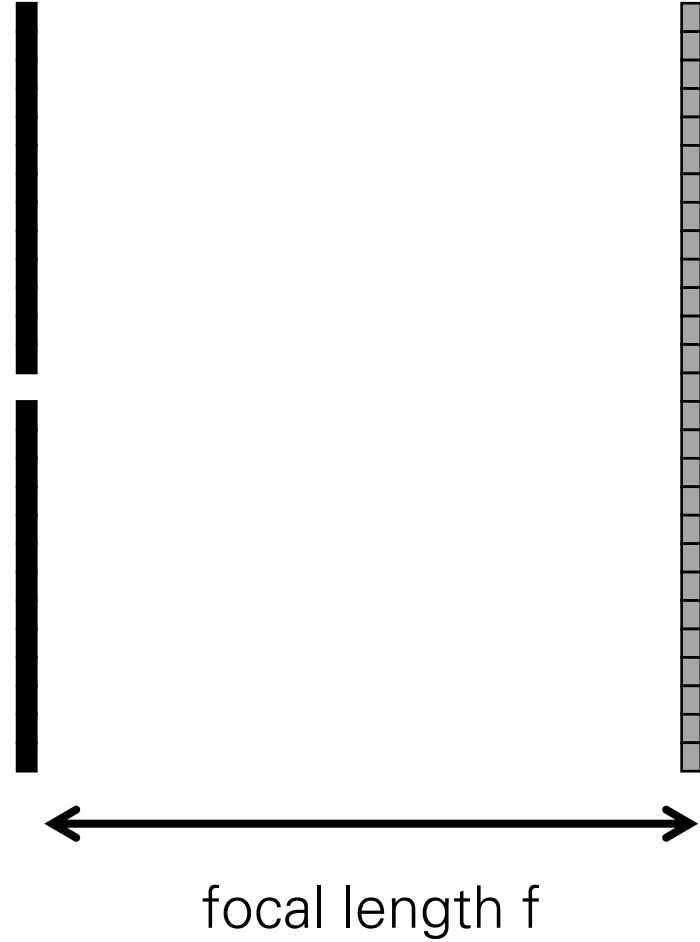
image plane



digital sensor
(CCD or CMOS)

Focal length

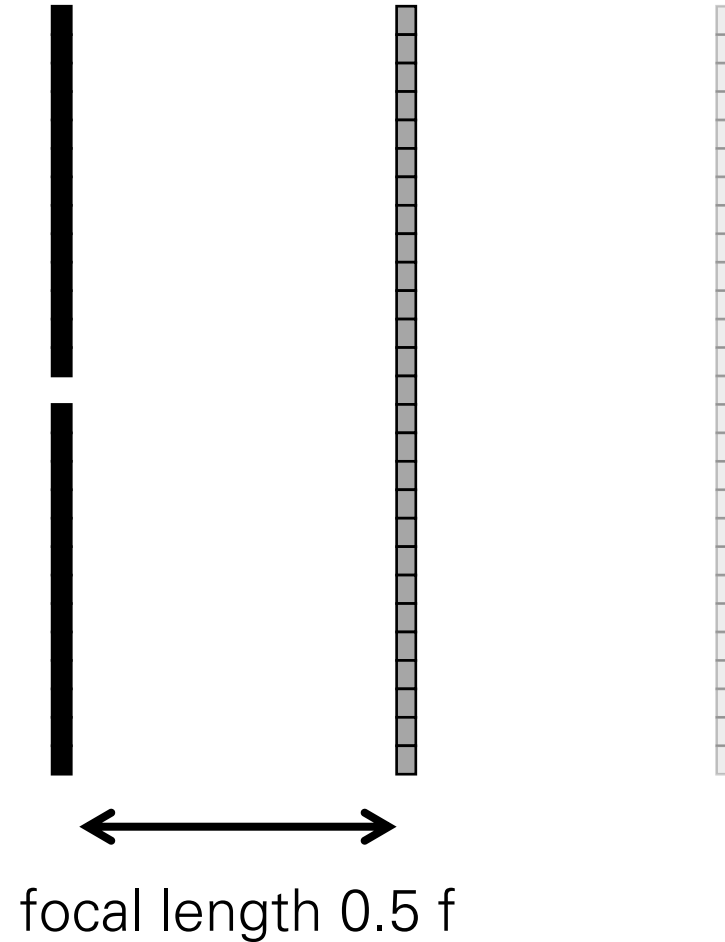
real-world
object



Focal length

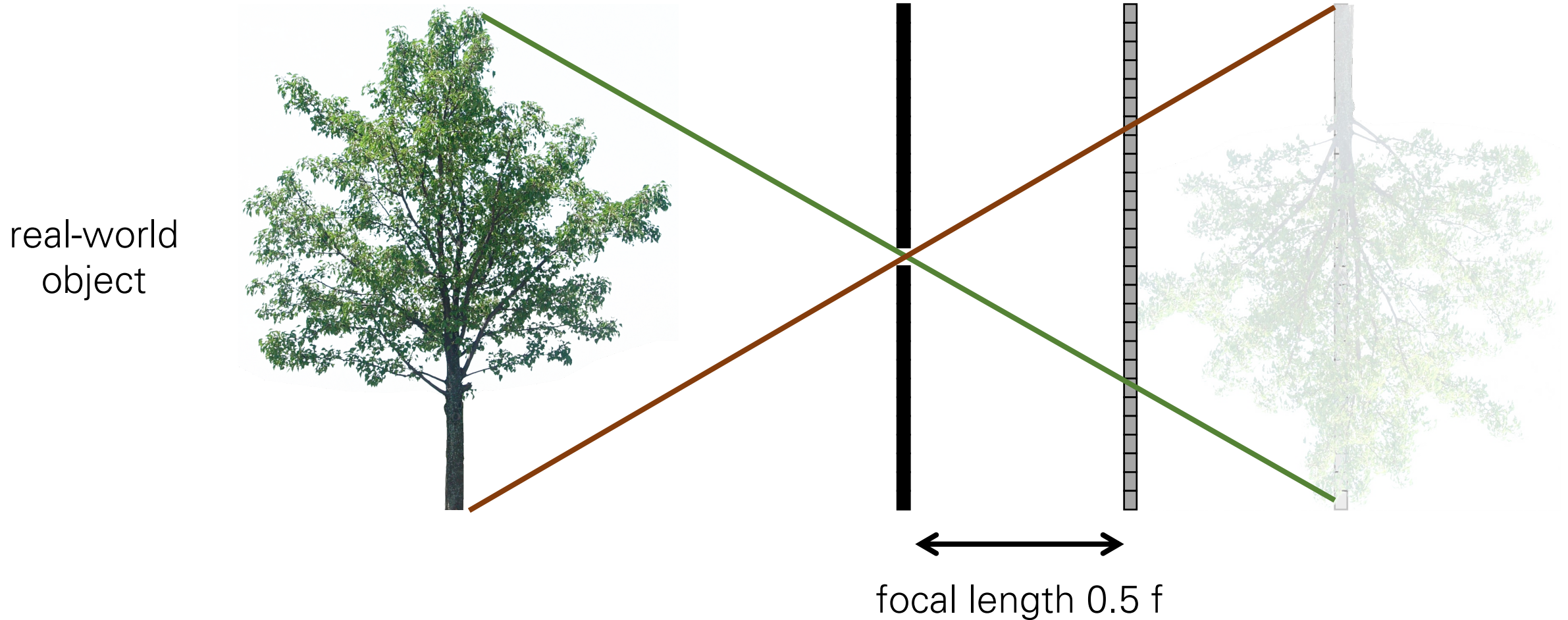
What happens as we change the focal length?

real-world
object



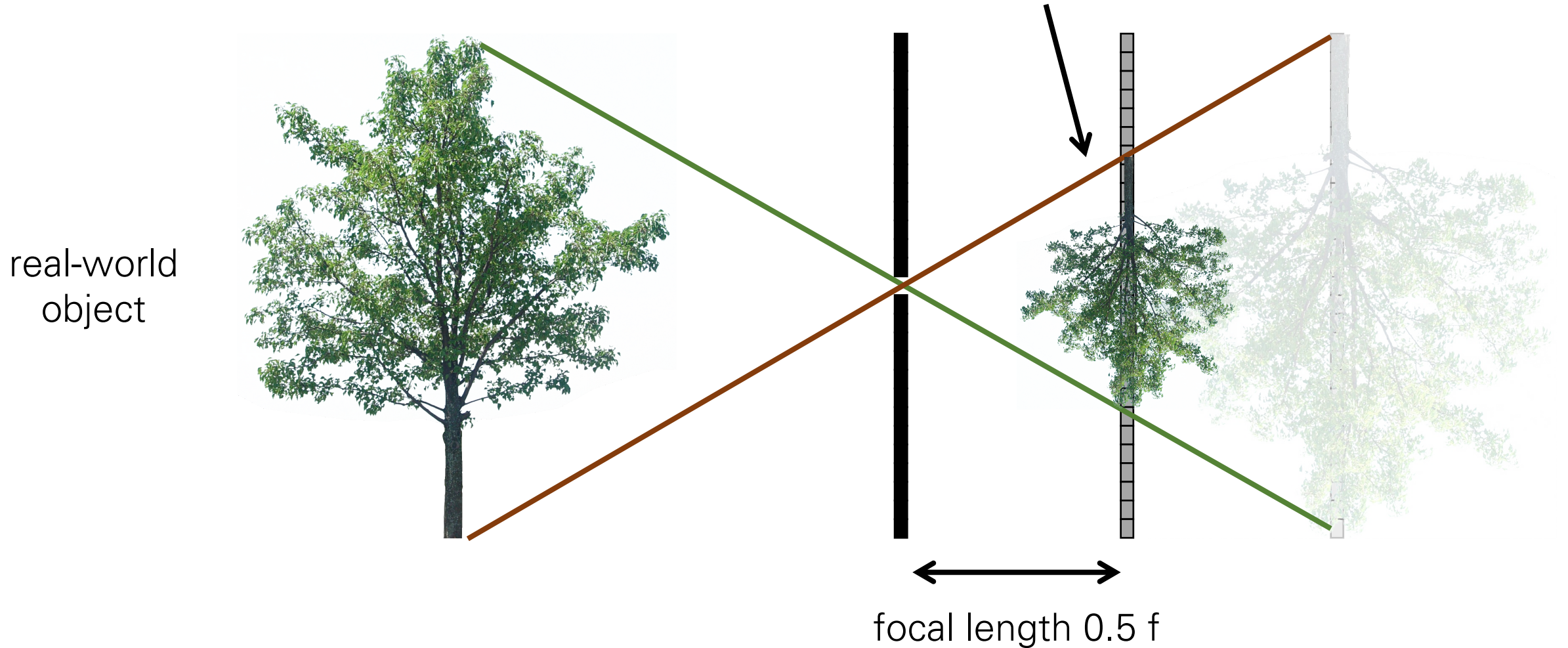
Focal length

What happens as we change the focal length?

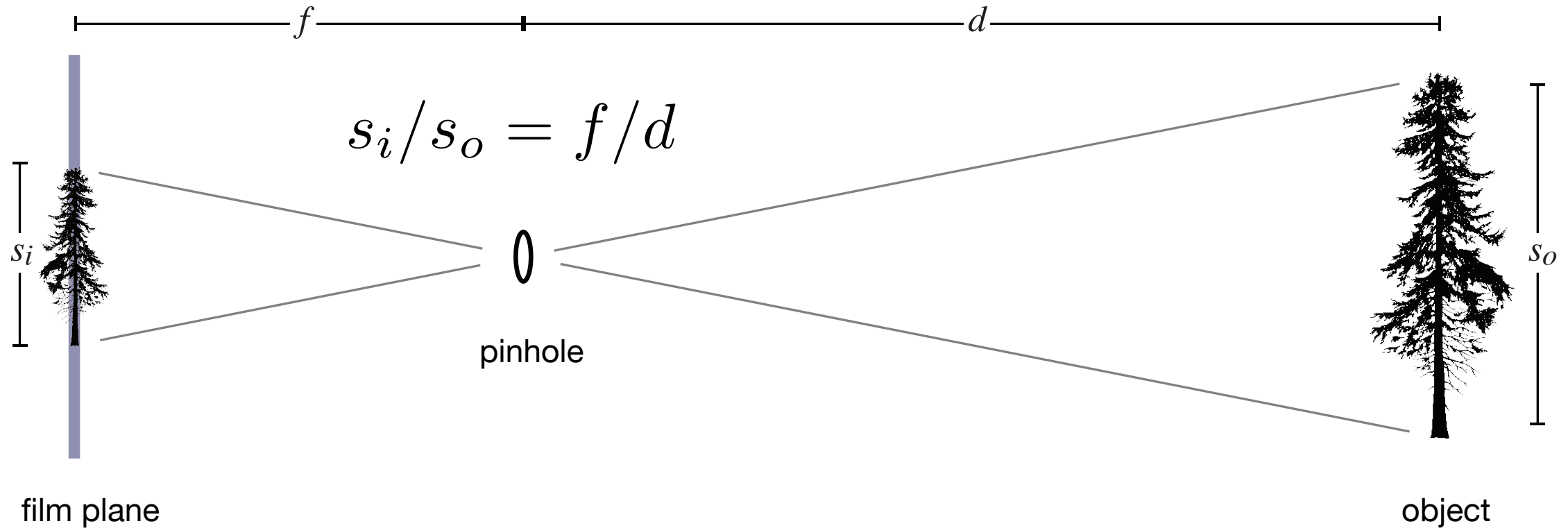


Focal length

What happens as we change the focal length?



Focal length



- Double “focal length” leads to
 - image twice as large
 - one fourth as much illumination at image plane

Pinhole size

real-world
object



pinhole
diameter



- Ideal pinhole has infinitesimally small size
- In practice that is impossible.

Pinhole size

What happens as we change the pinhole diameter?

real-world
object



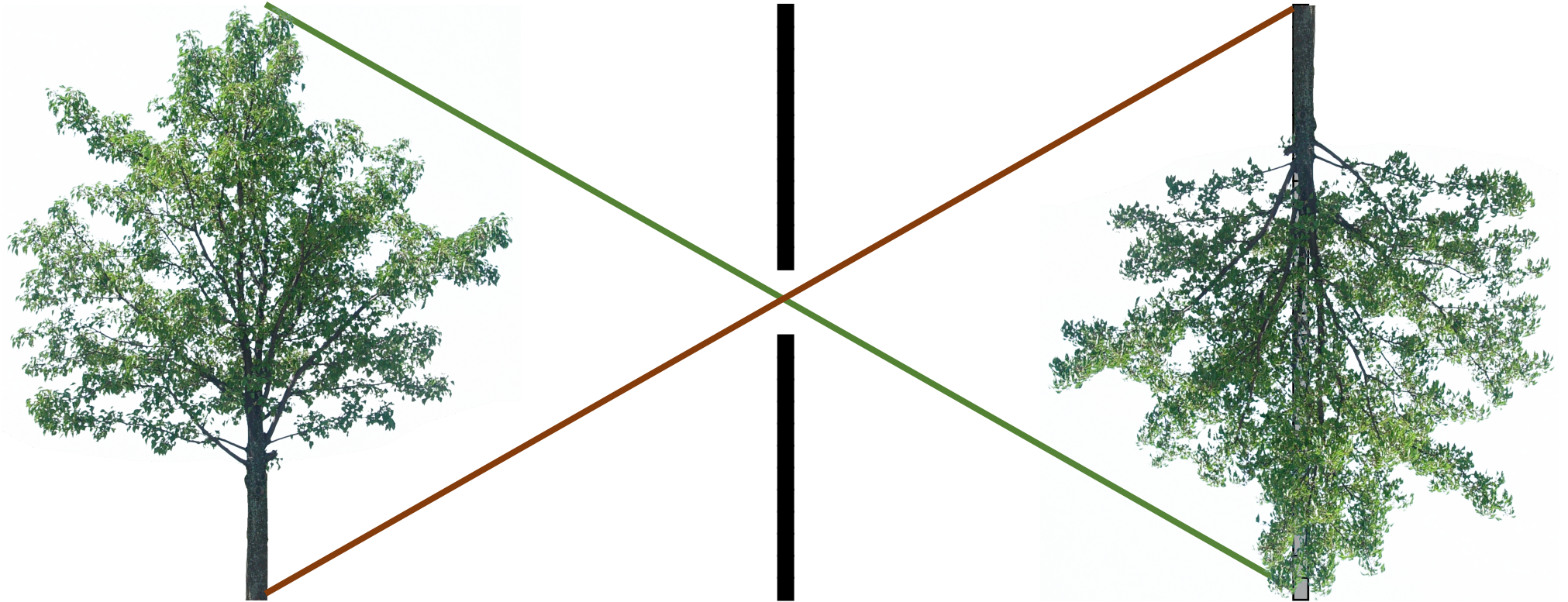
pinhole
diameter



Pinhole size

What happens as we change the pinhole diameter?

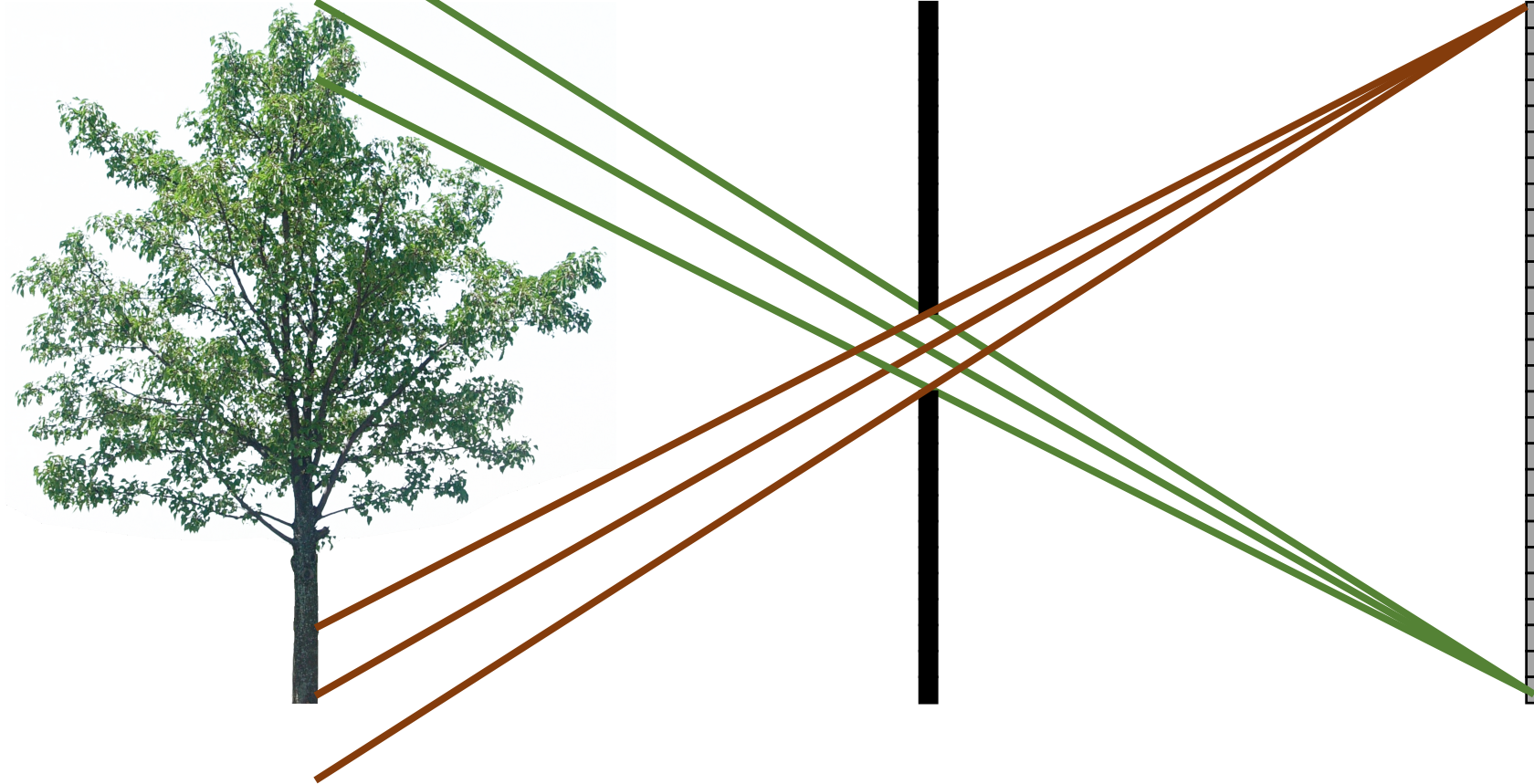
real-world
object



Pinhole size

What happens as we change the pinhole diameter?

real-world
object

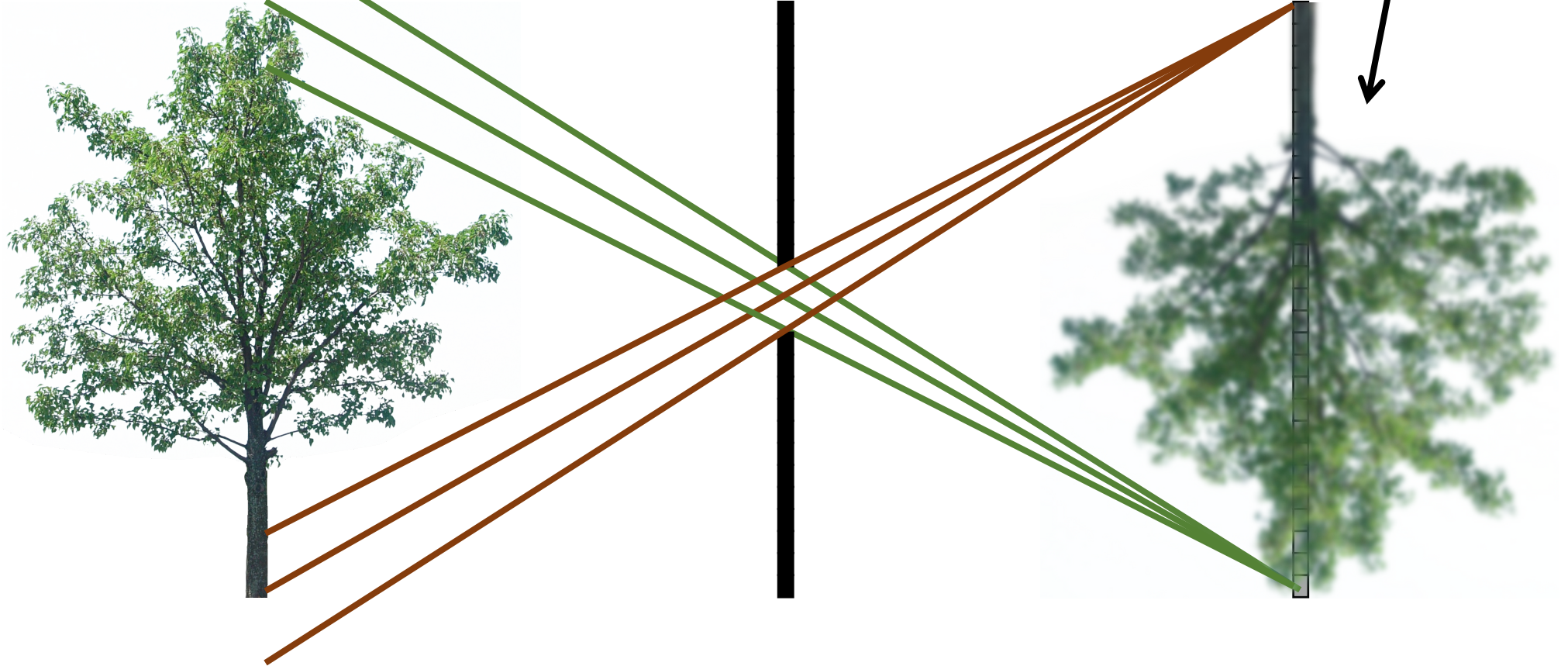


Pinhole size

What happens as we change the pinhole diameter?

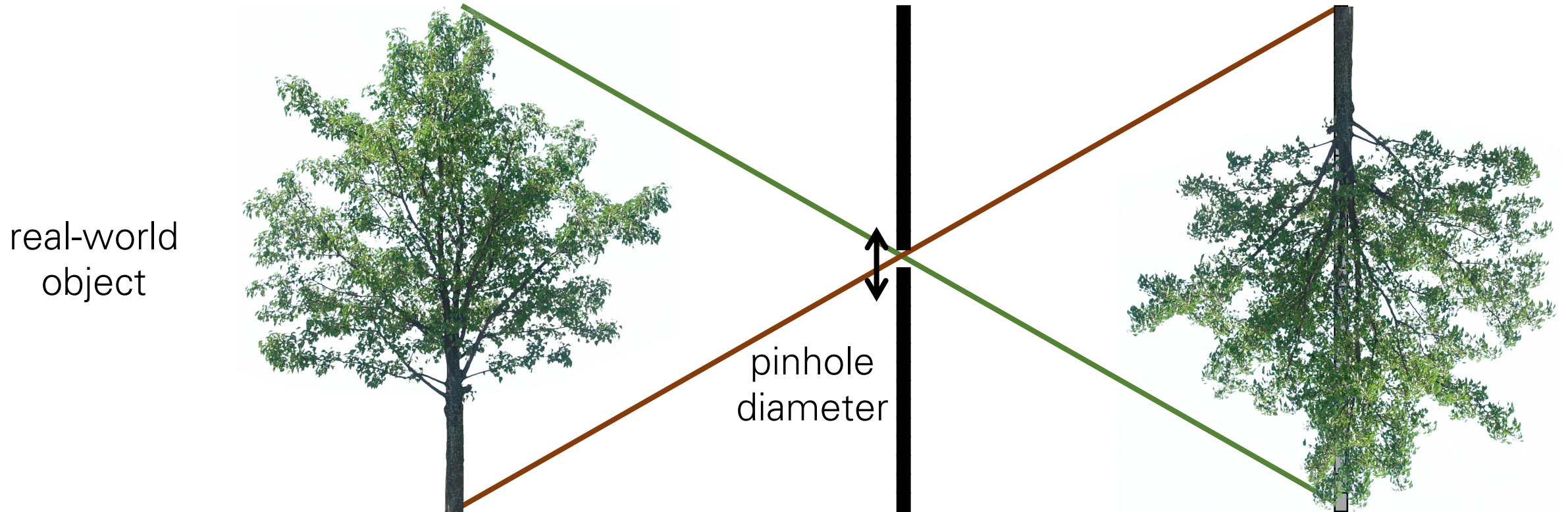
object projection becomes blurrier

real-world
object



Pinhole size

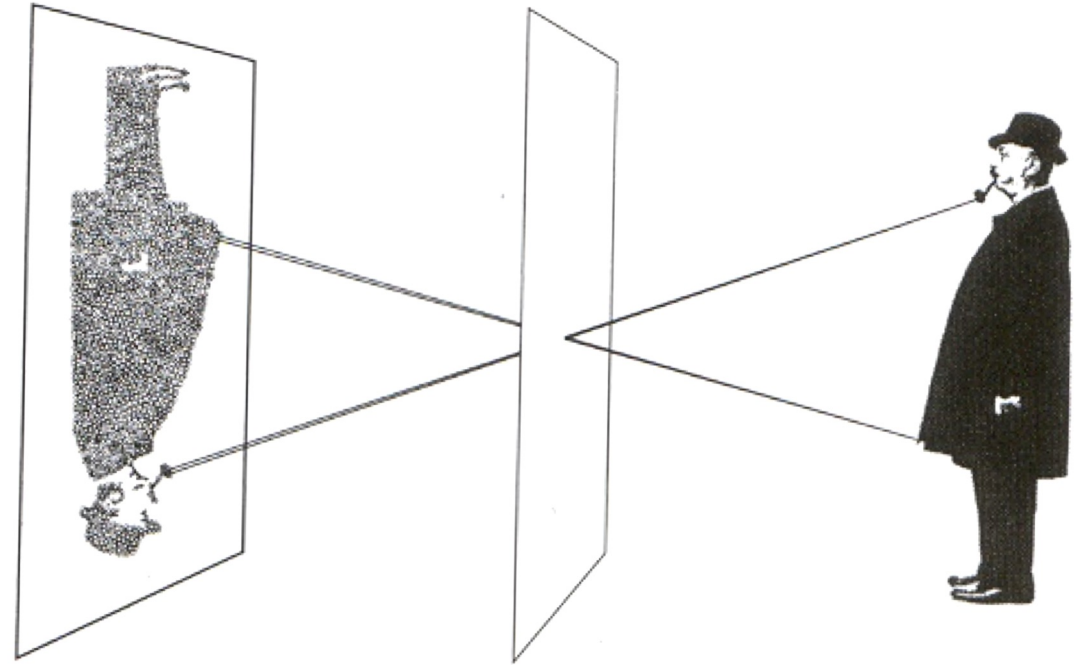
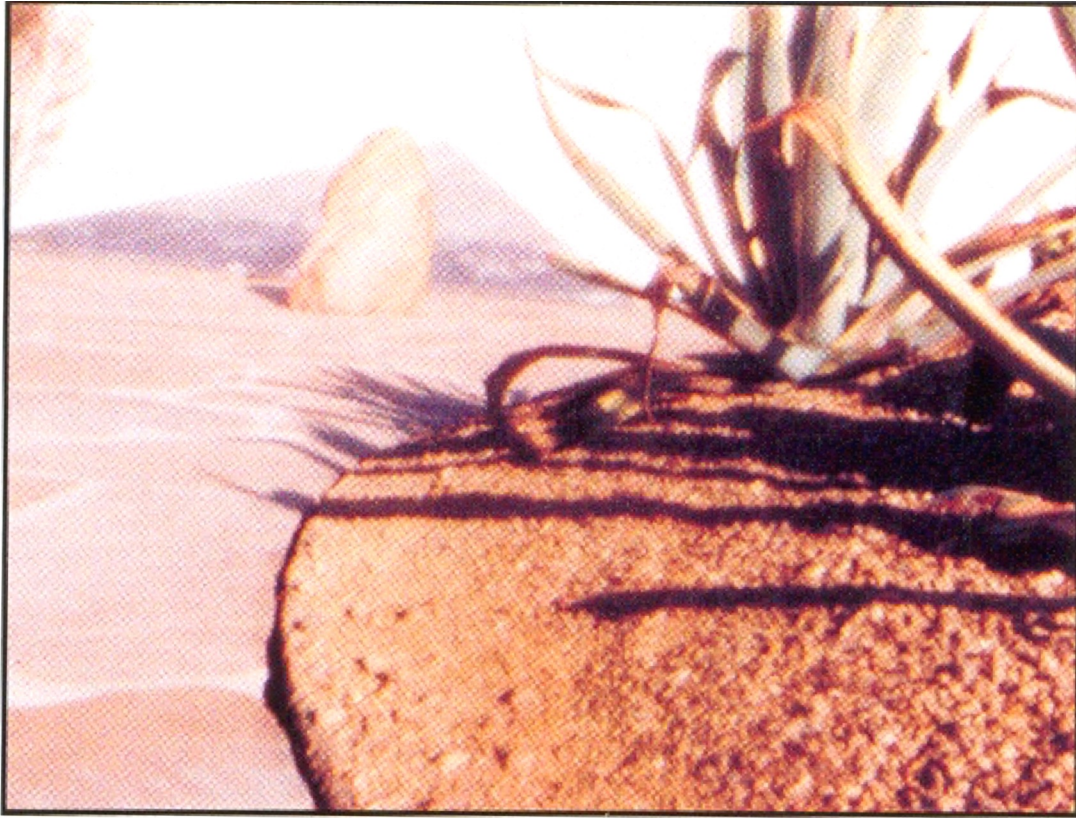
What happens as we change the pinhole diameter?



Will the image keep getting sharper the smaller we make the pinhole?

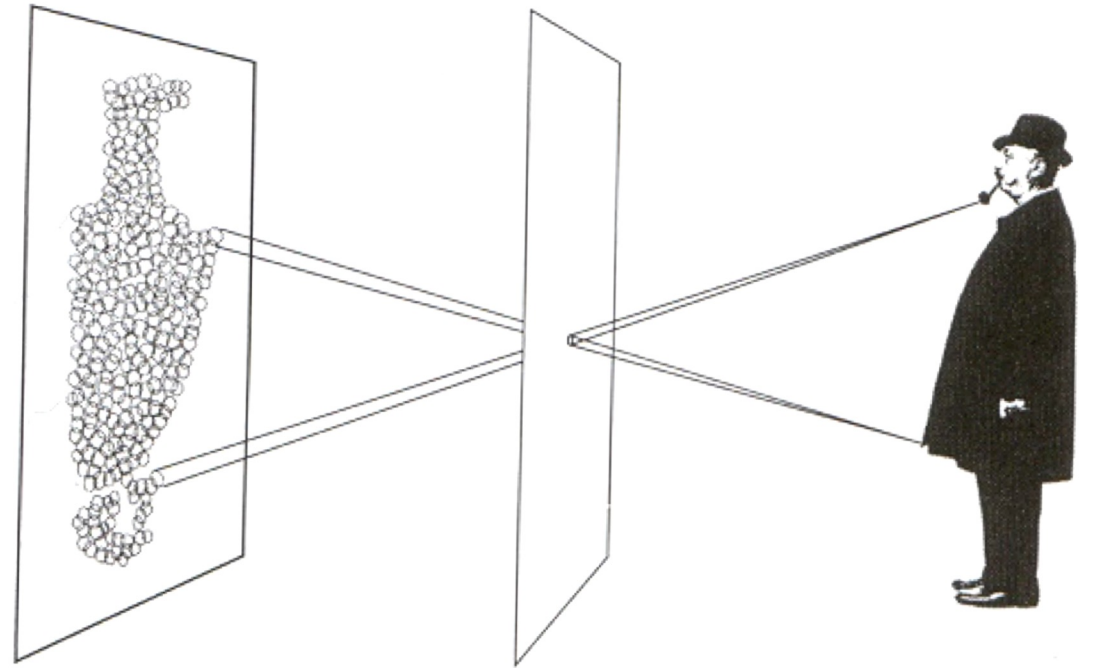
Effect of pinhole size

Photograph made with small pinhole

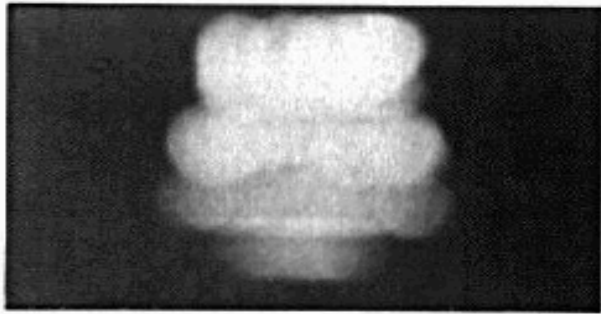


Effect of pinhole size

Photograph made with larger pinhole



Smaller pinhole is sharper



2 mm



1 mm

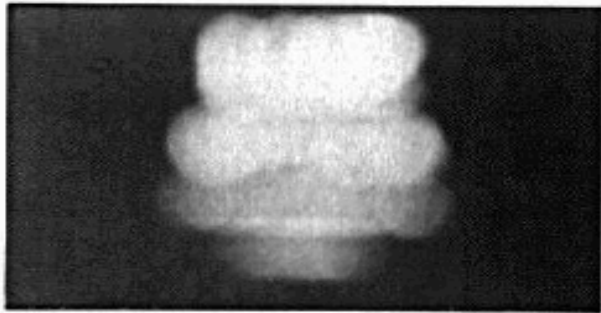


0.6mm

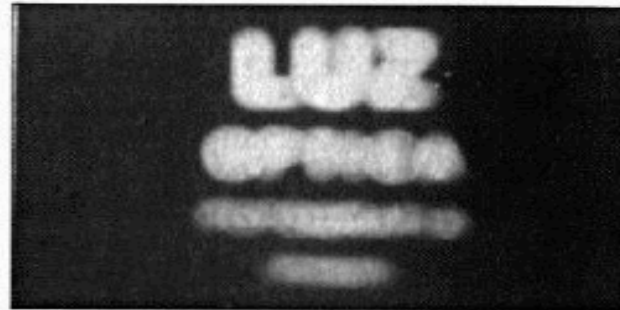


0.35 mm

Smaller pinhole is sharper ... to a point



2 mm



1 mm



0.6mm



0.35 mm



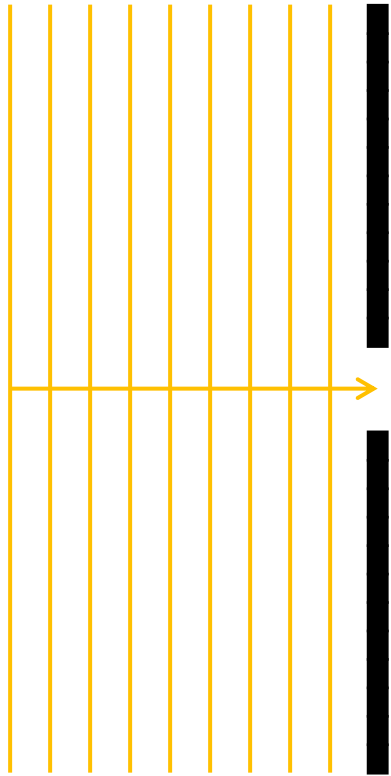
0.15 mm



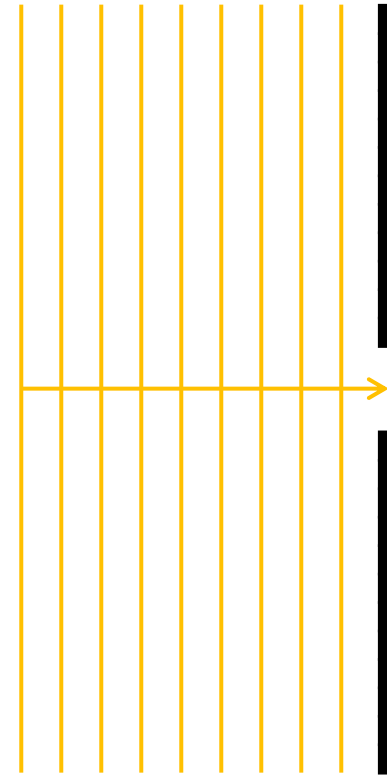
0.07 mm

Diffraction limit

A consequence of the wave nature of light



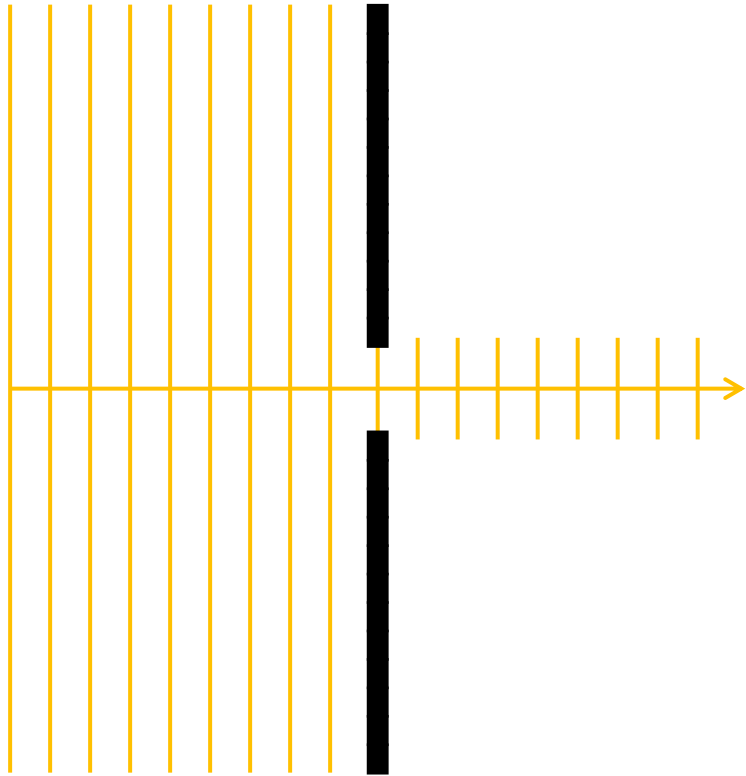
What do geometric optics predict will happen?



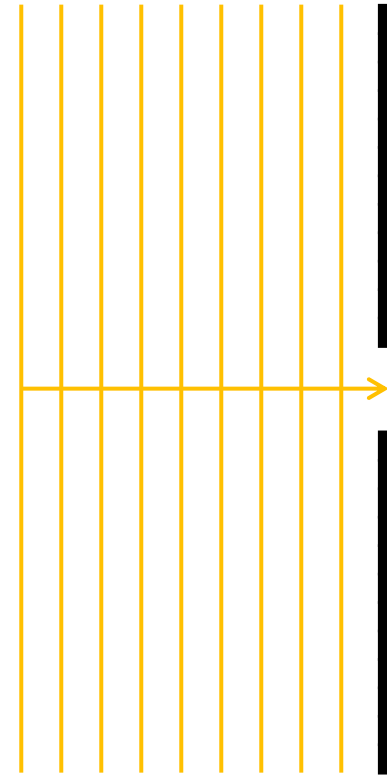
What do wave optics predict will happen?

Diffraction limit

A consequence of the wave nature of light



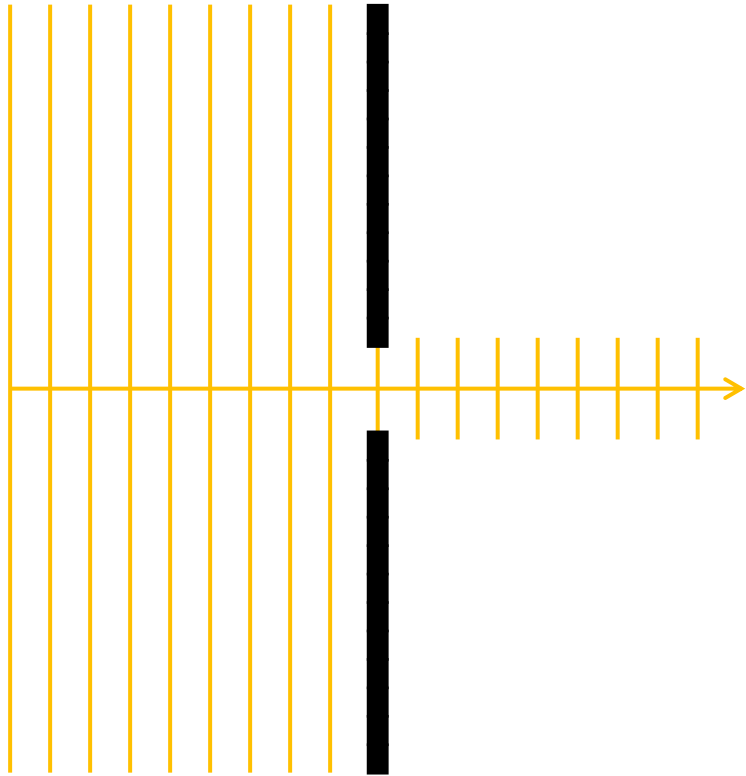
What do geometric optics predict will happen?



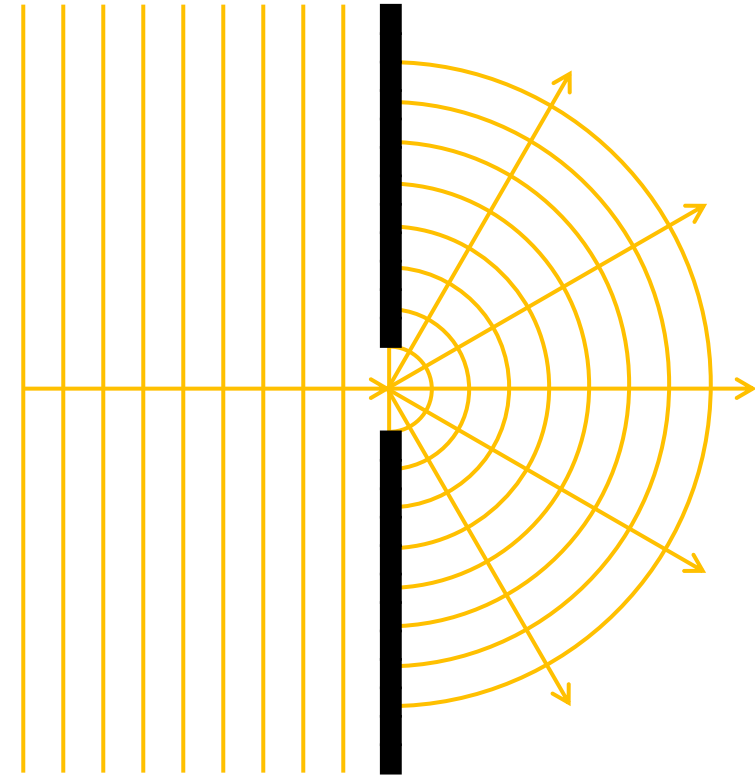
What do wave optics predict will happen?

Diffraction limit

A consequence of the wave nature of light



What do geometric optics predict will happen?

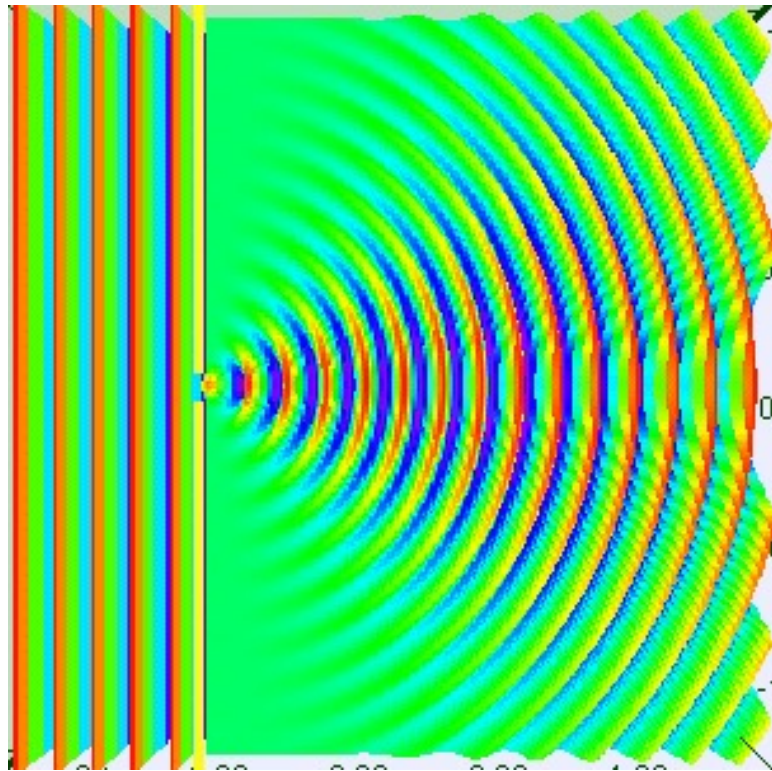


What do wave optics predict will happen?

Diffraction limit

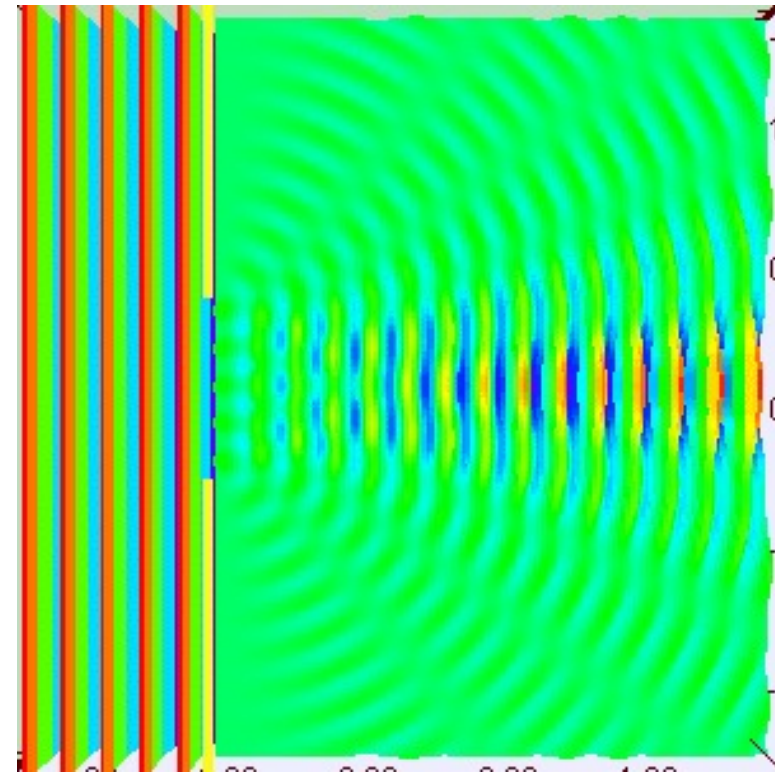
Diffraction pattern = Fourier transform of the pinhole.

- Smaller pinhole means bigger Fourier spectrum.
- Smaller pinhole means more diffraction.



small pinhole

wide
diffraction
pattern



large pinhole

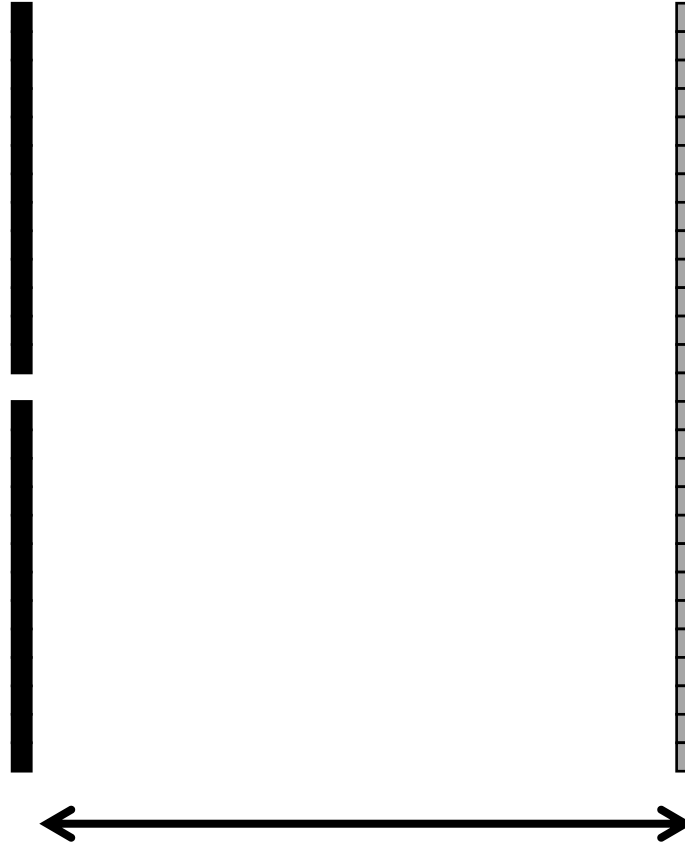
narrow
diffraction
pattern

What about light efficiency?

real-world
object



pinhole
diameter



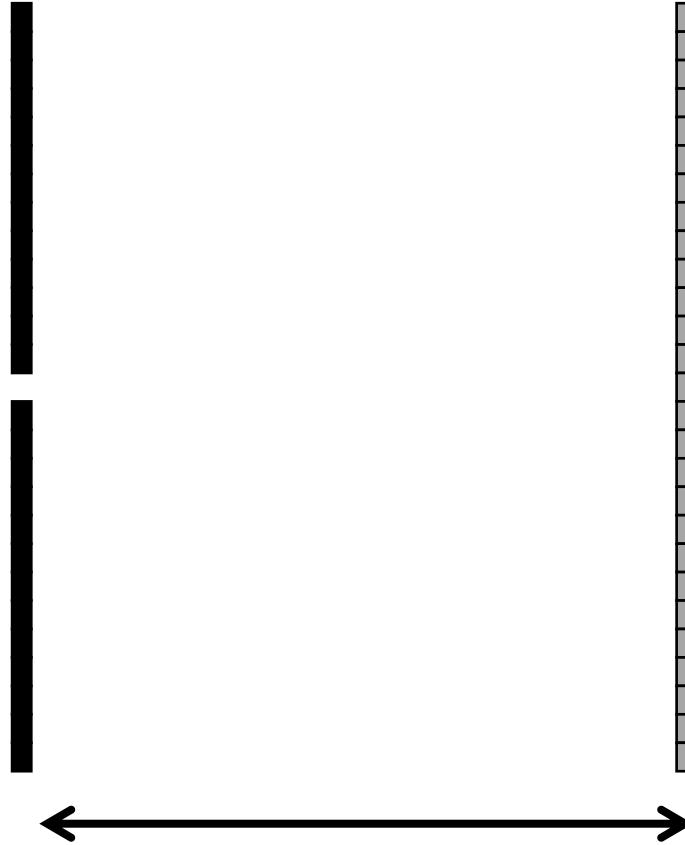
- What is the effect of doubling the pinhole diameter?
- What is the effect of doubling the focal length?

What about light efficiency?

real-world
object



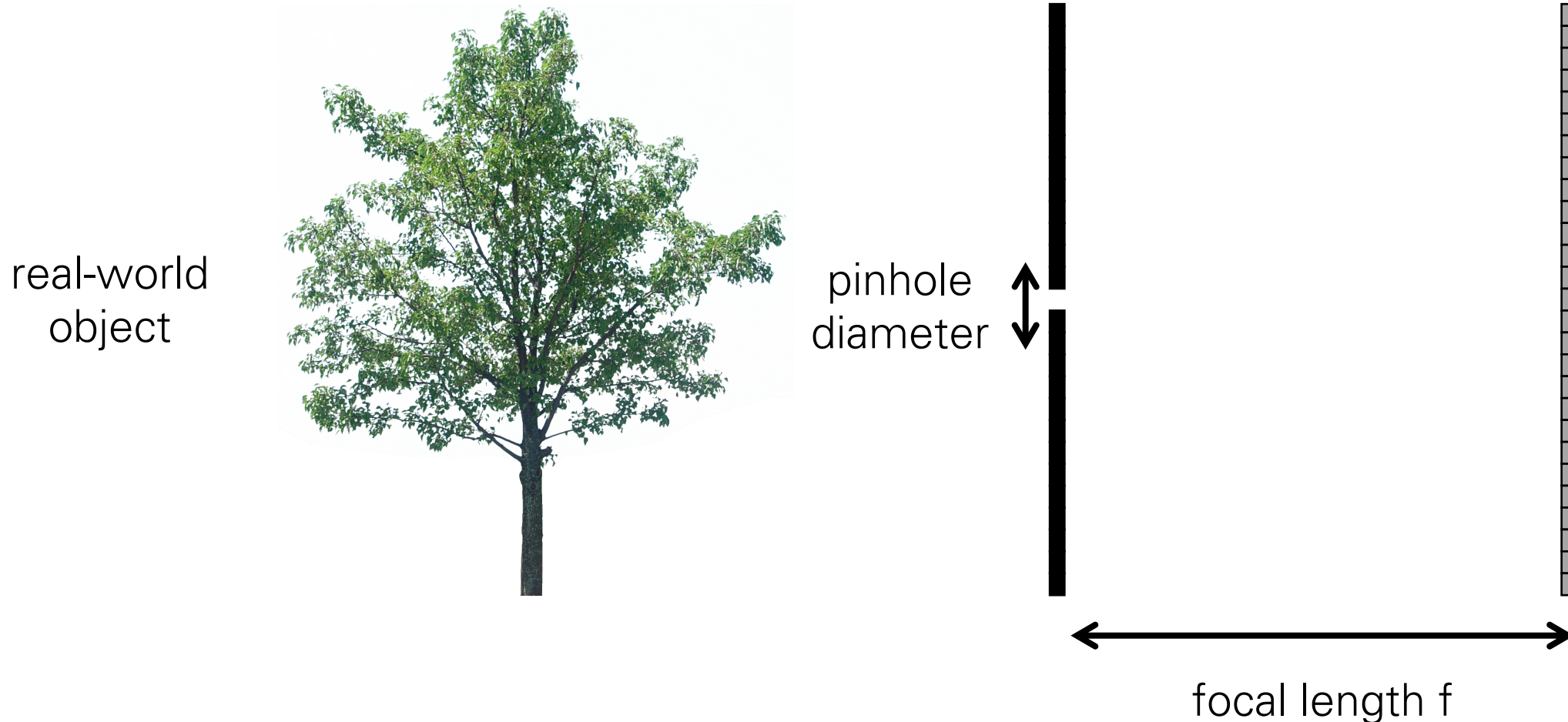
pinhole
diameter



- 2x pinhole diameter \rightarrow 4x light
- 2x focal length \rightarrow $\frac{1}{4}$ x light

Some terminology notes

A “stop” is a change in camera settings that changes amount of light by a factor of 2



The “f-number” is the ratio: focal length / pinhole diameter

Accidental pinholes

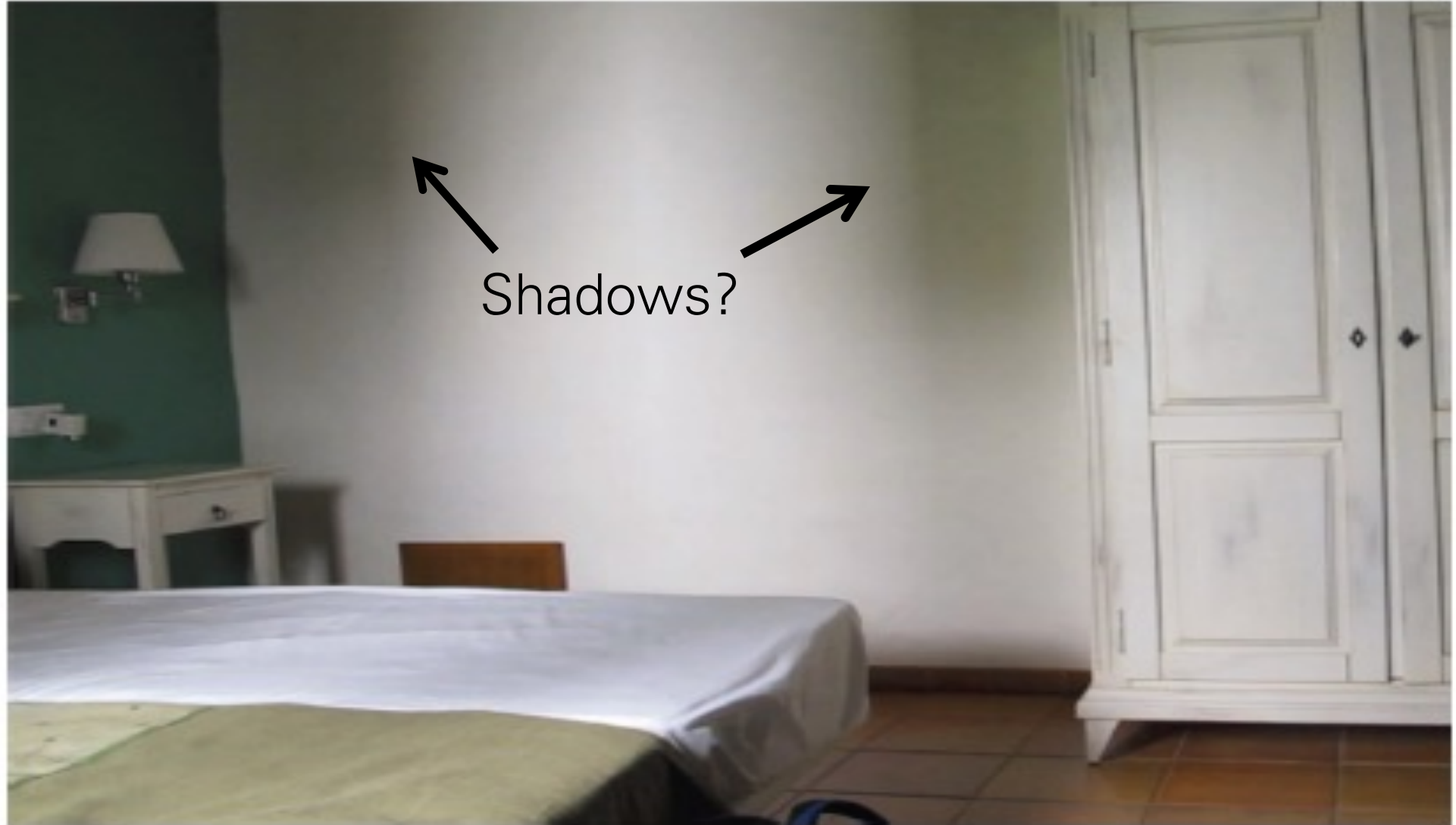
What does this image say about the world outside?



Accidental pinhole camera



Accidental pinhole camera



Accidental pinhole camera



Accidental pinhole camera



Accidental pinhole camera

Antonio Torralba, William T. Freeman
Computer Science and Artificial Intelligence Laboratory (CSAIL)
MIT
torralba@mit.edu, billf@mit.edu



Anti-pinhole or Pinspeck cameras

OPTICA ACTA, 1982, VOL. 29, NO. 1, 63-67

Anti-pinhole imaging

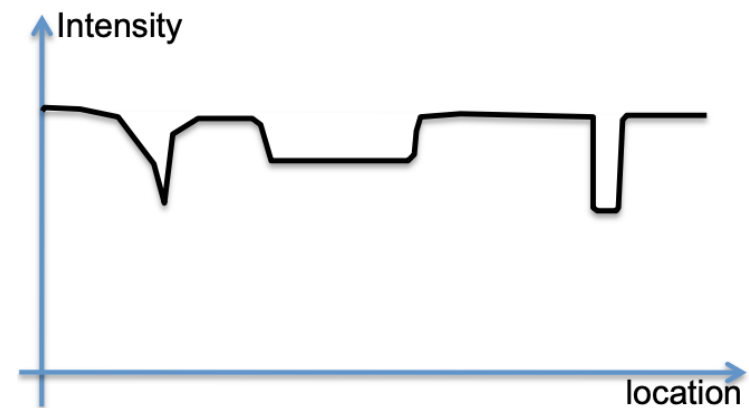
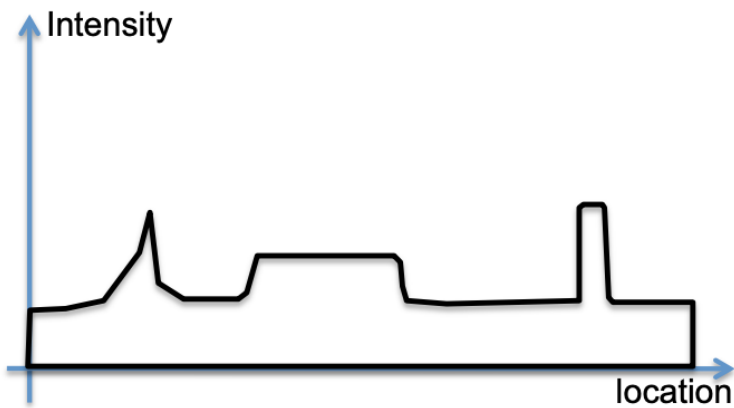
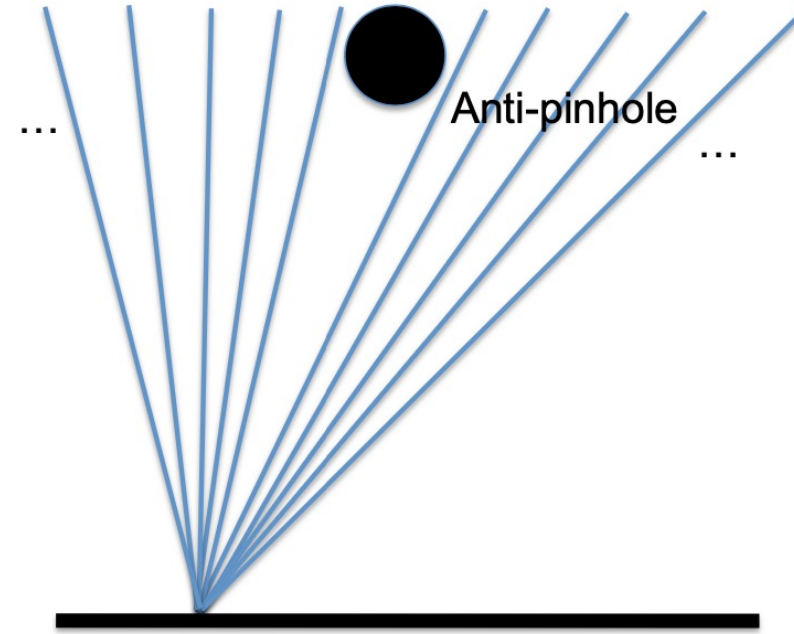
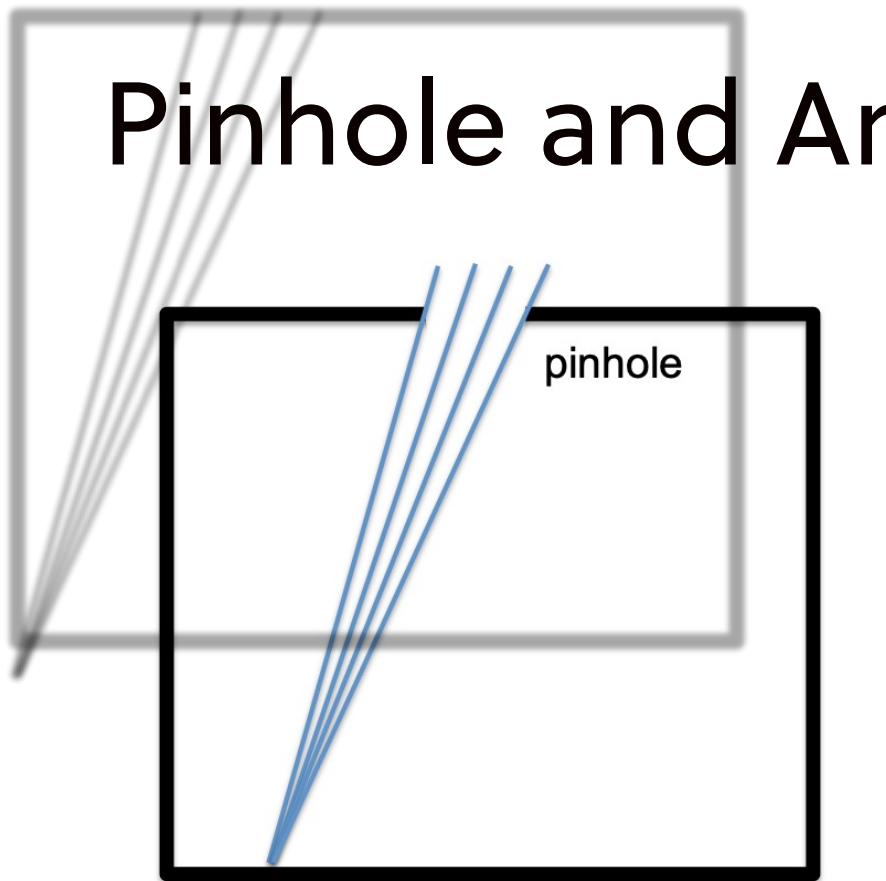
ADAM LLOYD COHEN

Parmly Research Institute, Loyola University of Chicago,
Chicago, Illinois 60626, U.S.A.

