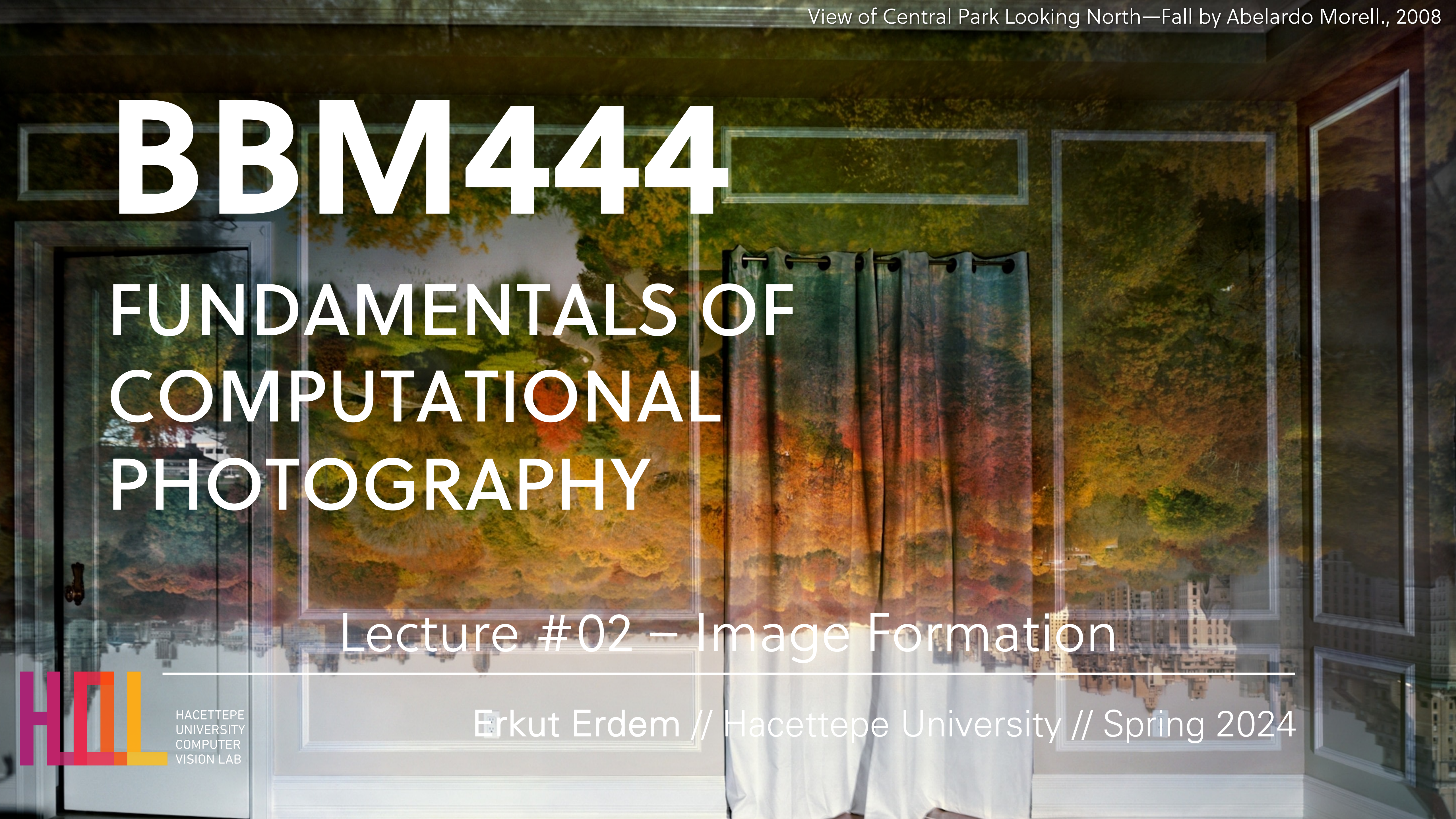


View of Central Park Looking North—Fall by Abelardo Morell., 2008

# BBM444

## FUNDAMENTALS OF COMPUTATIONAL PHOTOGRAPHY

### Lecture #02 – Image Formation


Erkut Erdem // Hacettepe University // Spring 2024

HACETTEPE UNIVERSITY COMPUTER VISION LAB

# Today's Lecture

- Pinhole camera
- Basics of geometric optics and lenses
- Field of view
- Magnification and perspective
- Zooming
- Orthographic camera and telecentric lenses

**Disclaimer:** The material and slides for this lecture were borrowed from

—Ioannis Gkioulekas' 15-463/15-663/15-862 "Computational Photography" class

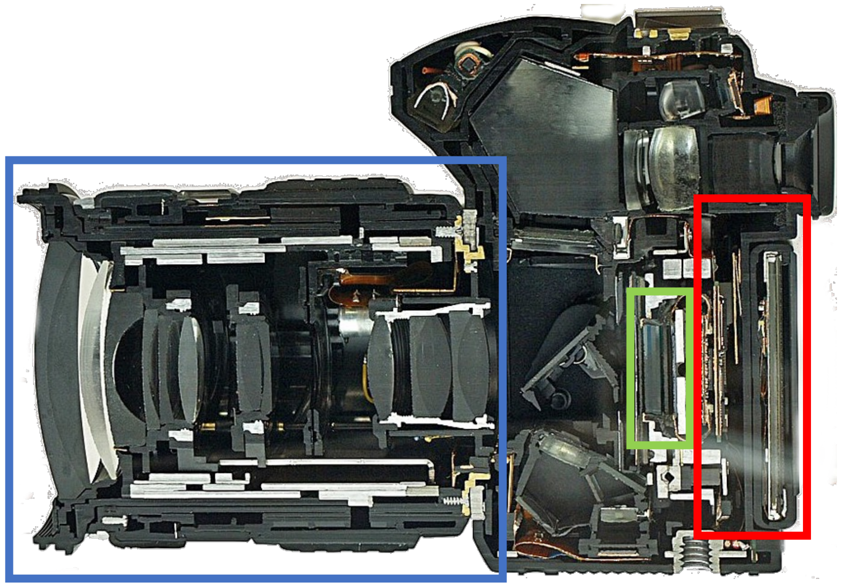
—Steve Marschner's CS6640 "Computational Photography" class

—David Lindell's CSC2529 "Computational Imaging" class

—James Tompkin's CSCI1290 "Computational Photography" class

# The modern photography pipeline

post-capture processing



optics and optical controls → sensor, analog front-end, and color filter array → in-camera image processing pipeline

# Some motivational imaging experiments

# Let’s say we have a sensor…

digital sensor
(CCD or CMOS)

# … and an object we like to photograph

real-world
object

digital sensor
(CCD or CMOS)

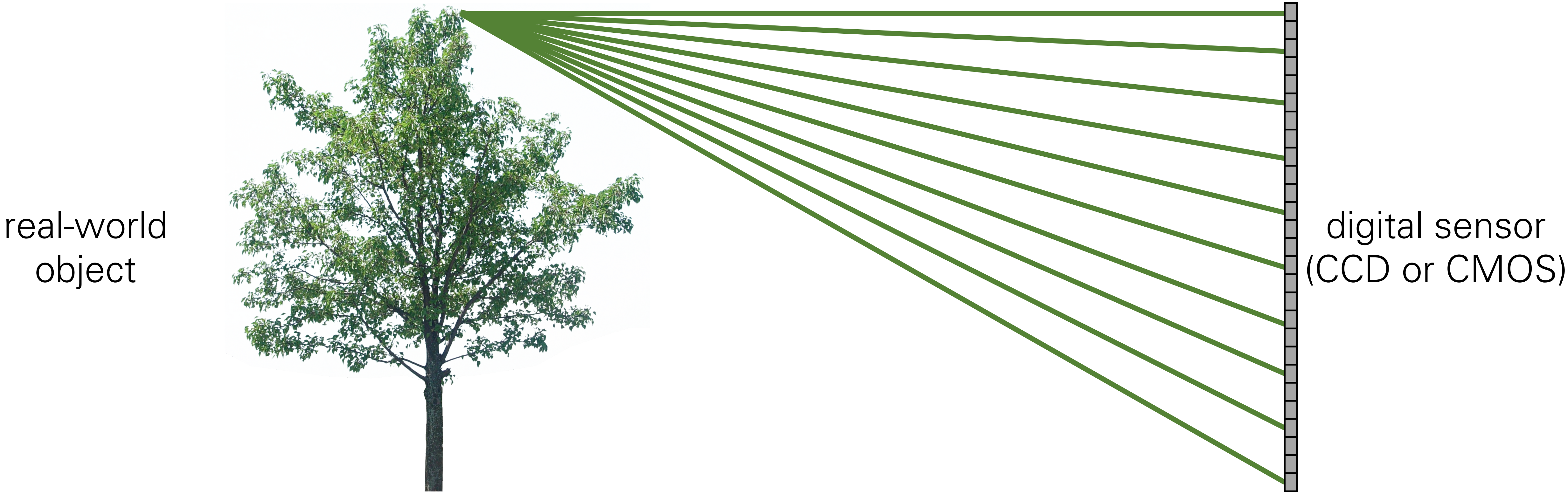
What would an image taken like this look like?

# Bare-sensor imaging

real-world object

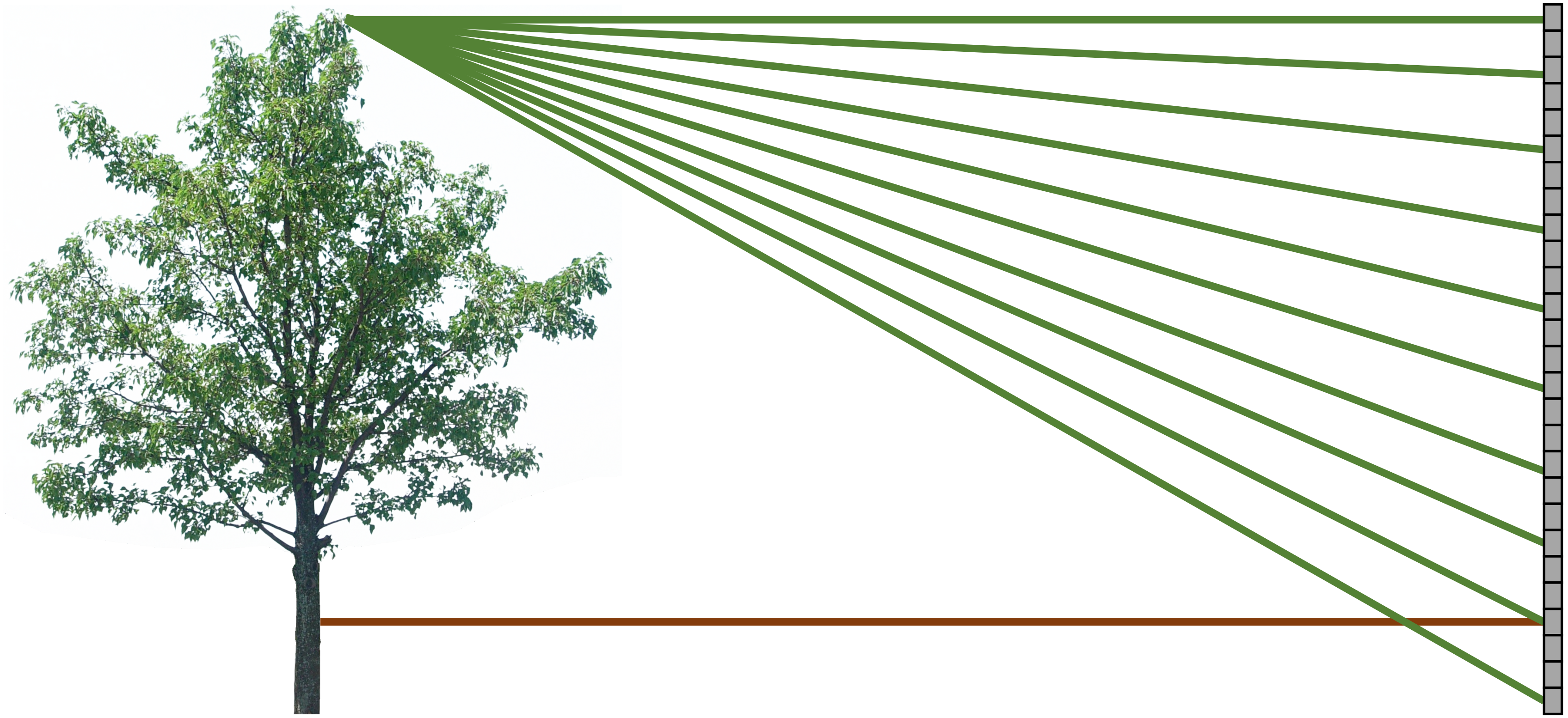
digital sensor (CCD or CMOS)

# Bare-sensor imaging



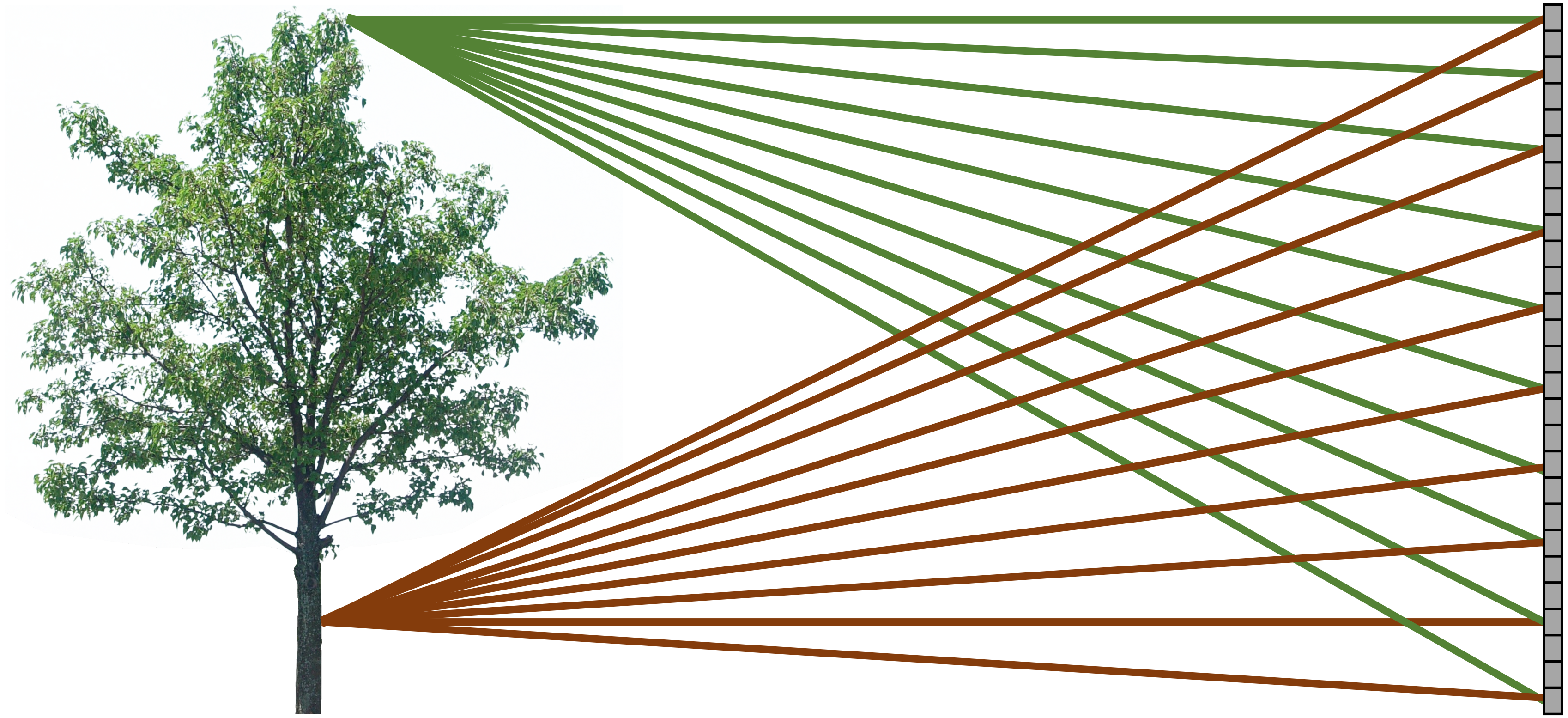
# Bare-sensor imaging

real-world
object



digital sensor
(CCD or CMOS)

# Bare-sensor imaging



real-world object

digital sensor (CCD or CMOS)

All scene points contribute to all sensor pixels

What does the image on the sensor look like?

# Bare-sensor imaging

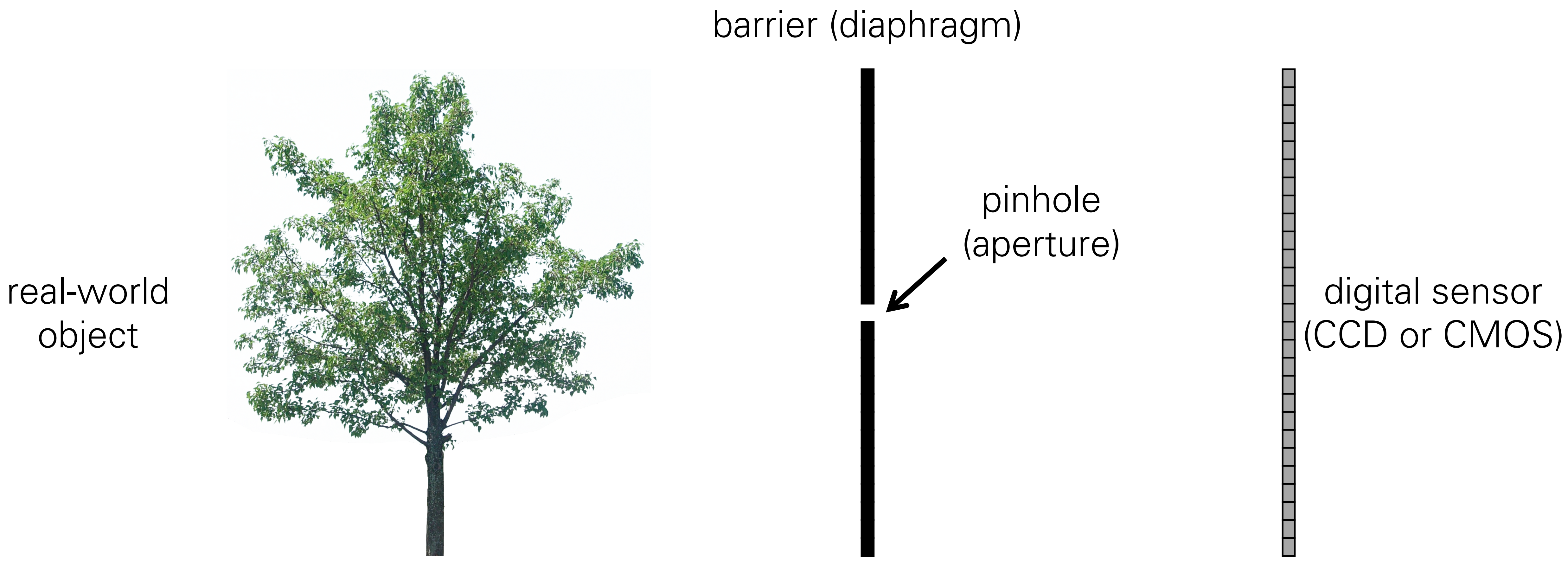
All scene points contribute to all sensor pixels

# What can we do to make our image look better?

real-world
object

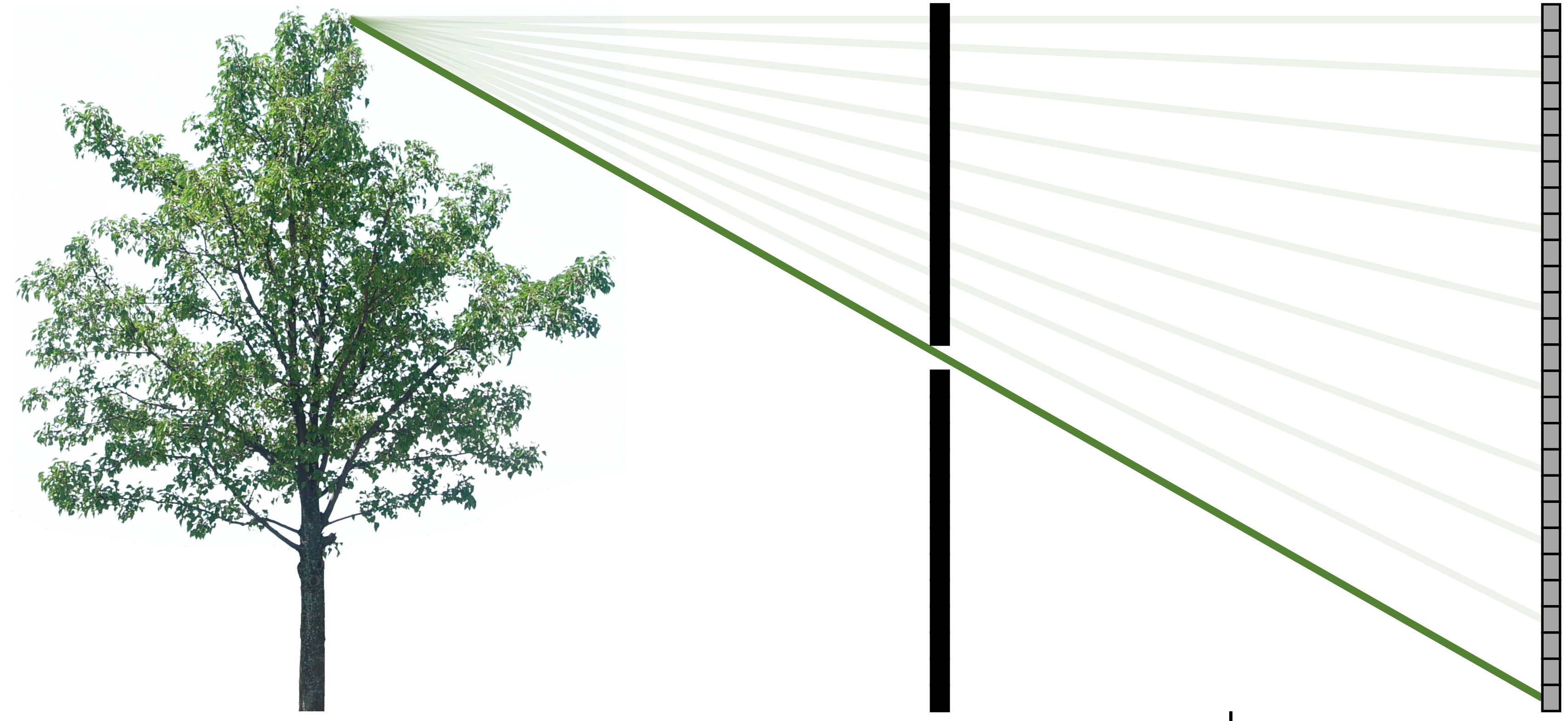
digital sensor
(CCD or CMOS)

# Let's add something to this scene

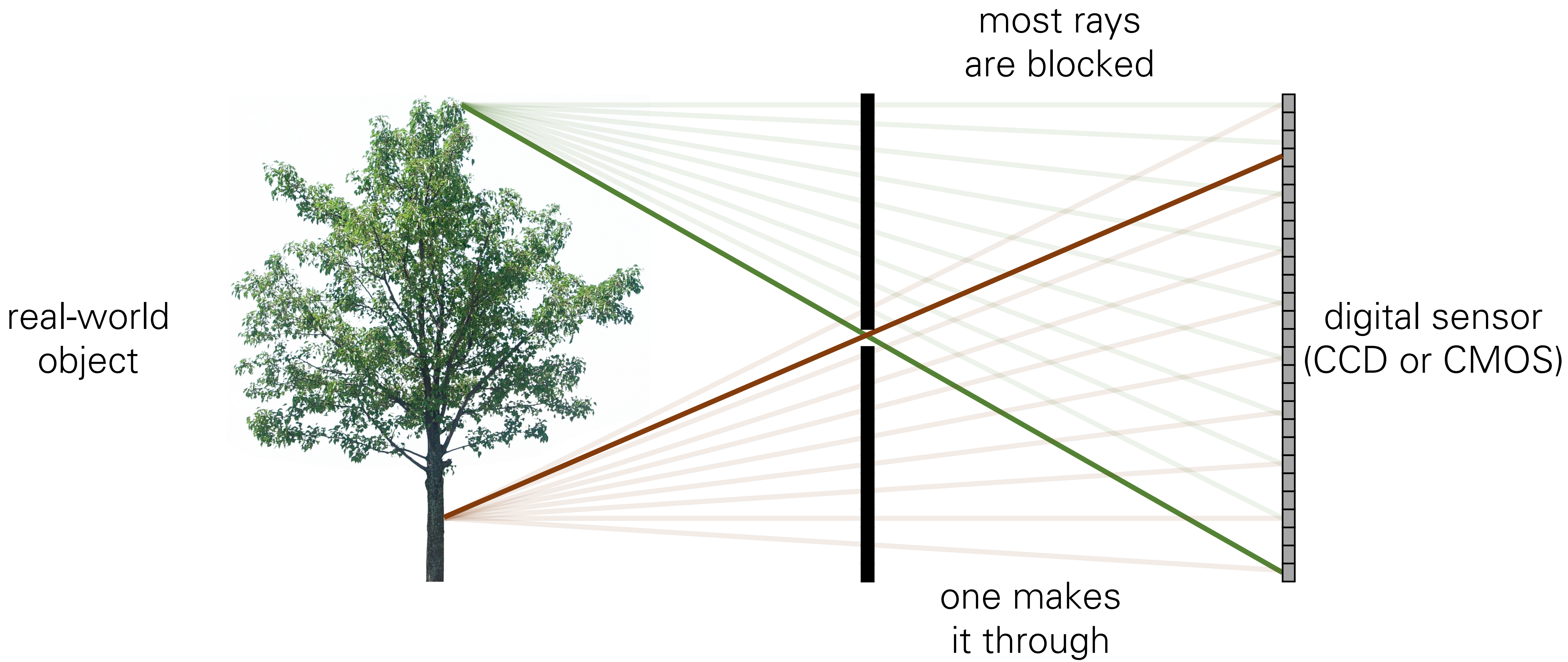



What would an image taken like this look like?

# Pinhole imaging

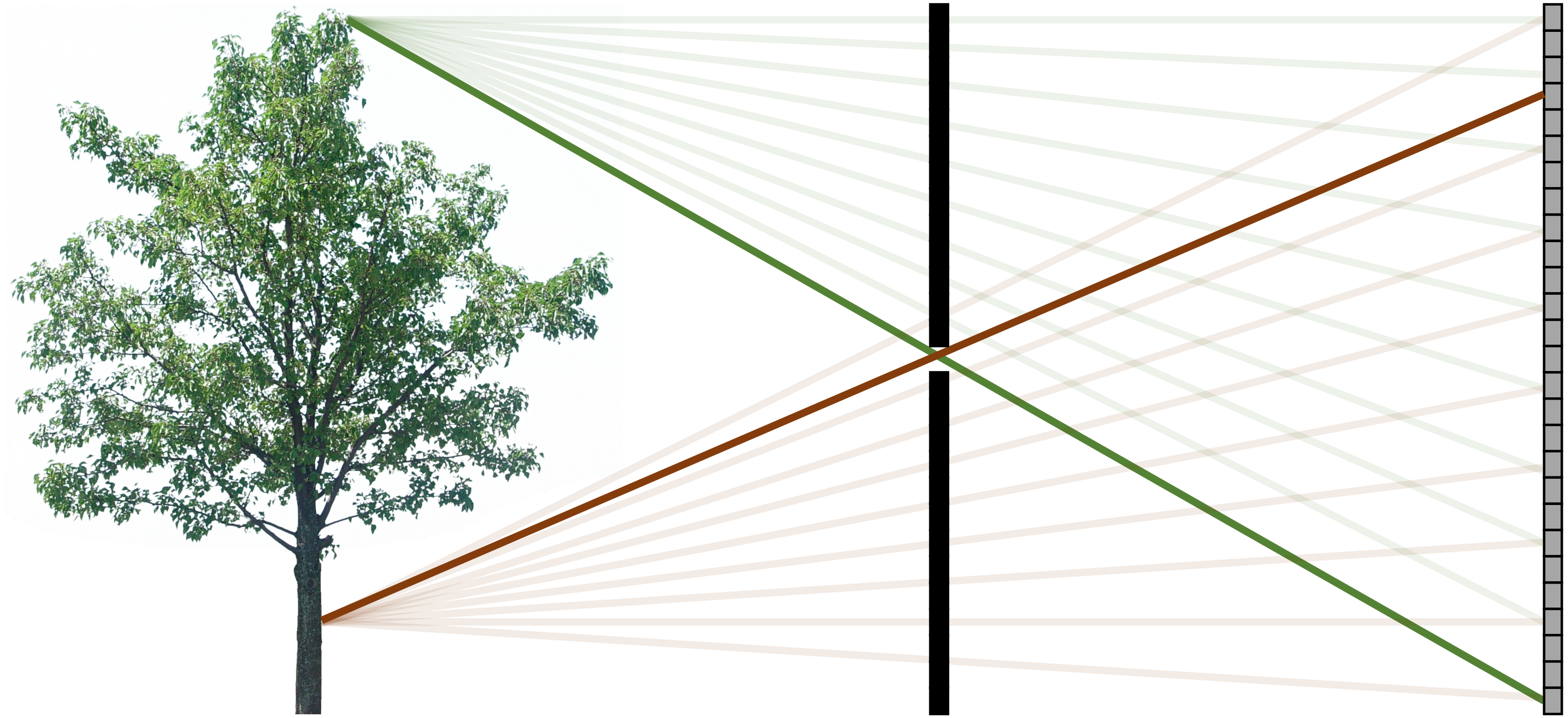


# Pinhole imaging



# Pinhole imaging

real-world
object



digital sensor
(CCD or CMOS)

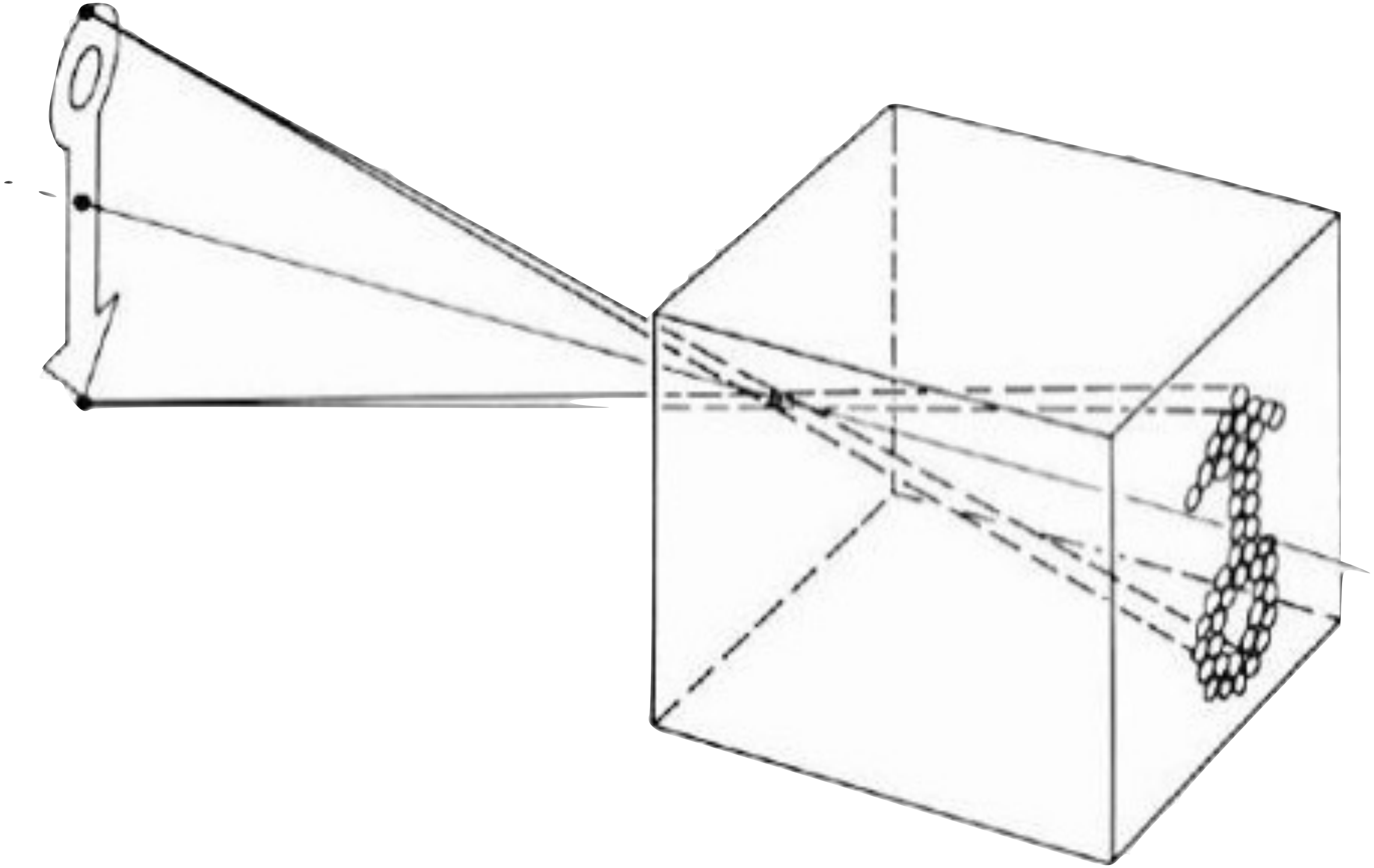
Each scene point contributes to only one sensor pixel

What does the image
on the sensor look like?

# Pinhole imaging

# Pinhole camera

# Pinhole camera a.k.a. camera obscura

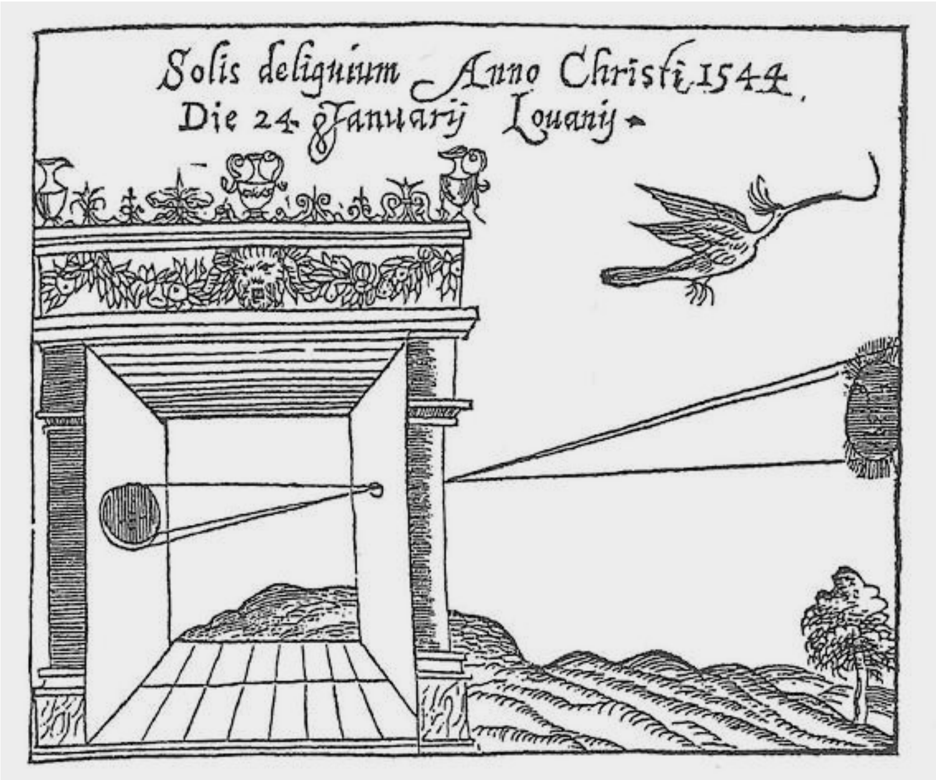


# Pinhole camera a.k.a. camera obscura

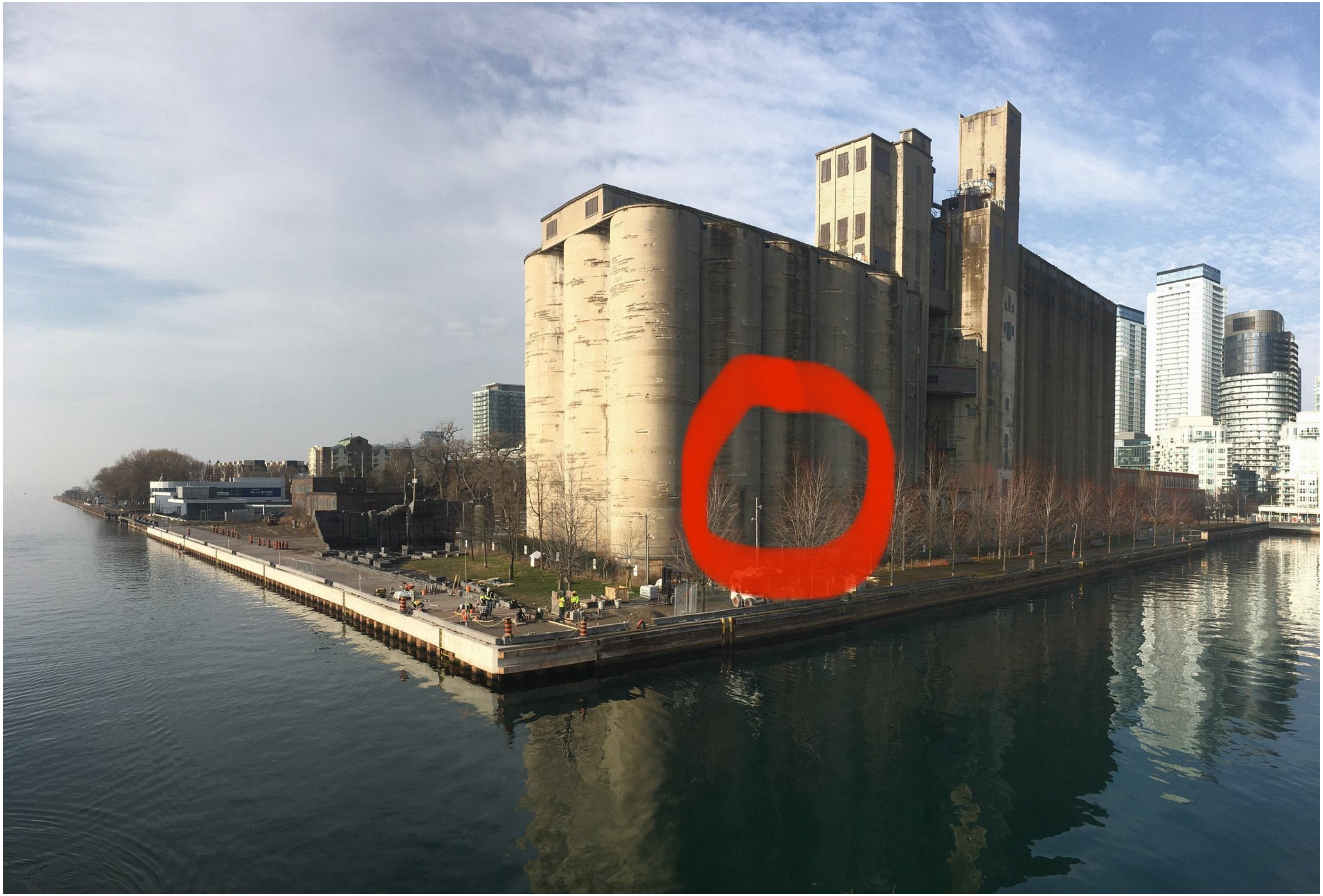
First mention …

Chinese philosopher Mozi
(470 to 390 BC)

First camera …

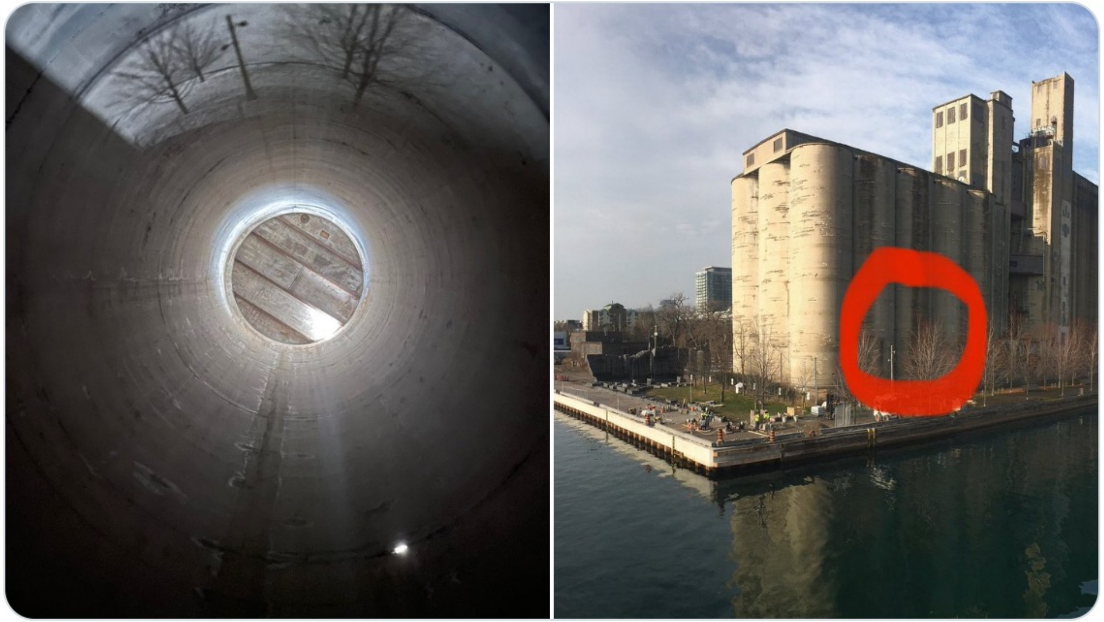



Greek philosopher Aristotle
(384 to 322 BC)



**Bryan Bowen**
@bryanmbowen

Fun discovery - a small crack in the eastern facade of the Canada Malting Co silos has created a perfect pinhole camera. The result: real time projection of Toronto's waterfront on the silo's interior curved surfaces. An unplugged projection show!



9:37 AM · Jan 27, 2022 · Twitter for iPhone

**656** Retweets **70** Quote Tweets **2,836** Likes

Abelardo Morell

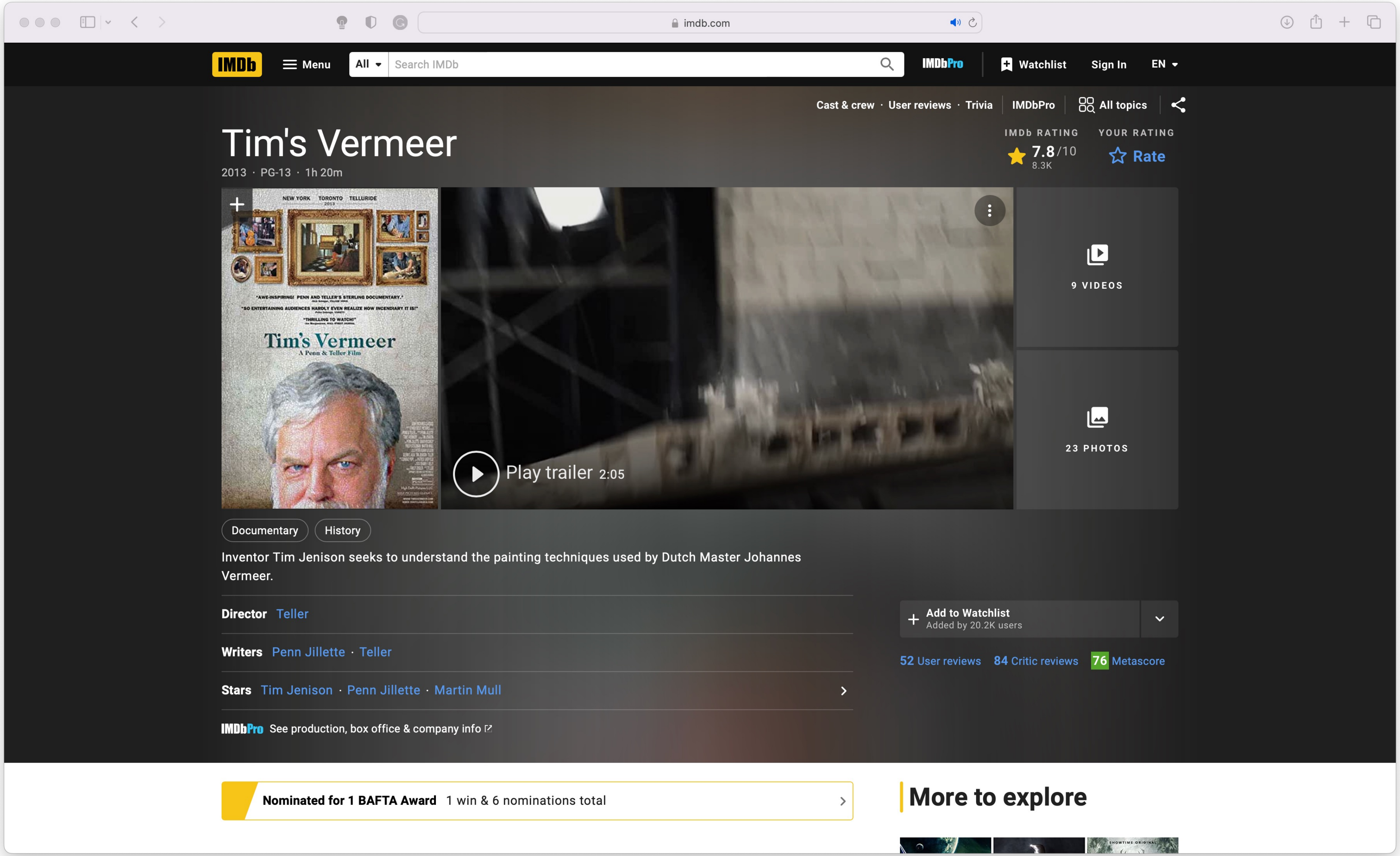

BBC FOUR

# Vermeer and The Camera Obscura

Officer and Laughing Girl, 1657

http://www.essentialvermeer.com/camera_obscura/co_one.html

# Vermeer and The Camera Obscura



THE FOLLOWING PREVIEW HAS BEEN **APPROVED** TO

**ACCOMPANY THIS FEATURE**

BY THE MOTION PICTURE ASSOCIATION OF AMERICA, INC.