(Received 16 April 1981; revision received 8 July 1981)

Abstract. By complementing a pinhole to produce an isolated opaque spot, the light ordinarily blocked from the pinhole image is transmitted, and the light ordinarily transmitted is blocked. A negative geometrical image is formed, distinct from the familiar 'bright-spot' diffraction image. Anti-pinhole, or 'pinspeck' images are visible during a solar eclipse, when the shadows of objects appear crescent-shaped. Pinspecks demonstrate unlimited depth of field, freedom from distortion and large angular field. Images of different magnification may be formed simultaneously. Contrast is poor, but is improvable by averaging to remove noise and subtraction of a d.c. bias. Pinspecks may have application in X-ray space optics, and might be employed in the eyes of simple organisms.

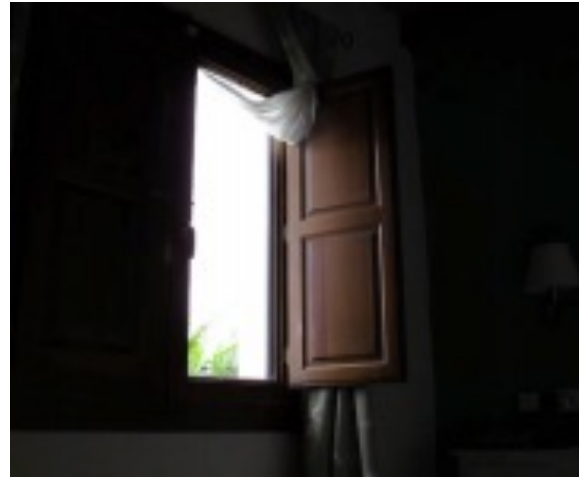
Pinhole and Anti-pinhole cameras



Adam L. Cohen, 1982

Accidental pinhole camera

projected pattern on the wall



window is an aperture

upside down



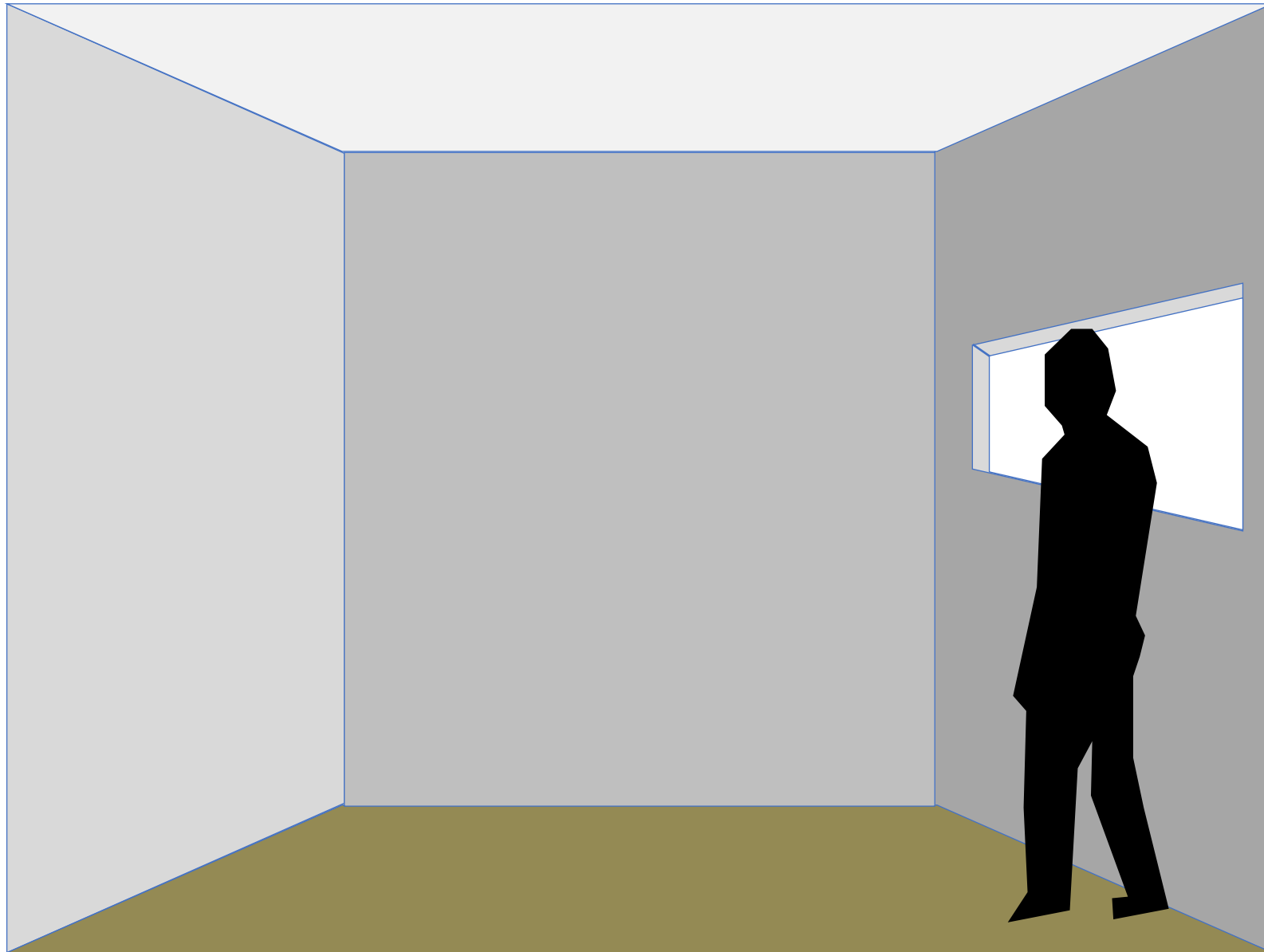
window with smaller gap



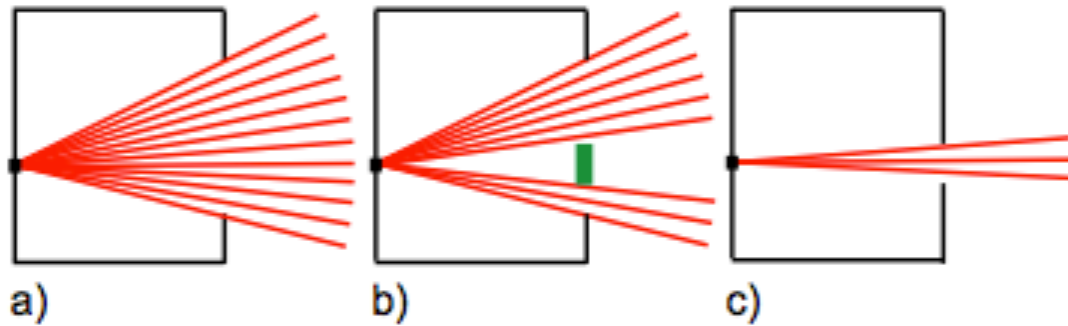
view outside window



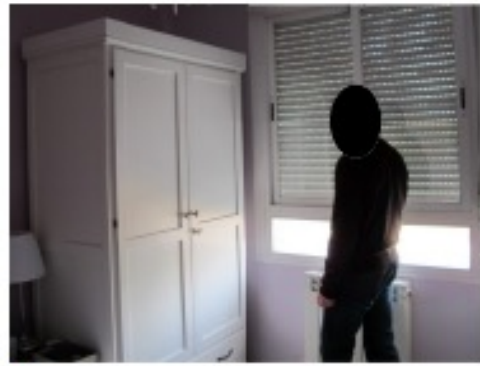
Mixed accidental pinhole and anti-pinhole cameras



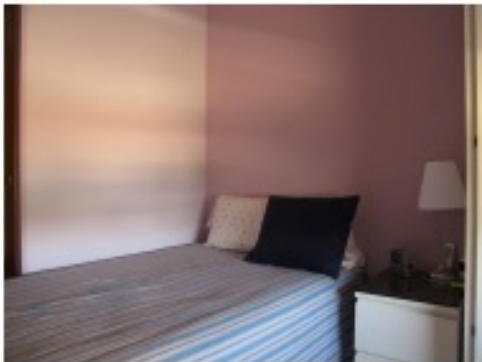
Accidental pinspeck camera



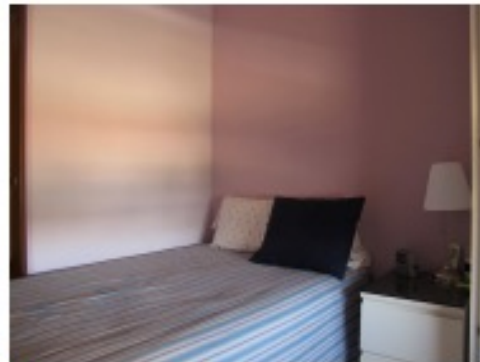
a)



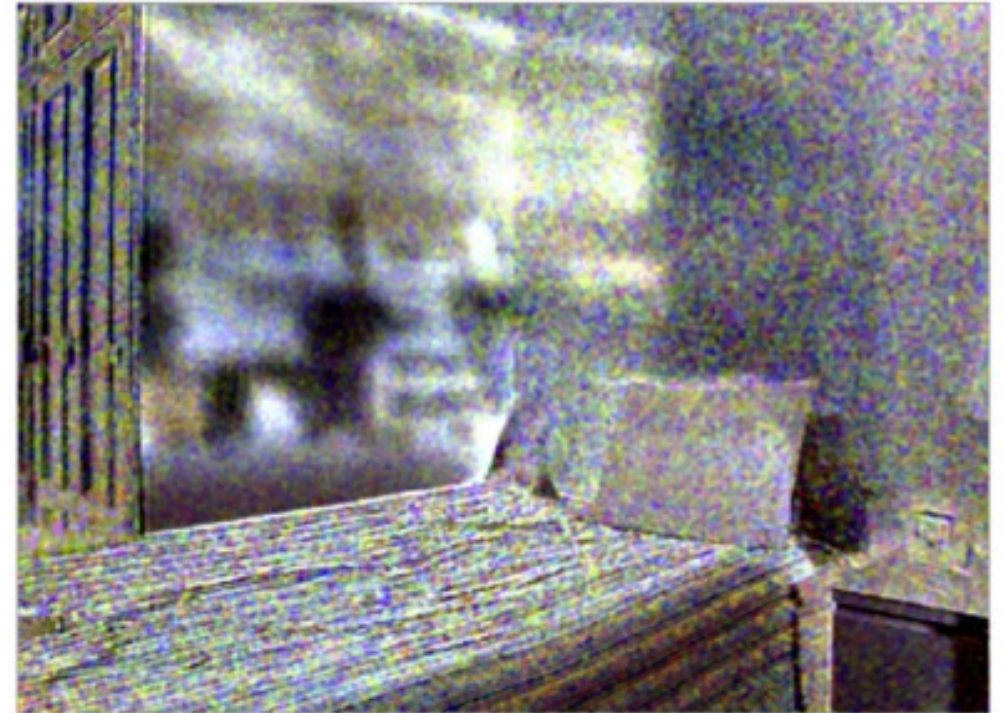
b)



c)



d)



a) Difference image

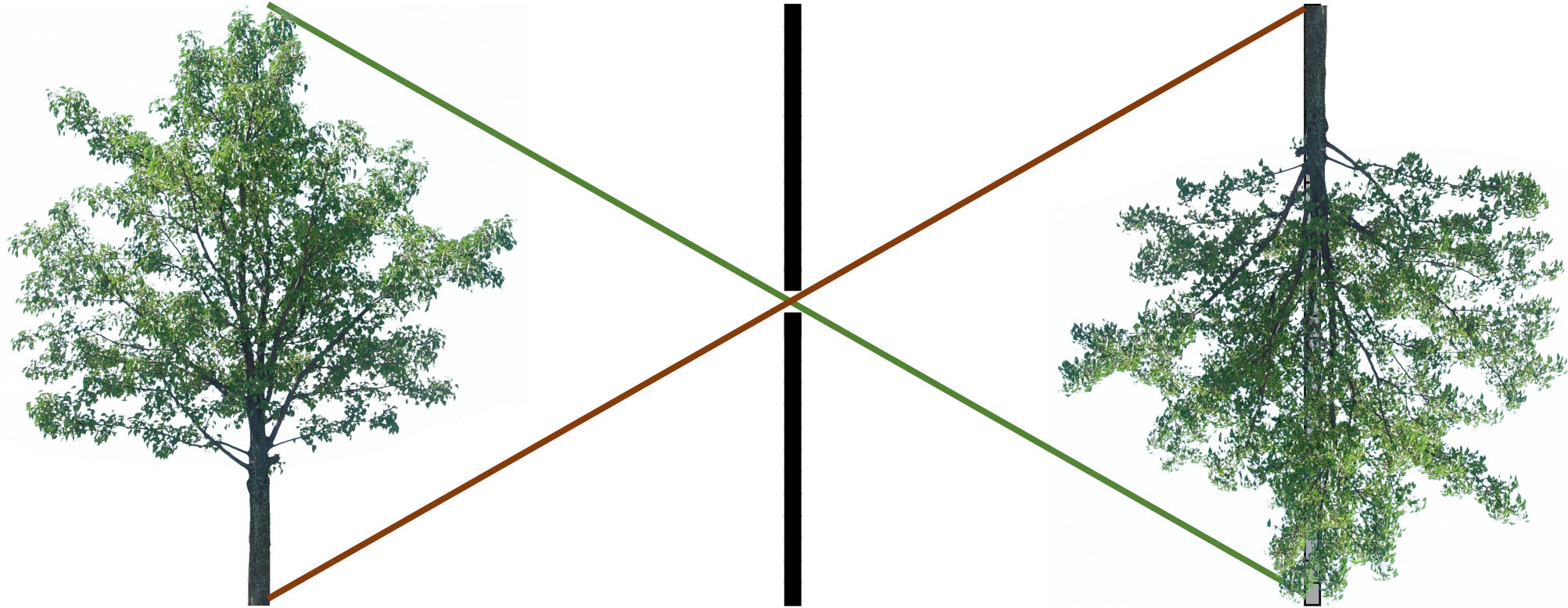


b) Difference upside down



c) True outdoor view

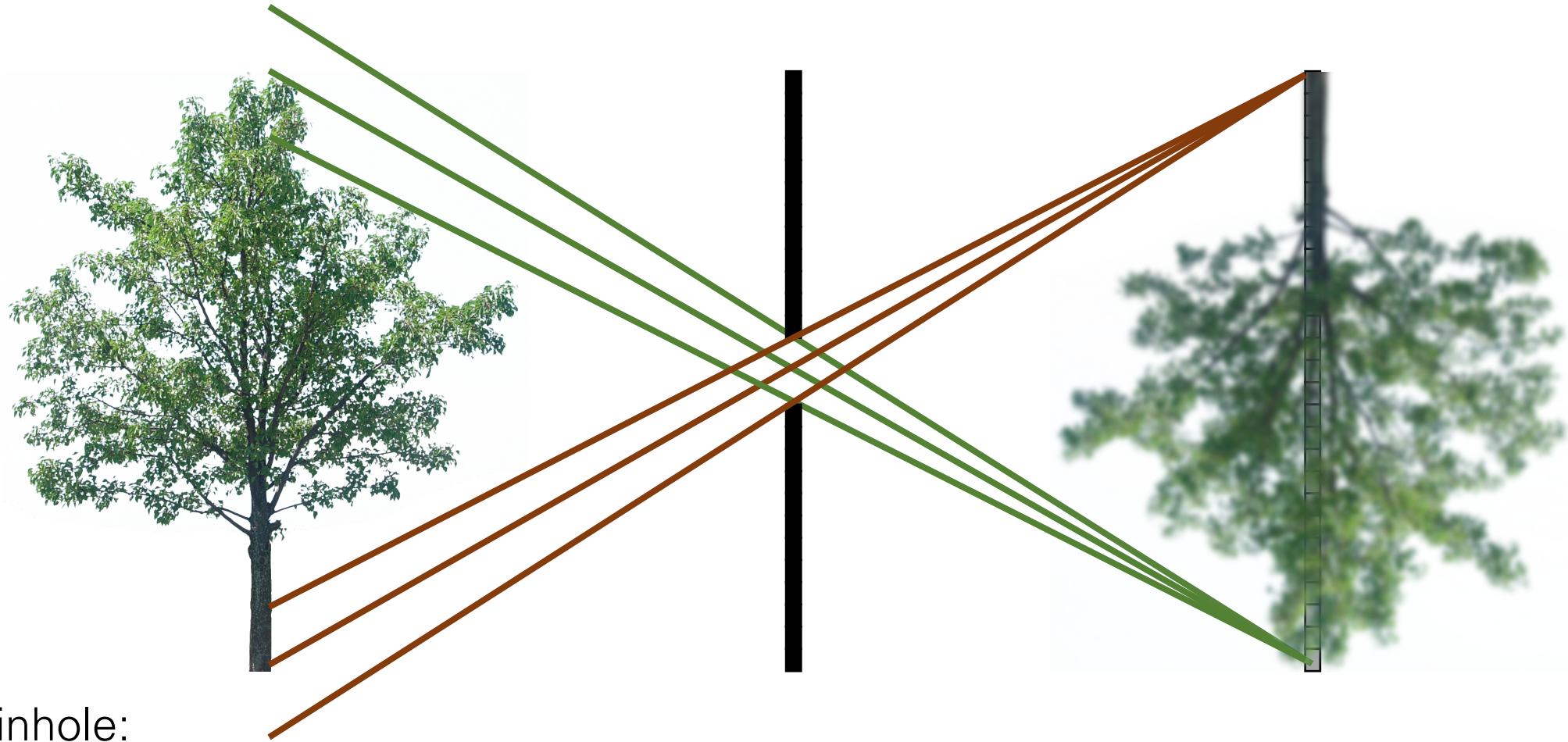
Pinhole camera trade-off



Small (ideal) pinhole:

1. Image is sharp.
2. Signal-to-noise ratio is low.

Pinhole camera trade-off

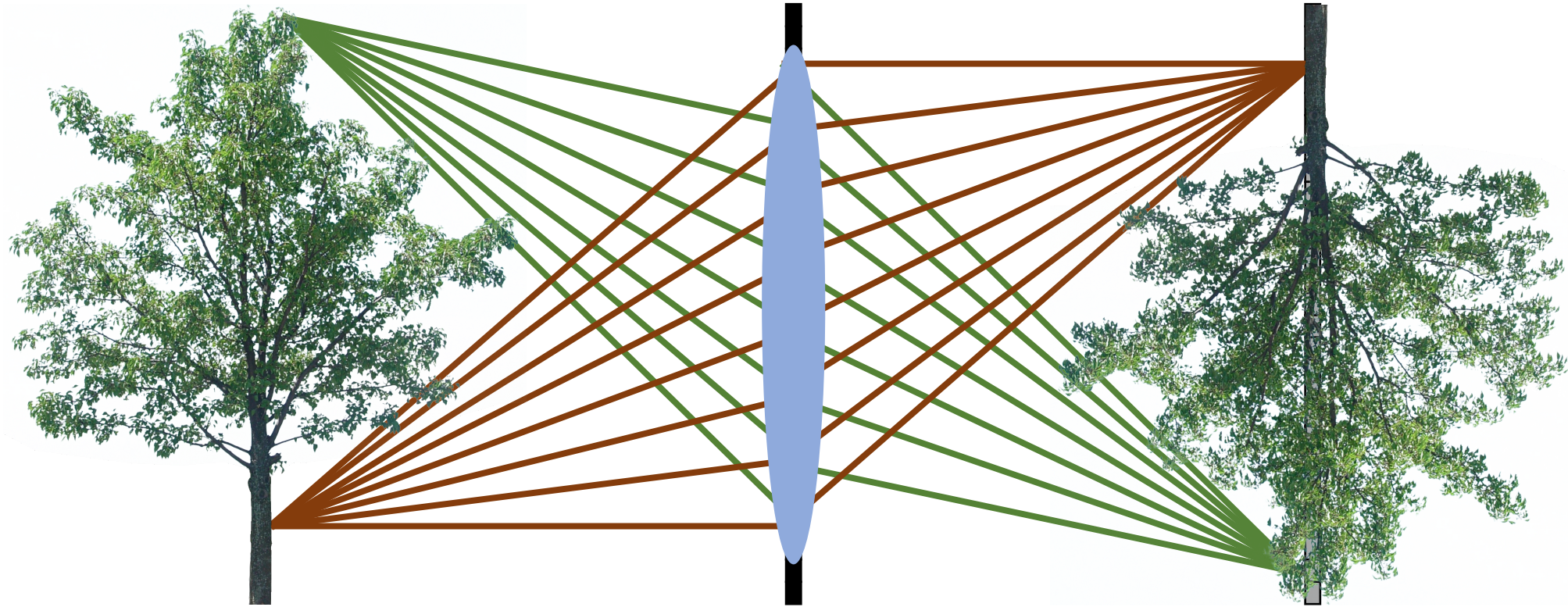


Large pinhole:

1. Image is blurry.
2. Signal-to-noise ratio is high.

Can we get best of both worlds?

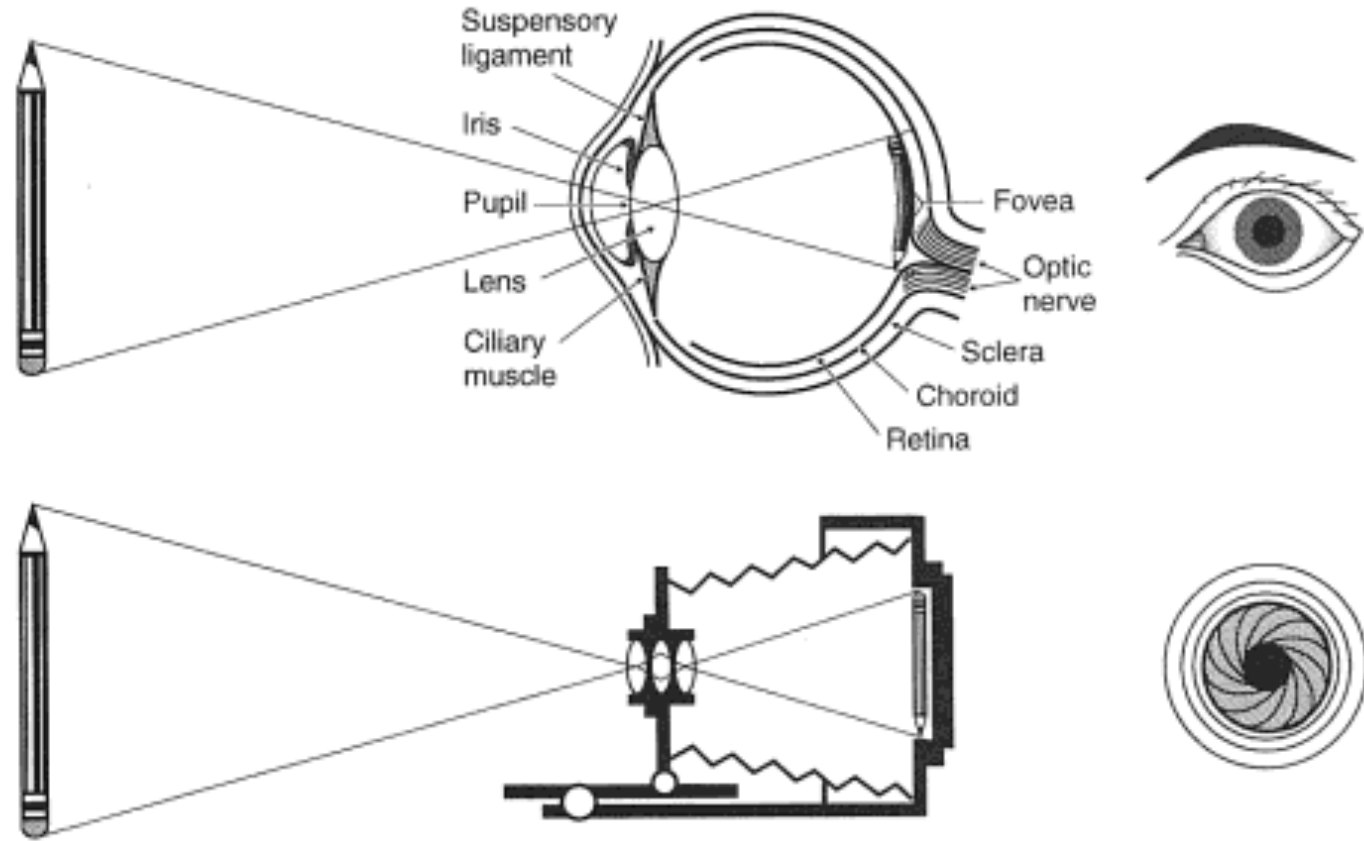
Almost, by using lenses



Lenses map “bundles” of rays from points on the scene to the sensor.

How does this mapping work exactly?

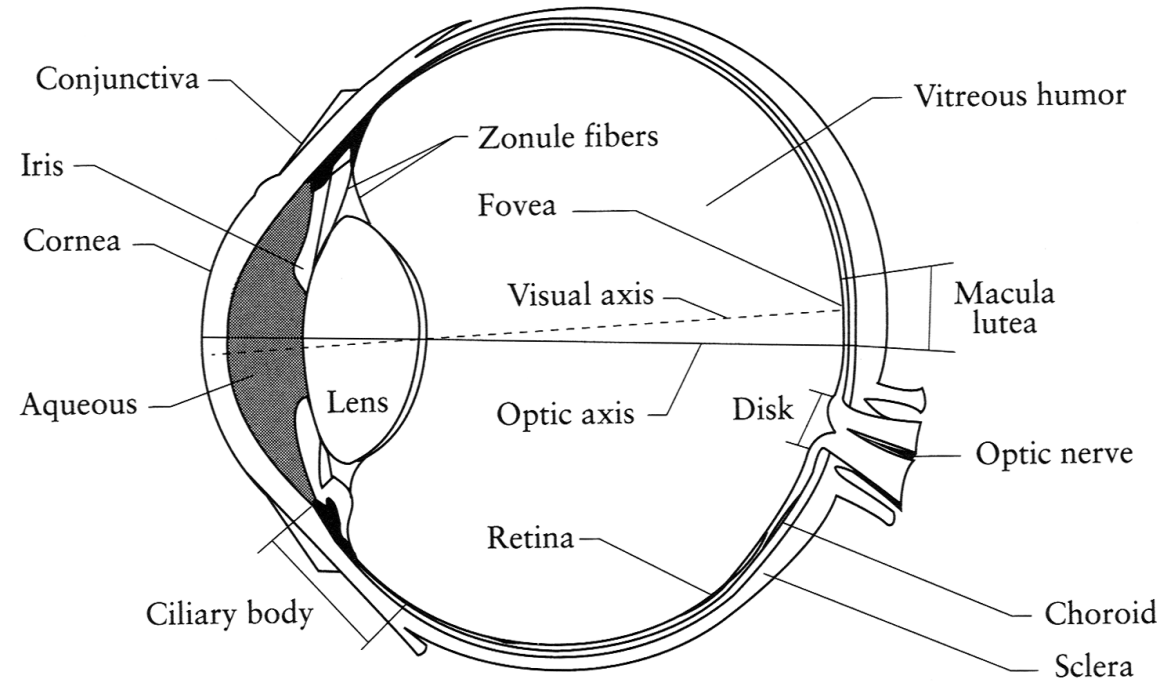
The Eye



Figures: Francis Crick, The Astonishing Hypothesis, 1995

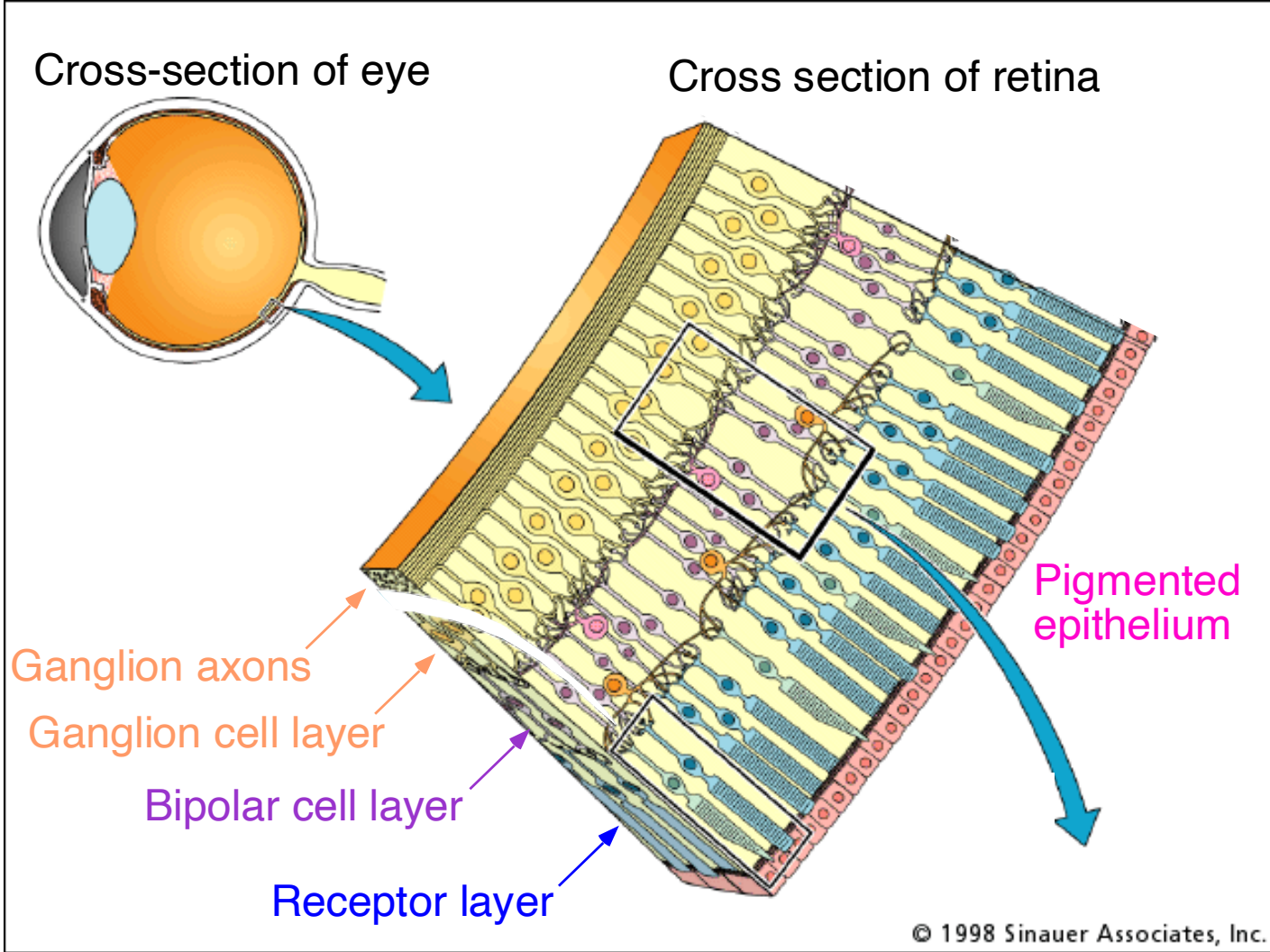
- The human eye is a camera

The Eye

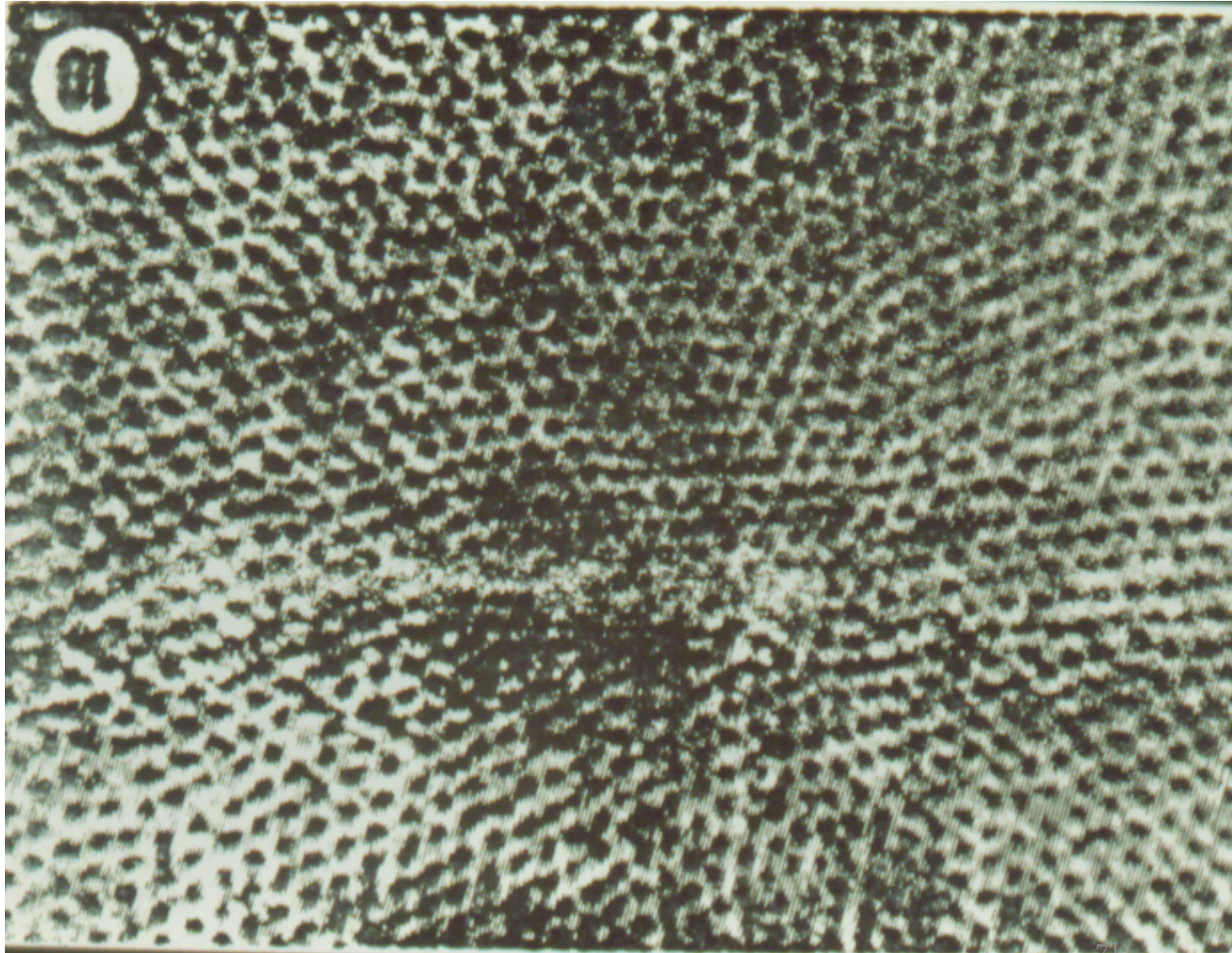


- The human eye is a camera!
 - Iris - colored annulus with radial muscles
 - Pupil - the hole (aperture) whose size is controlled by the iris
 - What's the "film"?
 - photoreceptor cells (rods and cones) in the retina

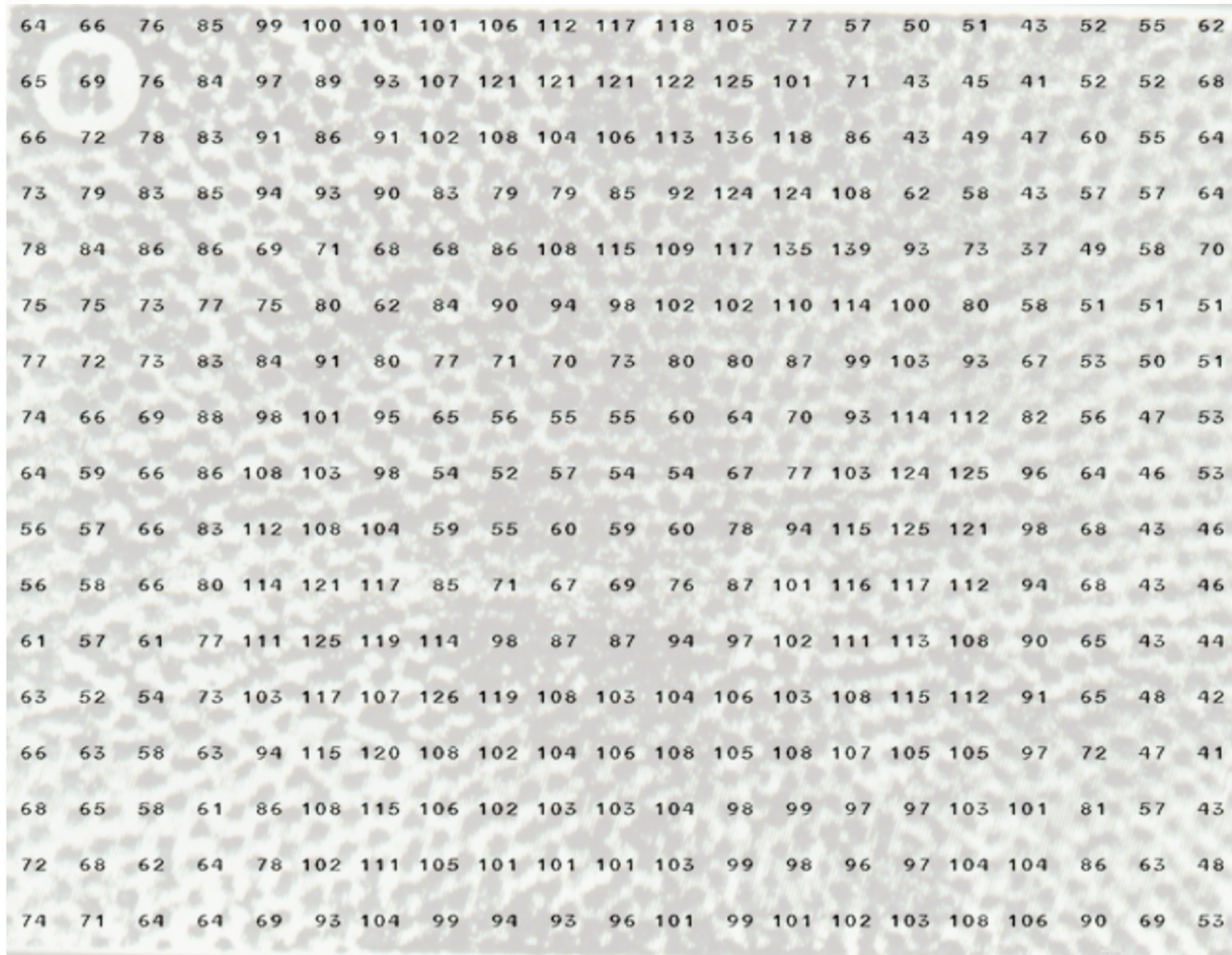
The Retina



Receptors Density - Fovea



Receptors Density - Fovea



Animal Eyes

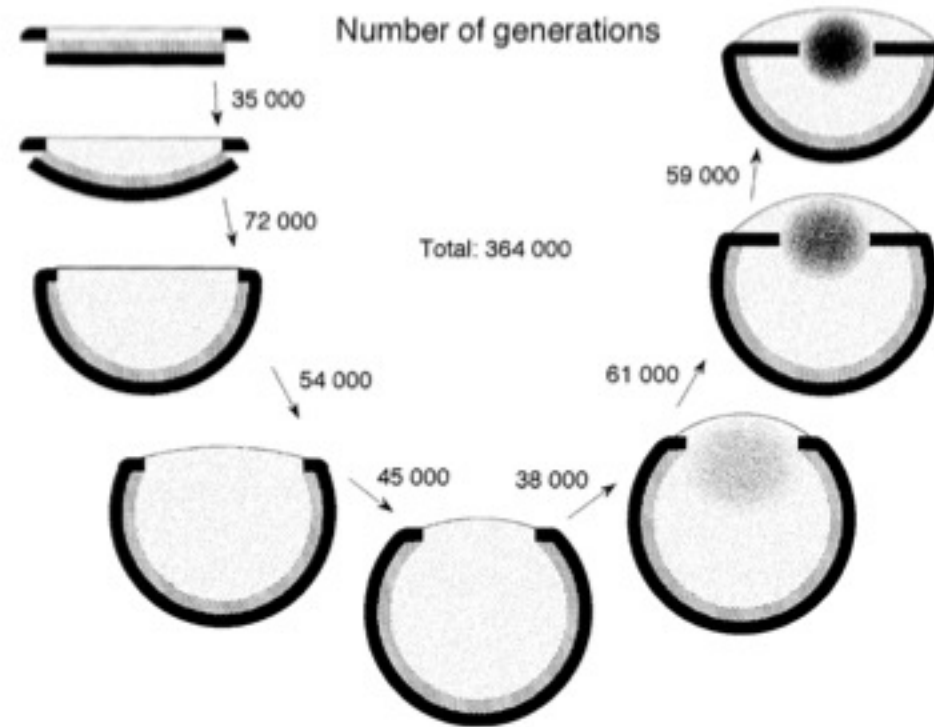
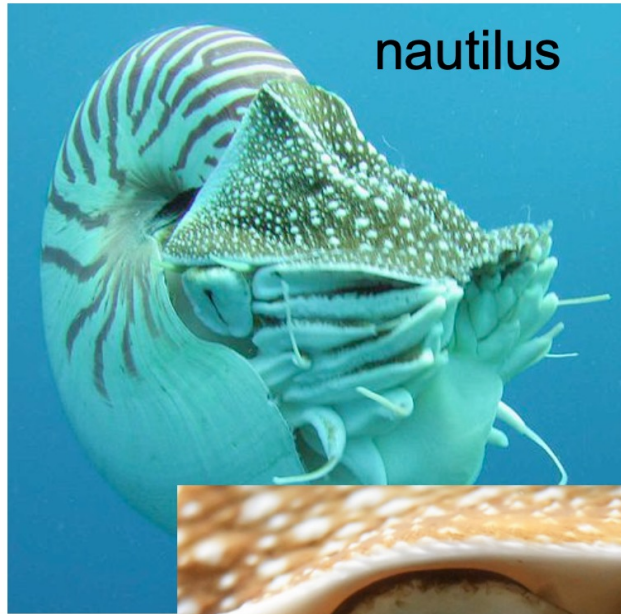
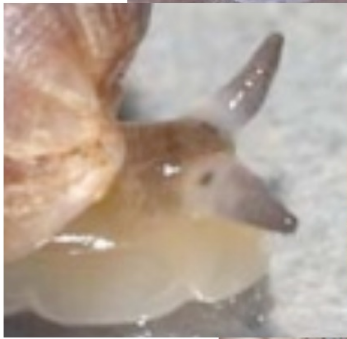


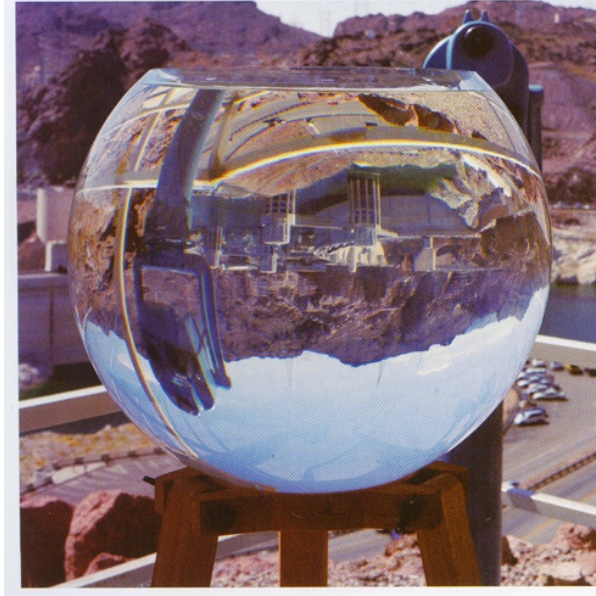
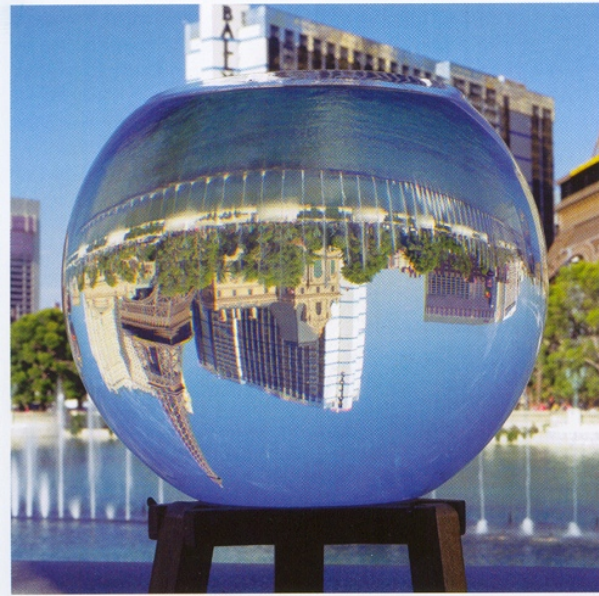
Fig. 1.6 A patch of light sensitive epithelium can be gradually turned into a perfectly focussed camera-type eye if there is a continuous selection for improved spatial vision. A theoretical model based on conservative assumptions about selection pressure and the amount of variation in natural populations suggest that the whole sequence can be accomplished amazingly fast, in less than 400 000 generations. The number of generations is also given between each of the consecutive intermediates that are drawn in the figure. The starting point is a flat piece of epithelium with an outer protective layer, an intermediate layer of receptor cells, and a bottom layer of pigment cells. The first half of the sequence is the formation of a pigment cup eye. When this principle cannot be improved any further, a lens gradually evolves. Modified from Nilsson and Pelger (1994).

Natural Eyes



Lens (very) basics

Replacing pinholes with lenses



Niepce "View from the Window at Le Gras", 1826



1826
8h exp

Daguerrotype



- invented in 1836 by Louis Daguerre
- lenses focus light, better chemicals!

Daguerre "Boulevard du Temple", 1838



exposure
10-12 mins

Lenses



Lenses

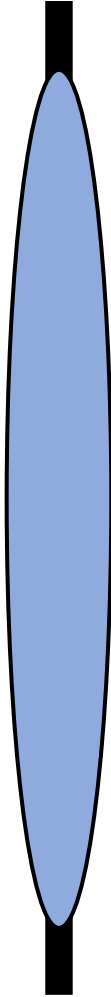


Lenses



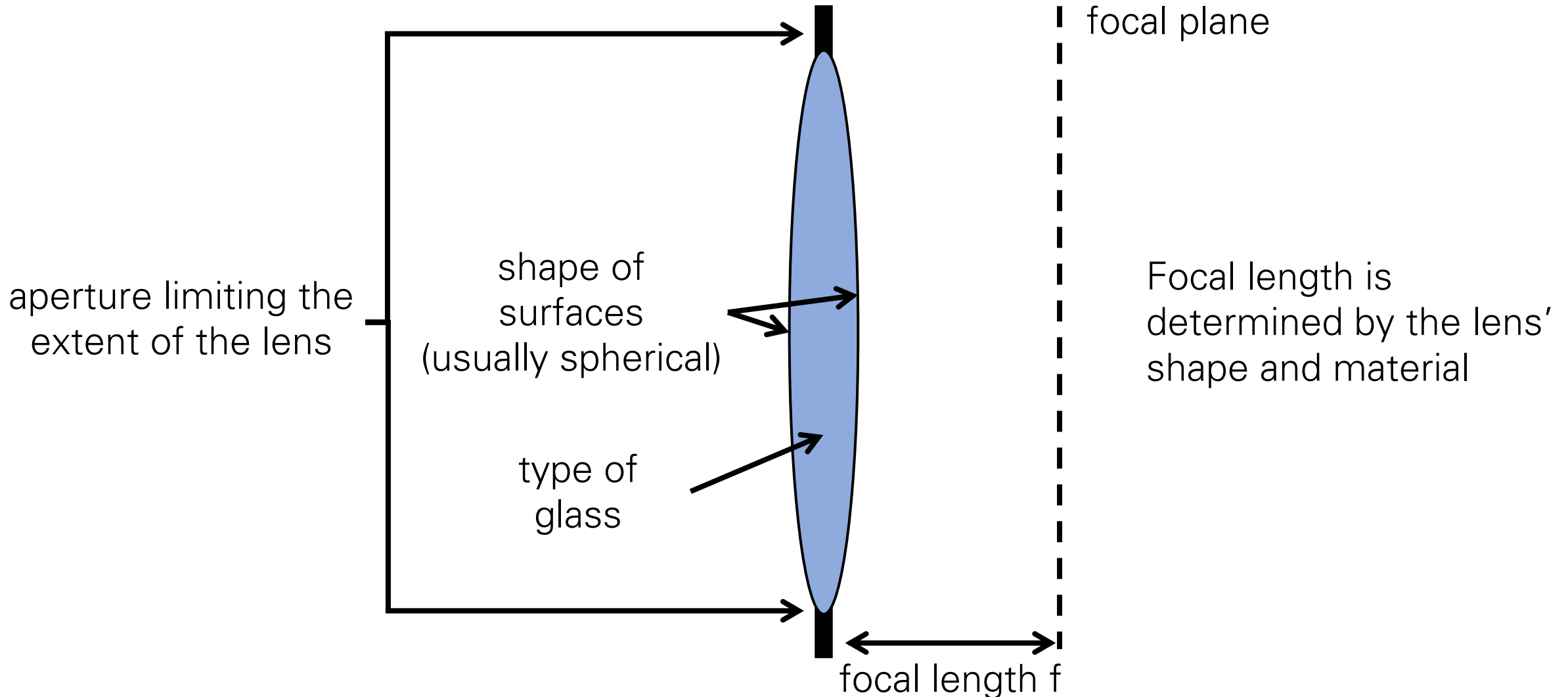
What is a lens?

A piece of glass manufactured to have a specific shape



What is a lens?

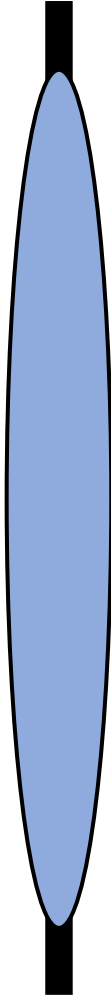
A piece of glass manufactured to have a specific shape



The lens on your camera

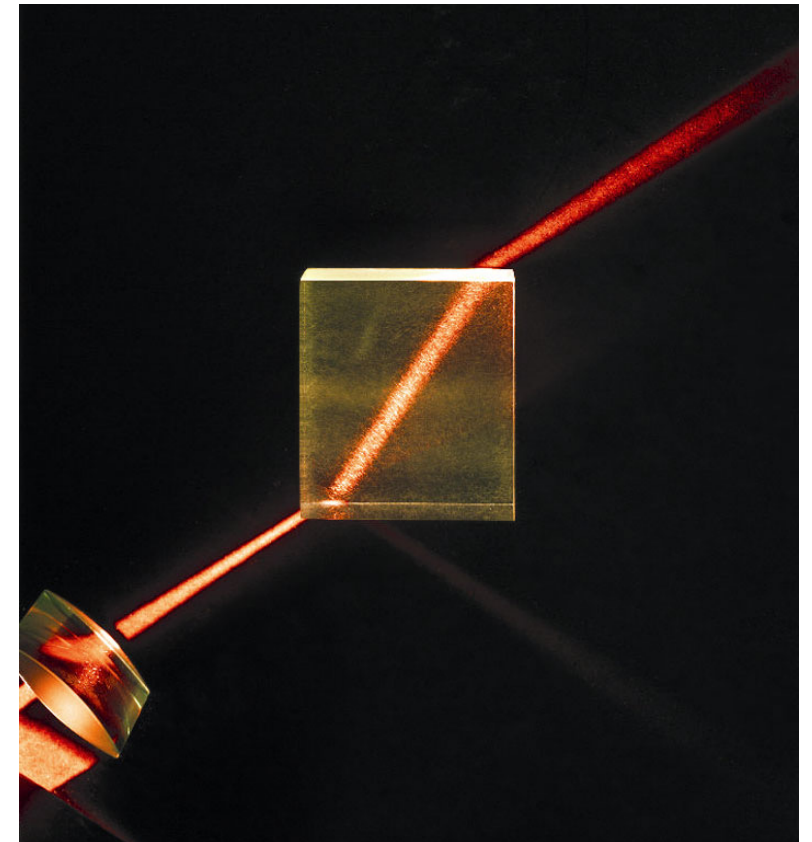
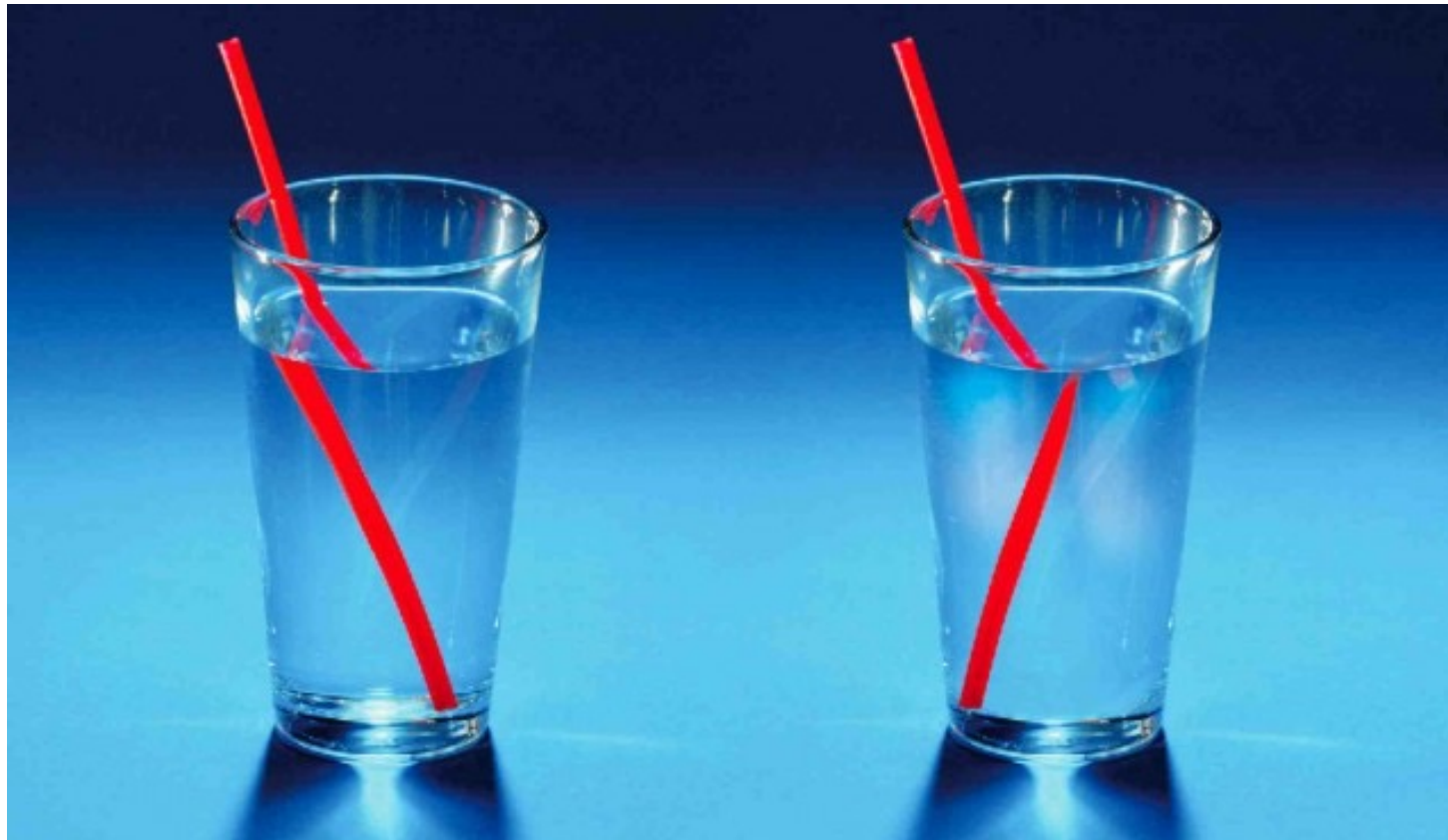


How does a lens work?



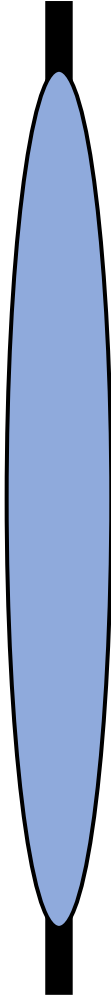
Refraction

Refraction is the bending of rays of light when they move from one material to another



How does a lens work?

Lenses are designed so that their refraction makes light rays bend in a very specific way.



The thin lens model

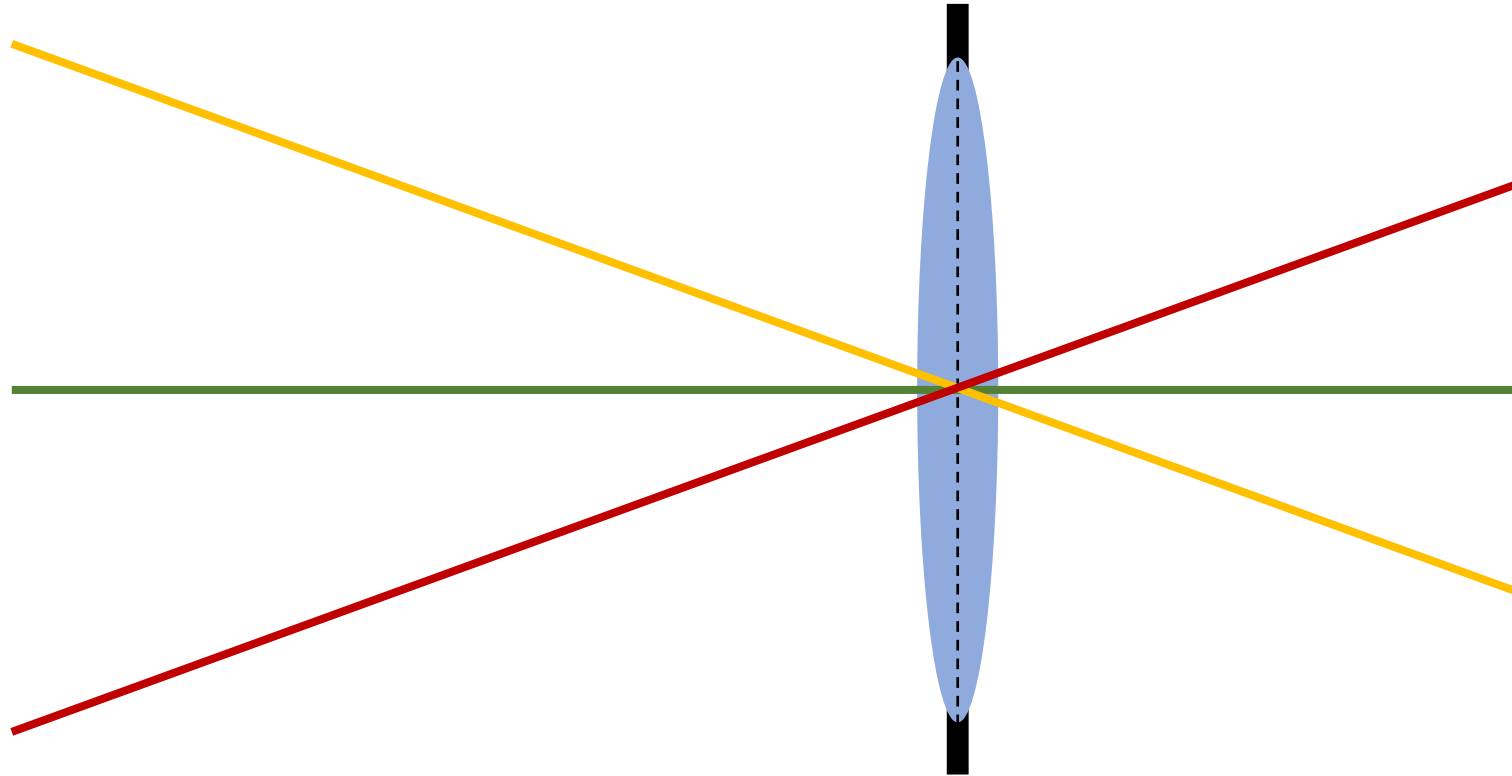
Thin lens model

Simplification of geometric optics for well-designed lenses.



Thin lens model

Simplification of geometric optics for well-designed lenses.

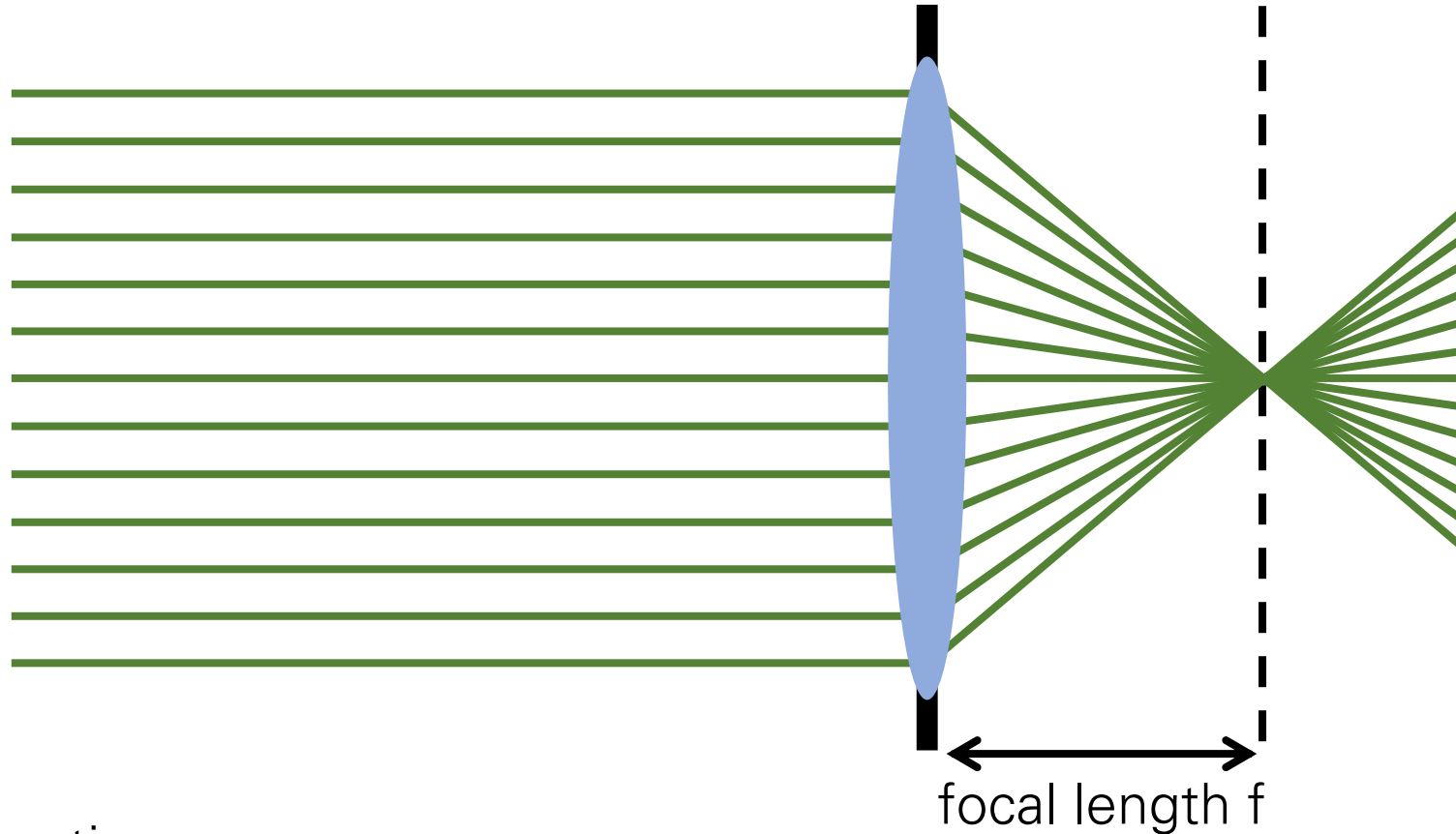


Two assumptions:

1. Rays passing through lens center are unaffected.

Thin lens model

Simplification of geometric optics for well-designed lenses.

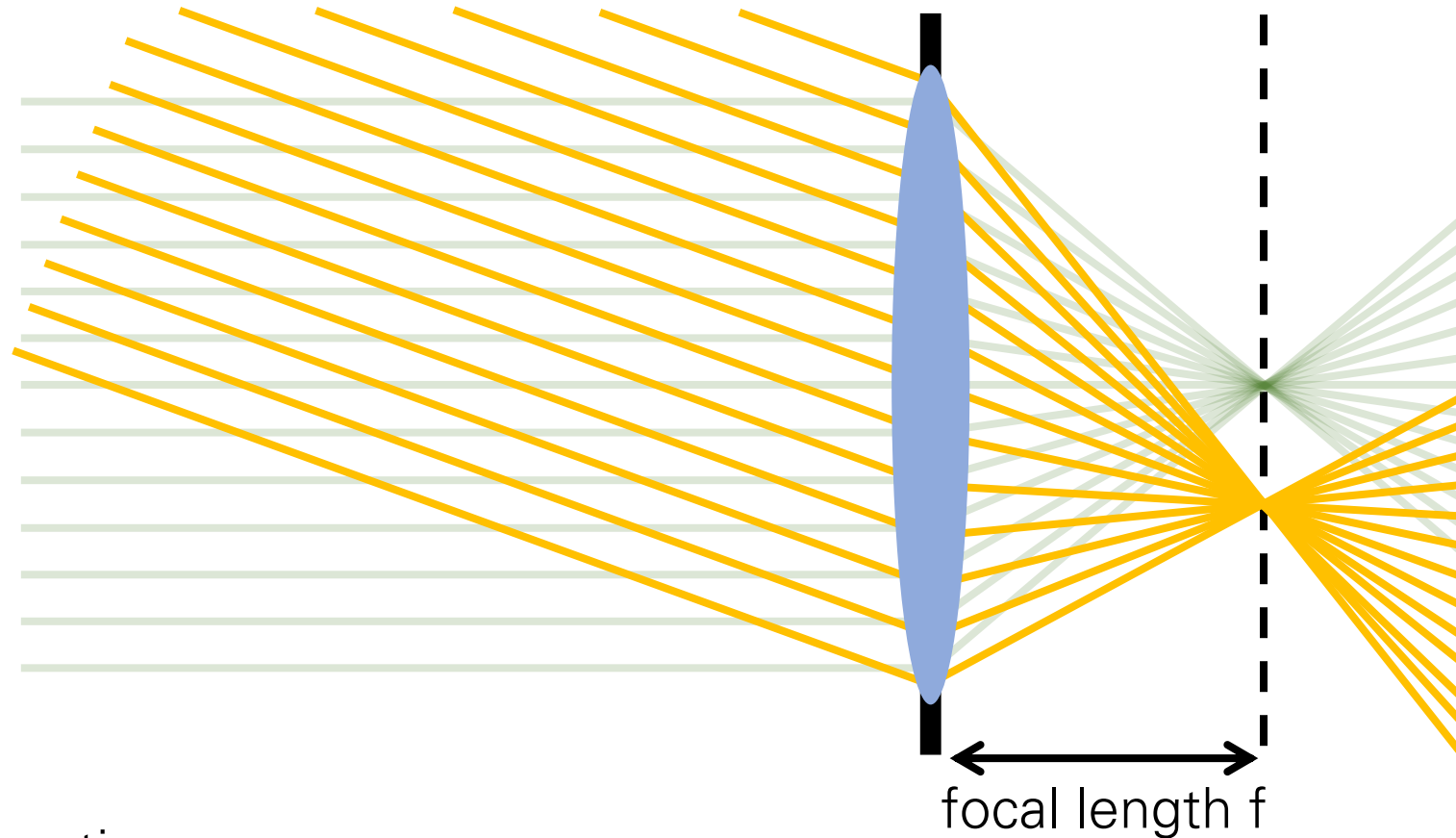


Two assumptions:

1. Rays passing through lens center are unaffected.
2. Parallel rays converge to a single point located on focal plane.

Thin lens model

Simplification of geometric optics for well-designed lenses.