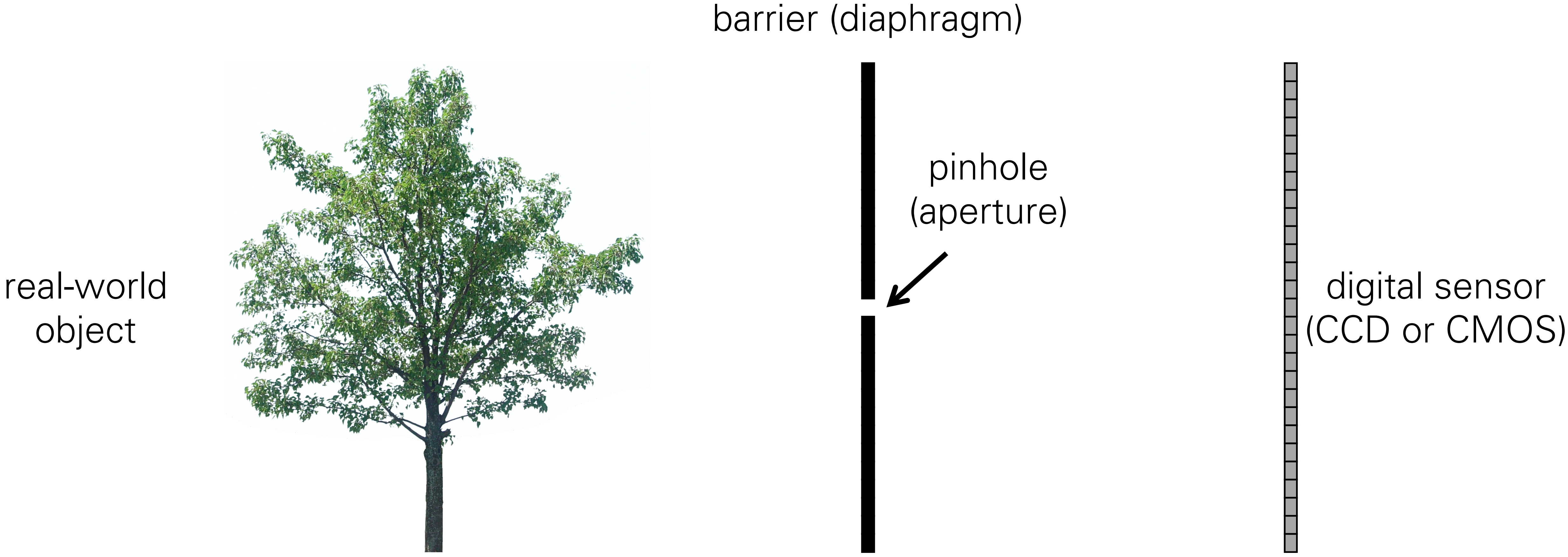
THE FILM ADVERTISED HAS BEEN RATED

| PARENTS STRONGLY CAUTIONED<br>**PG-13** | SOME STRONG LANGUAGE |
| --- | --- |
| Some Material May Be Inappropriate for Children Under 13 ® | |

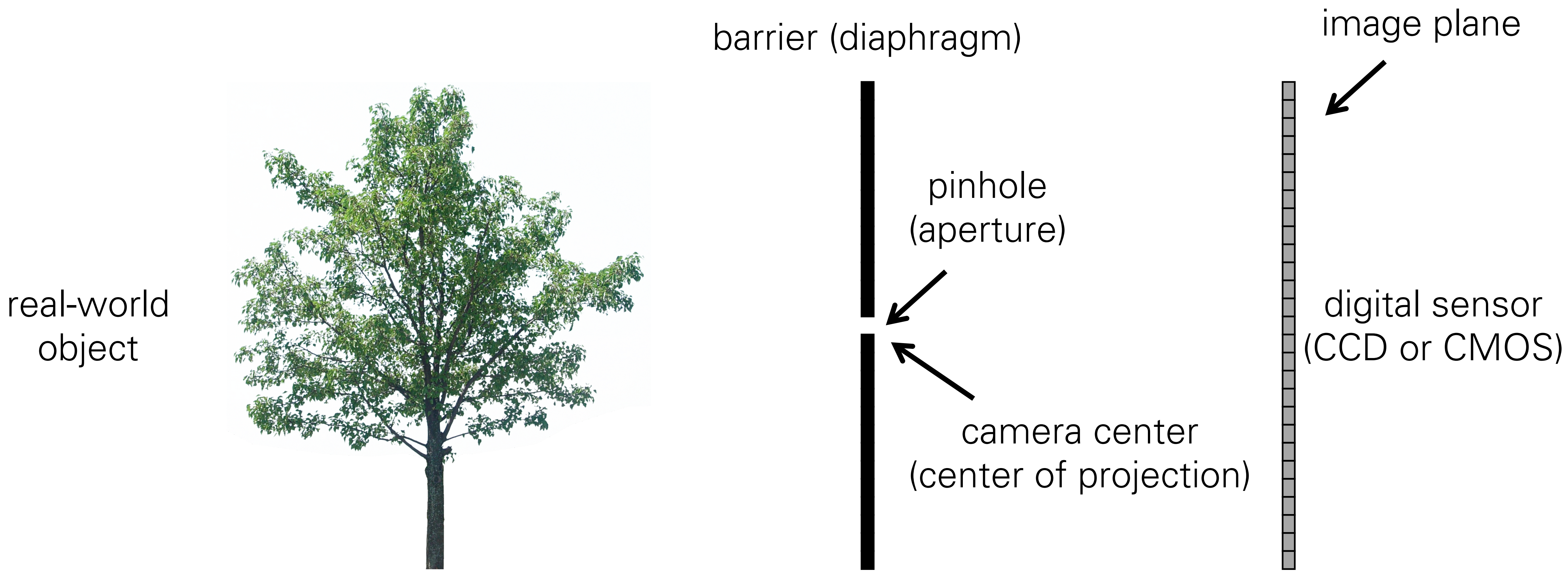
www.filmratings.com

www.mpaa.org

# Pinhole camera terms

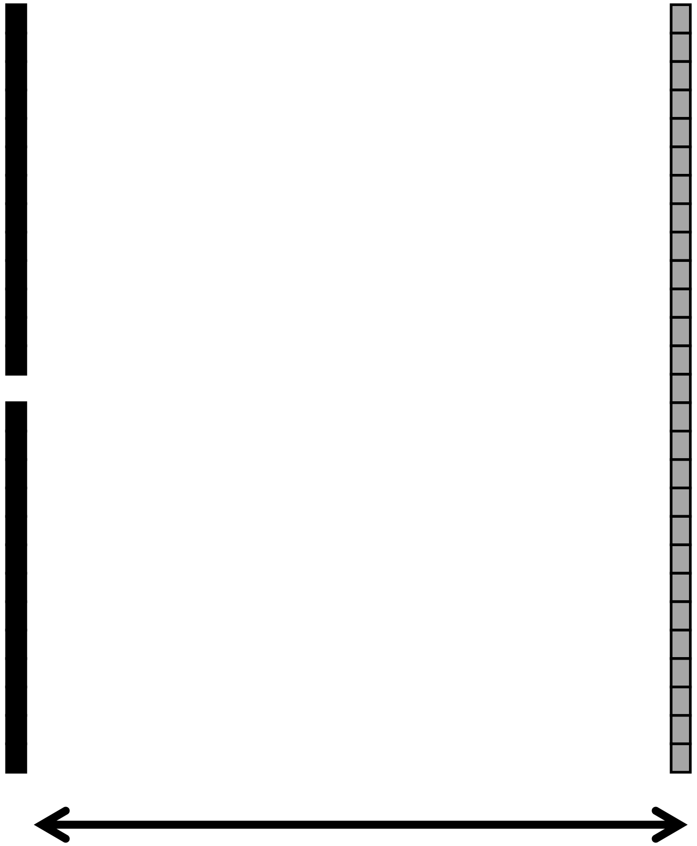


# Pinhole camera terms



# Focal length

real-world
object

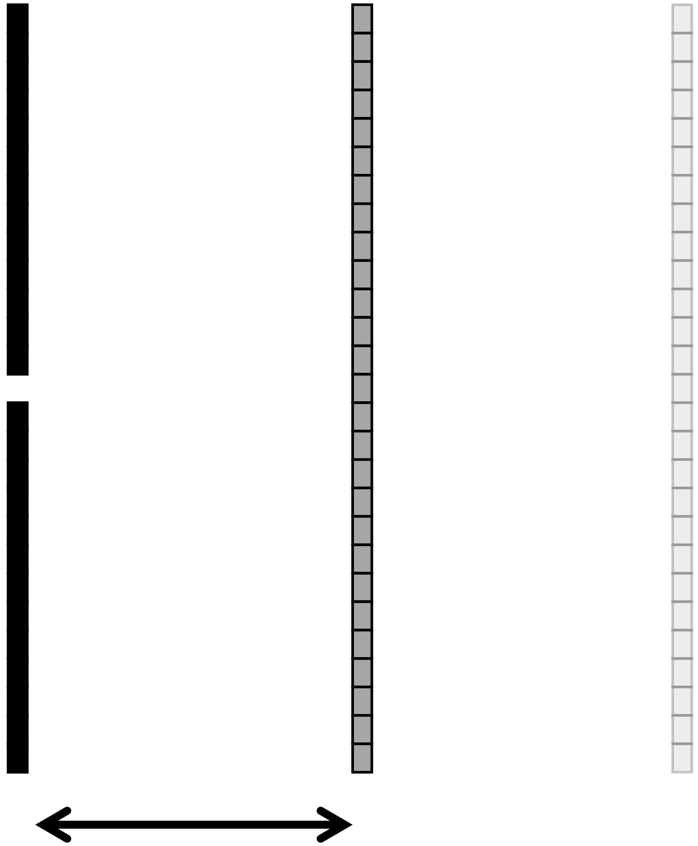


focal length f

# Focal length

What happens as we change the focal length?

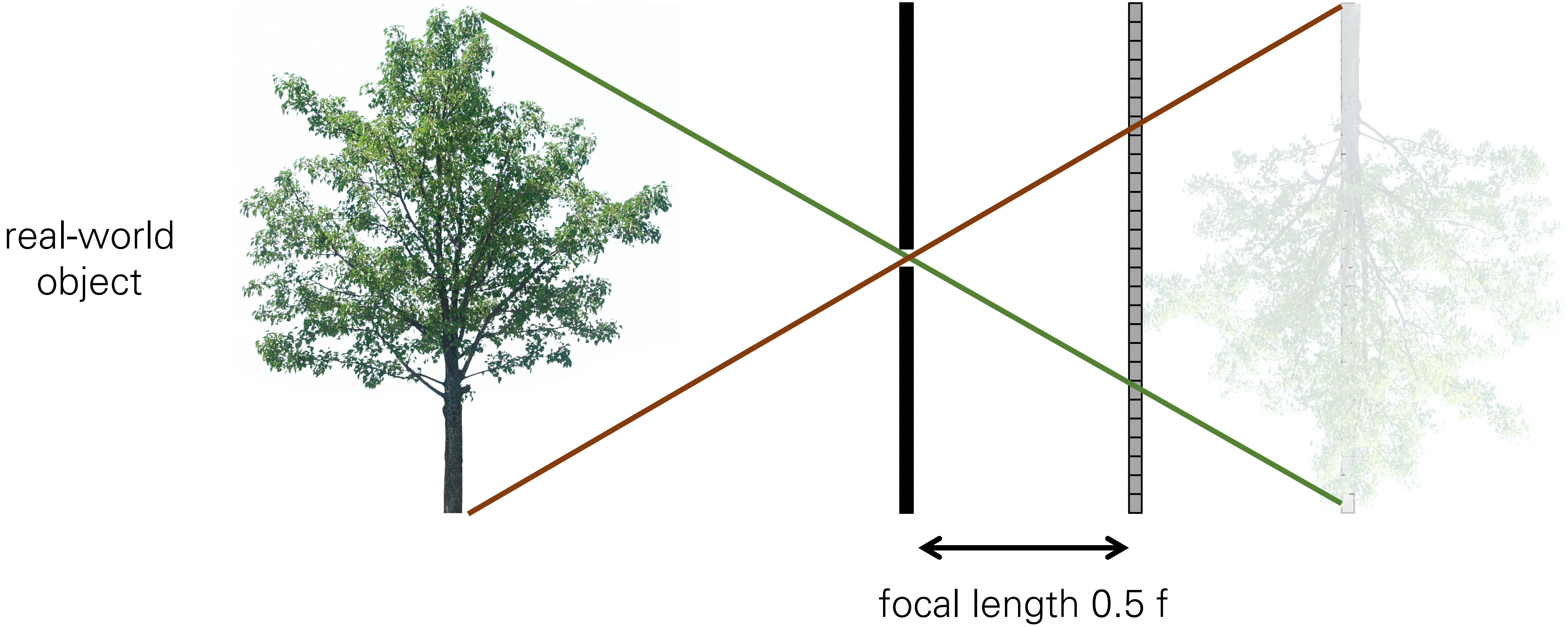
real-world
object



focal length 0.5 f

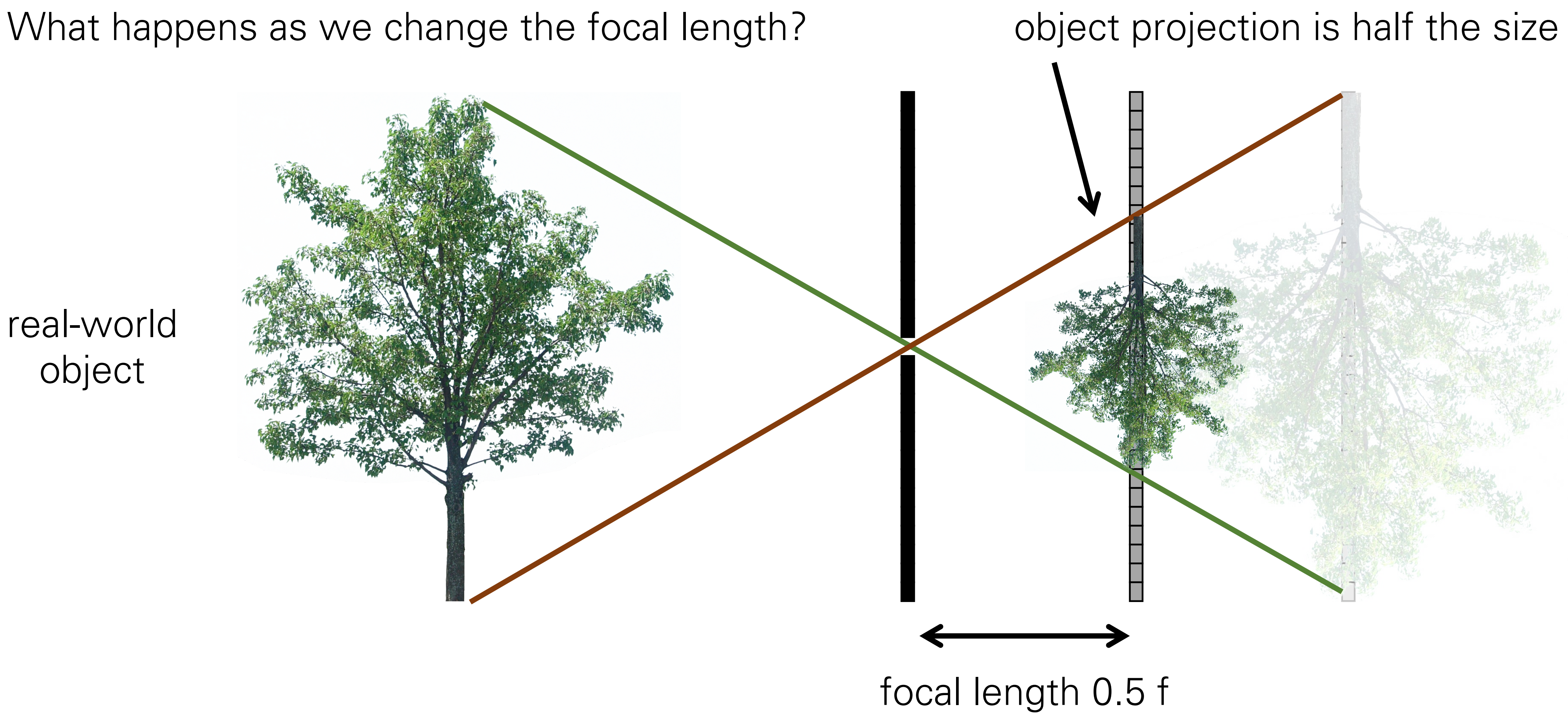
# Focal length

What happens as we change the focal length?

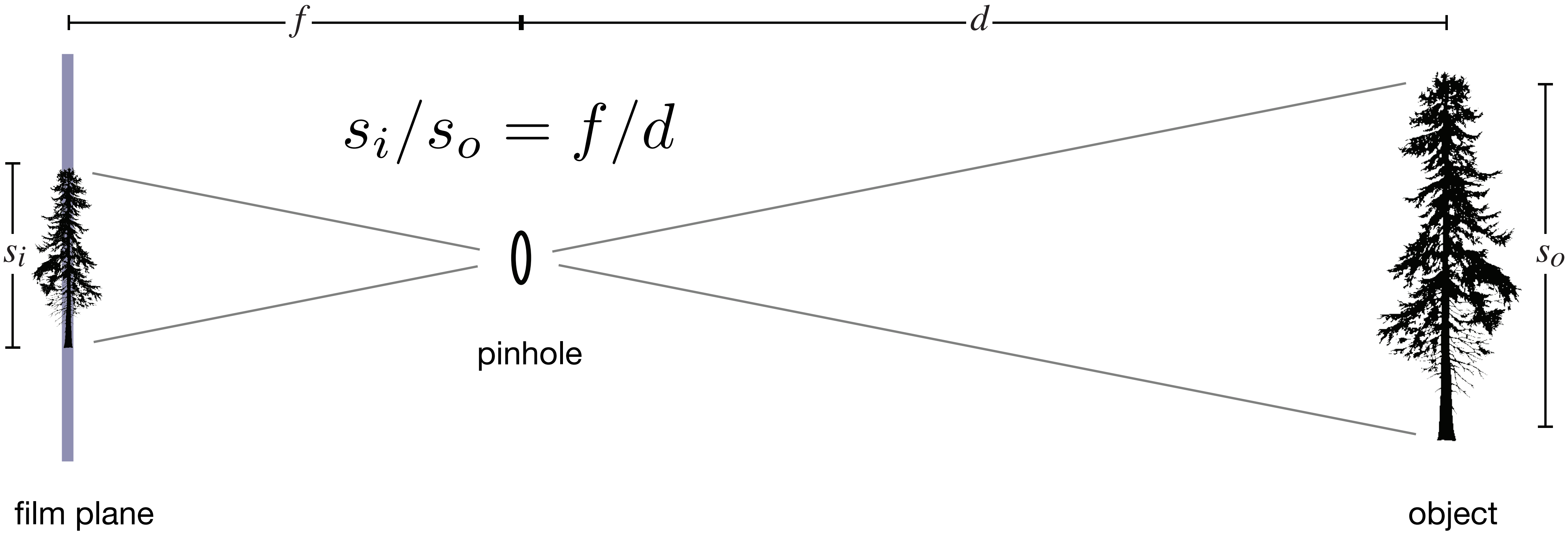


# Focal length

What happens as we change the focal length?

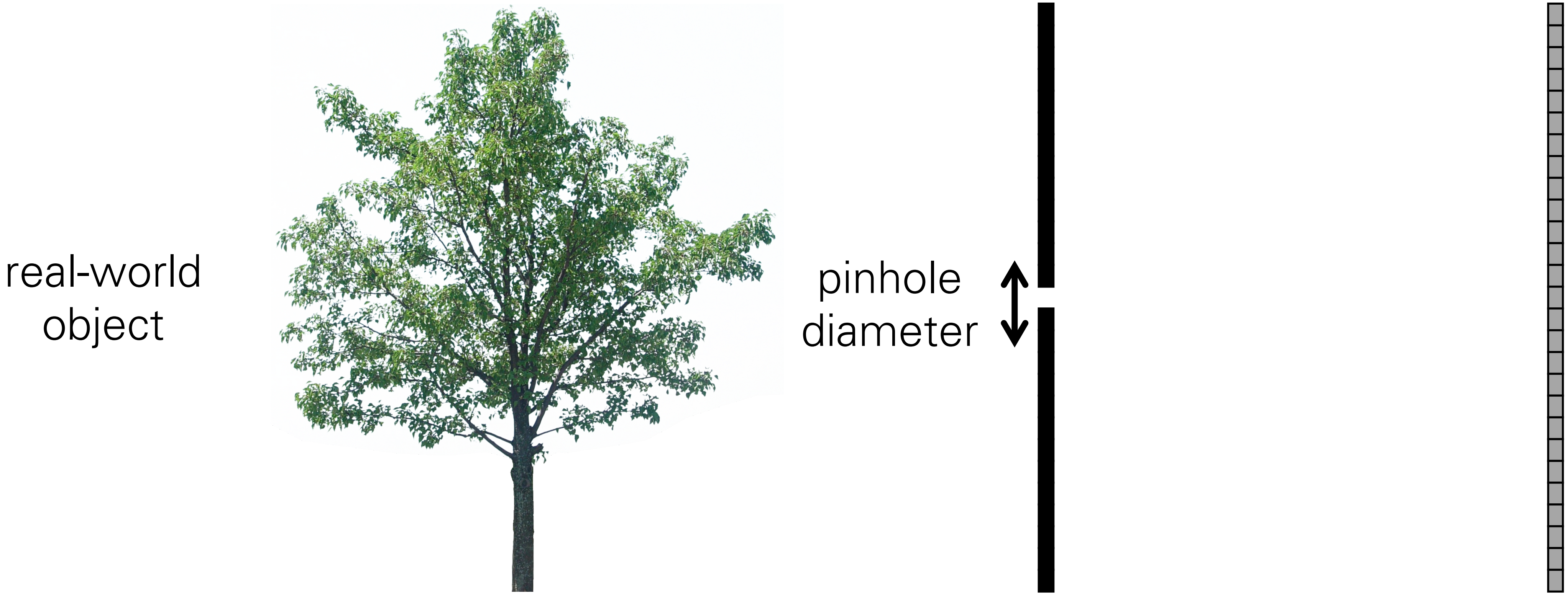


# Focal length




- Double "focal length" leads to
  - image twice as large
  - one fourth as much illumination at image plane

# Pinhole size

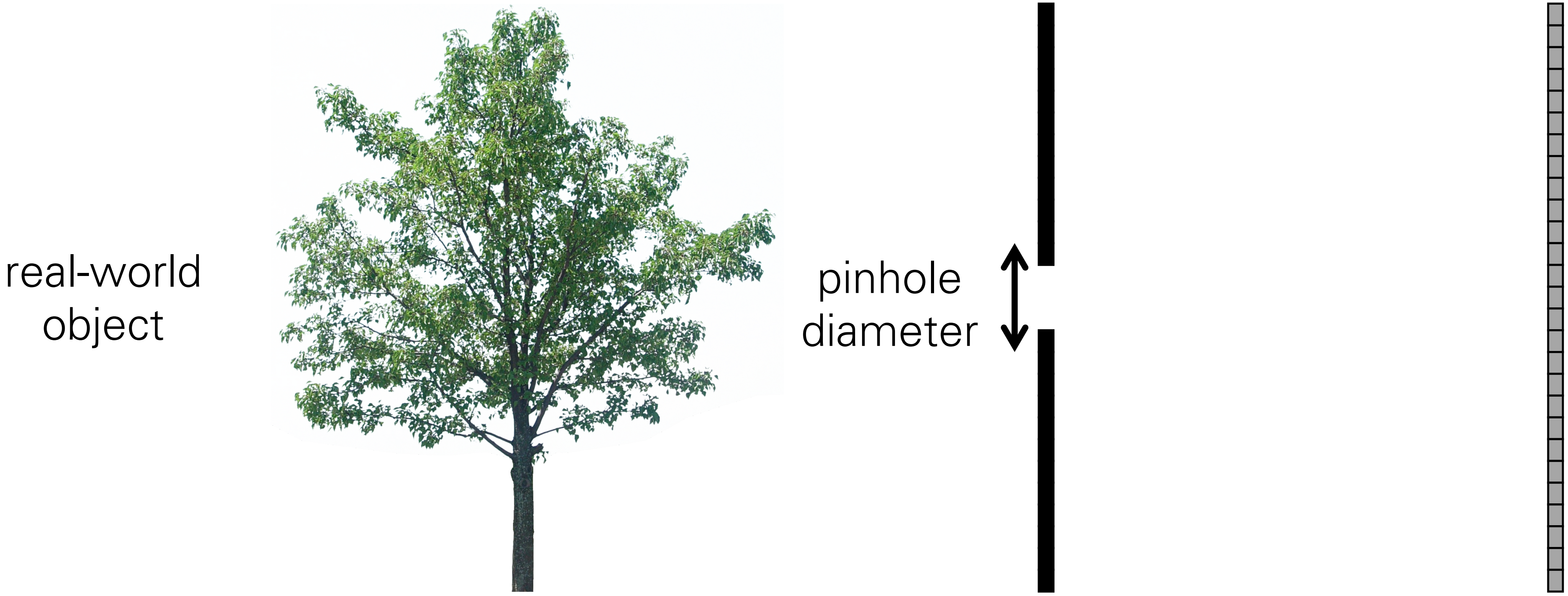



Ideal pinhole has infinitesimally small size

- In practice that is impossible.

# Pinhole size

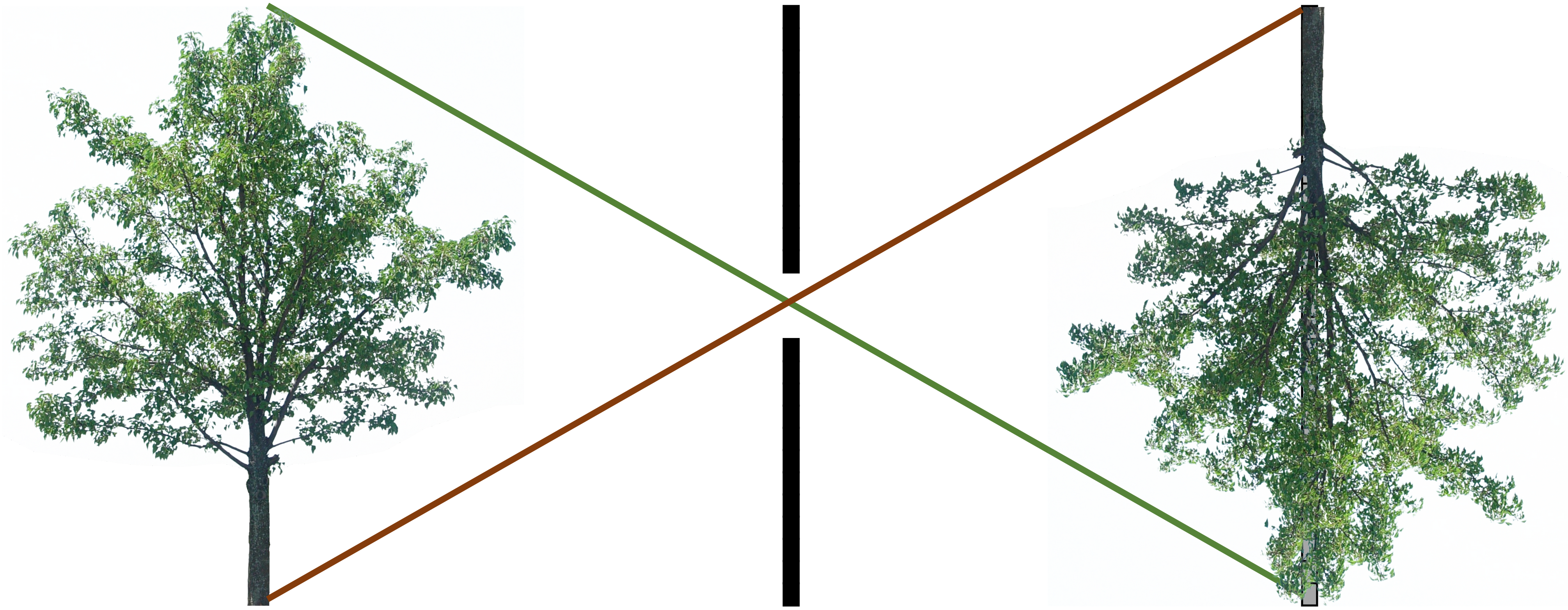
What happens as we change the pinhole diameter?



# Pinhole size

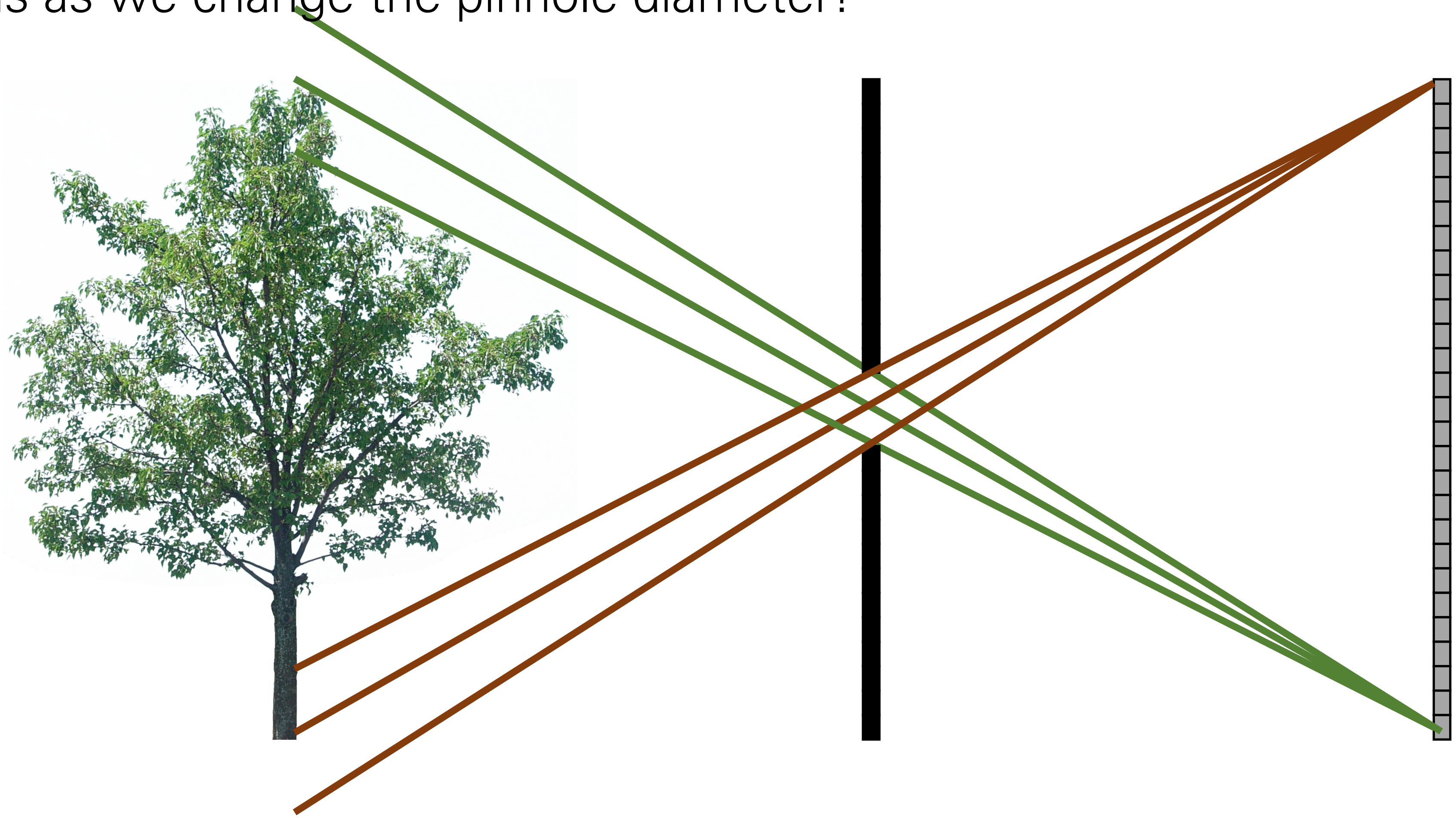
What happens as we change the pinhole diameter?

real-world
object



# Pinhole size

What happens as we change the pinhole diameter?



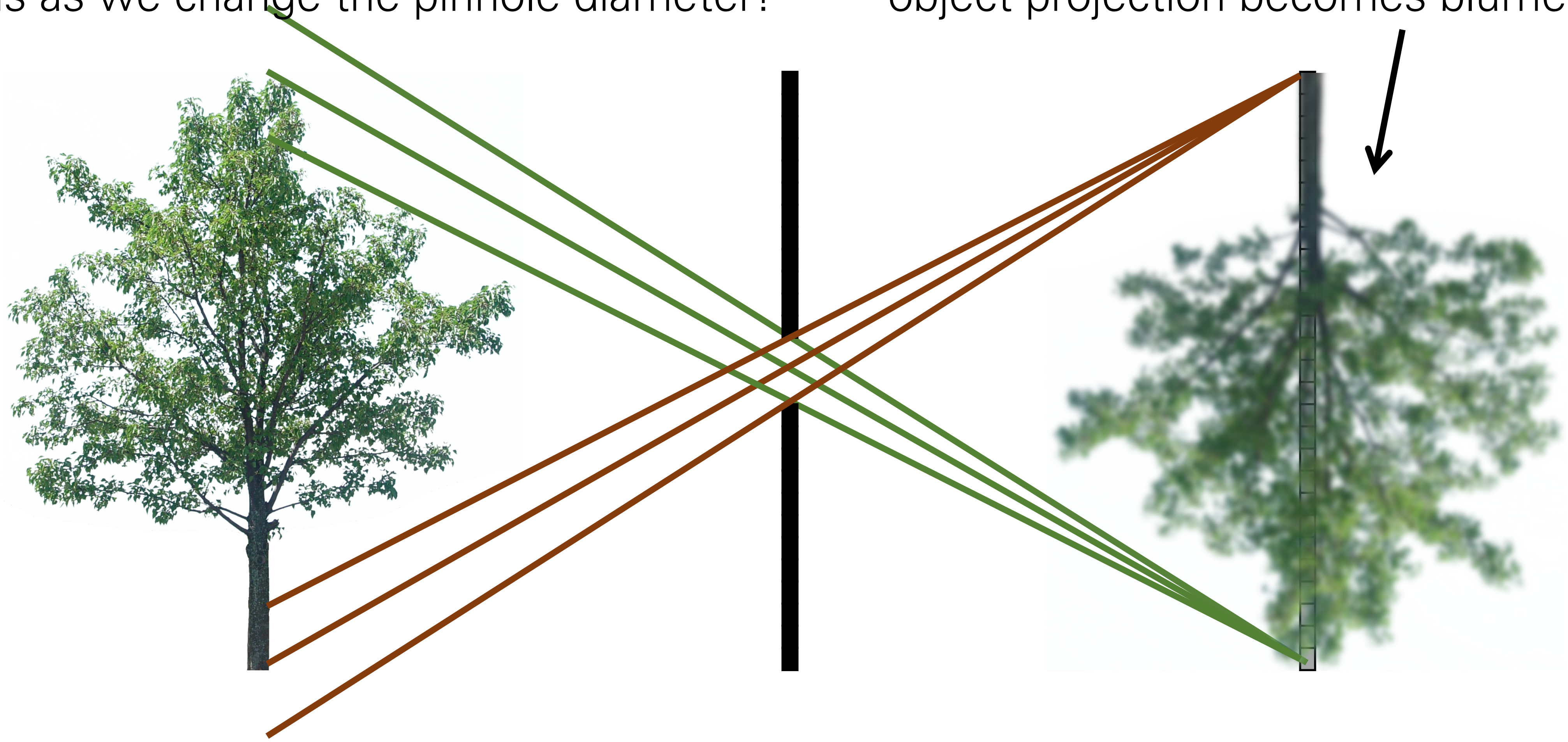
real-world object

# Pinhole size

What happens as we change the pinhole diameter?

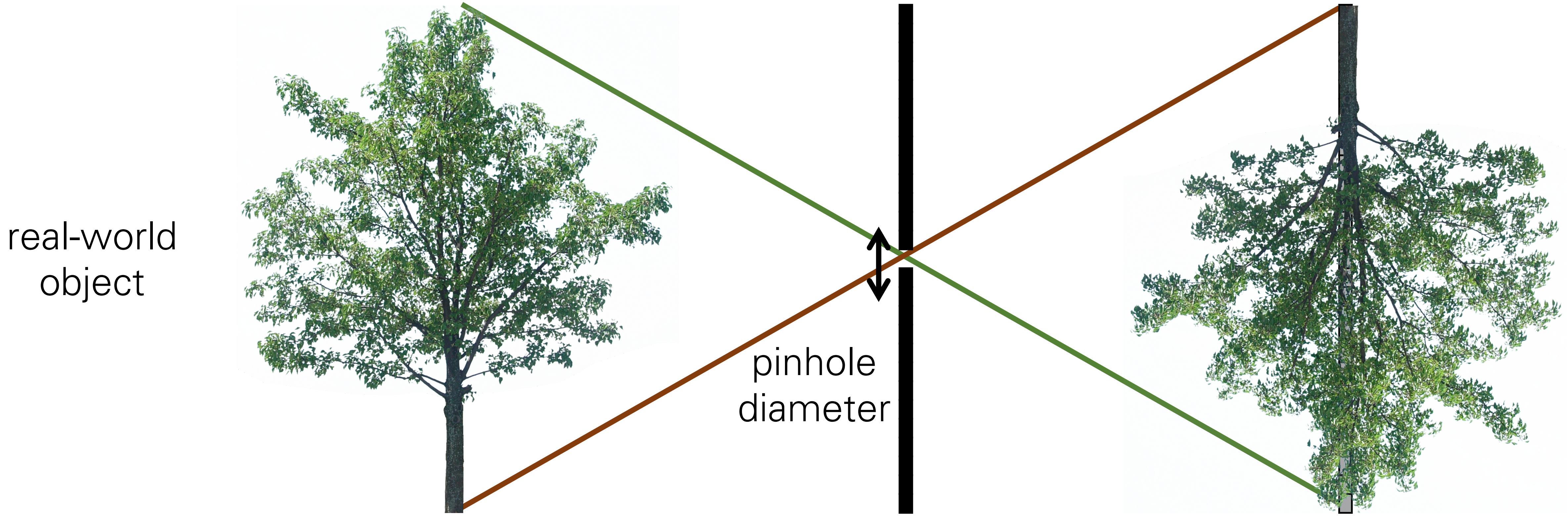
object projection becomes blurrier

real-world object



# Pinhole size

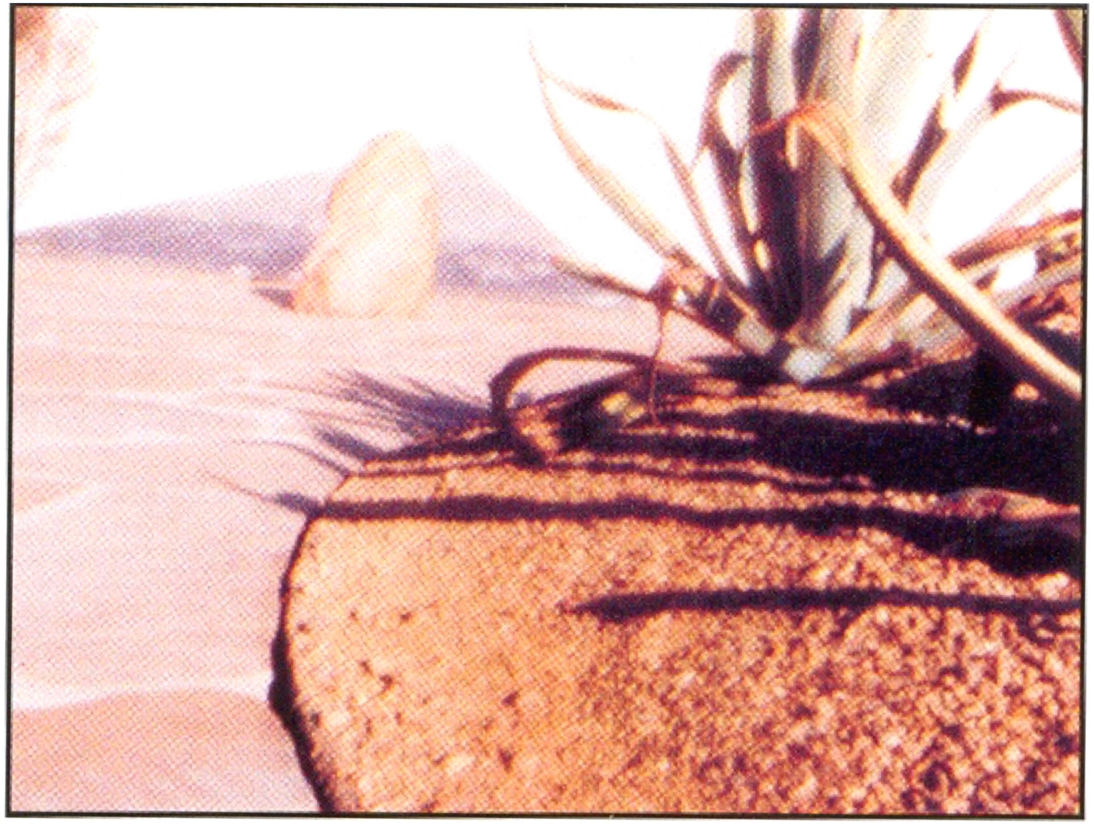
What happens as we change the pinhole diameter?

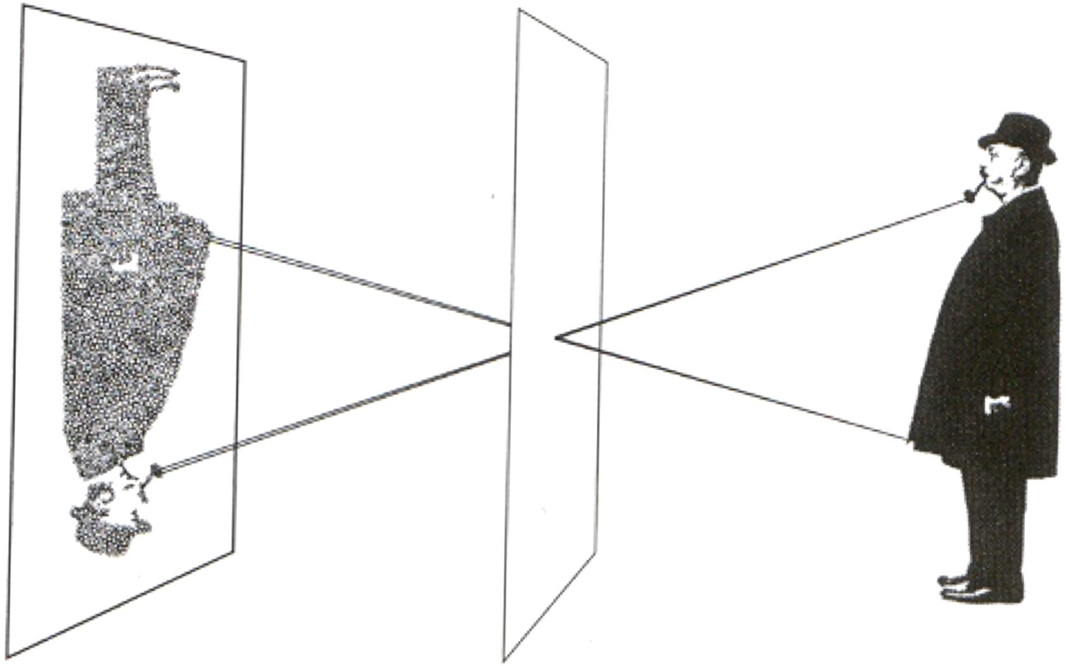



Will the image keep getting sharper the smaller we make the pinhole?