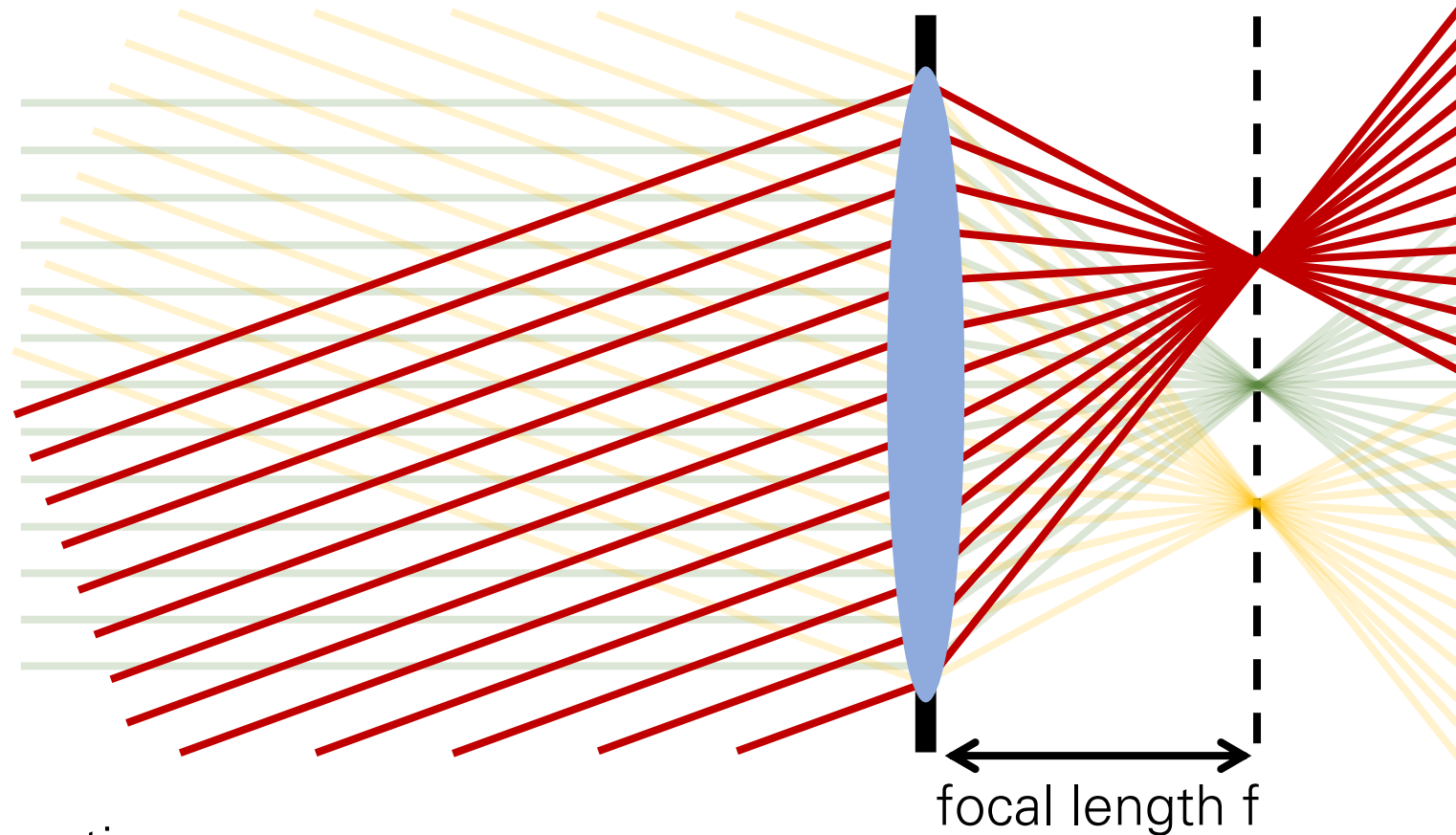


Two assumptions:

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Thin lens model

Simplification of geometric optics for well-designed lenses.

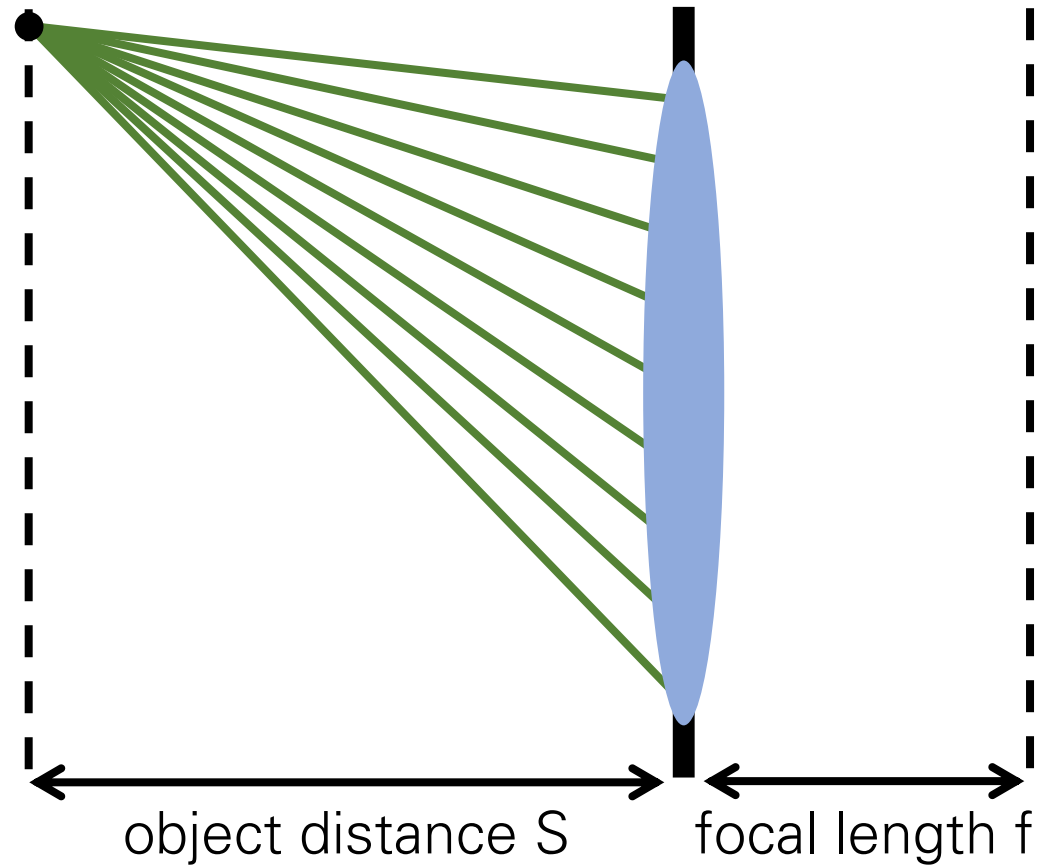


Two assumptions:

1. Rays passing through lens center are unaffected.
2. Parallel rays converge to a single point located on focal plane.

Tracing rays through a thin lens

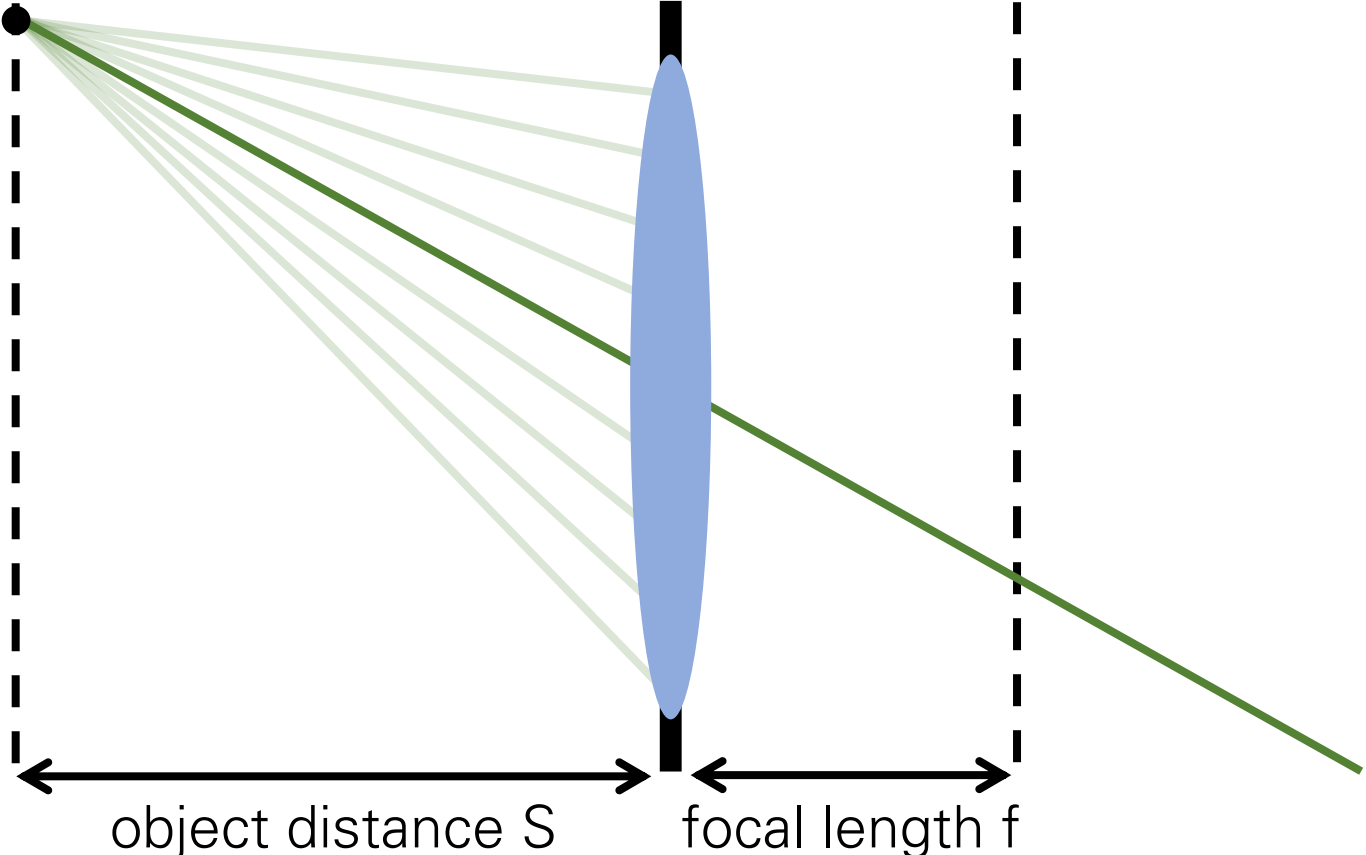
Consider an object emitting a bundle of rays. How do they propagate through the lens?



Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

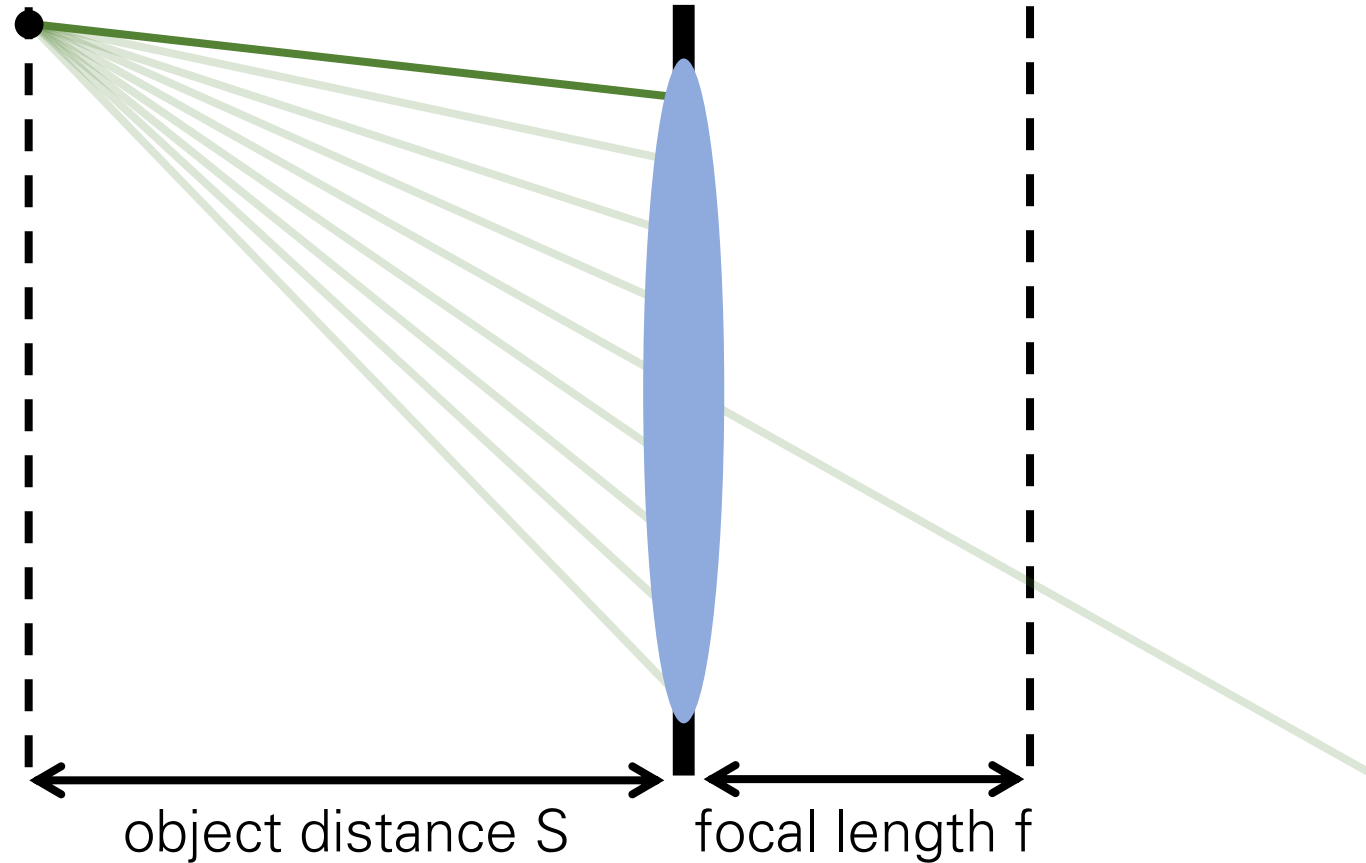
- 1. Trace rays through lens center.



Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

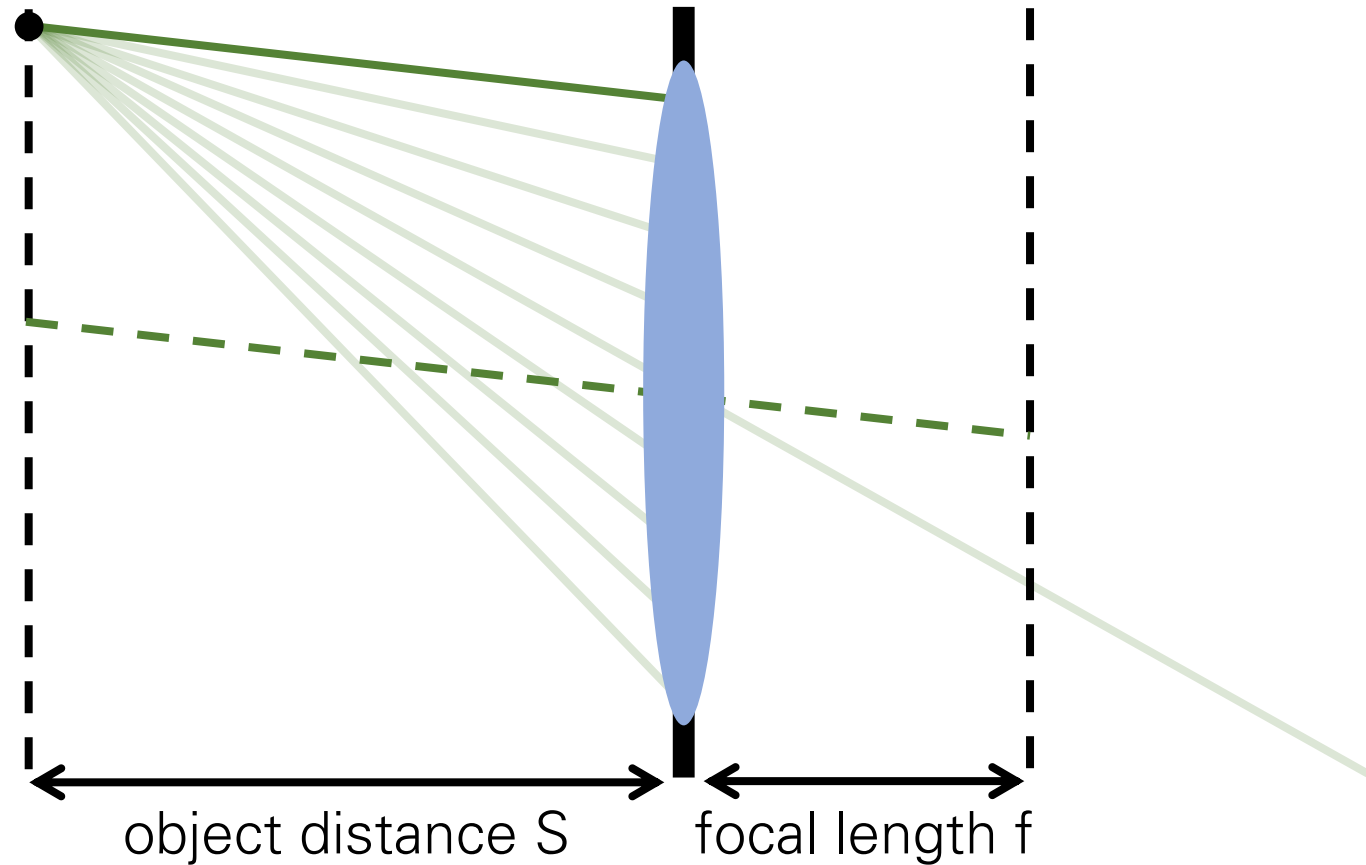
1. Trace rays through lens center.
2. For all other rays:



Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

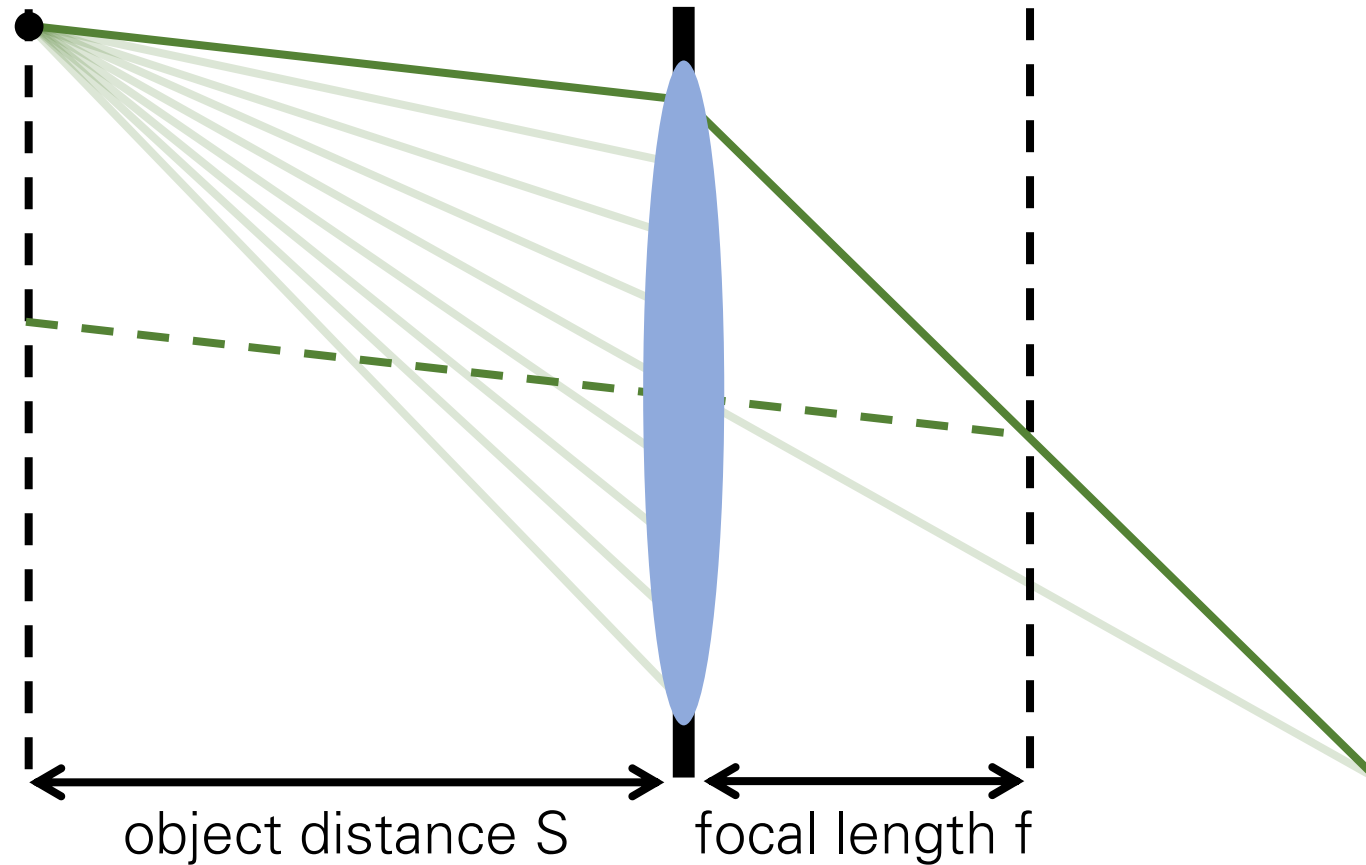
1. Trace rays through lens center.
2. For all other rays:
 - a. Trace their parallel through lens center.



Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

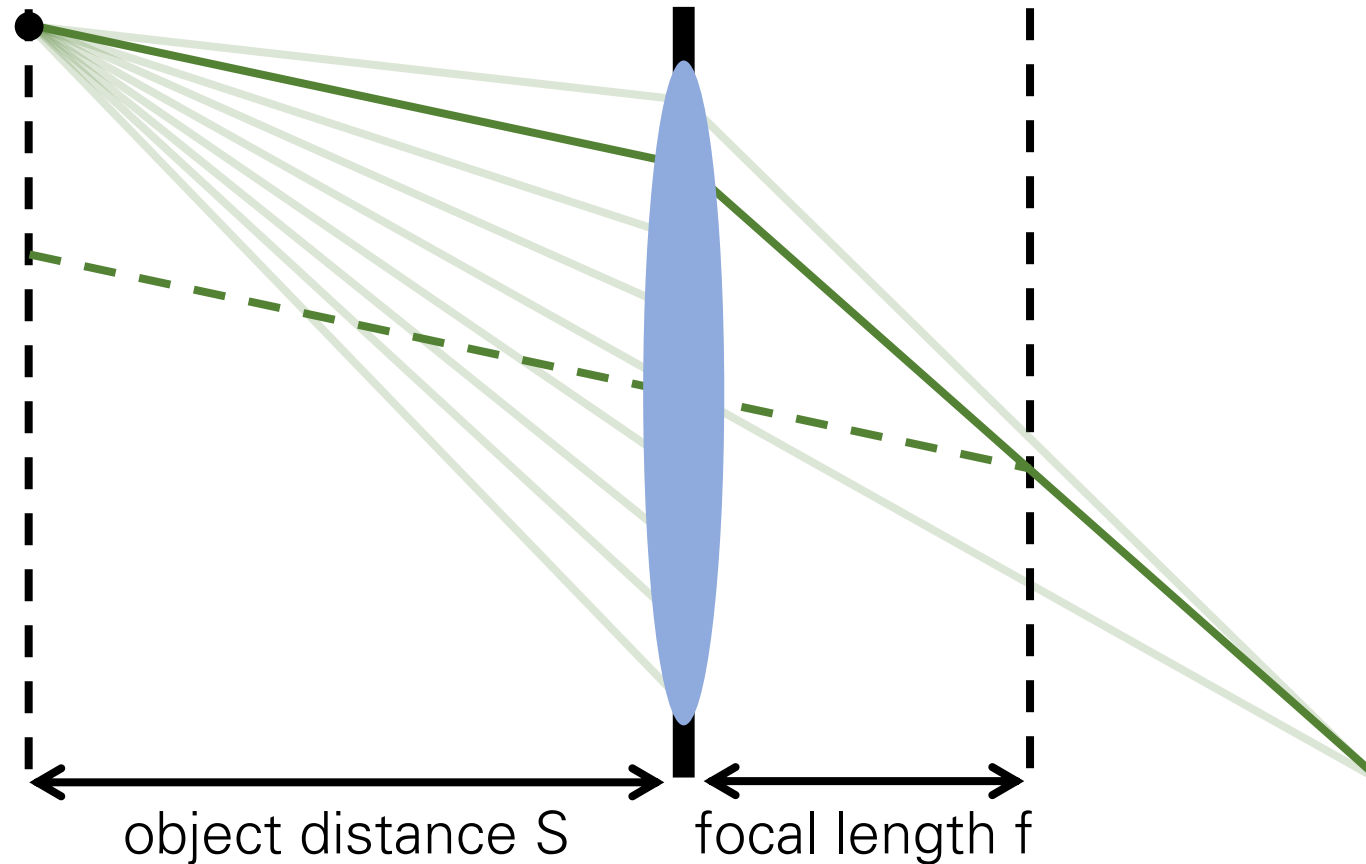
1. Trace rays through lens center.
2. For all other rays:
 - a. Trace their parallel through lens center.
 - b. Connect on focal plane.



Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

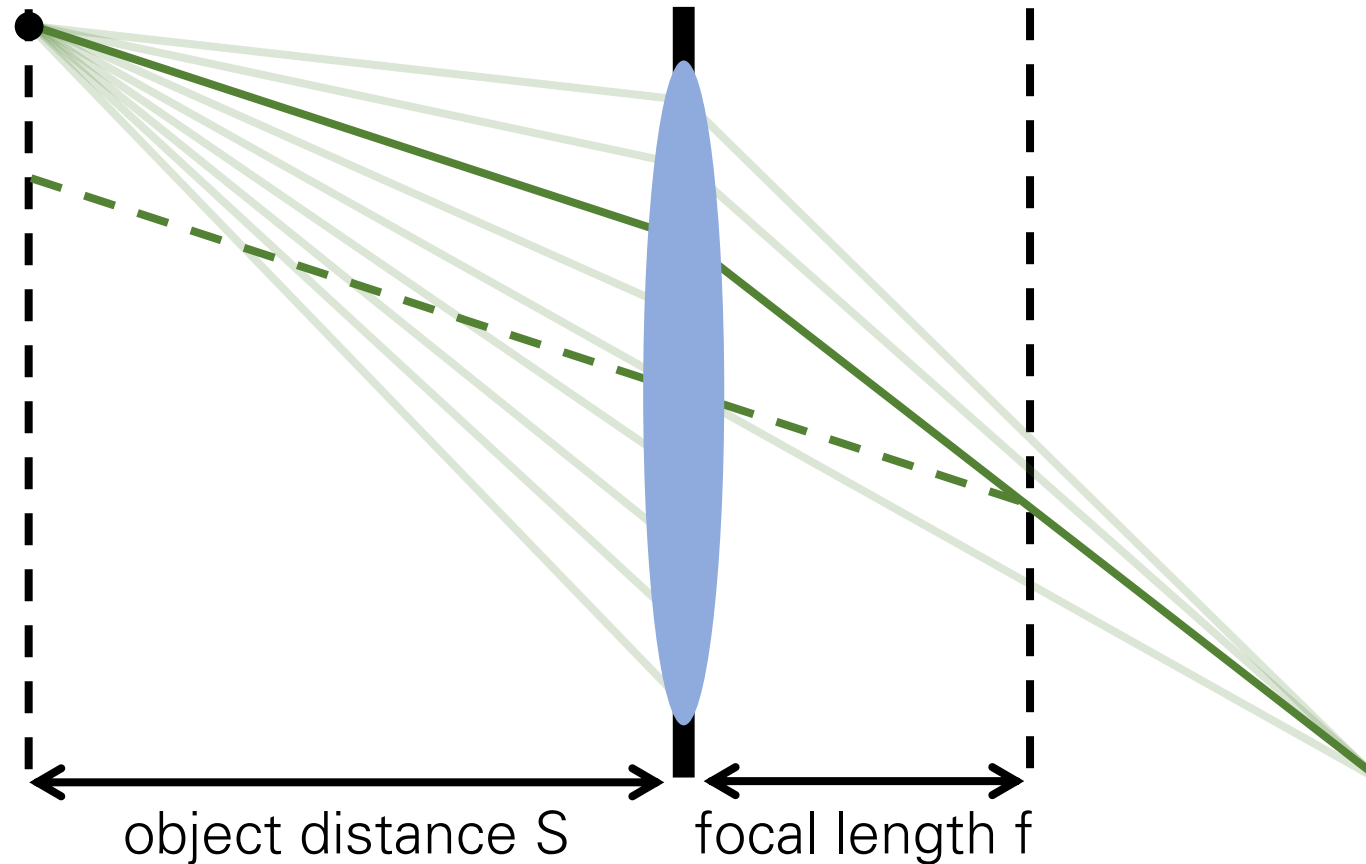
1. Trace rays through lens center.
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 - b. Connect on focal plane.



Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

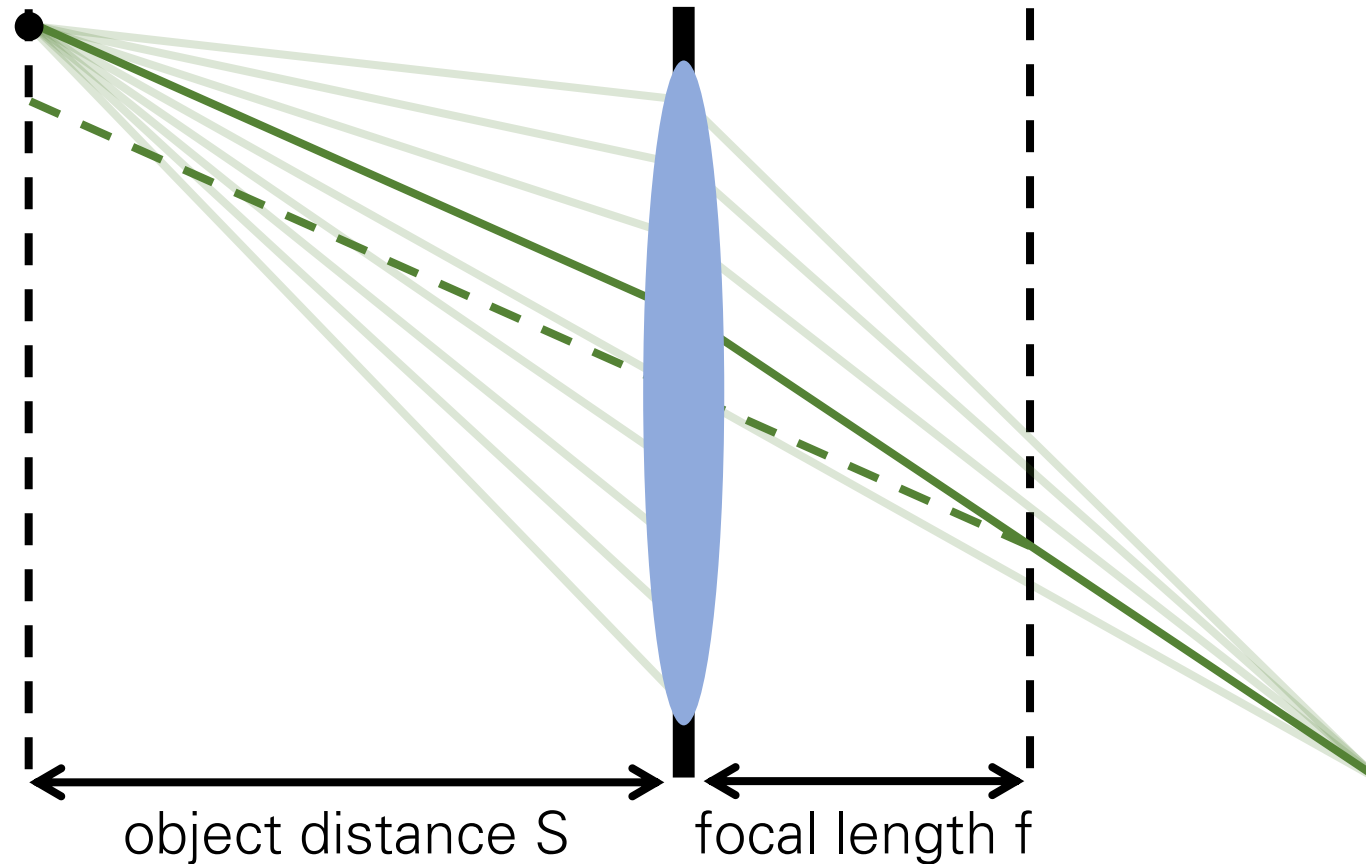
1. Trace rays through lens center.
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Tracing rays through a thin lens

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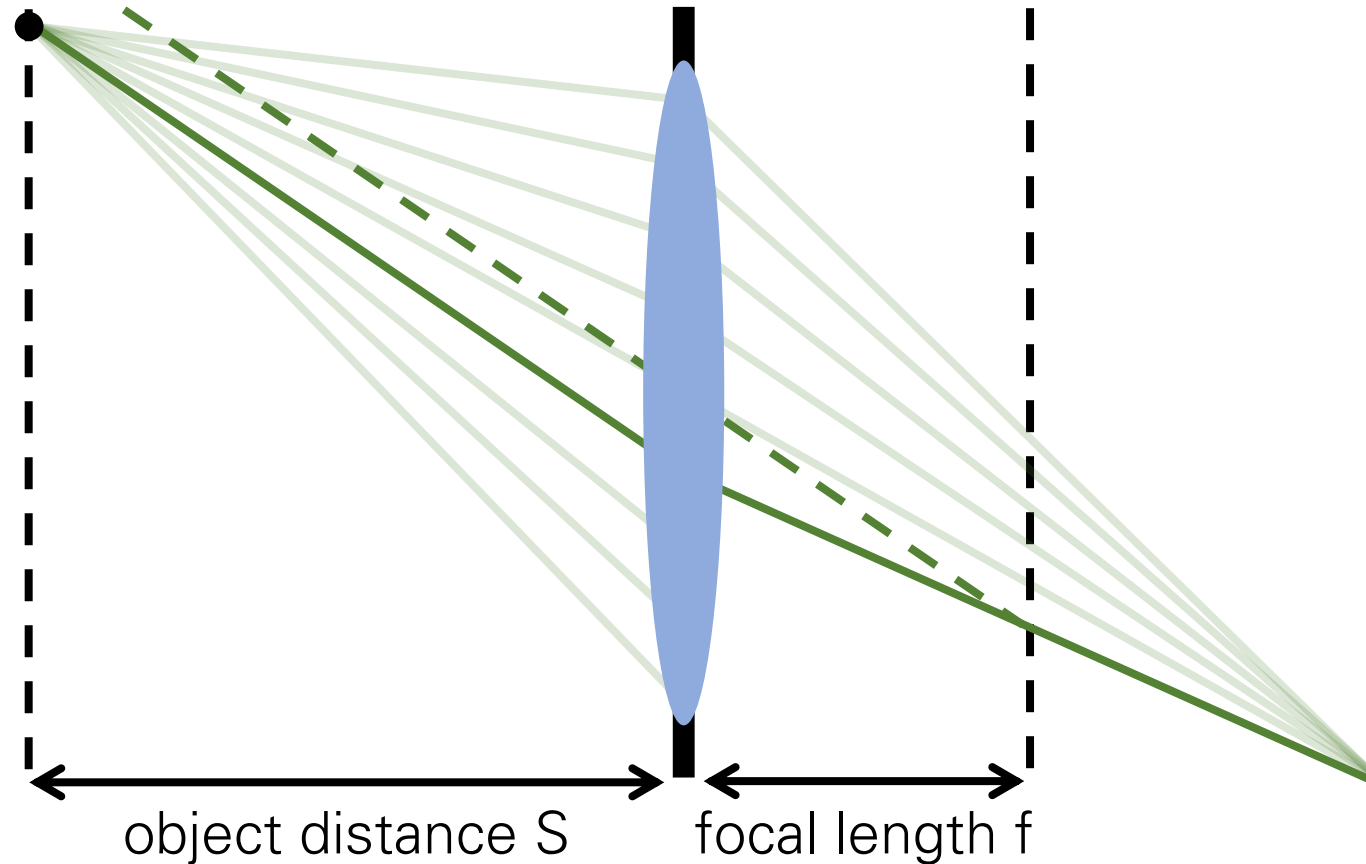
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 - a. Trace their parallel through lens center.
 - b. Connect on focal plane.



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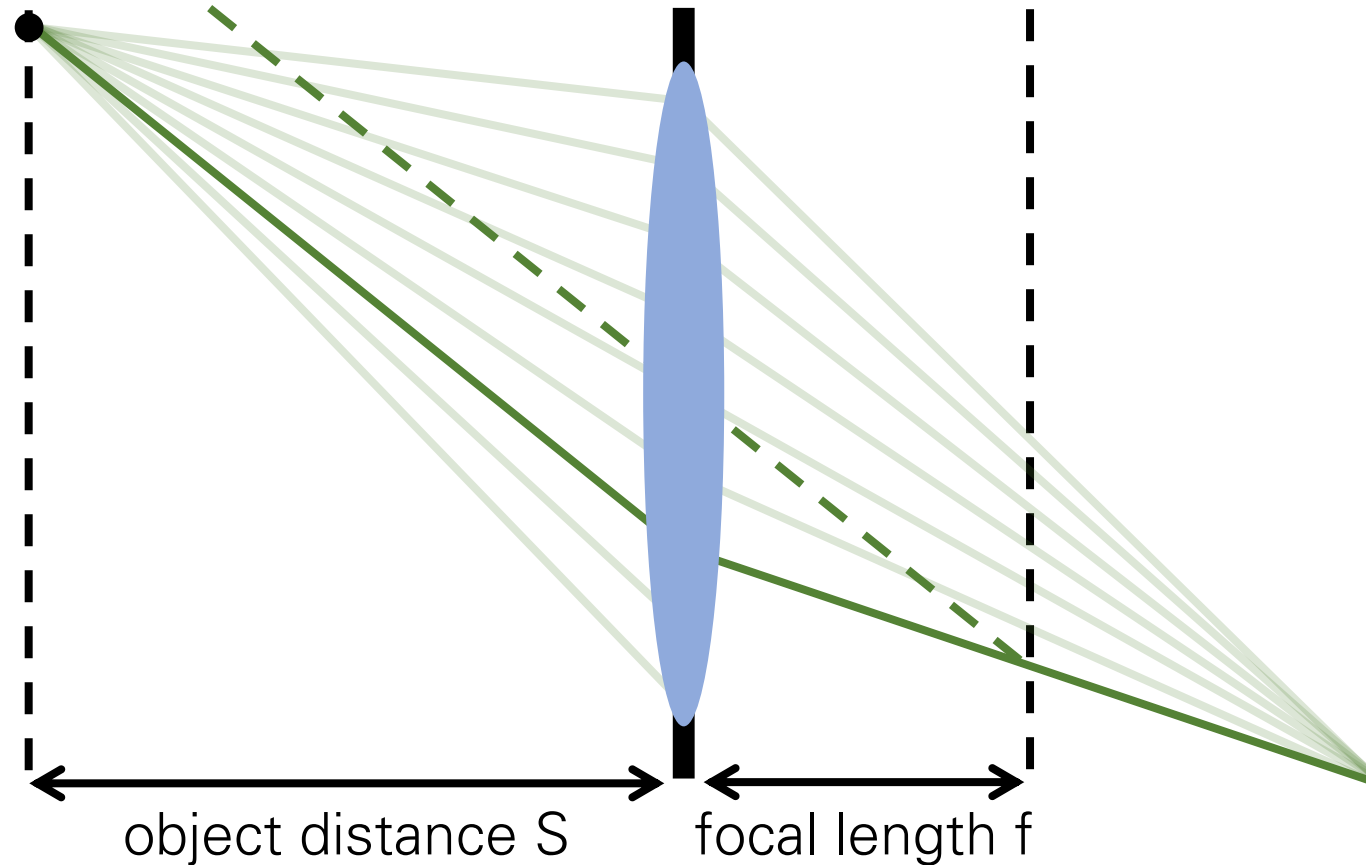
1. Trace rays through lens center.
2. For all other rays:
 - a. Trace their parallel through lens center.
 - b. Connect on focal plane.



Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

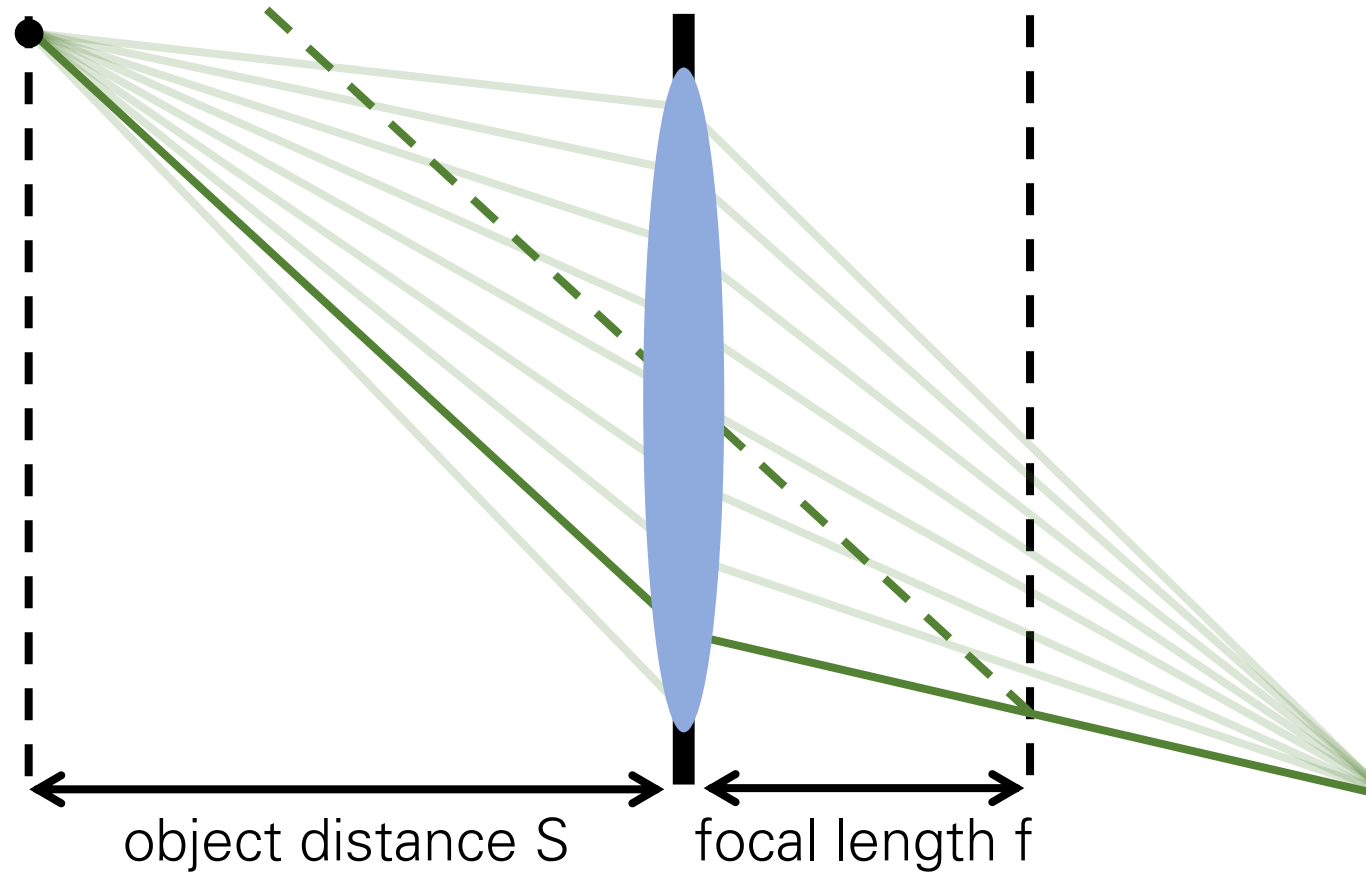
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 - a. Trace their parallel through lens center.
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Tracing rays through a thin lens

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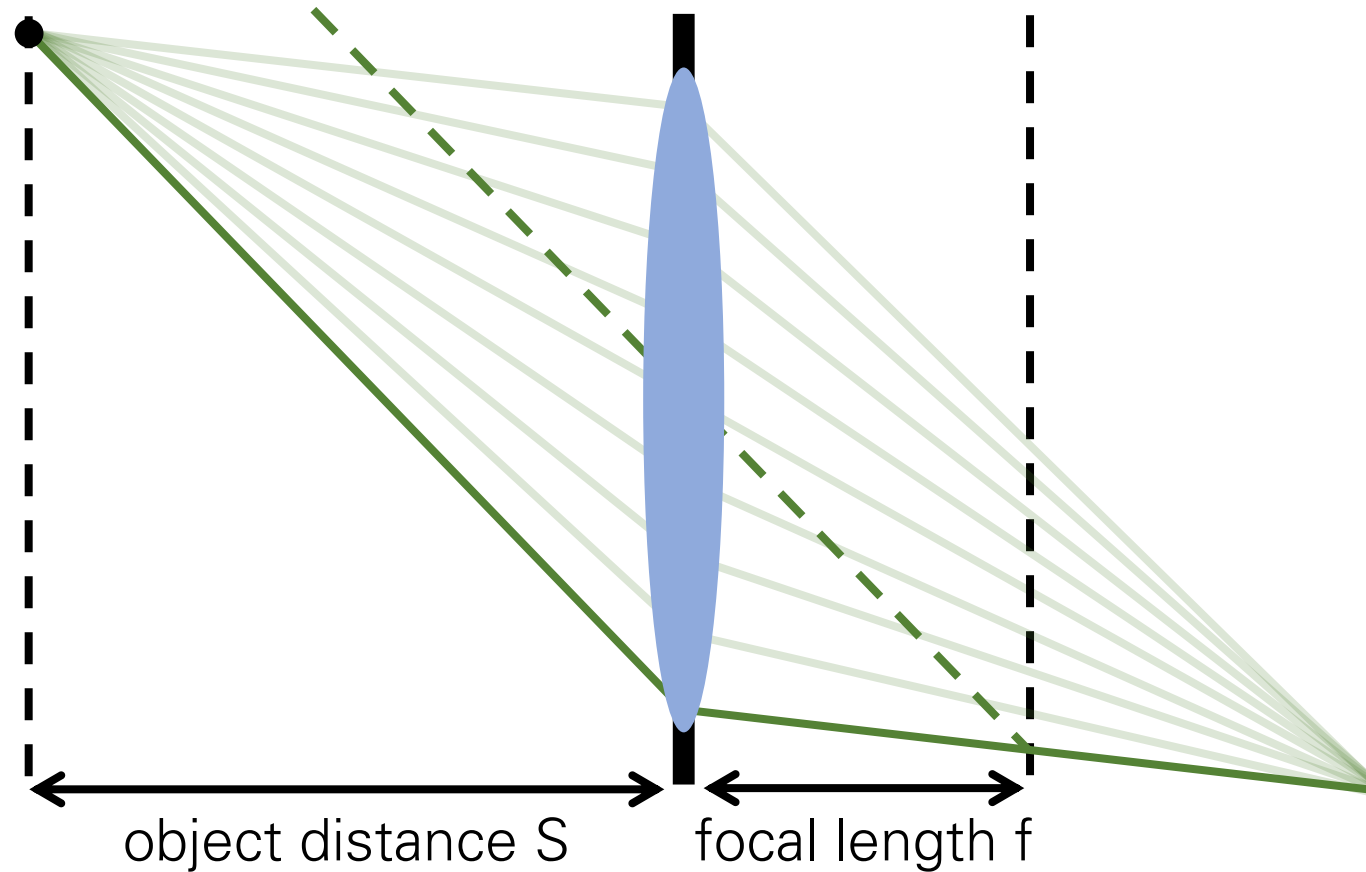
1. Trace rays through lens center.
2. For all other rays:
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 - b. Connect on focal plane.



Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

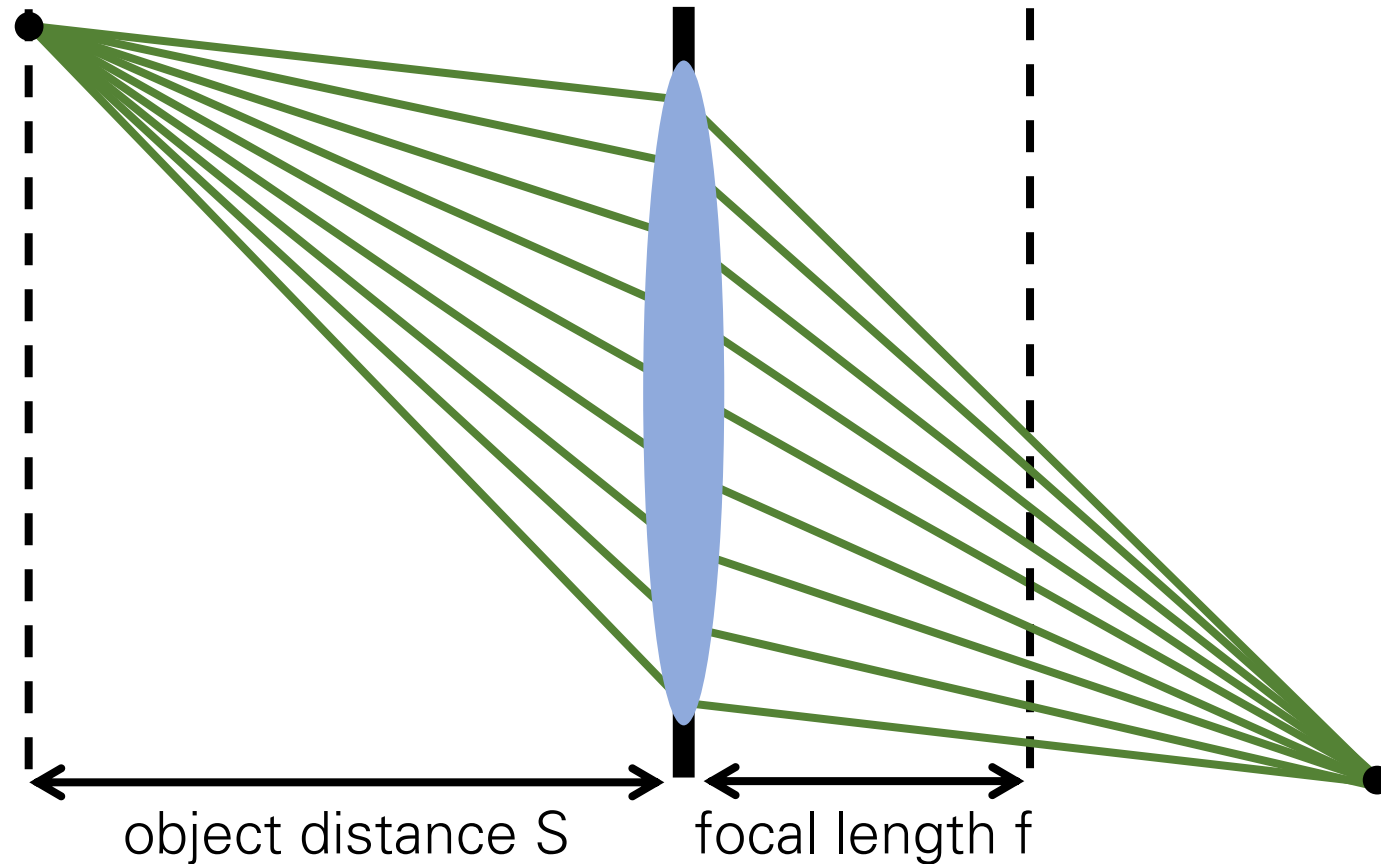
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1. Trace rays through lens center.
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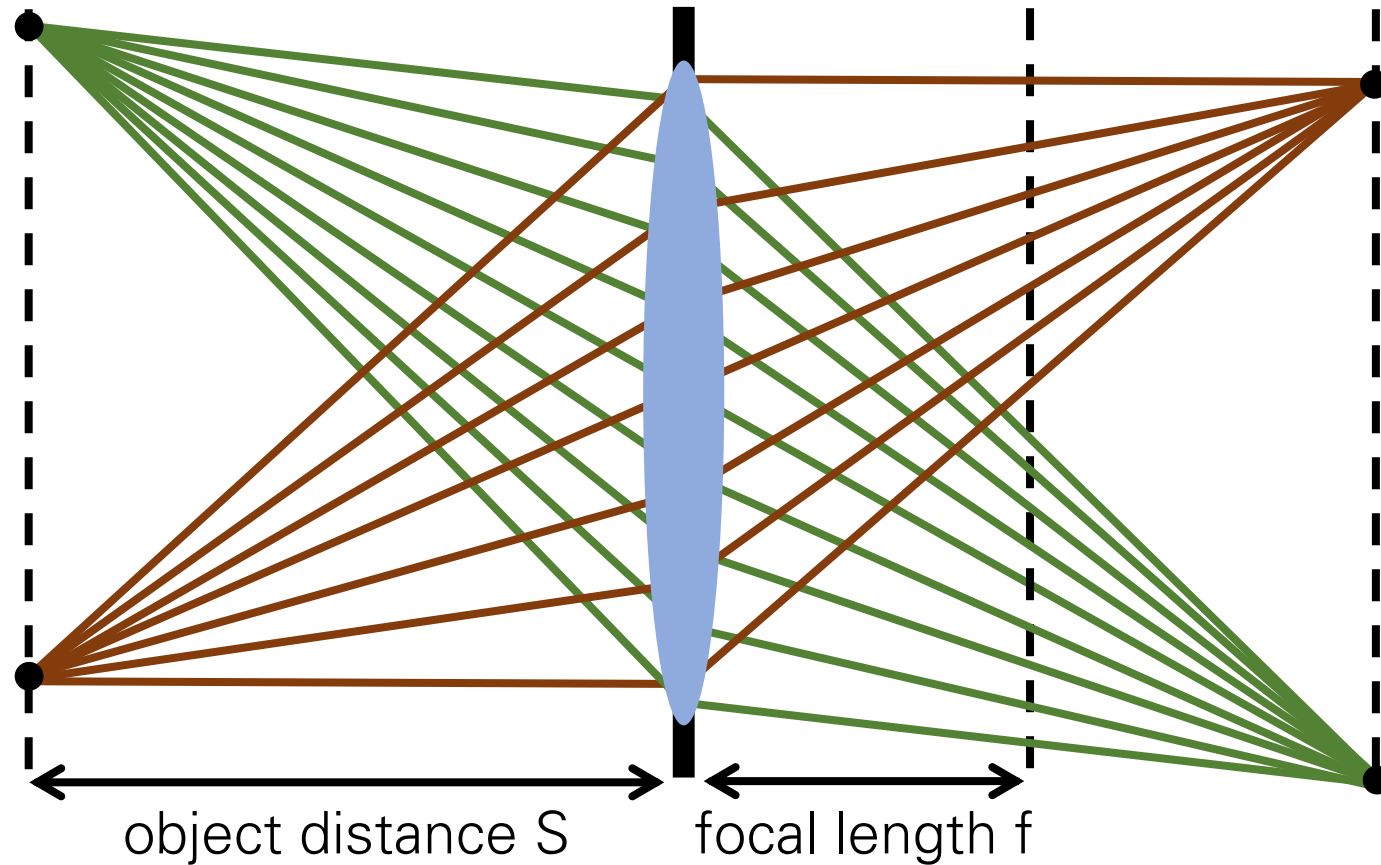
Focusing property:

1. Rays emitted from a point on one side converge to a point on the other side.

Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
 - a. Trace their parallel through lens center.
 - b. Connect on focal plane.

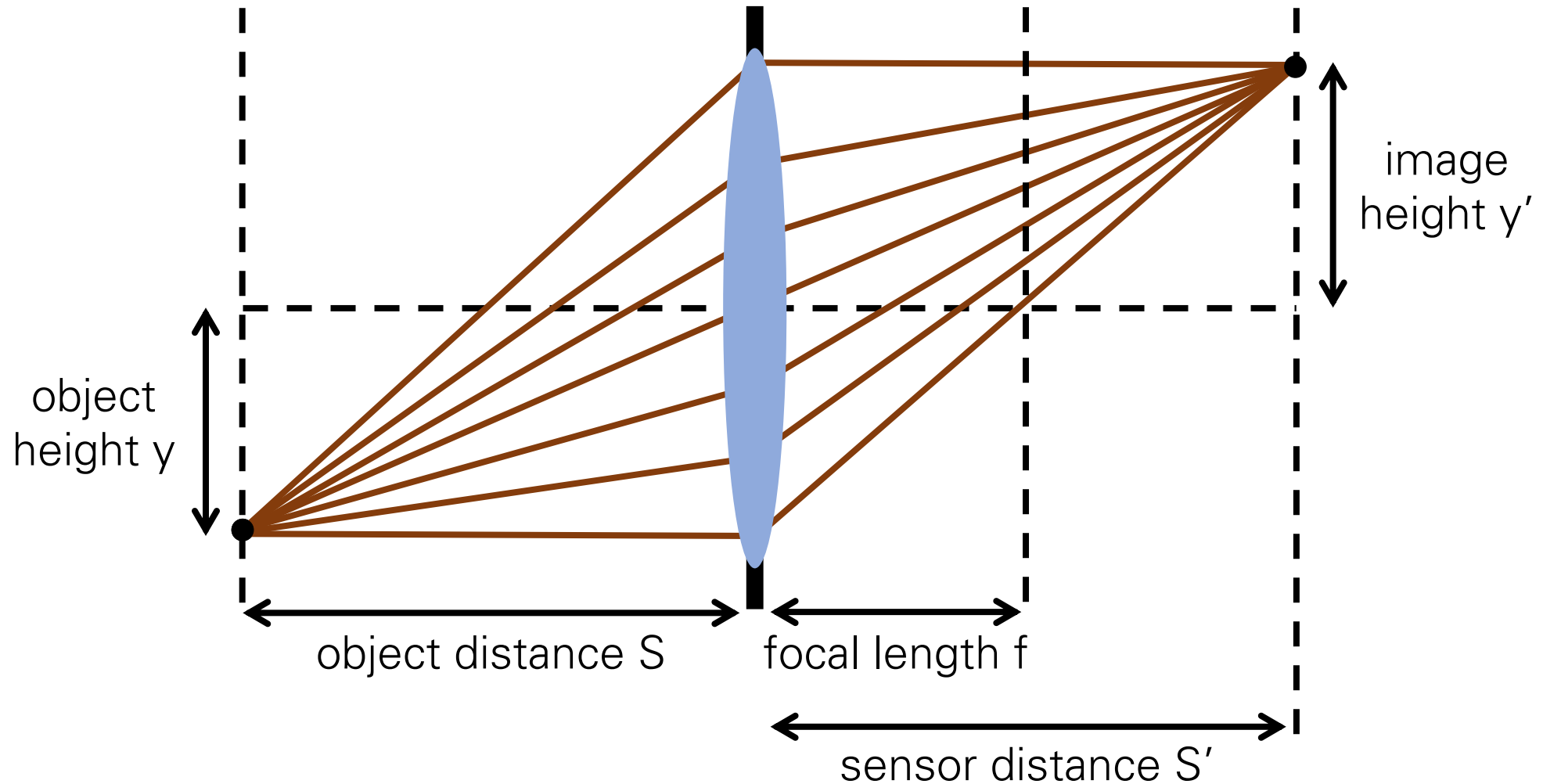


Focusing property:

1. Rays emitted from a point on one side converge to a point on the other side.
2. Bundles emitted from a plane parallel to the lens converge on a common plane.

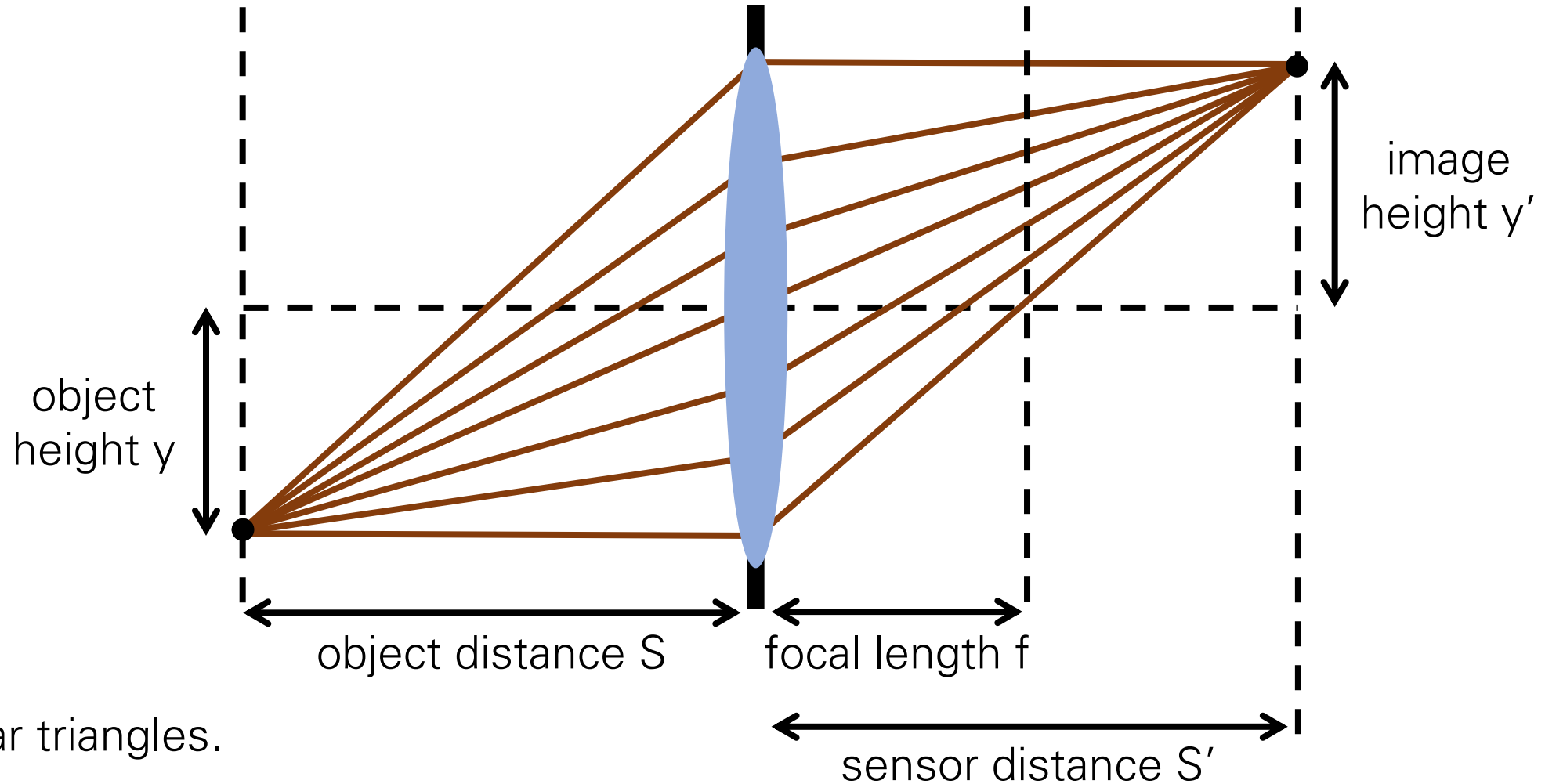
Gaussian lens formula

How can we relate scene-space (S, y) and image space (S', y') quantities?



Gaussian lens formula

How can we relate scene-space (S, y) and image space (S', y') quantities?