# Effect of pinhole size

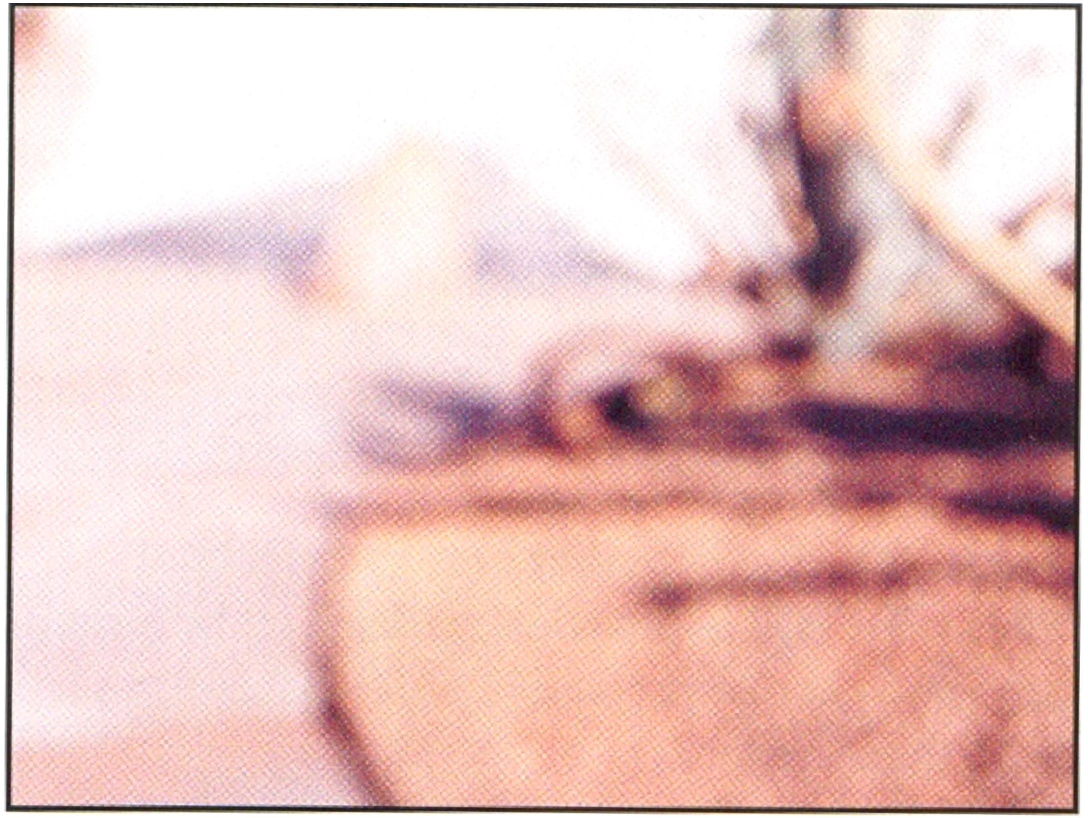
Photograph made with small pinhole

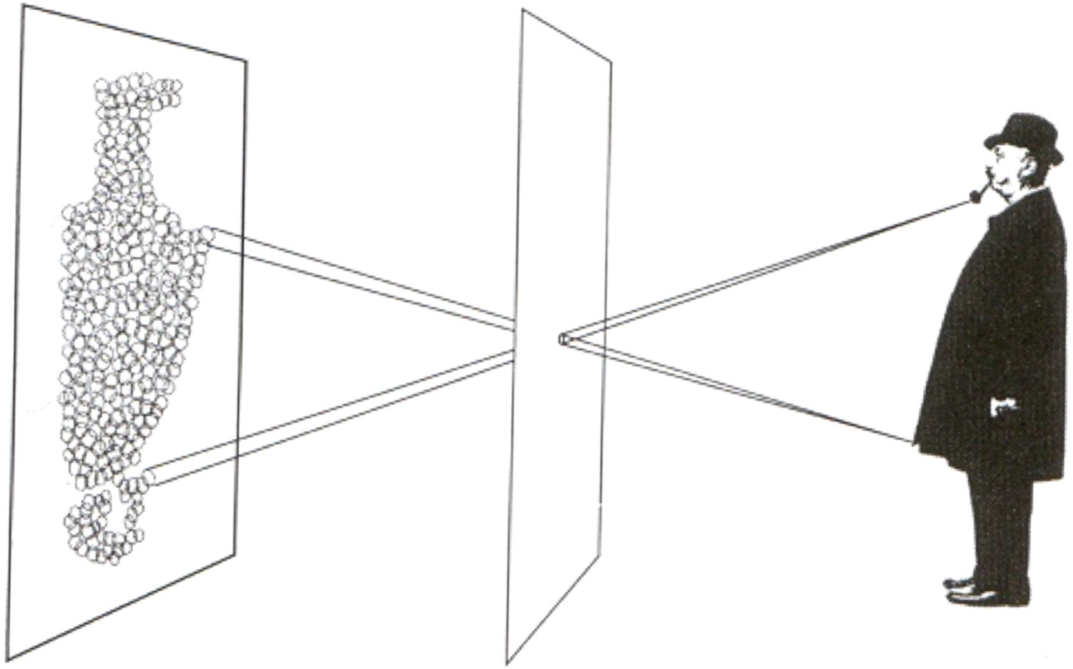




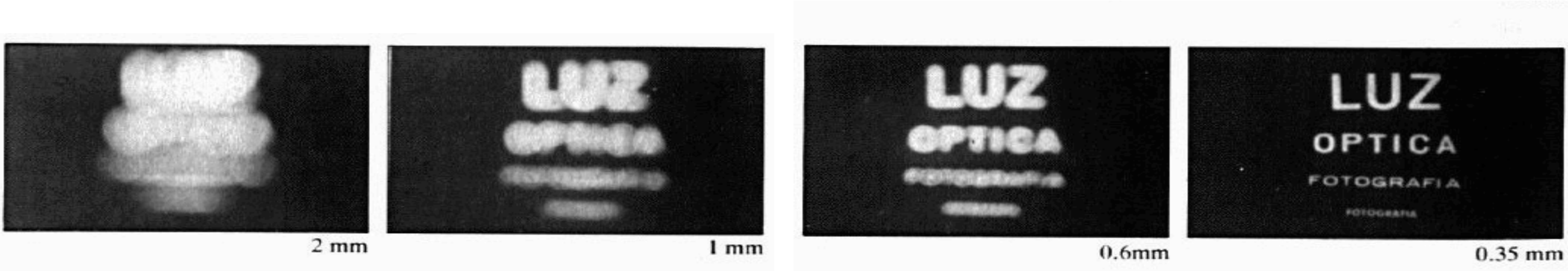
# Effect of pinhole size

Photograph made with larger pinhole

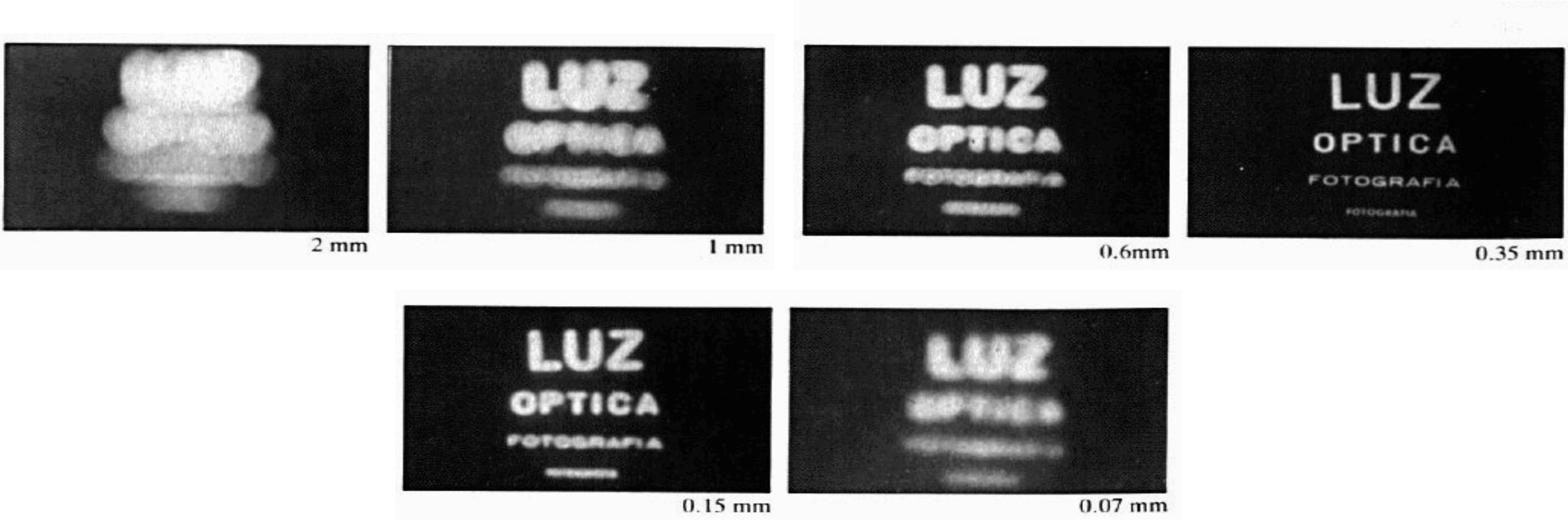




# Smaller pinhole is sharper

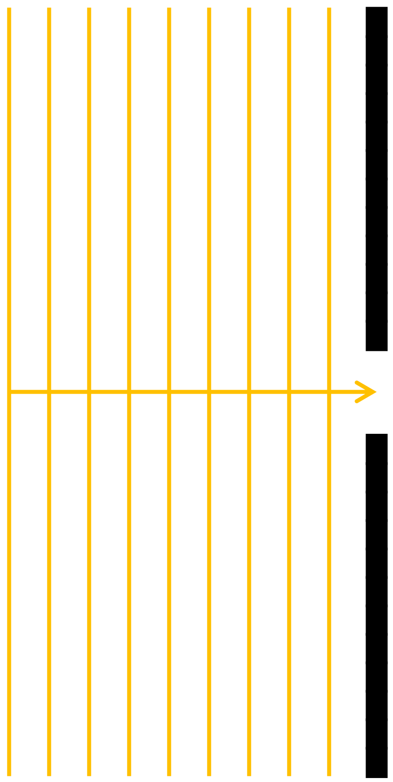


# Smaller pinhole is sharper … to a point

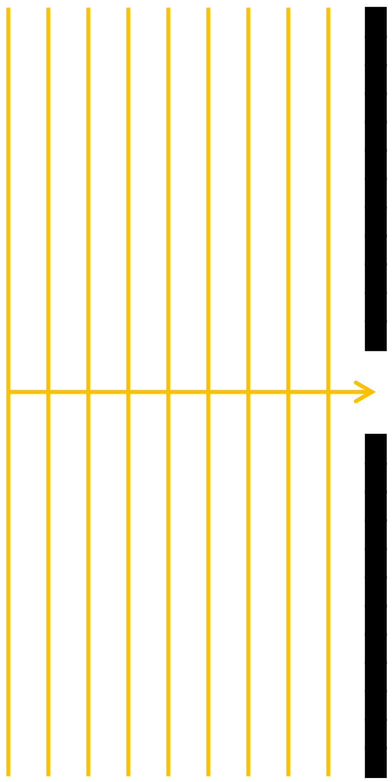


# Diffraction limit

A consequence of the wave nature of light



What do geometric optics
predict will happen?



What do wave optics
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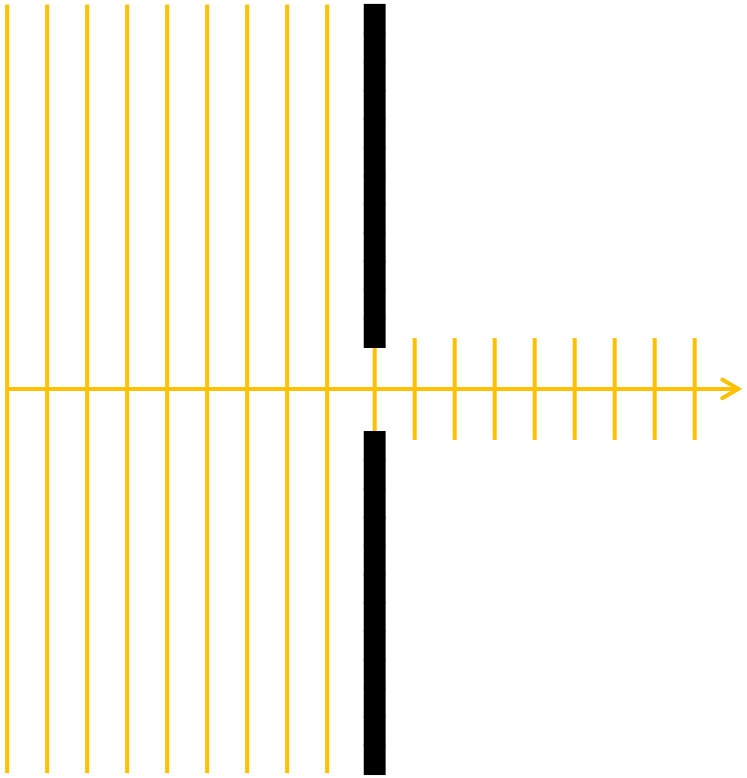
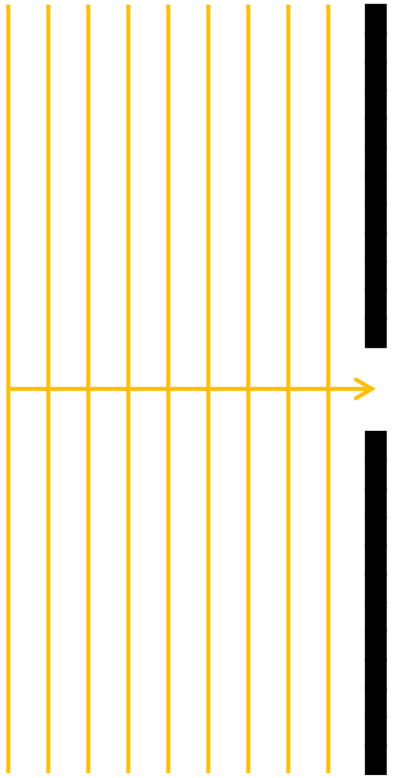
# Diffraction limit

A consequence of the wave nature of light



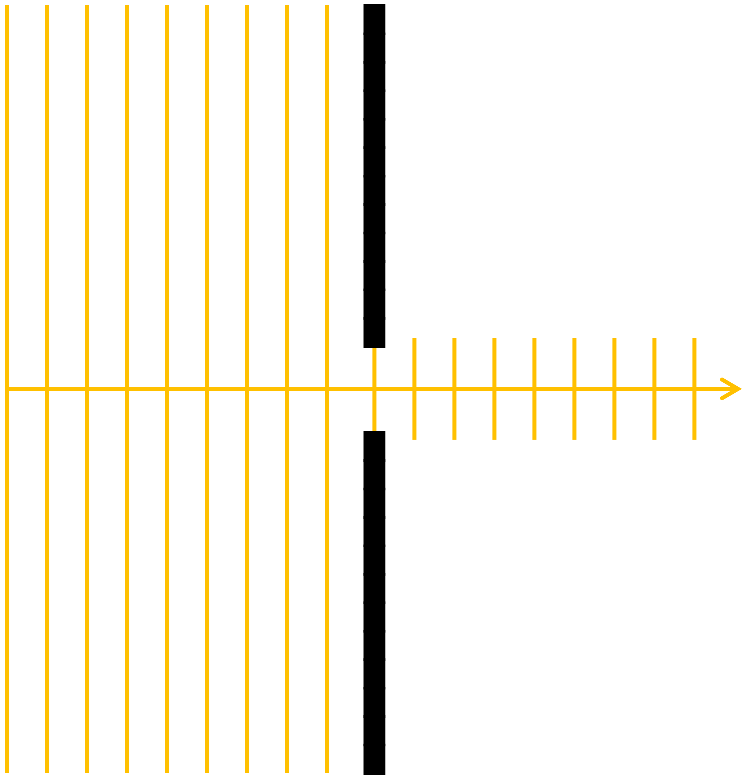
What do geometric optics predict will happen?



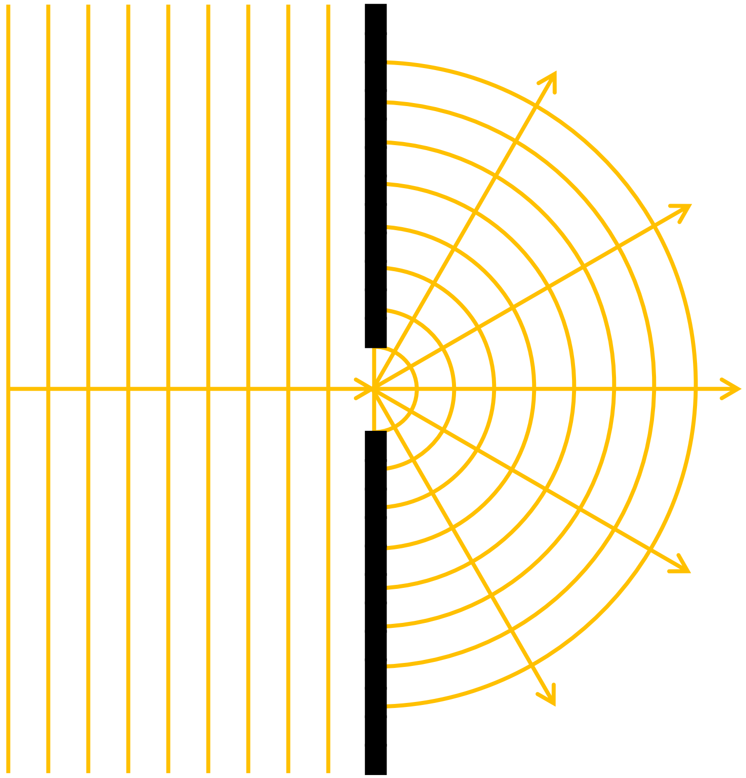
What do wave optics predict will happen?

# Diffraction limit

A consequence of the wave nature of light



What do geometric optics predict will happen?



What do wave optics predict will happen?

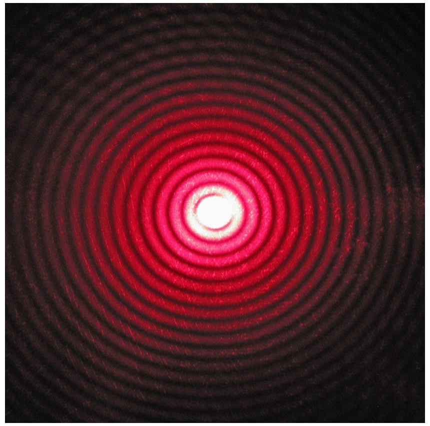
# Diffraction

Reminder: Physical water wave diffracting through an aperture to create circular patterns.

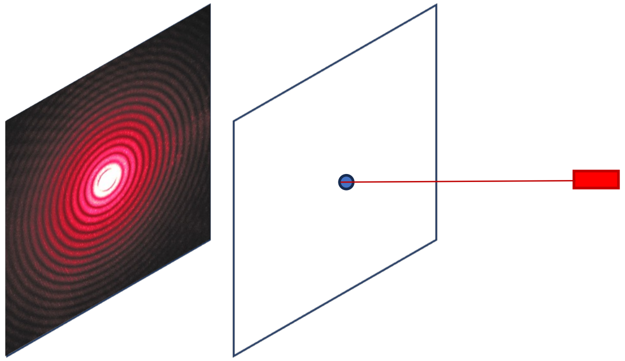
[Verbcatcher; Wikipedia]

## Light as a wave (particle-wave duality)

[Wisky; Wikipedia]



Diffraction pattern

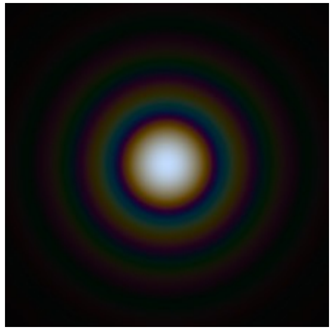



Airy pattern
As above. Diffraction-limited smallest pattern that a circular aperture (with perfect lens) makes from a point light source.

Airy disc – central disc or lobe.

Important in understanding limits of imaging:
Point sources with overlapping Airy discs cannot be clearly separated anymore.

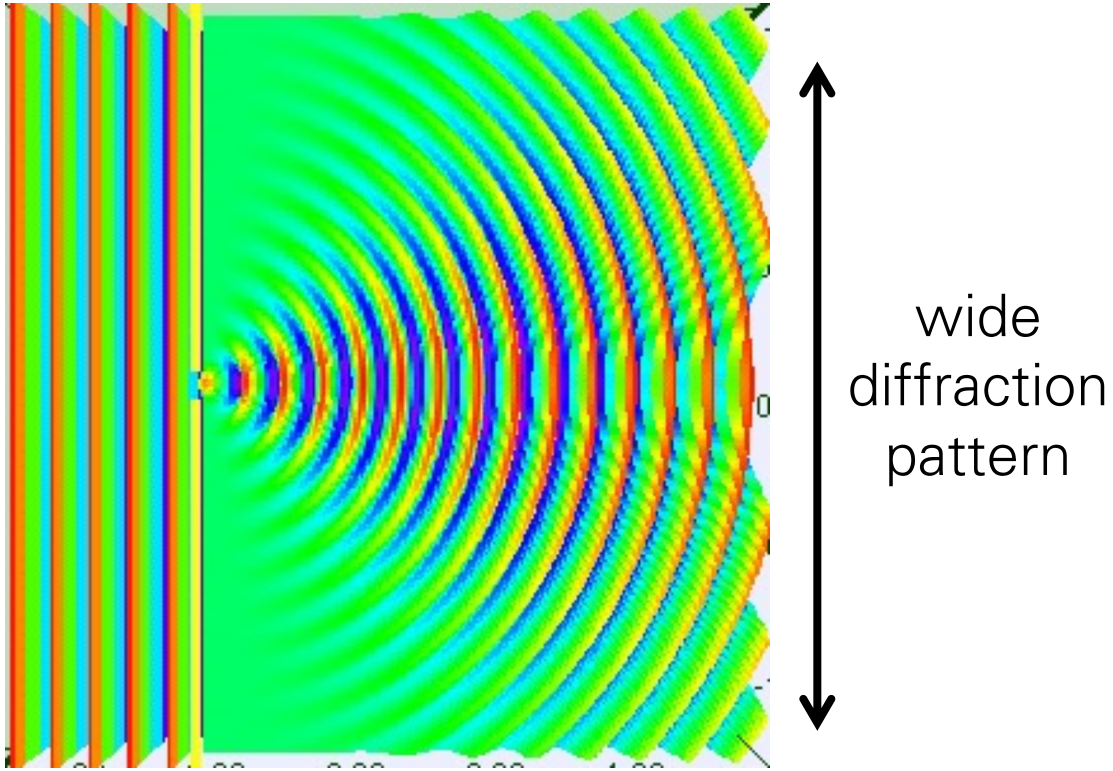
Simulated Airy pattern – note diffraction spectral (red/blue) shift



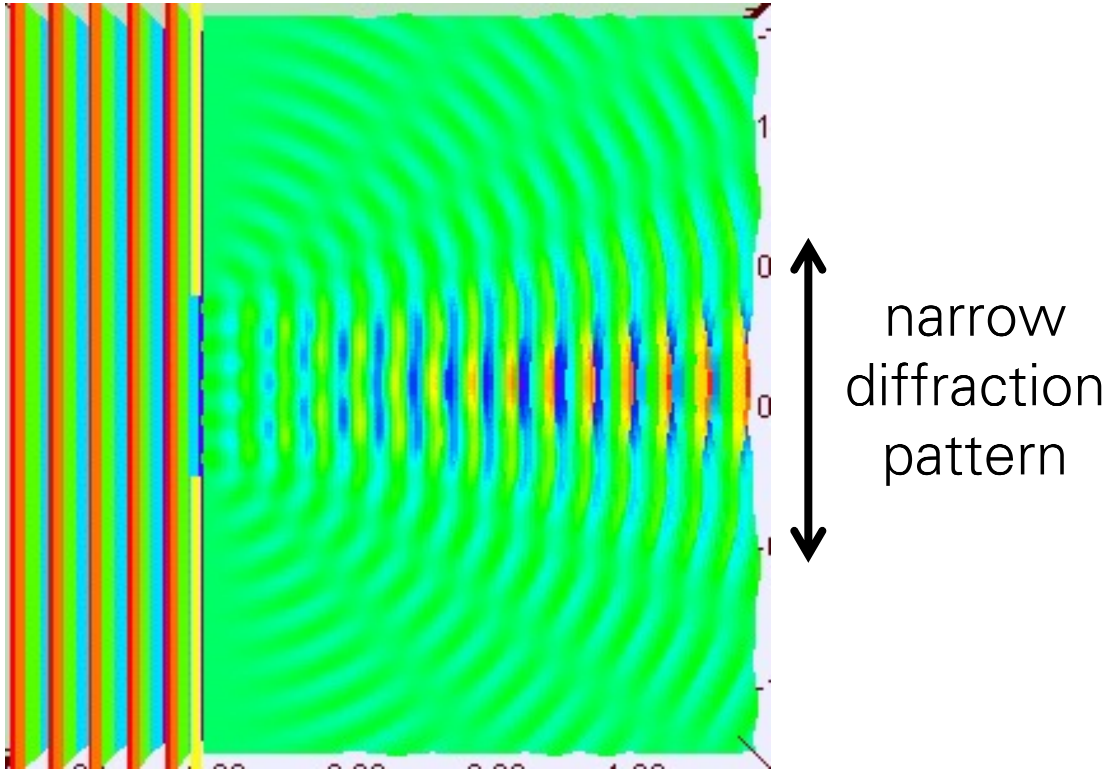
# Diffraction limit

Diffraction pattern = Fourier transform of the pinhole.

- Smaller pinhole means bigger Fourier spectrum.
- Smaller pinhole means more diffraction.

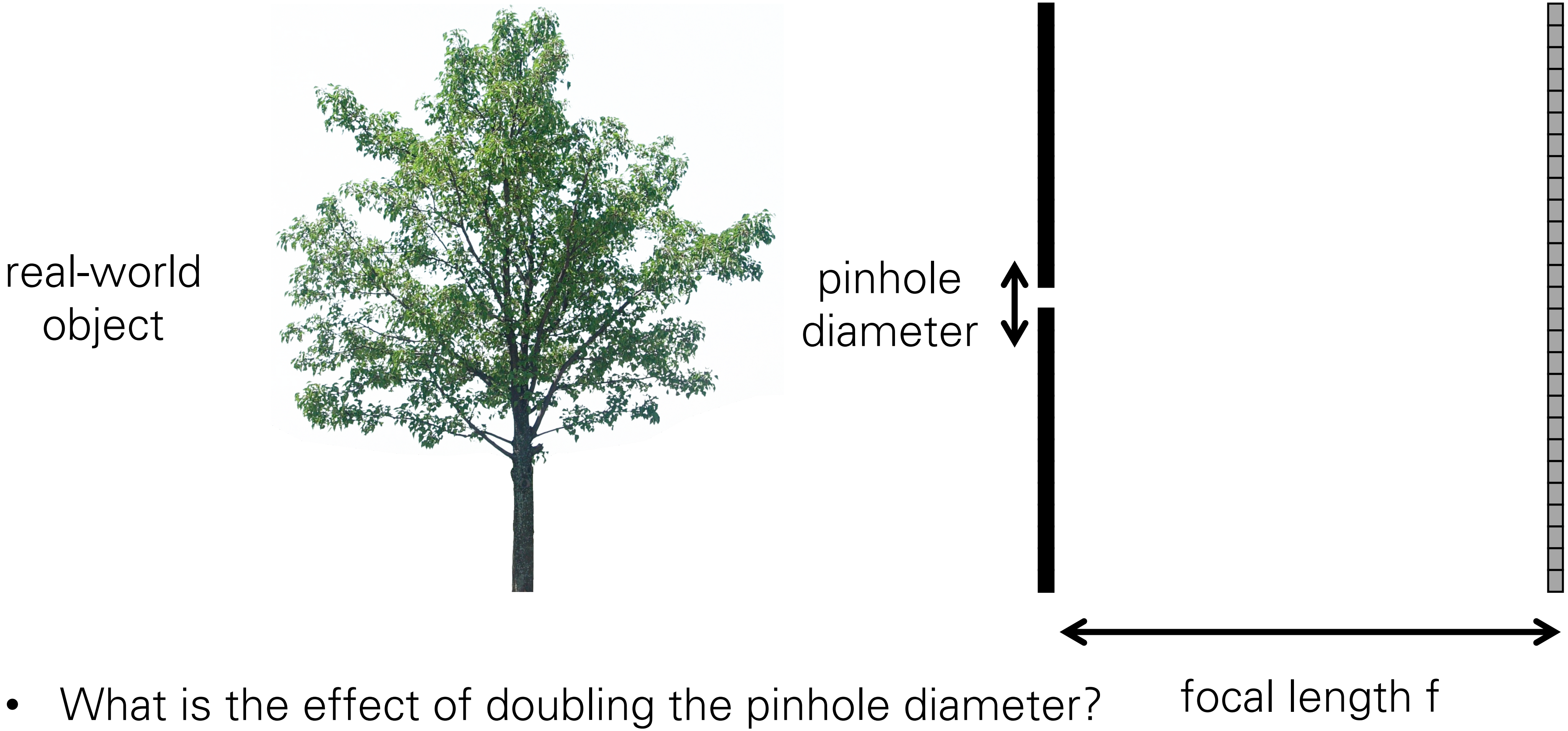



small pinhole



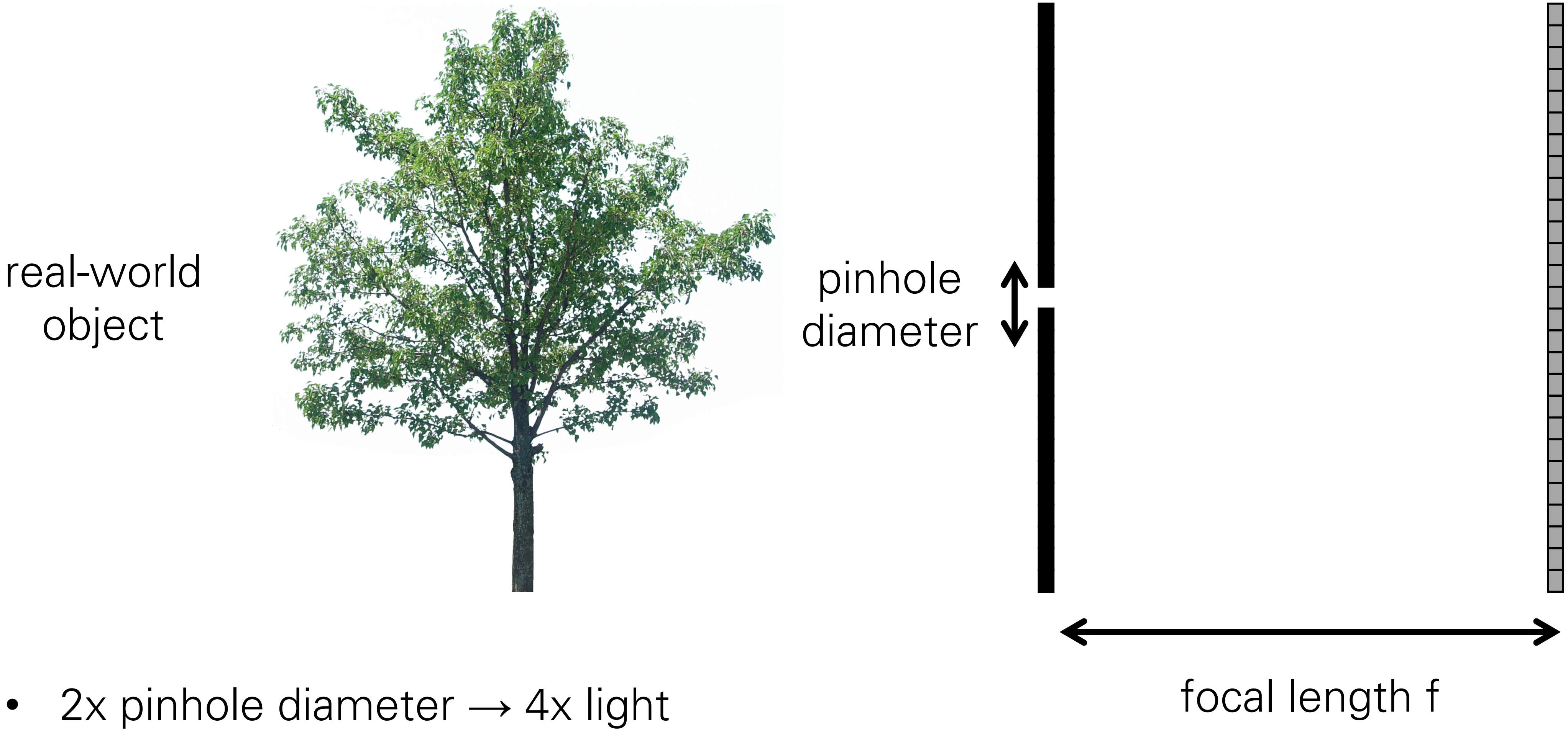

large pinhole

# What about light efficiency?




- What is the effect of doubling the pinhole diameter?
- What is the effect of doubling the focal length?

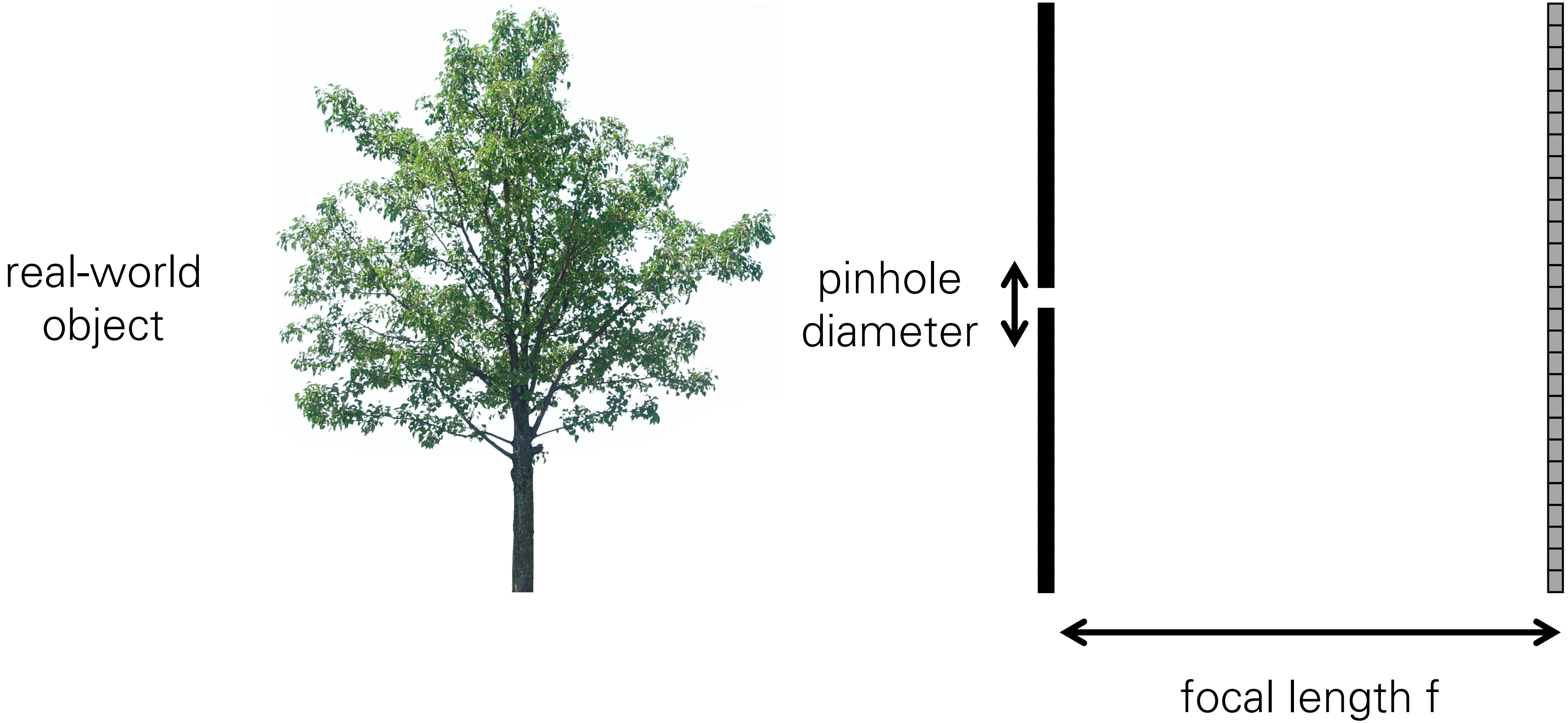
# What about light efficiency?




- 2x pinhole diameter → 4x light
- 2x focal length → ¼x light

# Some terminology notes

A "stop" is a change in camera settings that changes amount of light by a factor of 2



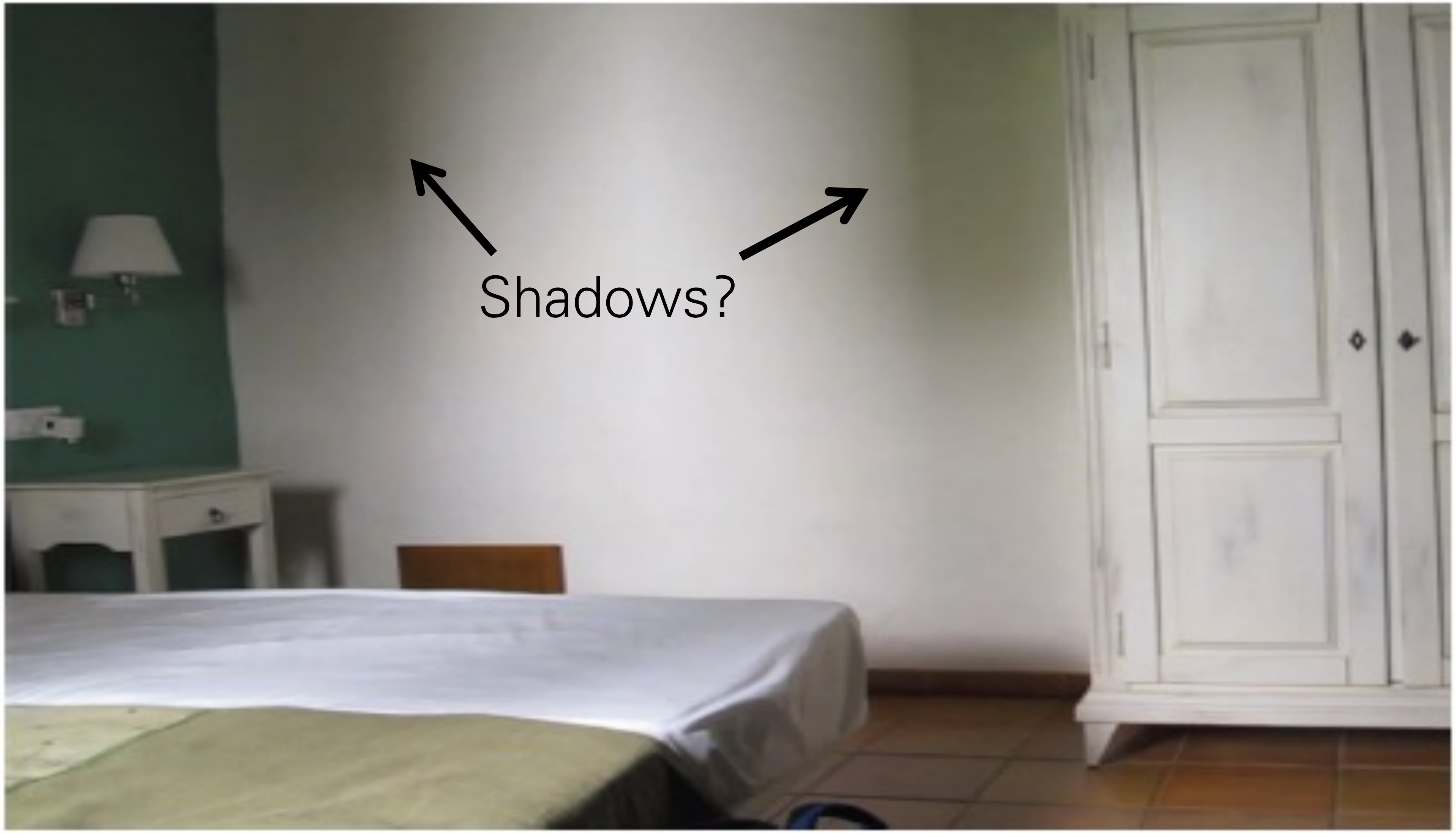

The "f-number" is the ratio: focal length / pinhole diameter

# Accidental pinholes

# What does this image say about the world outside?

# Accidental pinhole camera

# Accidental pinhole camera



# Accidental pinhole camera

# Accidental pinhole camera

# Accidental pinhole camera

**Accidental pinhole and pinspeck cameras: revealing the scene outside the picture**

Antonio Torralba, William T. Freeman
Computer Science and Artificial Intelligence Laboratory (CSAIL)
MIT
torralba@mit.edu, billf@mit.edu

# Anti-pinhole or Pinspeck cameras

OPTICA ACTA, 1982, VOL. 29, NO. 1, 63–67

## Anti-pinhole imaging

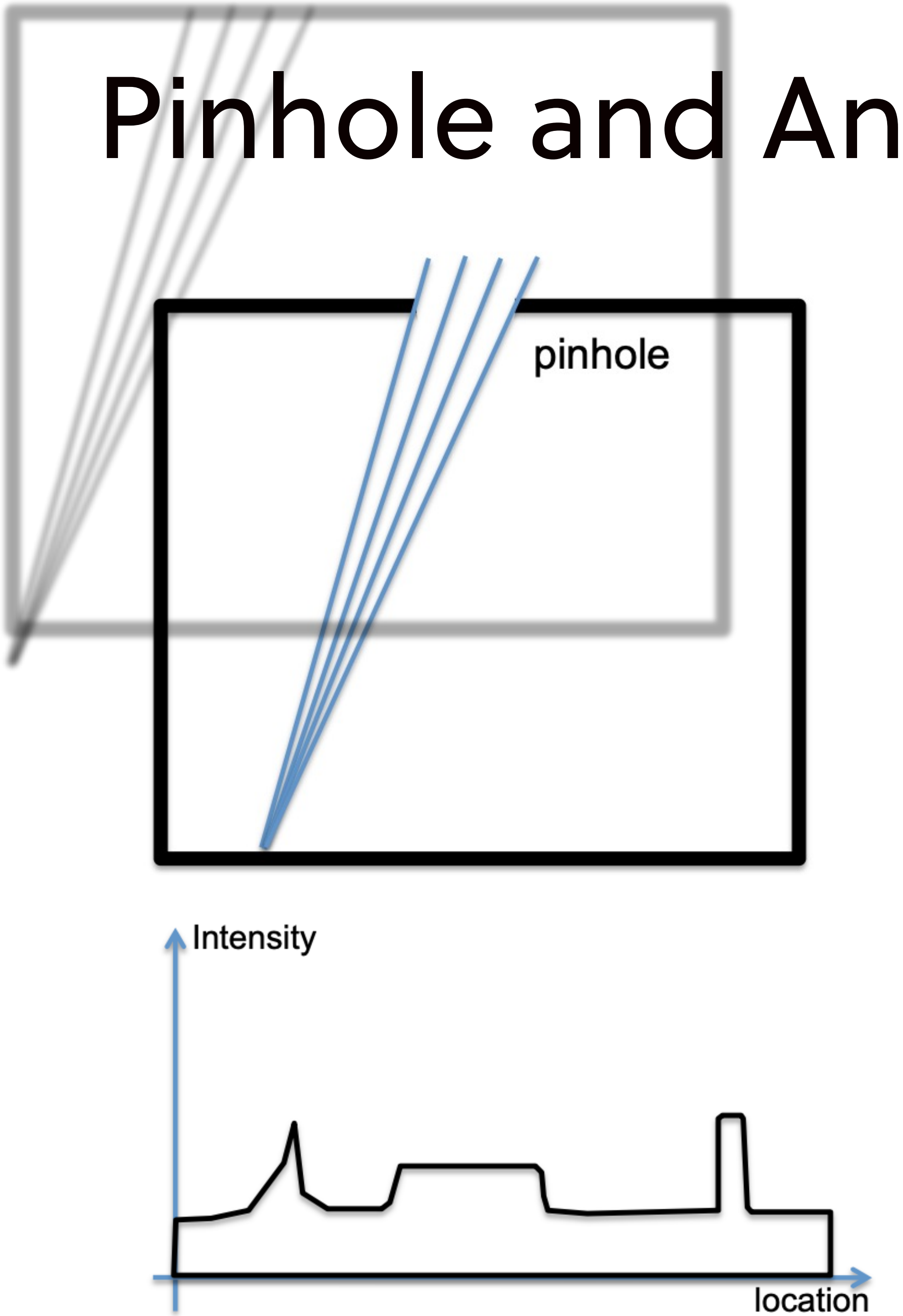
ADAM LLOYD COHEN
Parmly Research Institute, Loyola University of Chicago,
Chicago, Illinois 60626, U.S.A.

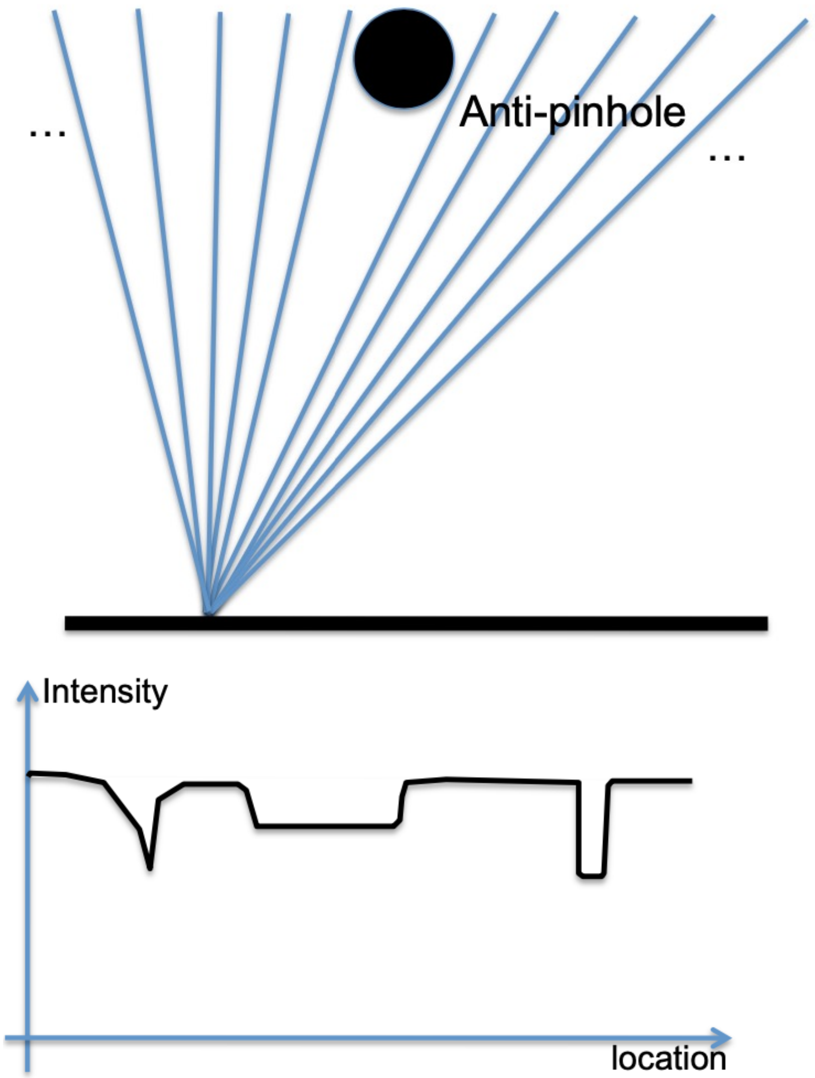
*(Received 16 April 1981; revision received 8 July 1981)*

**Abstract.** By complementing a pinhole to produce an isolated opaque spot, the light ordinarily blocked from the pinhole image is transmitted, and the light ordinarily transmitted is blocked. A negative geometrical image is formed, distinct from the familiar 'bright-spot' diffraction image. Anti-pinhole, or 'pinspeck' images are visible during a solar eclipse, when the shadows of objects appear crescent-shaped. Pinspecks demonstrate unlimited depth of field, freedom from distortion and large angular field. Images of different magnification may be formed simultaneously. Contrast is poor, but is improvable by averaging to remove noise and subtraction of a d.c. bias. Pinspecks may have application in X-ray space optics, and might be employed in the eyes of simple organisms.

Adam L. Cohen, 1982

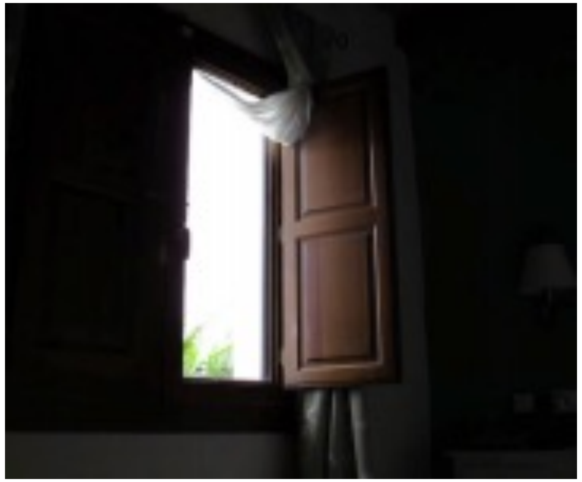
# Pinhole and Anti-pinhole cameras




Adam L. Cohen, 1982

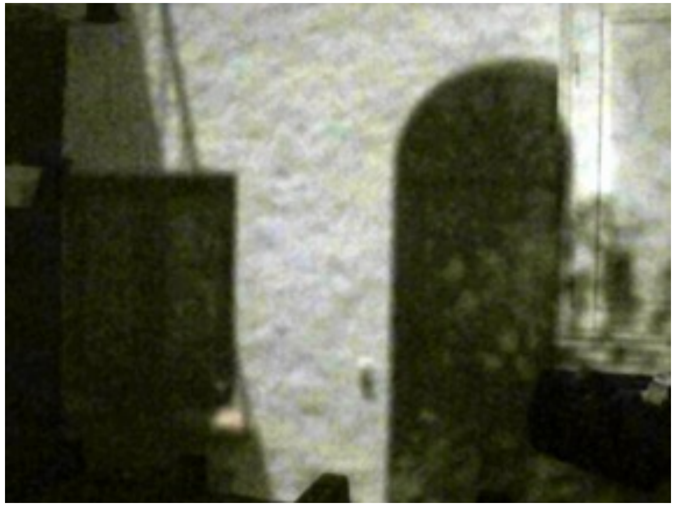
# Accidental pinhole camera



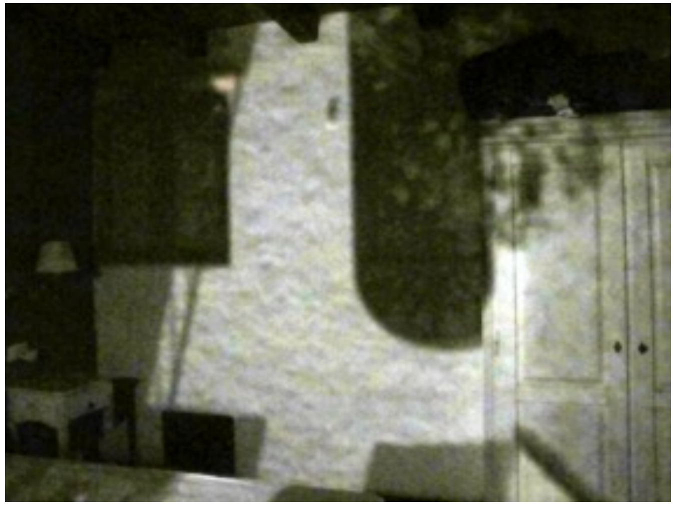
window is an aperture

projected pattern on the wall

upside down

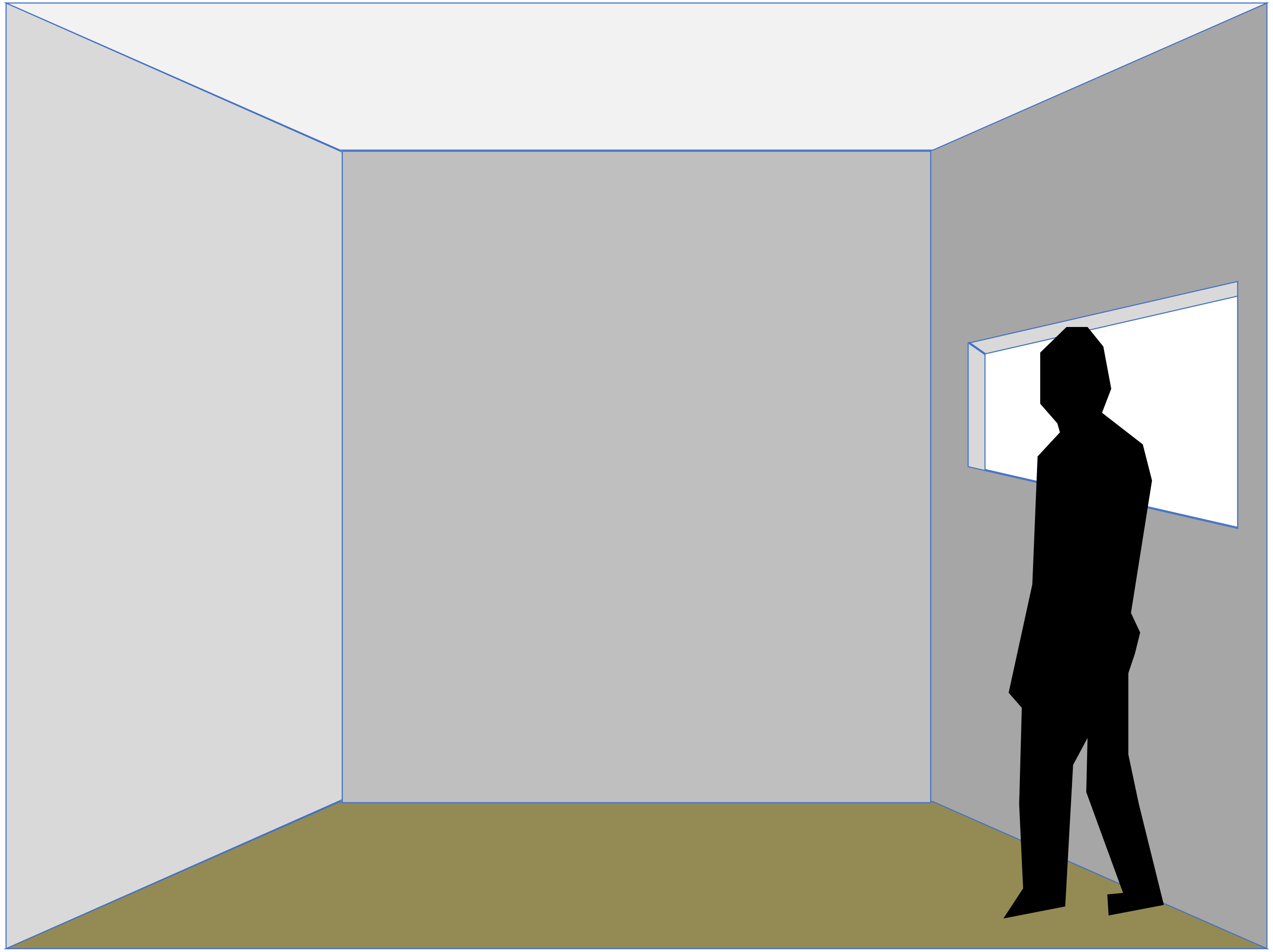


window with smaller gap

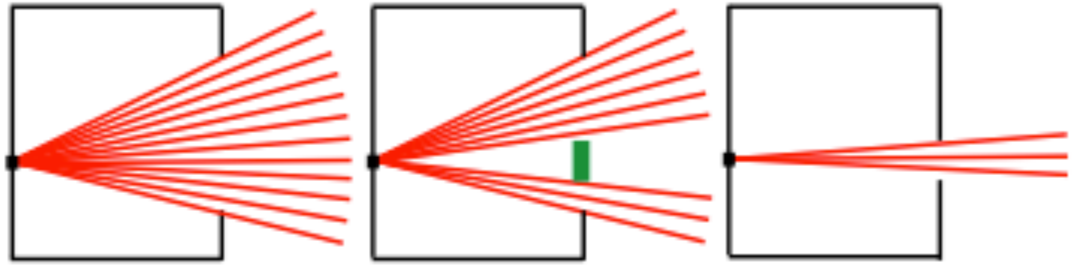


view outside window

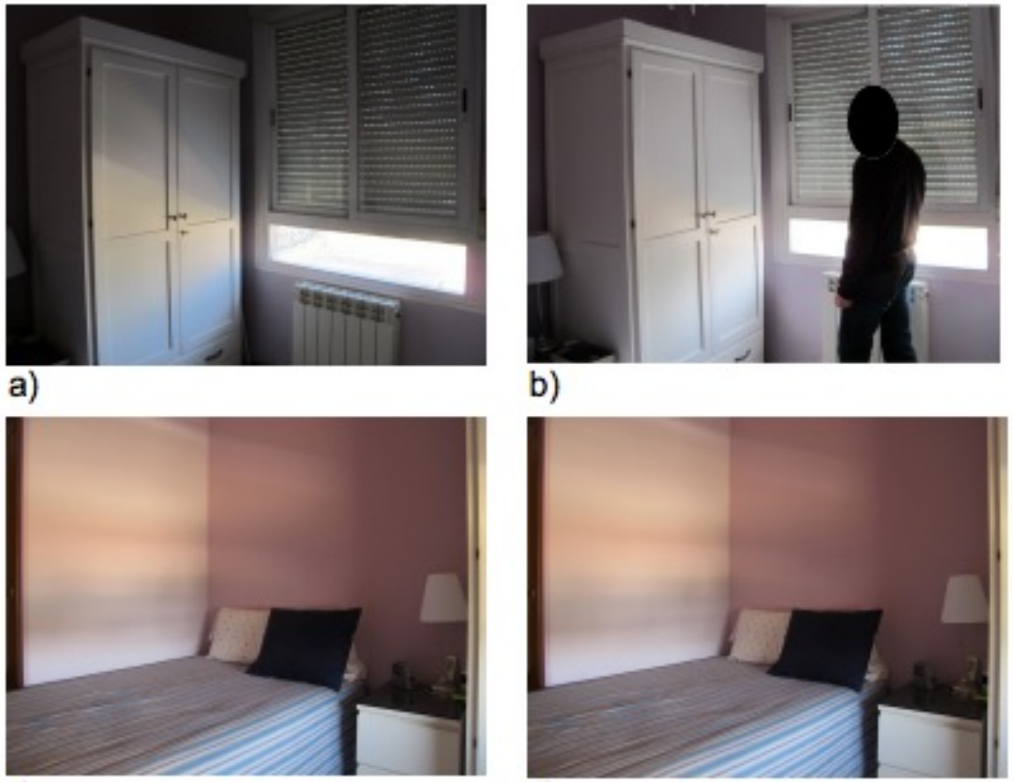
# Mixed accidental pinhole and anti-pinhole cameras



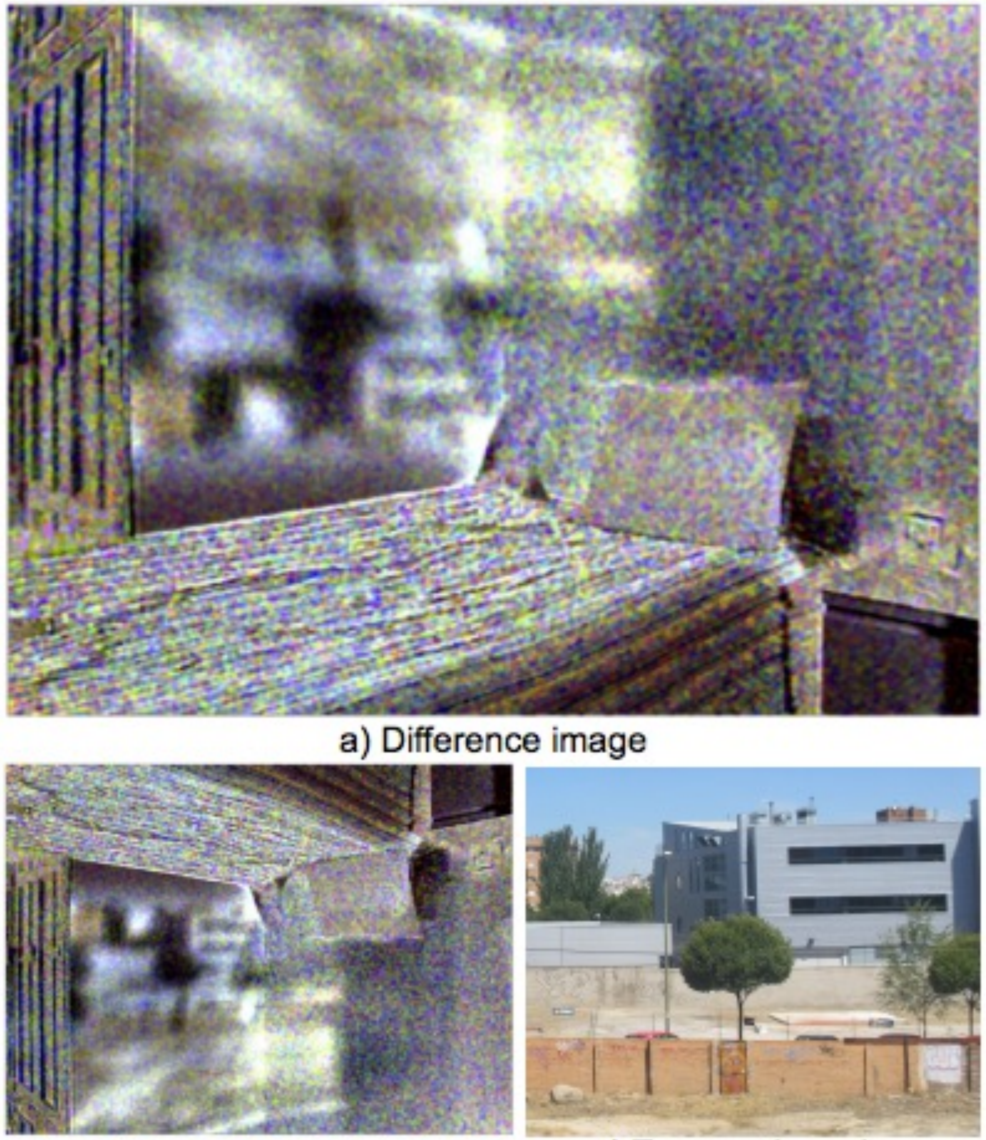
# Accidental pinspeck camera



a) b) c)



a) b) c) d)

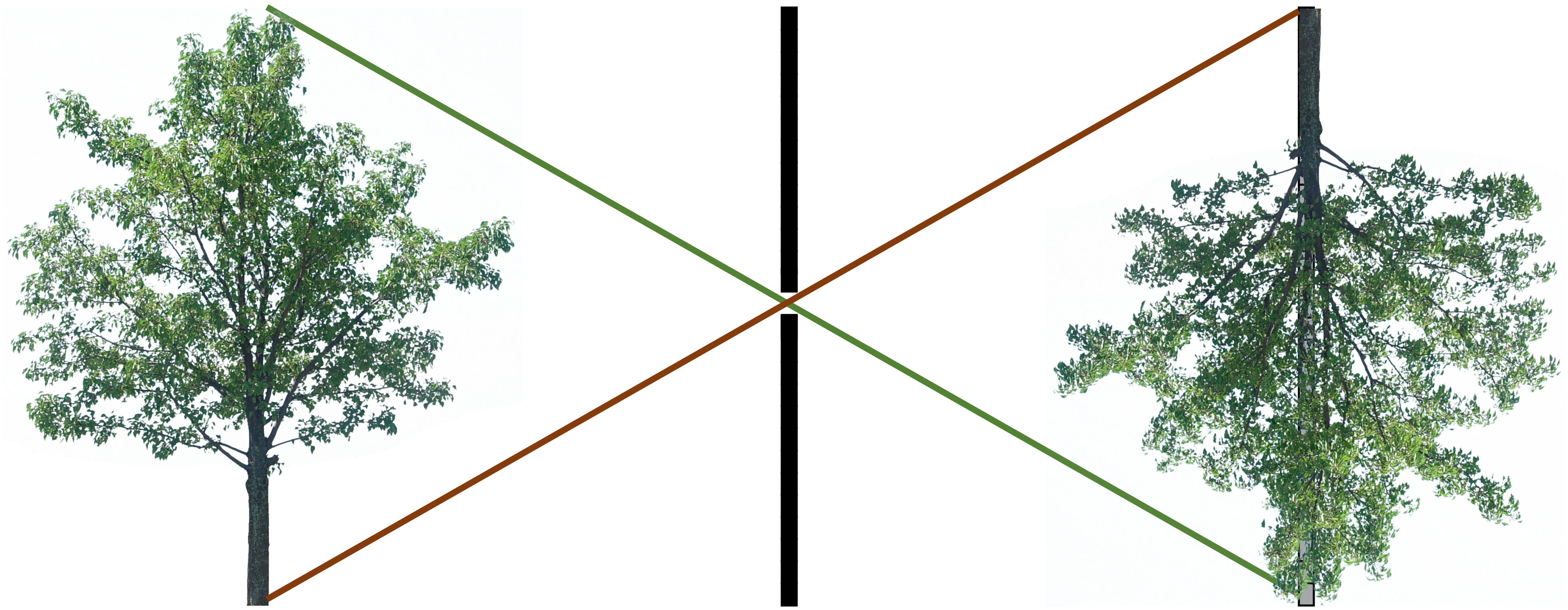


a) Difference image

b) Difference upside down

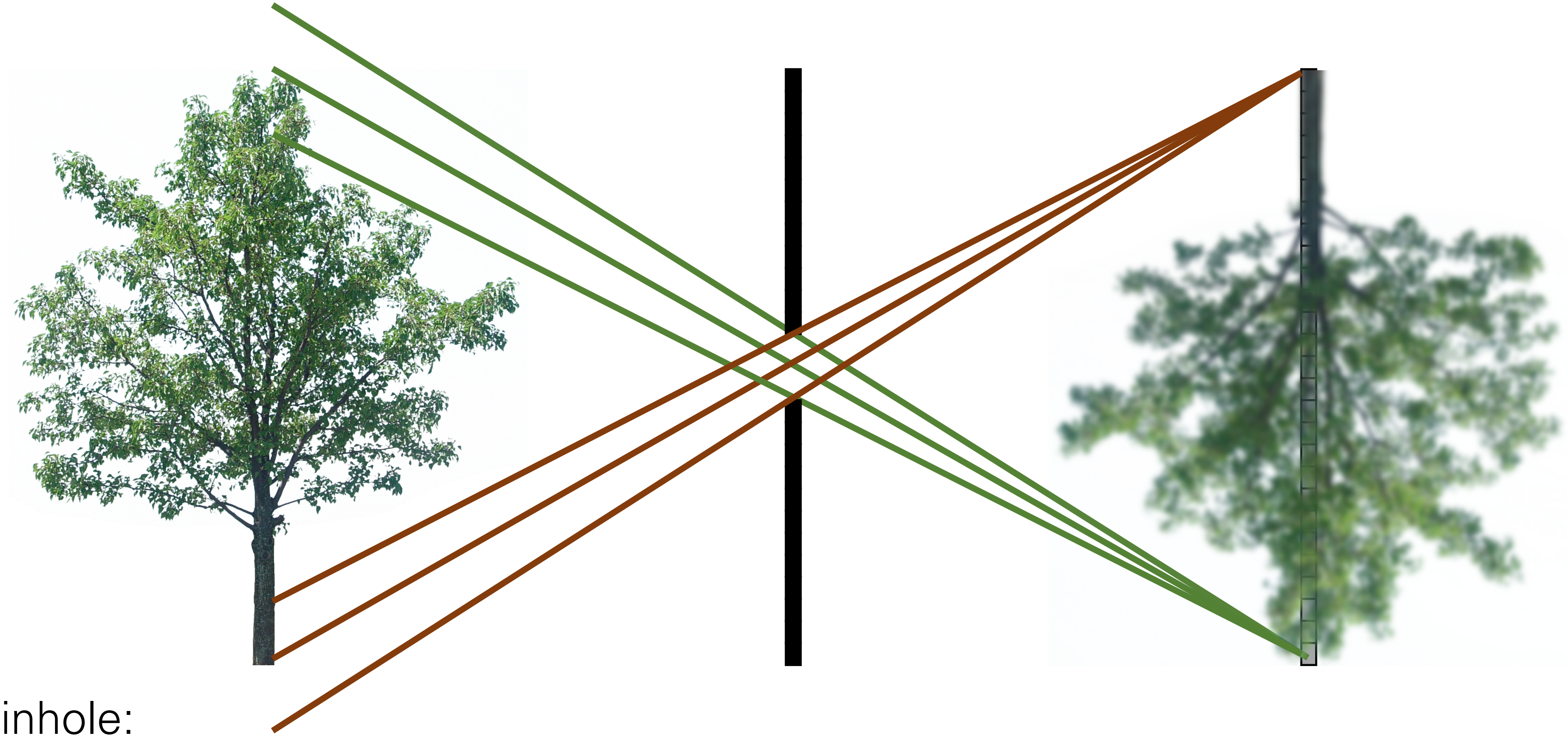
c) True outdoor view

# Pinhole camera trade-off



Small (ideal) pinhole:
1. Image is sharp.
2. Signal-to-noise ratio is low.

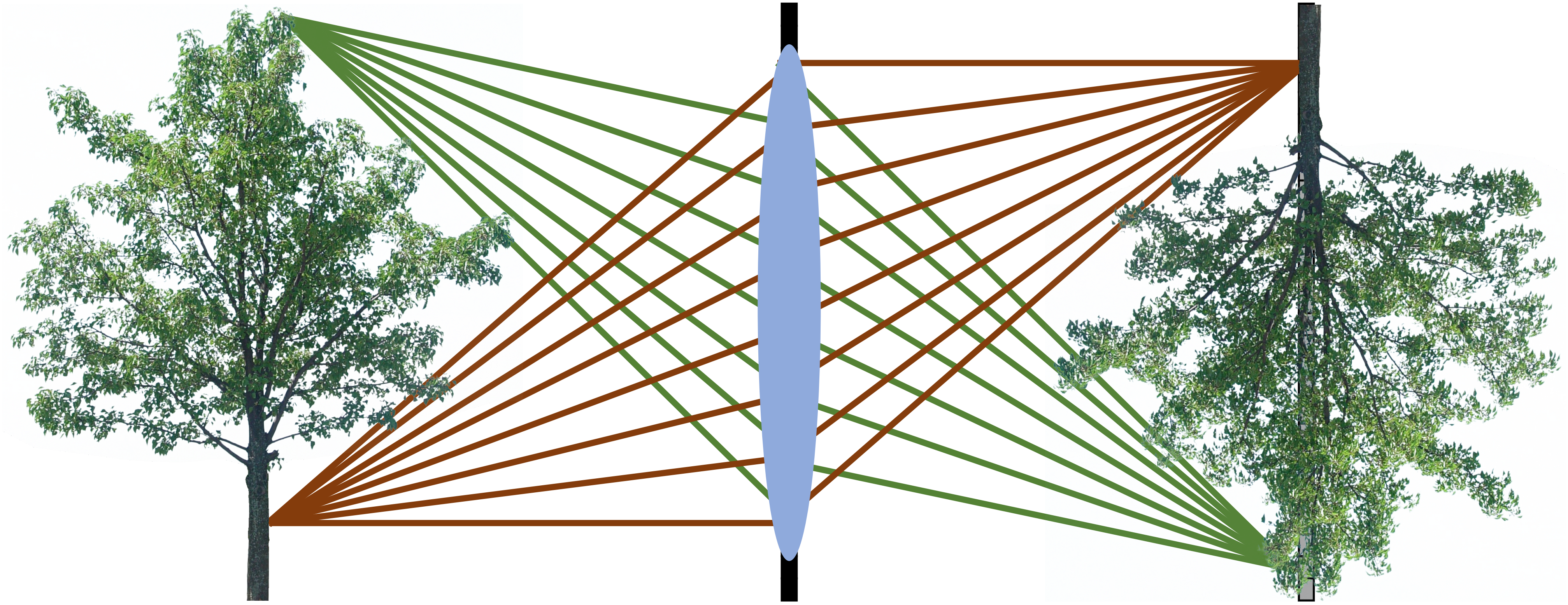
# Pinhole camera trade-off



Large pinhole:
1. Image is blurry.
2. Signal-to-noise ratio is high.

Can we get best of both worlds?

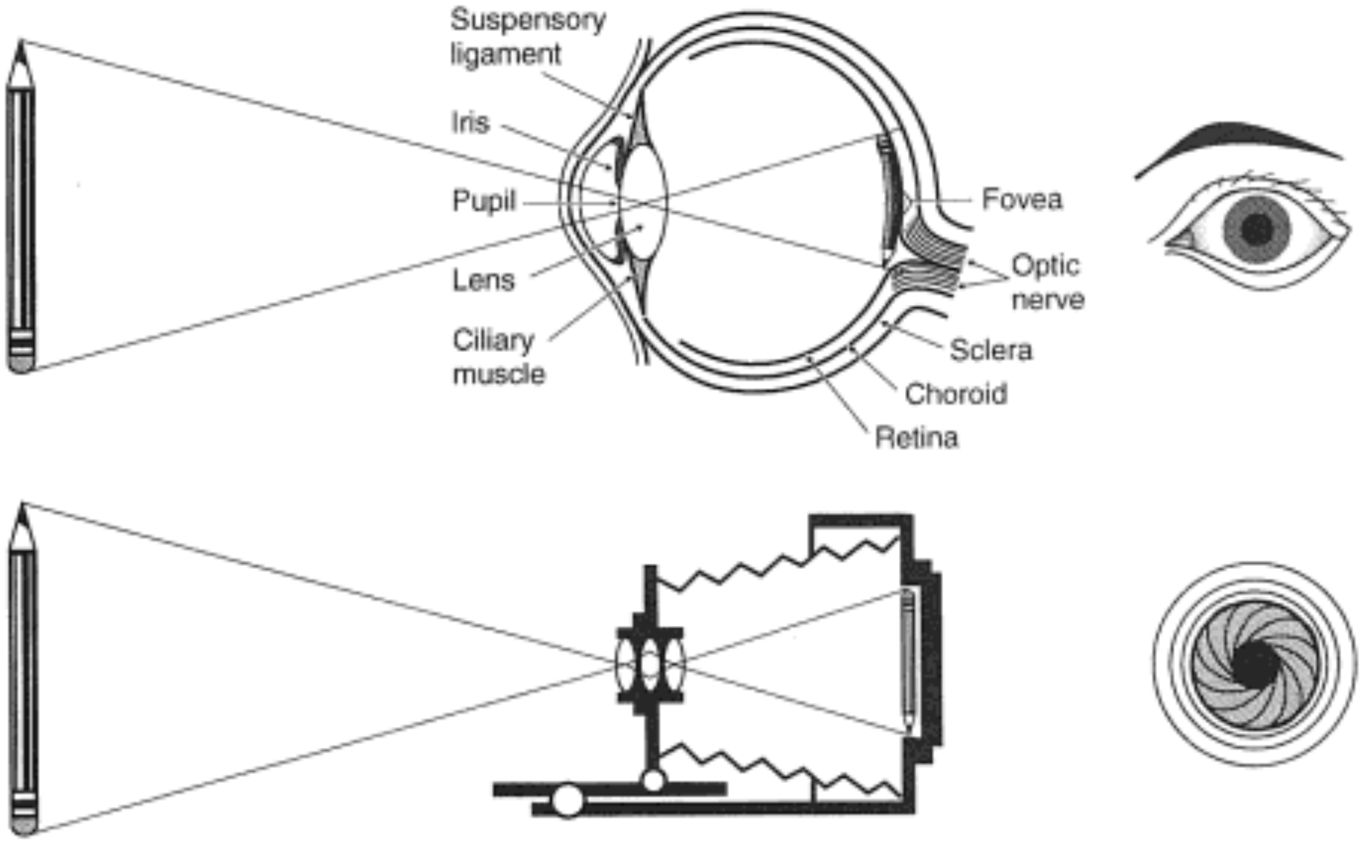
# Almost, by using lenses



Lenses map “bundles” of rays from points on the scene to the sensor.

How does this mapping work exactly?

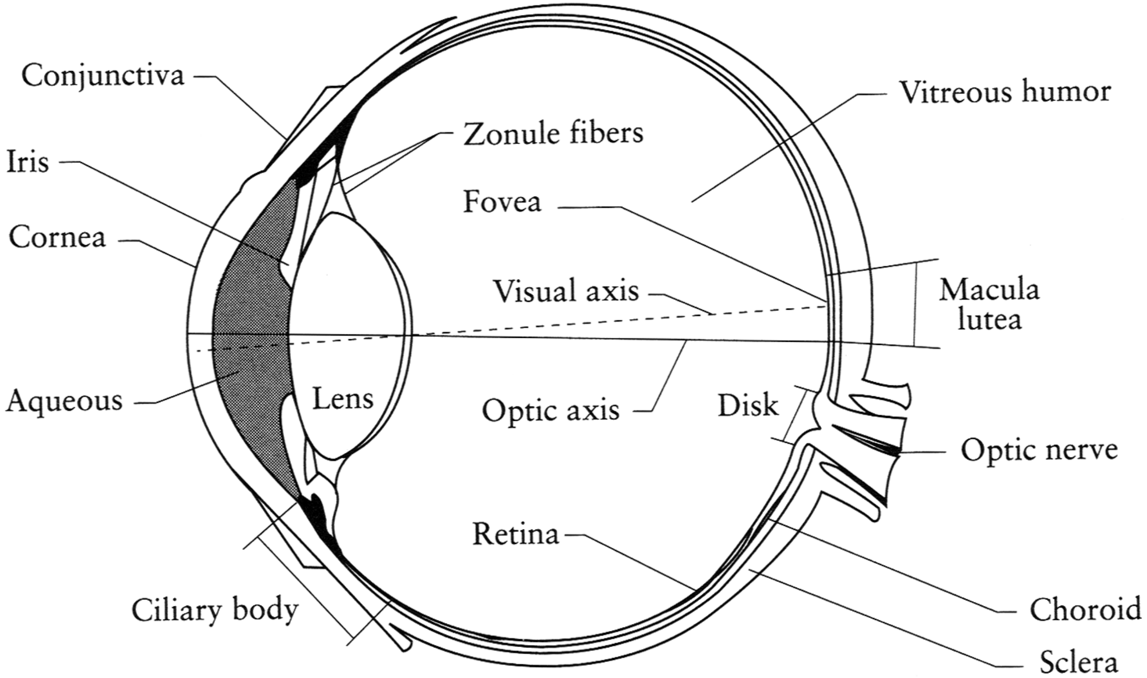
# The Eye




*Figures: Francis Crick, The Astonishing Hypothesis, 1995*

- The human eye is a camera

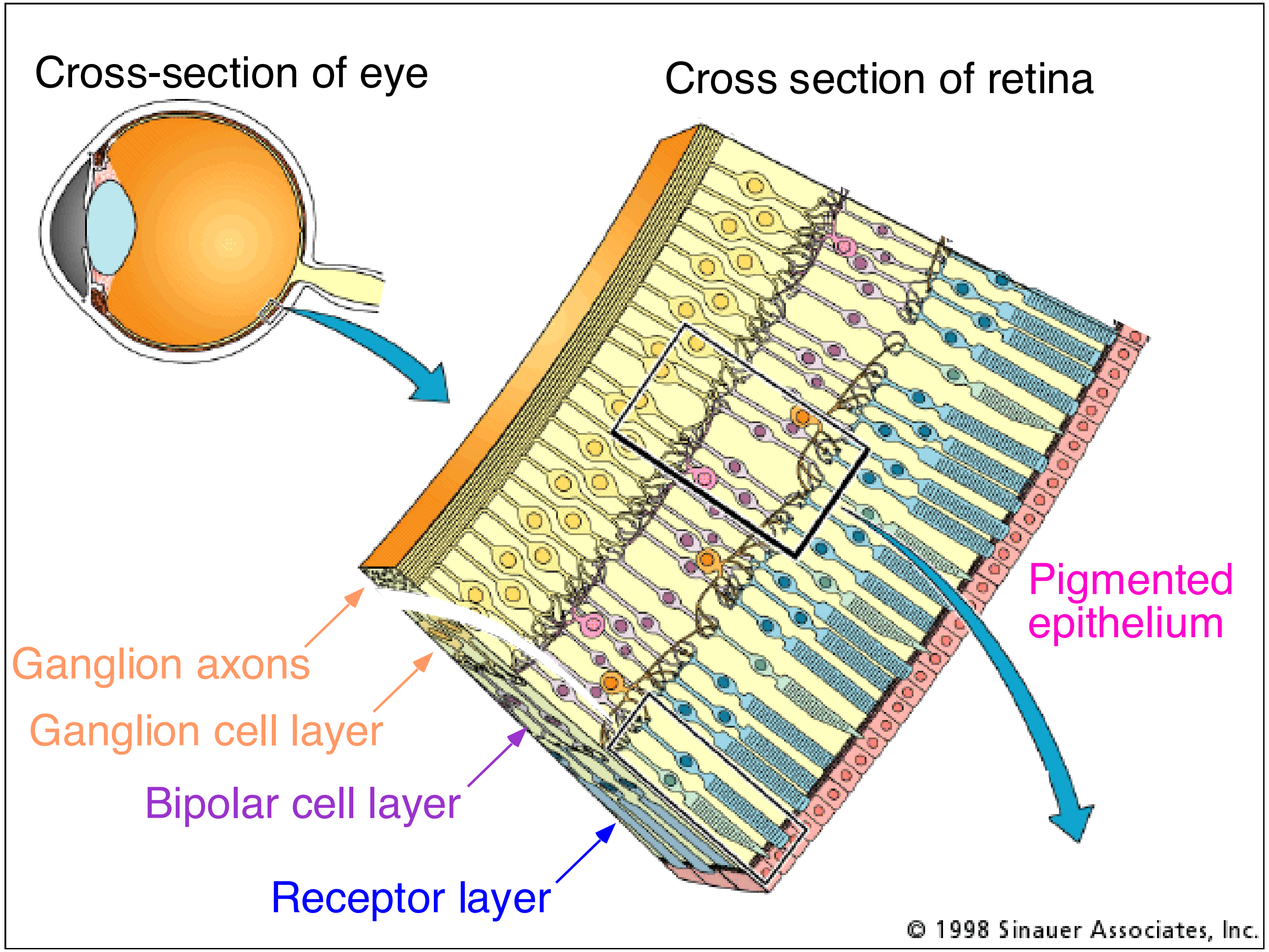
# The Eye




- The human eye is a camera!
  - Iris - colored annulus with radial muscles
  - Pupil - the hole (aperture) whose size is controlled by the iris
  - What’s the “film”?
    - photoreceptor cells (rods and cones) in the retina

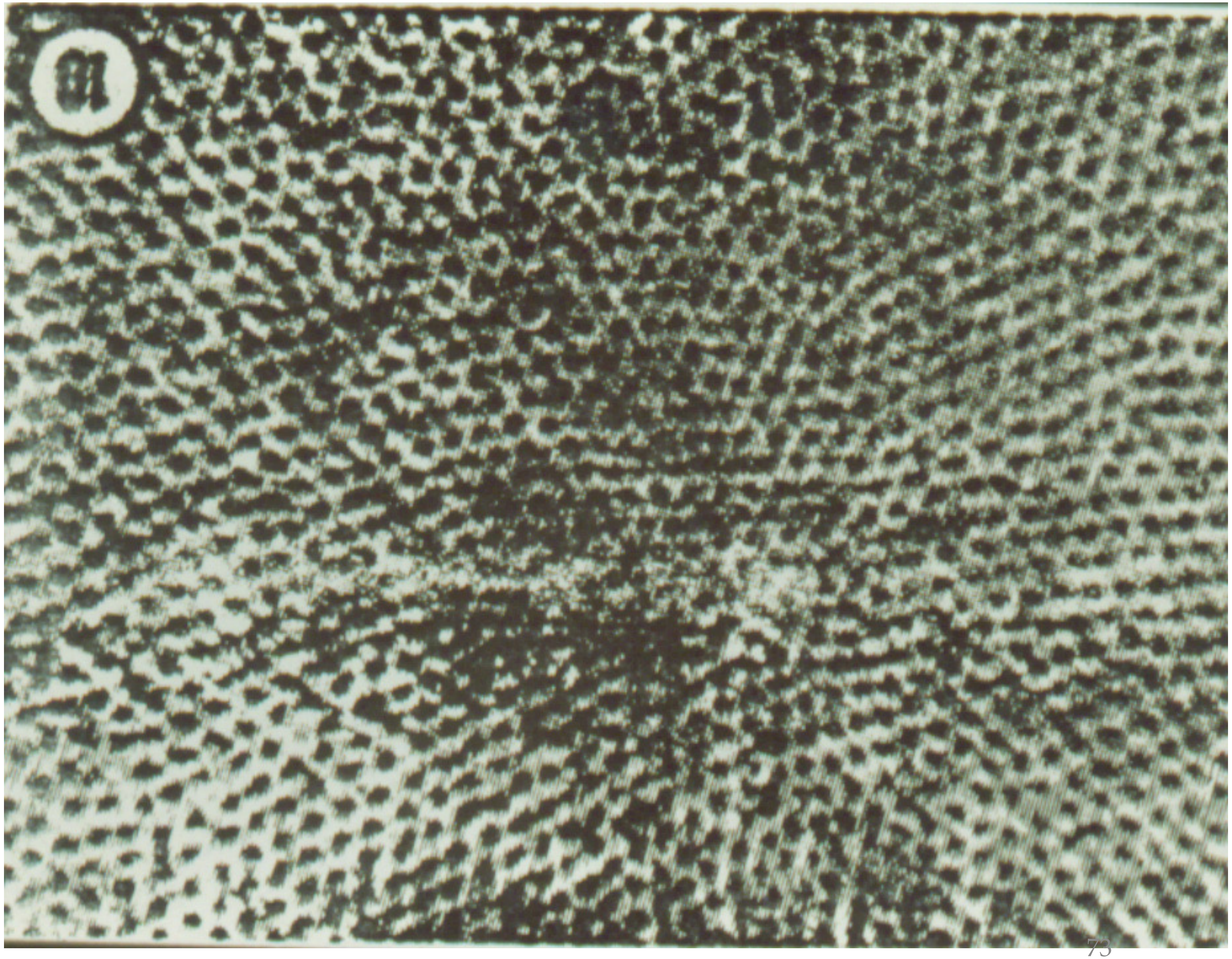
Slide credit: Steve Seitz

# The Retina



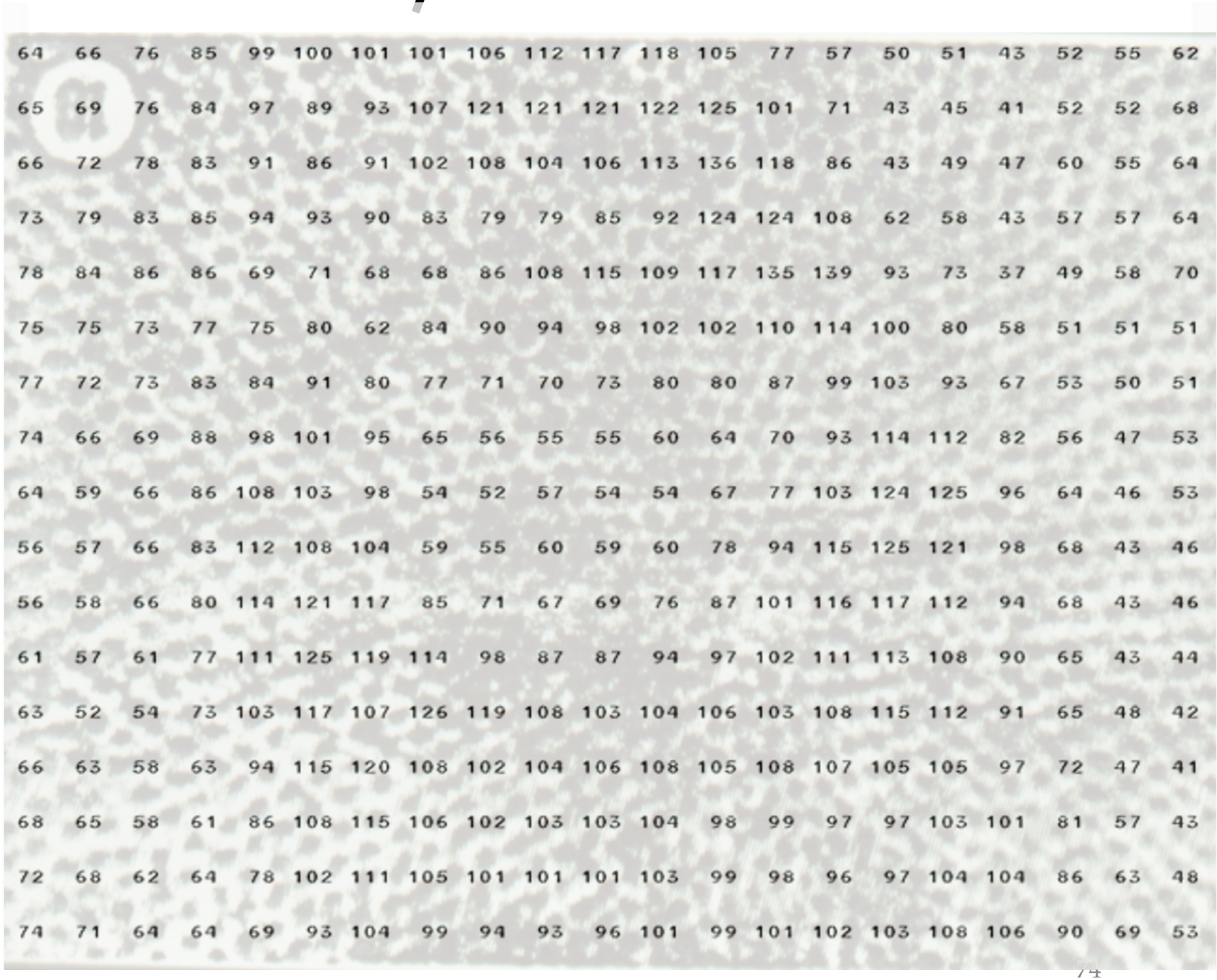

Slide credit: Alyosha Efros

# Receptors Density - Fovea



Slide credit: S. Ullman

# Receptors Density - Fovea



| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 64 | 66 | 76 | 85 | 99 | 100 | 101 | 101 | 106 | 112 | 117 | 118 | 105 | 77 | 57 | 50 | 51 | 43 | 52 | 55 | 62 |
| 65 | 69 | 76 | 84 | 97 | 89 | 93 | 107 | 121 | 121 | 121 | 122 | 125 | 101 | 71 | 43 | 45 | 41 | 52 | 52 | 68 |
| 66 | 72 | 78 | 83 | 91 | 86 | 91 | 102 | 108 | 104 | 106 | 113 | 136 | 118 | 86 | 43 | 49 | 47 | 60 | 55 | 64 |
| 73 | 79 | 83 | 85 | 94 | 93 | 90 | 83 | 79 | 79 | 85 | 92 | 124 | 124 | 108 | 62 | 58 | 43 | 57 | 57 | 64 |
| 78 | 84 | 86 | 86 | 69 | 71 | 68 | 68 | 86 | 108 | 115 | 109 | 117 | 135 | 139 | 93 | 73 | 37 | 49 | 58 | 70 |
| 75 | 75 | 73 | 77 | 75 | 80 | 62 | 84 | 90 | 94 | 98 | 102 | 102 | 110 | 114 | 100 | 80 | 58 | 51 | 51 | 51 |
| 77 | 72 | 73 | 83 | 84 | 91 | 80 | 77 | 71 | 70 | 73 | 80 | 80 | 87 | 99 | 103 | 93 | 67 | 53 | 50 | 51 |
| 74 | 66 | 69 | 88 | 98 | 101 | 95 | 65 | 56 | 55 | 55 | 60 | 64 | 70 | 93 | 114 | 112 | 82 | 56 | 47 | 53 |
| 64 | 59 | 66 | 86 | 108 | 103 | 98 | 54 | 52 | 57 | 54 | 54 | 67 | 77 | 103 | 124 | 125 | 96 | 64 | 46 | 53 |
| 56 | 57 | 66 | 83 | 112 | 108 | 104 | 59 | 55 | 60 | 59 | 60 | 78 | 94 | 115 | 125 | 121 | 98 | 68 | 43 | 46 |
| 56 | 58 | 66 | 80 | 114 | 121 | 117 | 85 | 71 | 67 | 69 | 76 | 87 | 101 | 116 | 117 | 112 | 94 | 68 | 43 | 46 |
| 61 | 57 | 61 | 77 | 111 | 125 | 119 | 114 | 98 | 87 | 87 | 94 | 97 | 102 | 111 | 113 | 108 | 90 | 65 | 43 | 44 |
| 63 | 52 | 54 | 73 | 103 | 117 | 107 | 126 | 119 | 108 | 103 | 104 | 106 | 103 | 108 | 115 | 112 | 91 | 65 | 48 | 42 |
| 66 | 63 | 58 | 63 | 94 | 115 | 120 | 108 | 102 | 104 | 106 | 108 | 105 | 108 | 107 | 105 | 105 | 97 | 72 | 47 | 41 |
| 68 | 65 | 58 | 61 | 86 | 108 | 115 | 106 | 102 | 103 | 103 | 104 | 98 | 99 | 97 | 97 | 103 | 101 | 81 | 57 | 43 |
| 72 | 68 | 62 | 64 | 78 | 102 | 111 | 105 | 101 | 101 | 101 | 103 | 99 | 98 | 96 | 97 | 104 | 104 | 86 | 63 | 48 |
| 74 | 71 | 64 | 64 | 69 | 93 | 104 | 99 | 94 | 93 | 96 | 101 | 99 | 101 | 102 | 103 | 108 | 106 | 90 | 69 | 53 |

Slide credit: S. Ullman

# Animal Eyes

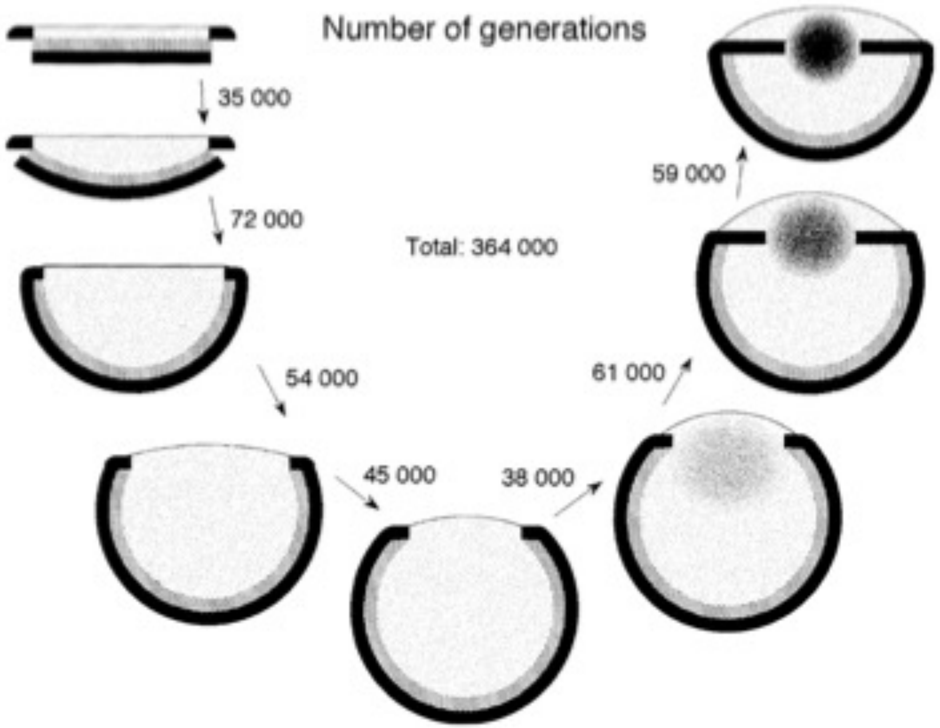



**Fig. 1.6** A patch of light sensitive epithelium can be gradually turned into a perfectly focussed camera-type eye if there is a continuous selection for improved spatial vision. A theoretical model based on conservative assumptions about selection pressure and the amount of variation in natural populations suggest that the whole sequence can be accomplished amazingly fast, in less than 400 000 generations. The number of generations is also given between each of the consecutive intermediates that are drawn in the figure. The starting point is a flat piece of epithelium with an outer protective layer, an intermediate layer of receptor cells, and a bottom layer of pigment cells. The first half of the sequence is the formation of a pigment cup eye. When this principle cannot be improved any further, a lens gradually evolves. Modified from Nilsson and Pelger (1994).