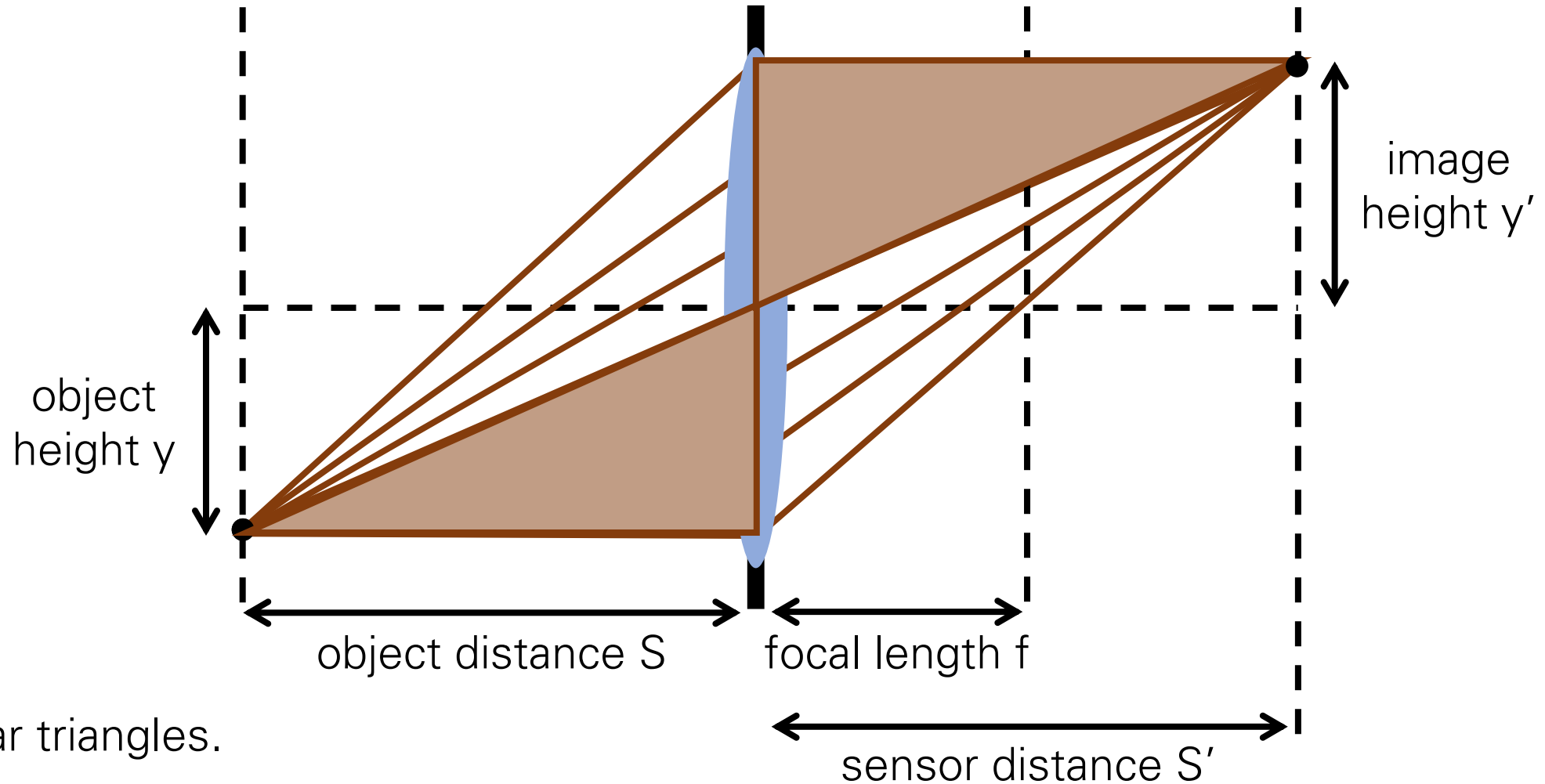


Use similar triangles.

Gaussian lens formula

How can we relate scene-space (S, y) and image space (S', y') quantities?

$$\frac{y'}{y} = ?$$

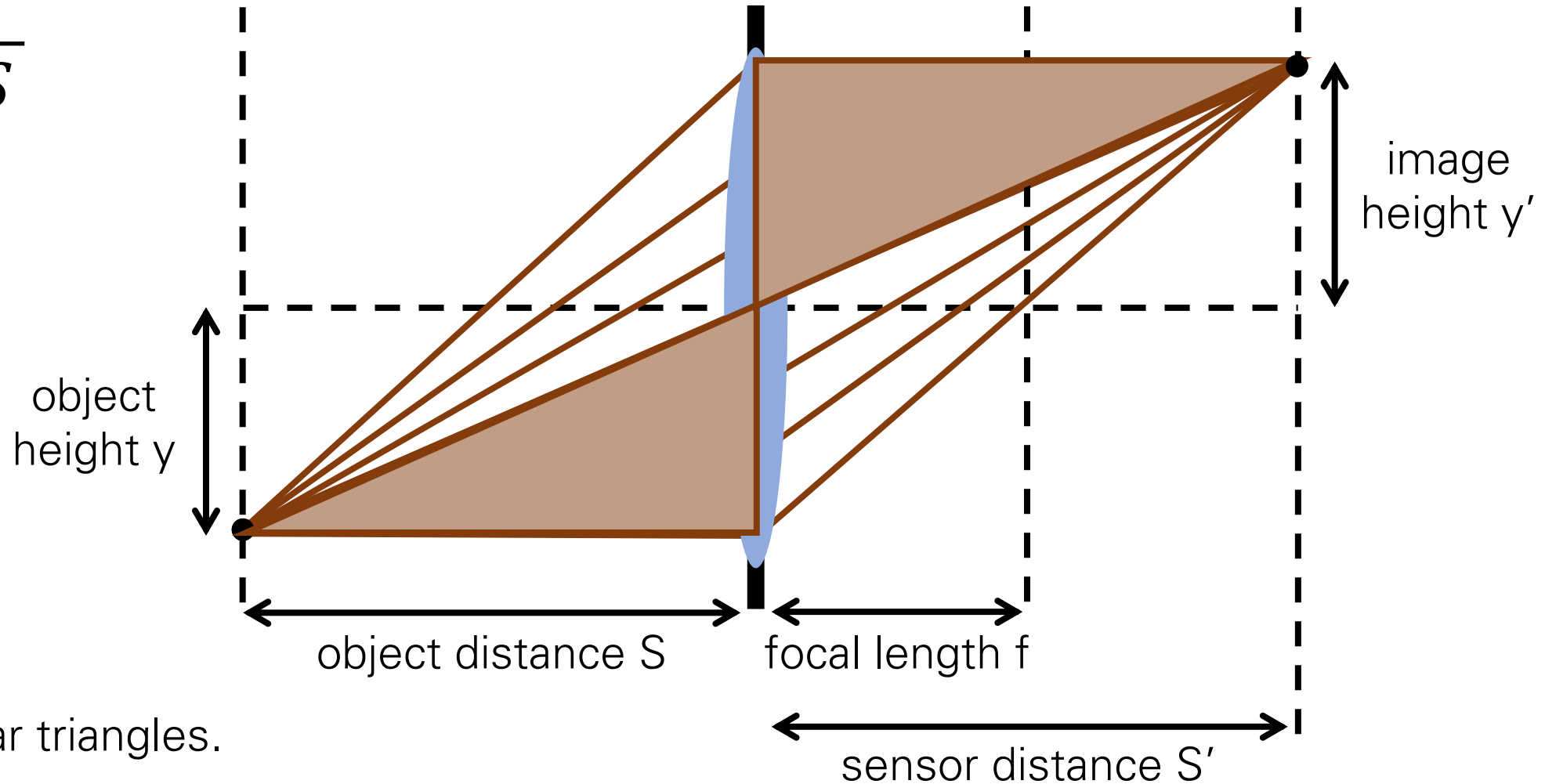


Use similar triangles.

Gaussian lens formula

How can we relate scene-space (S, y) and image space (S', y') quantities?

$$\frac{y'}{y} = \frac{S'}{S}$$



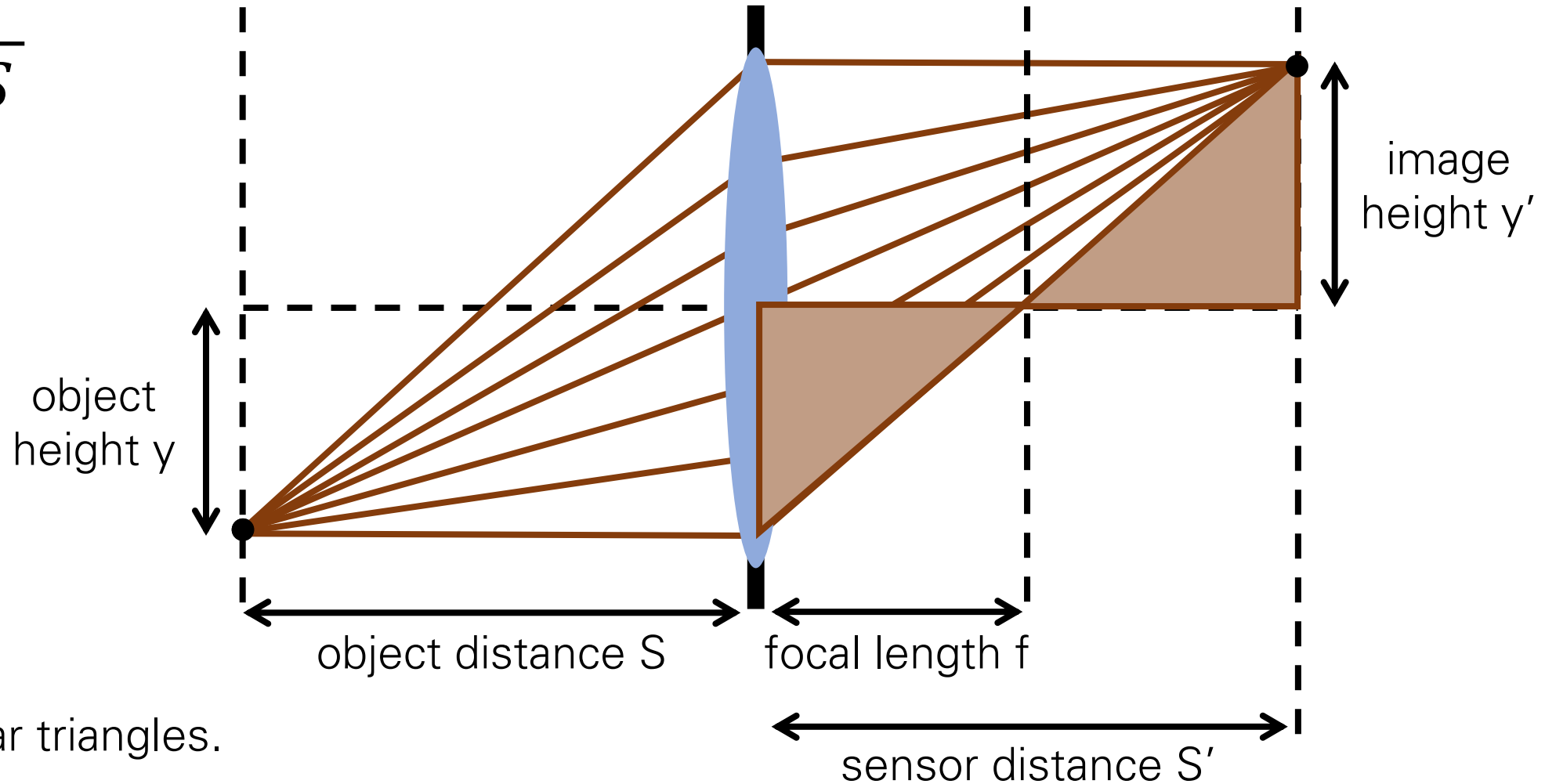
Use similar triangles.

Gaussian lens formula

How can we relate scene-space (S, y) and image space (S', y') quantities?

$$\frac{y'}{y} = \frac{S'}{S}$$

$$\frac{y'}{y} = ?$$



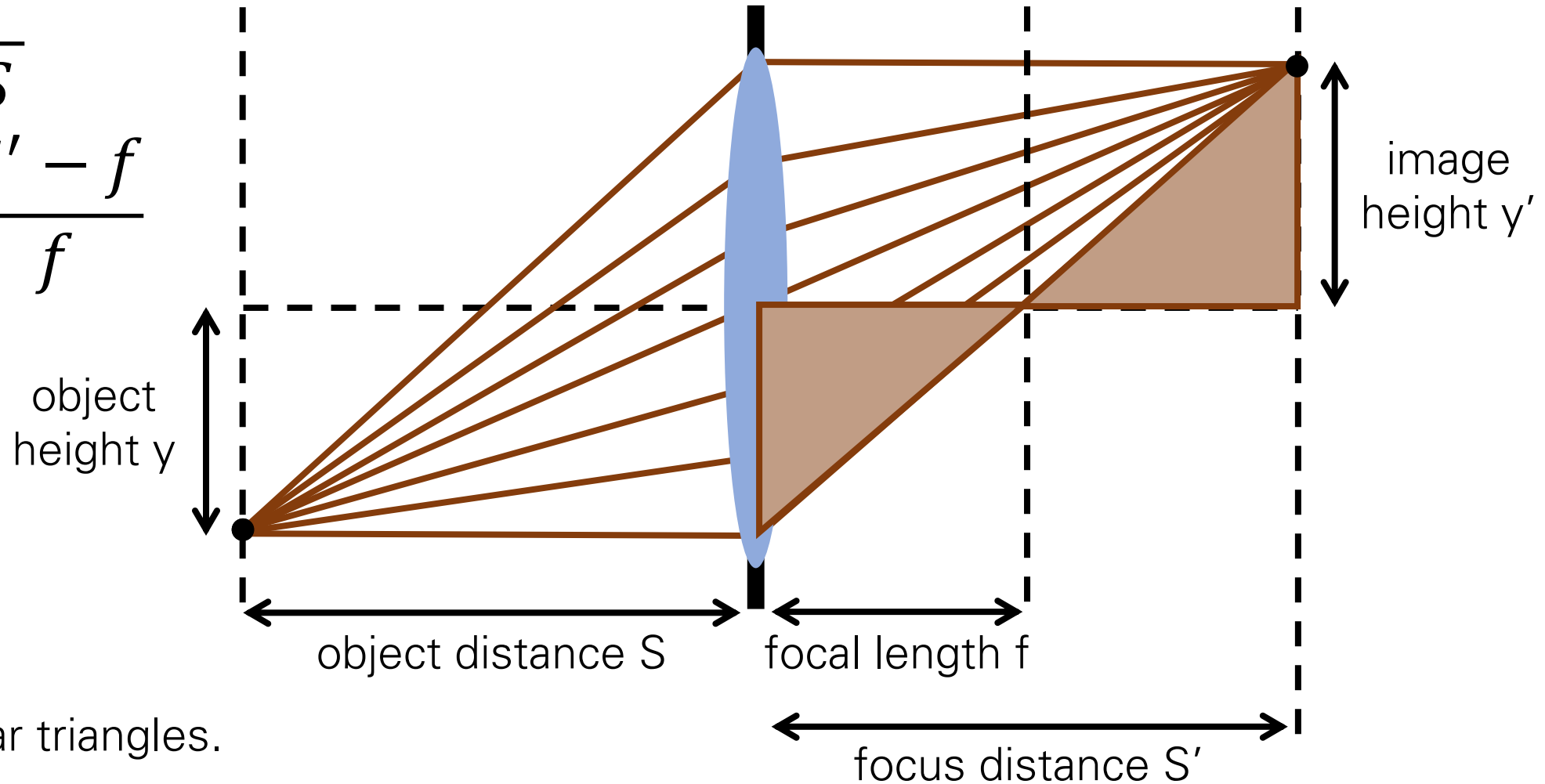
Use similar triangles.

Gaussian lens formula

How can we relate scene-space (S, y) and image space (S', y') quantities?

$$\frac{y'}{y} = \frac{S'}{S}$$

$$\frac{y'}{y} = \frac{S' - f}{f}$$

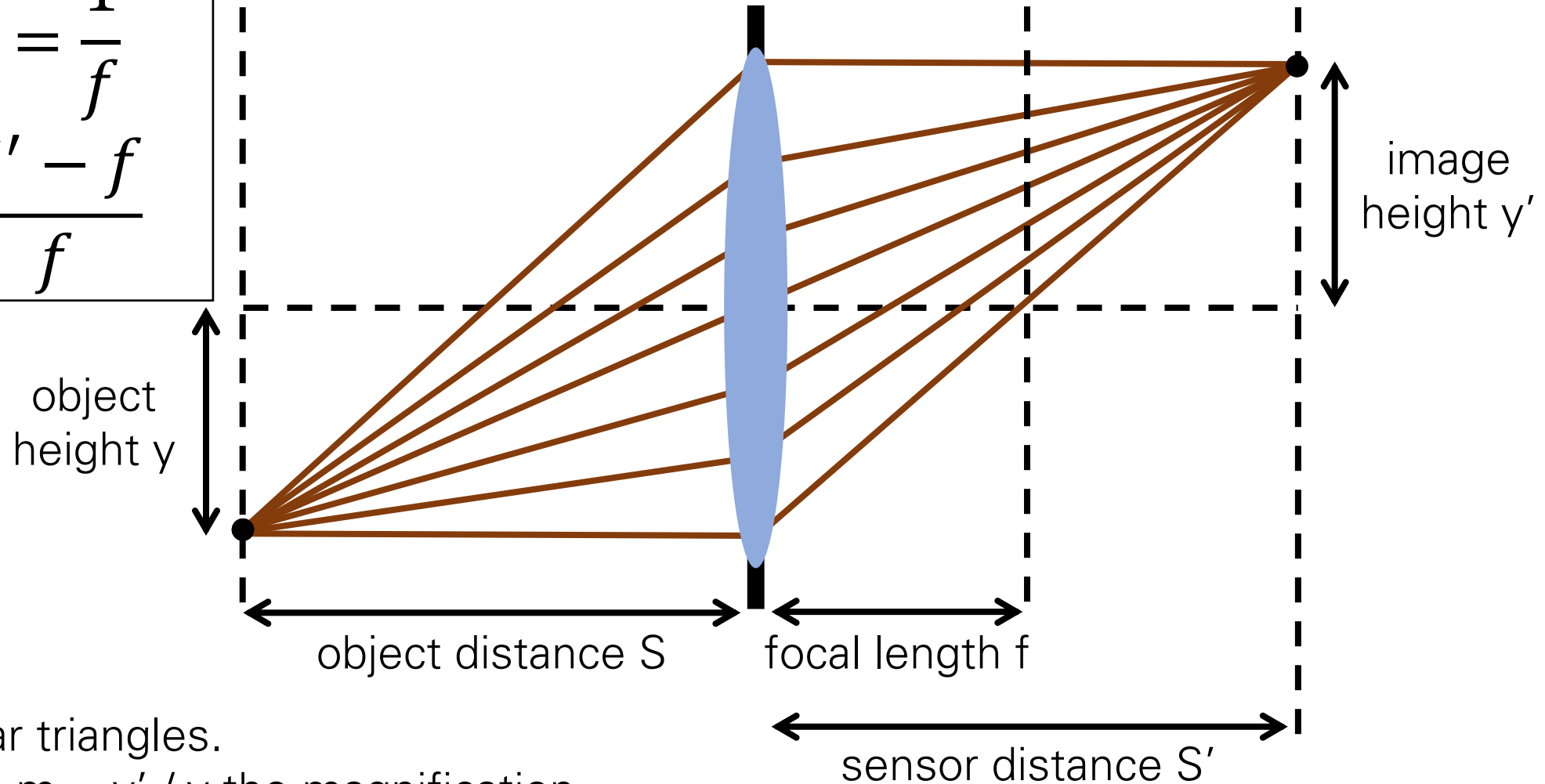


Use similar triangles.

Gaussian lens formula

How can we relate scene-space (S, y) and image space (S', y') quantities?

$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$
$$m = \frac{S' - f}{f}$$



Use similar triangles.

- We call $m = y' / y$ the magnification.

Special focus distances

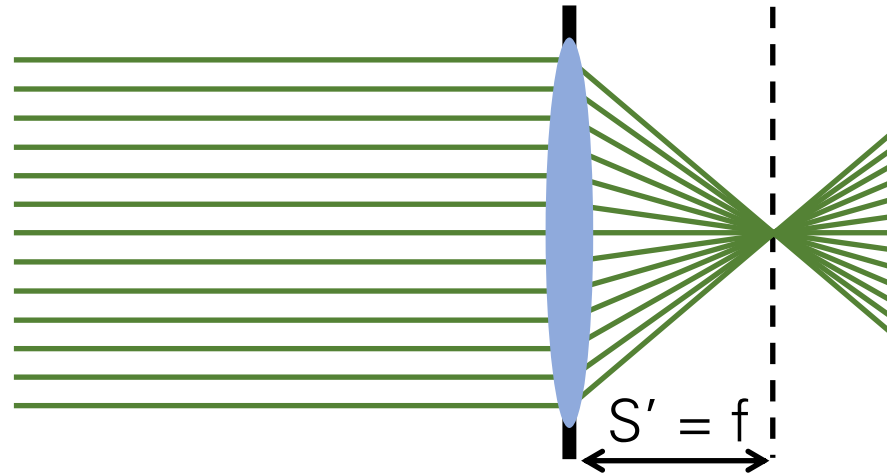
$$S' = f, S = ?, m = ?$$

$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$
$$m = \frac{S' - f}{f}$$

Special focus distances

$S' = f, S = \infty, m = 0 \rightarrow$ infinity focus (parallel rays)

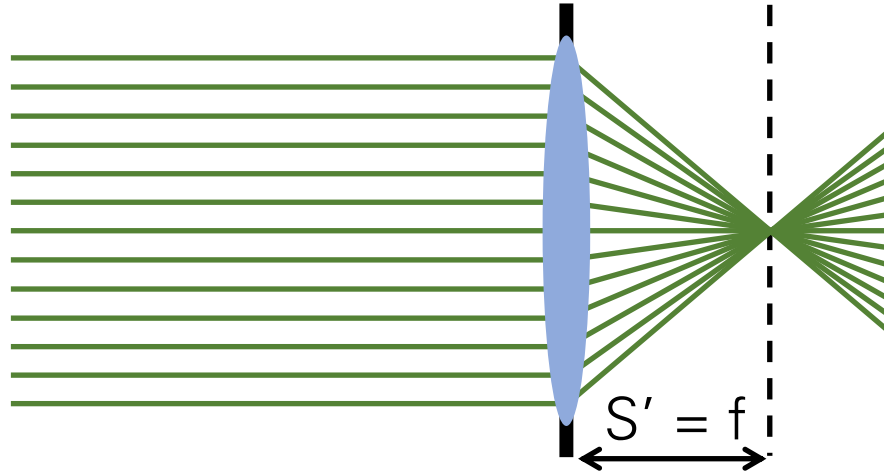
$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$
$$m = \frac{S' - f}{f}$$



$S' = S = ?, m = ?$

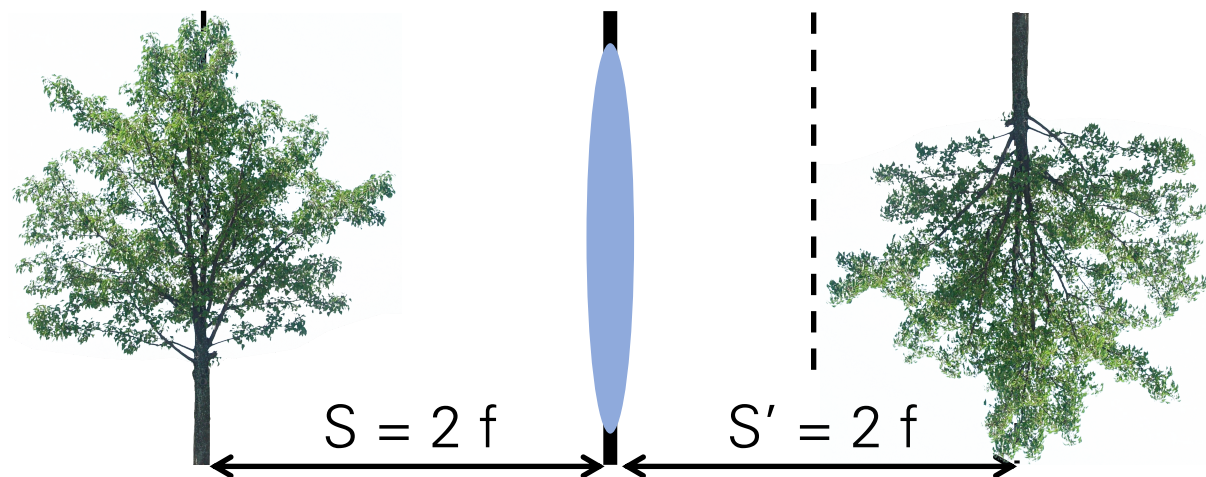
Special focus distances

$S' = f, S = \infty, m = 0 \rightarrow$ infinity focus (parallel rays)

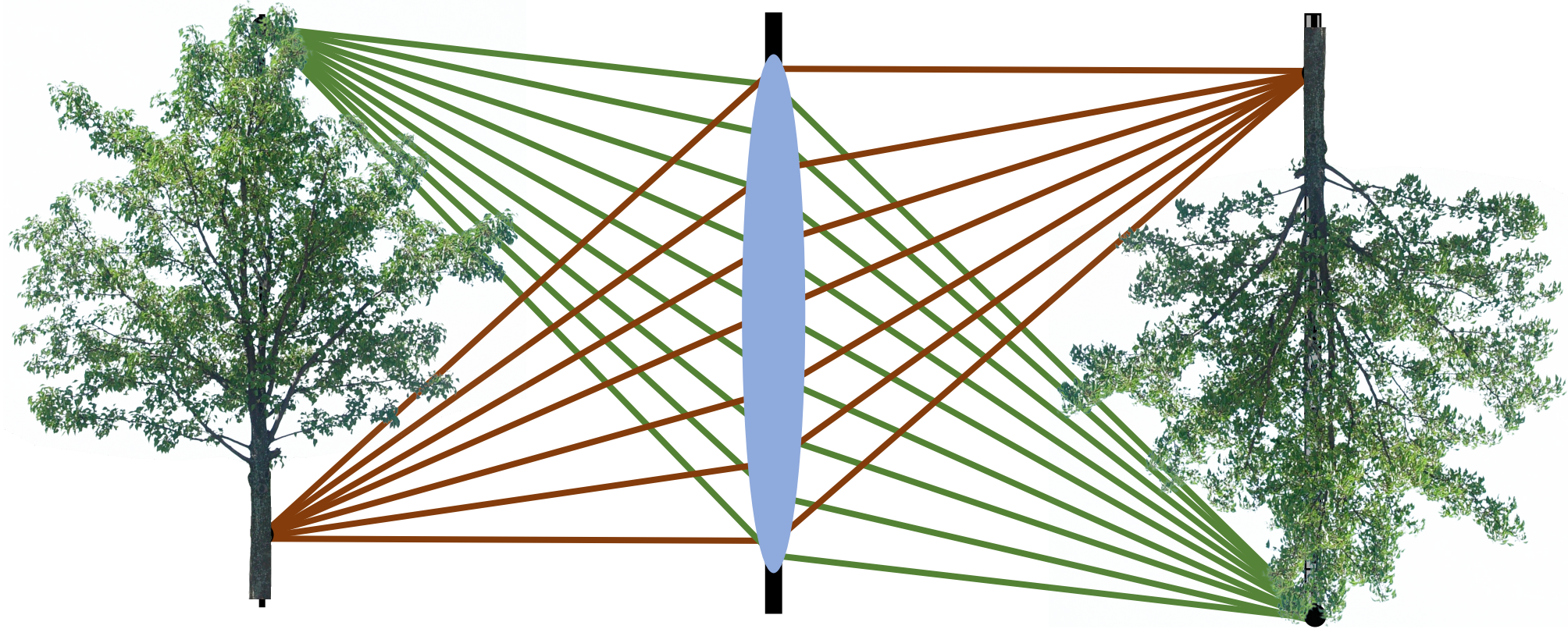


$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$
$$m = \frac{S' - f}{f}$$

$S' = S = 2f, m = 1 \rightarrow$ object is reproduced in real-life size



Free lunch?



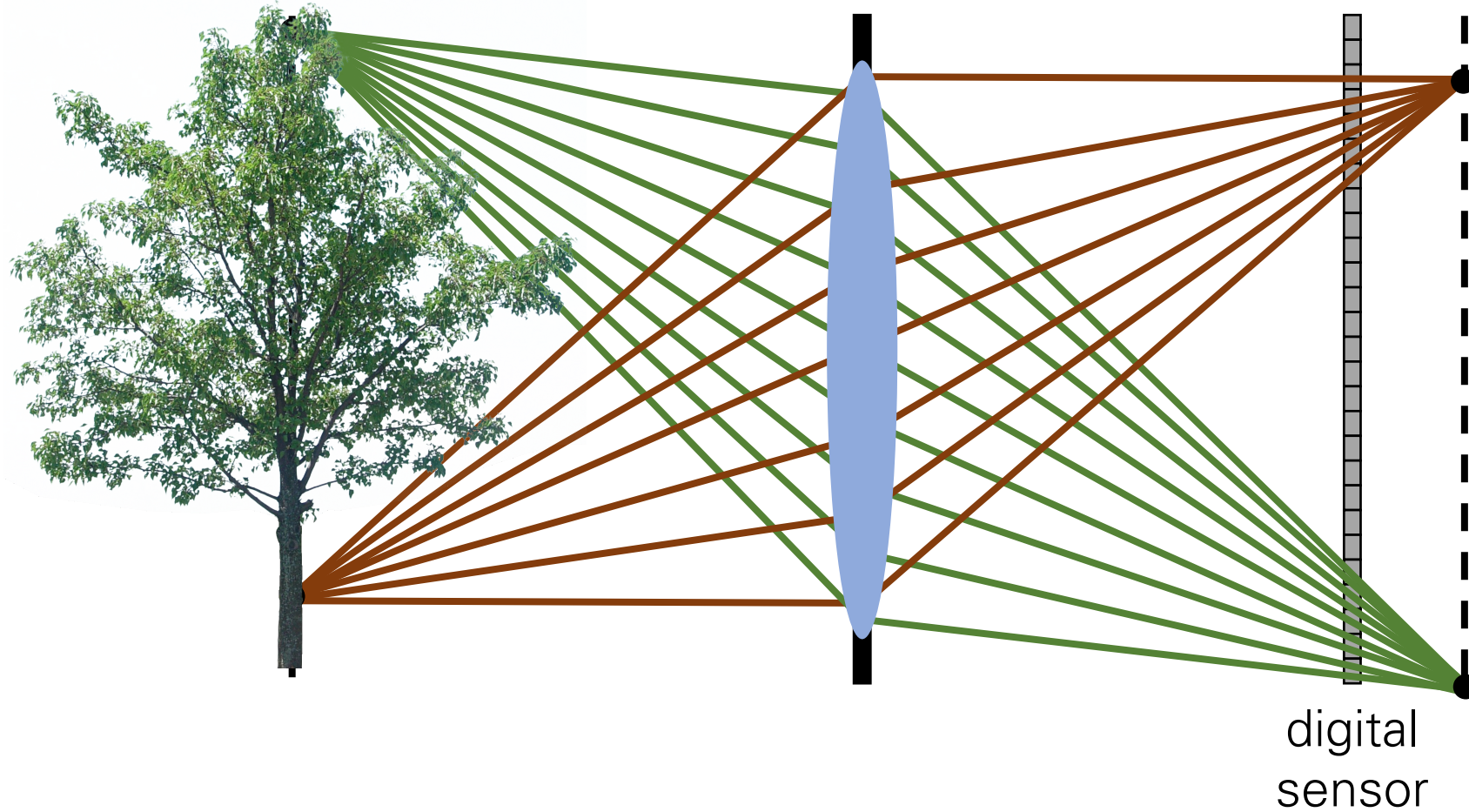
By using a lens, we simultaneously achieve:

1. Image is sharp.
2. Signal-to-noise ratio is high.

Do we lose anything by using a lens?

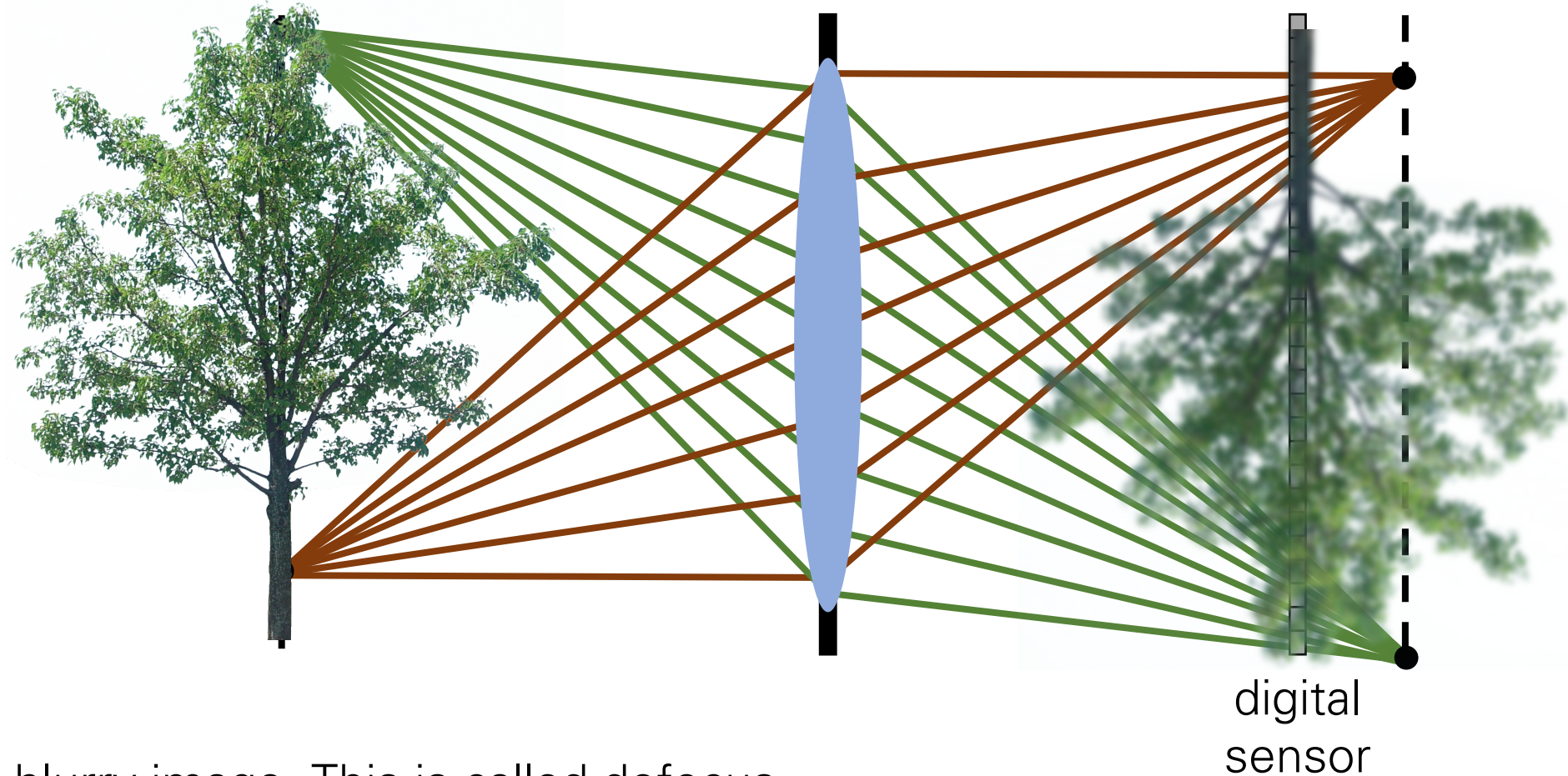
Defocus

What happens if we don't place the sensor at the focus distance?



Defocus

What happens if we don't place the sensor at the focus distance?



We get a blurry image. This is called defocus.

- Defocus never happens with an ideal pinhole camera.

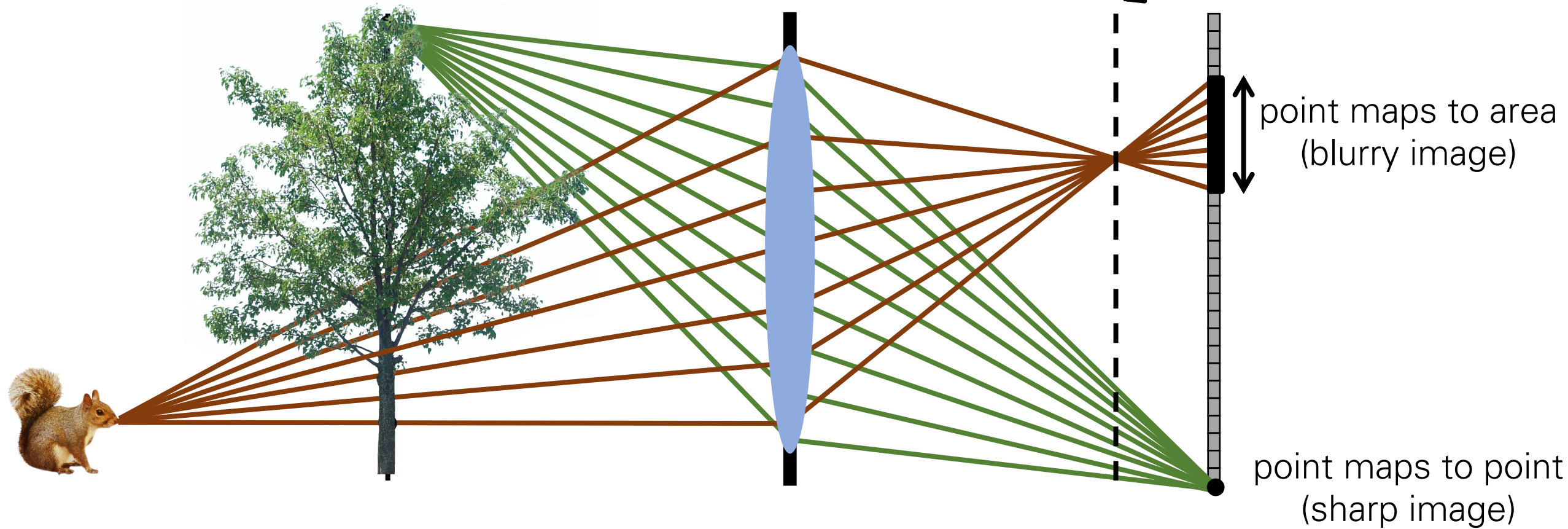
Defocus

Can't we just move the sensor to the correct distance?

Defocus

Can't we just move the sensor to the correct distance?

where we need to move sensor
for squirrel to be in focus

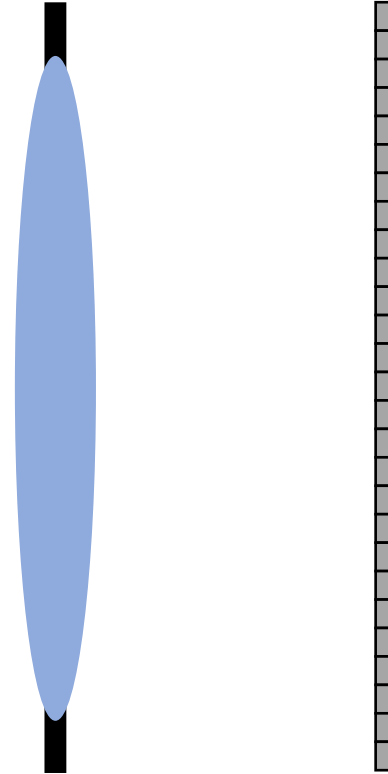


Unless our scene is just one plane, part of it will always be out of focus.

How do we control what is in focus?

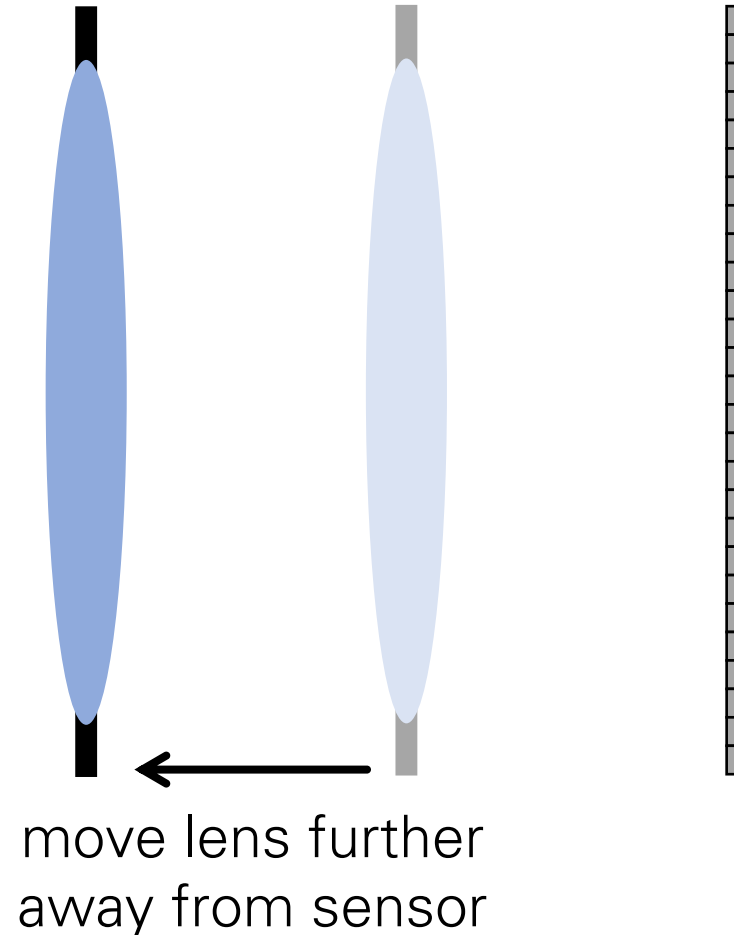
How do we control what is in focus?

We change the distance between the sensor and the lens



How do we control what is in focus?

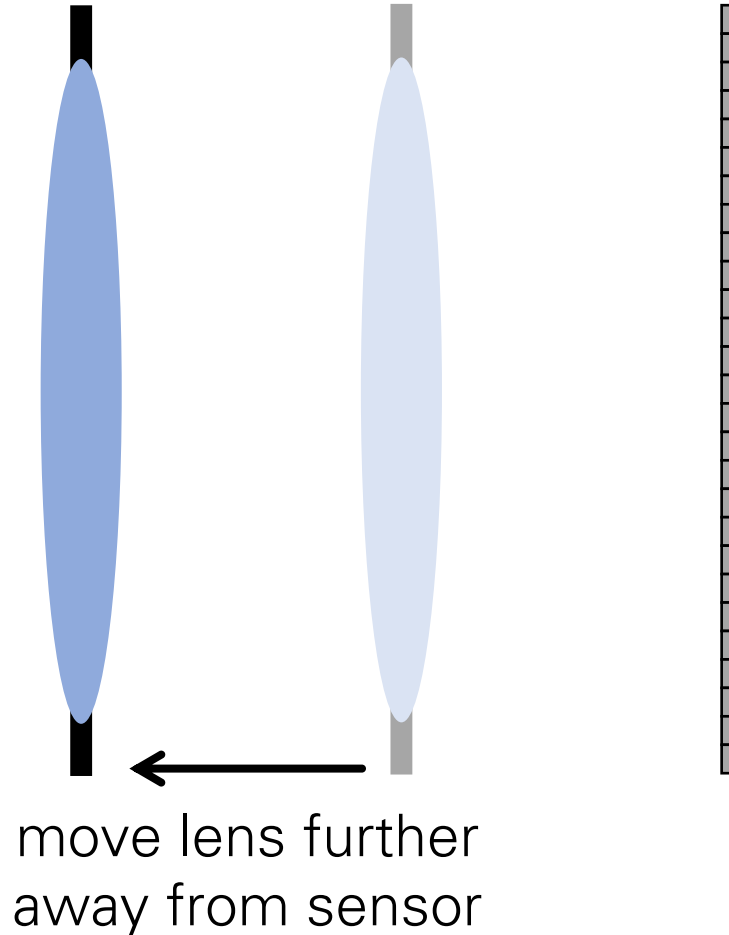
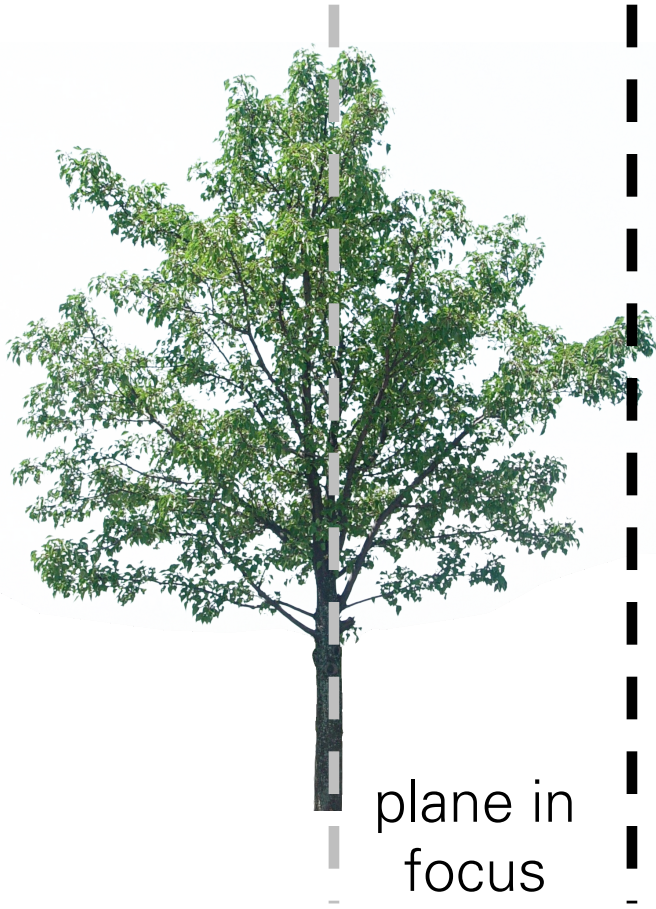
We change the distance between the sensor and the lens



- What happens to plane in focus?

How do we control what is in focus?

We change the distance between the sensor and the lens



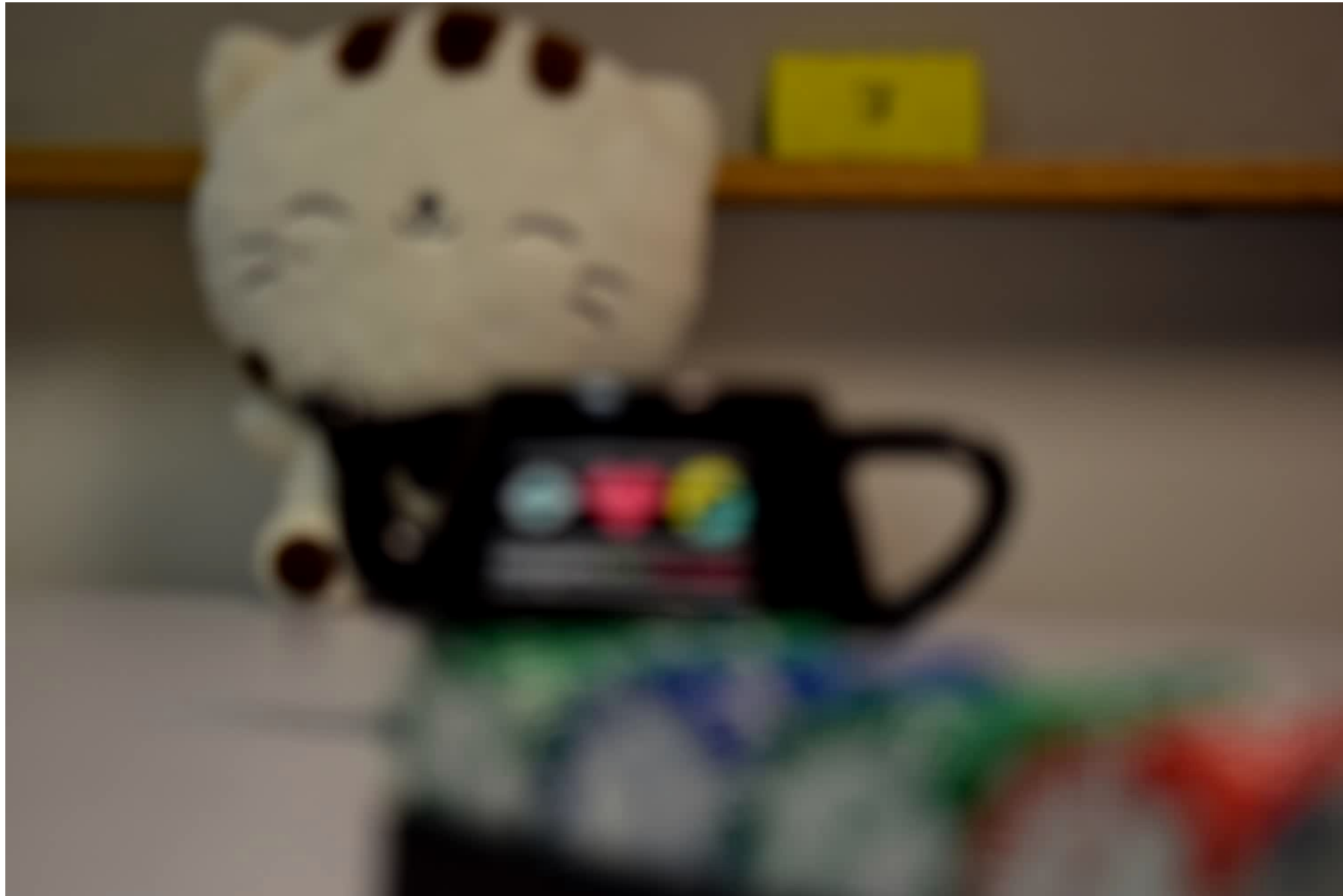
- What happens to plane in focus? → It moves closer.

The lens on your camera

Focus ring: controls distance of lens from sensor



Demonstration

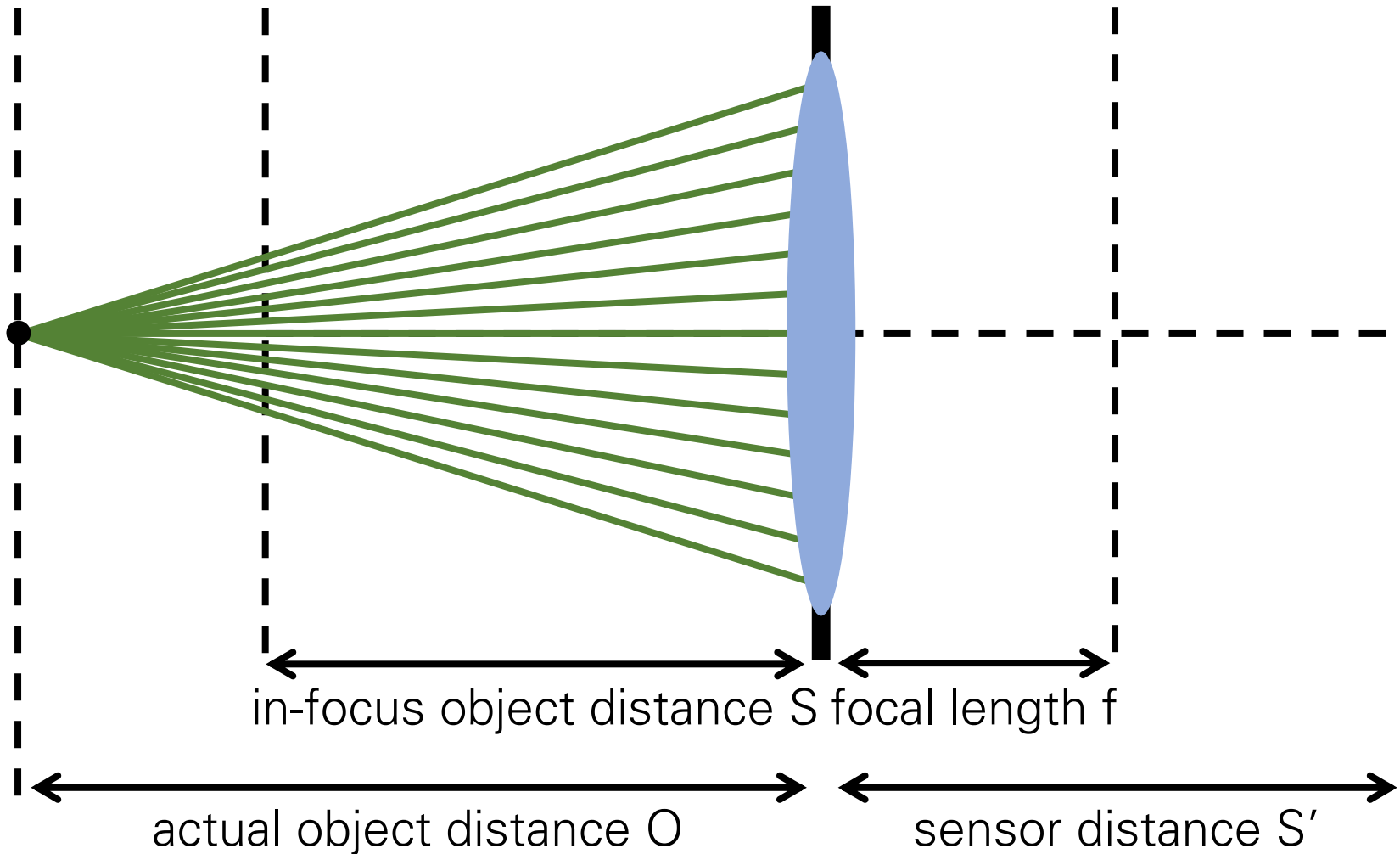


Defocus

Does the mean that lenses are only good for planar scenes?

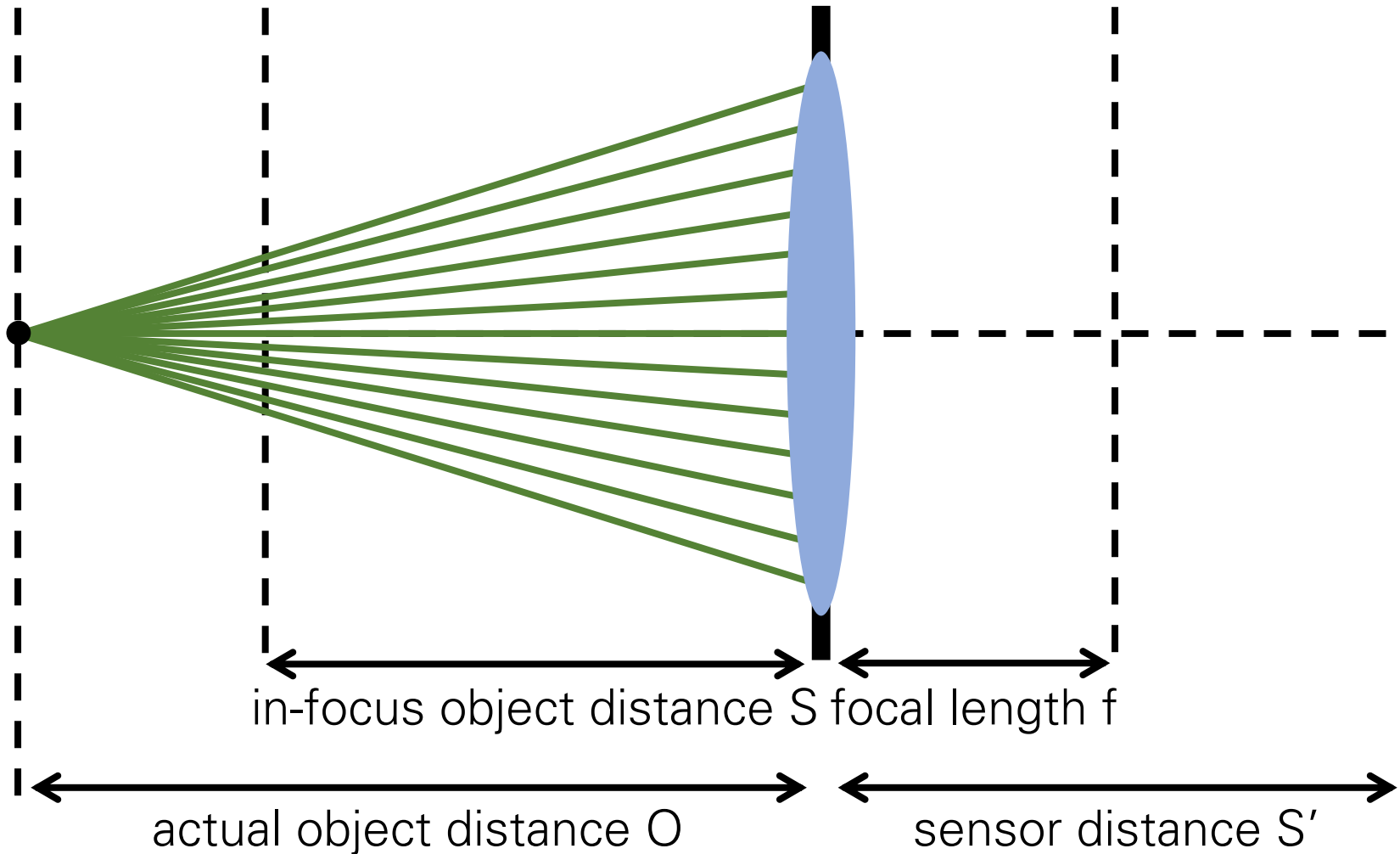
Circle of confusion

How do we find where the point will focus?



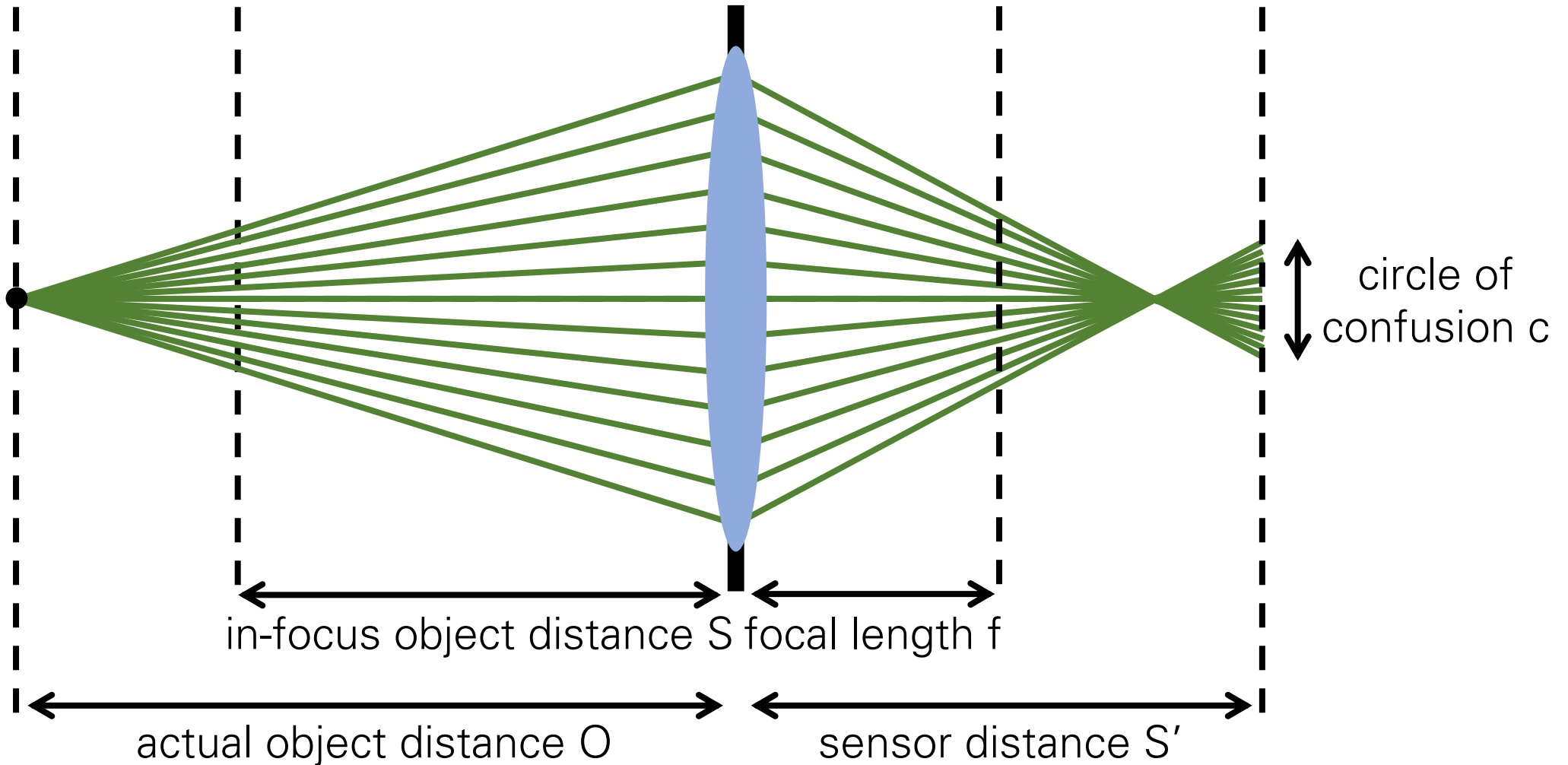
Circle of confusion

Will the point focus at a distance smaller or larger than S' ?



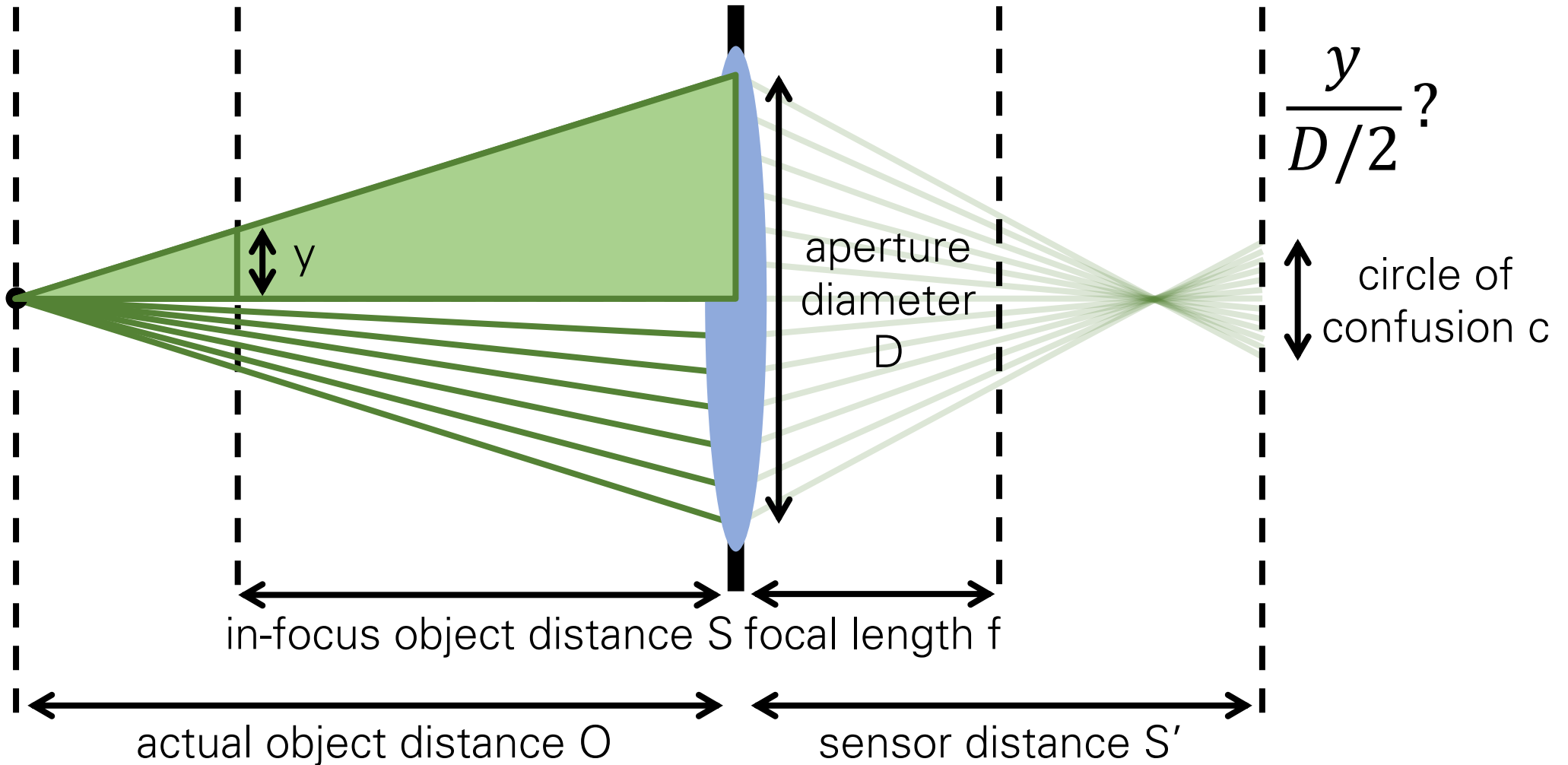
Circle of confusion

How can we compute the diameter of the circle of confusion?



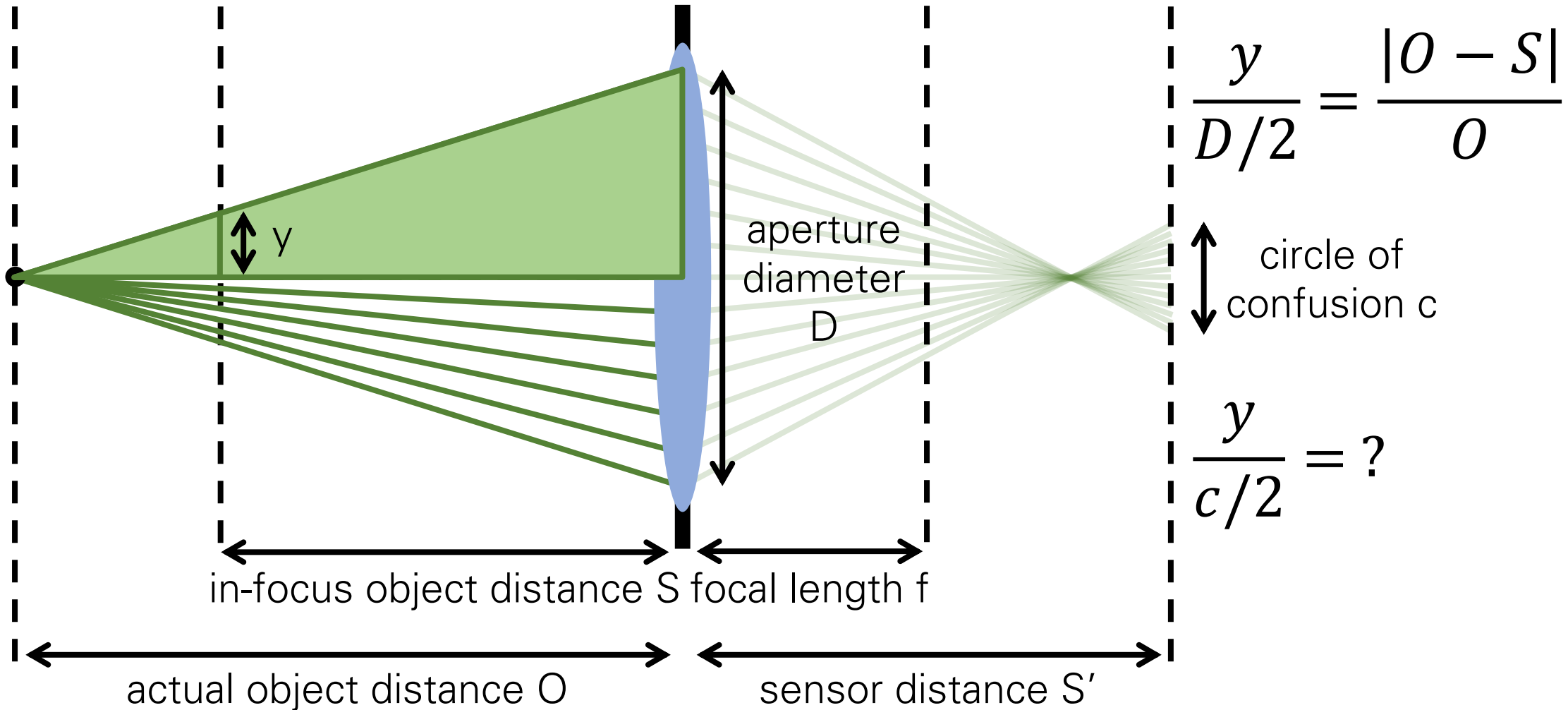
Circle of confusion

How can we compute the diameter of the circle of confusion? → Use similar triangles.