Animal Eyes. Land & Nilsson. Oxford Univ. Press

# Natural Eyes


# Lens (very) basics

# Replacing pinholes with lenses

Niepce “View from the Window at Le Gras”, 1826

1826
8h exp

# Daguerrotype

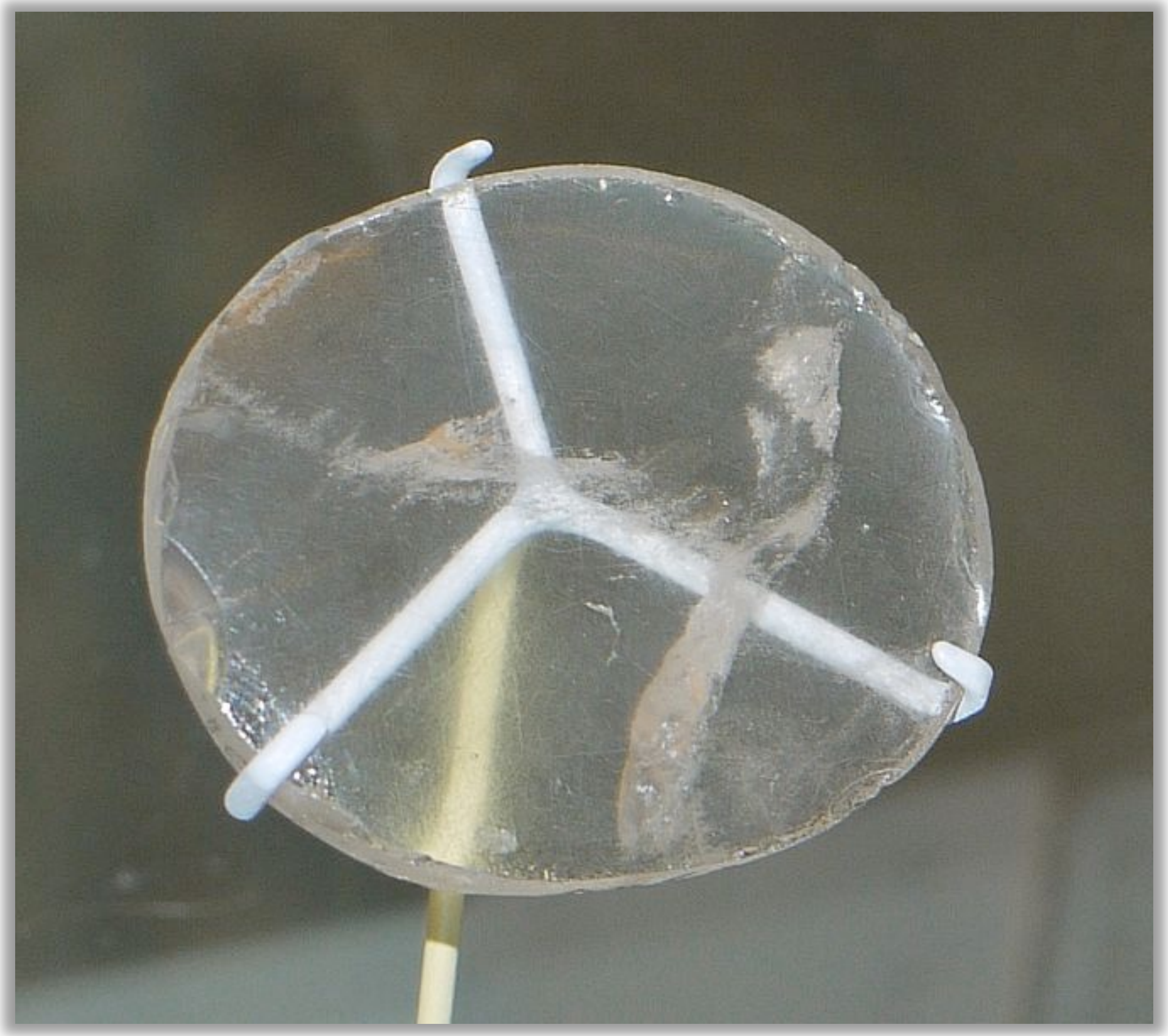
- invented in 1836 by Louis Daguerre
- lenses focus light, better chemicals!

Daguerre “Boulevard du Temple”, 1838

exposure
10-12 mins

# Lenses

- focus light
- magnify objects



Nimrud lens - 2700 years old

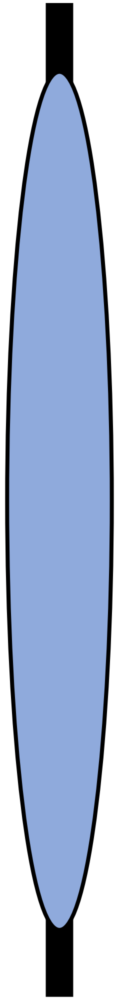
# Lenses

# Lenses

# Lenses

# What is a lens?

A piece of glass manufactured to have a specific shape



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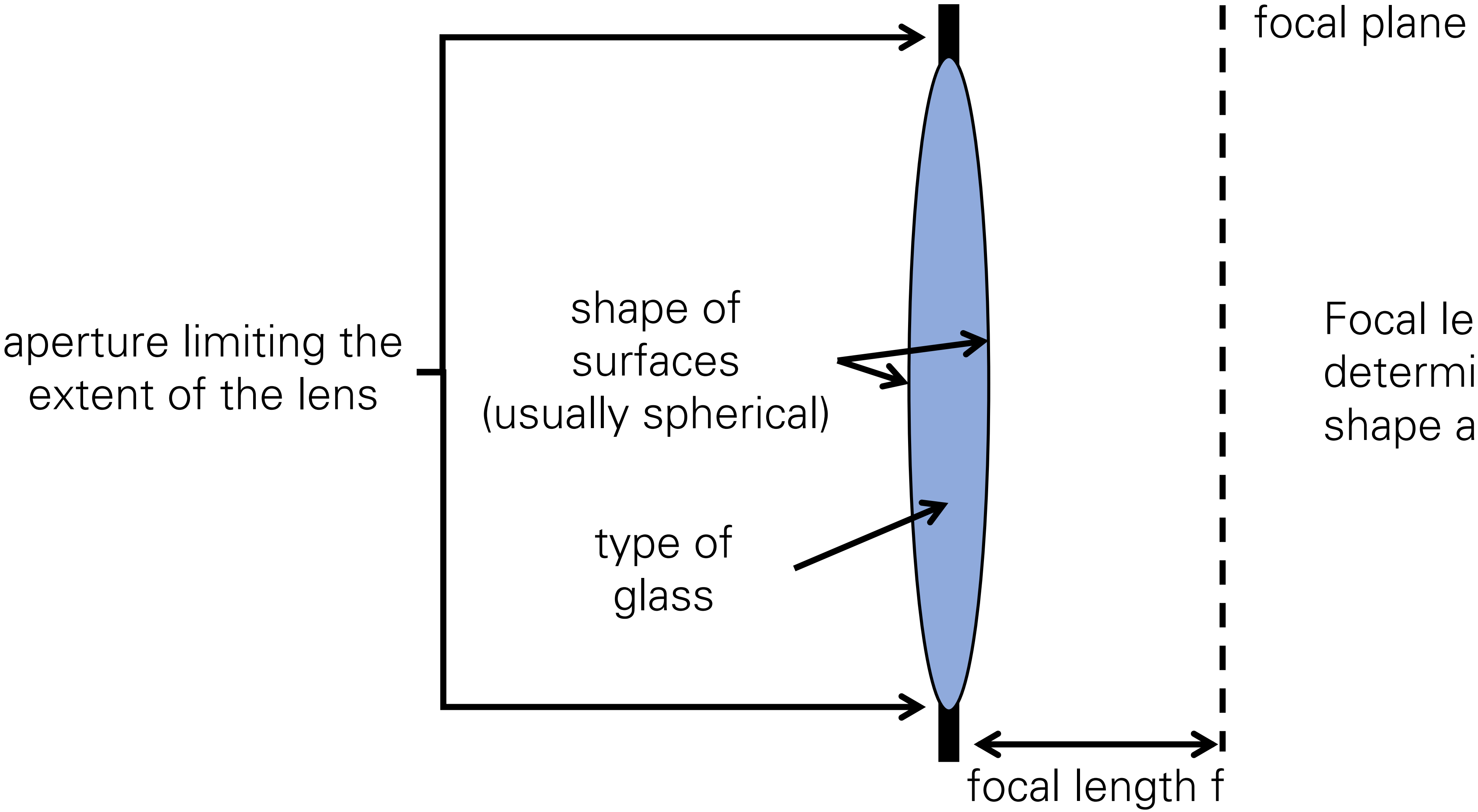
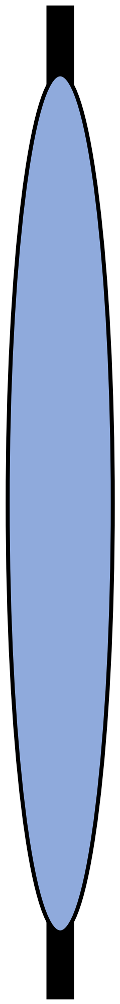



Focal length is determined by the lens' shape and material

# The lens on your camera

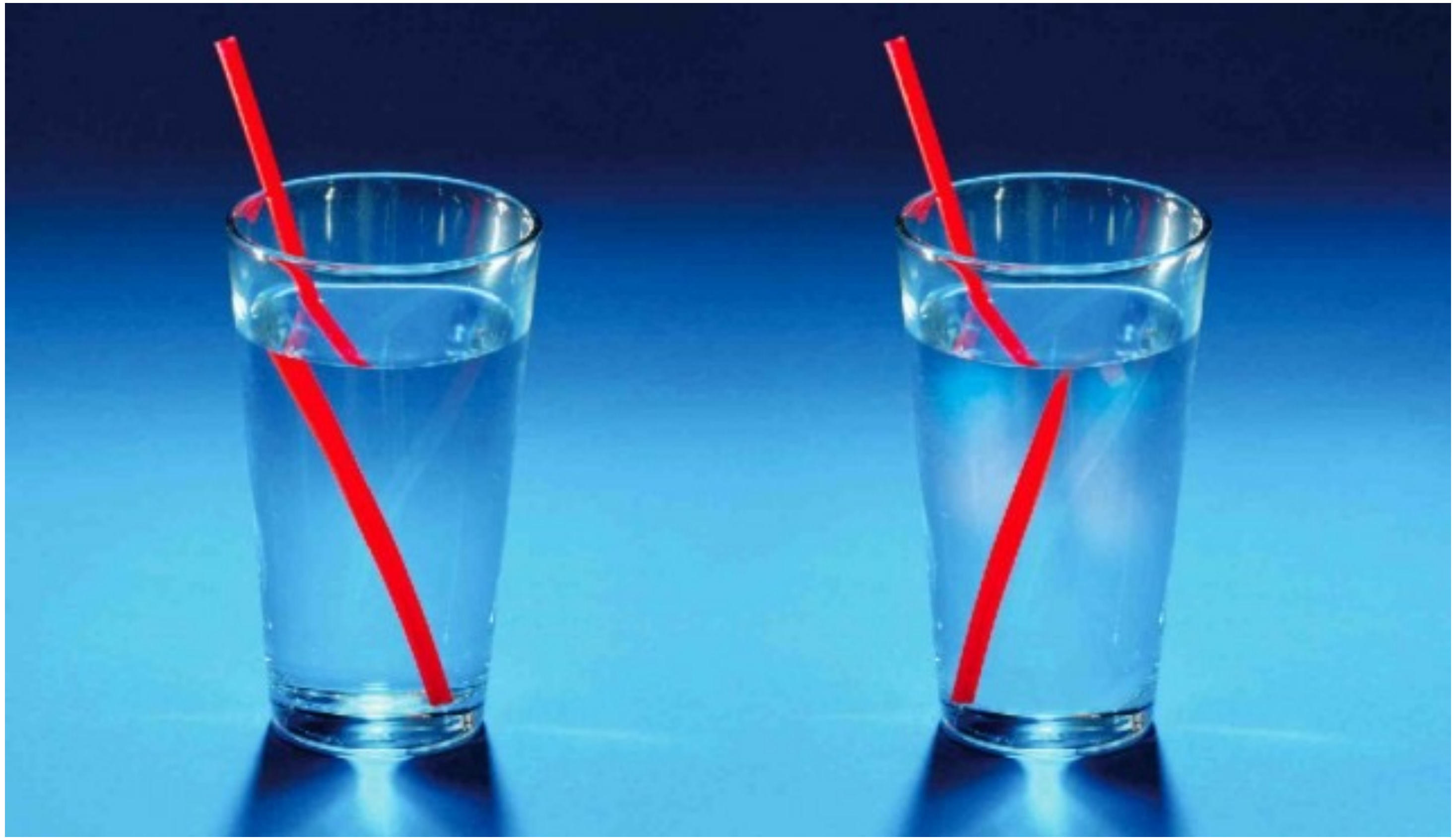


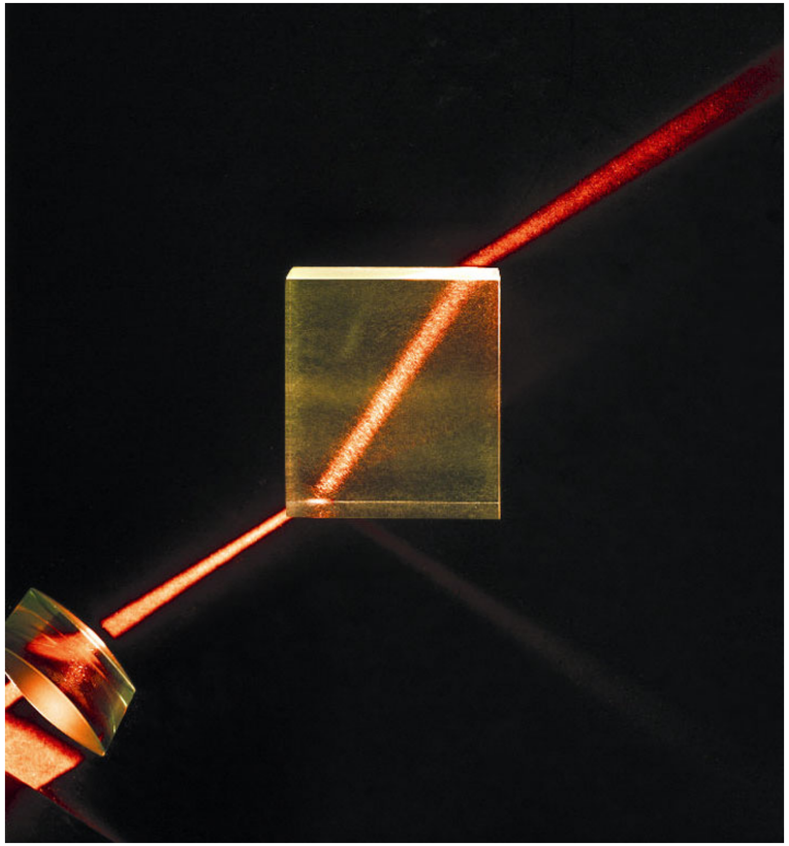
# How does a lens work?



# Refraction

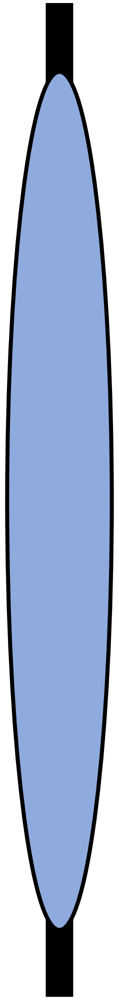
Refraction is the bending of rays of light when they move from one material to another





# How does a lens work?

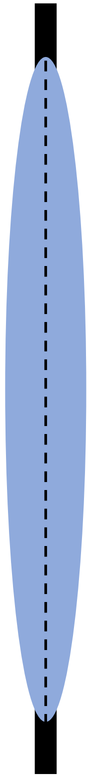
Lenses are designed so that their refraction makes light rays bend in a very specific way.



# The thin lens model

# Thin lens model

Simplification of geometric optics for <u>well-designed</u> lenses.



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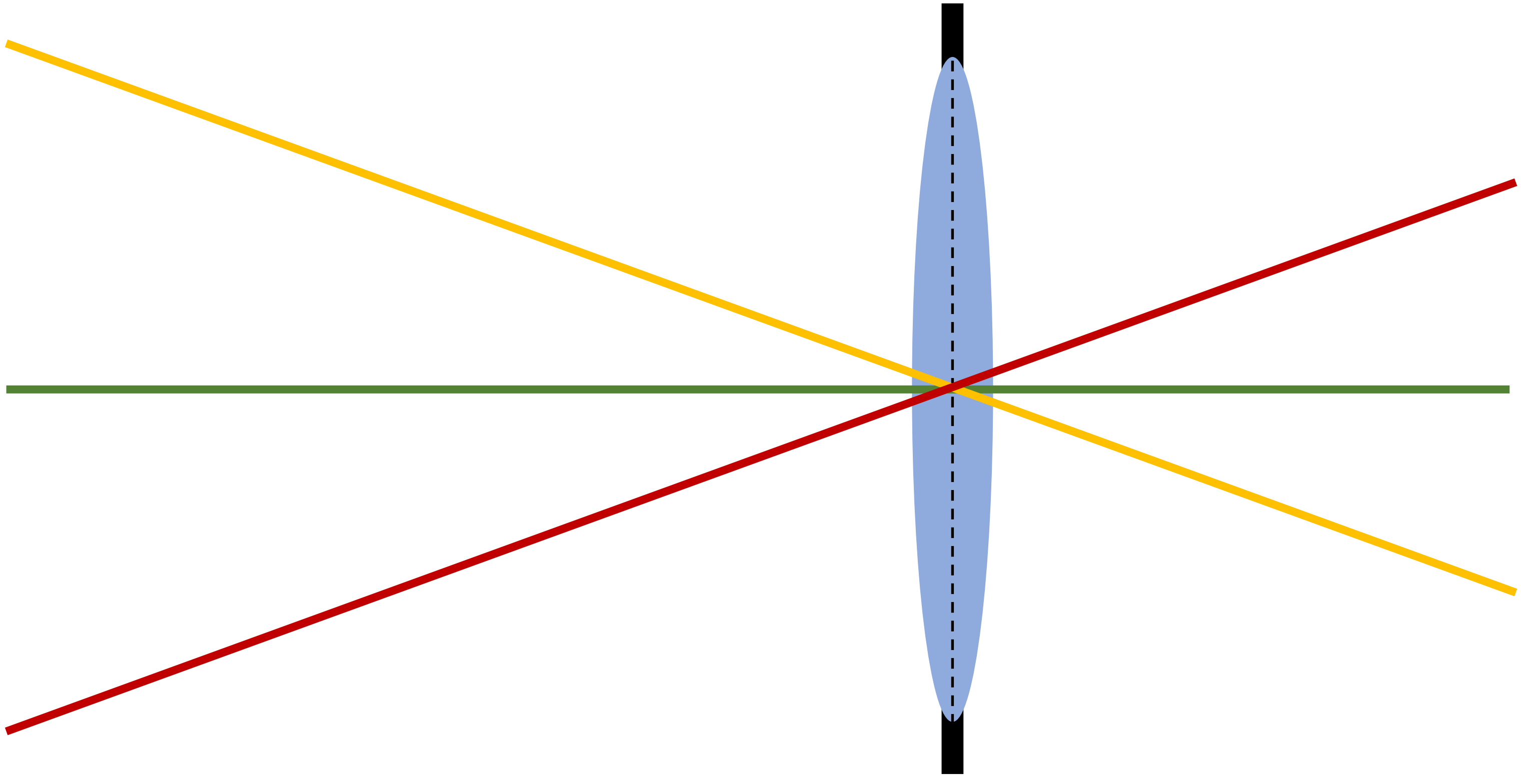

Two assumptions:

1. Rays passing through lens center are unaffected.

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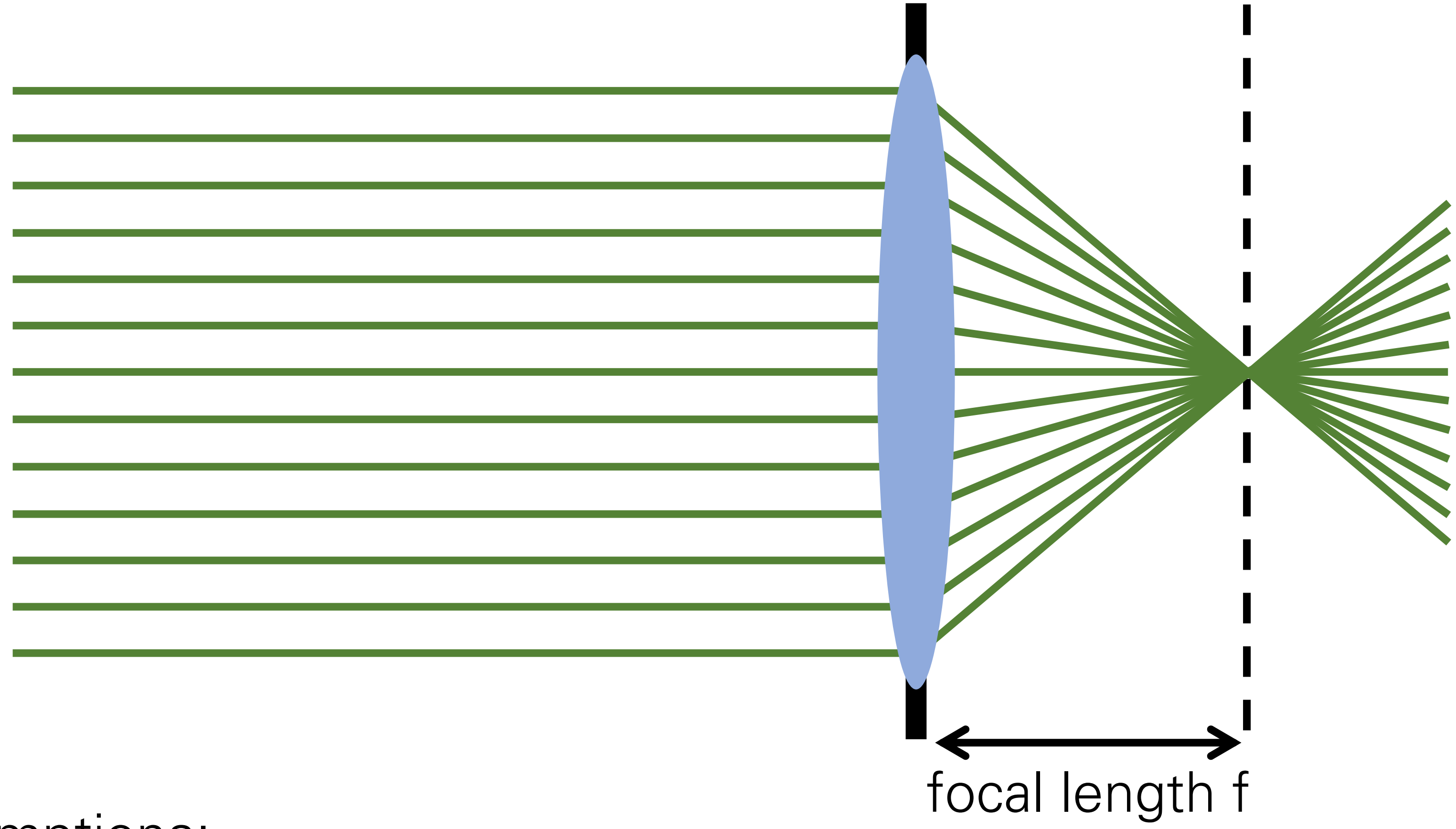


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2. Parallel rays converge to a single point located on focal plane.

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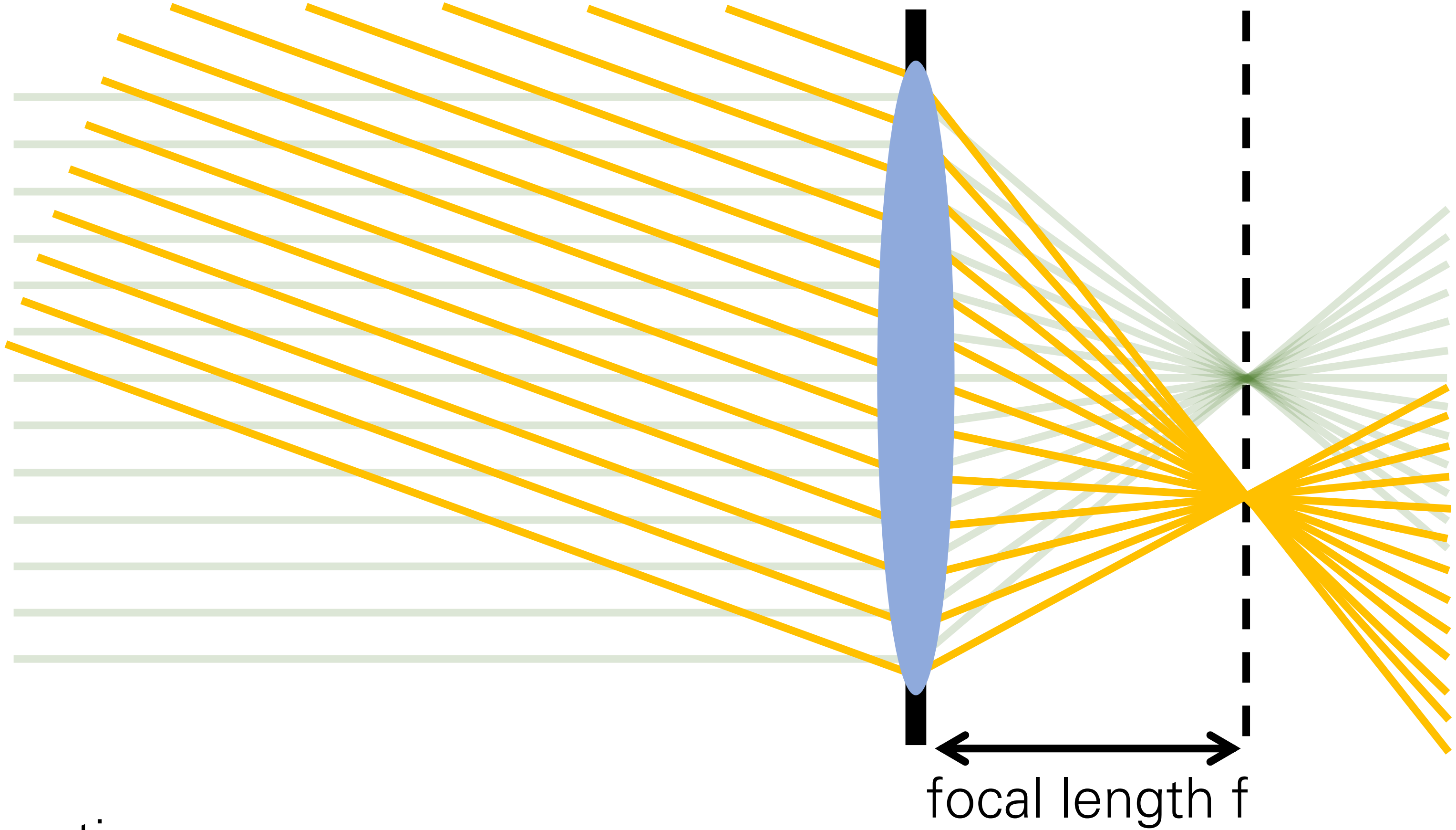


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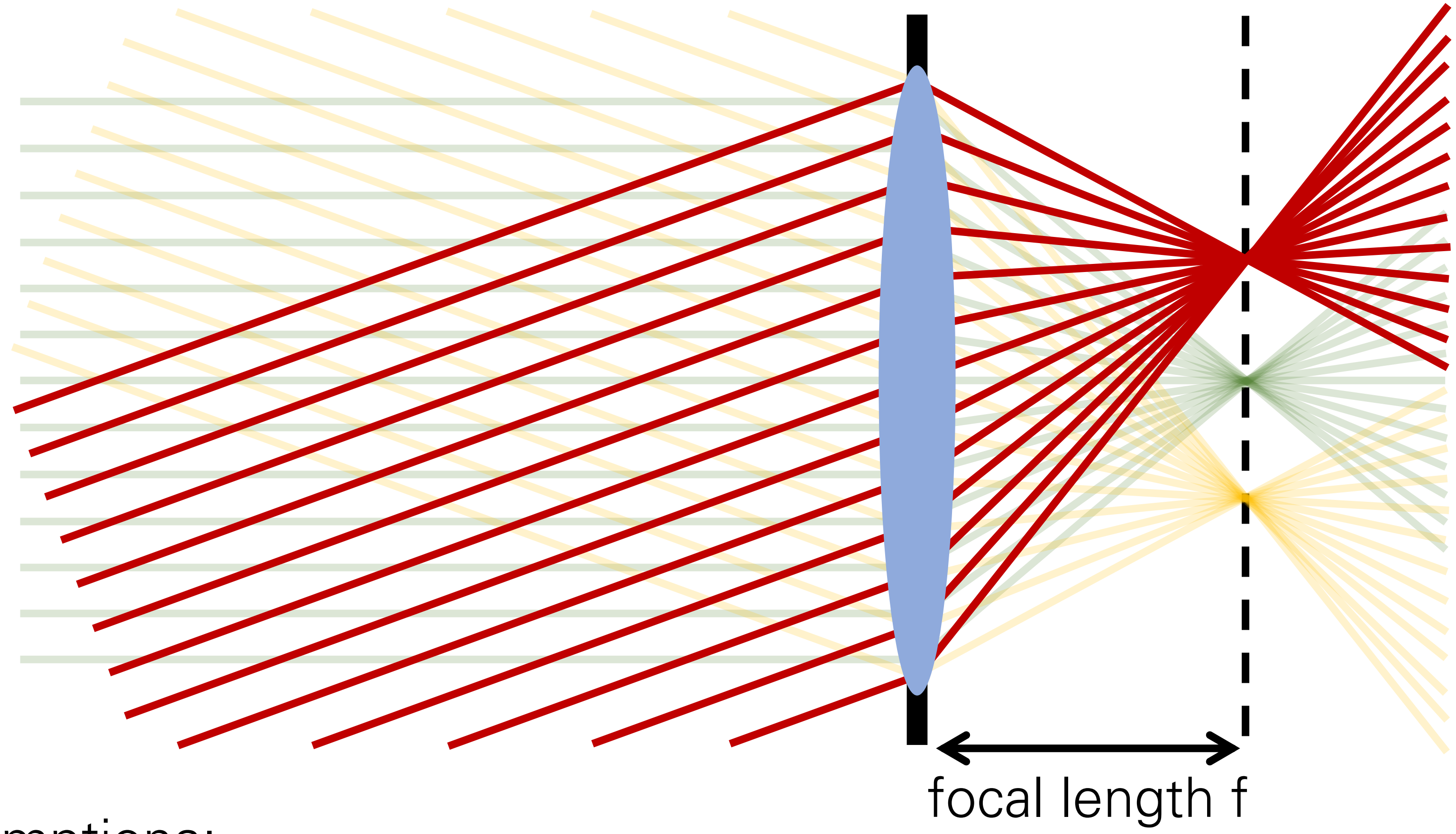


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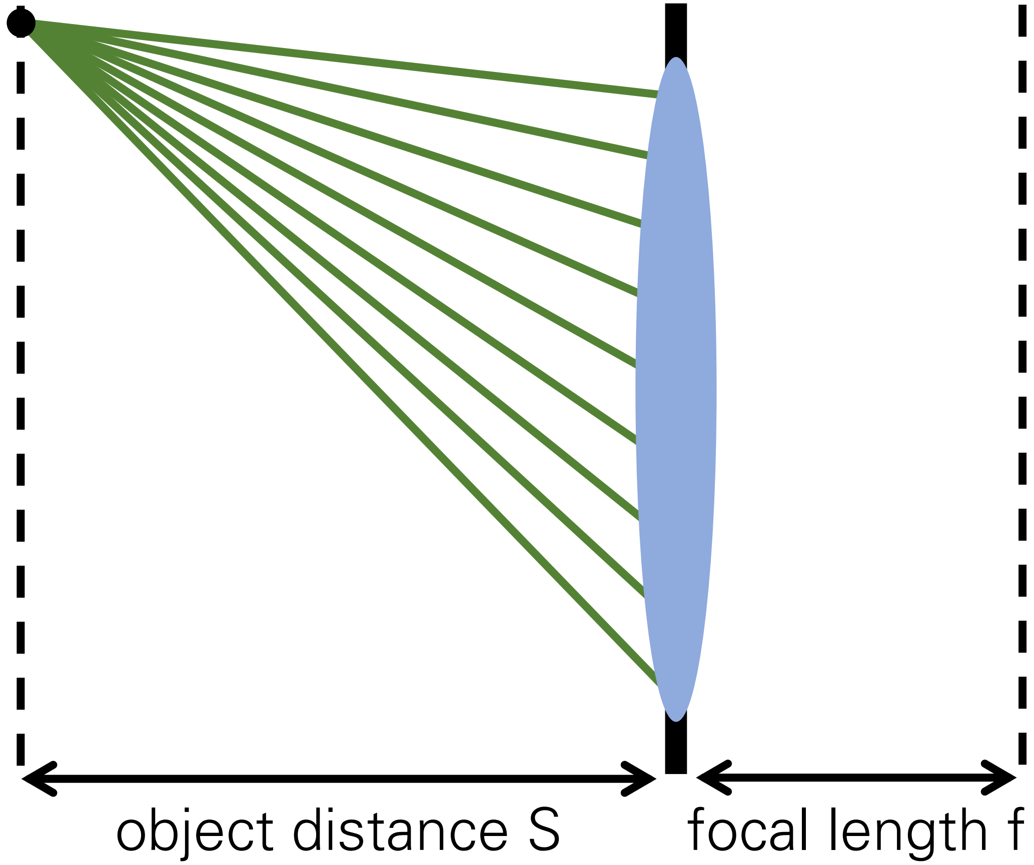
# Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?



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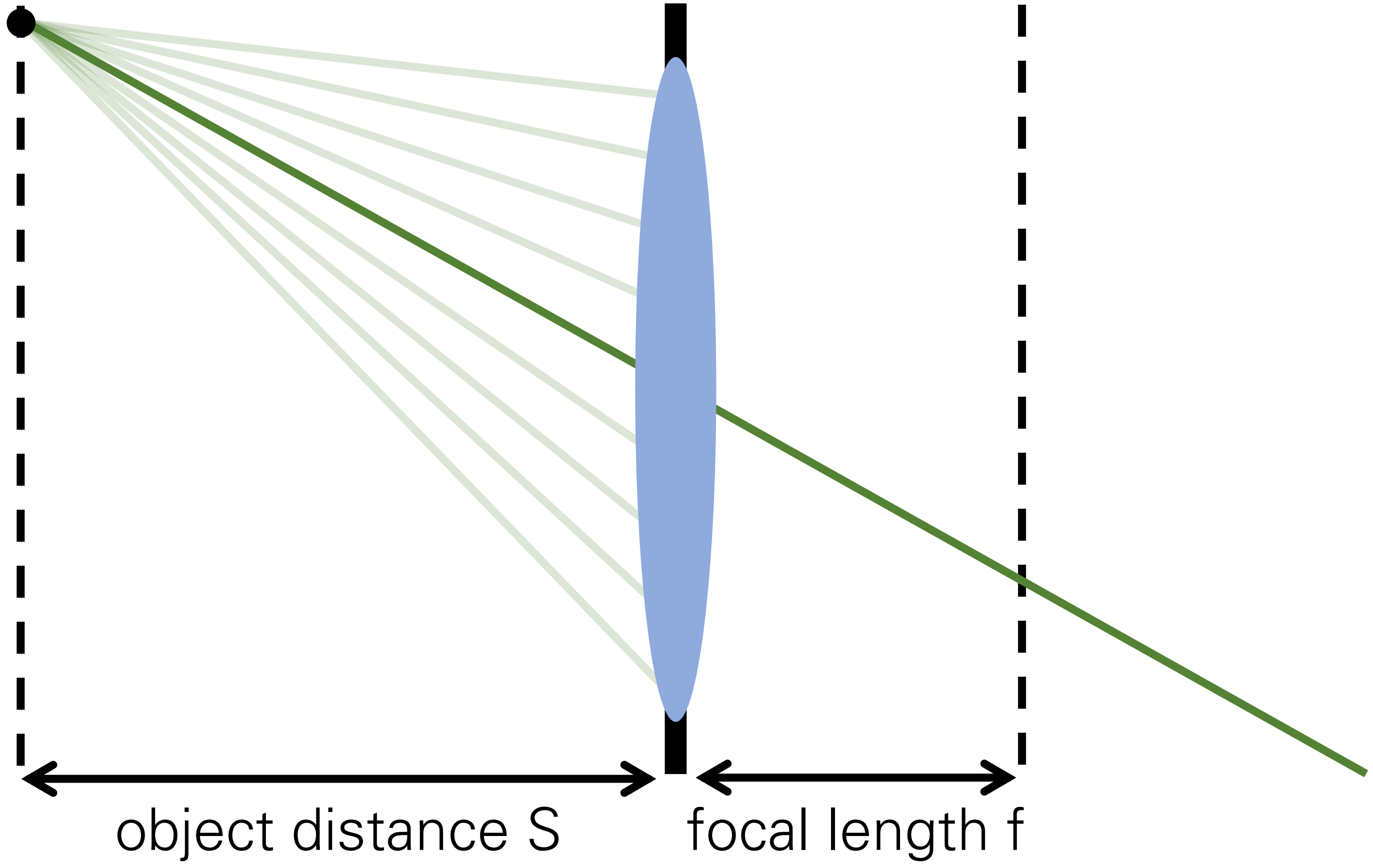

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Consider an object emitting a bundle of rays. How do they propagate through the lens?

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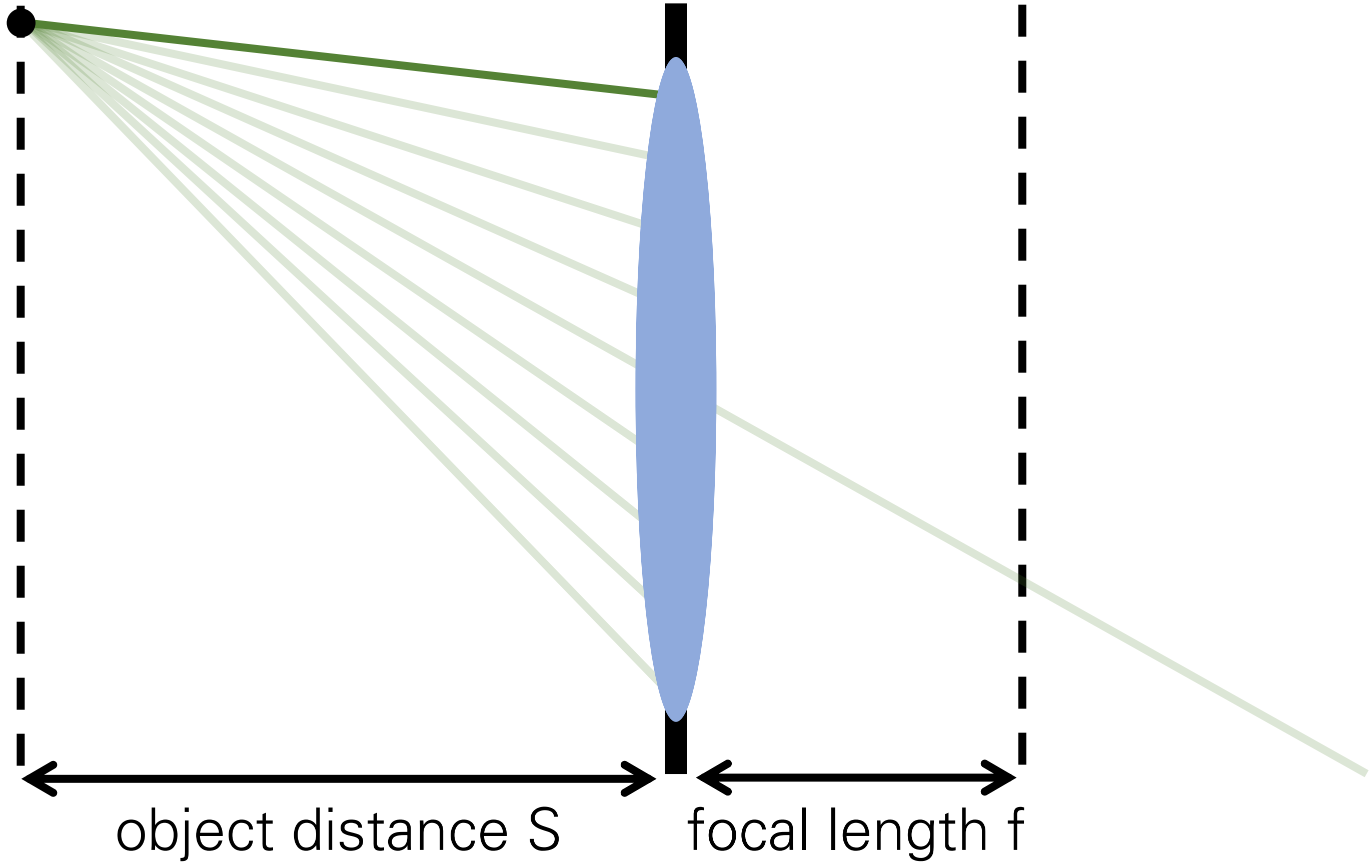
2. For all other rays:



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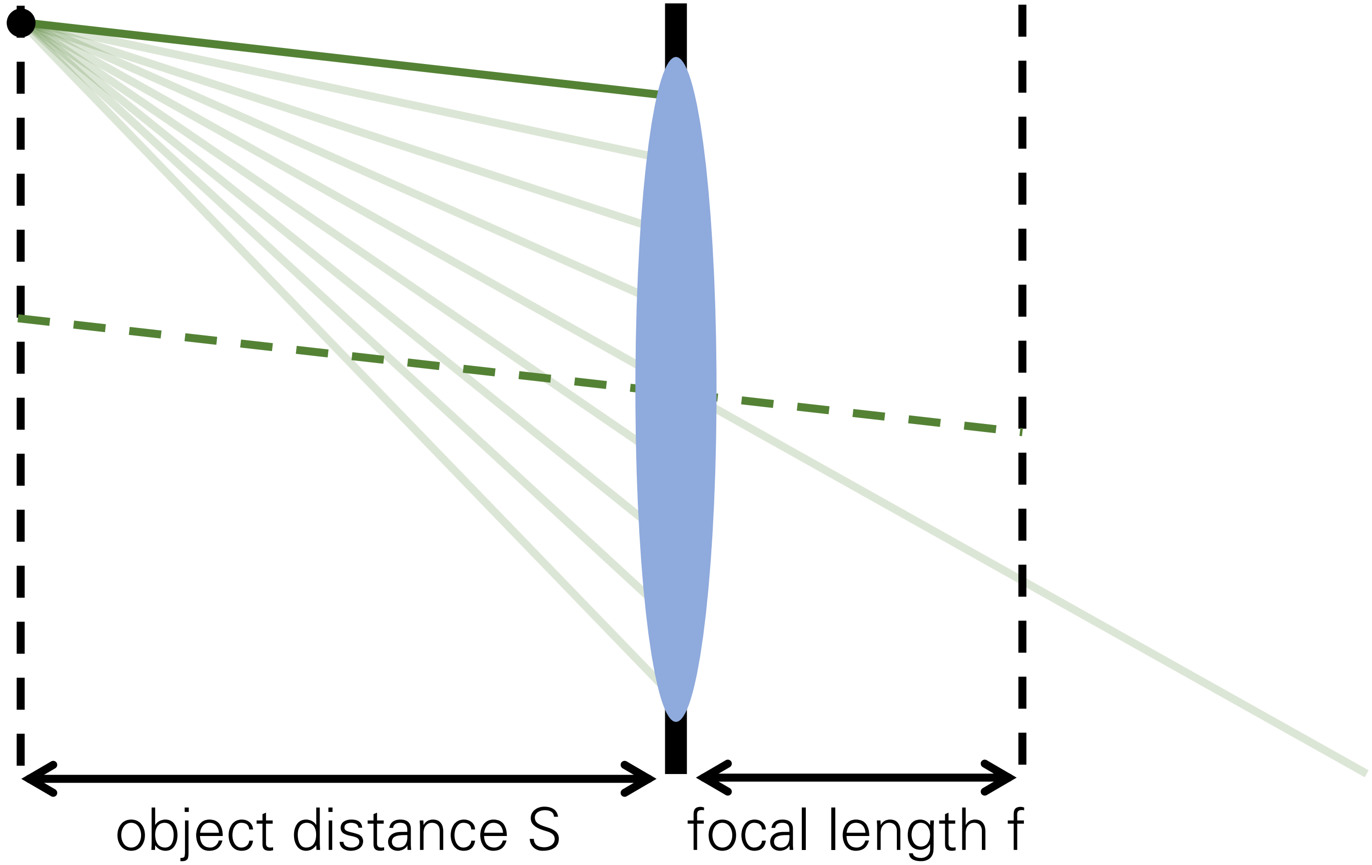

# Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

1. Trace rays through lens center.
2. For all other rays:
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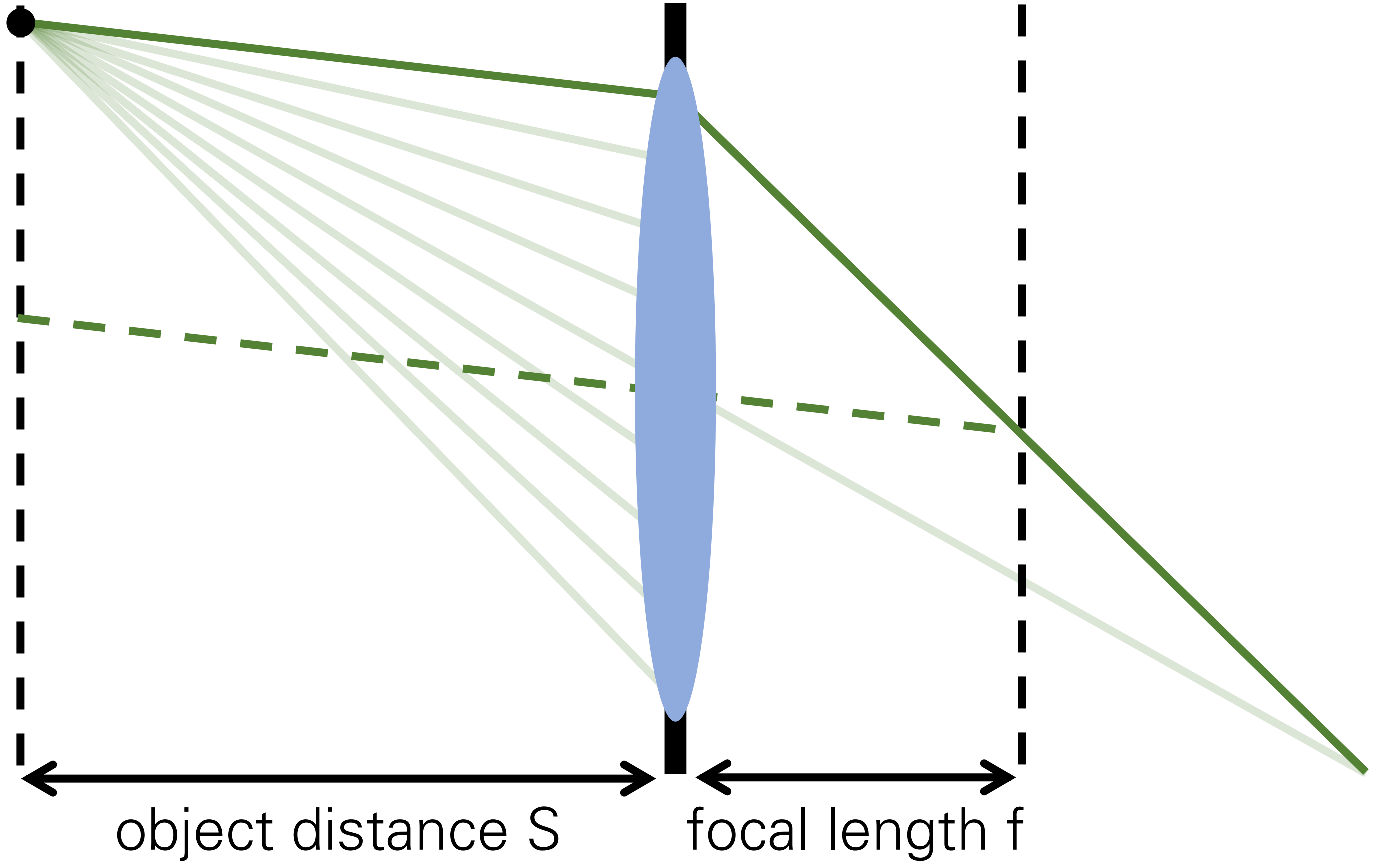
   b. Connect on focal plane.



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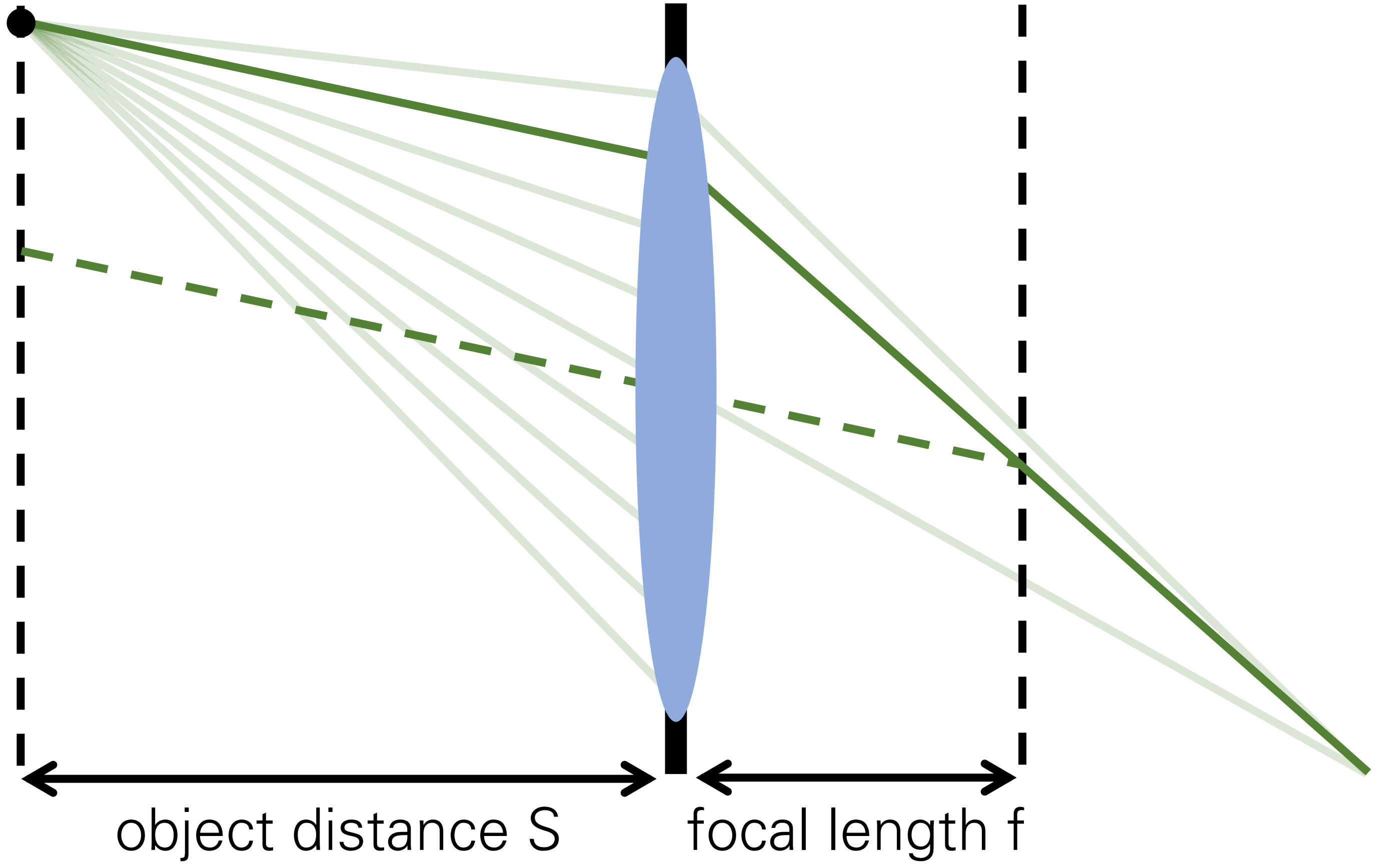

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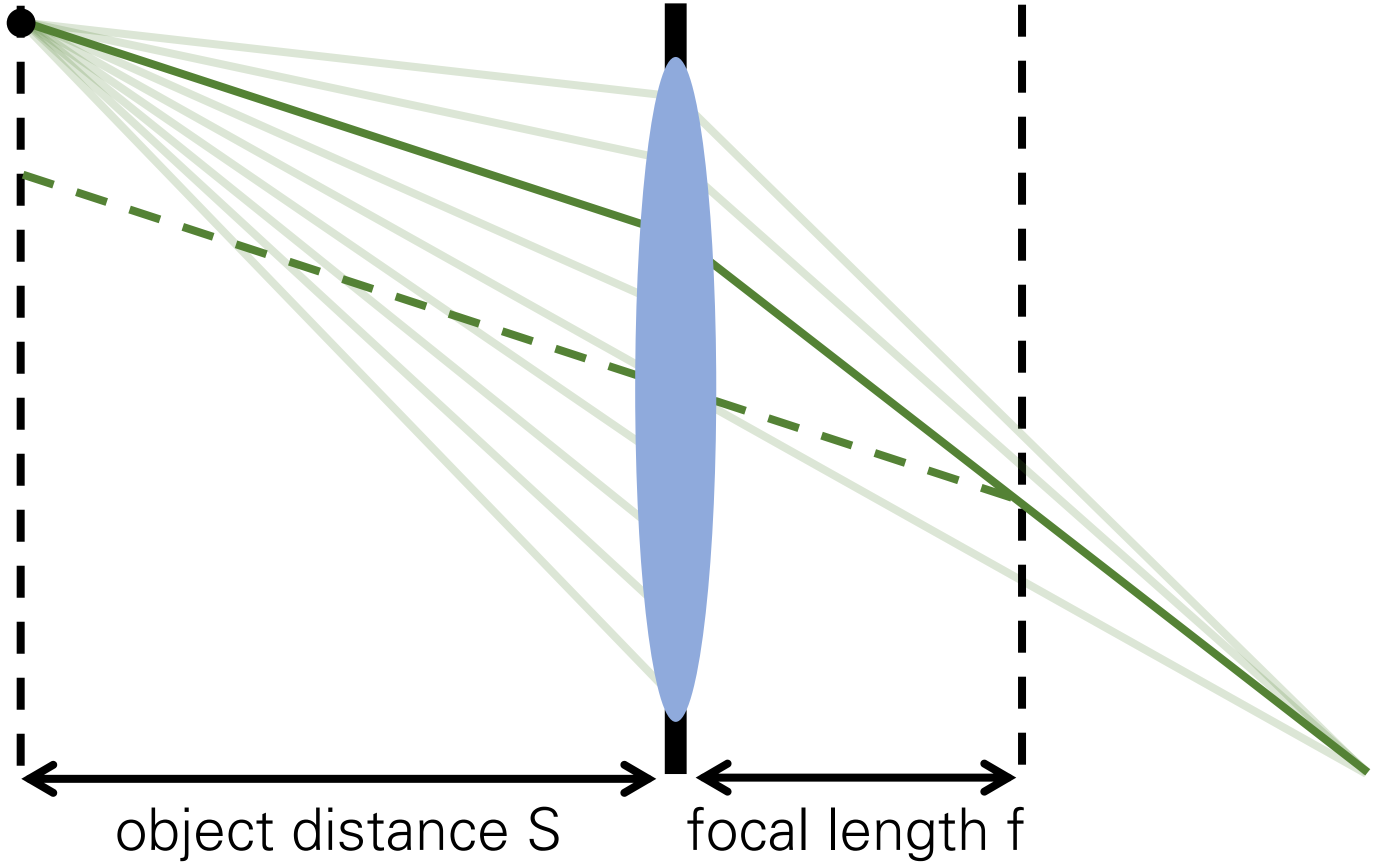

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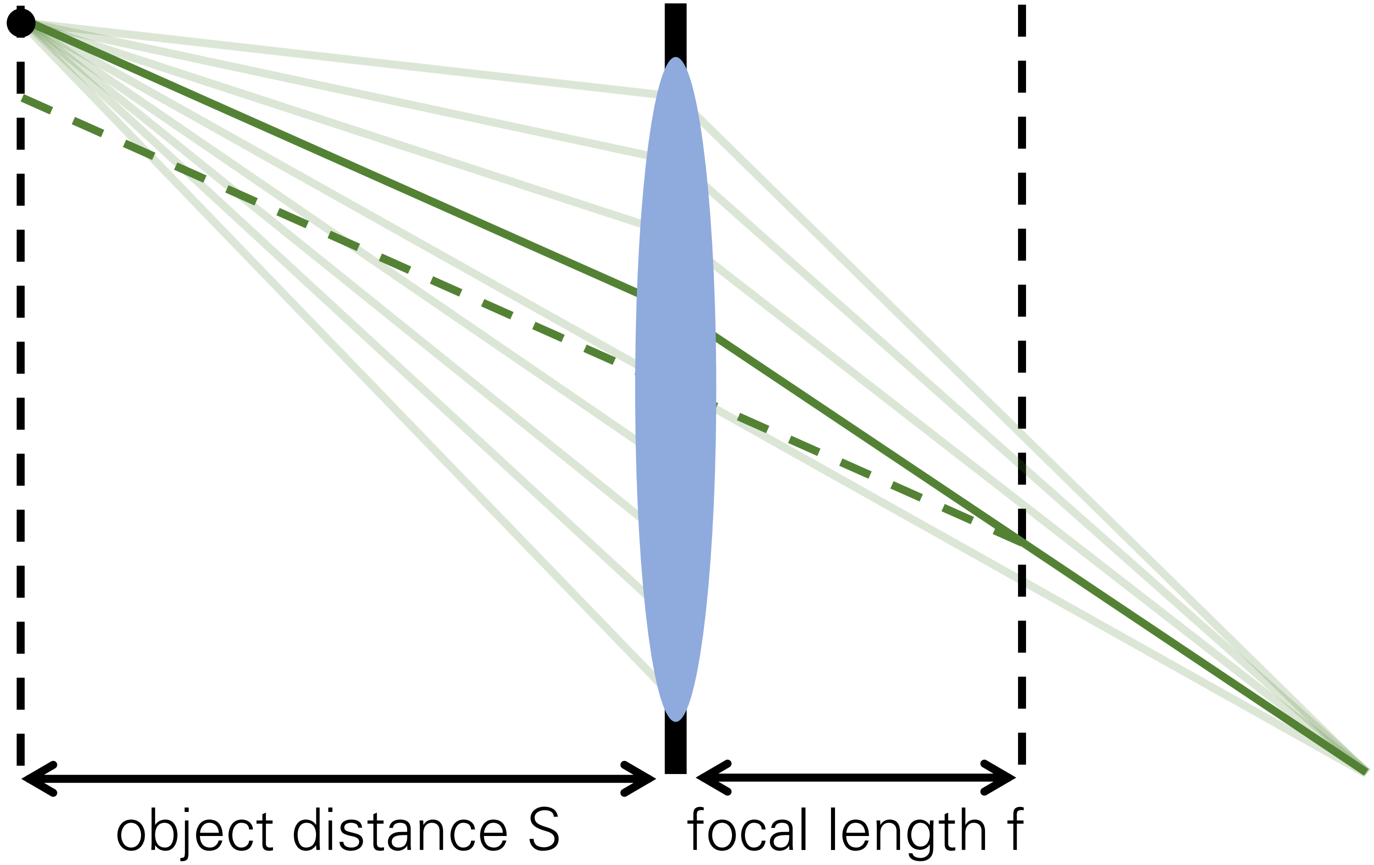

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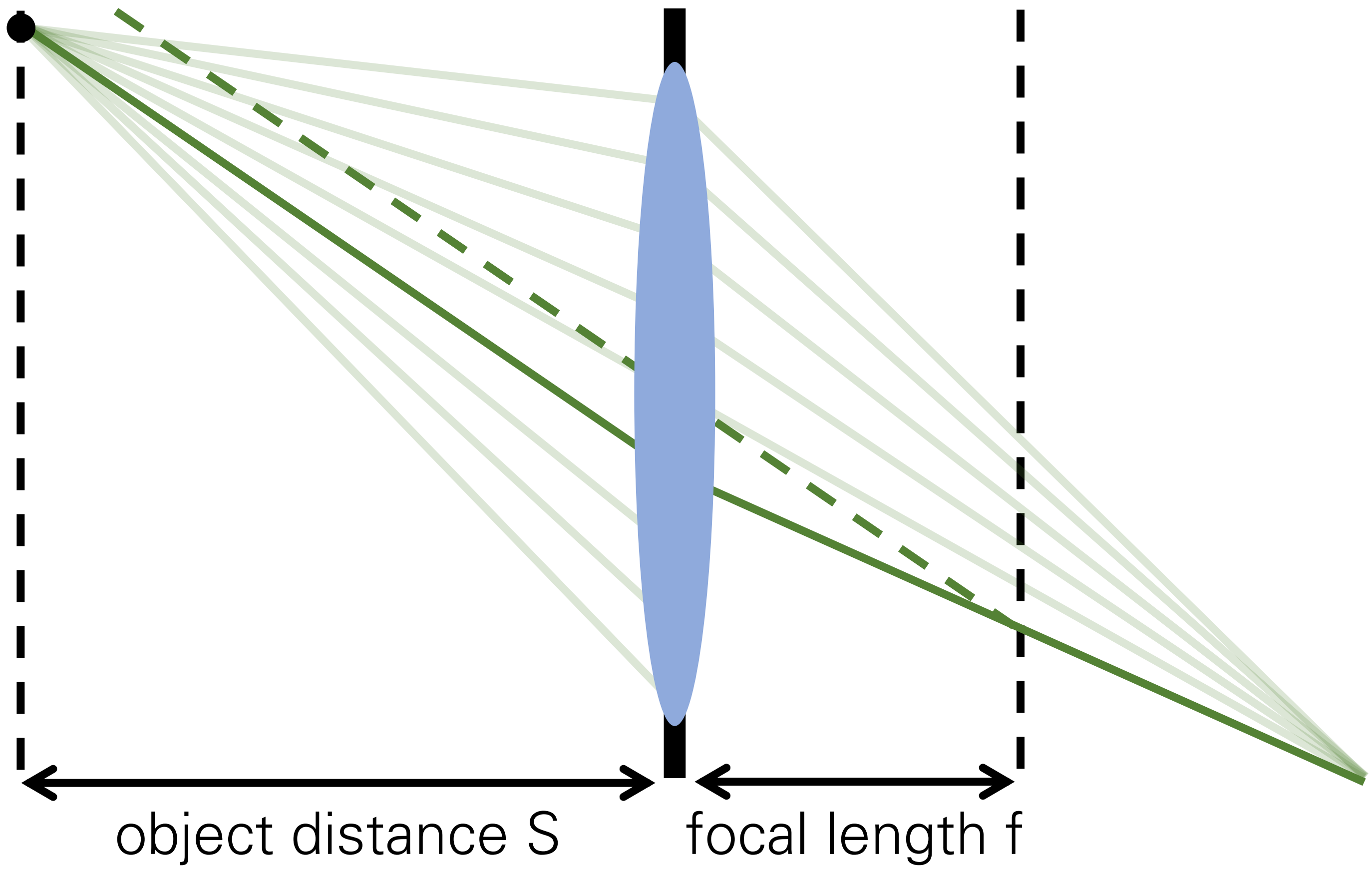

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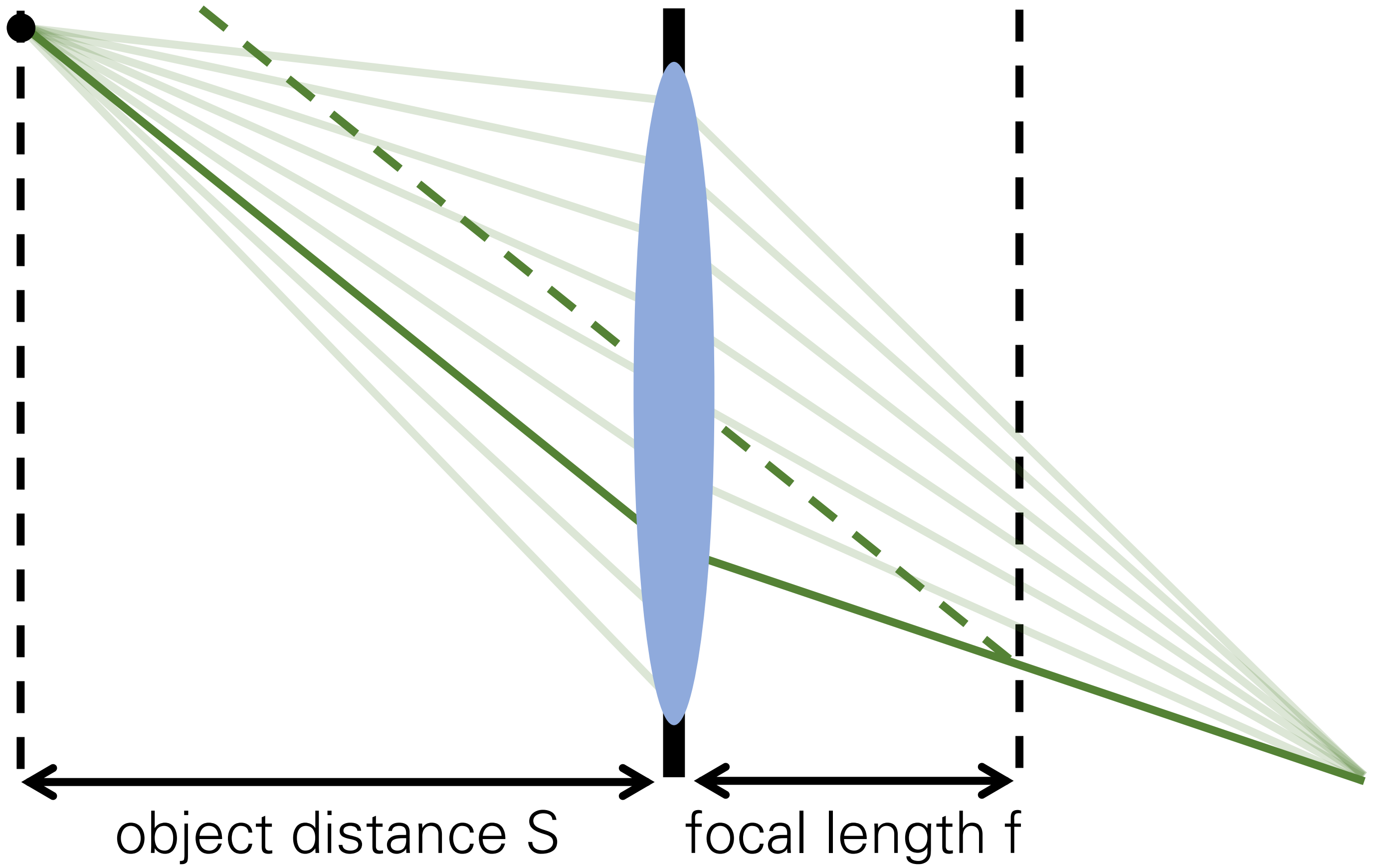

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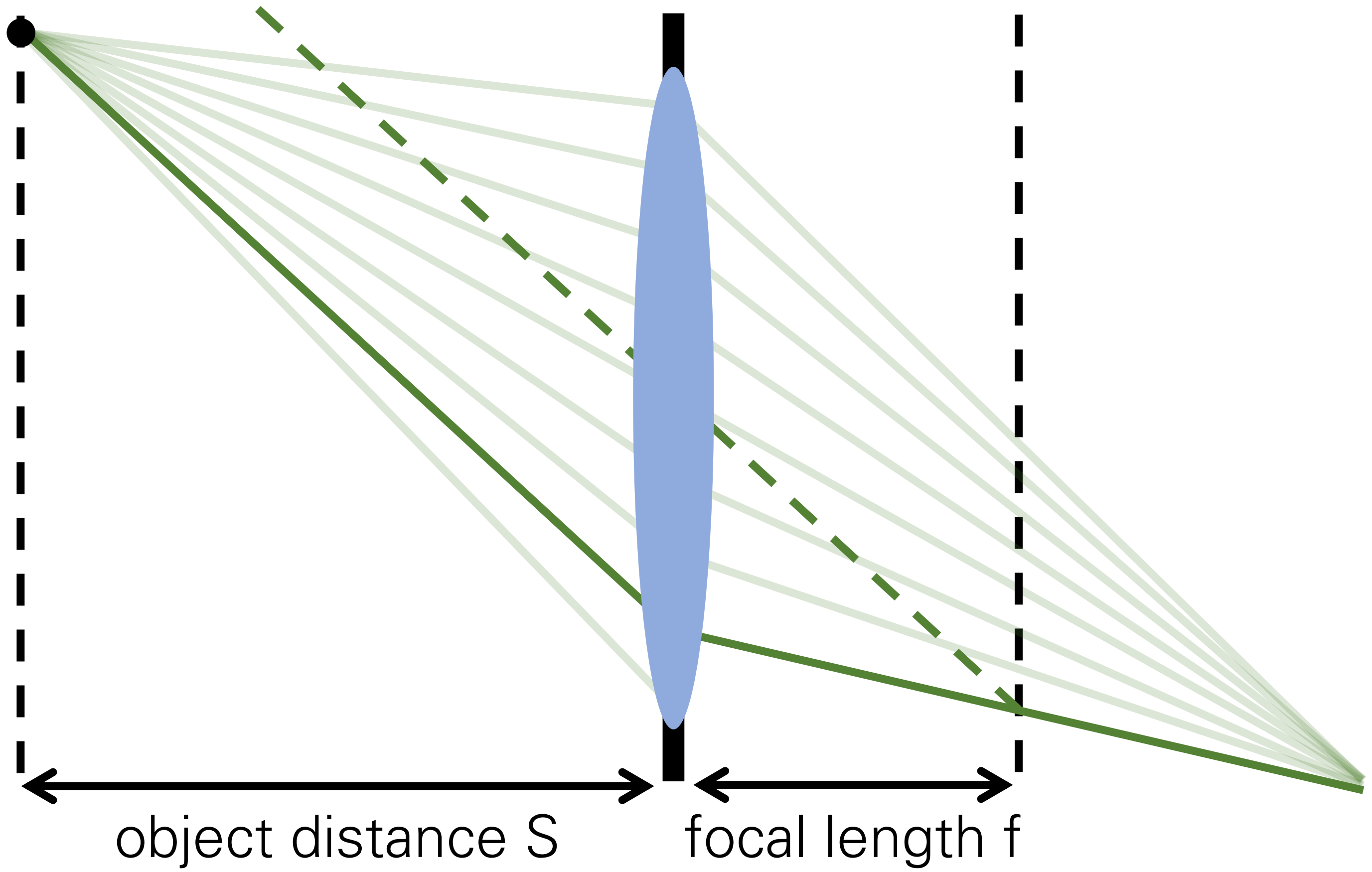

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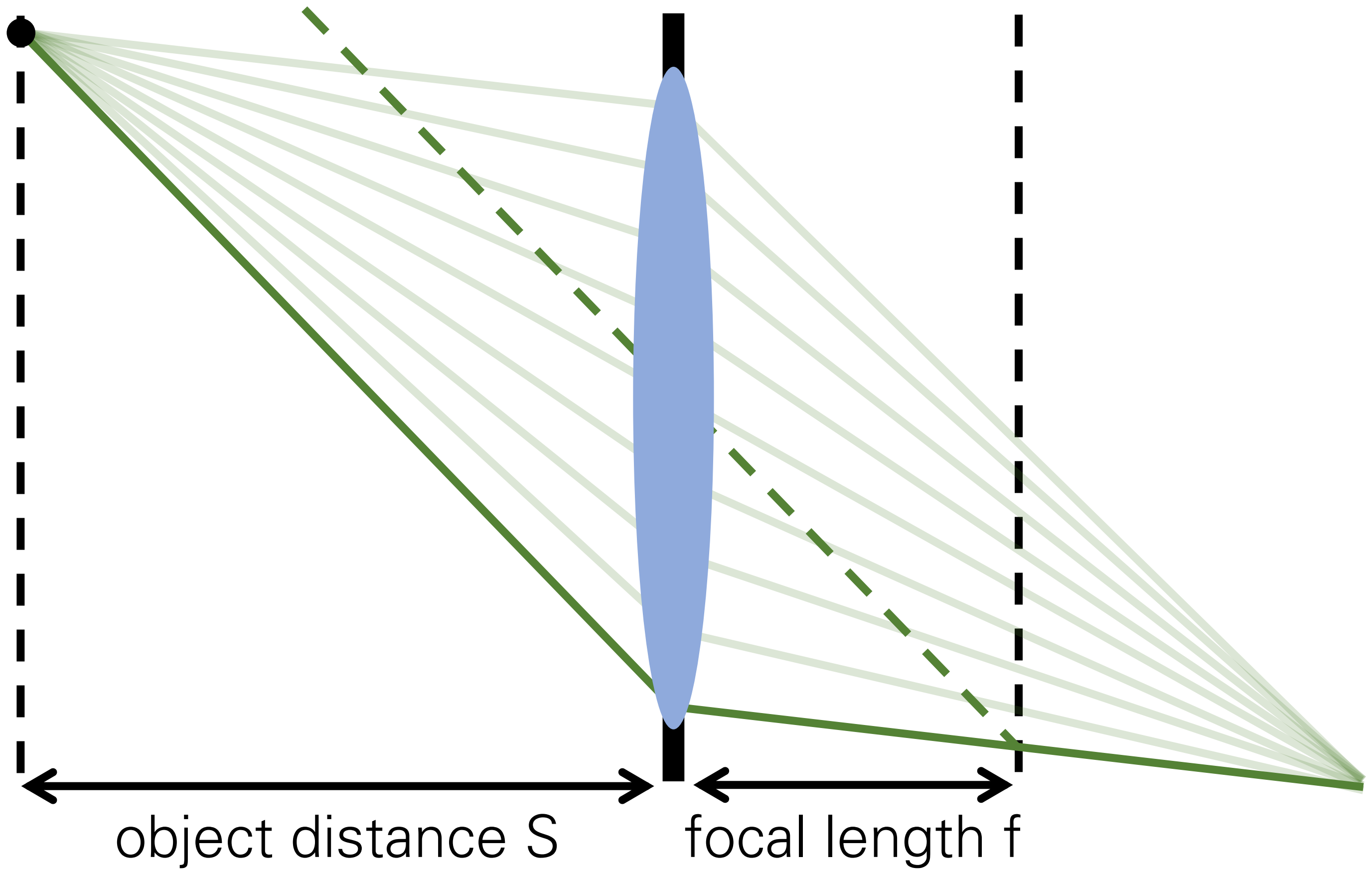

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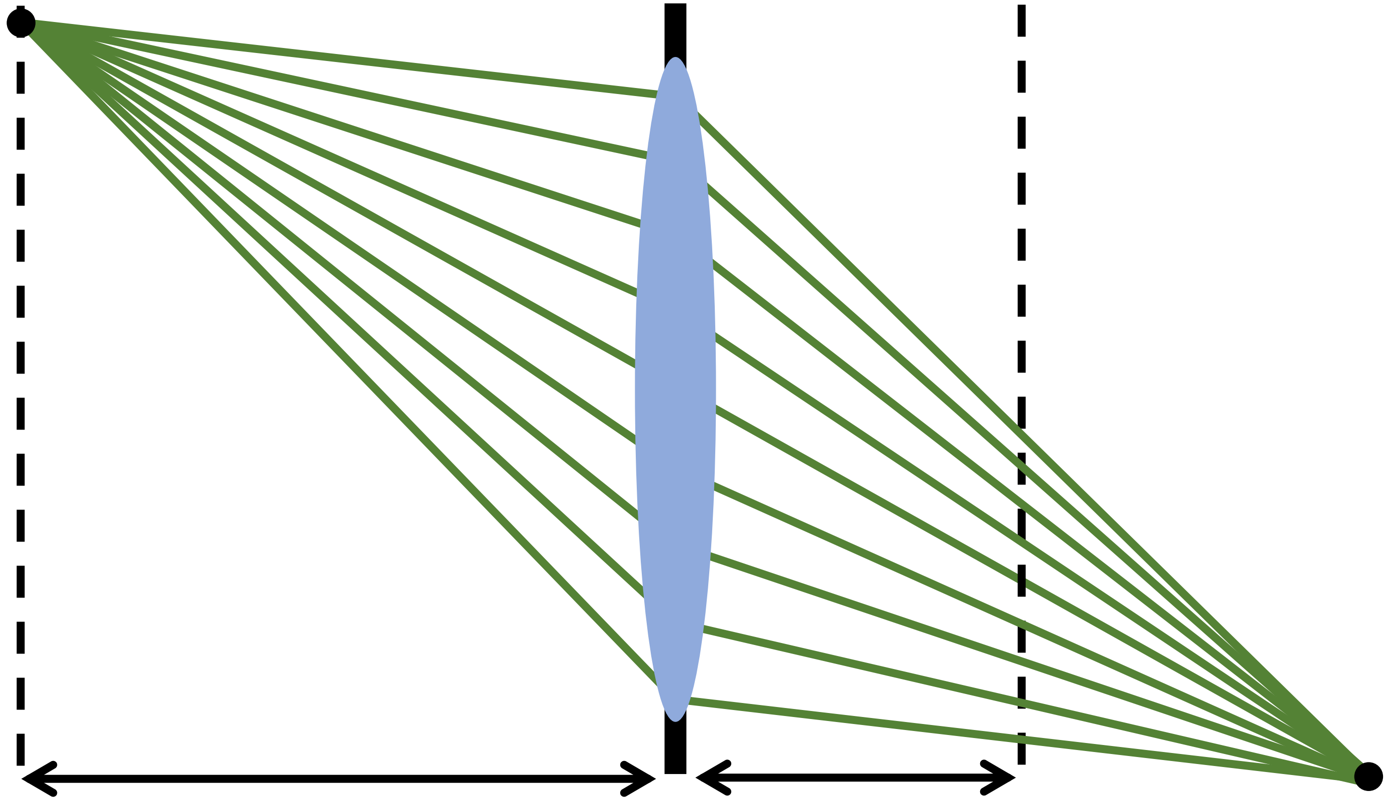


Focusing property:

1. Rays emitted from a point on one side converge to a point on the other side.

# Tracing rays through a thin lens

Consider an object emitting a bundle of rays. How do they propagate through the lens?

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2. For all other rays:
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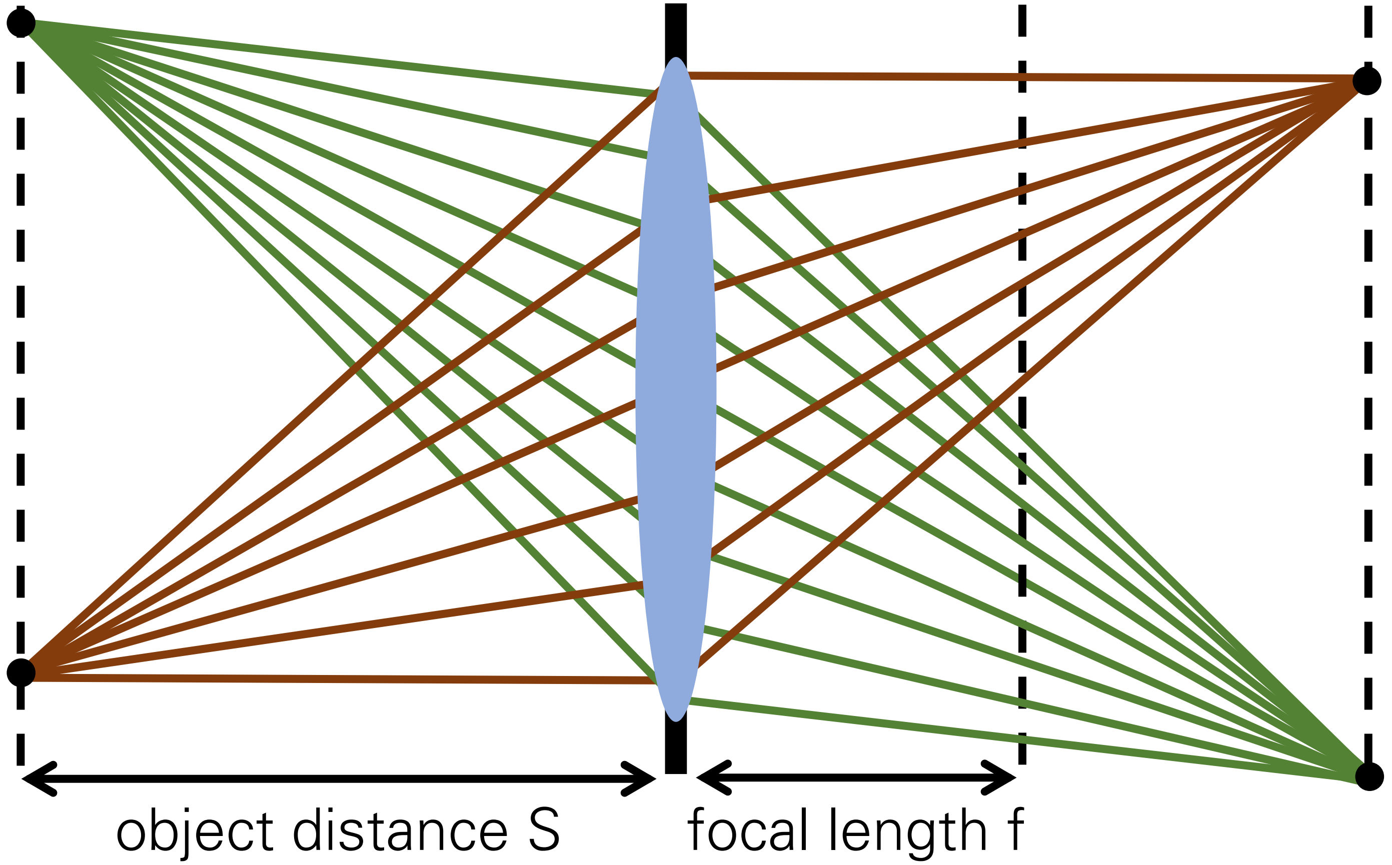
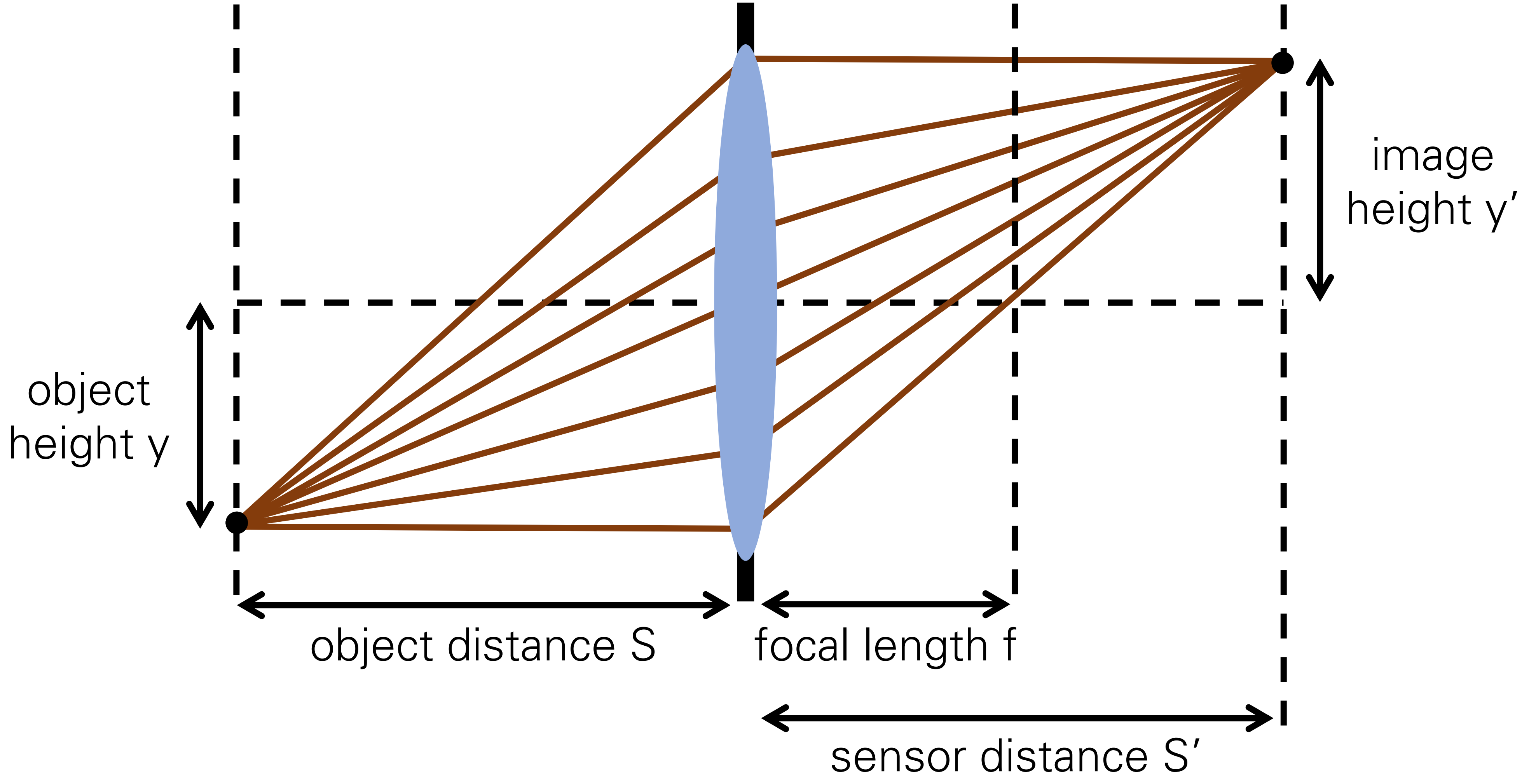



Focusing property:

1. Rays emitted from a point on one side converge to a point on the other side.
2. Bundles emitted from a plane parallel to the lens converge on a common plane.

# Gaussian lens formula

How can we relate scene-space (S, y) and image space (S′, y′) quantities?



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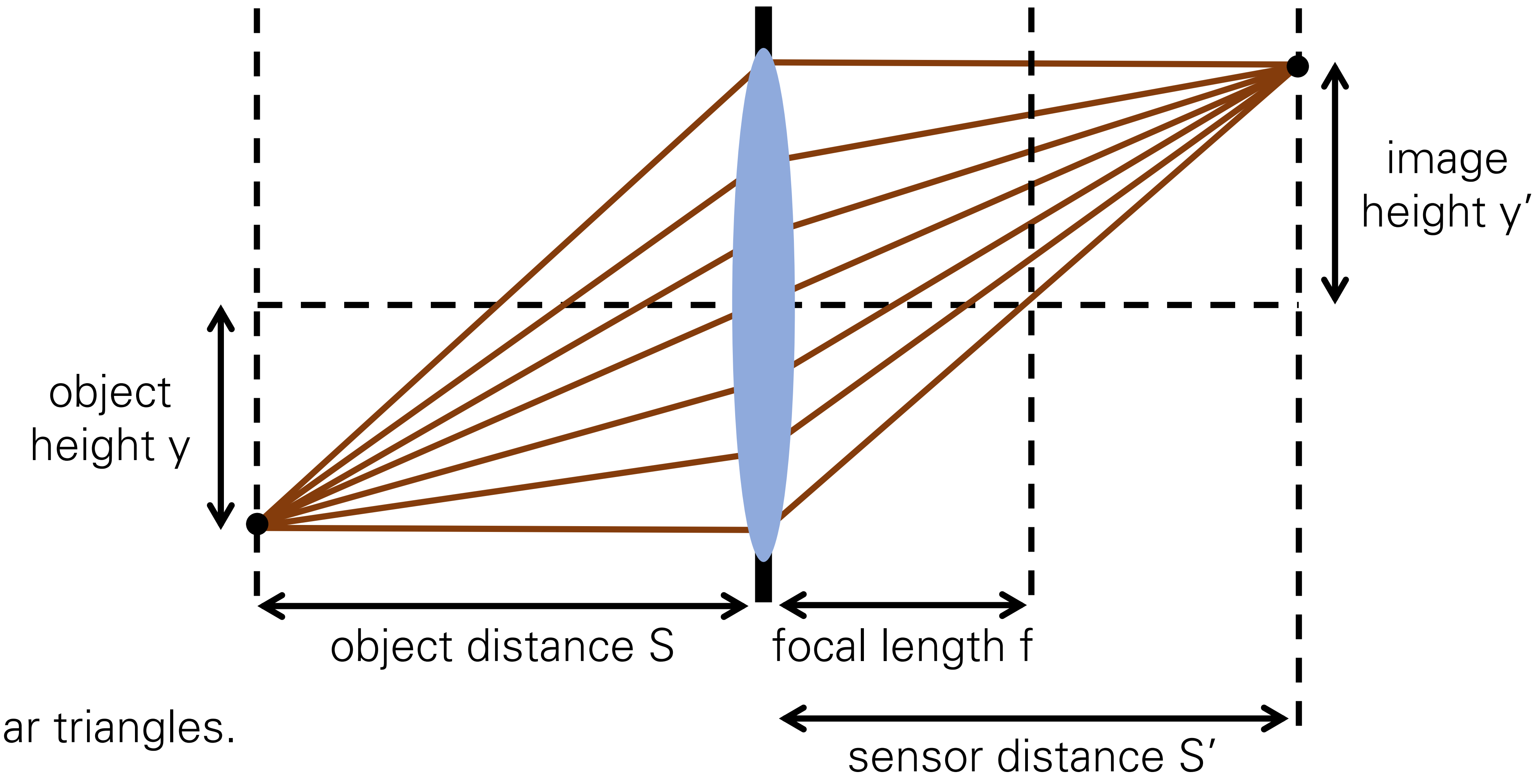
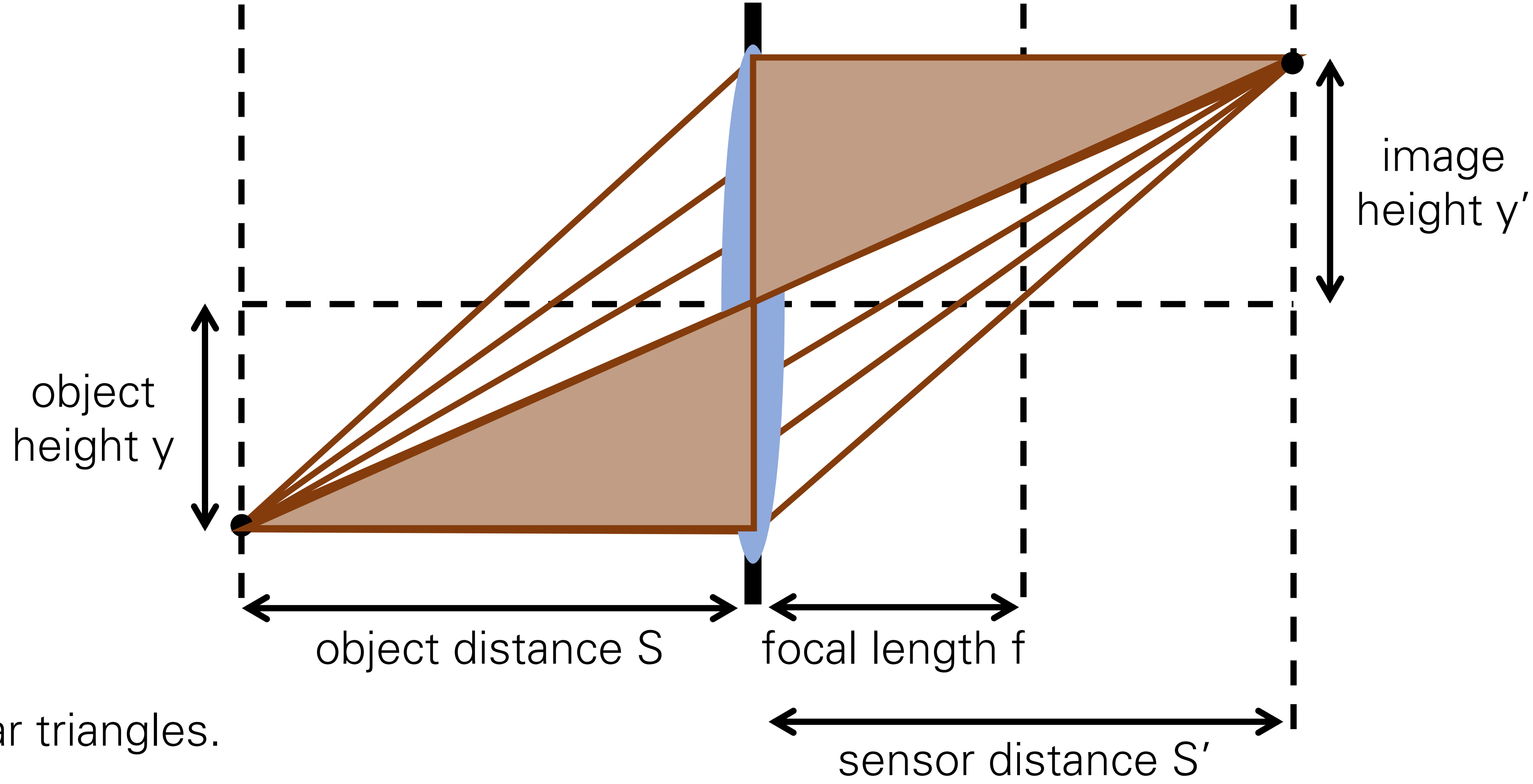



Use similar triangles.

# Gaussian lens formula

How can we relate scene-space (S, y) and image space (S′, y′) quantities?

$$\frac{y'}{y} = ?$$

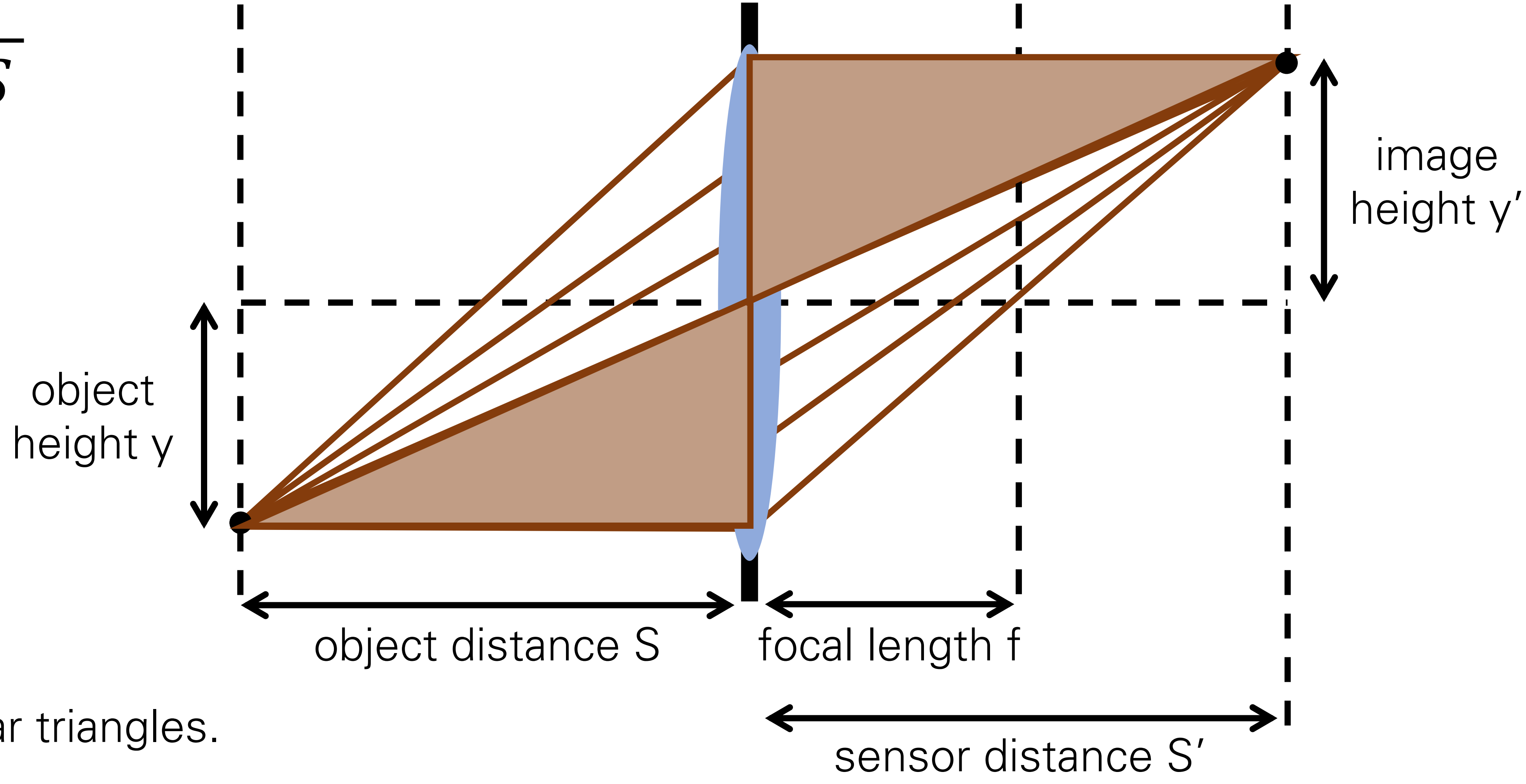



Use similar triangles.