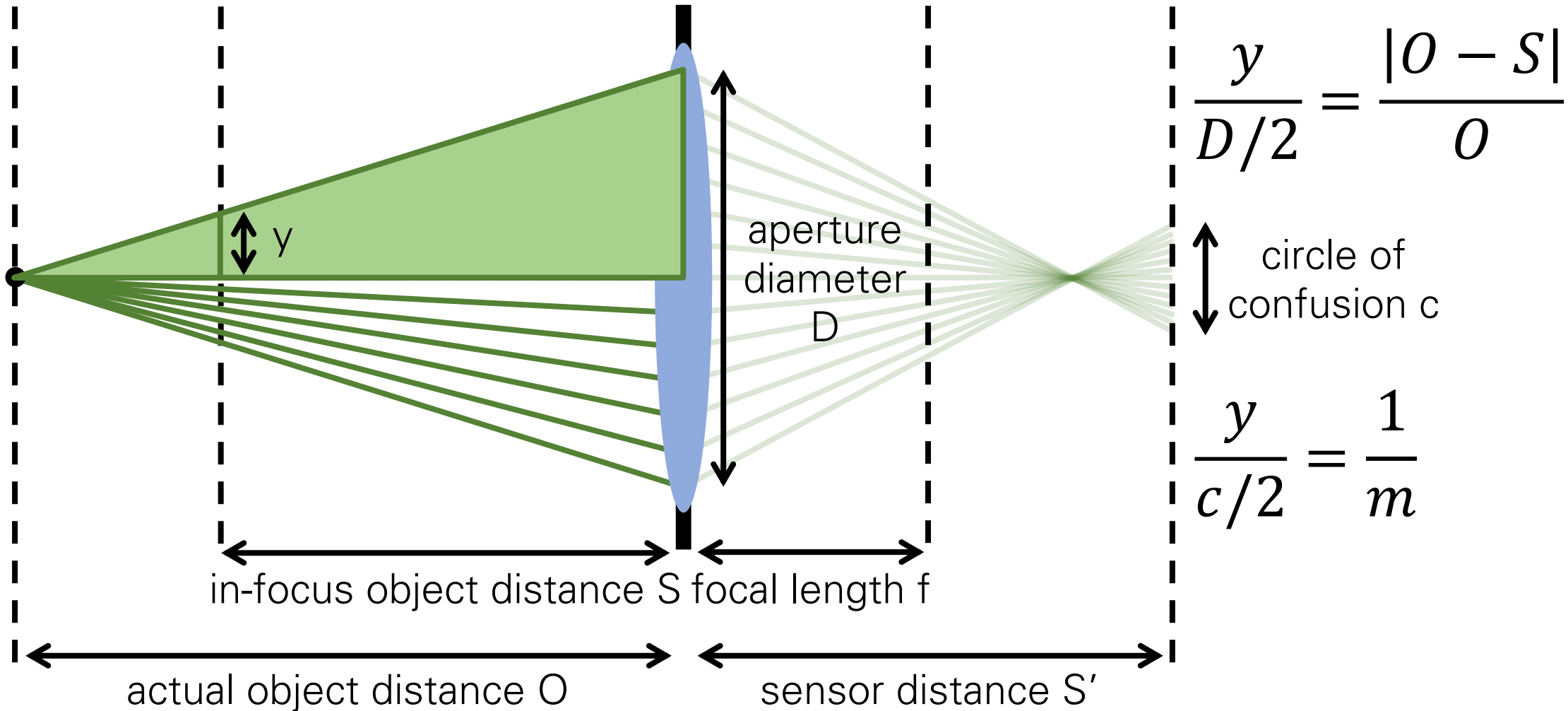
Circle of confusion

How can we compute the diameter of the circle of confusion? → Use similar triangles.



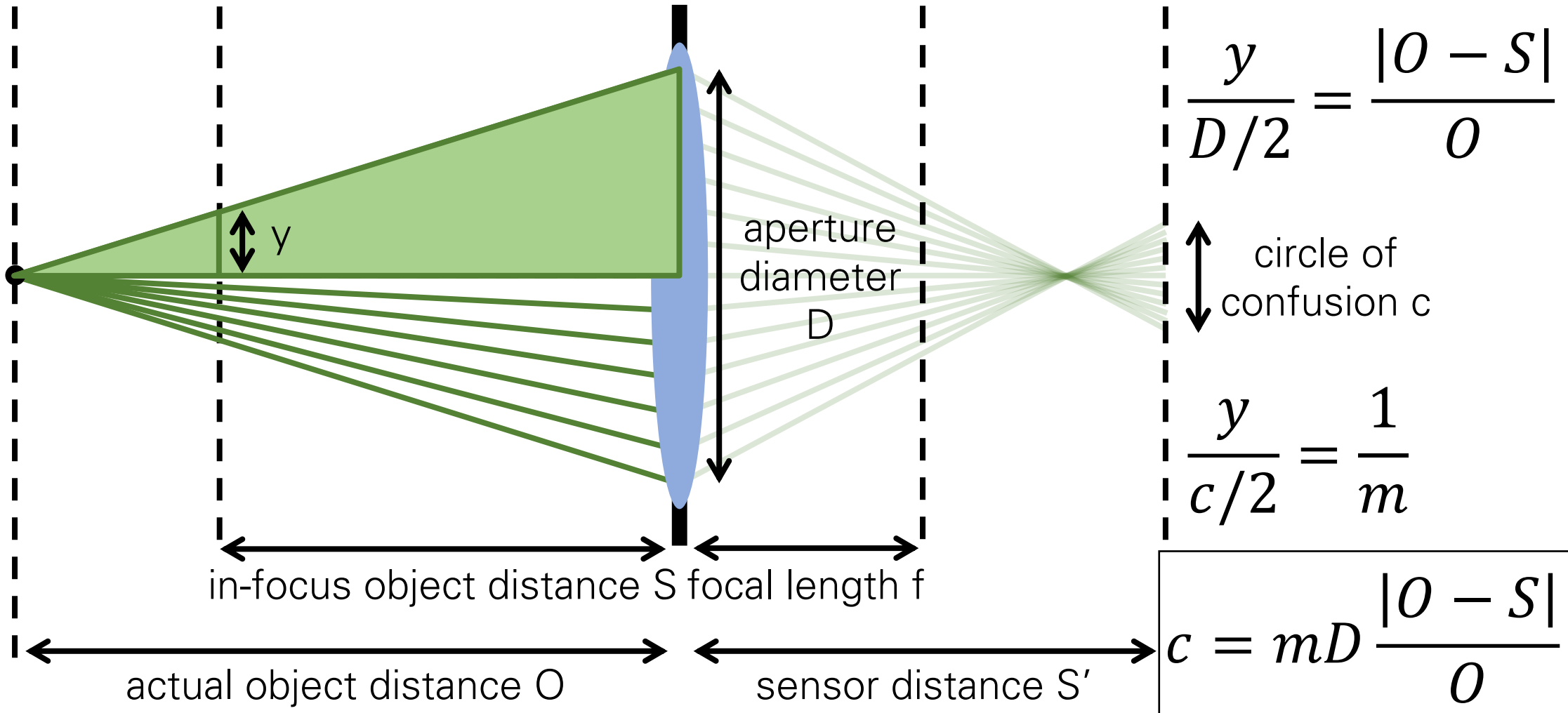
Circle of confusion

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Circle of confusion

How can we compute the diameter of the circle of confusion? → Use similar triangles.



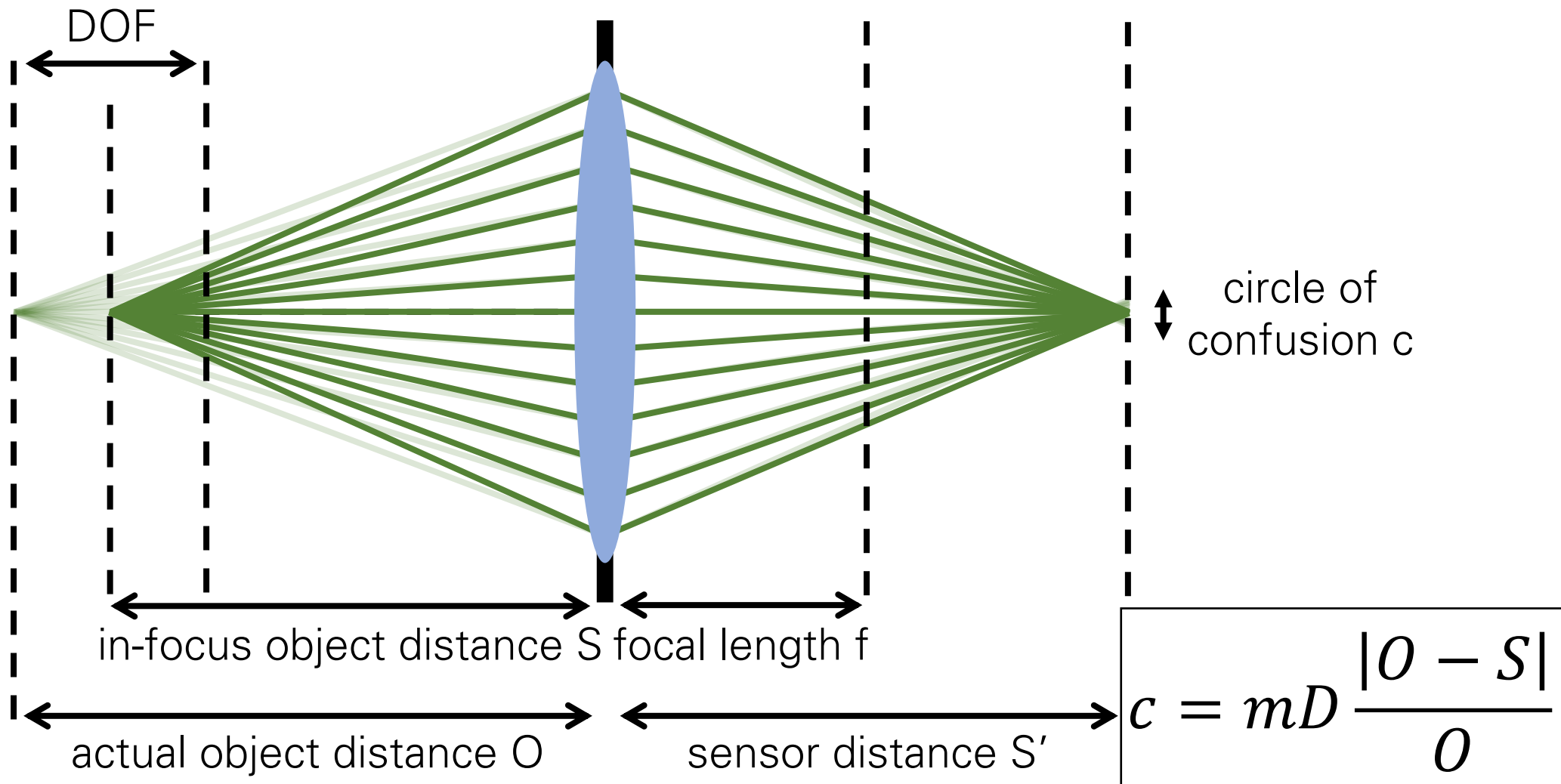
Depth of field

Distance from the in-focus object plane where the circle of confusion is acceptably small.

e.g., equal to
4-5 pixels

$$c < \varepsilon \Rightarrow$$

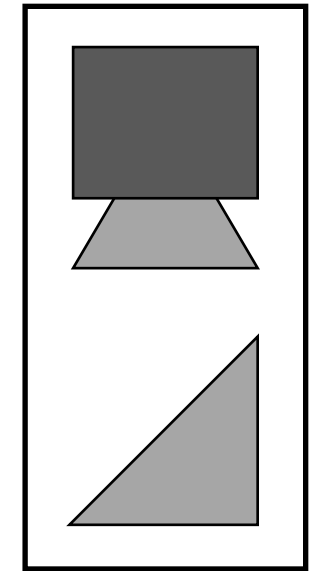
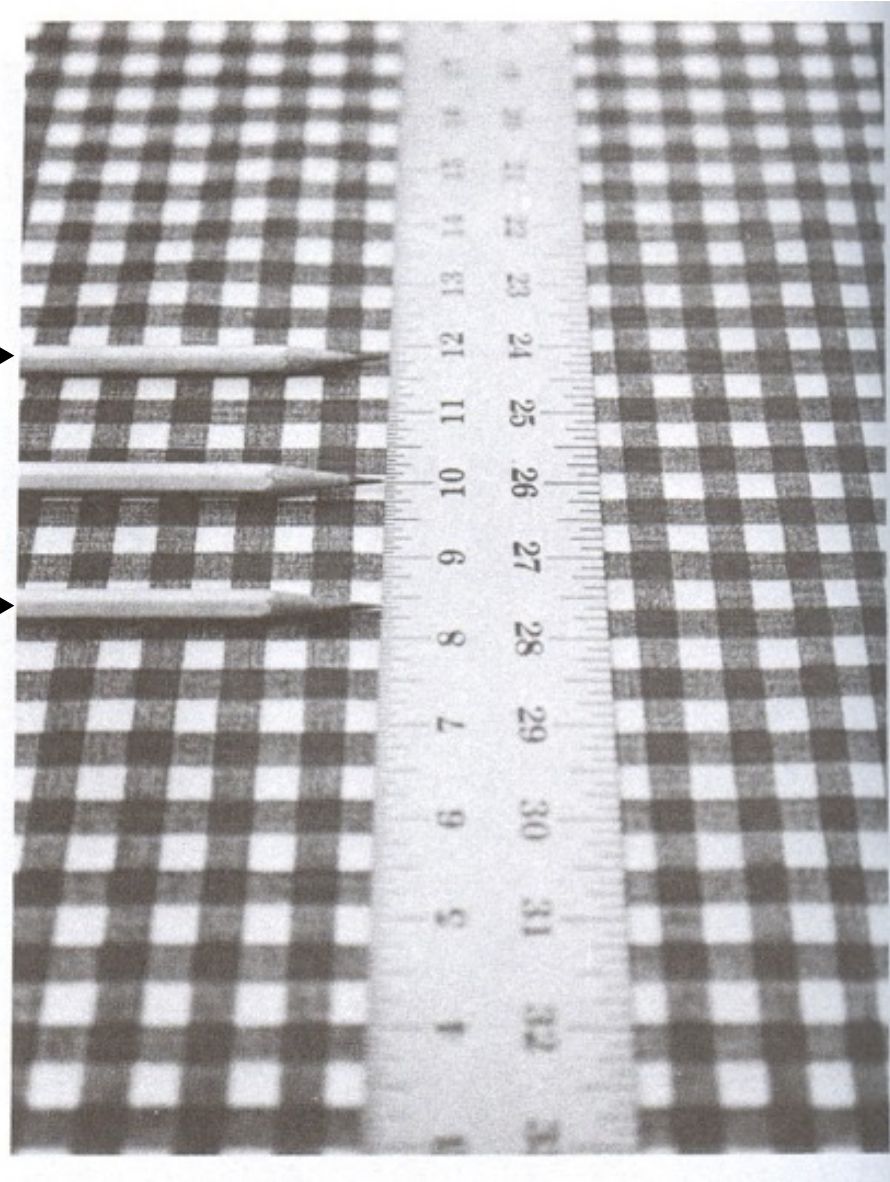
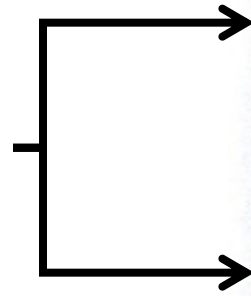
$$\text{DOF} = \frac{2\varepsilon O}{mD}$$



Note: in reality,
DOF is slightly
asymmetrical.

Depth of field

depth of field



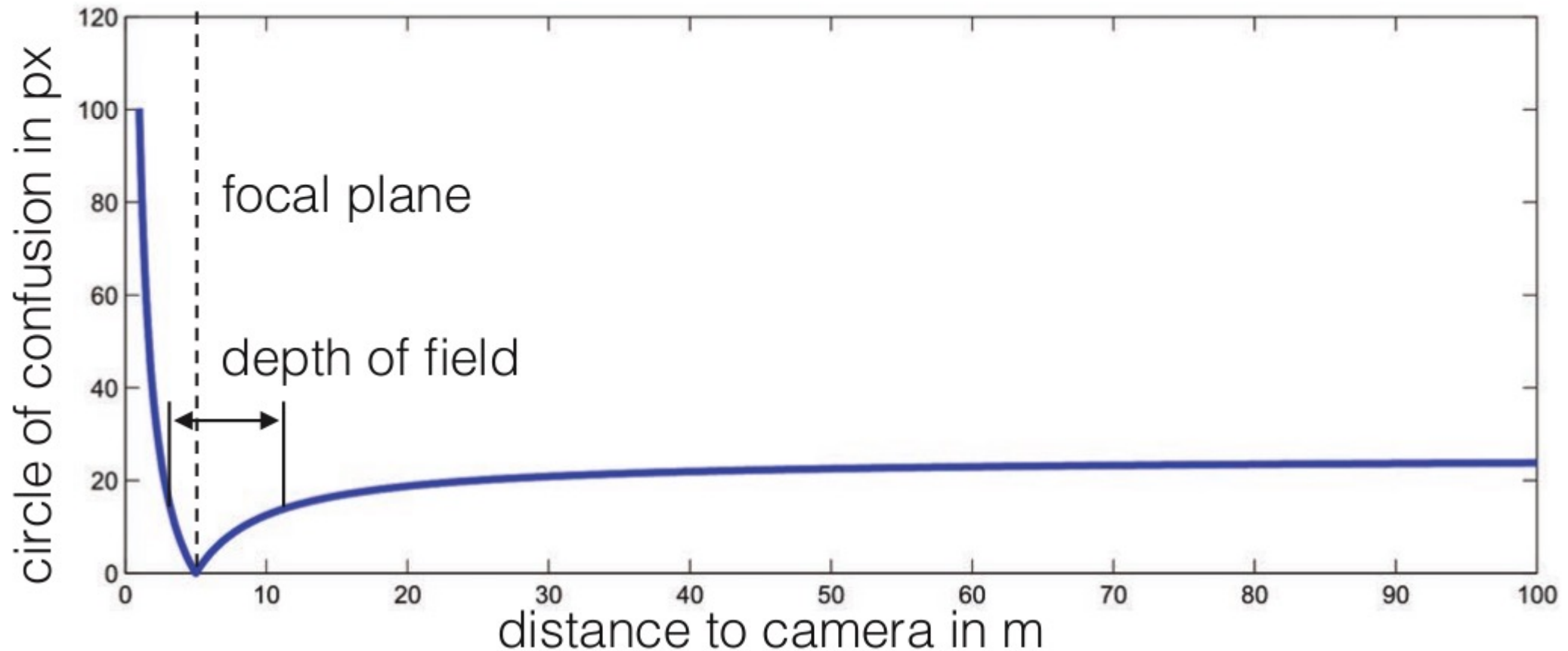
scene

Circle of confusion

$$c = M \cdot D \cdot \frac{|S - S_1|}{S}$$

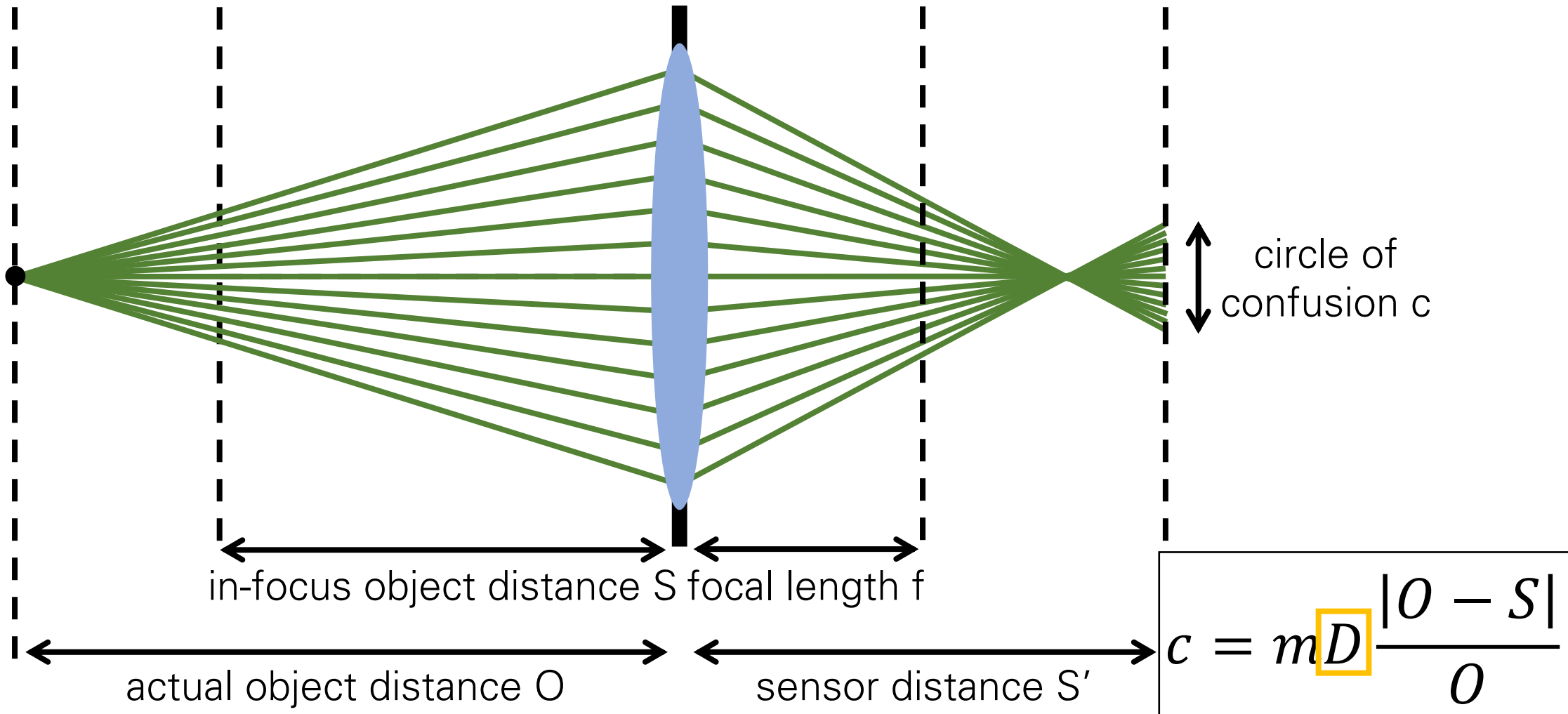
Canon 5D Mark III: $f=50\text{mm}$, $f/2.8$ ($N=2.8$),

focused at 5m , pixel size= $7.5\mu\text{m}$



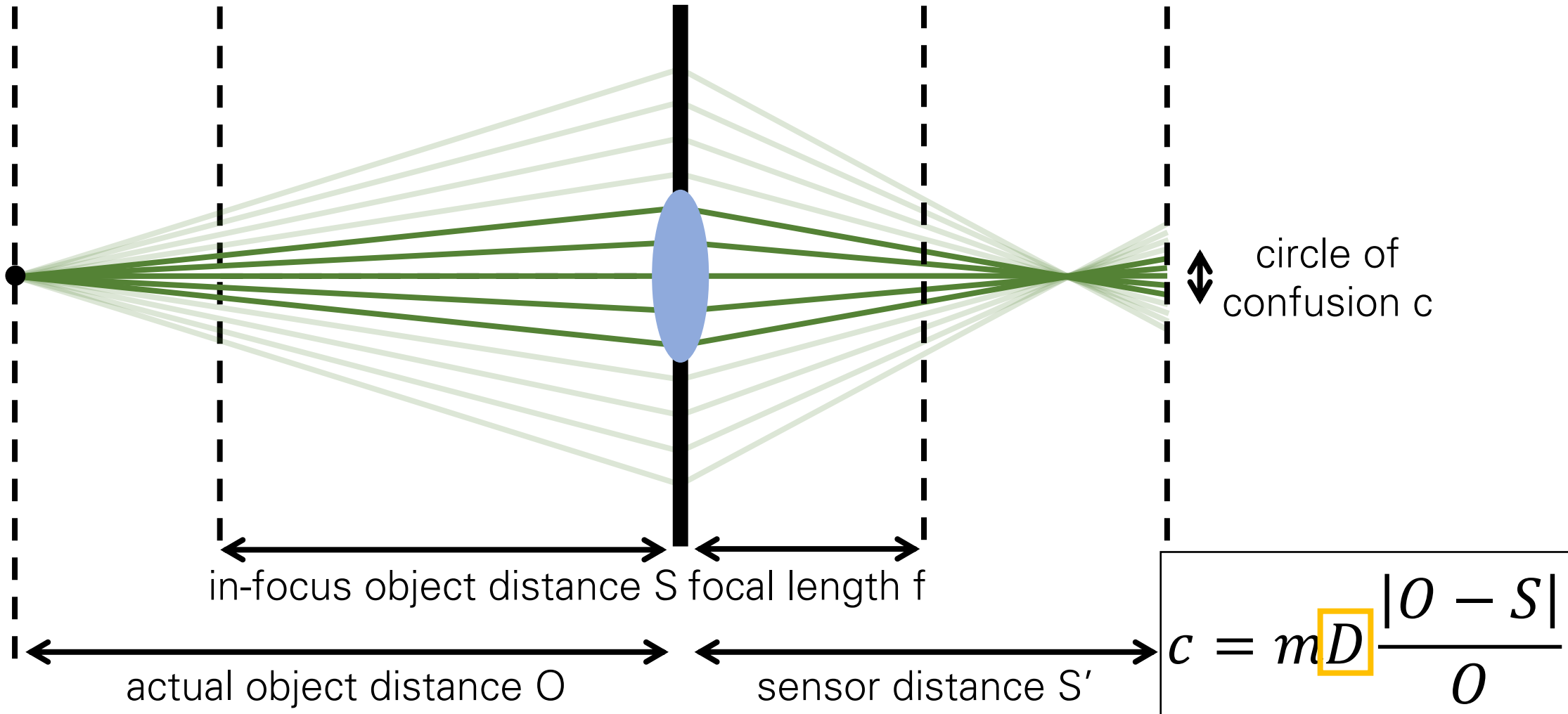
Defocus depends on aperture diameter

What happens to the circle of confusion as the aperture diameter is reduced?



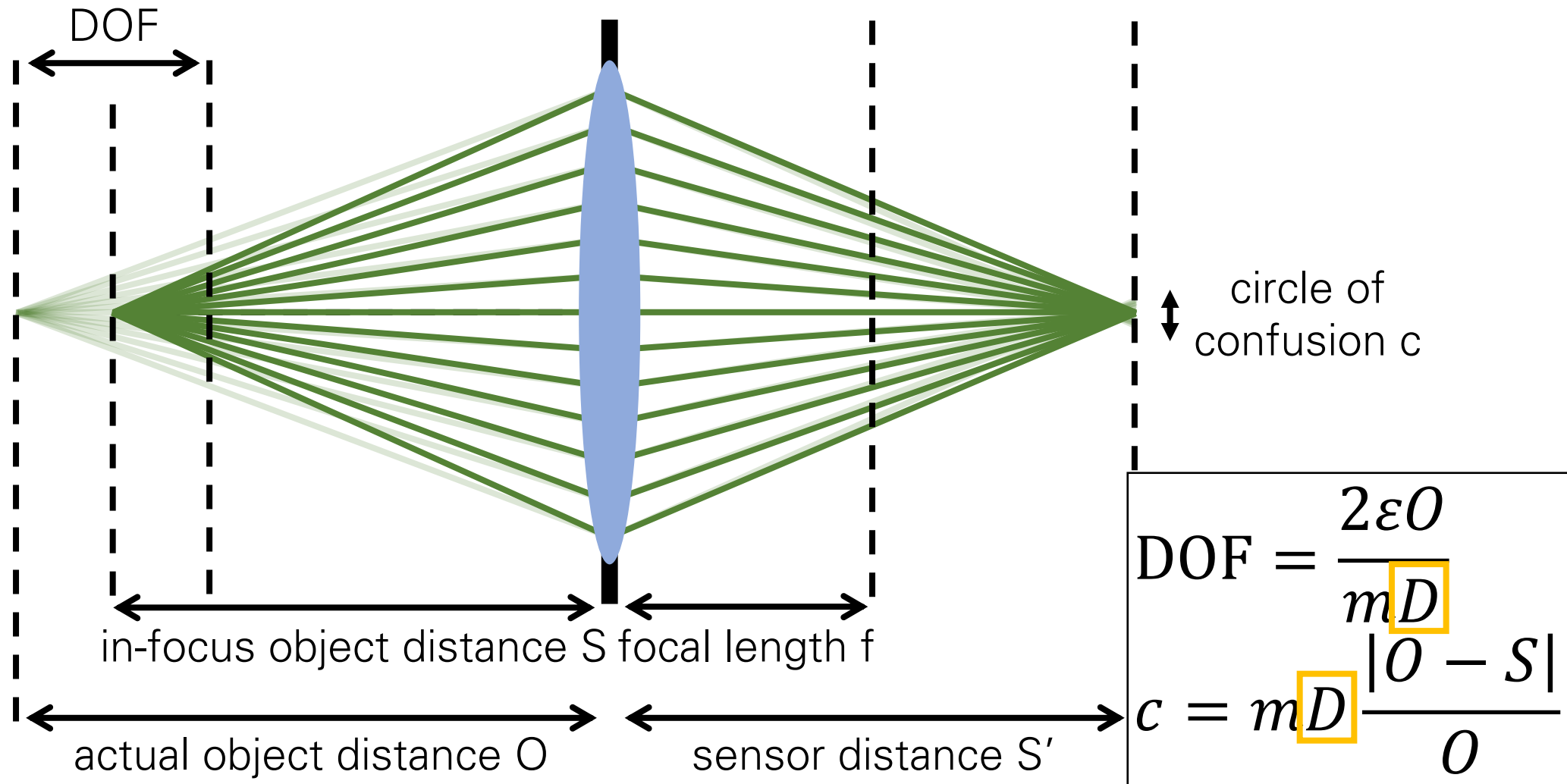
Defocus depends on aperture diameter

What happens to the circle of confusion as the aperture diameter is reduced? → It shrinks.



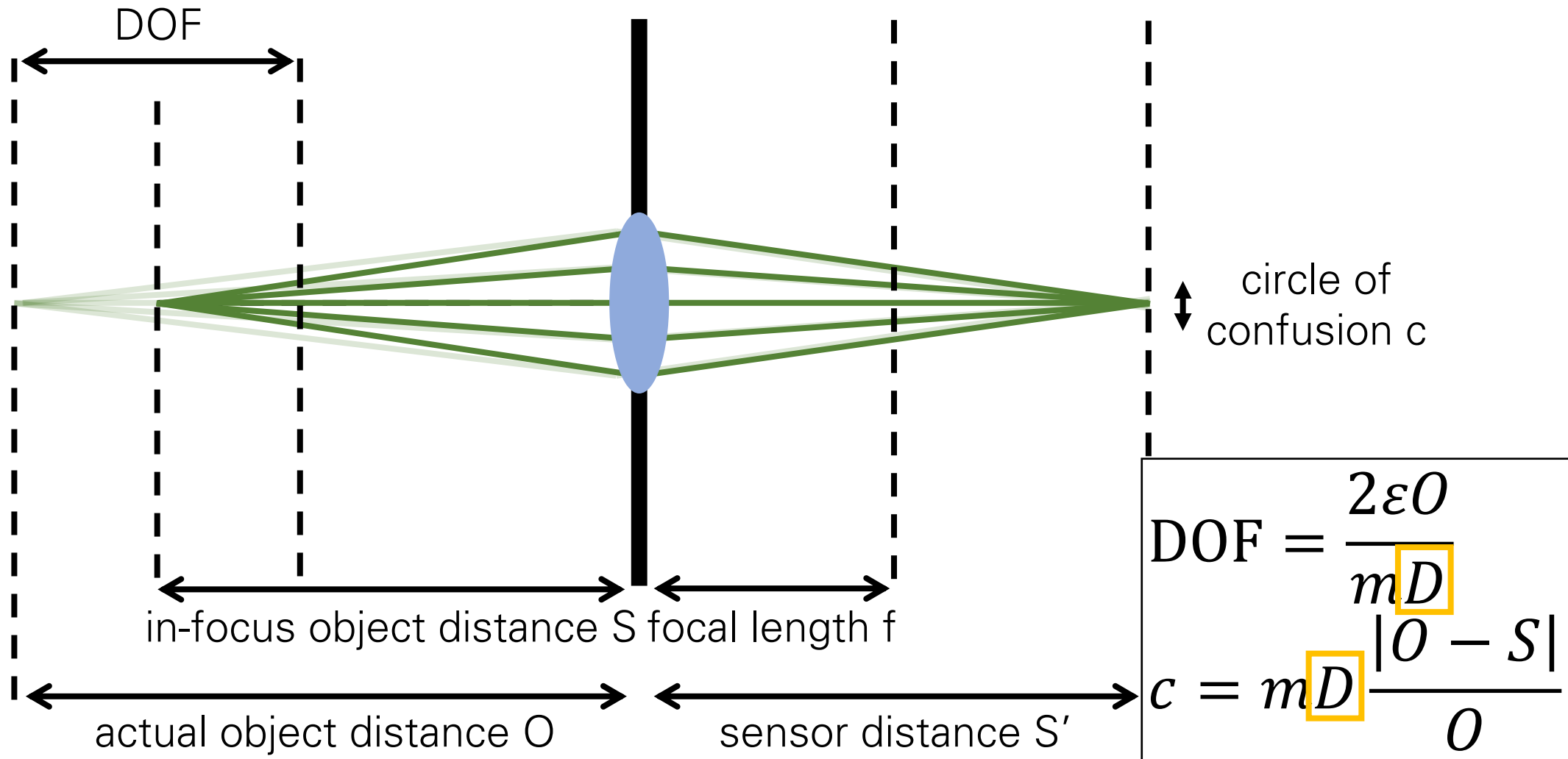
Defocus depends on aperture diameter

What happens to the depth of field as the aperture diameter is reduced?



Defocus depends on aperture diameter

What happens to the depth of field as the aperture diameter is reduced? → It expands.



Aperture size

Most lenses have apertures of variable size.

- The size of the aperture is expressed as the “f-number”: The bigger this number, the smaller the aperture.



f / 1.4



f / 2.8



f / 4



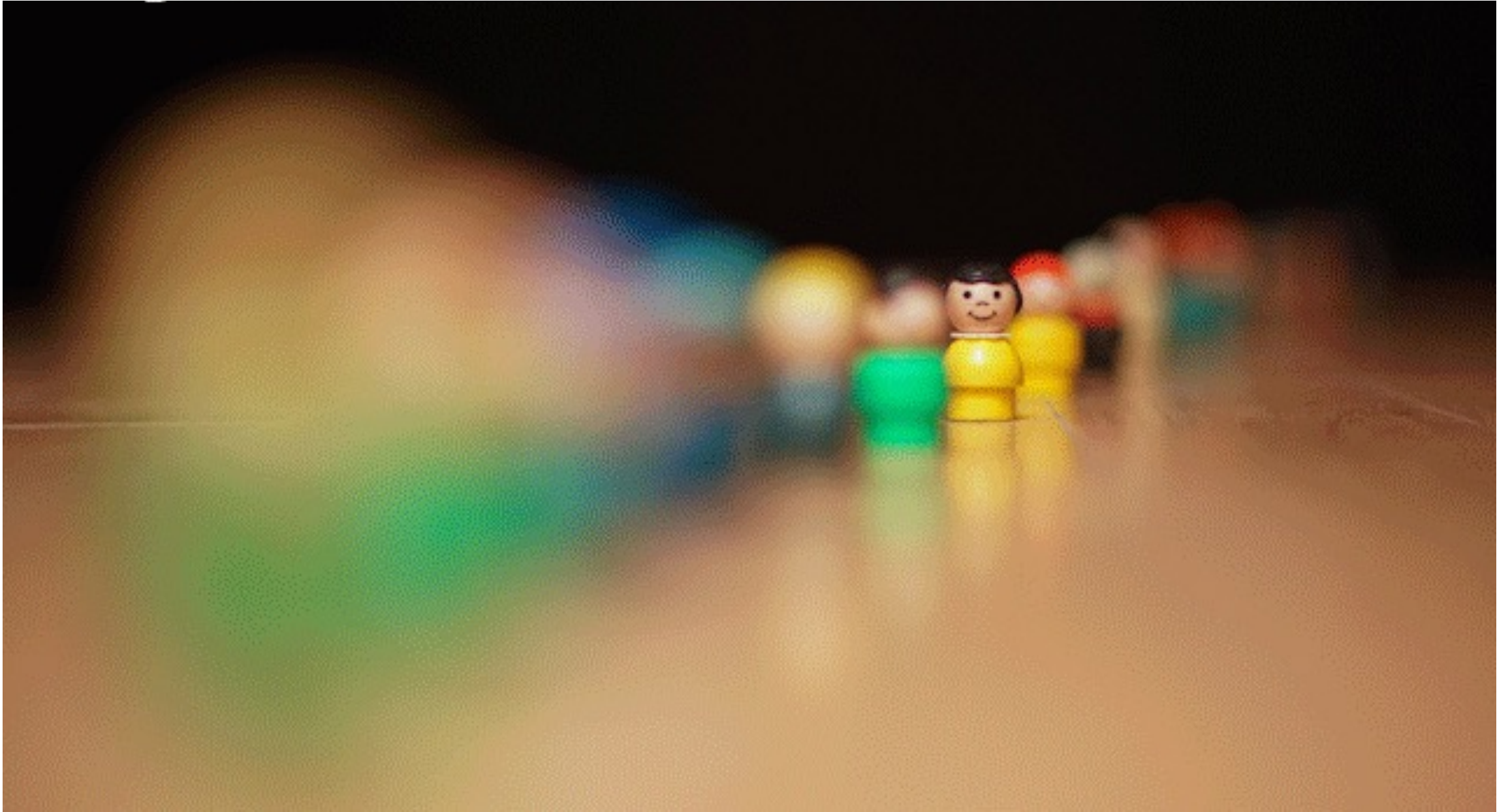
f / 8



f / 16

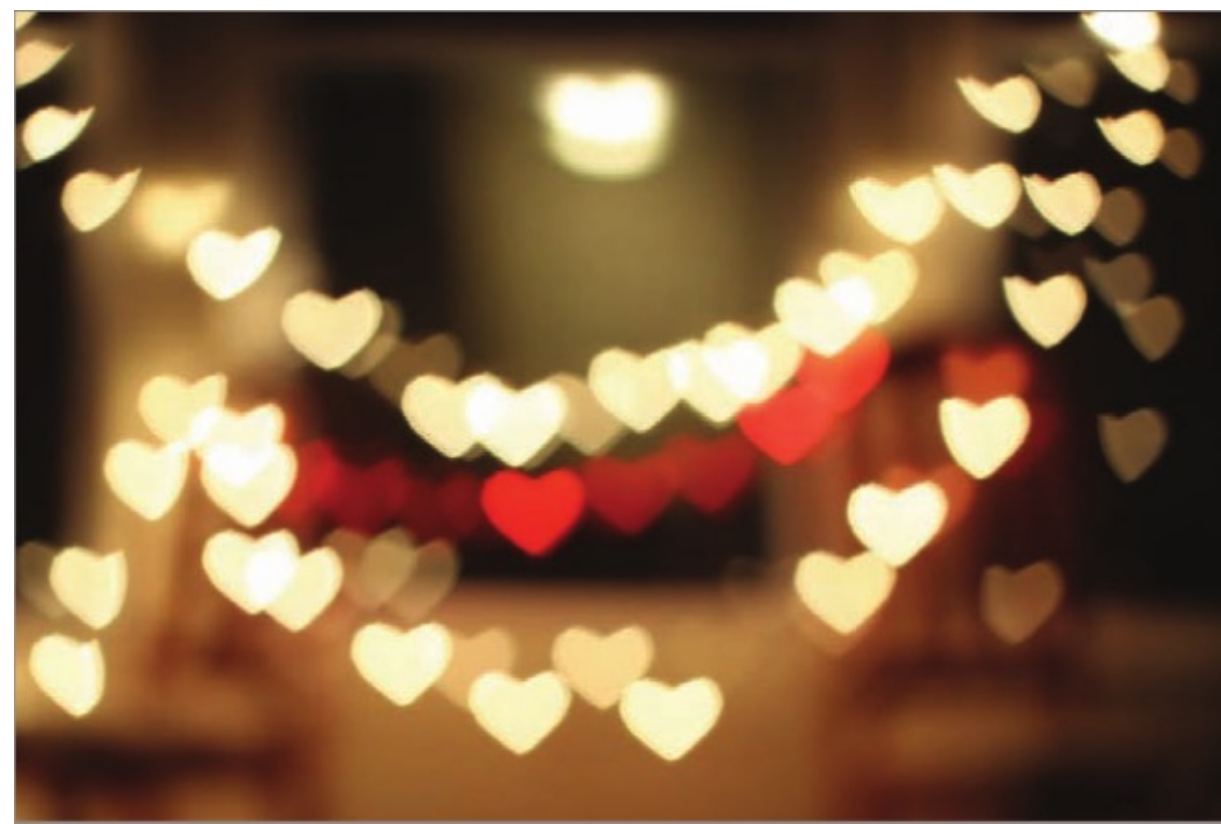
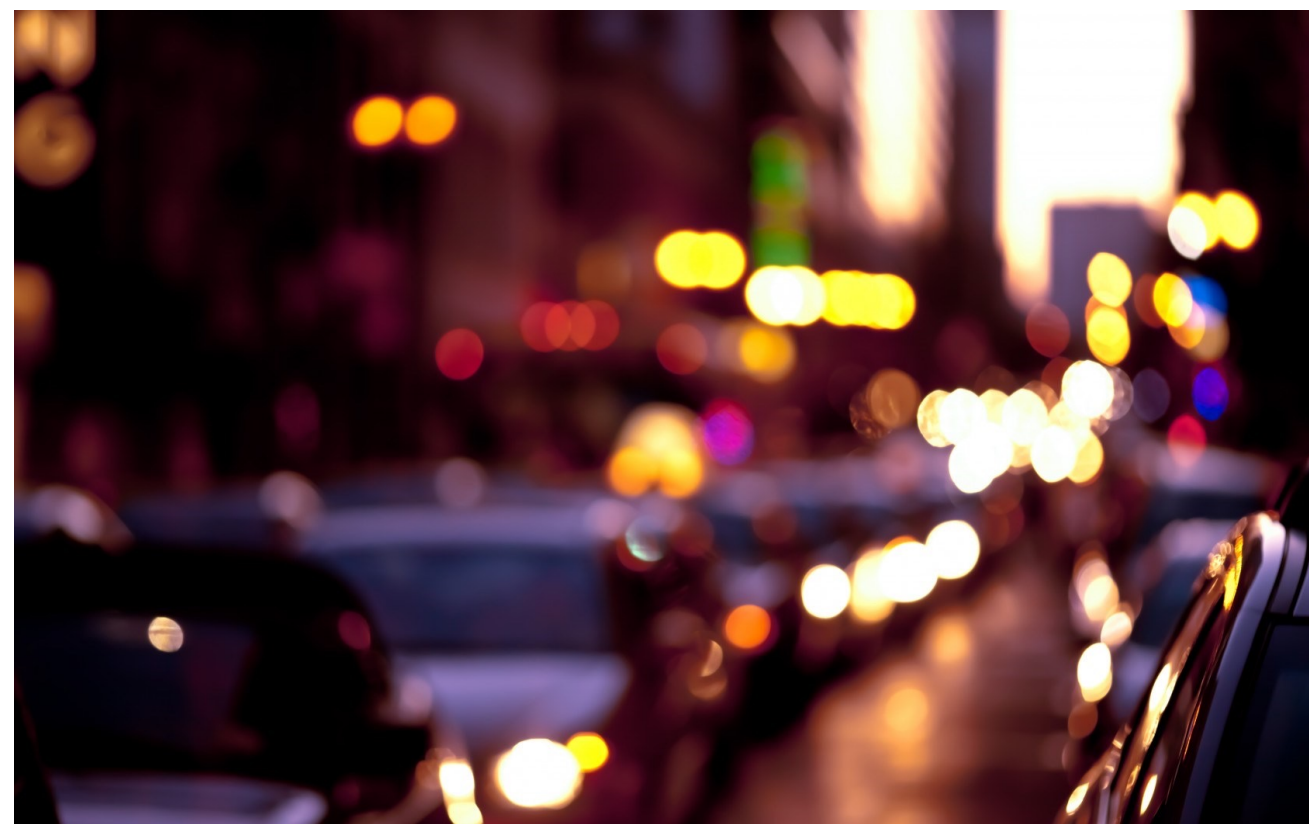
You can see the aperture by removing the lens and looking inside it.

Demonstration



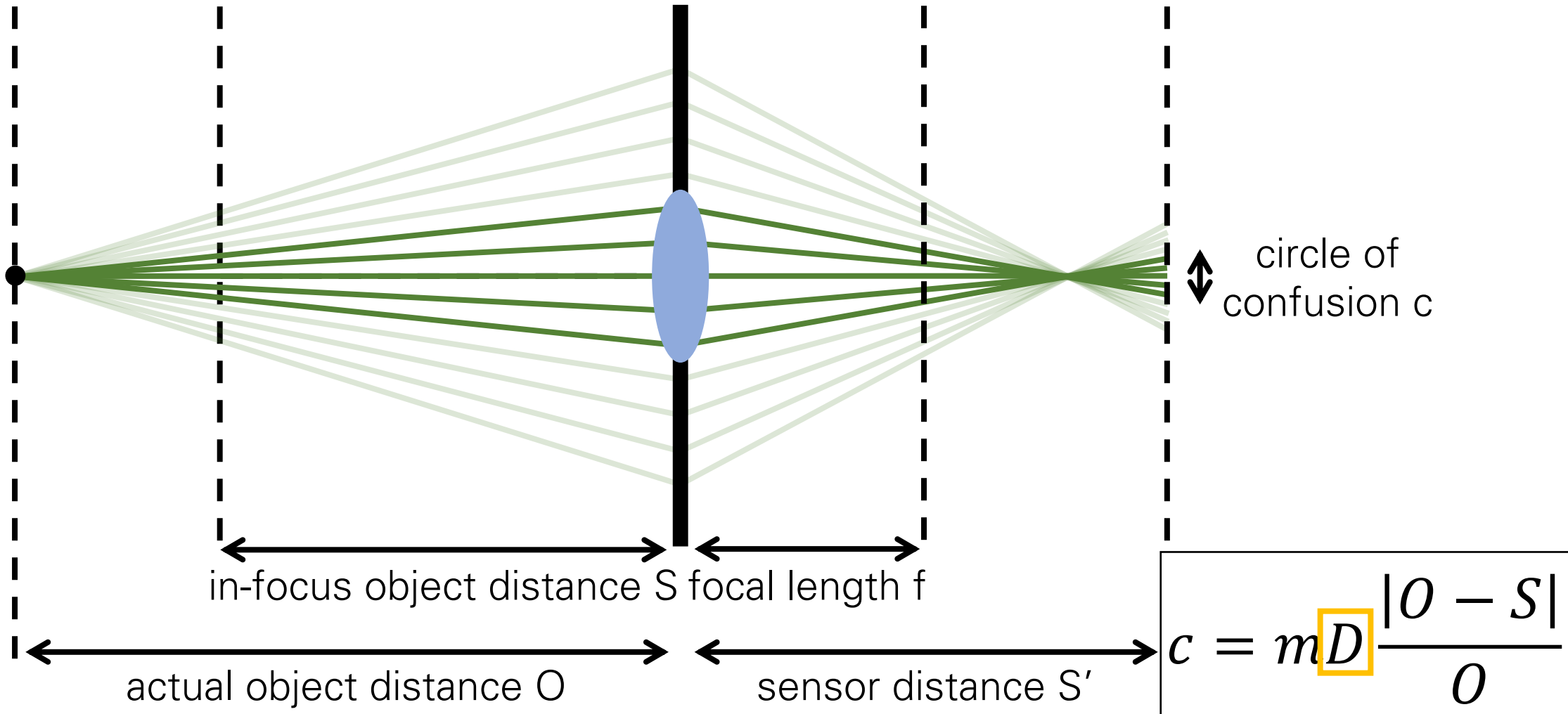
Depth of Field

Form of defocus blur is determined by shape of aperture.



Defocus depends on aperture diameter

If small aperture sizes reduce defocus blur, should we always use the smallest aperture?

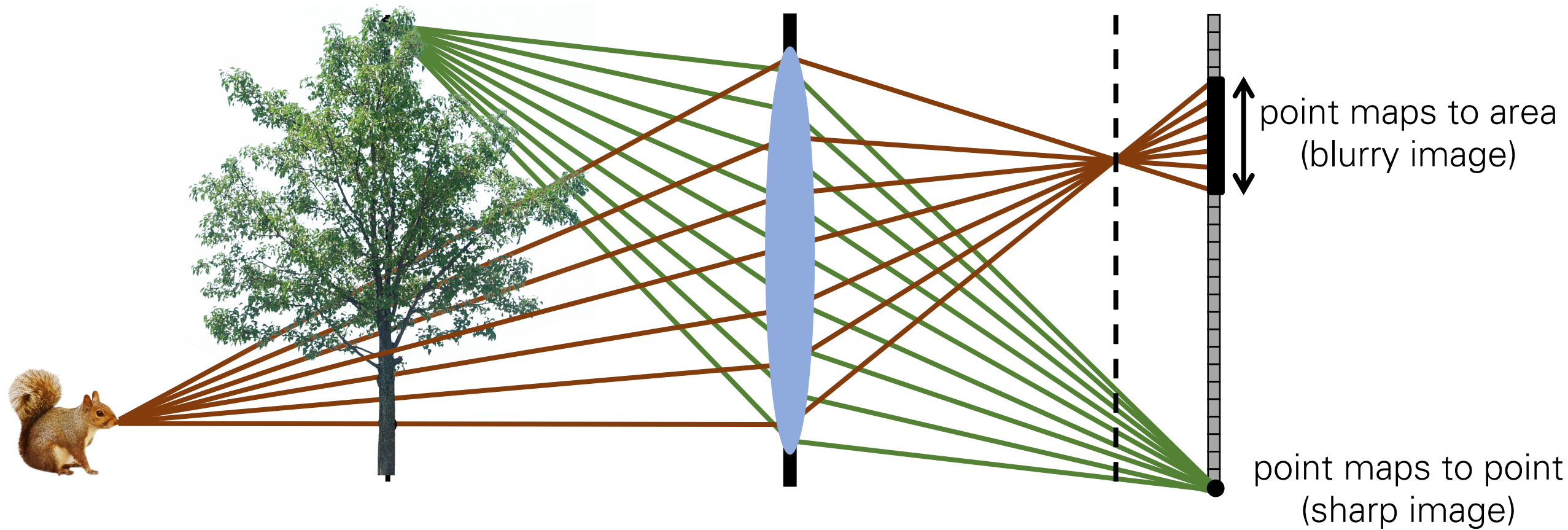


Bokeh

Sharp depth of field (“bokeh”) is often desirable.

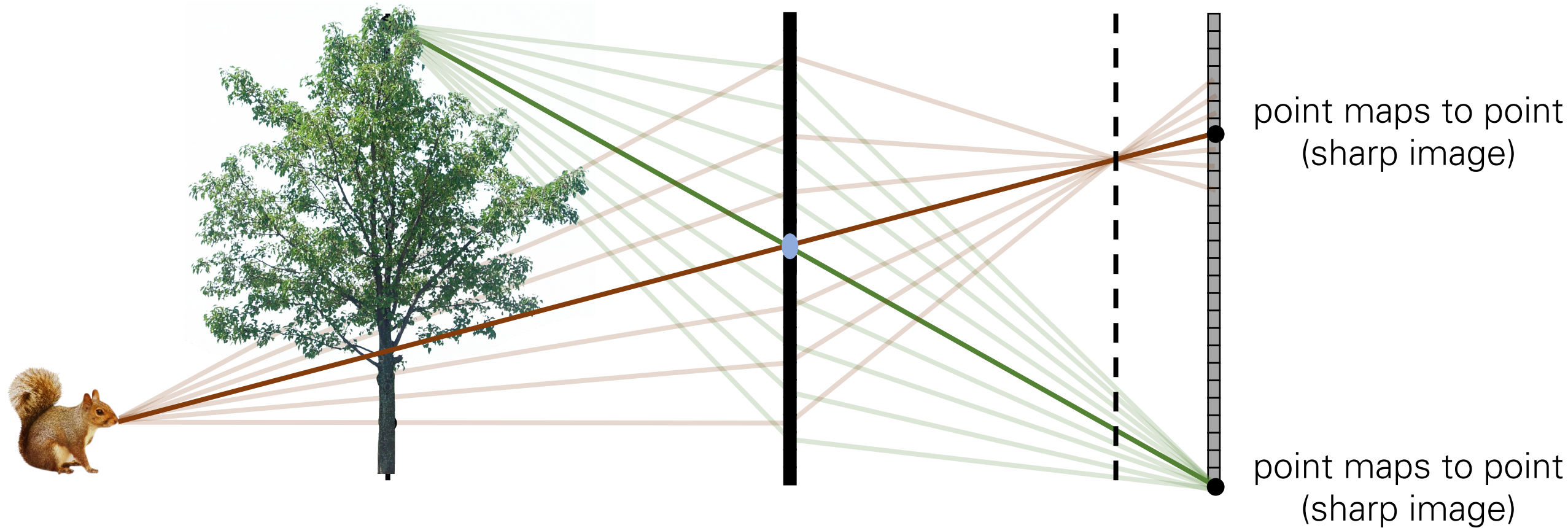


What happens as the aperture keeps getting smaller?



What happens as the aperture keeps getting smaller?

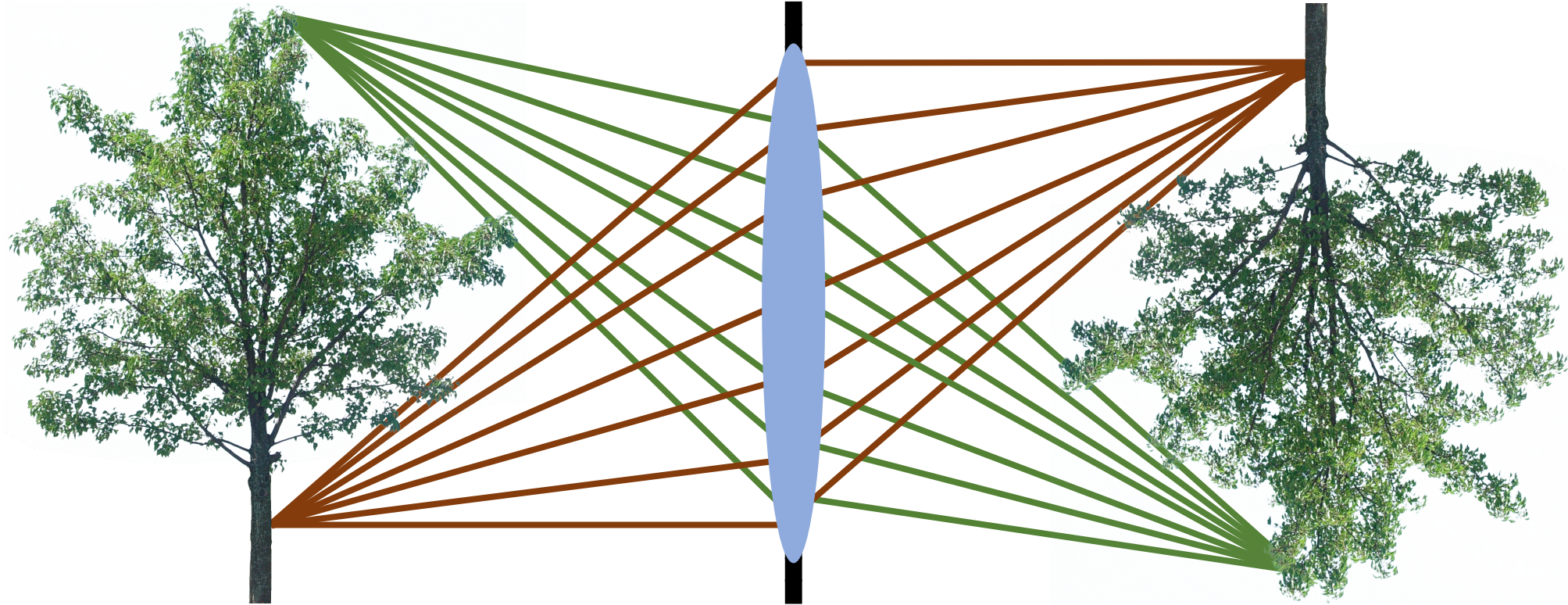
Lens becomes equivalent to a pinhole.



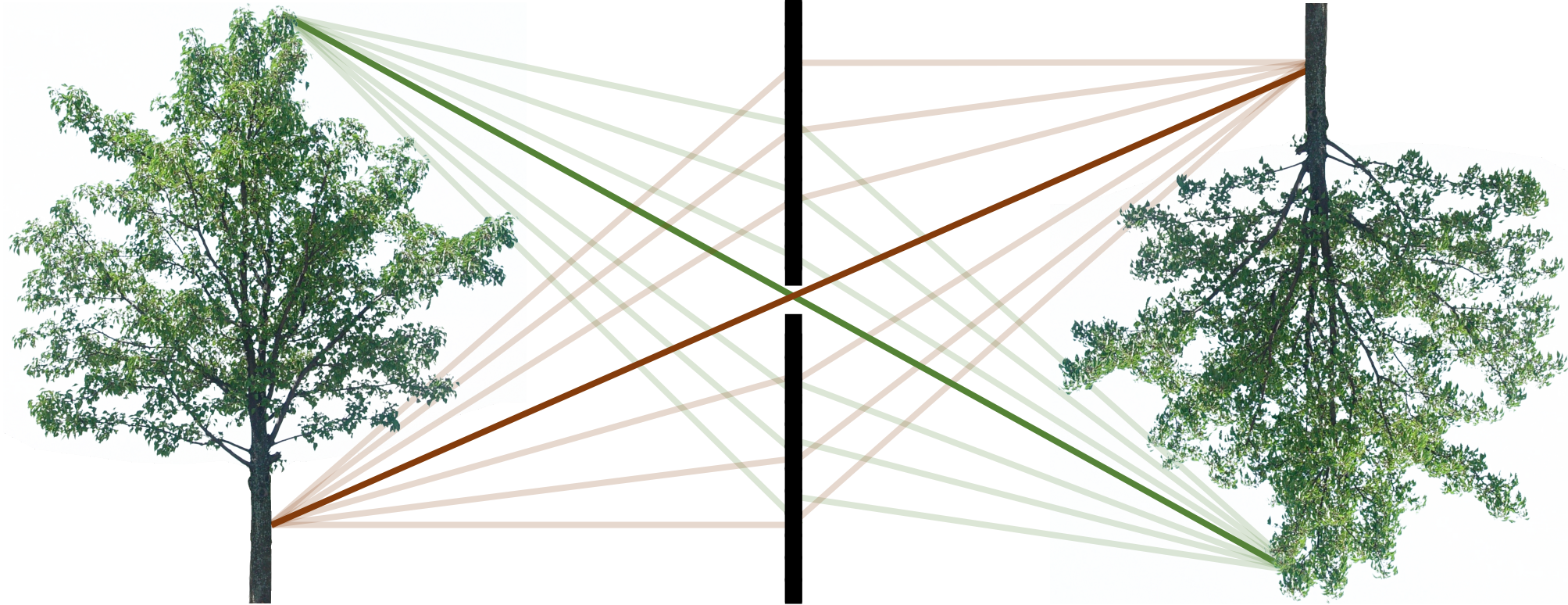
- No defocus, everything is sharp regardless of depth.
- Very little light, signal-to-noise ratio is just as bad as pinhole.

Lens camera and pinhole camera

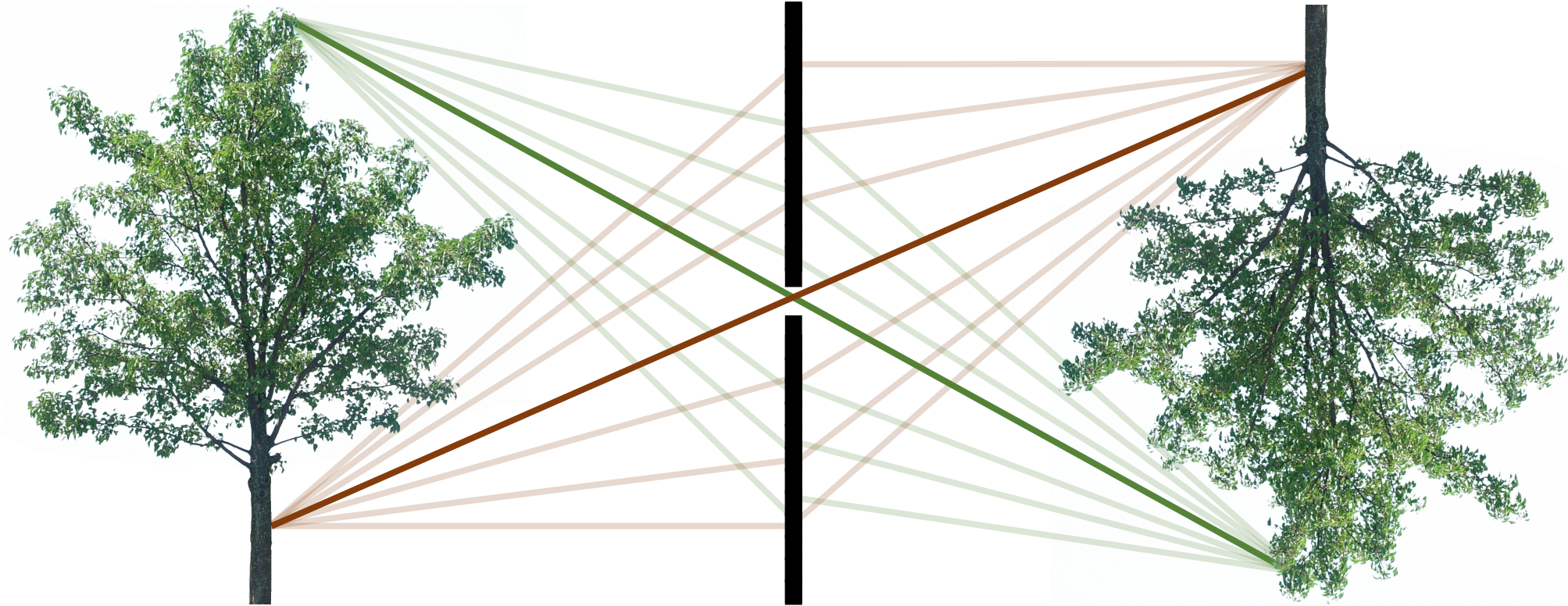
The lens camera



The pinhole camera

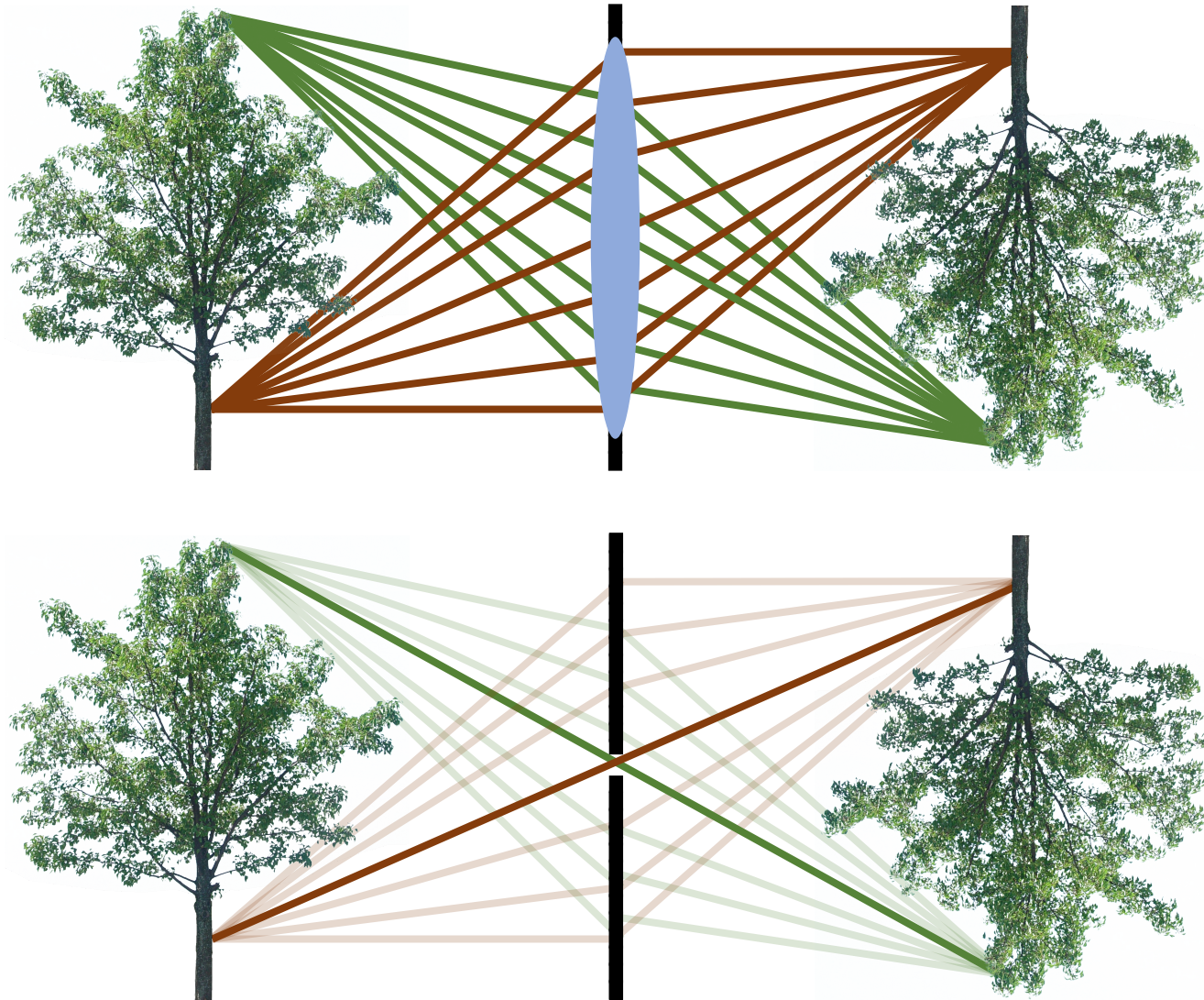


The pinhole camera



Central rays propagate in the same way for both models!

Describing both lens and pinhole cameras



We can derive properties and descriptions that hold for both camera models if:

- We consider only central rays.
- We assume that everything of interest in the scene is within the depth of field.