# Gaussian lens formula

How can we relate scene-space (S, y) and image space (S′, y′) quantities?

$$\frac{y'}{y} = \frac{S'}{S}$$

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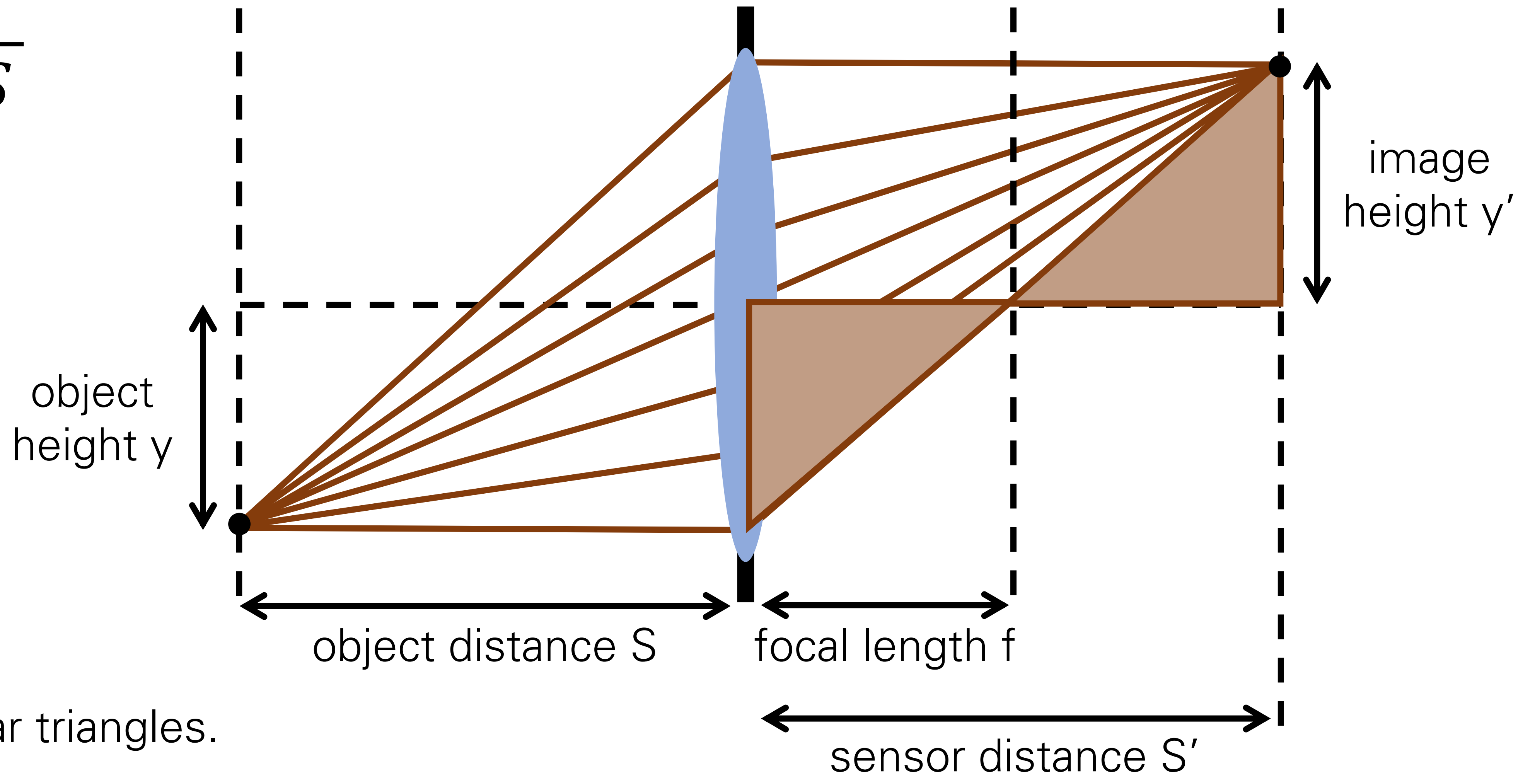
$$\frac{y'}{y} = ?$$



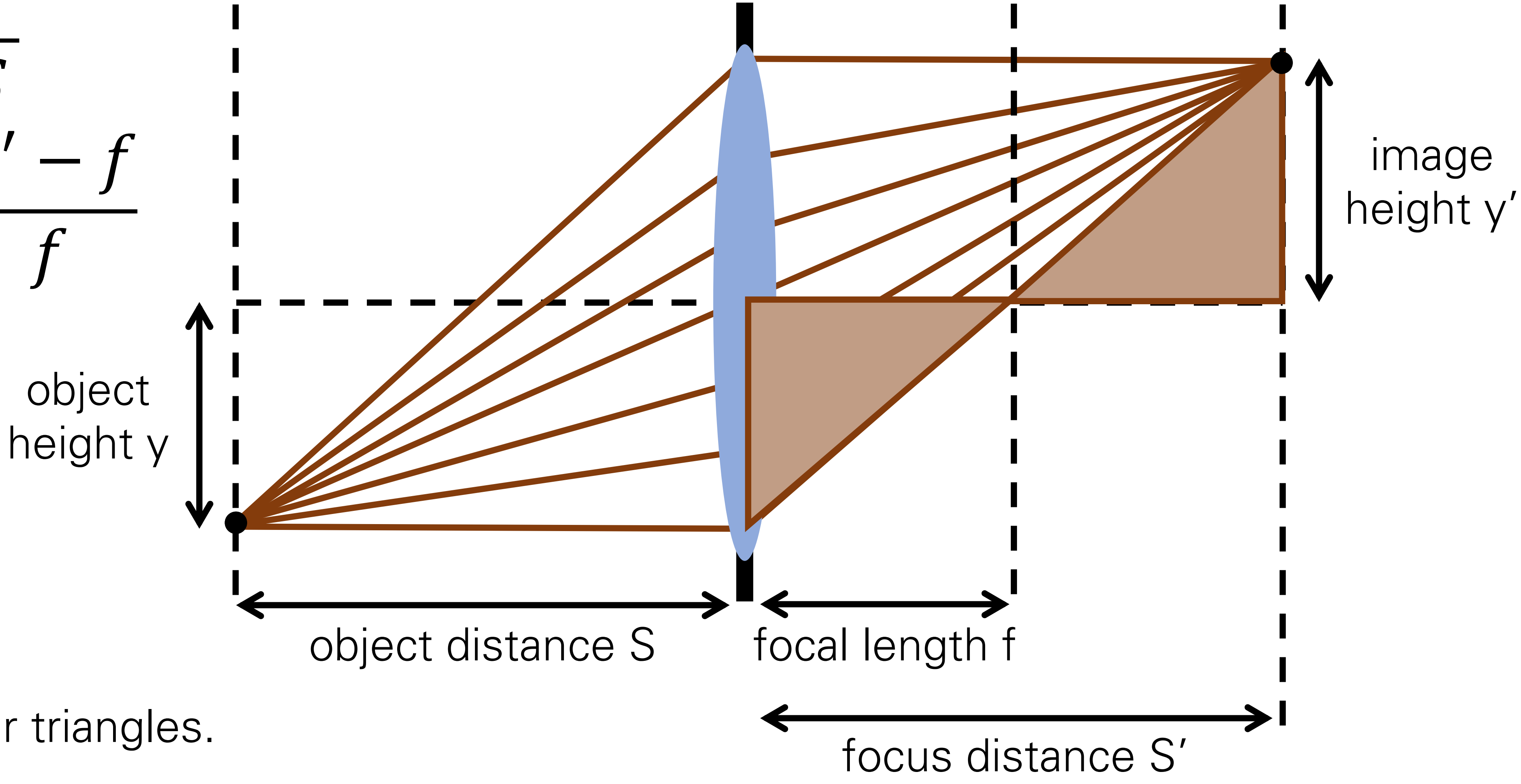

Use similar triangles.

# Gaussian lens formula

How can we relate scene-space (S, y) and image space (S′, y′) quantities?

$$\frac{y'}{y} = \frac{S'}{S}$$

$$\frac{y'}{y} = \frac{S'-f}{f}$$



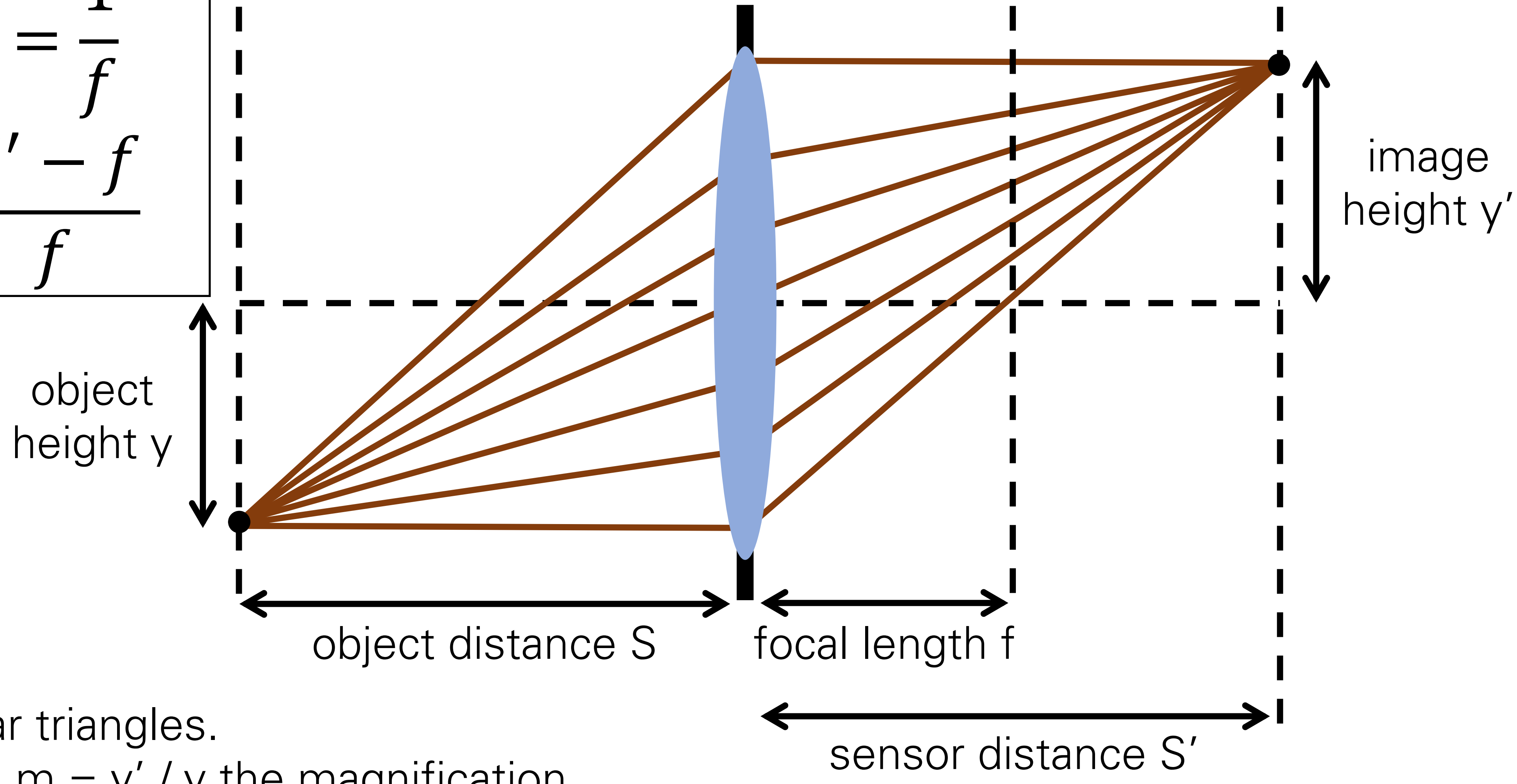

Use similar triangles.

# Gaussian lens formula

How can we relate scene-space (S, y) and image space (S′, y′) quantities?

$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$

$$m = \frac{S' - f}{f}$$




Use similar triangles.

- We call m = y′ / y the magnification.

# Special focus distances

S’ = f, S = ?, m = ?

$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$

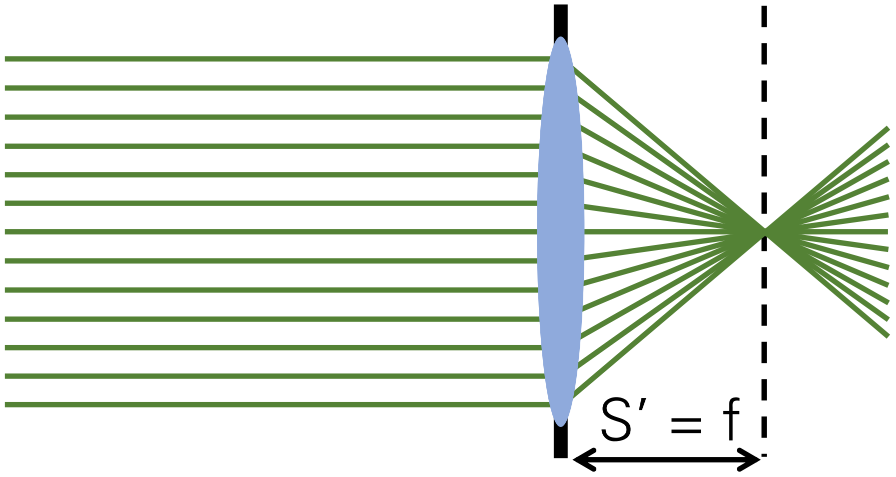
$$m = \frac{S' - f}{f}$$

# Special focus distances

$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$

$$m = \frac{S' - f}{f}$$

S′ = f, S = ∞, m = 0 → infinity focus (parallel rays)



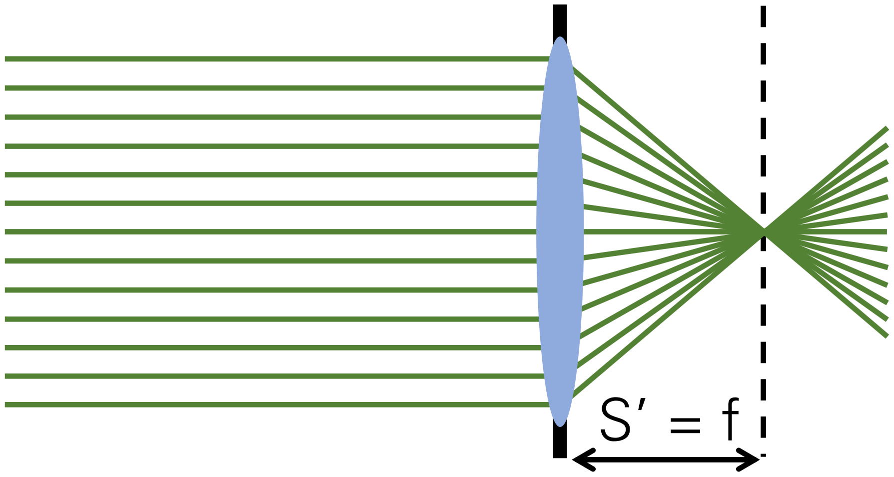

S′ = S = ?, m = ?

# Special focus distances

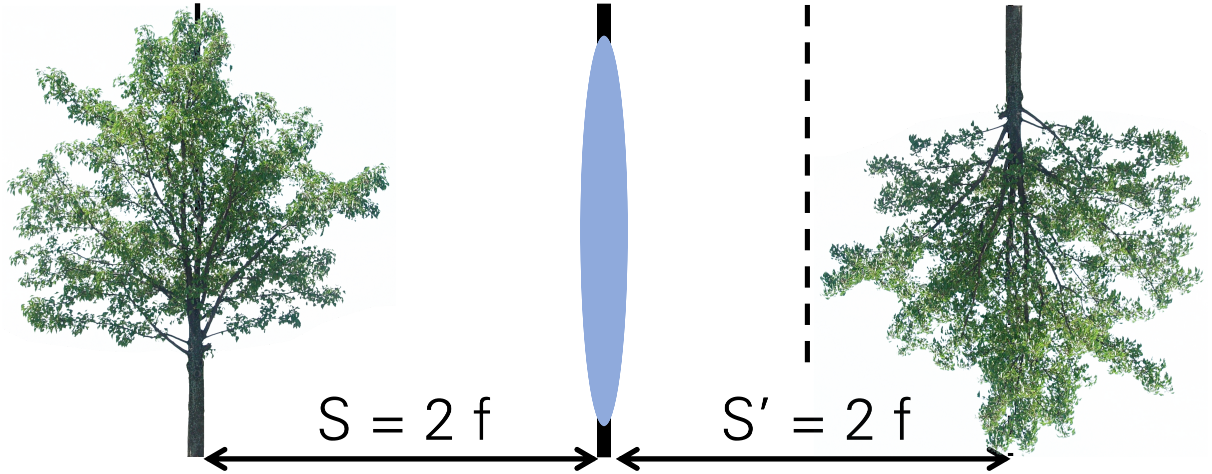
$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$

$$m = \frac{S' - f}{f}$$

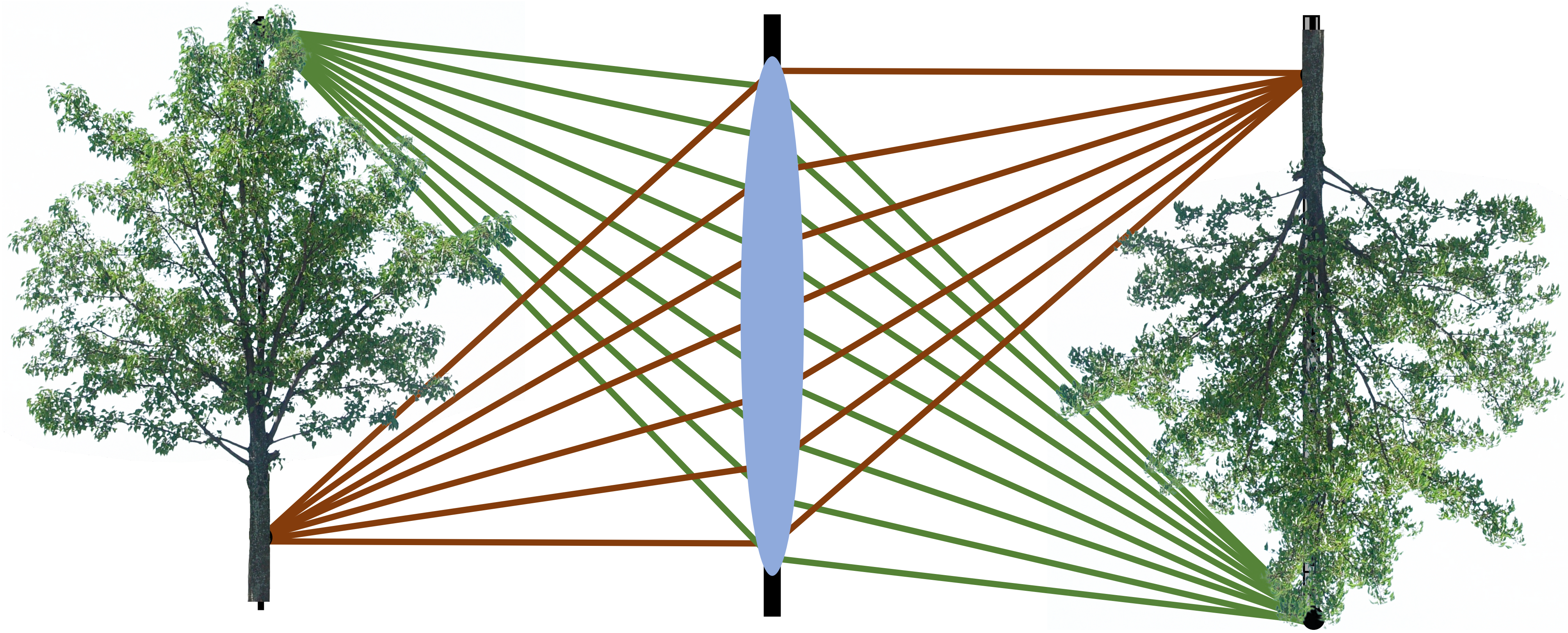
S′ = f, S = ∞, m = 0 → infinity focus (parallel rays)




S′ = S = 2 f, m = 1 → object is reproduced in real-life size



# Free lunch?



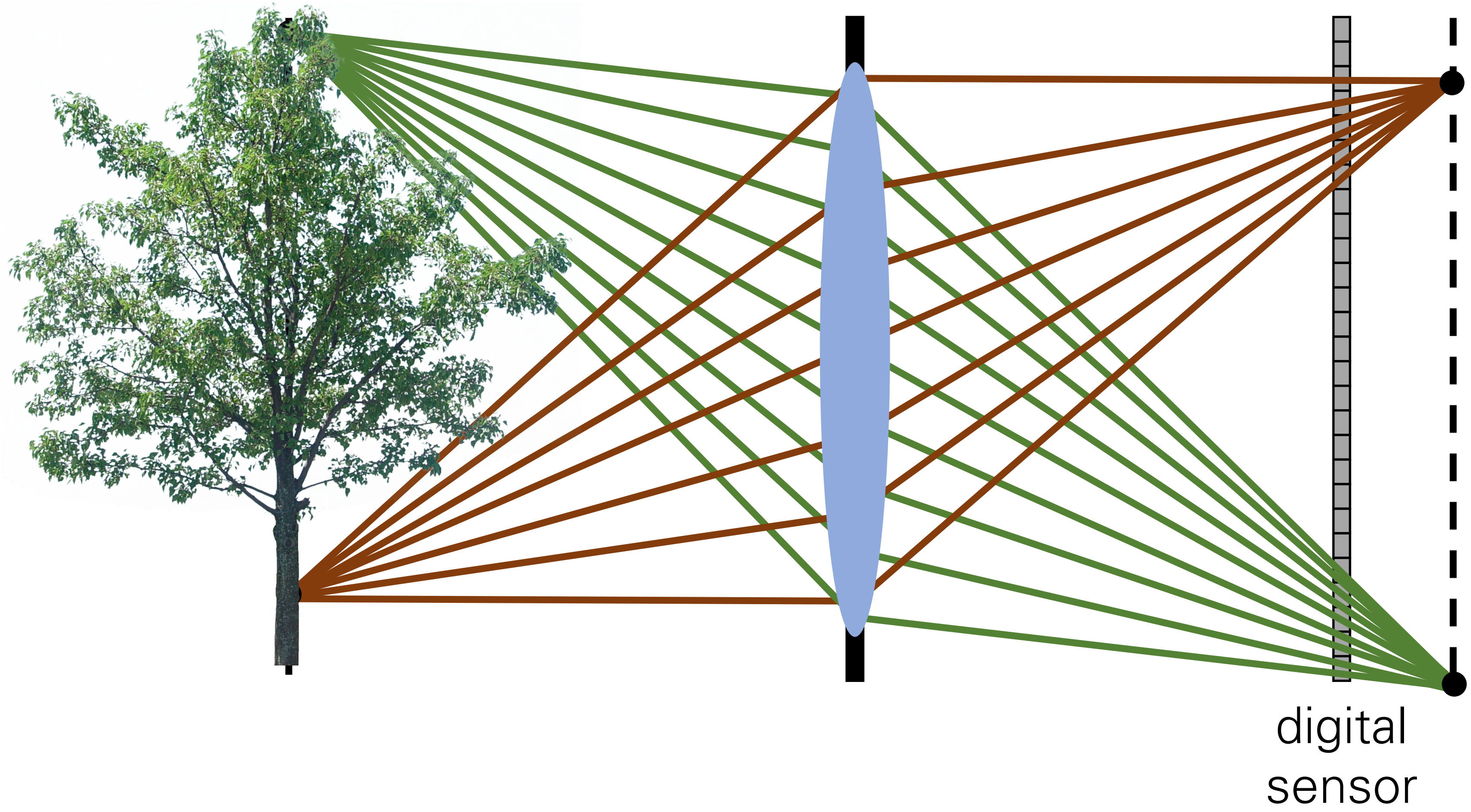
By using a lens, we simultaneously achieve:

1. Image is sharp.
2. Signal-to-noise ratio is high.

Do we lose anything by using a lens?

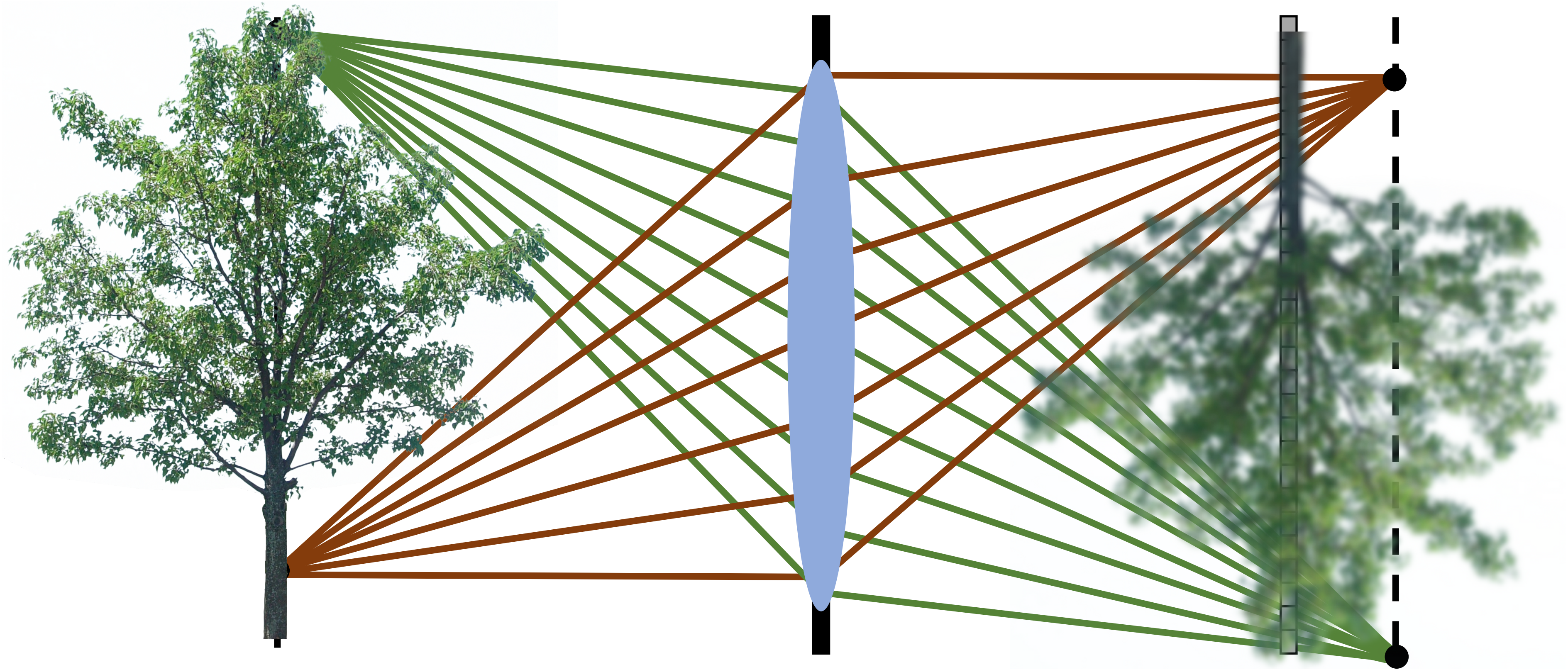
# Defocus

What happens if we don’t place the sensor at the focus distance?



# Defocus

What happens if we don’t place the sensor at the focus distance?




We get a blurry image. This is called defocus.

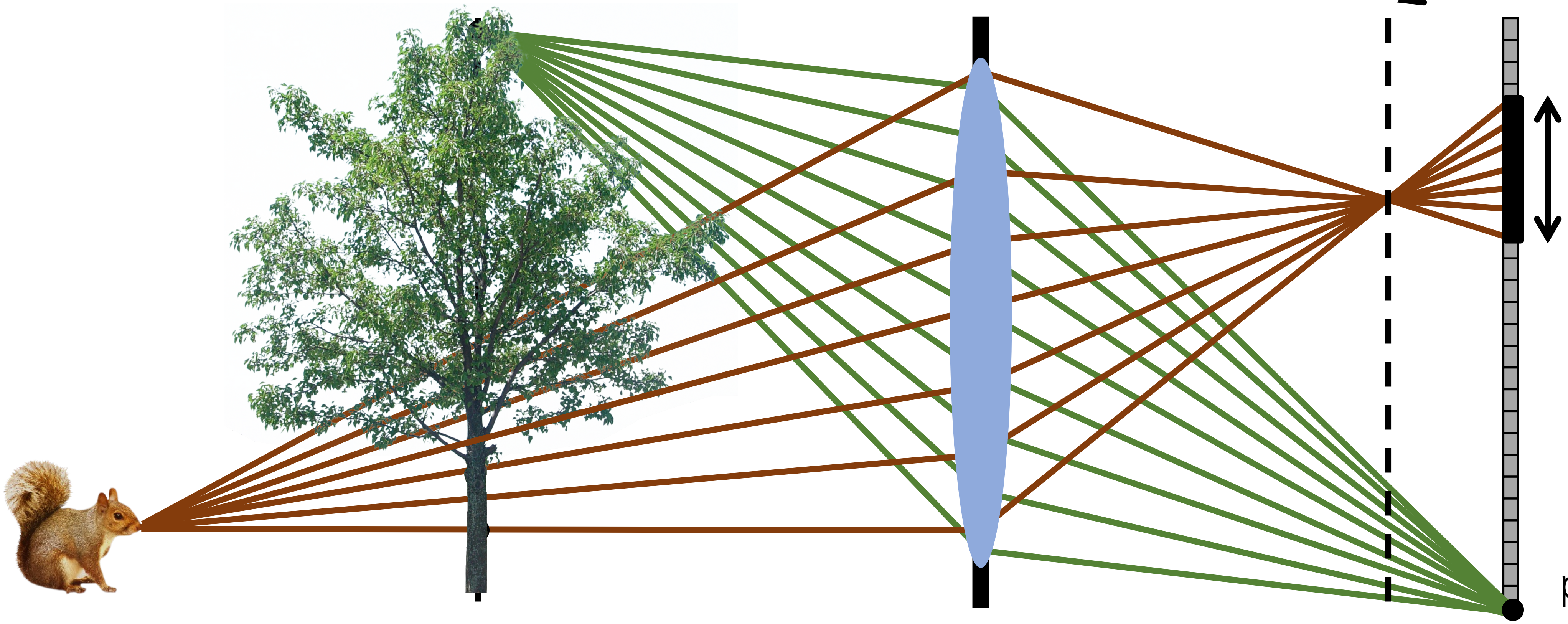
- Defocus never happens with an ideal pinhole camera.

# Defocus

Can’t we just move the sensor to the correct distance?

# Defocus

Can’t we just move the sensor to the correct distance?




Unless our scene is just one plane, part of it will always be out of focus.

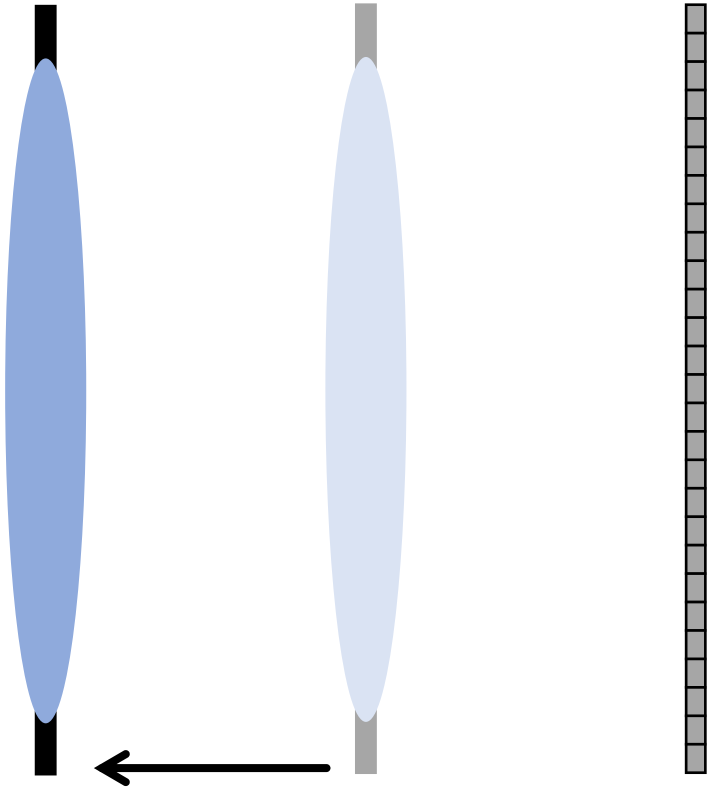
# How do we control what is in focus?

# How do we control what is in focus?

We change the distance between the sensor and the lens

# How do we control what is in focus?

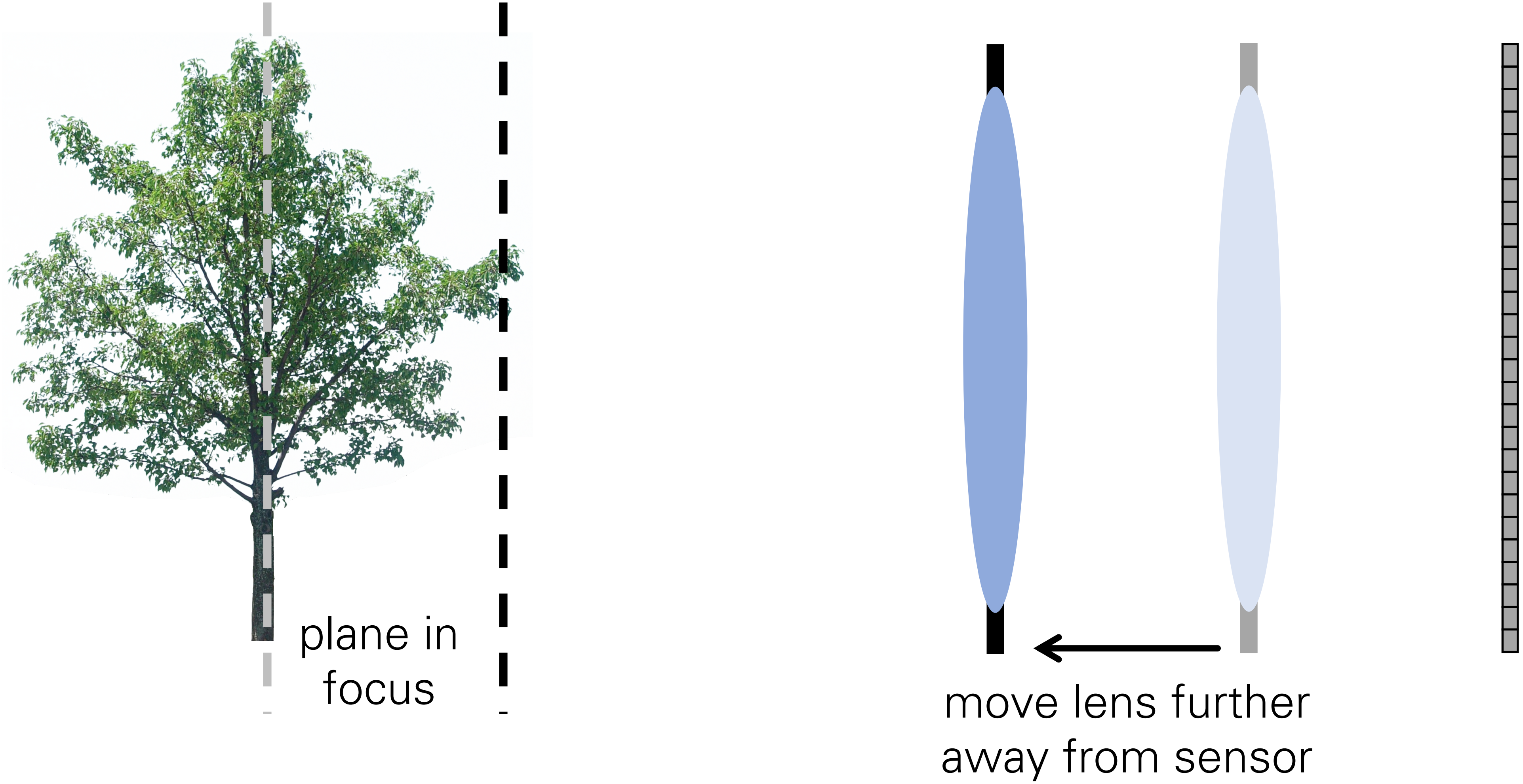
We change the distance between the sensor and the lens


- What happens to plane in focus?

# How do we control what is in focus?

We change the distance between the sensor and the lens




- What happens to plane in focus? → It moves closer.

# The lens on your camera

Focus ring: controls distance of lens from sensor

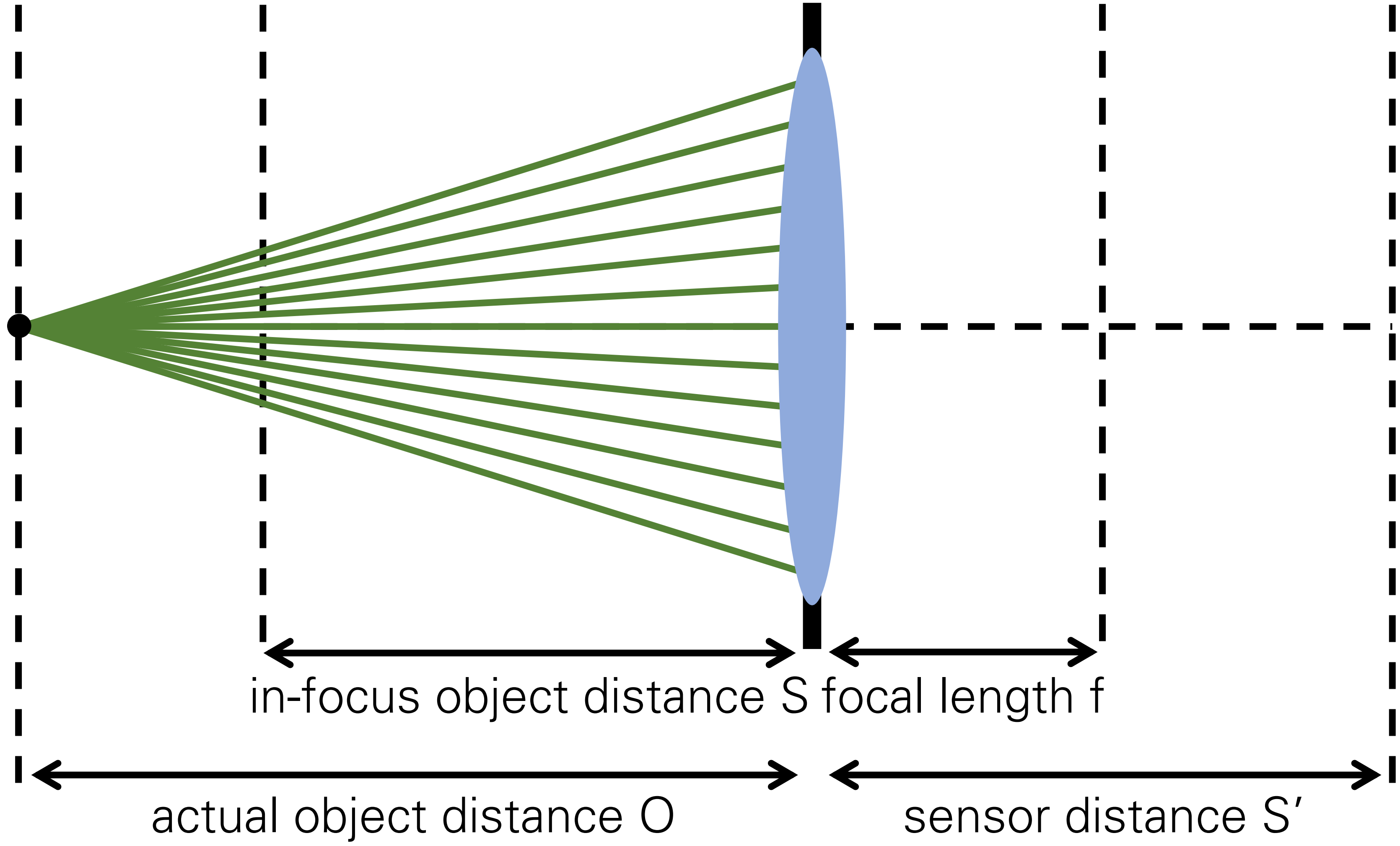
# Demonstration

# Defocus

Does the mean that lenses are only good for planar scenes?

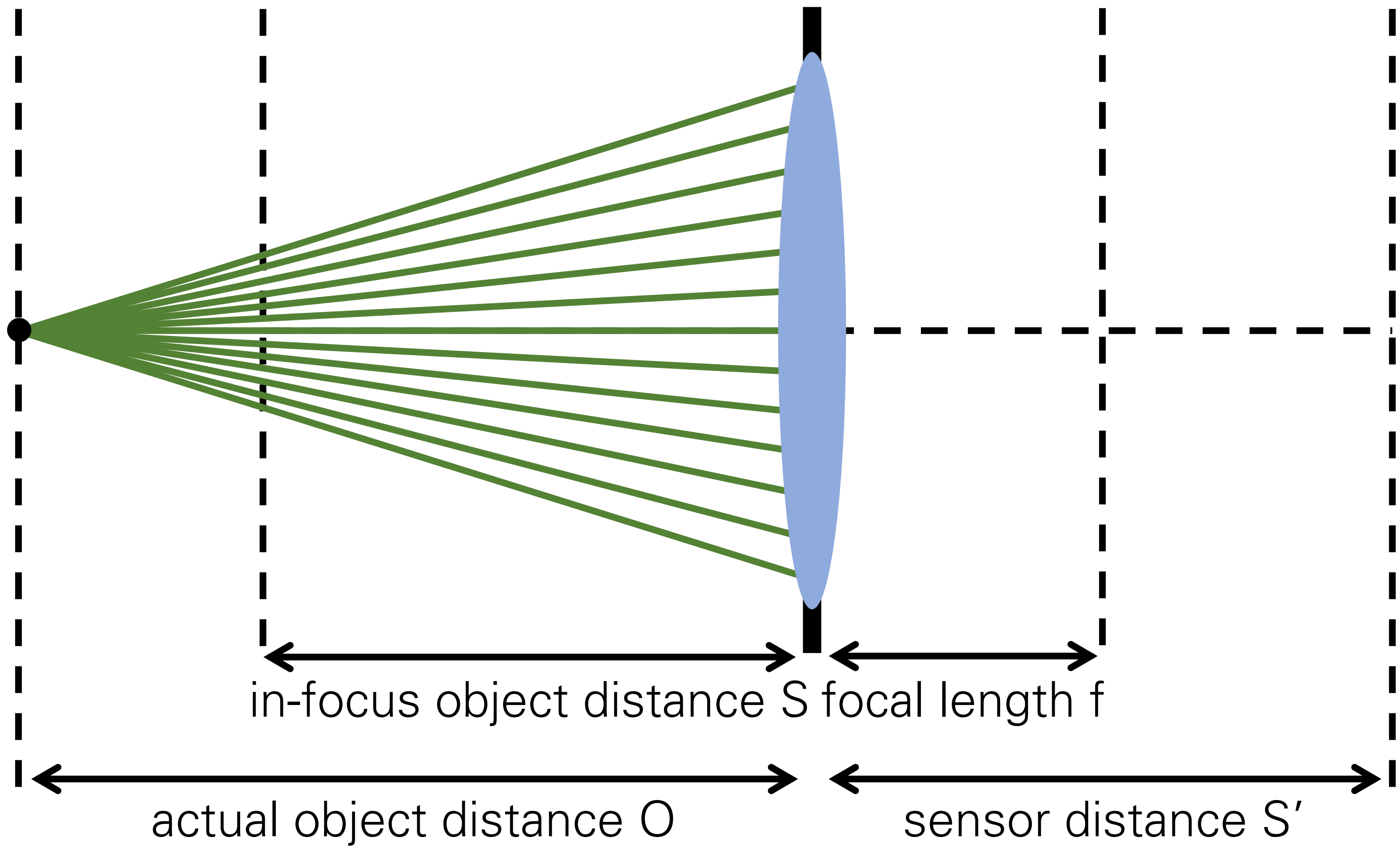
# Circle of confusion

How do we find where the point will focus?