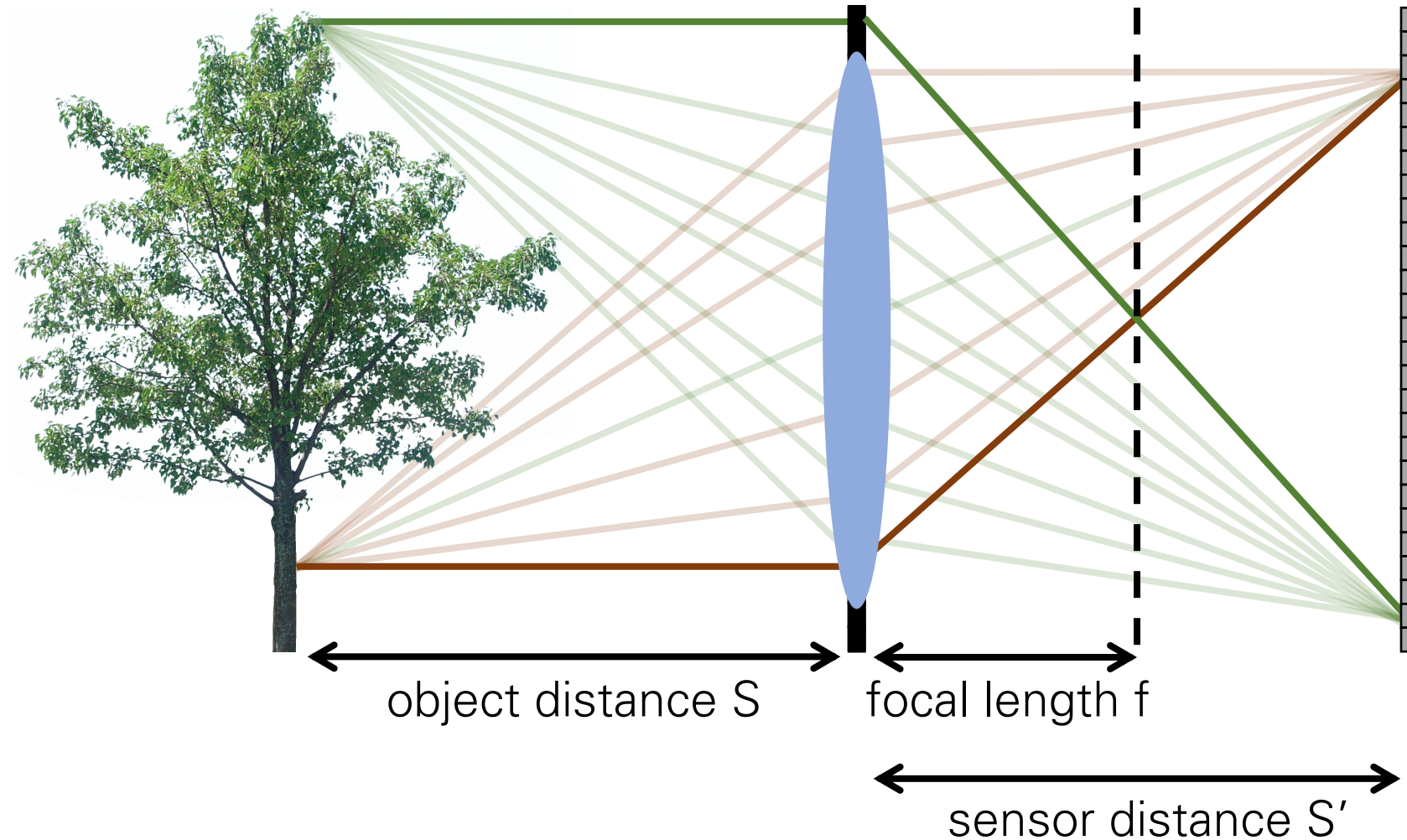
Important difference: focal length

In a pinhole camera, focal length is distance between aperture and sensor

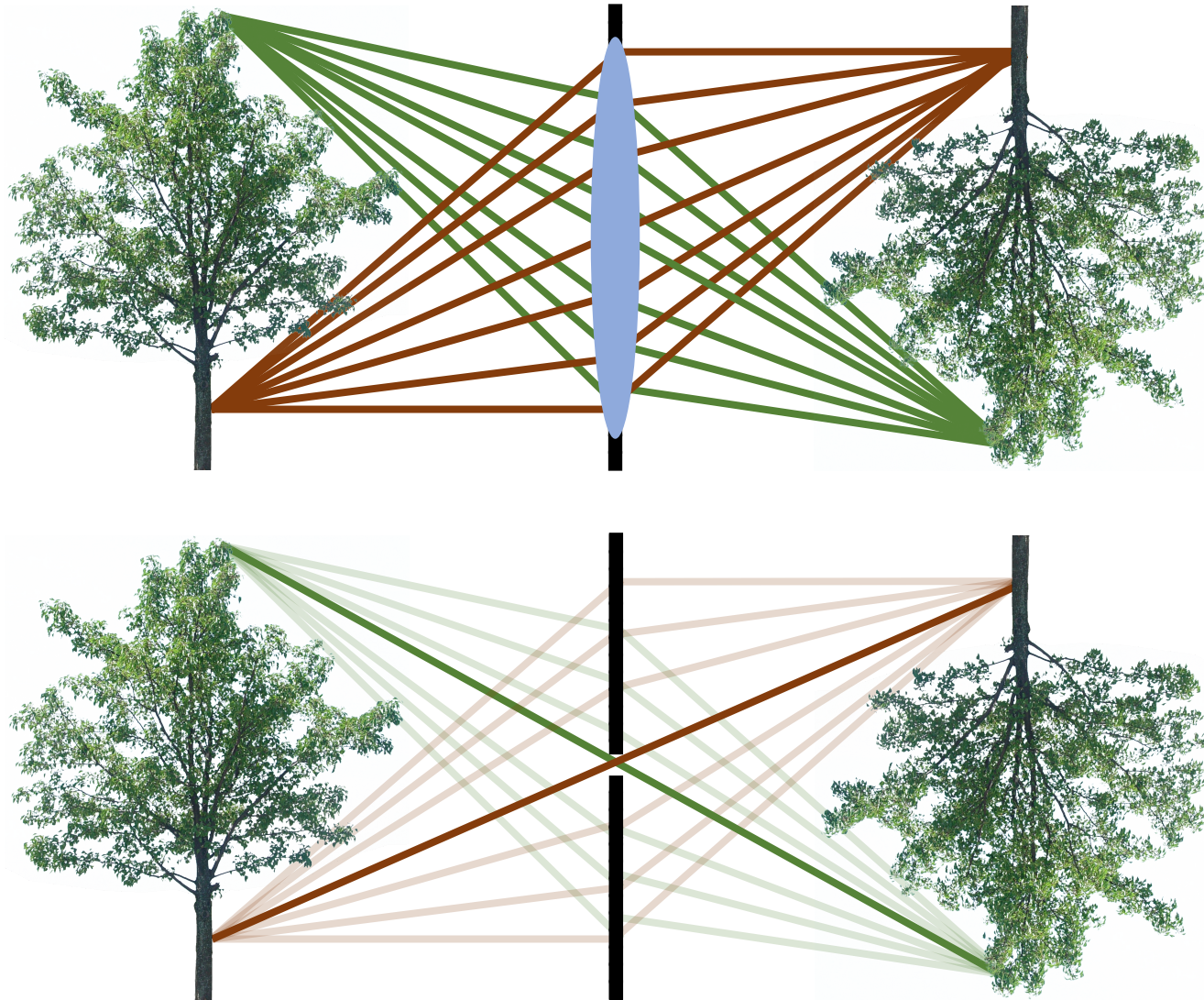


Important difference: focal length

In a lens camera, focal length is distance where parallel rays intersect



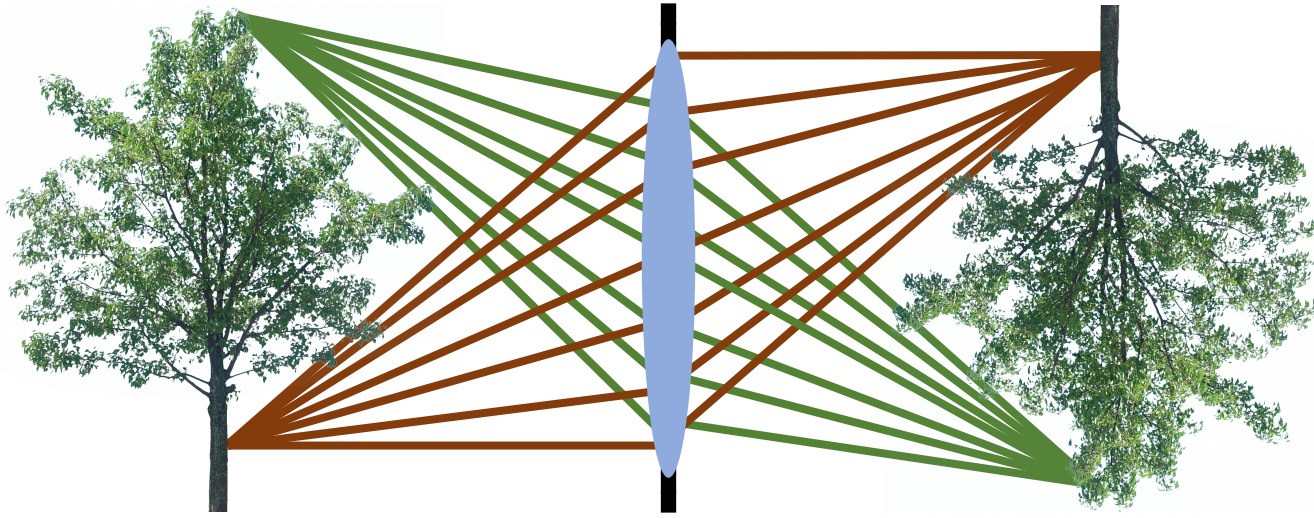
Describing both lens and pinhole cameras



We can derive properties and descriptions that hold for both camera models if:

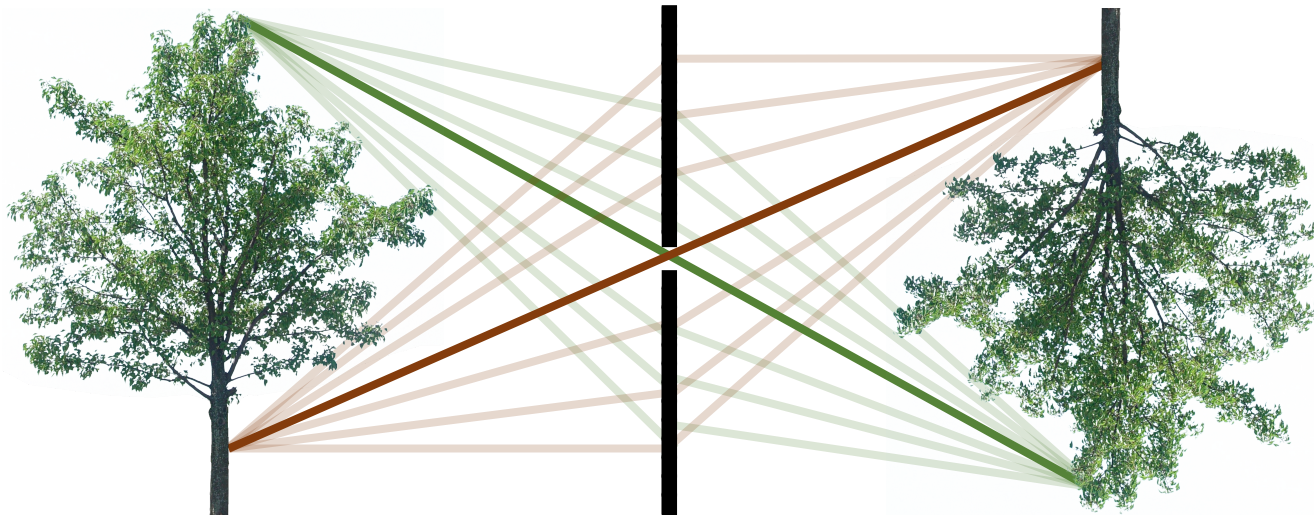
- We consider only central rays.
- We assume everything of interest in the scene is within the depth of field.
- We assume that the focus distance of the lens camera is equal to the focal length of the pinhole camera.

Effect of aperture size on lens and pinhole cameras



Doubling the aperture diameter:

- Increases light throughput by four times.
- Increases circle of confusion for out-of-focus plane by two times.
- Decreases depth of field by two times.



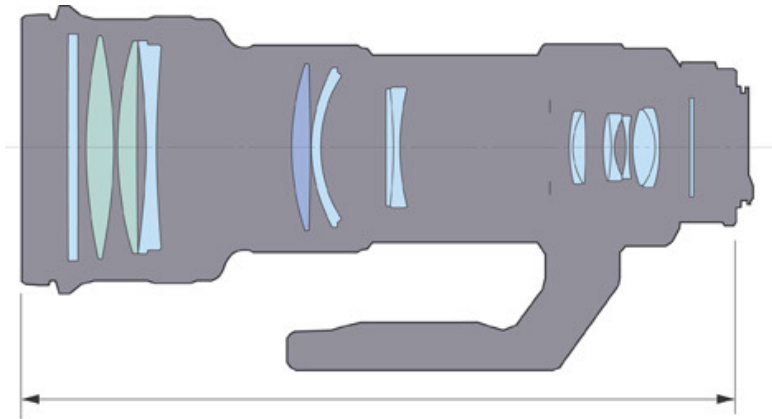
Doubling the aperture diameter:

- Increases light throughput by four times.
- Increases circle of confusion for all planes by two times.

Thin lenses are fiction!

Thin lenses are a fiction

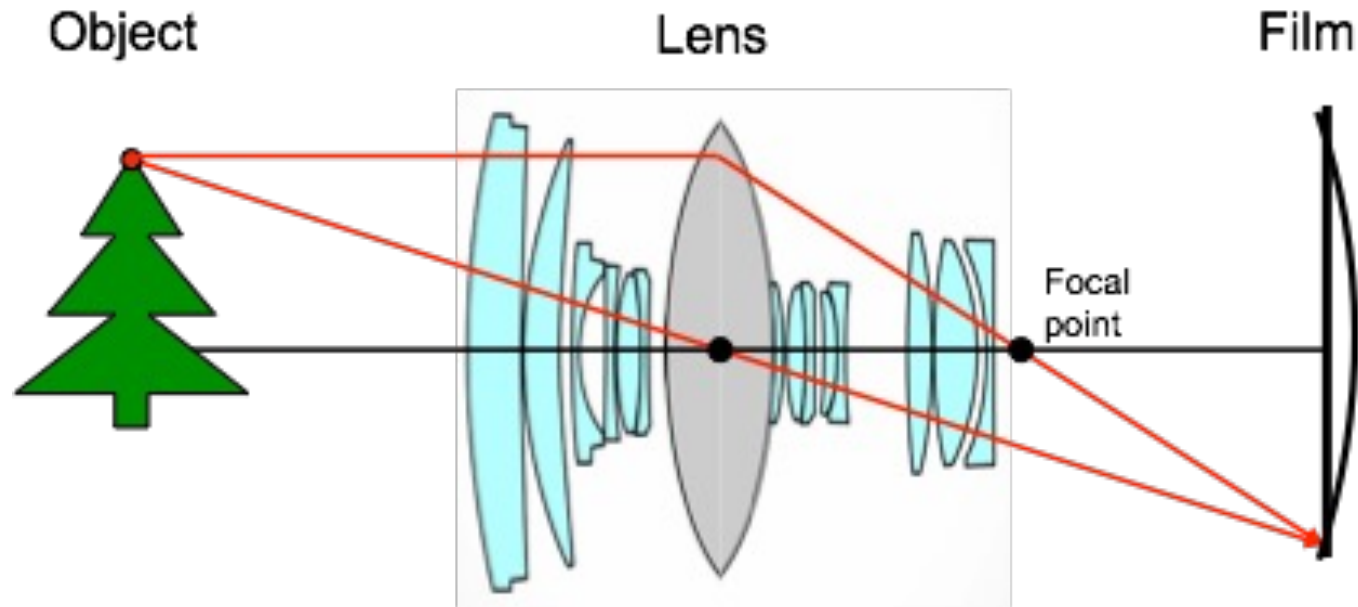
The thin lens model assumes that the lens has no thickness, but this is rarely true...



To make real lenses behave like ideal thin lenses, we have to use combinations of multiple lens elements (compound lenses).

Thin lenses are a fiction

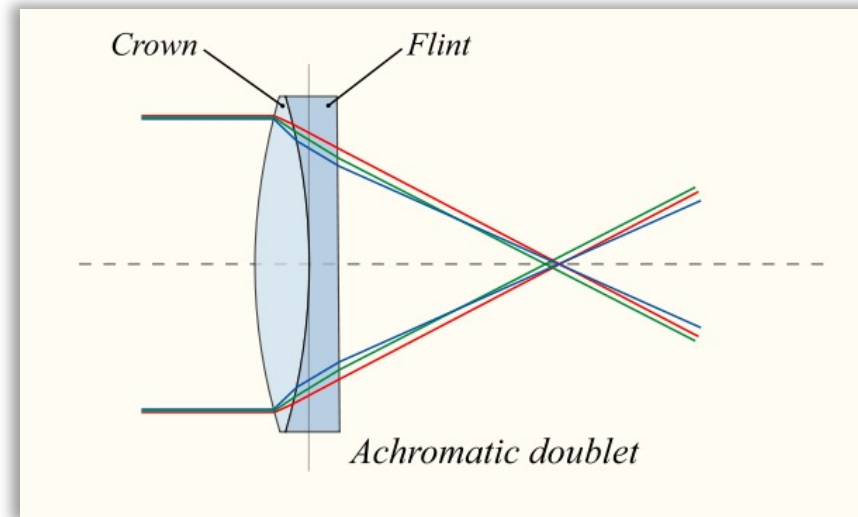
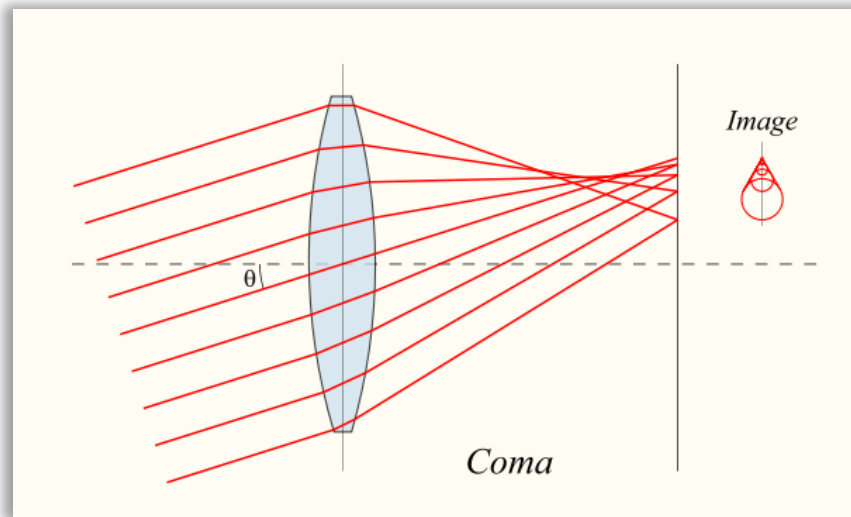
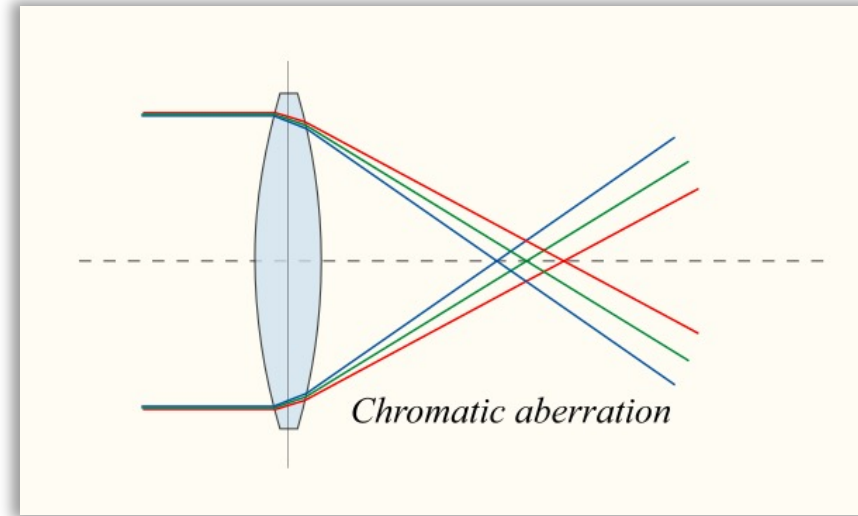
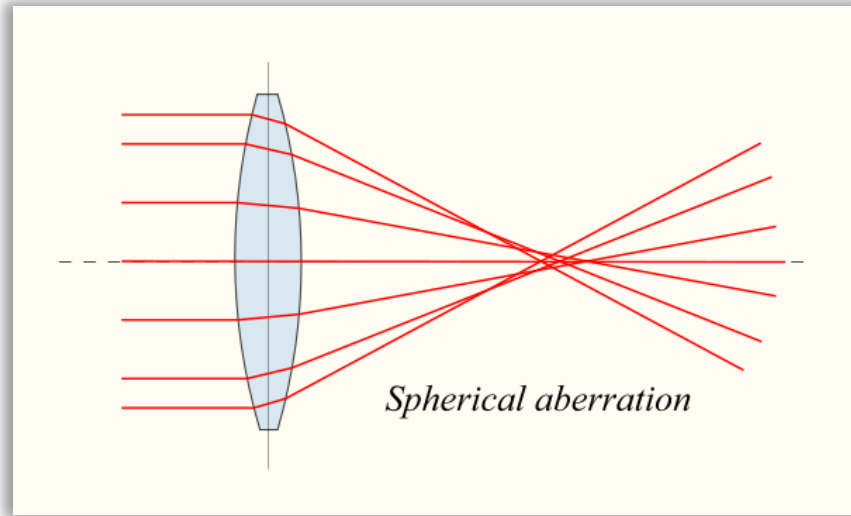
The thin lens model assumes that the lens has no thickness, but this is rarely true...



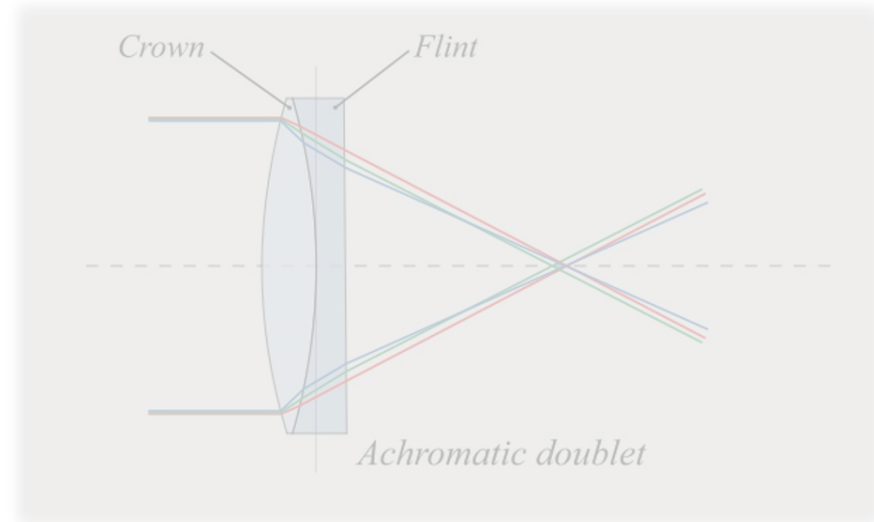
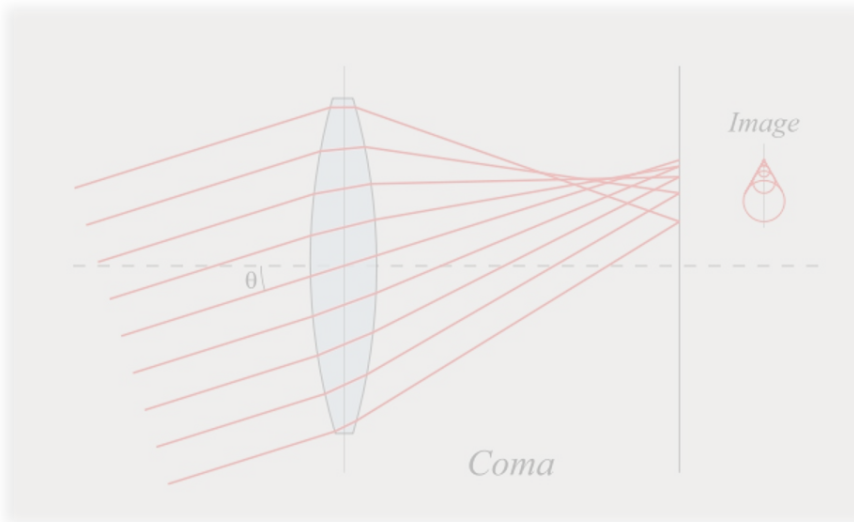
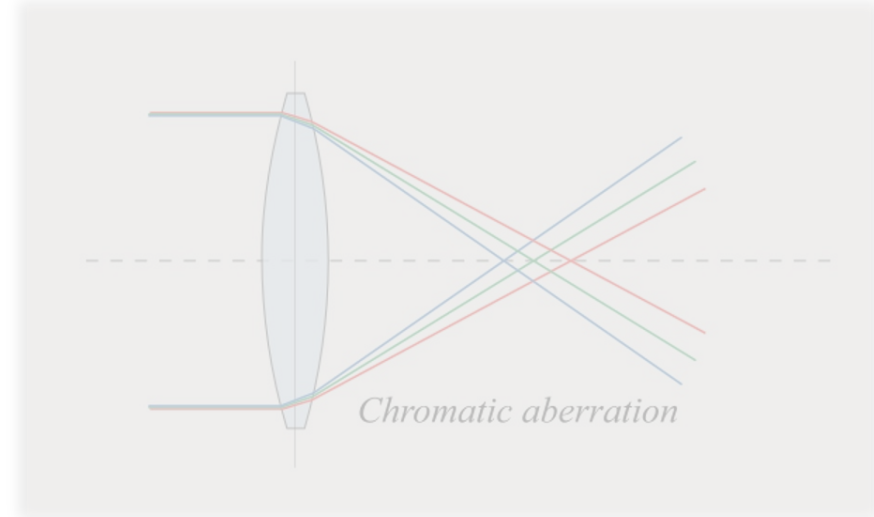
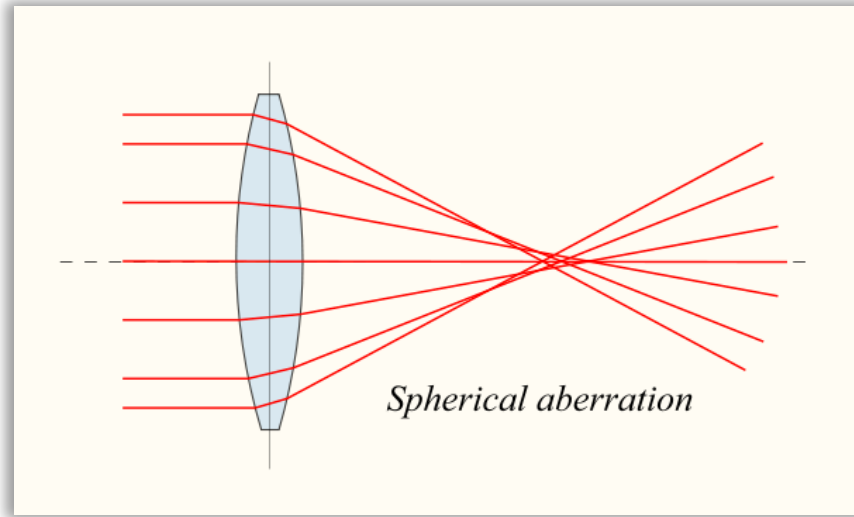
Even though we have multiple lenses, the entire optical system can be (paraxially) described using a single thin lens of some equivalent focal length and aperture number.

To make real lenses behave like ideal thin lenses, we have to use combinations of multiple lens elements (compound lenses).

Lenses - Aberrations

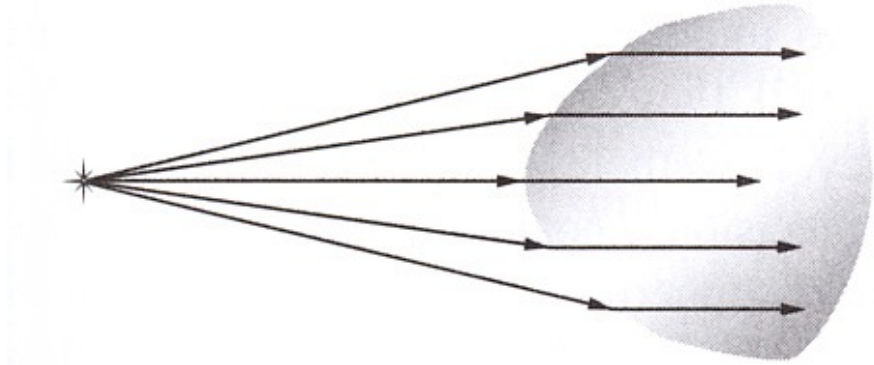


Lenses - Aberrations



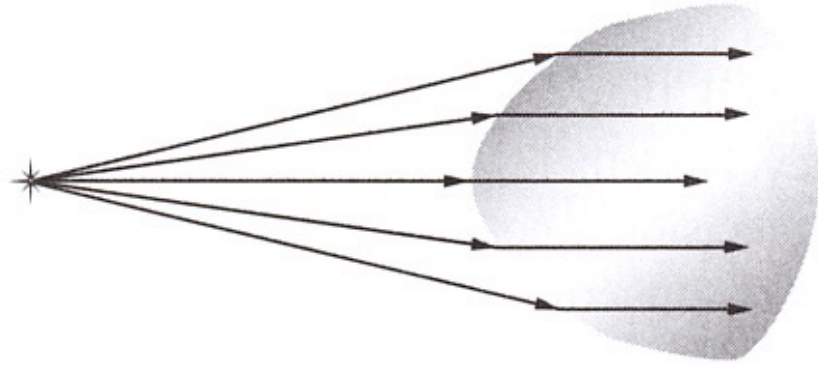
Refraction at interfaces of complicated shapes

What shape should an interface have to make parallel rays converge to a point?



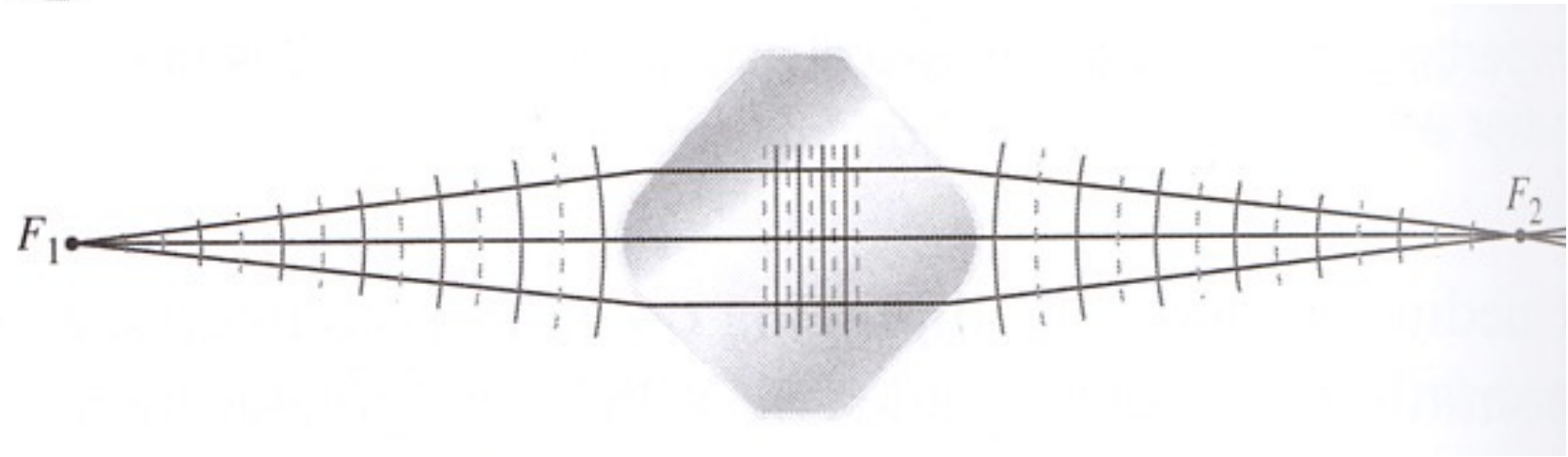
Refraction at interfaces of complicated shapes

What shape should an interface have to make parallel rays converge to a point?



Single hyperbolic interface:
point to parallel rays

Double hyperbolic interface:
point to point rays

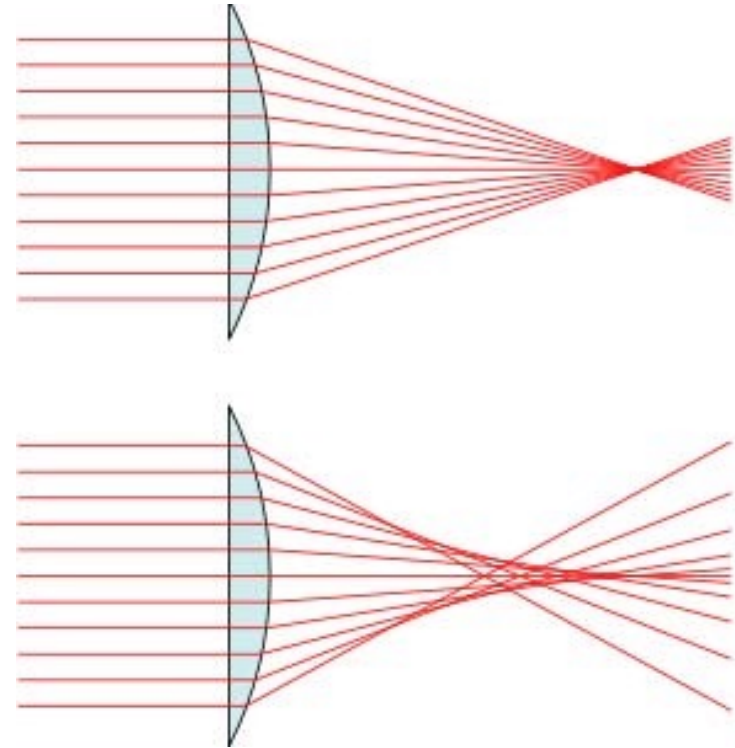
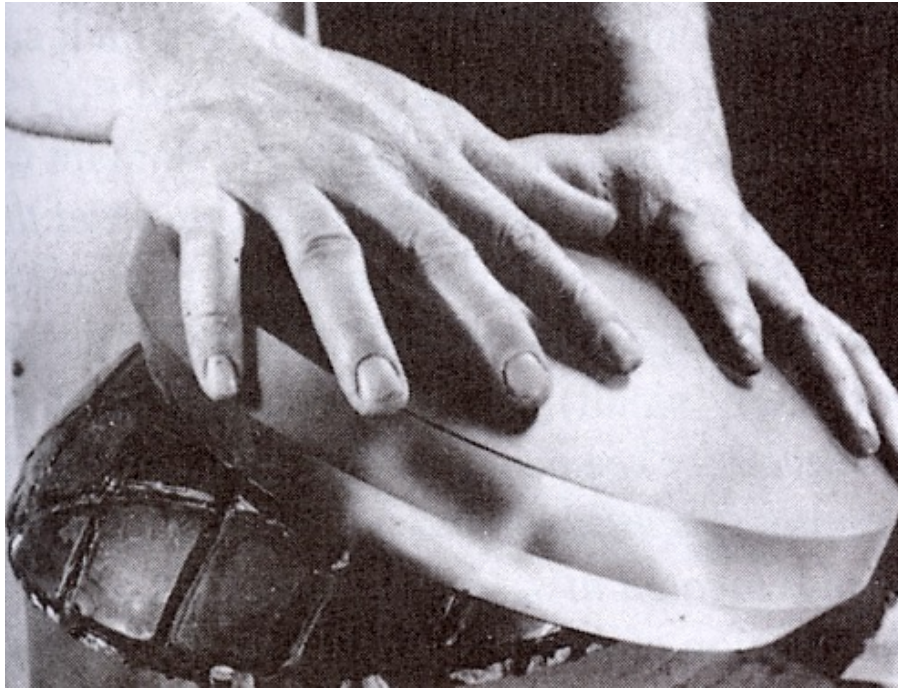


What shape should an interface have to make parallel rays converge to a point?

Spherical lenses

In practice, lenses are often made to have spherical interfaces for ease of fabrication.

- Two roughly fitting curved surfaces ground together will eventually become spherical.



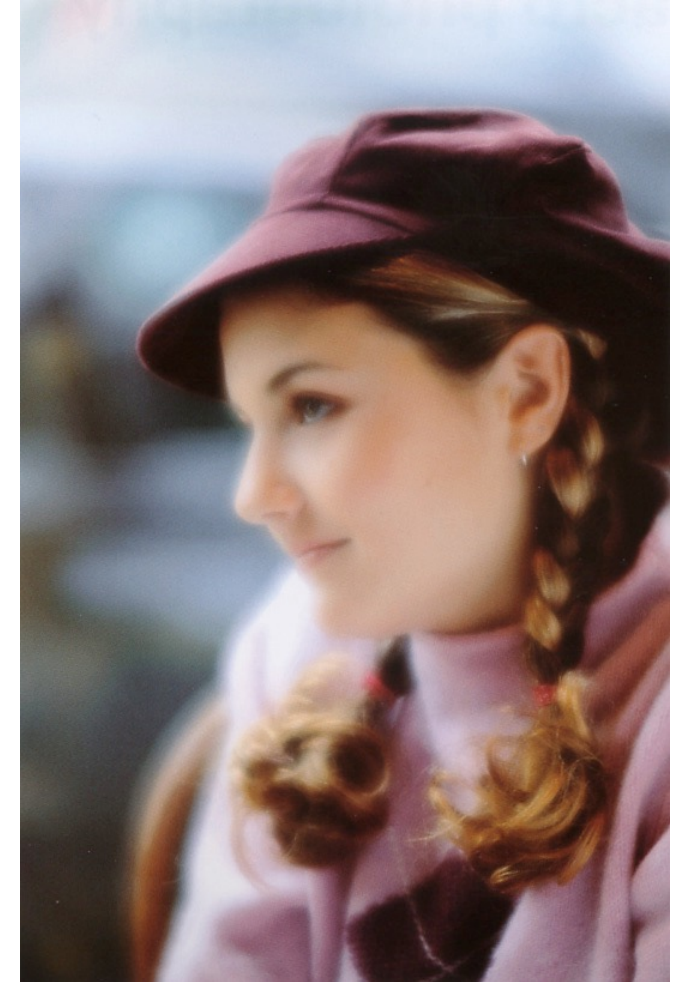
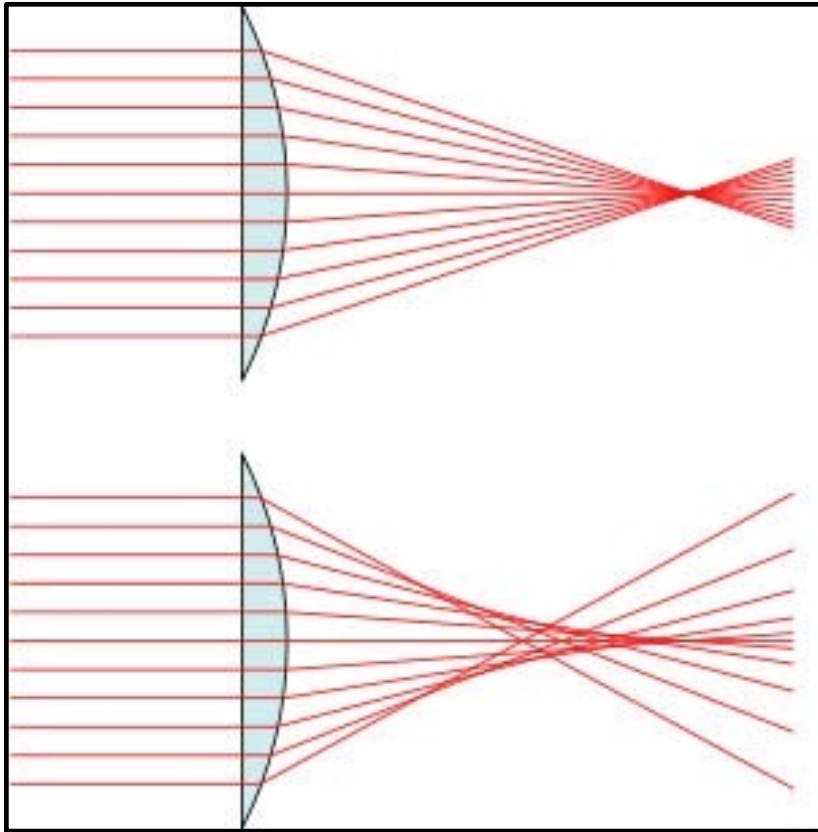
Spherical lenses don't bring parallel rays to a point.

- This is called spherical aberration.
- Approximately axial (i.e., paraxial) rays behave better.

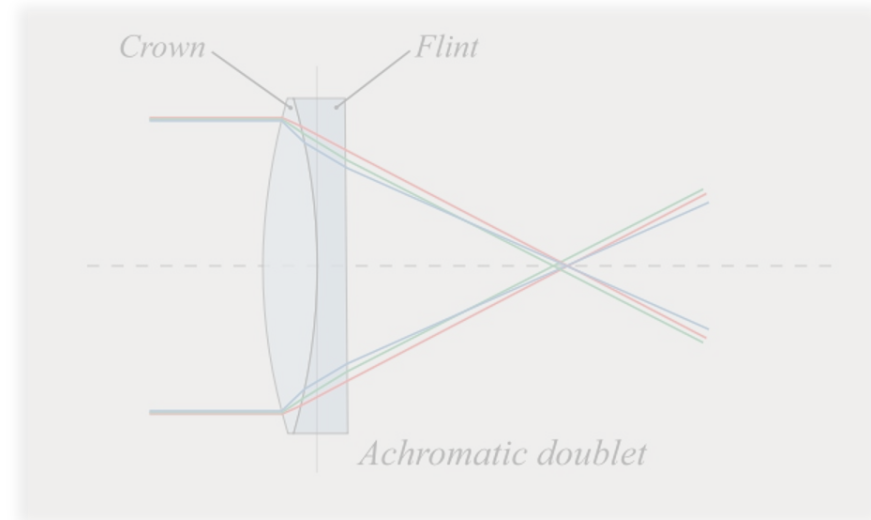
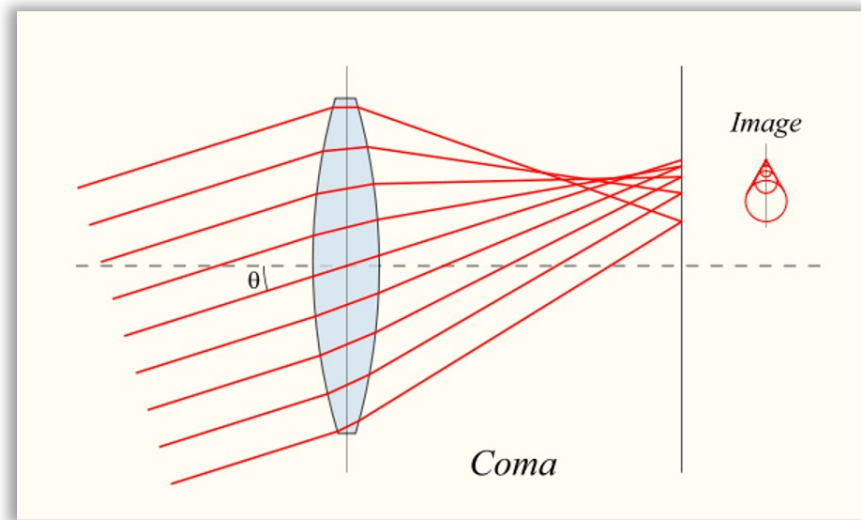
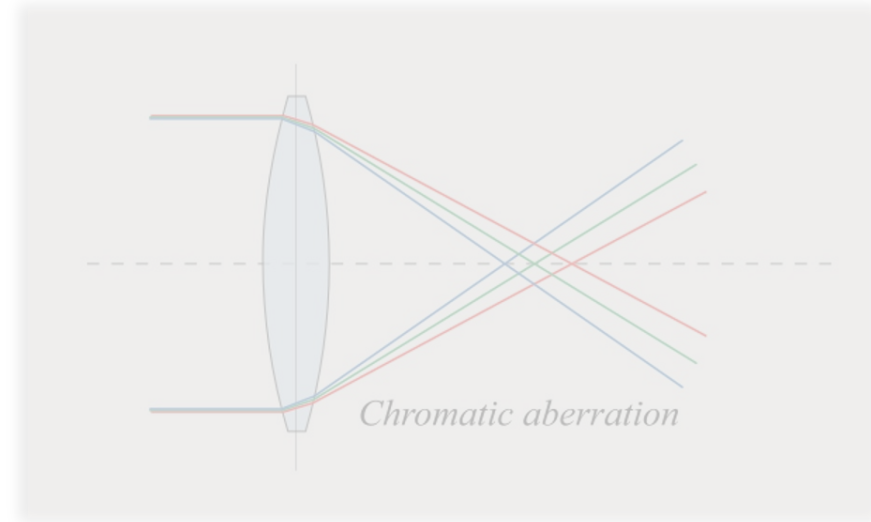
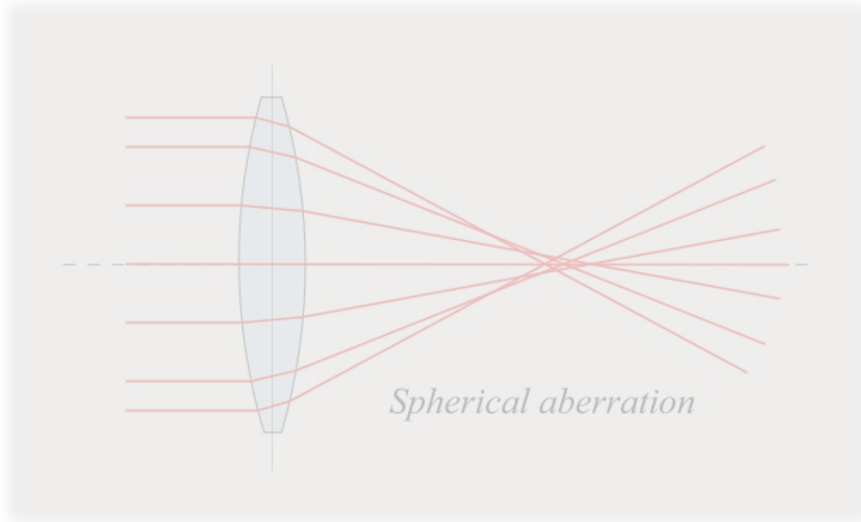
Aberrations

Deviations from ideal thin lens behavior (e.g., imperfect focus).

- Example: spherical aberration.



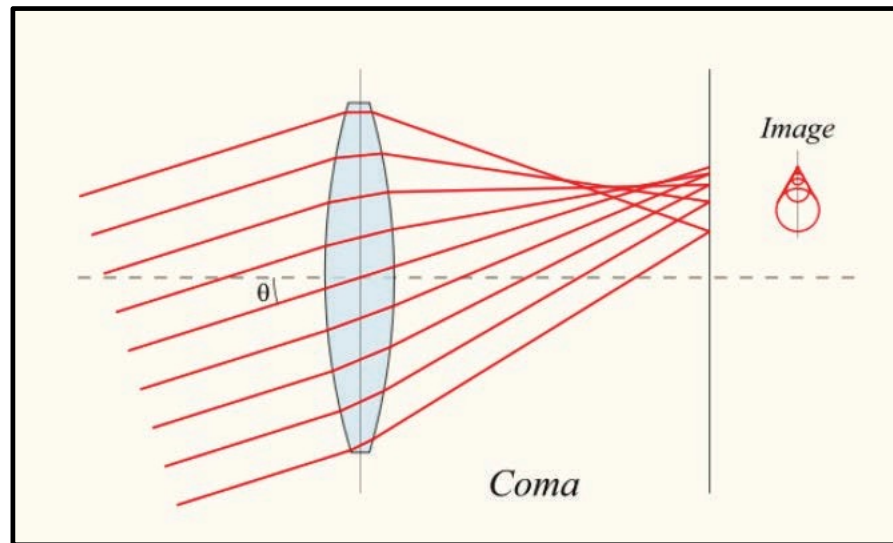
Lenses - Aberrations



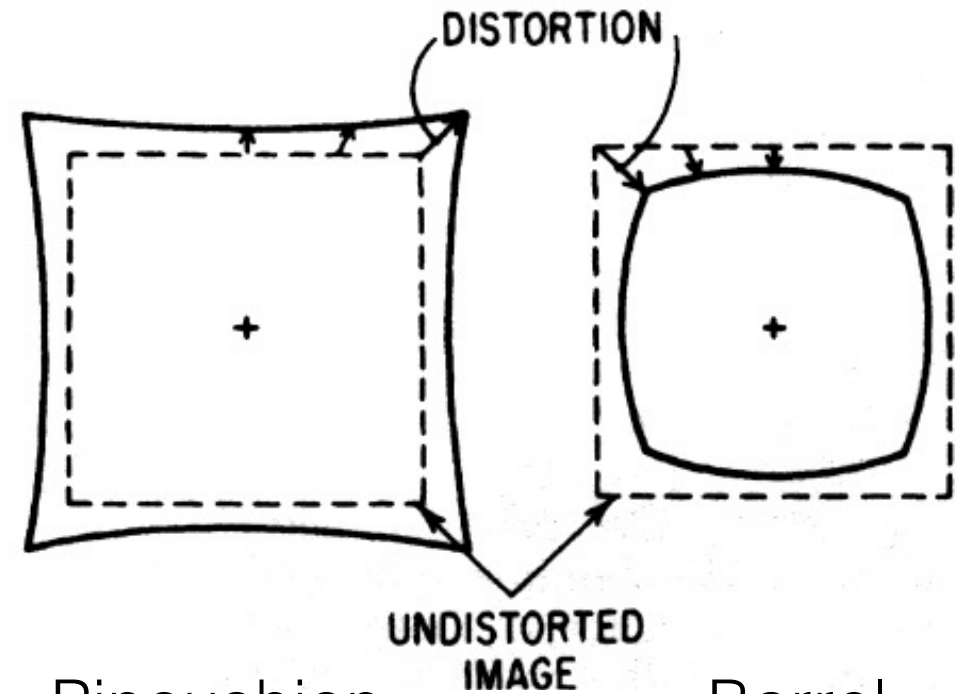
Oblique aberrations

These appear only as we move further from the center of the field of view.

- Contrast with spherical and chromatic, which appear everywhere.
- Many other examples (astigmatism, field curvature, etc.).



Coma



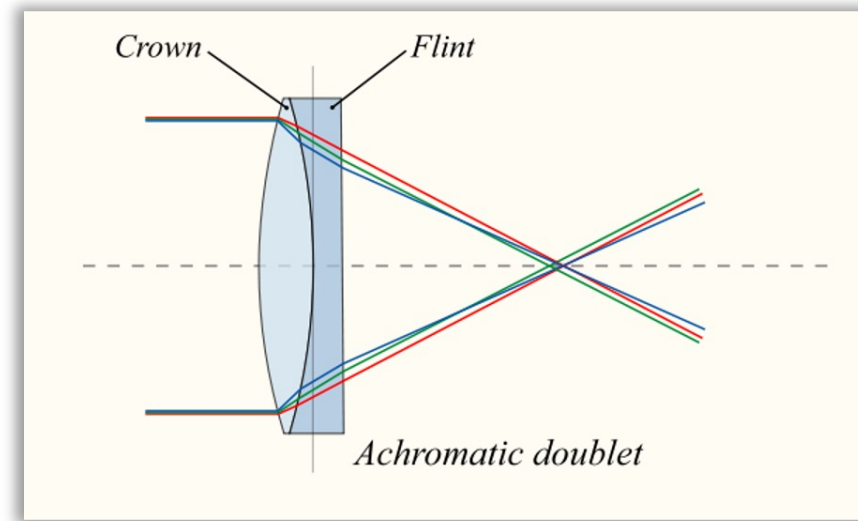
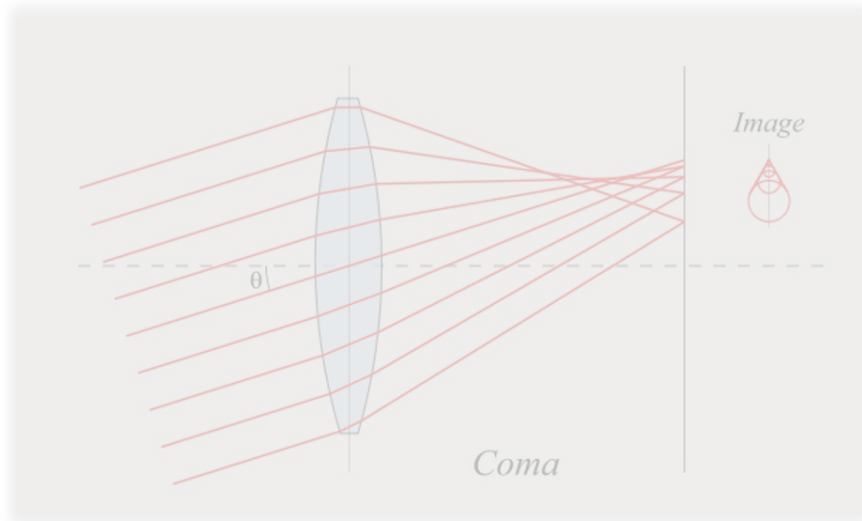
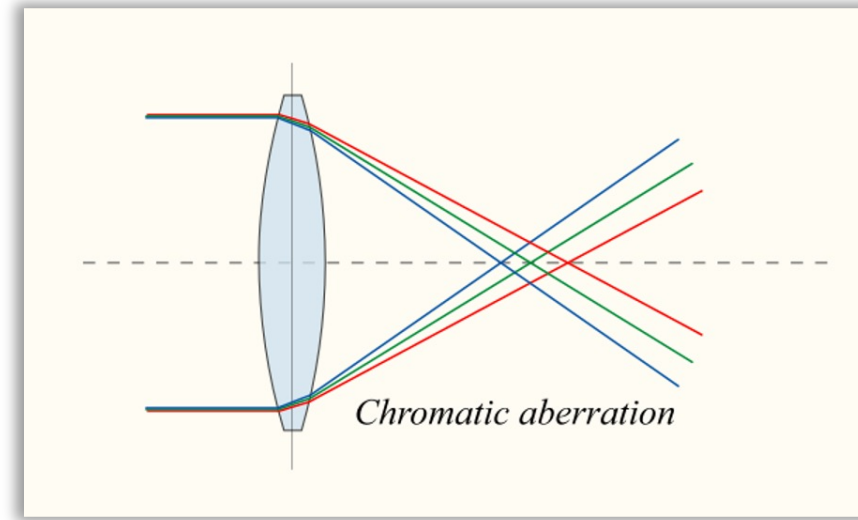
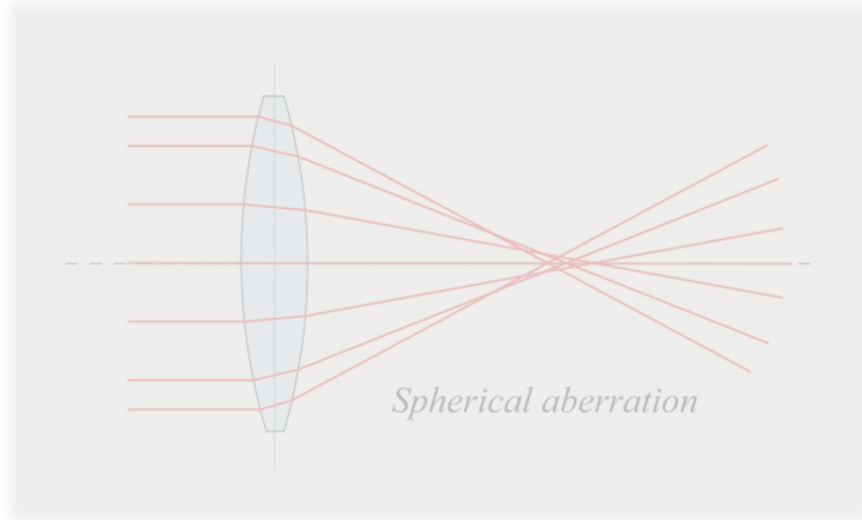
Pincushion

Barrel

Distortion example



Lenses - Aberrations

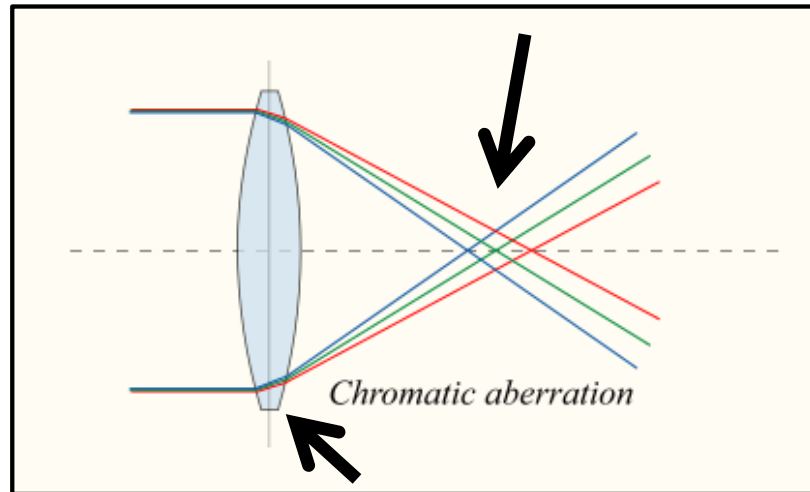


Aberrations

Deviations from ideal thin lens behavior (e.g., imperfect focus).

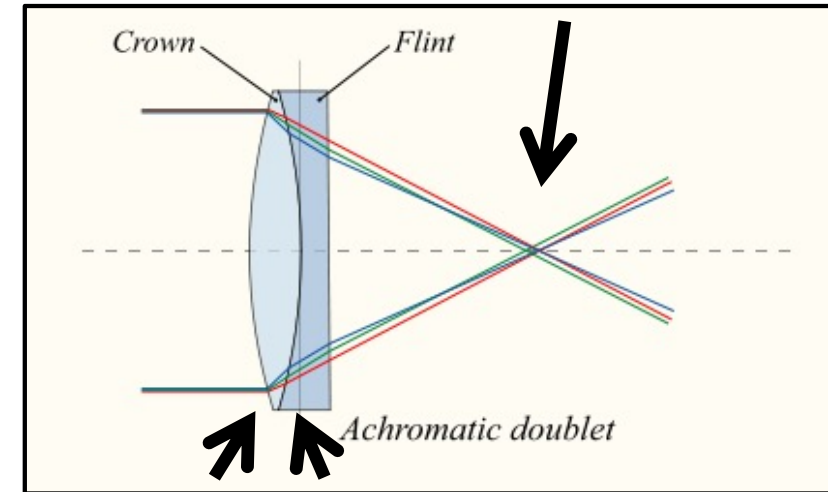
- Example: chromatic aberration.

focal length shifts with wavelength



glass has dispersion (refractive index changes with wavelength)

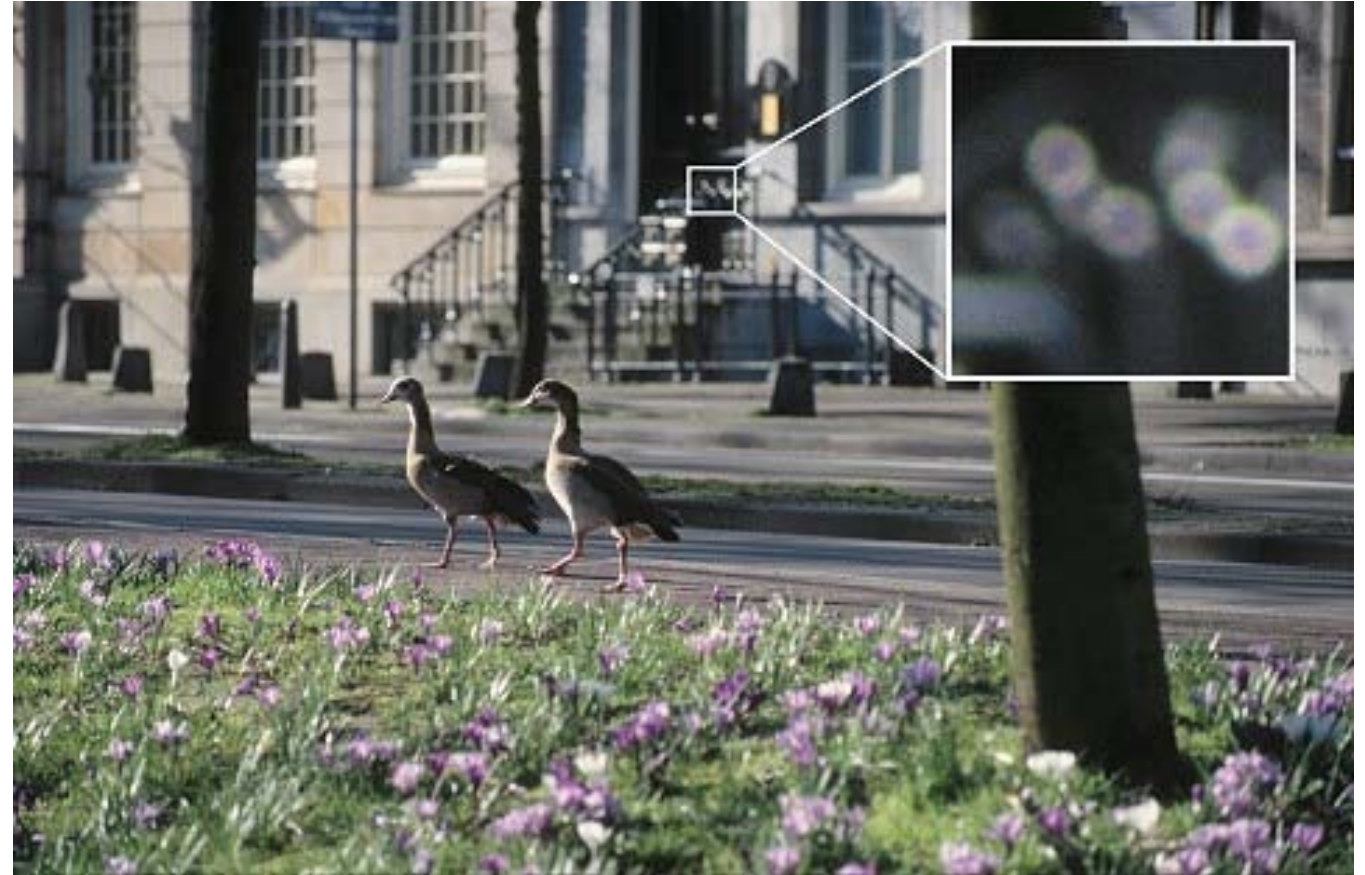
one lens cancels out dispersion of other



glasses of different refractive index

Using a doublet (two-element compound lens), we can reduce chromatic aberration.

Chromatic aberration examples

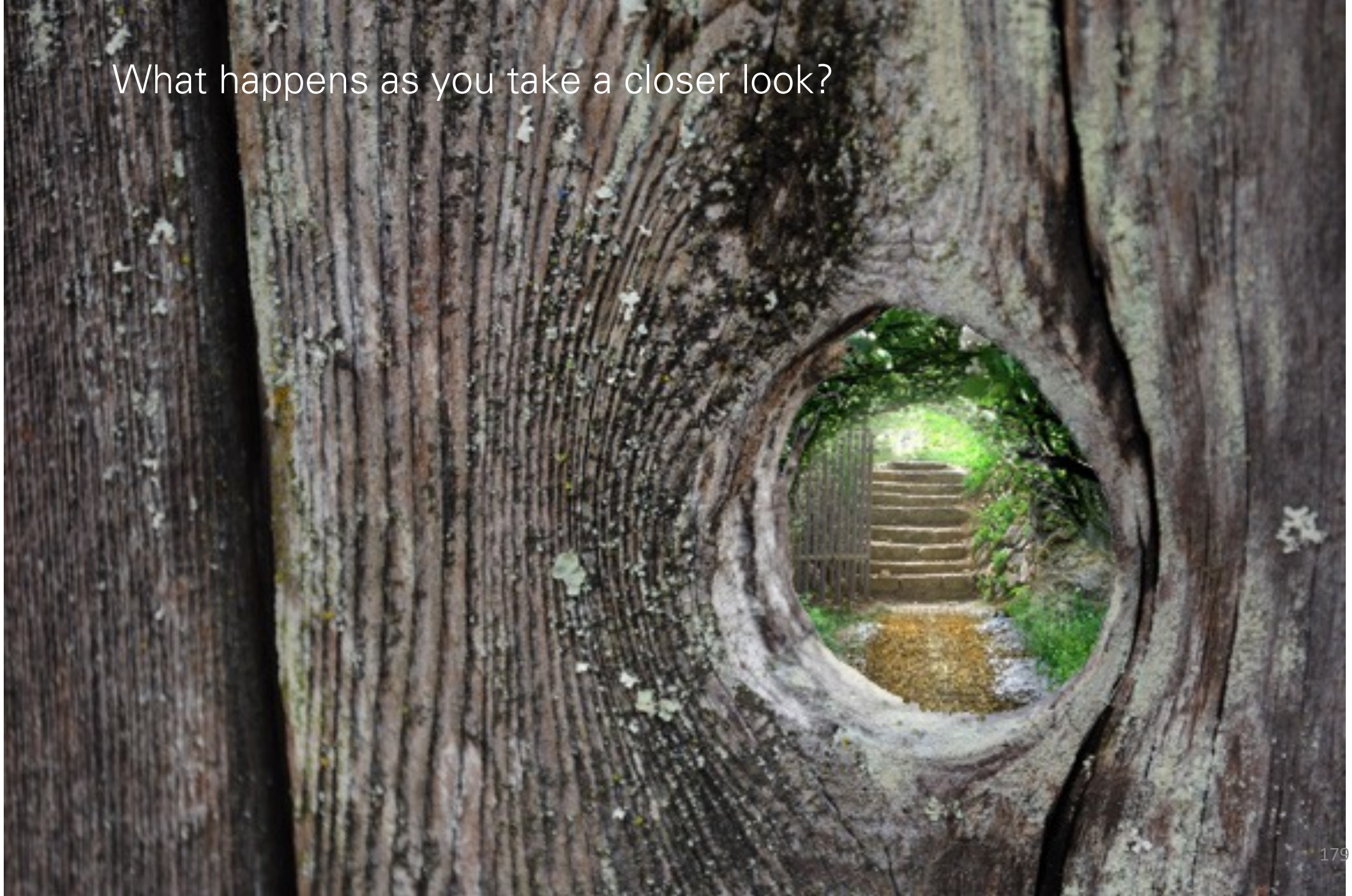


Field of view

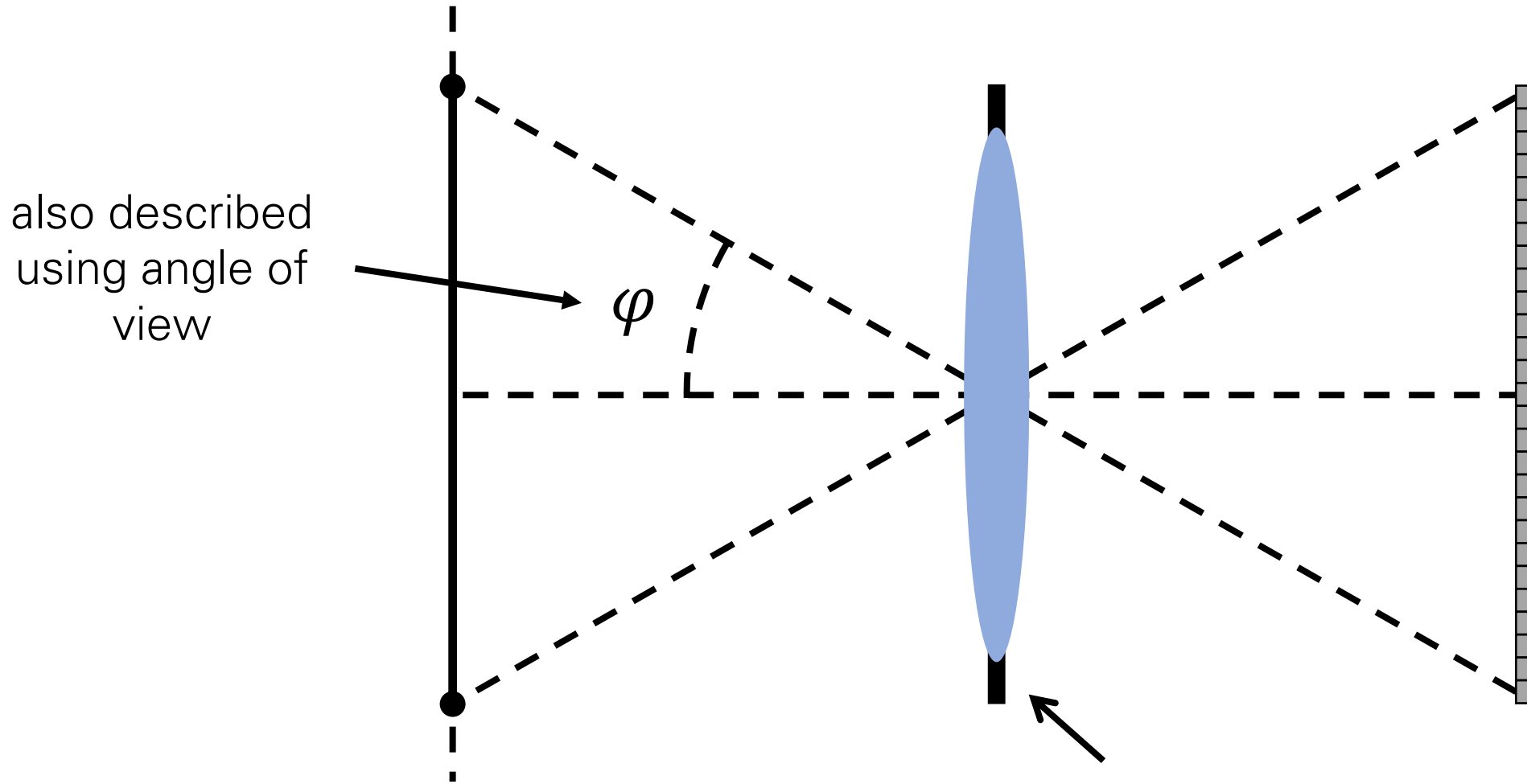
Field of view

- Determines how much of scene is in frame
- Traditionally specified by focal length
 - but interpreting this number requires considering the “format,” or size of the film or sensor
- **After decades of 35mm, that format is stuck in our heads**
 - fields of view are usually discussed using the numbers that would be written on a lens for the 35mm format
- **Changing FOV while keeping the camera fixed**
 - strictly “crops” the image: relationships between objects are fixed corresponds to turning the zoom control on a modern camera

What happens as you take a closer look?

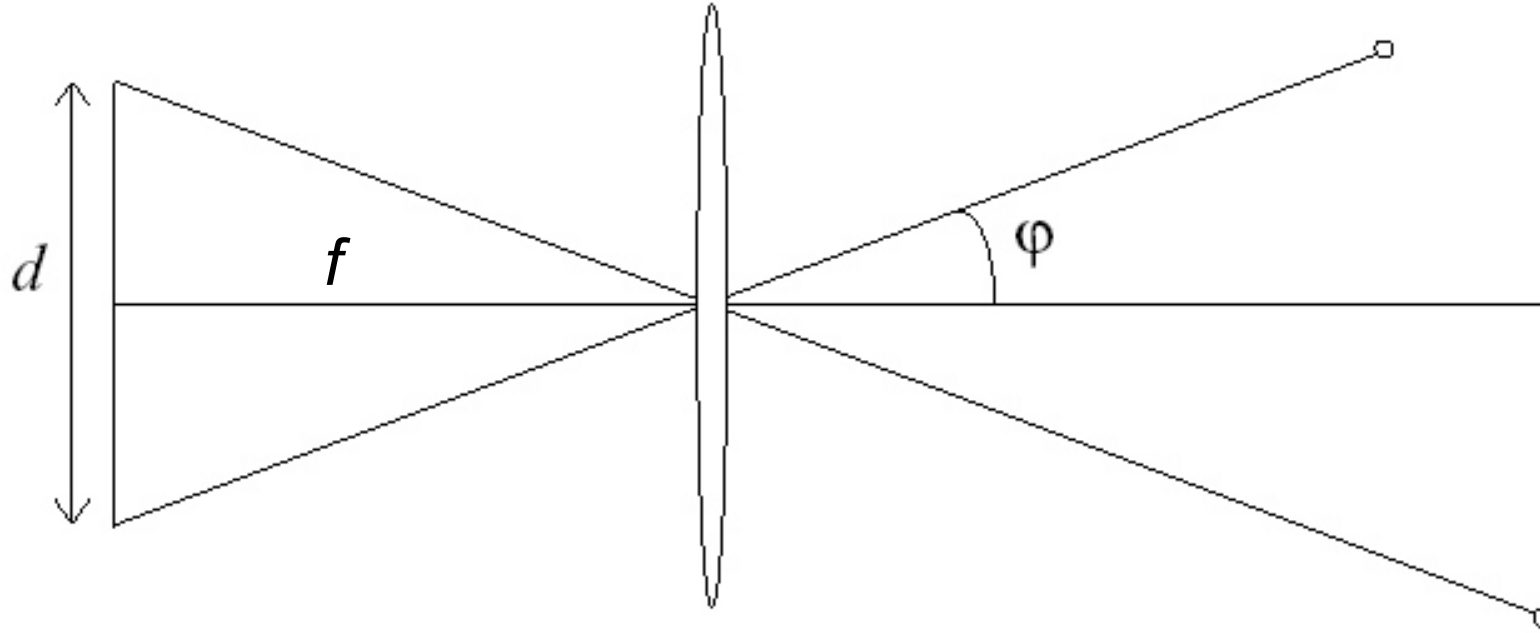


Field of view



Note: here I drew a lens, but I could have just as well drawn a pinhole

Field of view depends of focal length



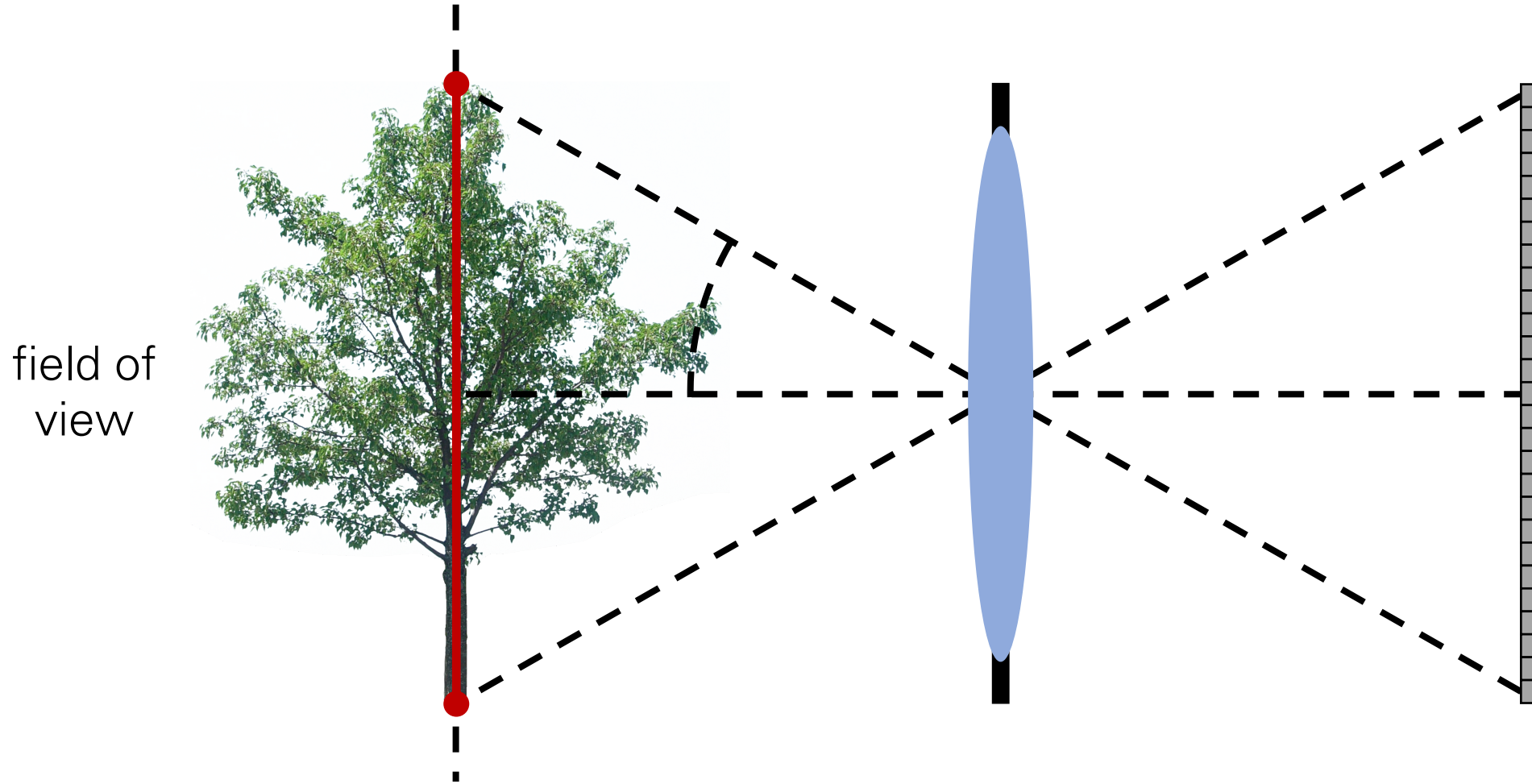
Size of field of view governed by size of the camera retina:

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Smaller field of view = larger focal length

Field of view

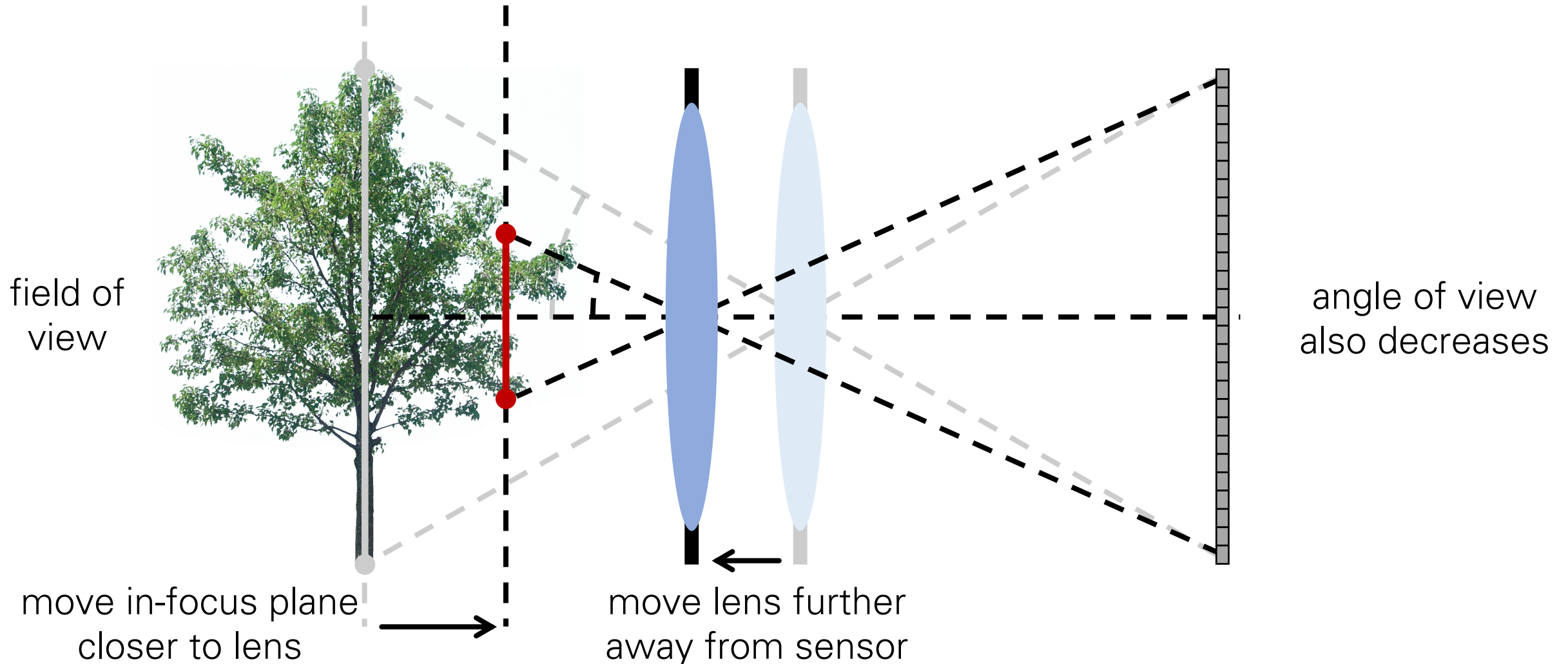
The part of the in-focus plane that gets mapped on the sensor.



- What happens to field of view as we focus closer?

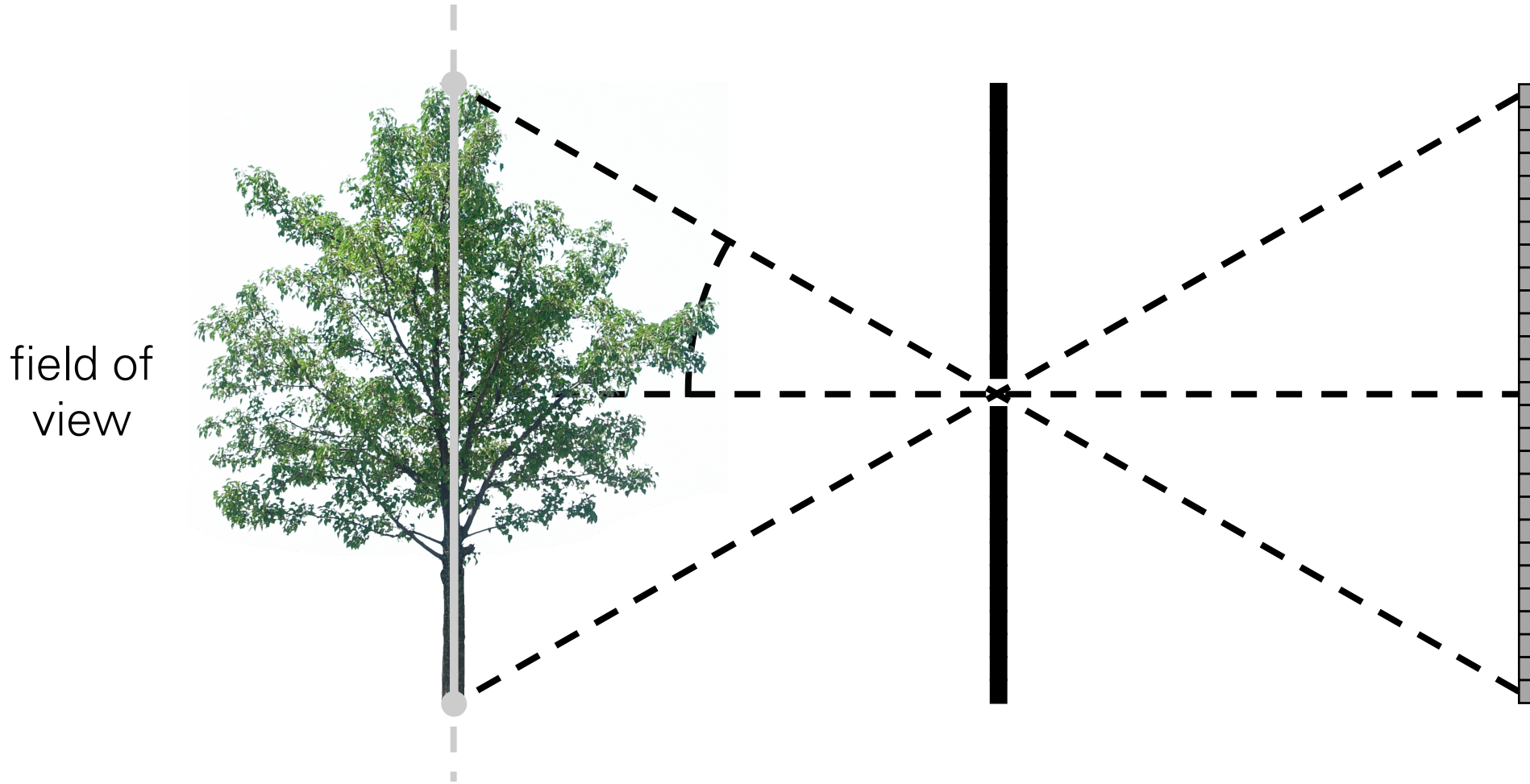
Field of view

The part of the in-focus plane that gets mapped on the sensor.



- What happens to field of view as we focus closer? → It becomes smaller.

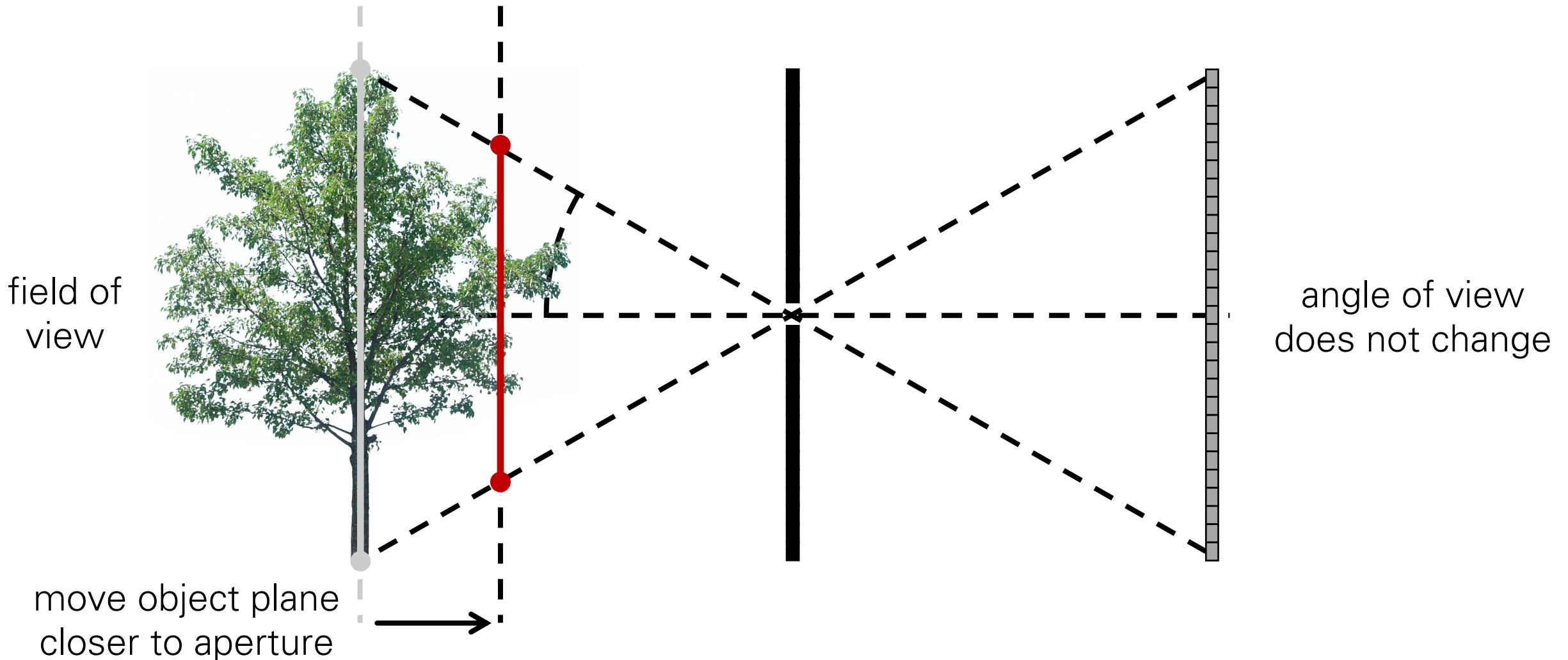
Comparison with pinhole camera



- What happens to field of view as we move closer?

Comparison with pinhole camera

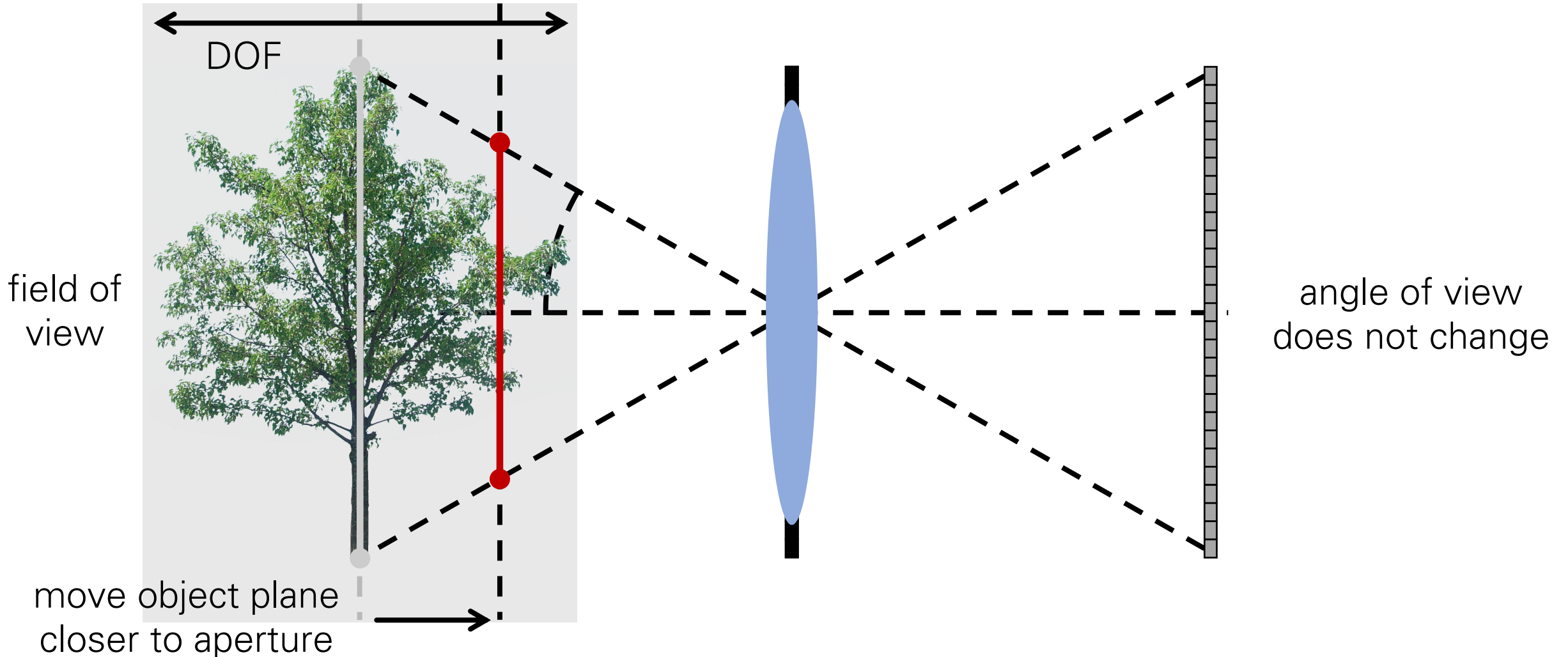
No need to refocus: we can move object closer without changing aperture-sensor distance.



- What happens to field of view as we move closer? → It becomes smaller, but amount differs.

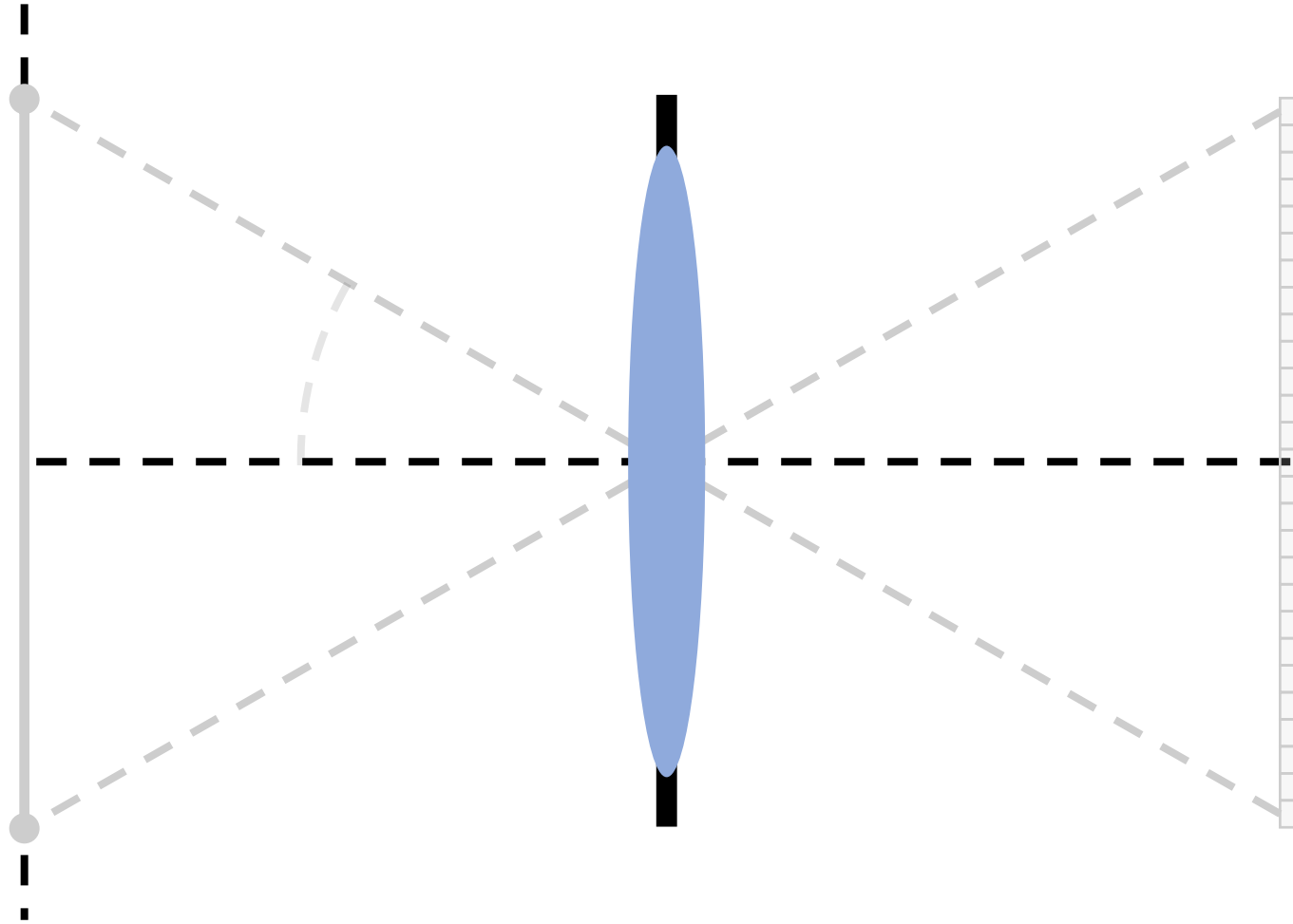
Comparison with pinhole camera

No need to refocus: we can move object closer without changing aperture-sensor distance.



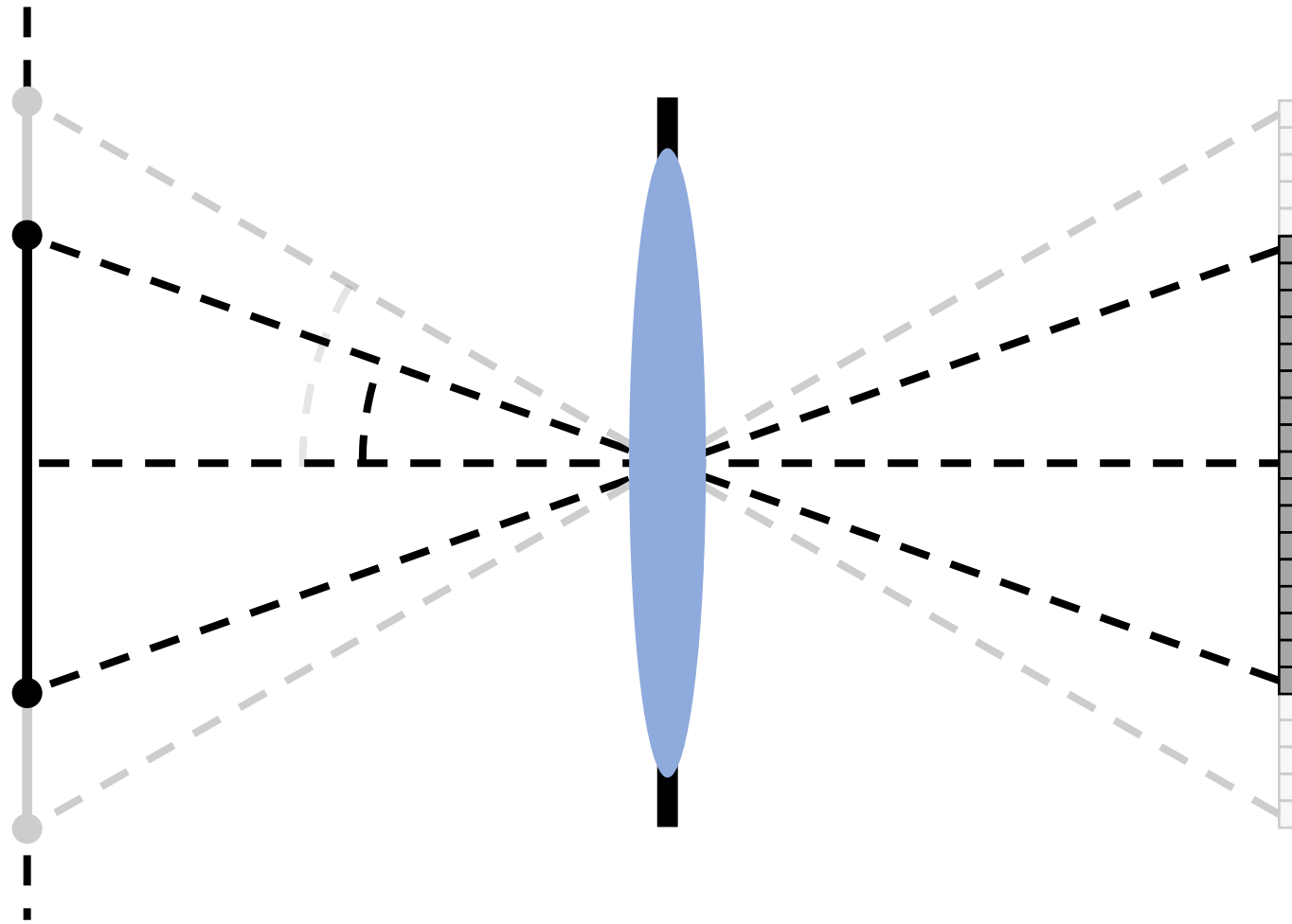
This can be done with a lens only if depth of field is large enough. Then the two behave the same.

Field of view also depends on sensor size



- What happens to field of view when we reduce sensor size?

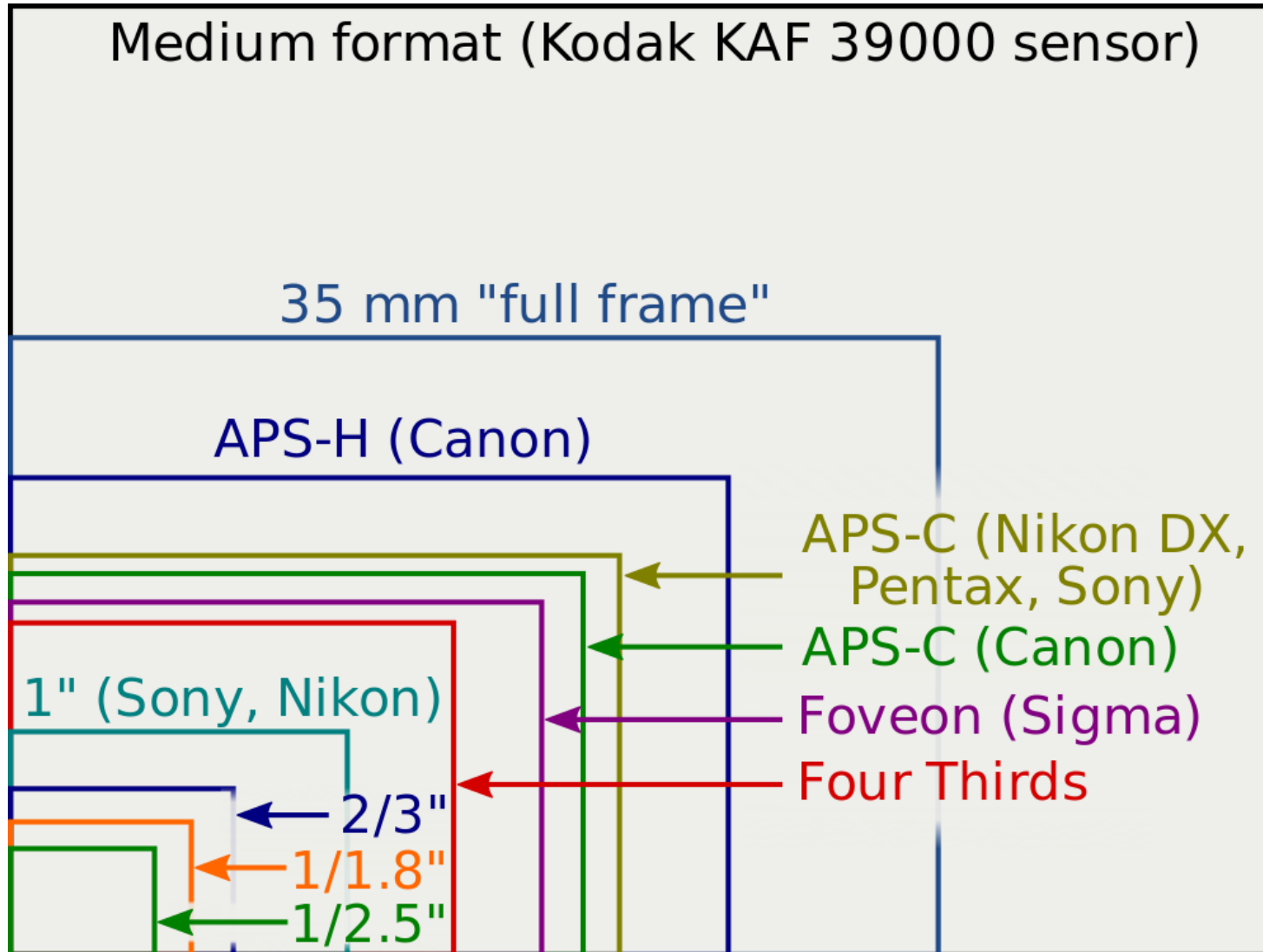
Field of view also depends on sensor size



Lens and pinhole cameras behave the same in this case.

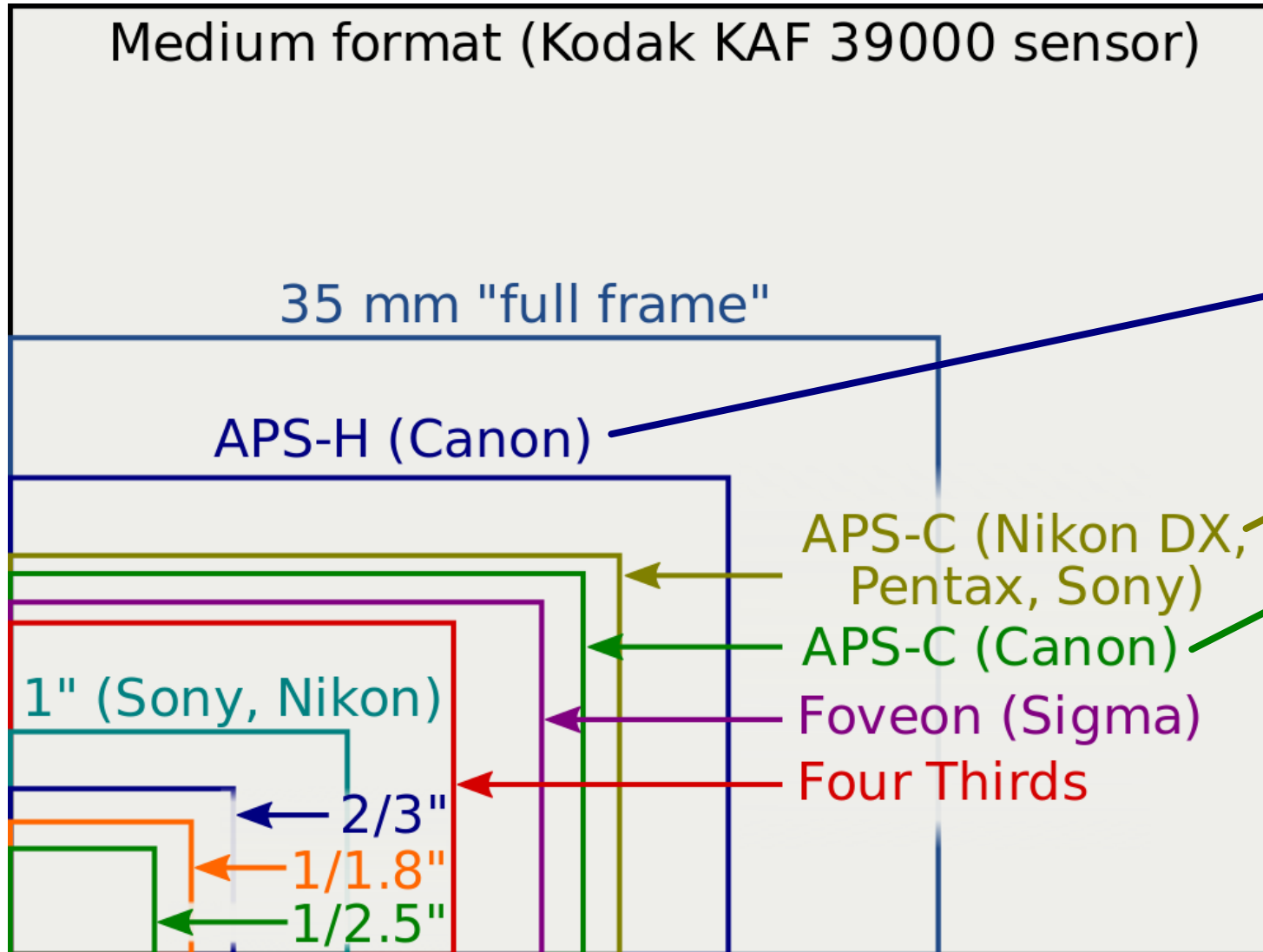
- What happens to field of view when we reduce sensor size? → It decreases.

Field of view also depends on sensor size



- "Full frame" corresponds to standard film size.
- Digital sensors come in smaller formats due to manufacturing limitations (now mostly overcome).
- Lenses are often described in terms of field of view on film instead of focal length.
- These descriptions are invalid when not using full-frame sensor.

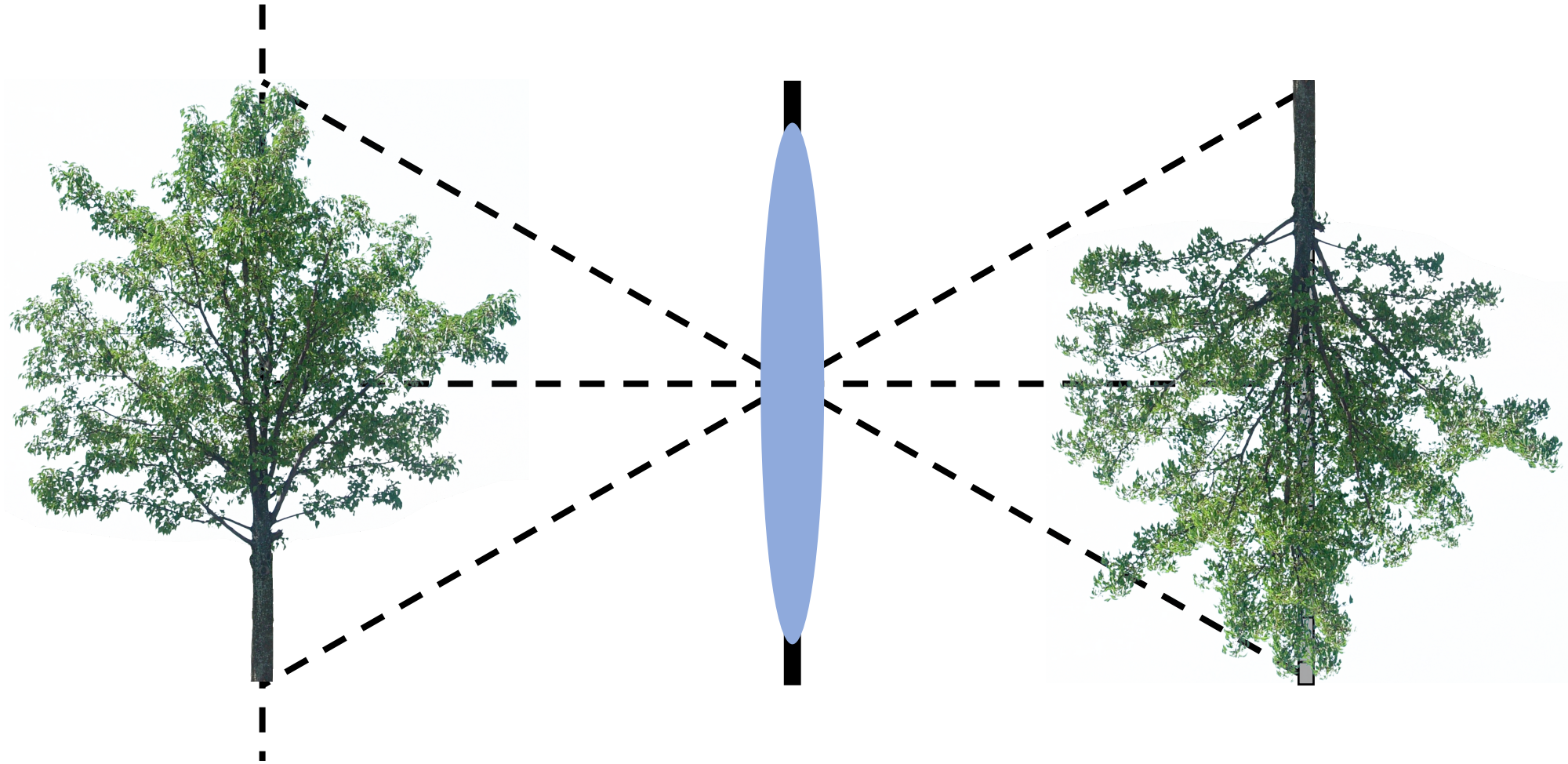
Crop factor



How much field of view is cropped when using a sensor smaller than full frame.

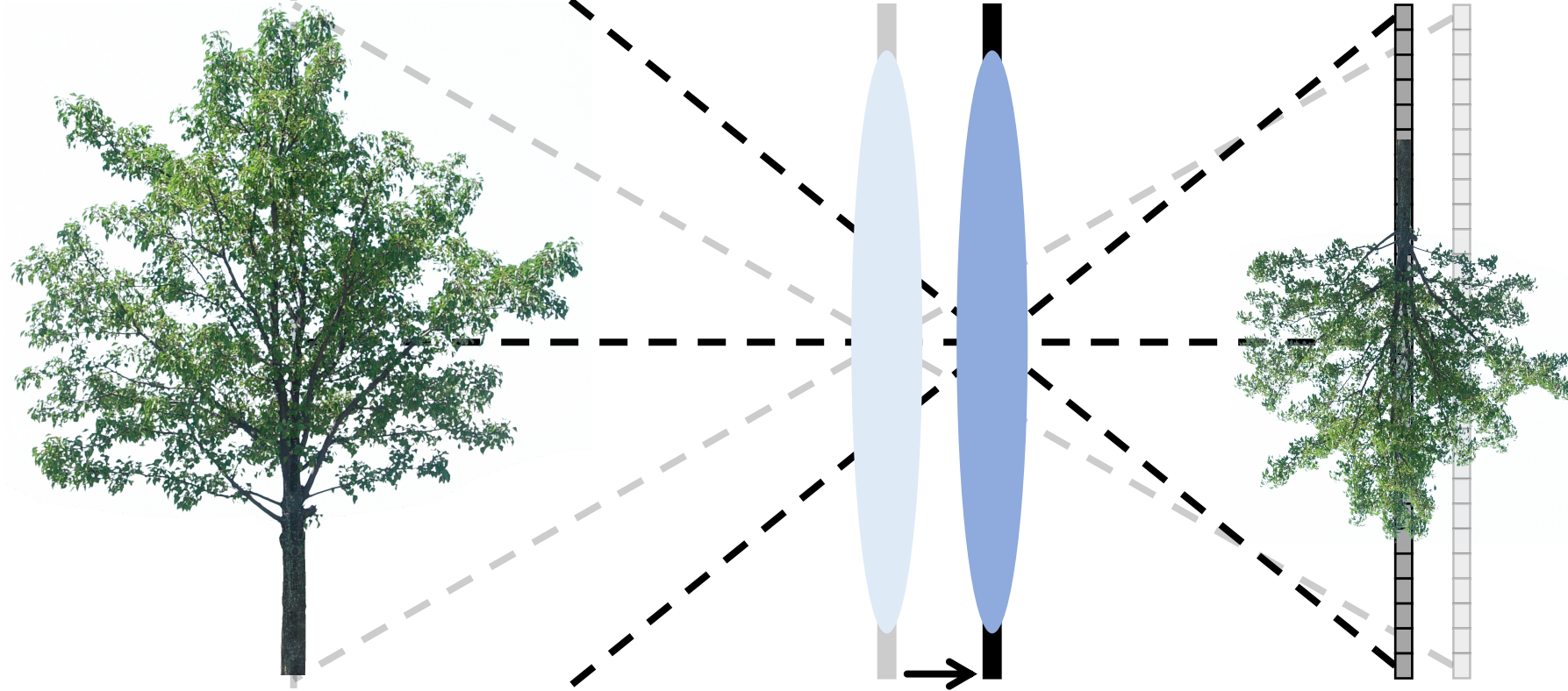
Magnification and perspective

Magnification depends on depth



- What happens to magnification as we focus further away?

Magnification depends on depth



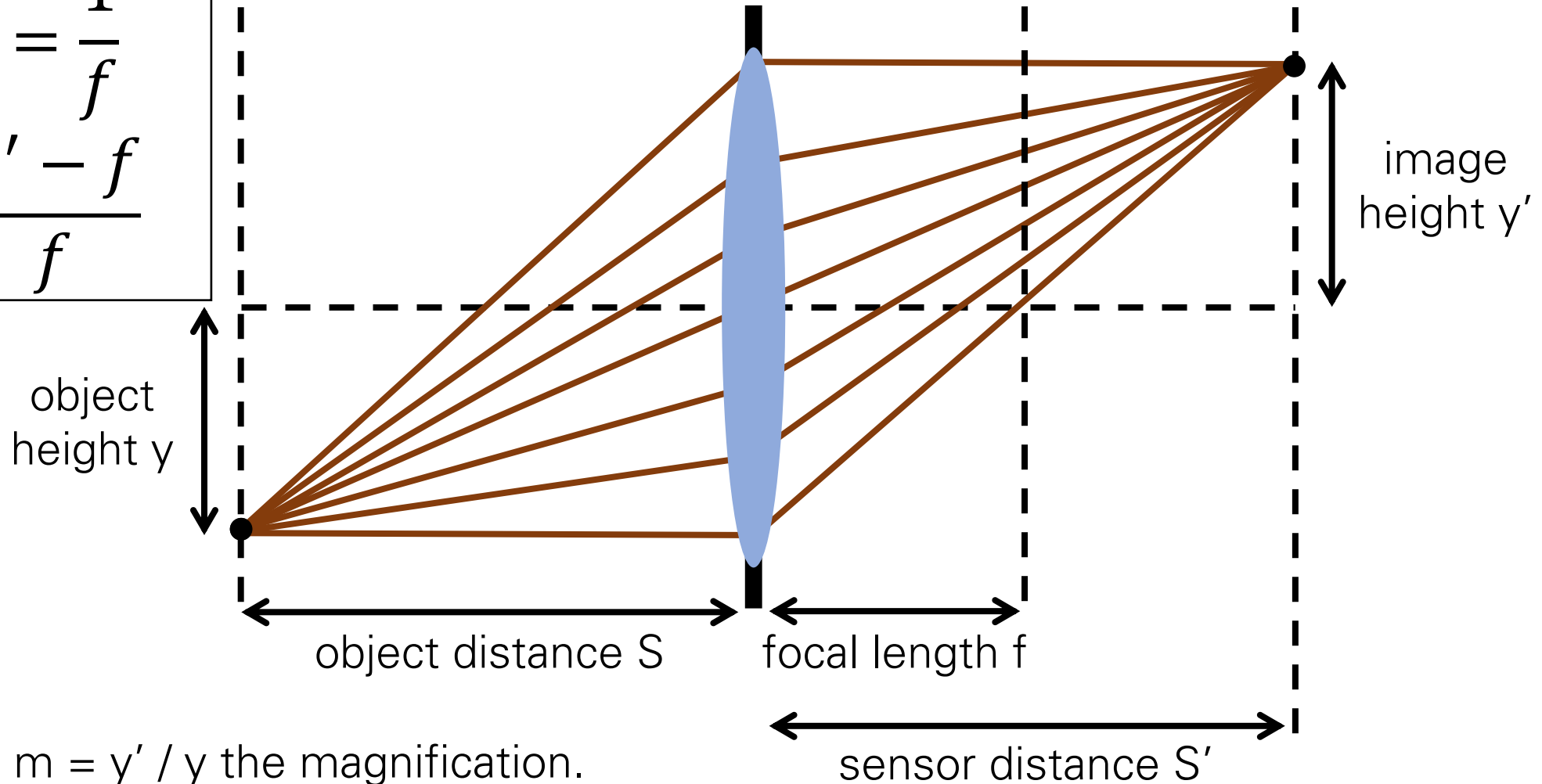
move object further
away from lens

move lens closer
to sensor

- What happens to magnification as we focus further? → It becomes smaller.

Magnification depends on depth

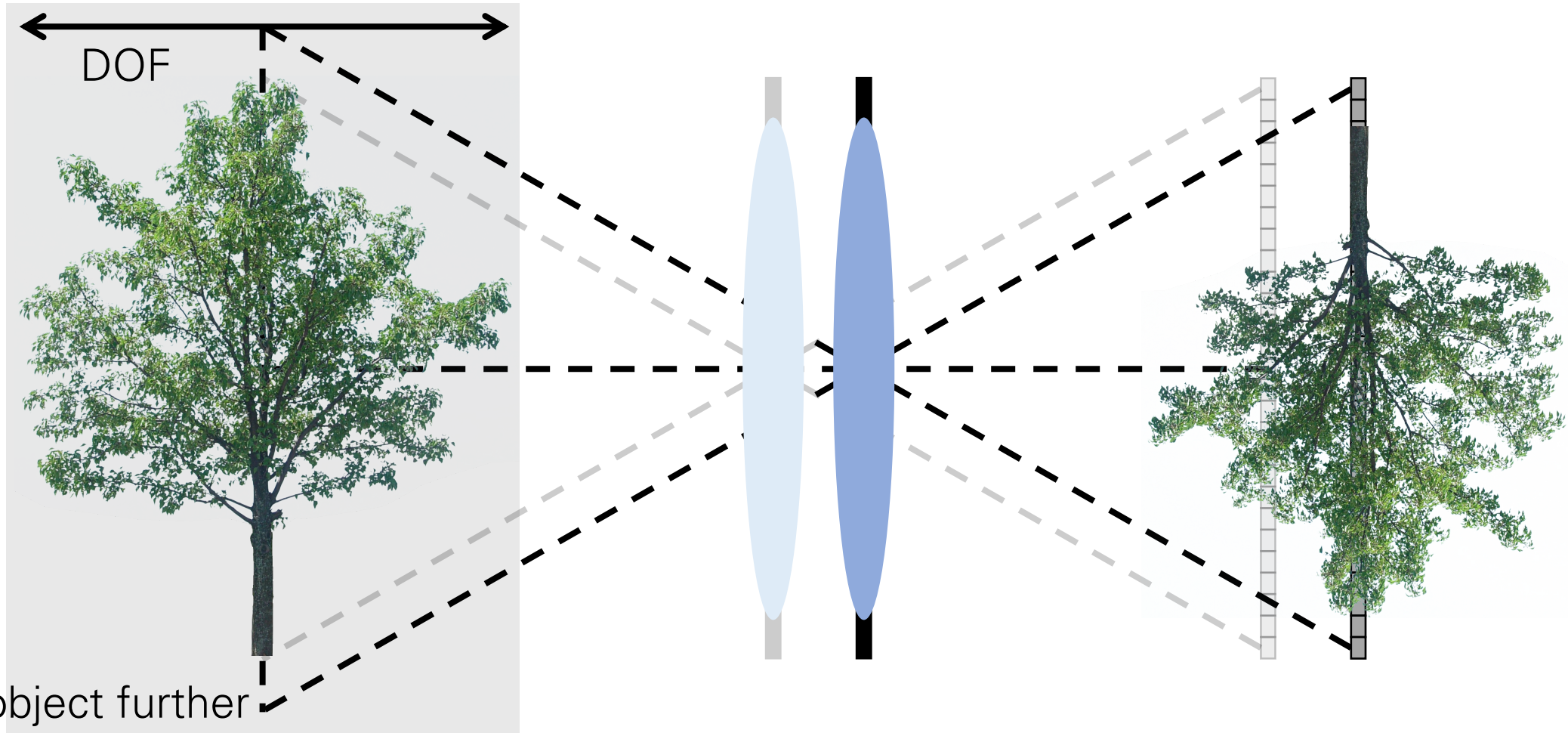
$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$
$$m = \frac{S' - f}{f}$$



- We call $m = y' / y$ the magnification.

Comparison with pinhole camera

No need to refocus: we can move object further without changing aperture-sensor distance.



move object further
away from aperture

This can be done with a lens only if depth of field is large enough. Then the two behave the same.

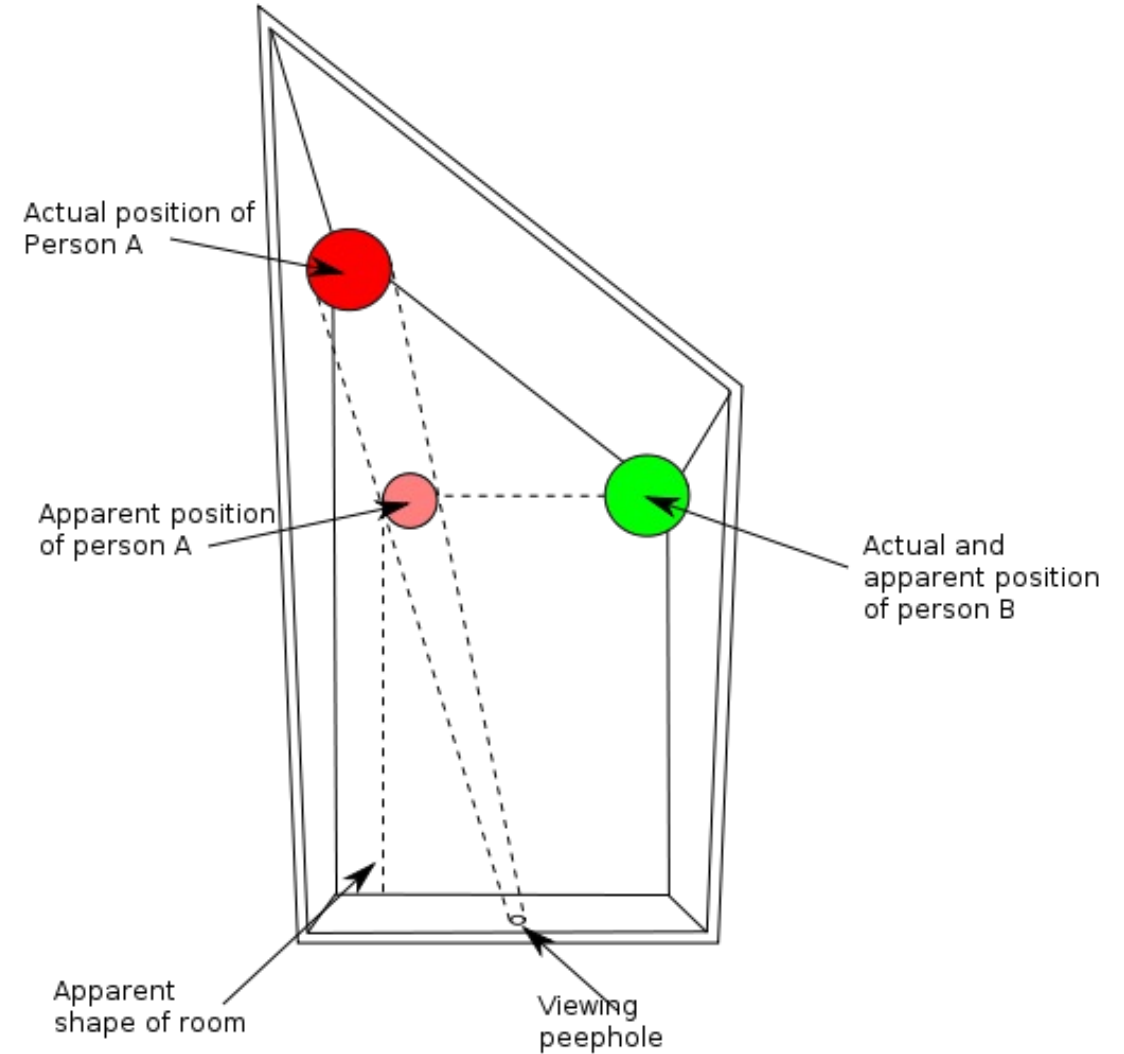
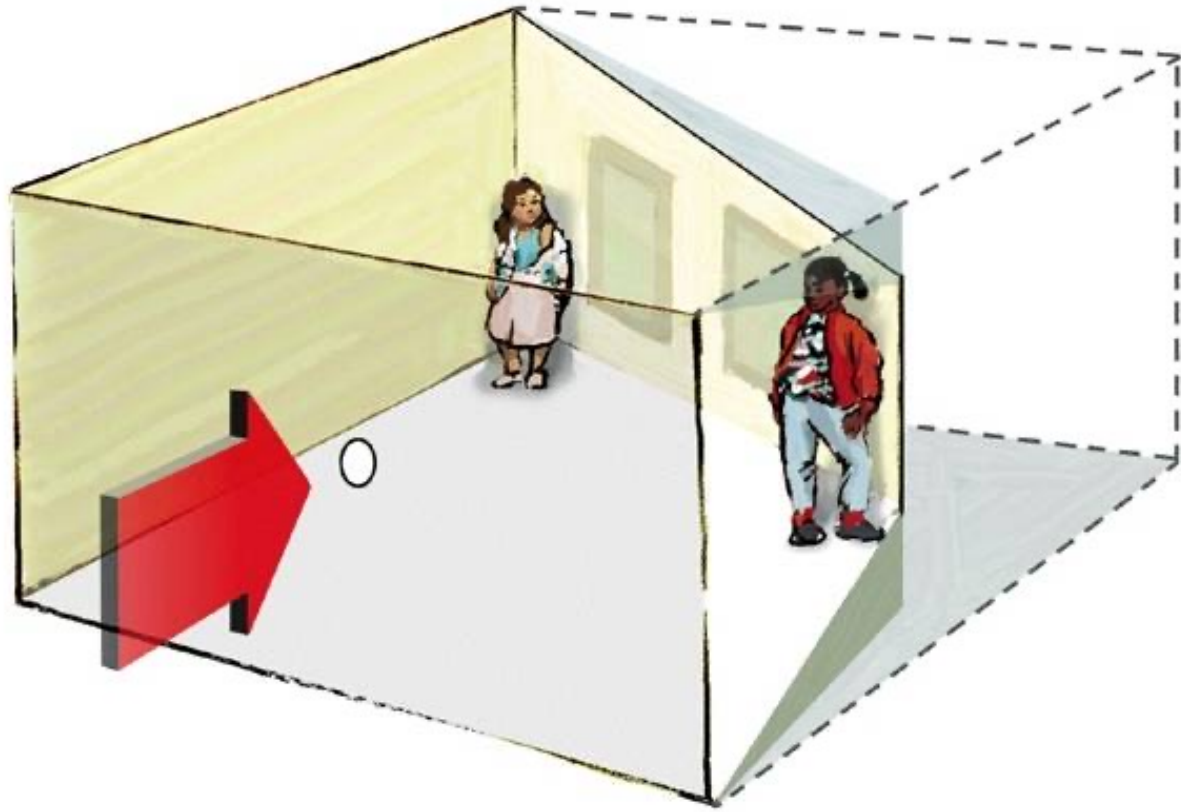
Forced perspective



The Ames room illusion



The Ames room illusion



The arrow illusion

Prof. Kokichi Sugihara has many other amazing illusions involving perspective distortion, check them out on YouTube or on his website:

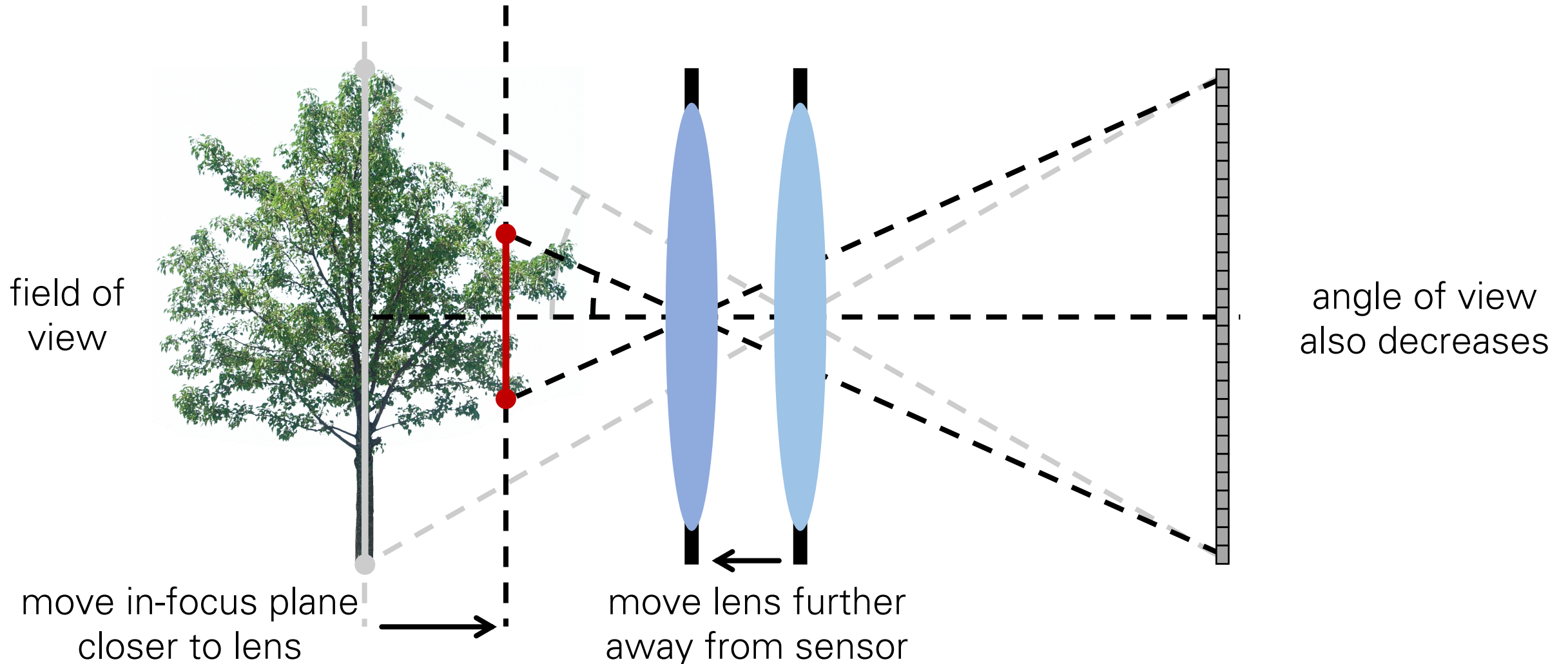
<http://www.isc.meiji.ac.jp/~kokichis/>



Zooming

Field of view

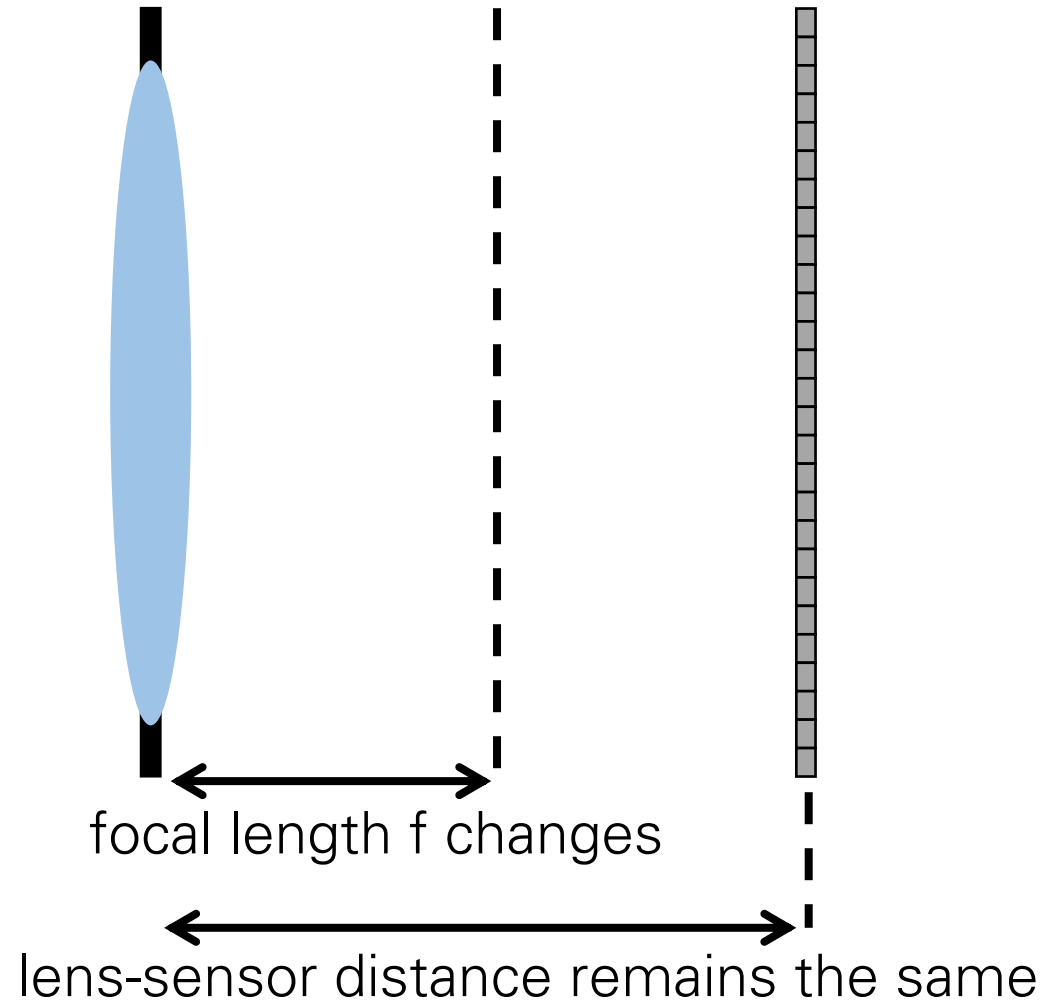
The part of the in-focus plane that gets mapped on the sensor.



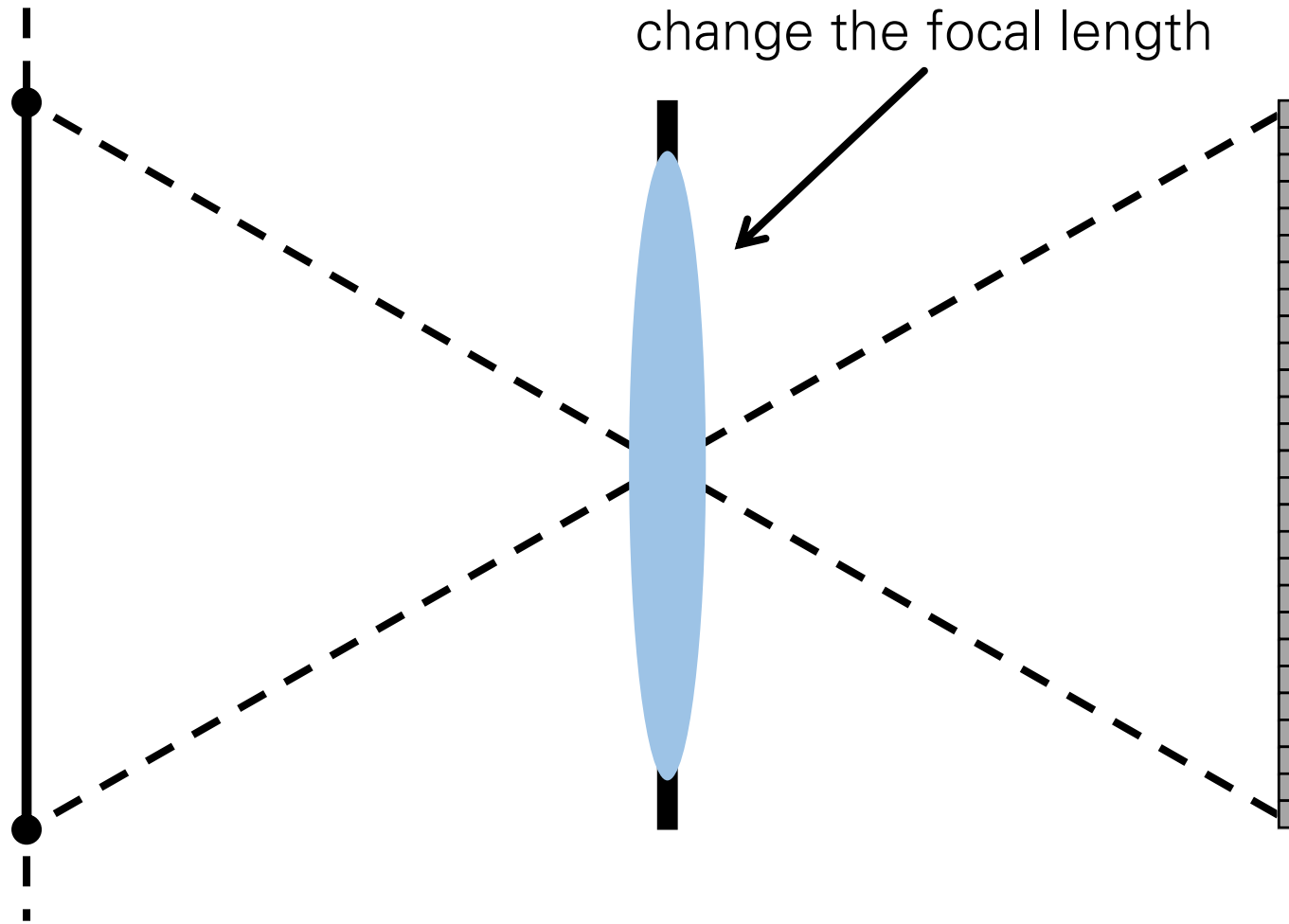
- What happens to field of view as we focus closer? → It becomes smaller.

Zooming means changing the focal length

Very different
process from
refocusing

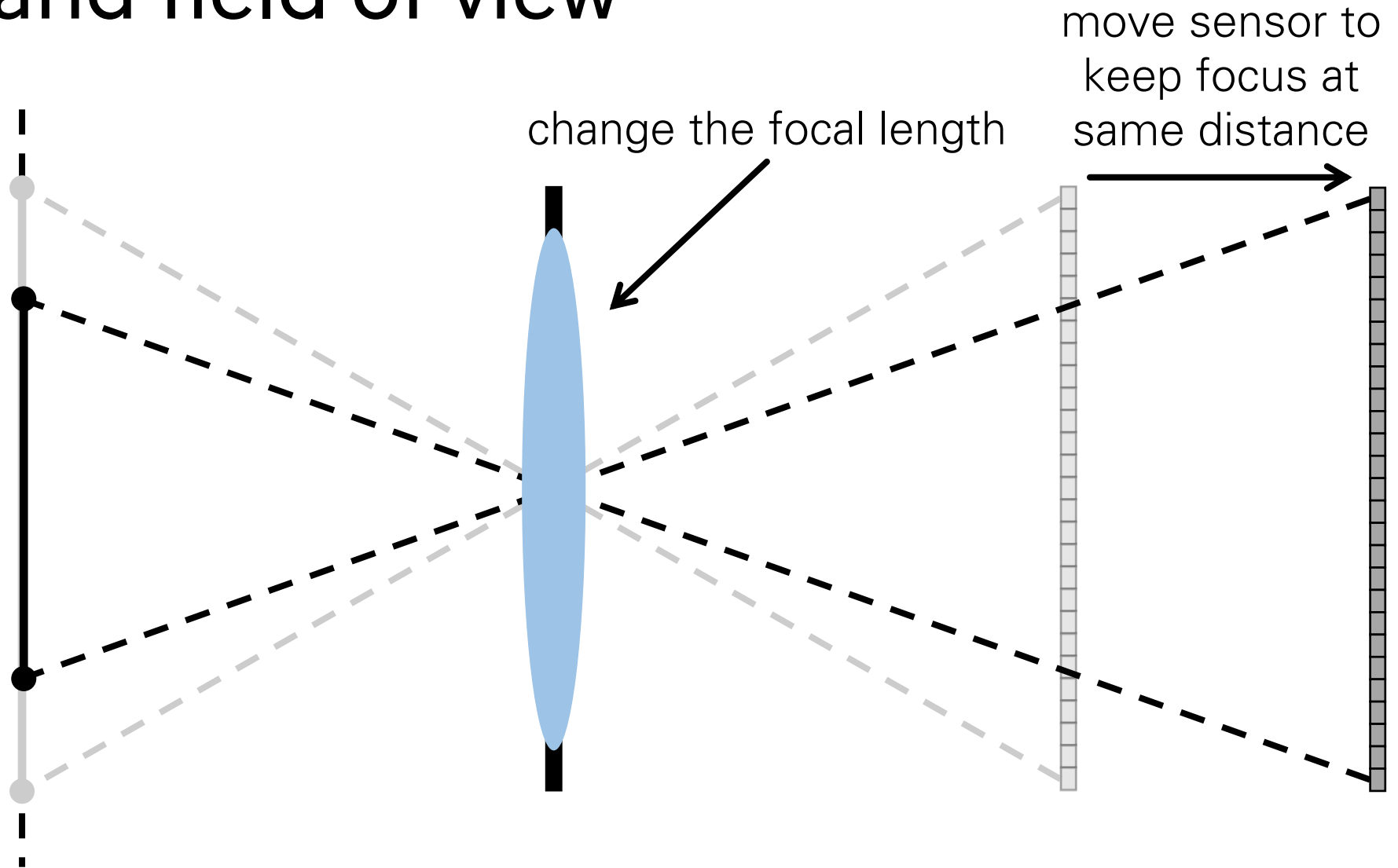


Zooming and field of view



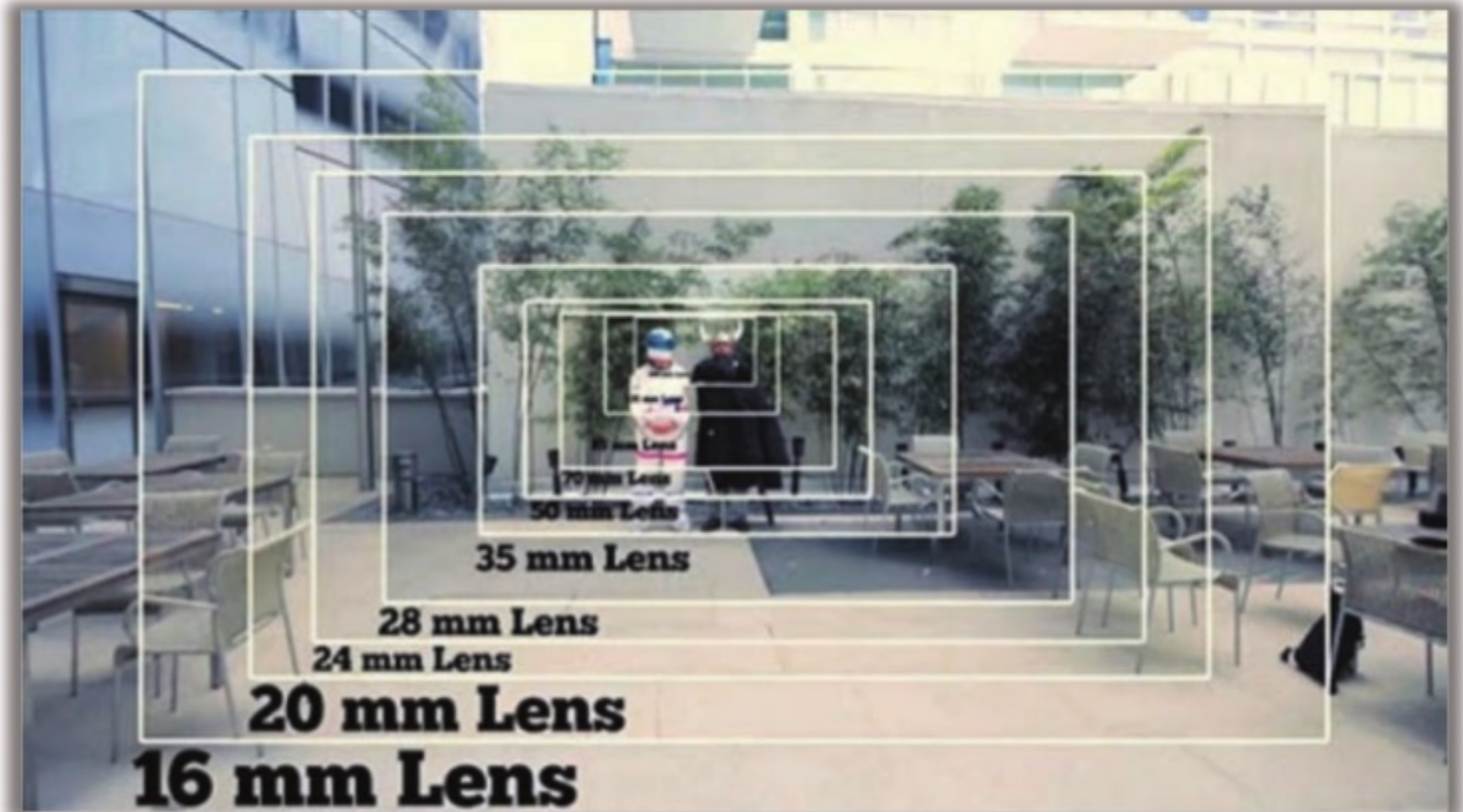
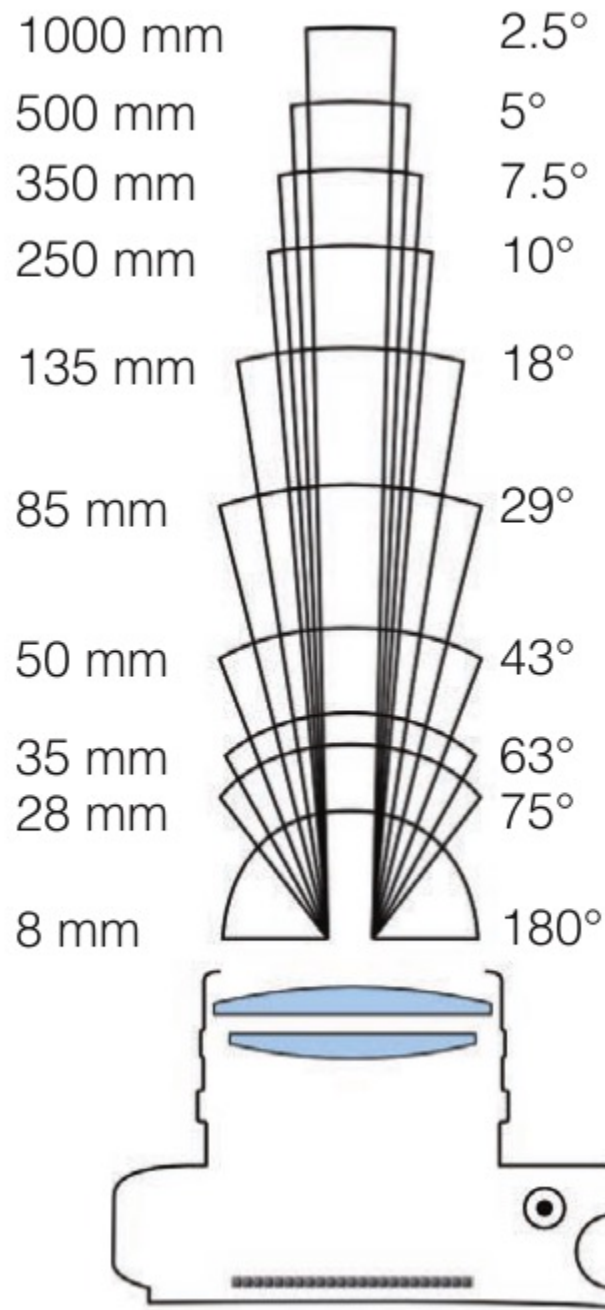
- What happens to field of view when we focus closer? → It decreases.
- What happens to field of view when we increase lens focal length?

Zooming and field of view



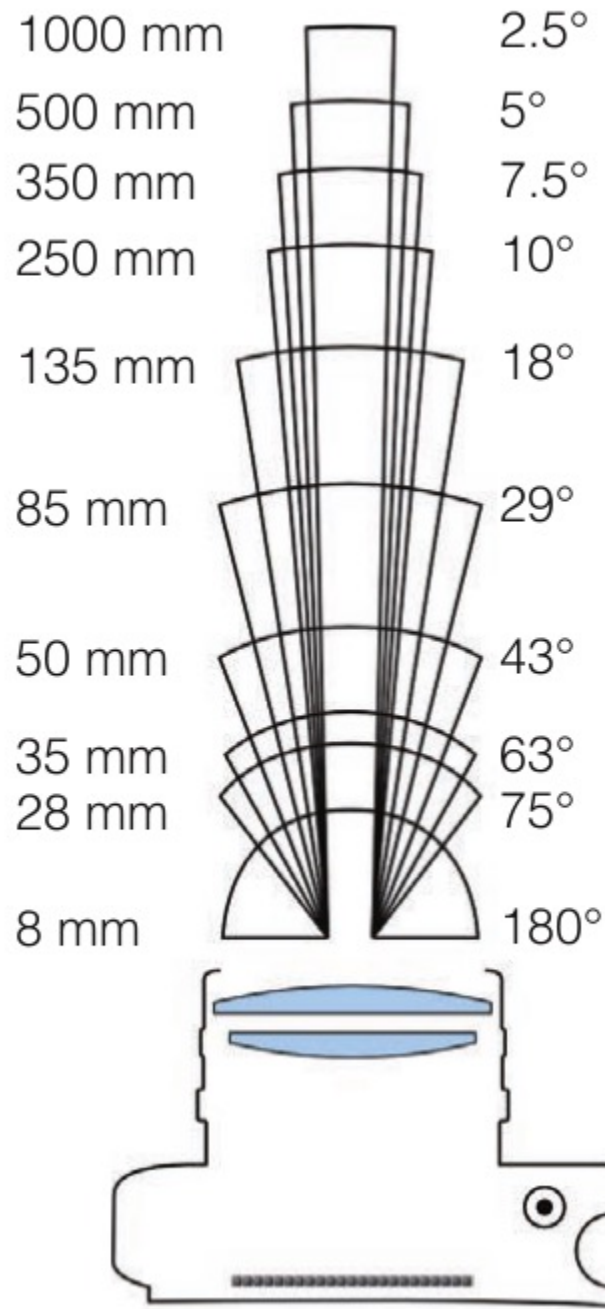
- When we increase lens focal length, field of view decreases (we "zoom in").

Field of view



Andrew McWilliams

Field of view



17mm



28mm

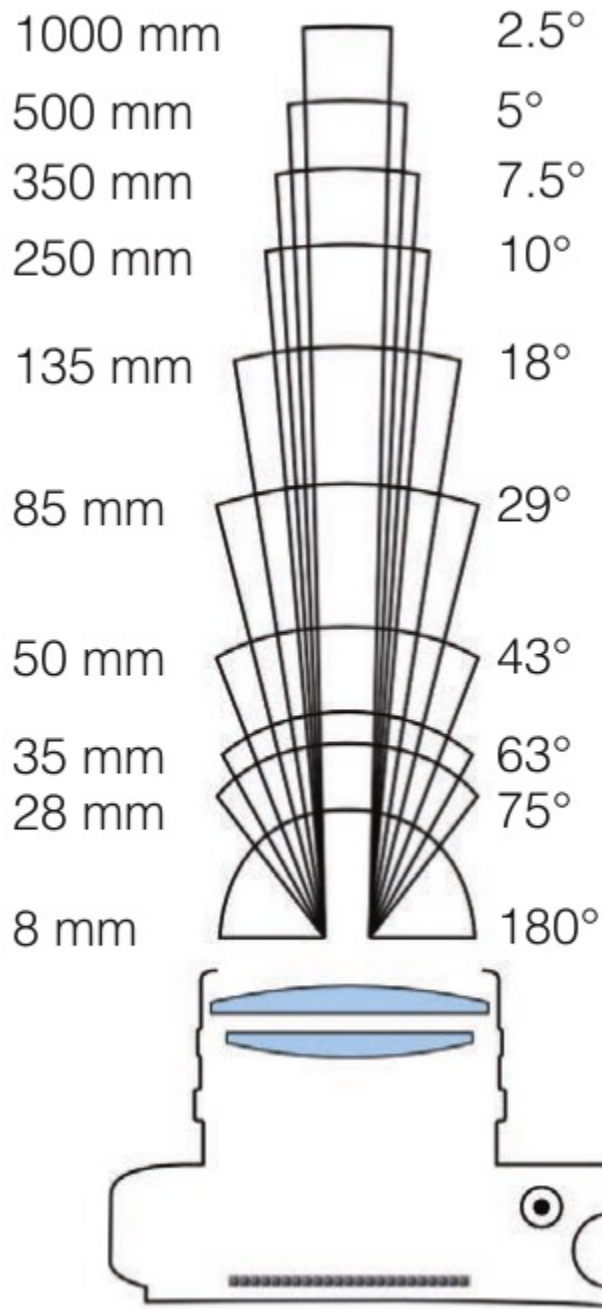


50mm

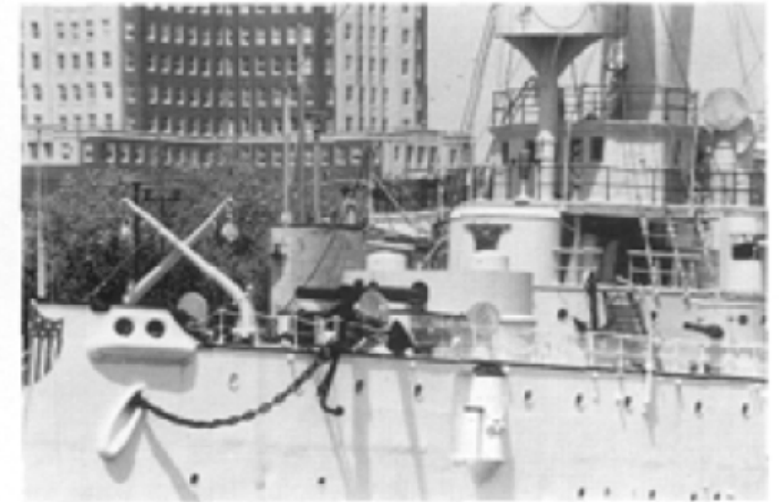


85mm

Field of view



135mm



300mm



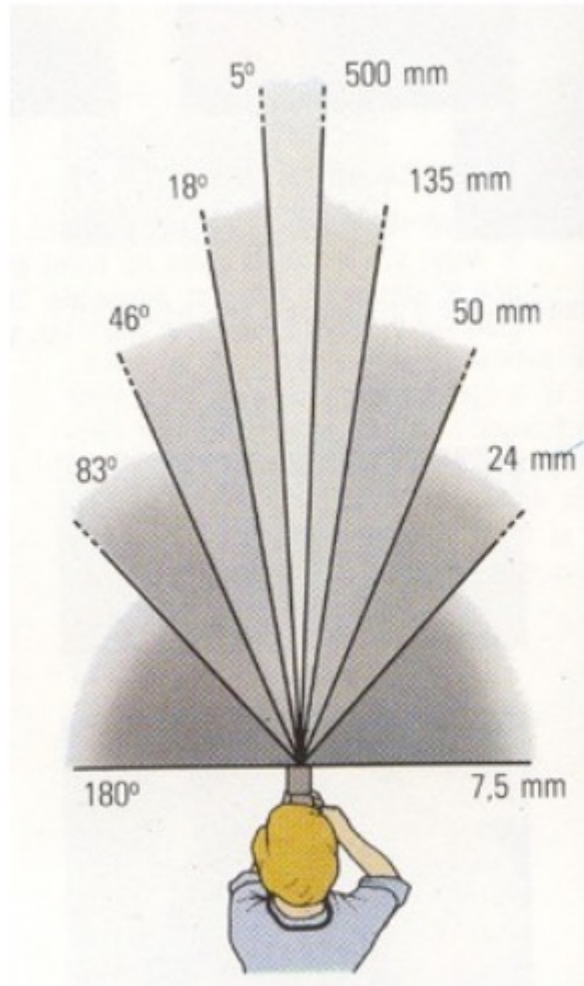
500mm



1000mm

Field of view

Increasing the lens focal length is similar to cropping



$f = 25 \text{ mm}$



$f = 50 \text{ mm}$



$f = 135 \text{ mm}$



Is this effect identical to cropping?

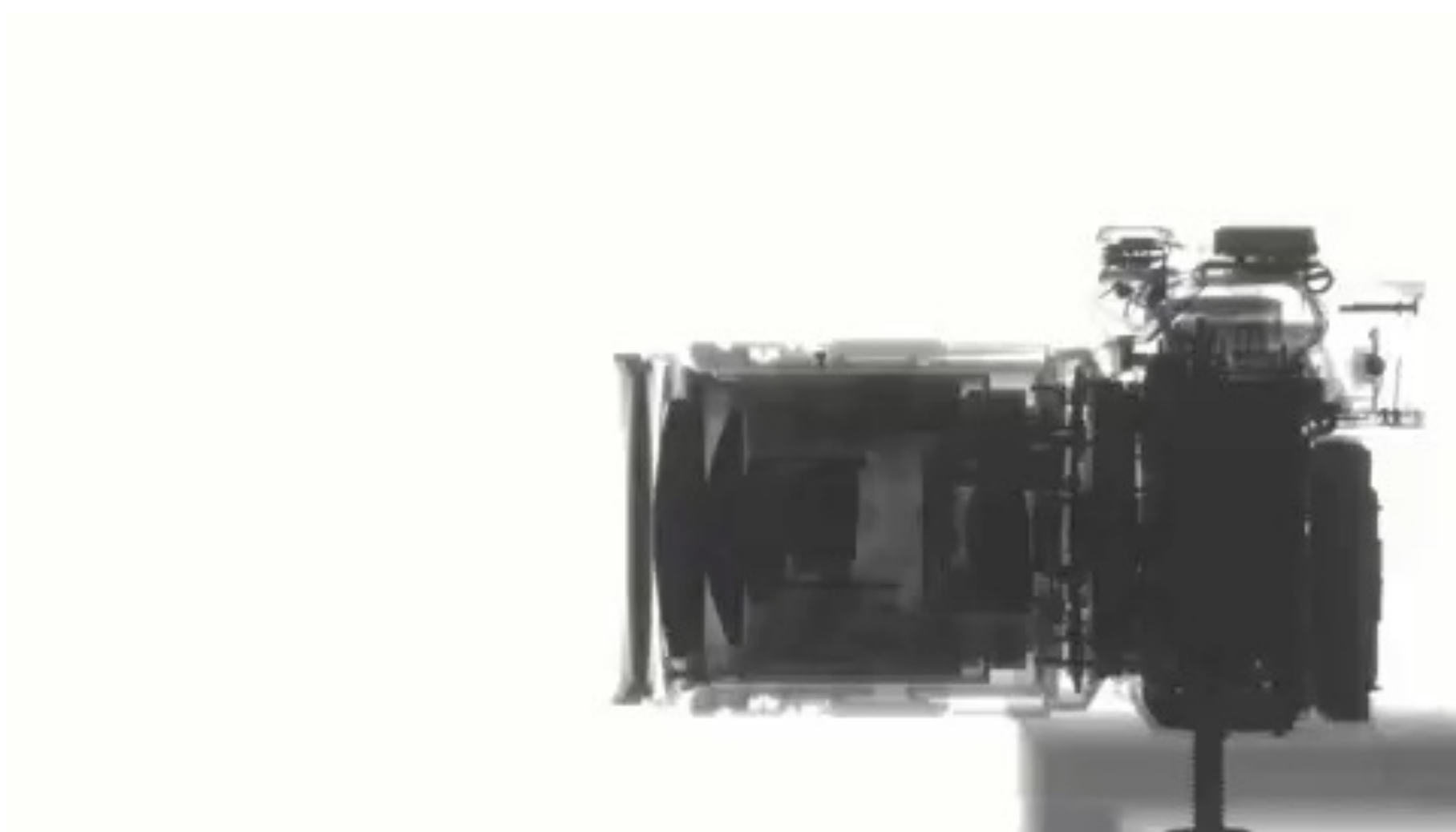
The lens on your camera

Focus ring: controls distance of lens from sensor



Zoom ring: controls focal length of lens

The lens on your camera



Focusing versus zooming

When you turn the focus ring to bring lens further-away from the sensor:

1. The in-focus distance decreases (you need to get closer to object).
2. The field of view decreases (you see a smaller part of the object).
3. The magnification increases (same part of the object is bigger on sensor).

When you turn the zoom ring to decrease the focal length of the lens:

1. The in-focus distance increases (you need to move away from the object).
2. The field of view increases (you see a larger part of the object).
3. The magnification decreases (same part of the object is smaller on sensor).

Focusing versus zooming

When you turn the focus ring to bring lens further-away from the sensor:

1. The in-focus distance decreases (you need to get closer to object).
2. The field of view decreases (you see a smaller part of the object).
3. The magnification increases (same part of the object is bigger on sensor).

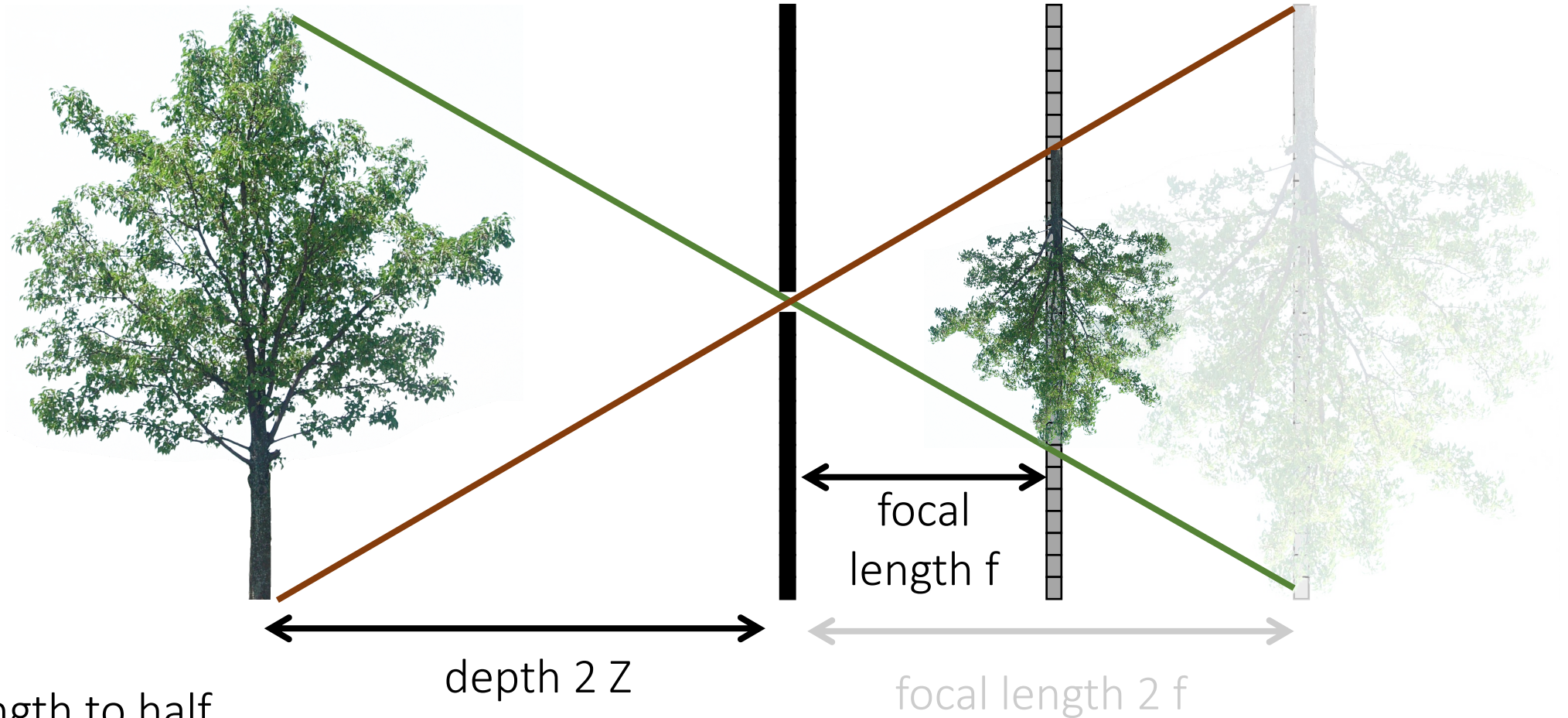
When you turn the zoom ring to decrease the focal length of the lens:

1. The in-focus distance increases (you need to move away from the object).
2. The field of view increases (you see a larger part of the object).
3. The magnification decreases (same part of the object is smaller on sensor).

We can use both focus and zoom to cancel out their effects.

What if...

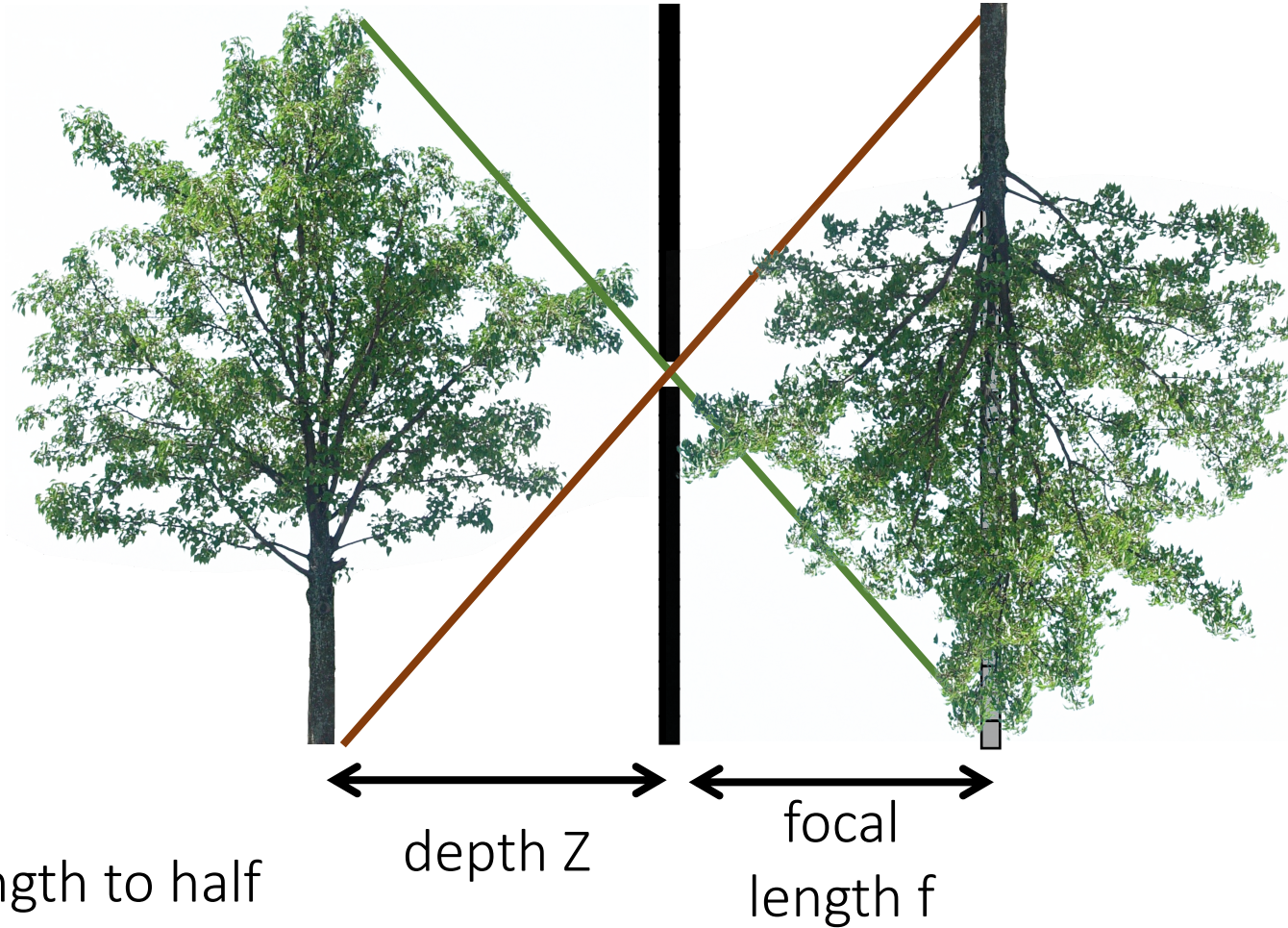
real-world
object



1. Set focal length to half

What if...

real-world
object



Is this the same image as
the one we had at focal
length $2f$ and distance $2Z$?

Similar construction can
be done with lenses, after
taking care of refocusing.

1. Set focal length to half
2. Set depth to half

Perspective distortion



long focal length



mid focal length



short focal length

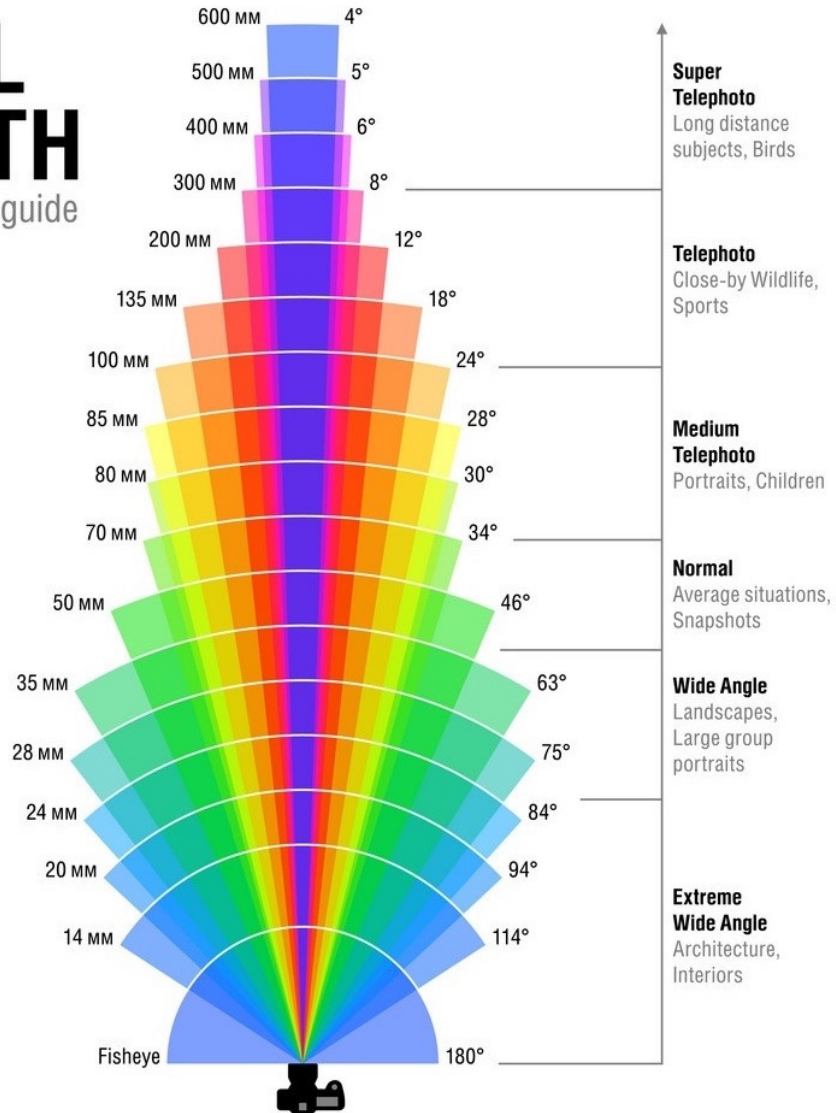
Perspective distortion



Perspective distortion

FOCAL LENGTH

& angle of view guide



What is the best focal length for portraits?

That's like asking which is better, vi or emacs...



long focal length



mid focal length



short focal length

Vertigo effect

Named after Alfred Hitchcock's movie

- also known as "dolly zoom"



Vertigo effect



How would you
create this effect?

Long focal length

When the focal length is long, the field of view becomes very small and the resulting image appears more flat.



Long focal length

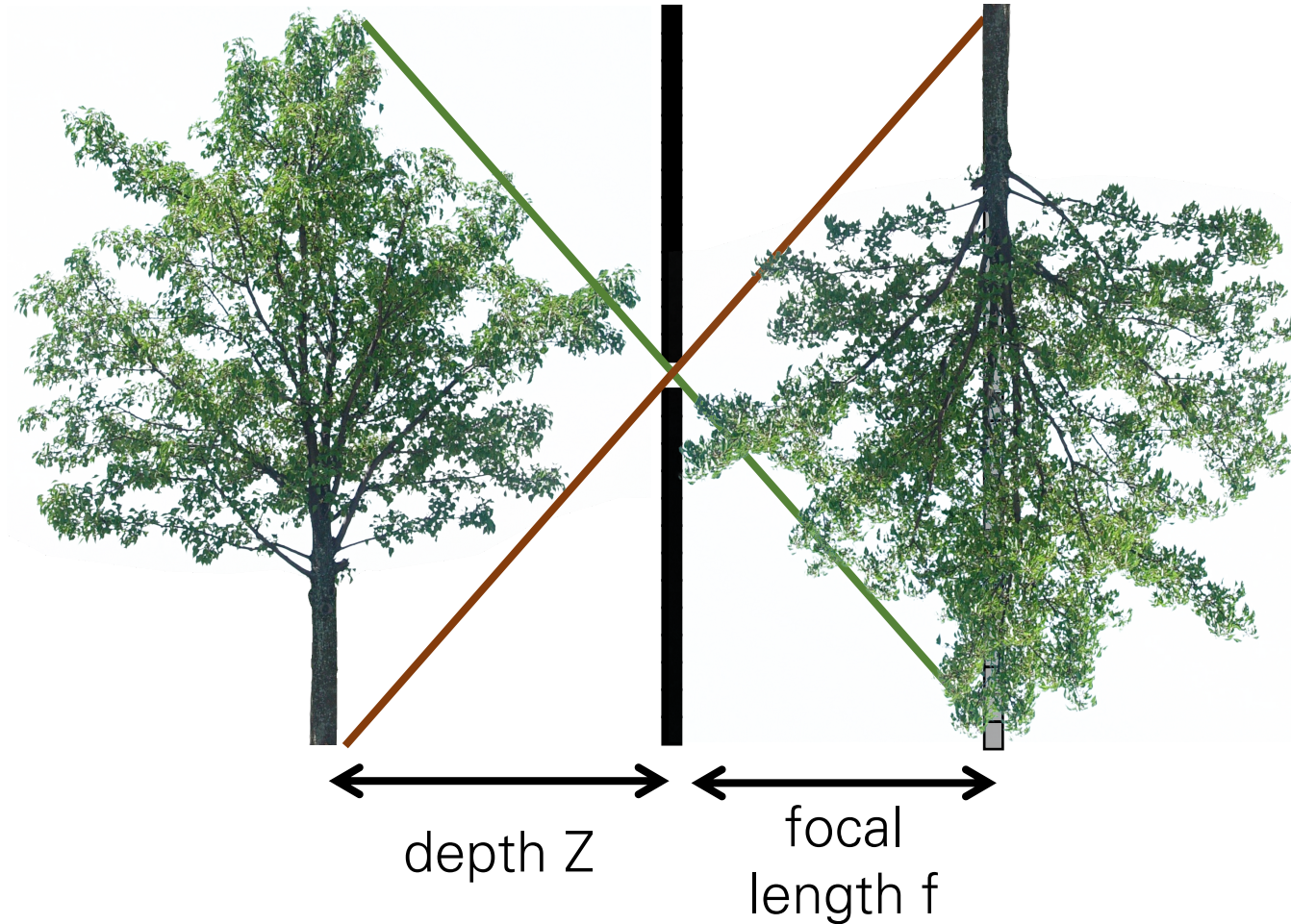
Here's another example:
Empire State building and the Statue
of Liberty are about 4.5 miles apart,
and the former is 5x taller.



Orthographic camera and telecentric lenses

What if...

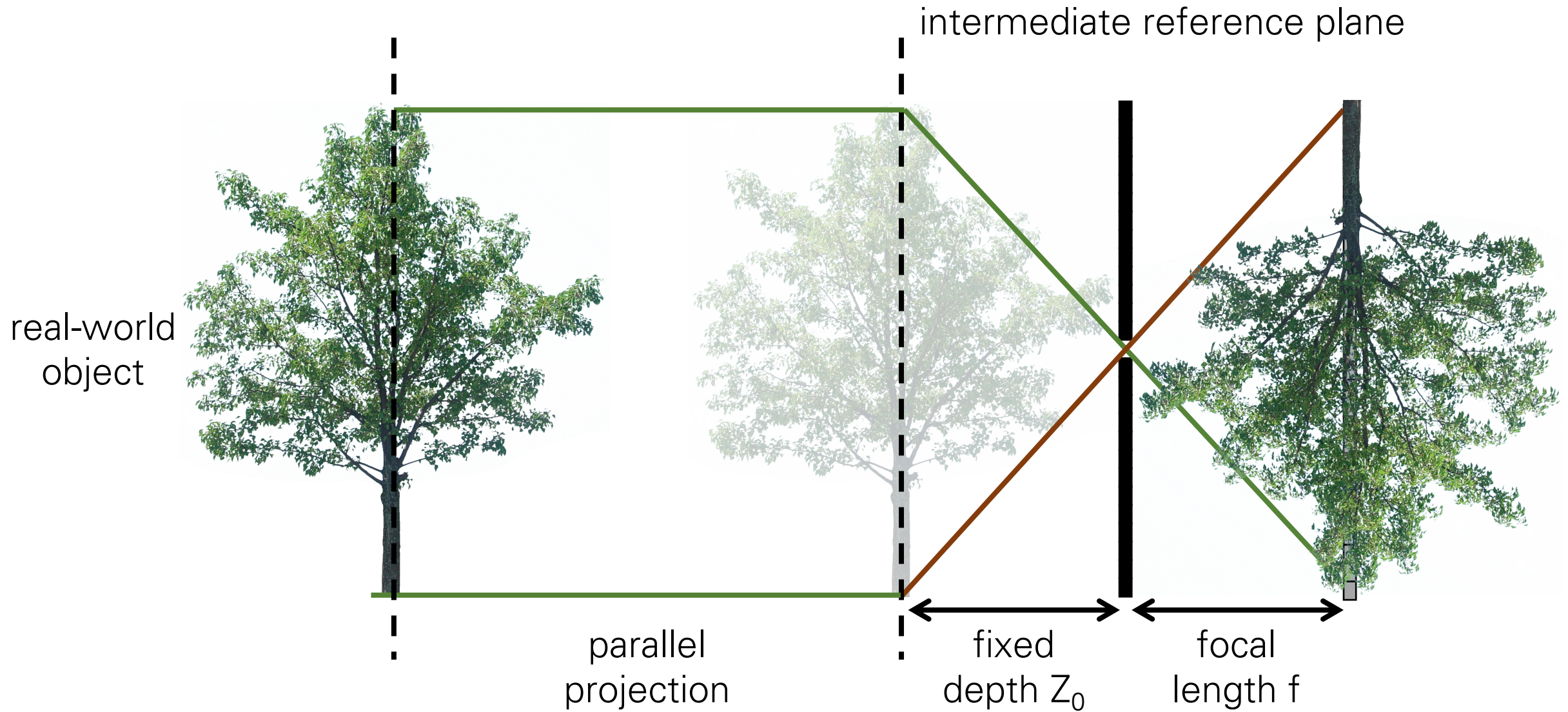
real-world
object



Continue increasing Z and f while maintaining same magnification?

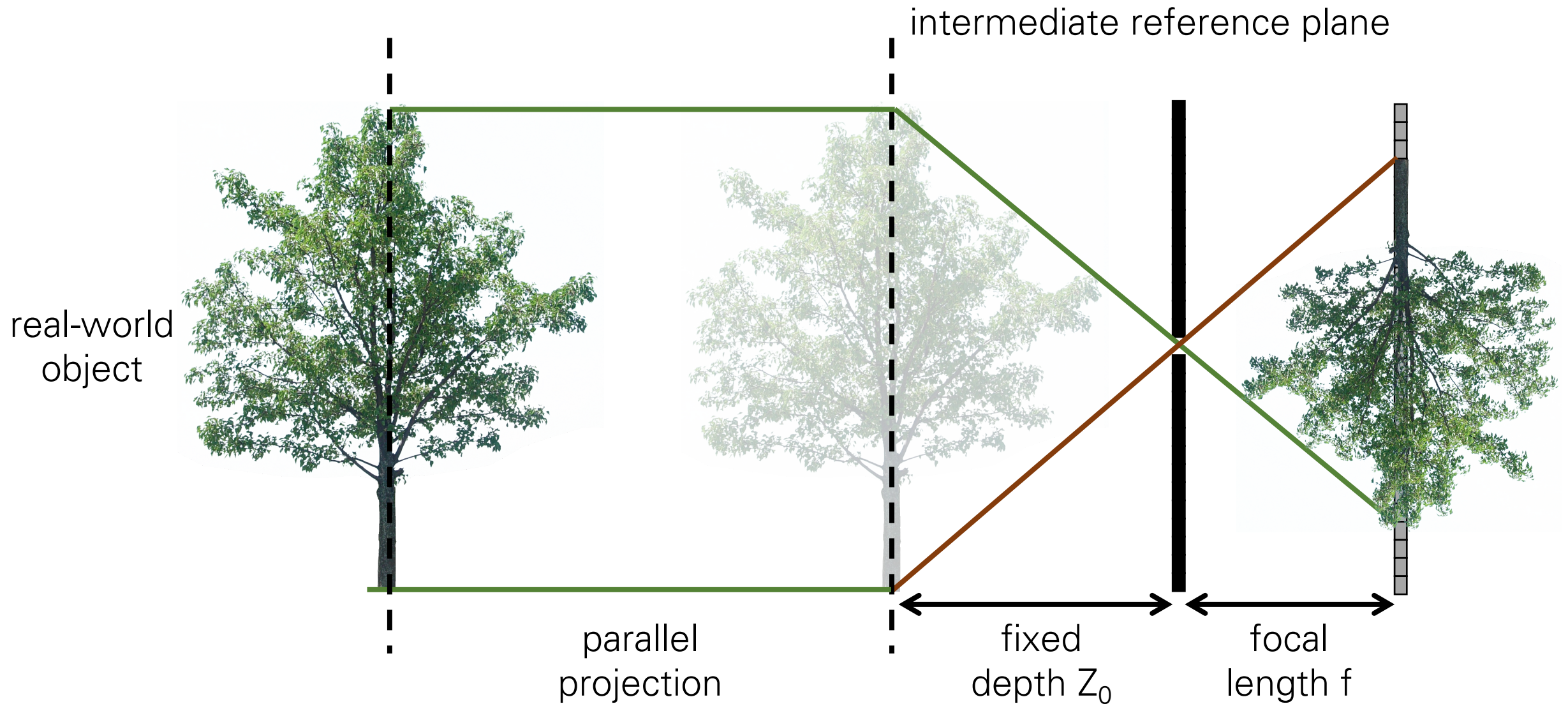
$$f \rightarrow \infty \text{ and } \frac{f}{Z} = \text{constant}$$

Orthographic camera



Depth-independent magnification $m = 1$ (real-life size).

Weak-perspective camera

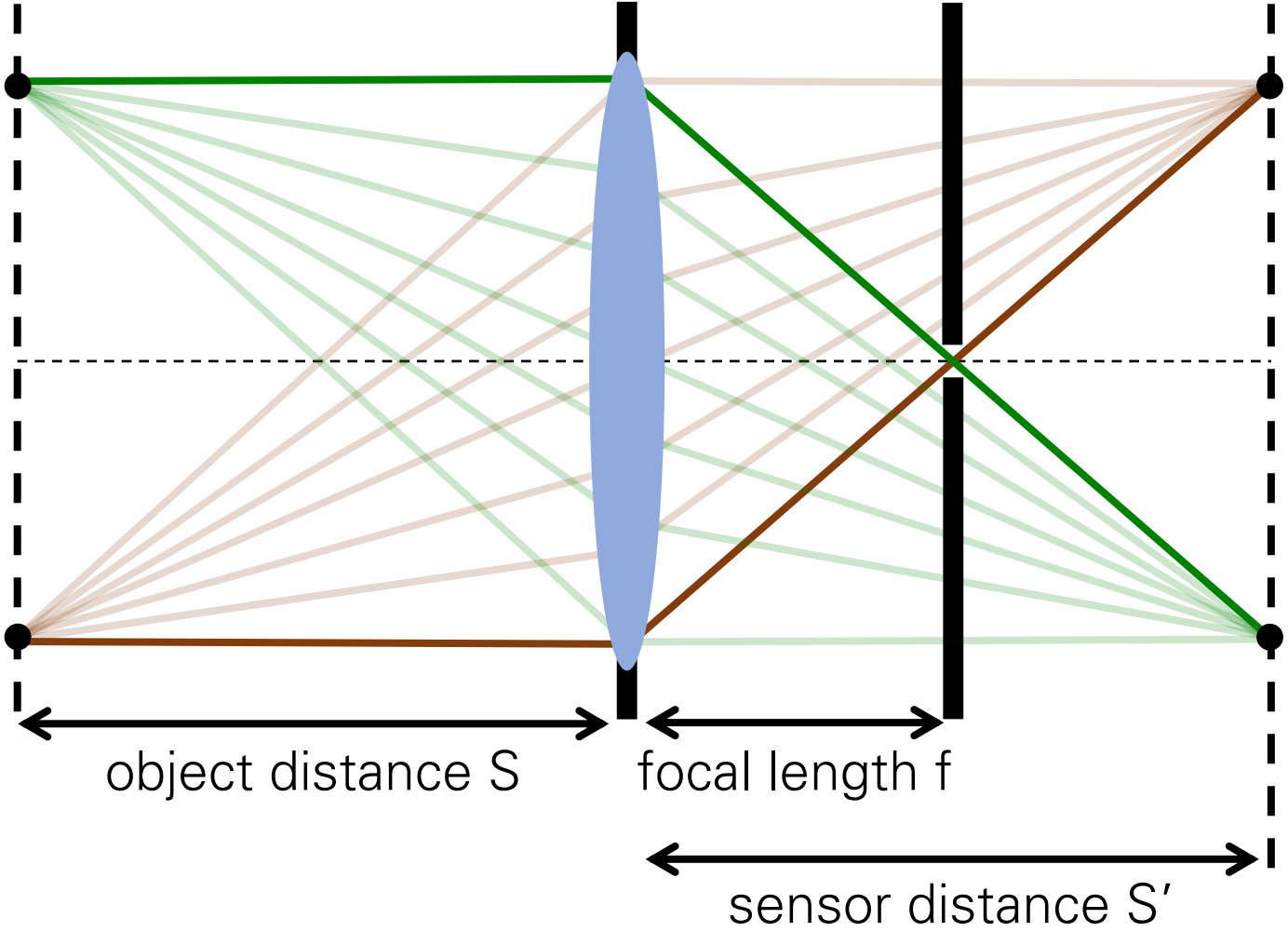


Depth-independent magnification $m = f / Z_0$.

How can we implement such a camera with lenses?

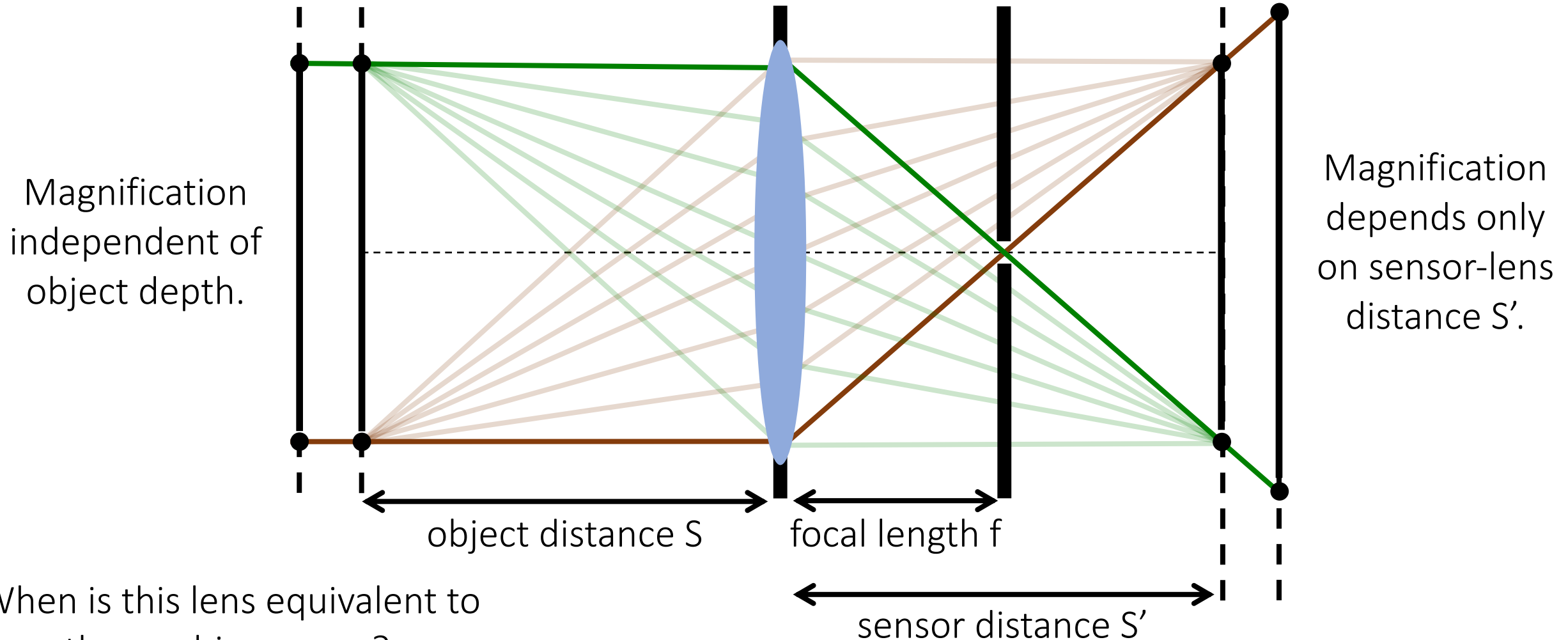
Telecentric lens

Place a pinhole at focal length, so that only rays parallel to primary ray pass through.

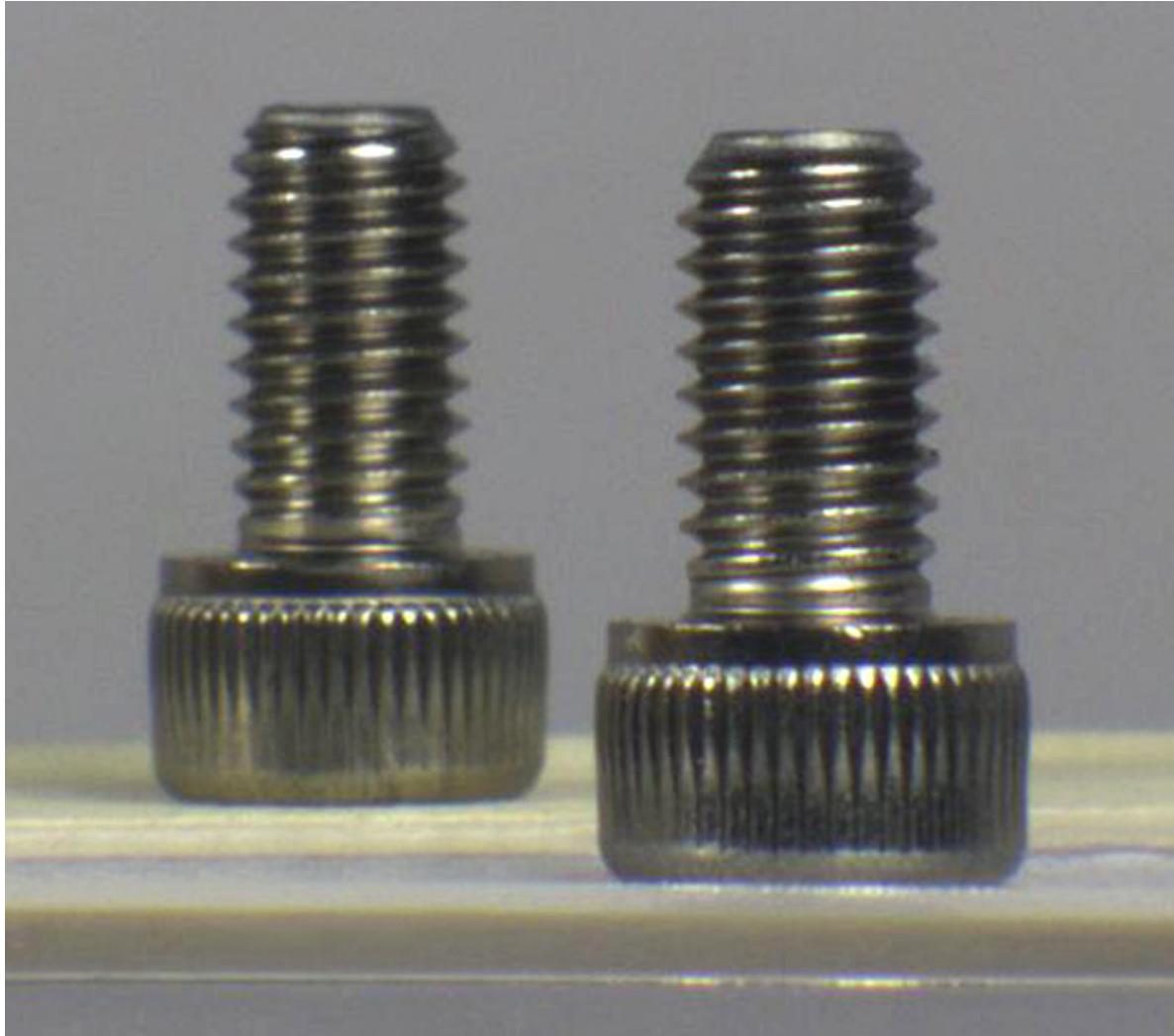


Telecentric lens

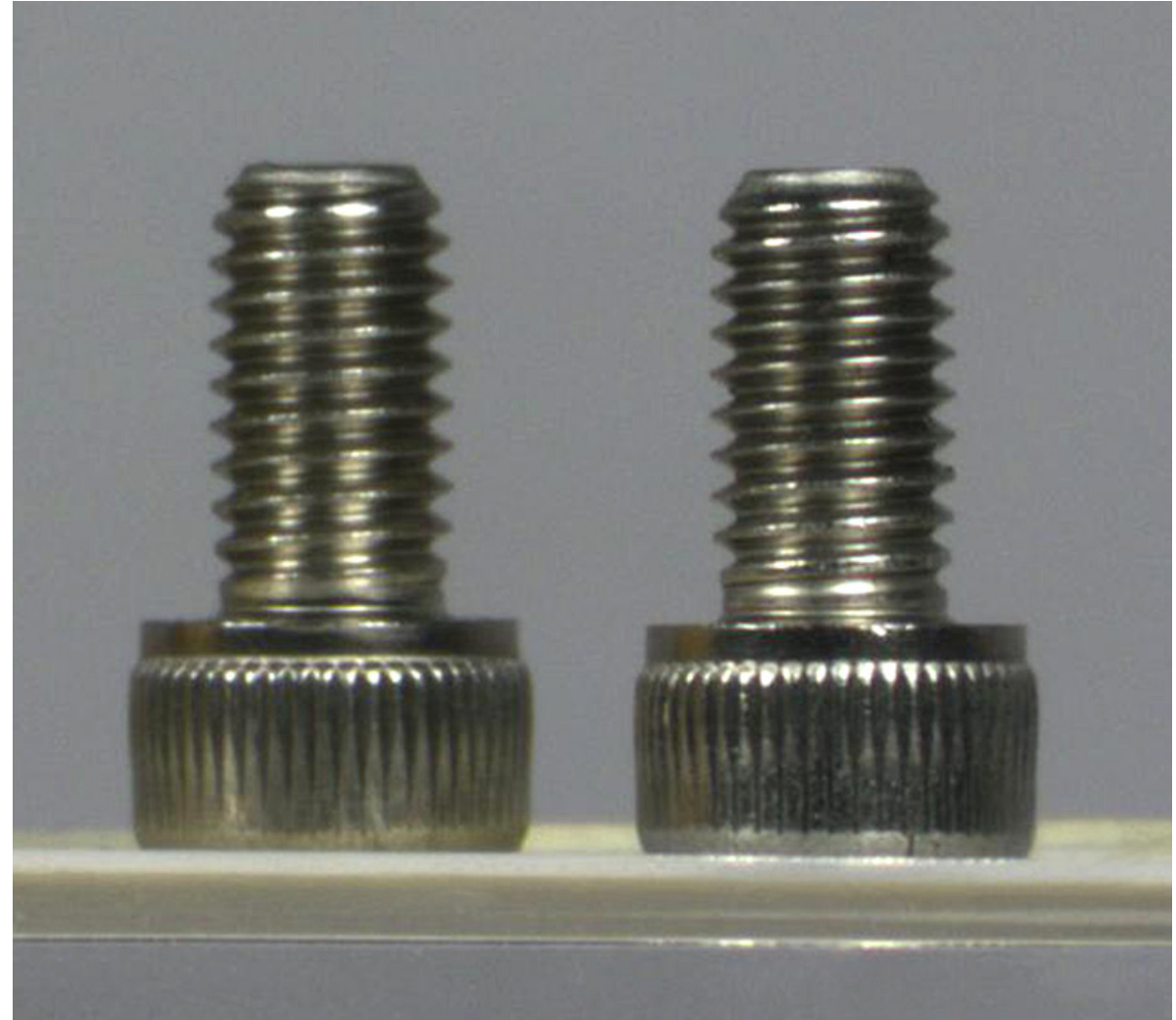
Place a pinhole at focal length, so that only rays parallel to primary ray pass through.



Regular vs telecentric lens



regular lens



telecentric lens

Next Lecture: Noise and Color