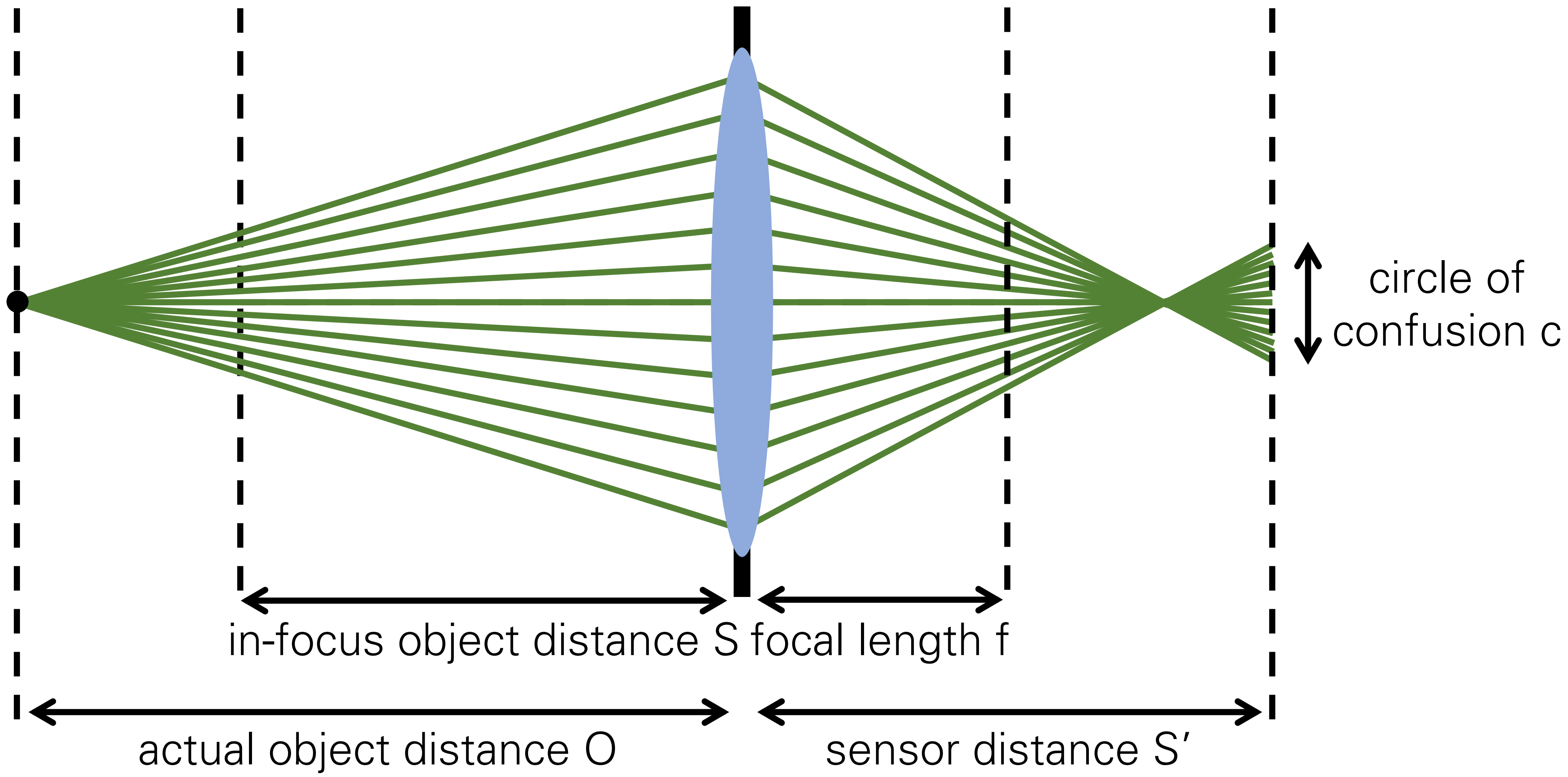
# Circle of confusion

Will the point focus at a distance smaller or larger than S′?



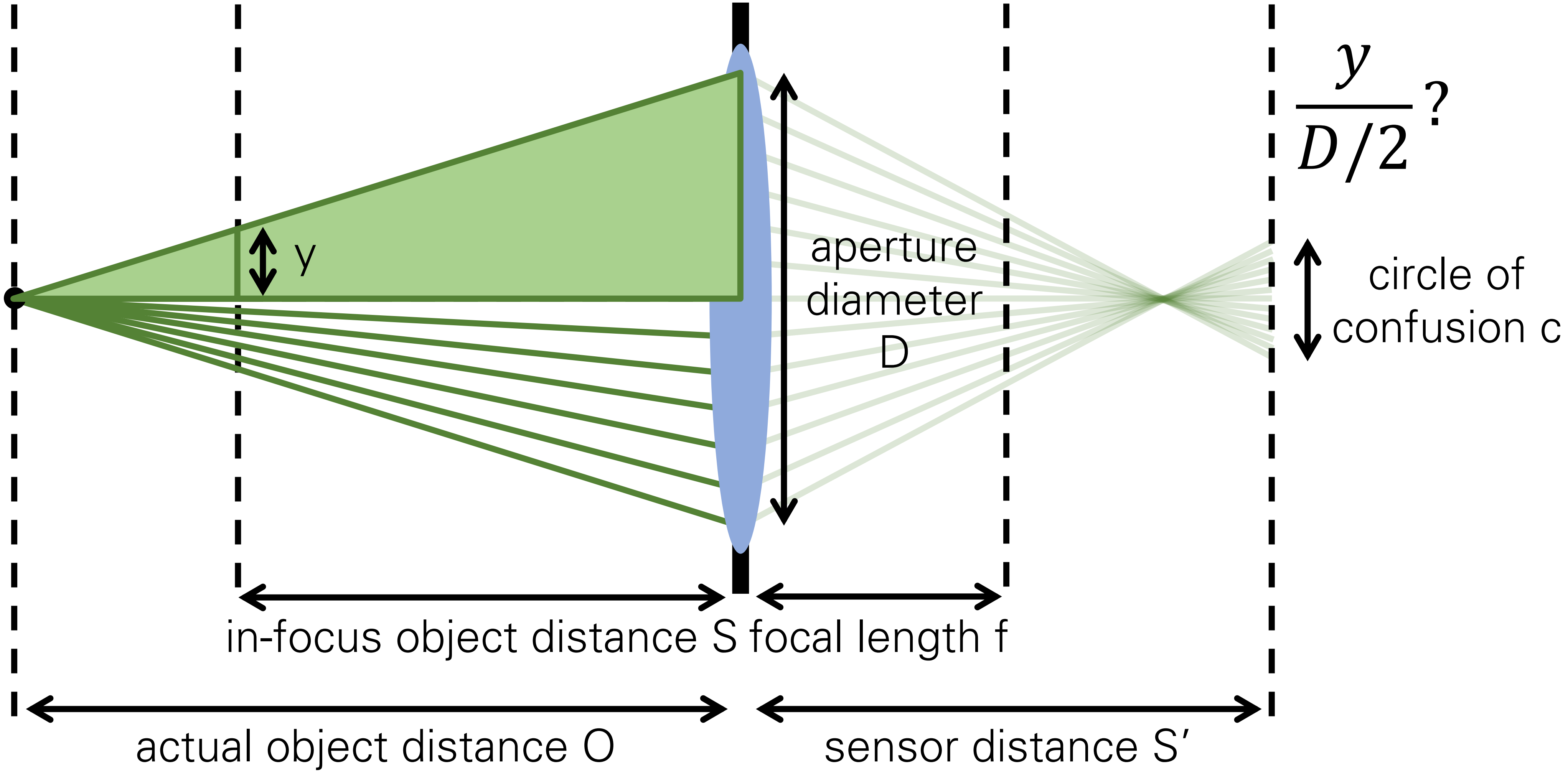
# Circle of confusion

How can we compute the diameter of the circle of confusion?



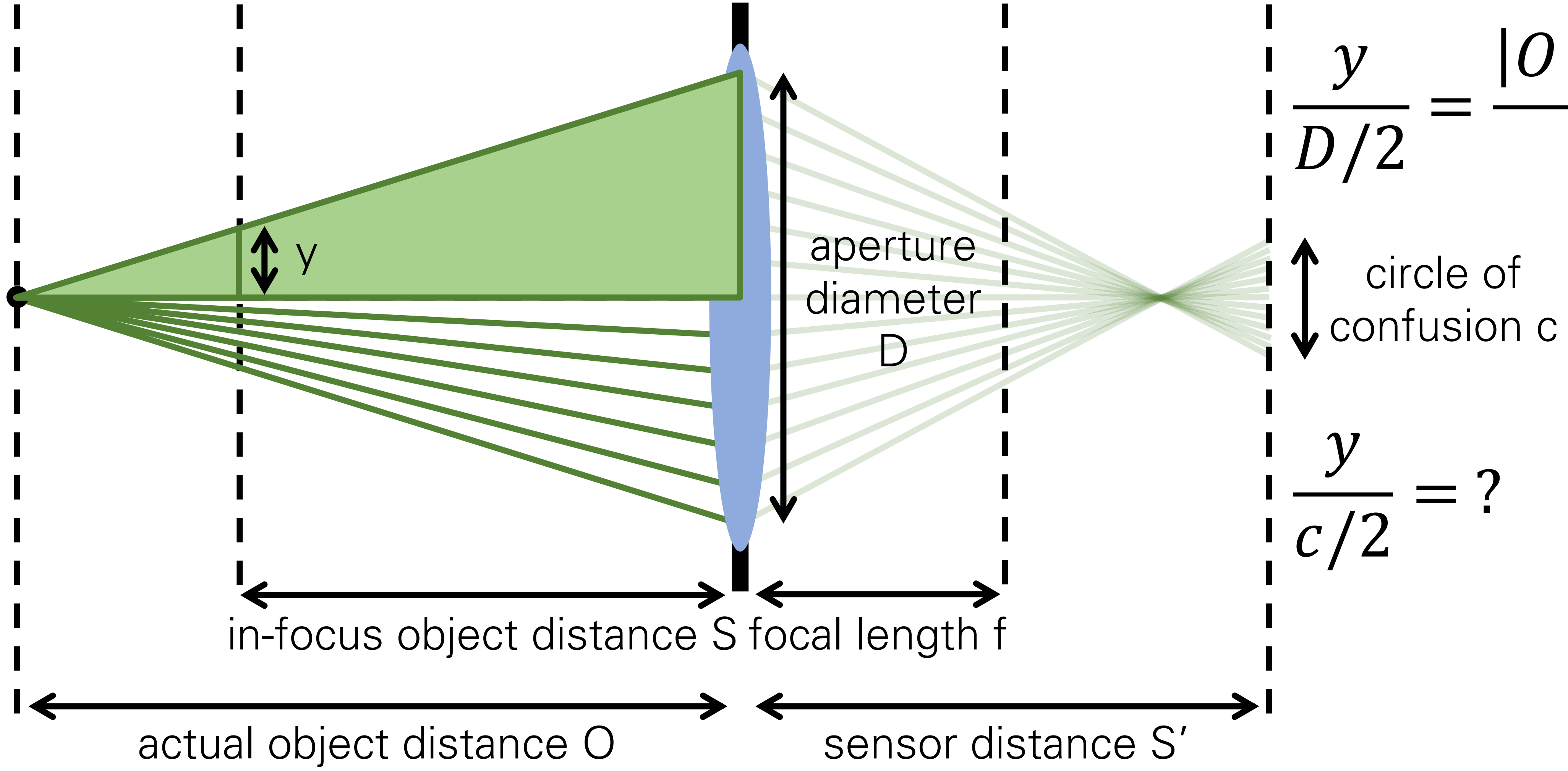
# Circle of confusion

How can we compute the diameter of the circle of confusion? → Use similar triangles.



# Circle of confusion

How can we compute the diameter of the circle of confusion? → Use similar triangles.

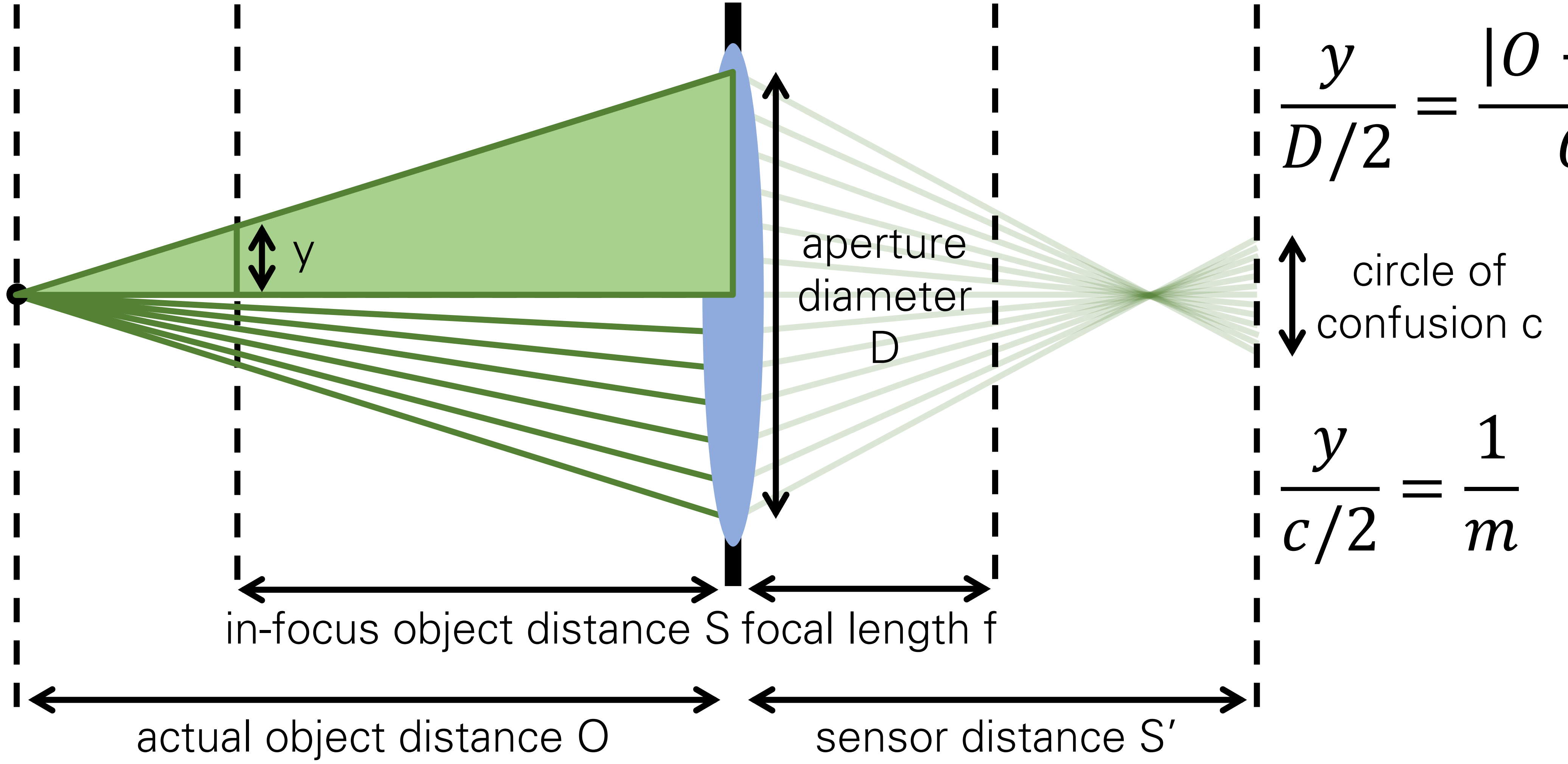



$$\frac{y}{D/2} = \frac{|O - S|}{O}$$

$$\frac{y}{c/2} = ?$$

# Circle of confusion

How can we compute the diameter of the circle of confusion? → Use similar triangles.

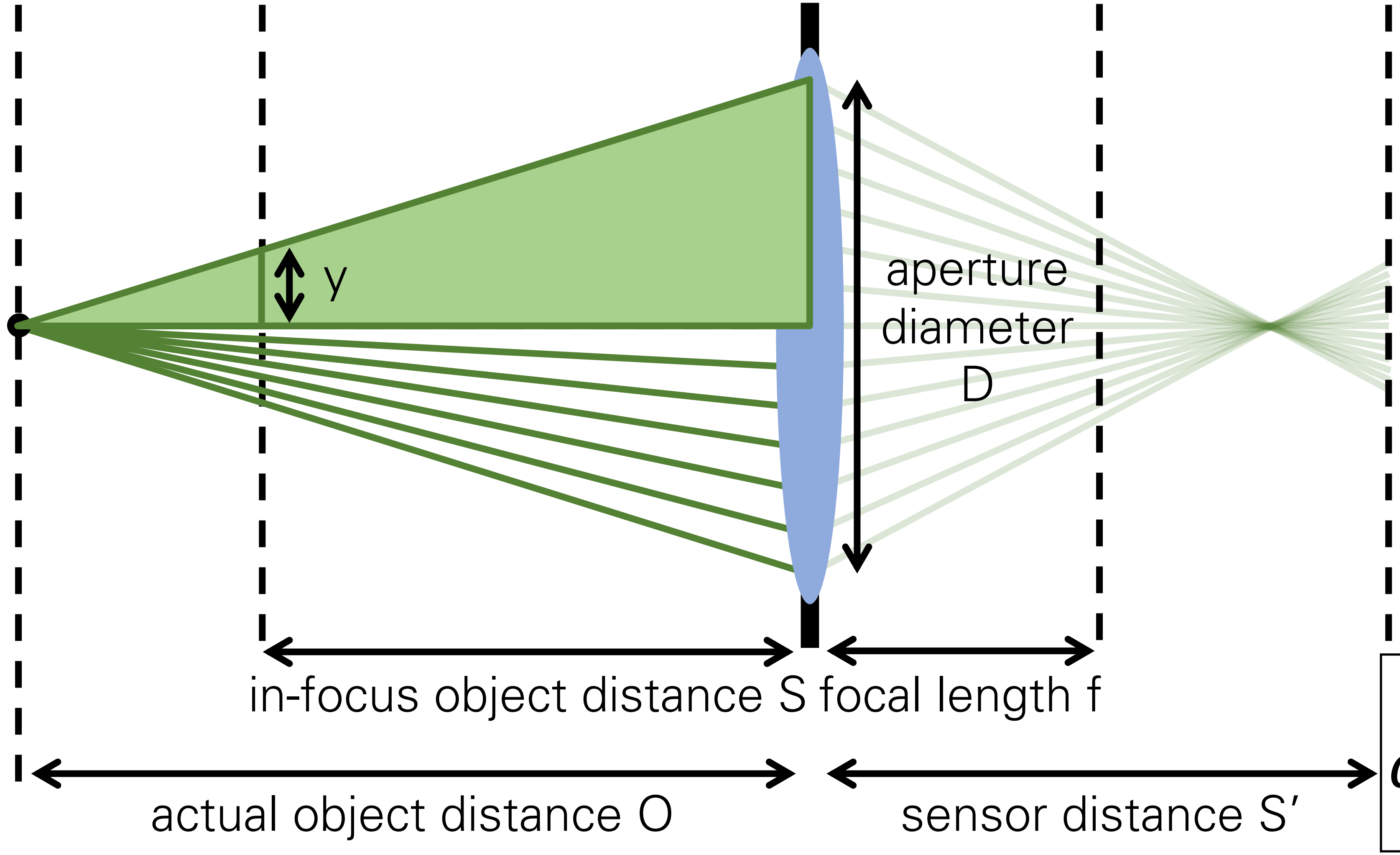



$$\frac{y}{D/2} = \frac{|O - S|}{O}$$

$$\frac{y}{c/2} = \frac{1}{m}$$

# Circle of confusion

How can we compute the diameter of the circle of confusion? → Use similar triangles.




$$\frac{y}{D/2} = \frac{|O - S|}{O}$$

$$\frac{y}{c/2} = \frac{1}{m}$$

$$c = mD\,\frac{|O - S|}{O}$$

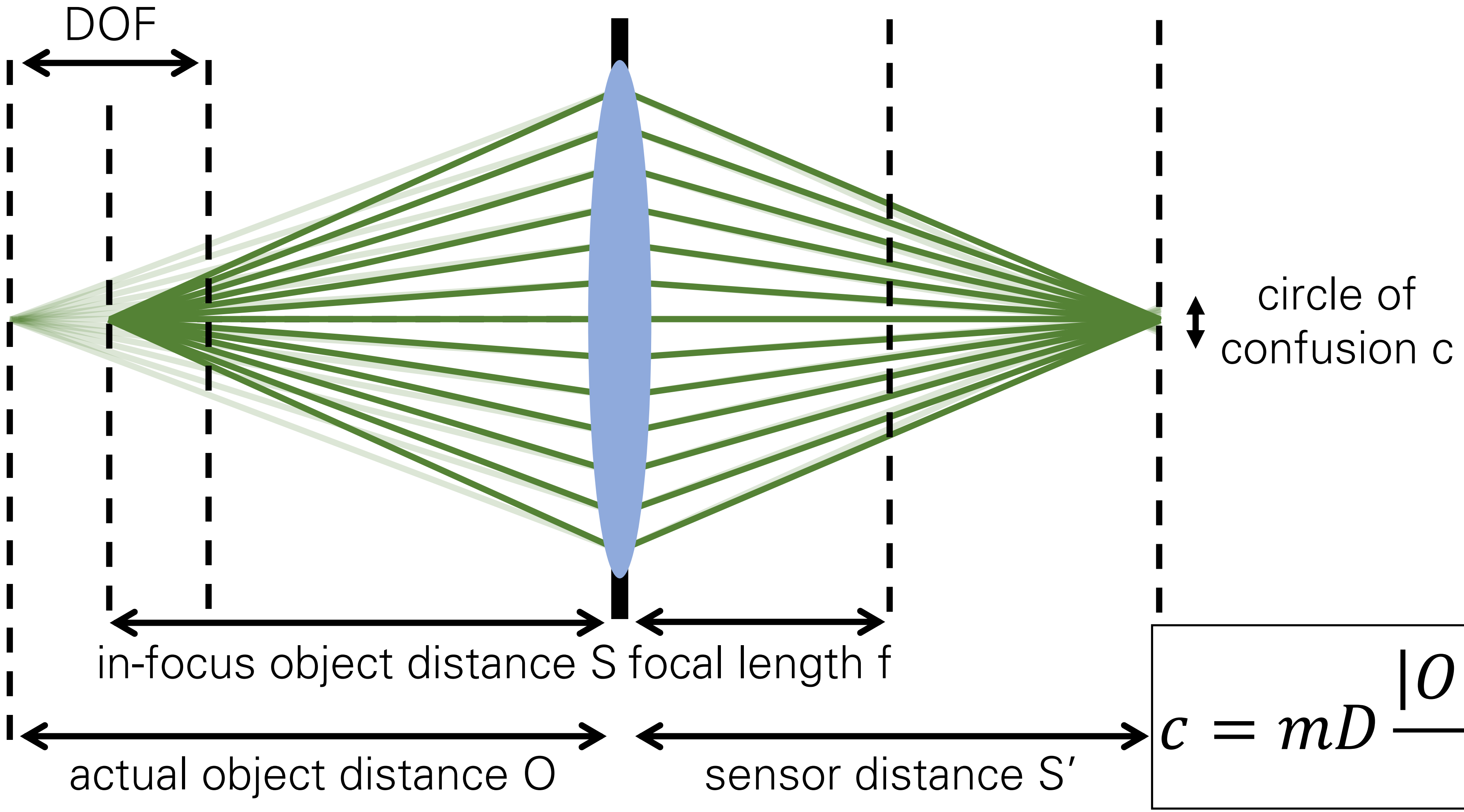
# Depth of field

Distance from the in-focus object plane where the circle of confusion is acceptably small.

e.g., equal to 4-5 pixels

$$c < \varepsilon \Rightarrow$$

$$\mathrm{DOF} = \frac{2\varepsilon O}{mD}$$

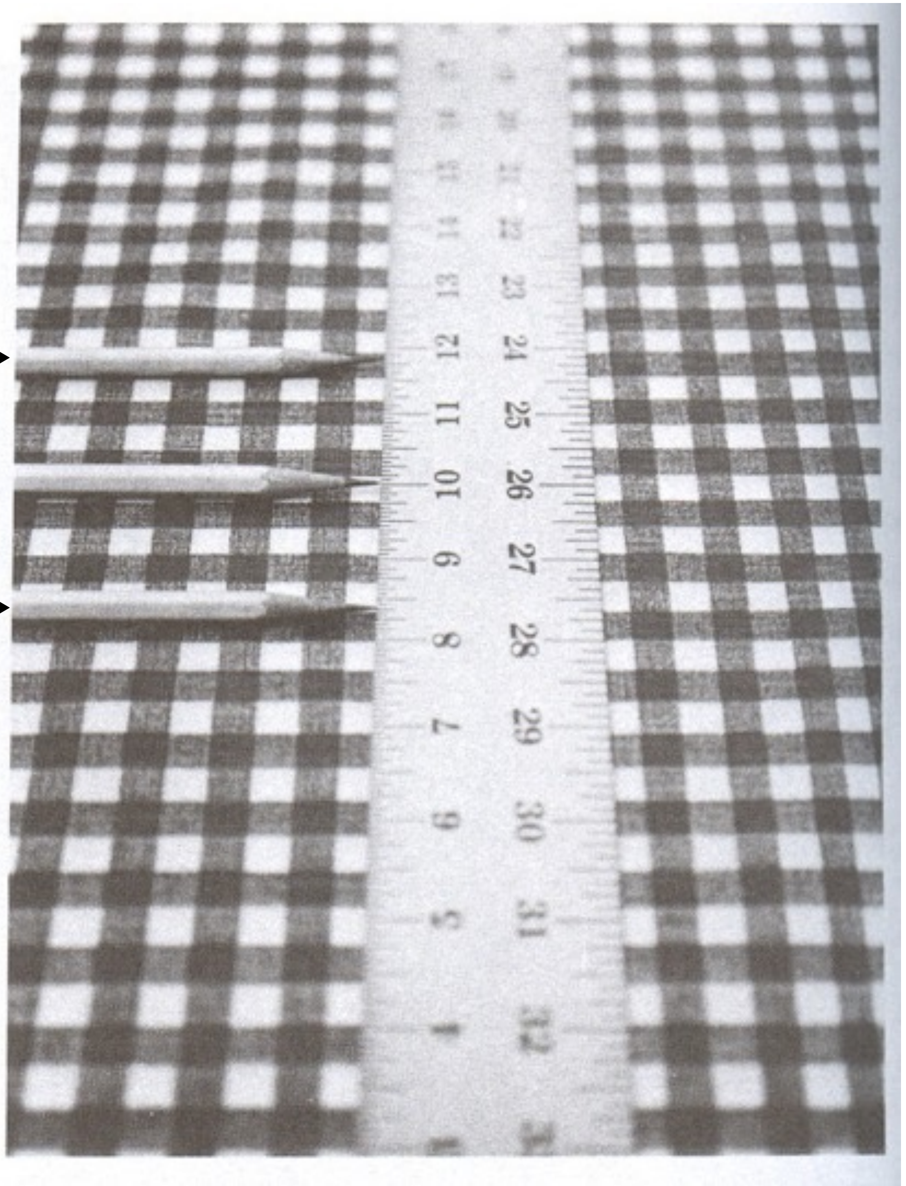


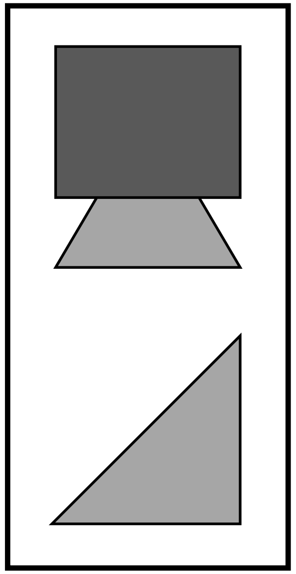

$$c = mD\,\frac{|O - S|}{O}$$

Note: in reality, DOF is slightly asymmetrical.

# Depth of field

depth of field





scene

# Depth of field

Defocus blur behind focal plane

Focal plane (a line from it)



Defocus blur in front of focal plane

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

# Depth of field

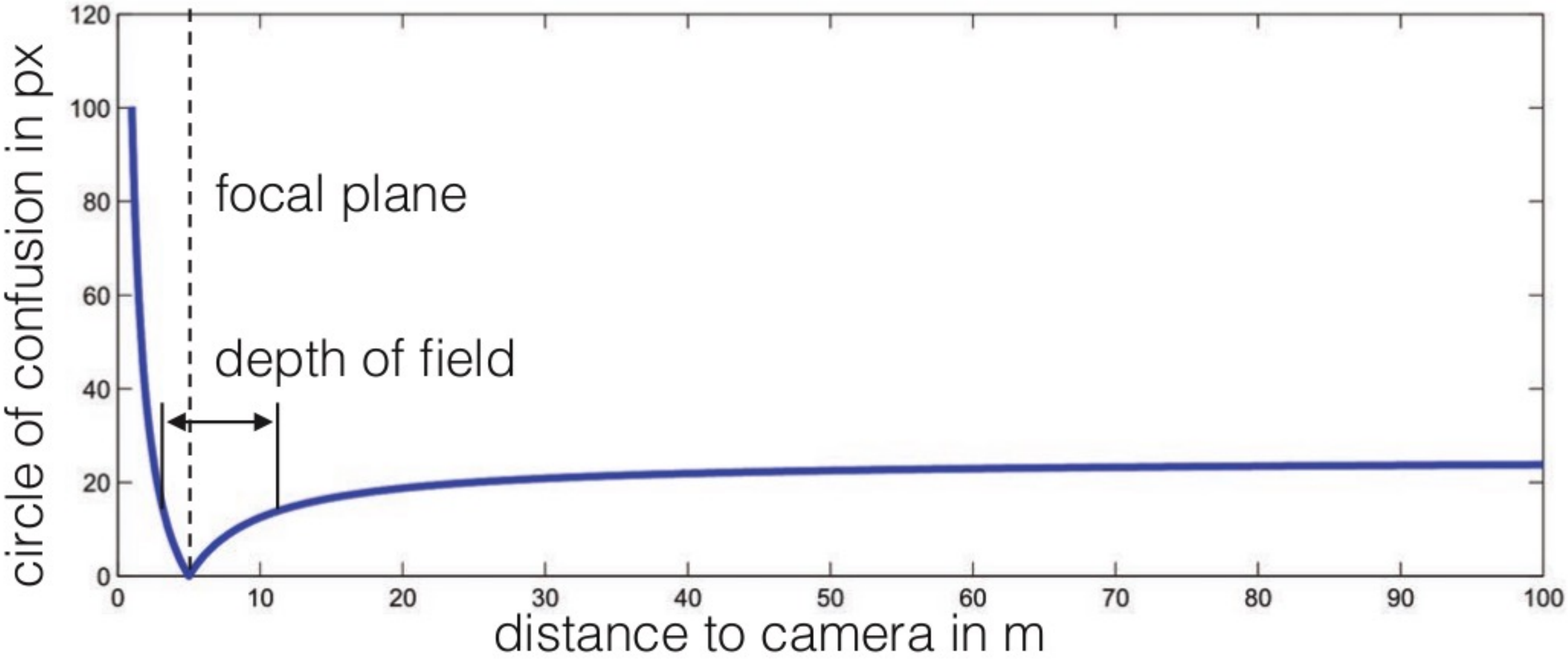
Focal plane
(a line from it)

Depth of field

# Circle of confusion

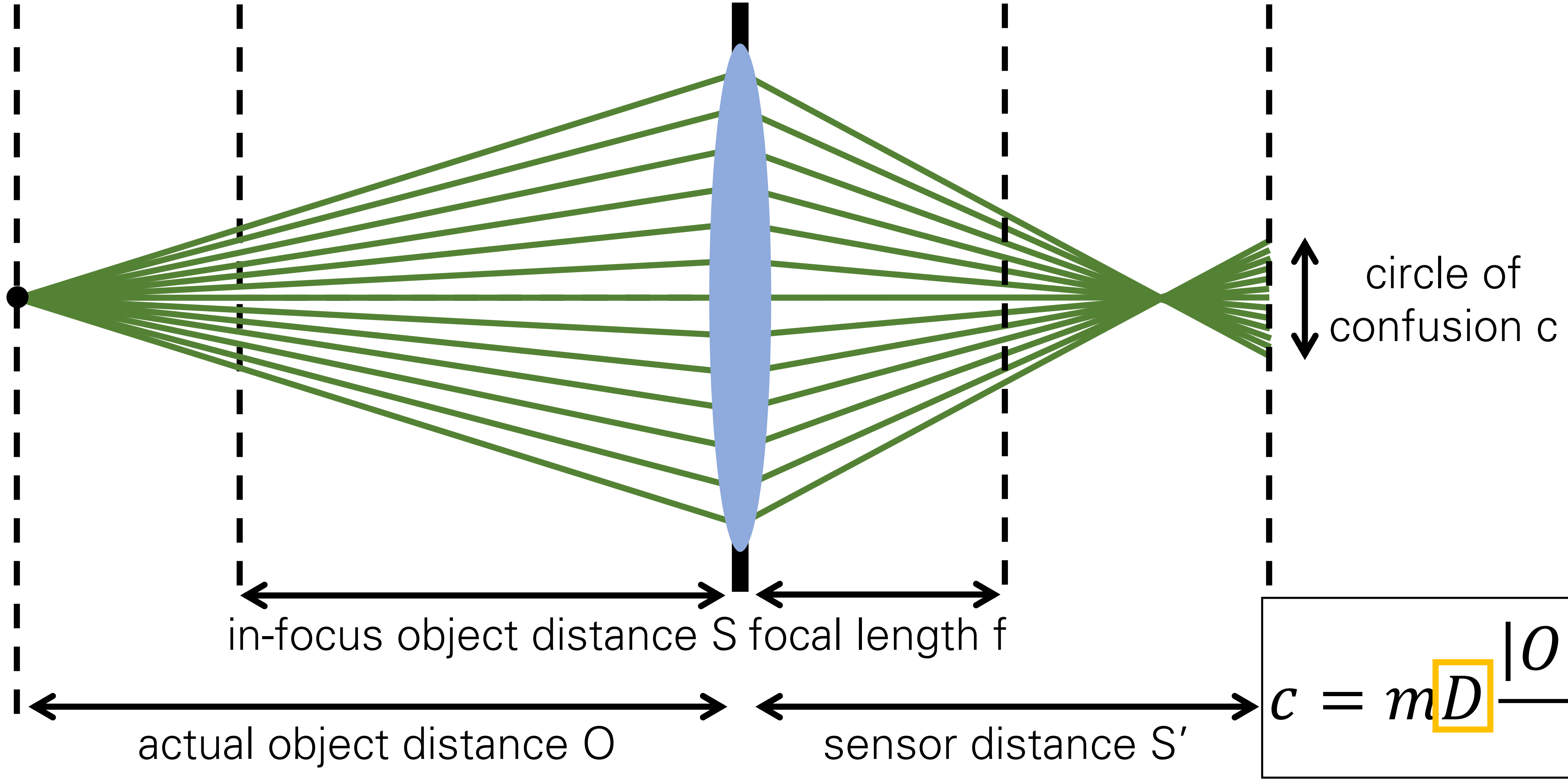
$$c = M \cdot D \cdot \frac{|S - S_1|}{S}$$

Canon 5D Mark III: f=50mm, f/2.8 (N=2.8), focused at 5m, pixel size=7.5um



# Defocus depends on aperture diameter

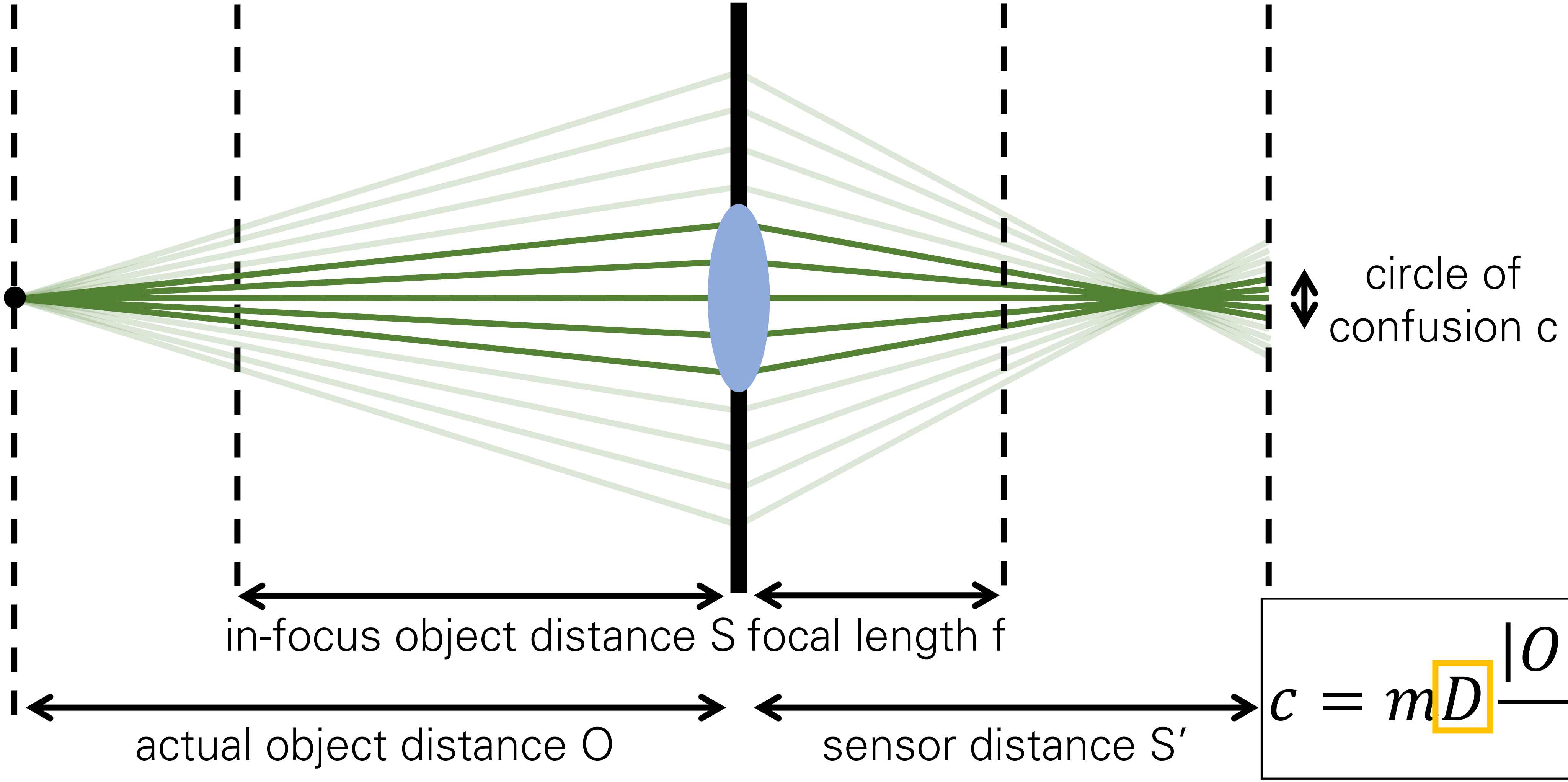
What happens to the circle of confusion as the aperture diameter is reduced?




$$c = mD\frac{|O - S|}{O}$$

# Defocus depends on aperture diameter

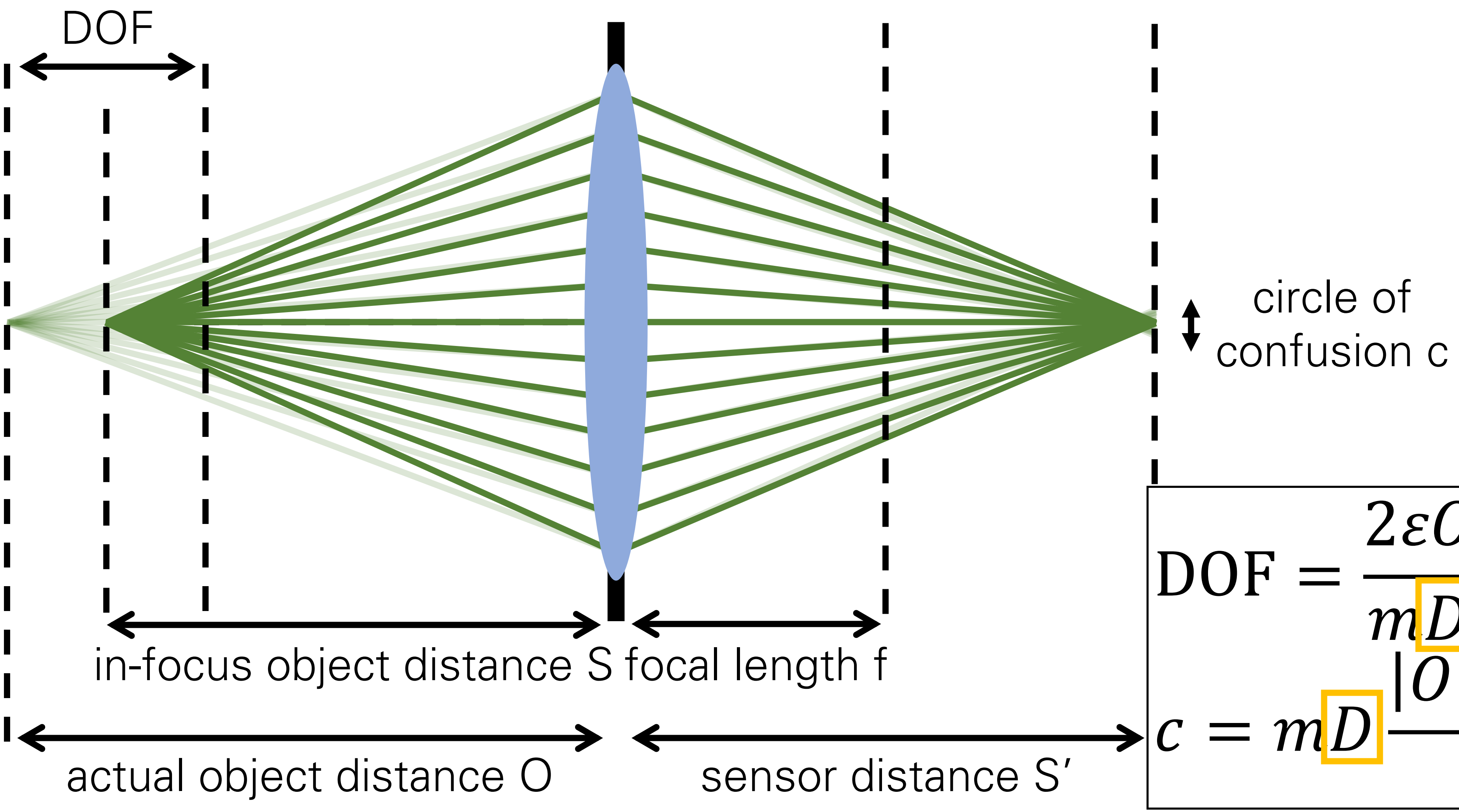
What happens to the circle of confusion as the aperture diameter is reduced? → It shrinks.




$$c = mD\frac{|O - S|}{O}$$

# Defocus depends on aperture diameter

What happens to the depth of field as the aperture diameter is reduced?

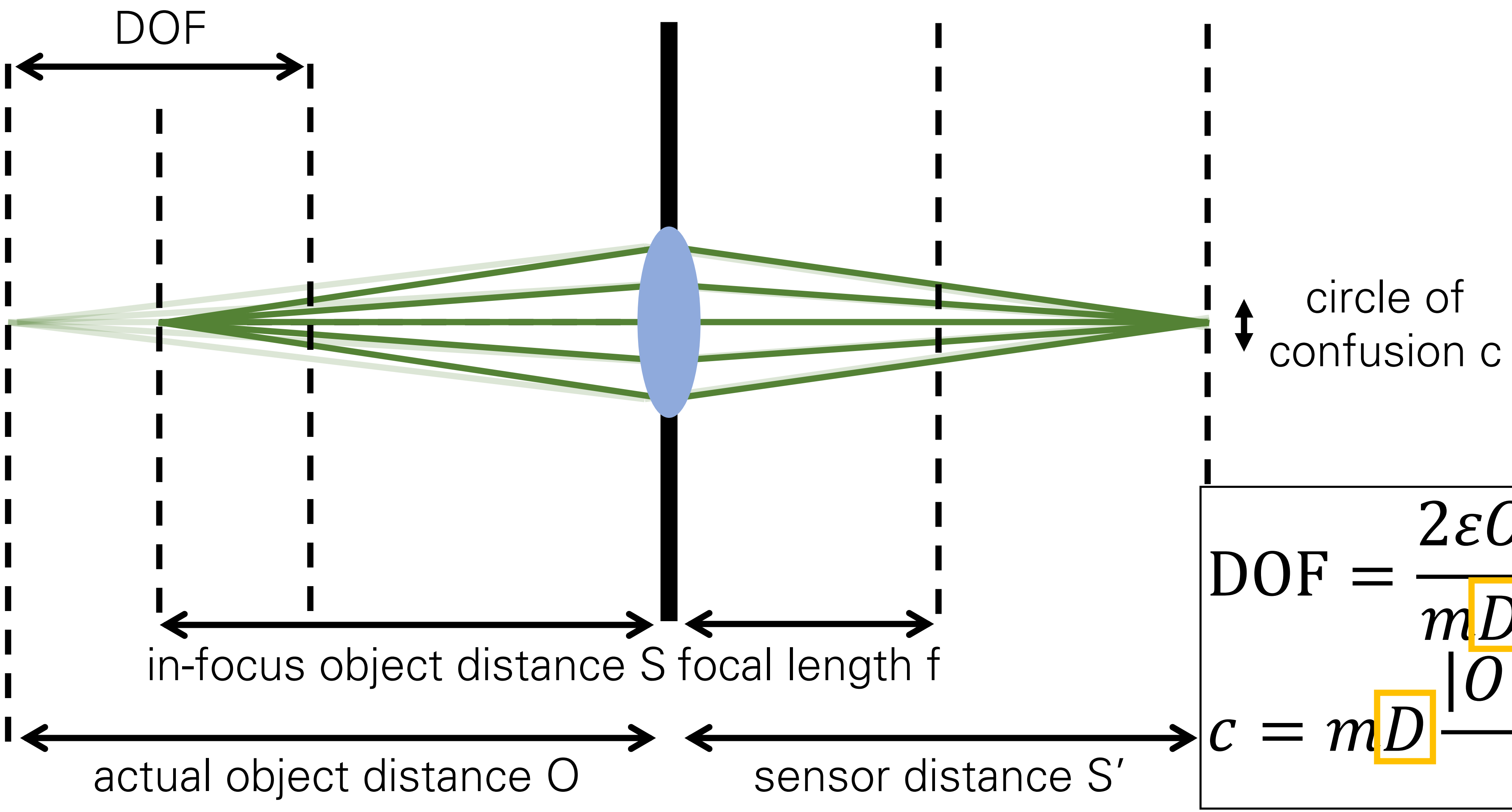



$$\mathrm{DOF} = \frac{2\varepsilon O}{mD}$$

$$c = mD\frac{|O - S|}{O}$$

# Defocus depends on aperture diameter

What happens to the depth of field as the aperture diameter is reduced? → It expands.




$$\mathrm{DOF} = \frac{2\varepsilon O}{mD}$$

$$c = mD\frac{|O - S|}{O}$$

# Aperture size

Most lenses have apertures of variable size.

- The size of the aperture is expressed as the "f-number": The bigger this number, the smaller the aperture.


f / 1.4 f / 2.8 f / 4 f / 8 f / 16

You can see the aperture by removing the lens and looking inside it.

# Demonstration

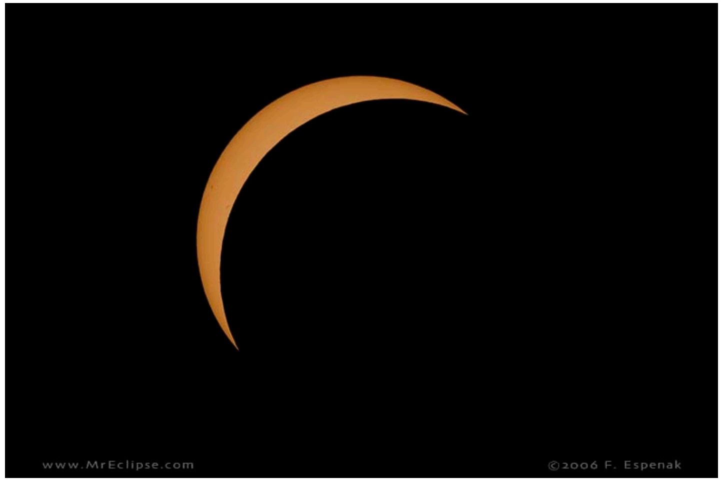
# Aperture size

Plane of focus is ≈ at the depth of the front dark blue chair.


# Depth of Field

Form of defocus blur is determined by shape of aperture.

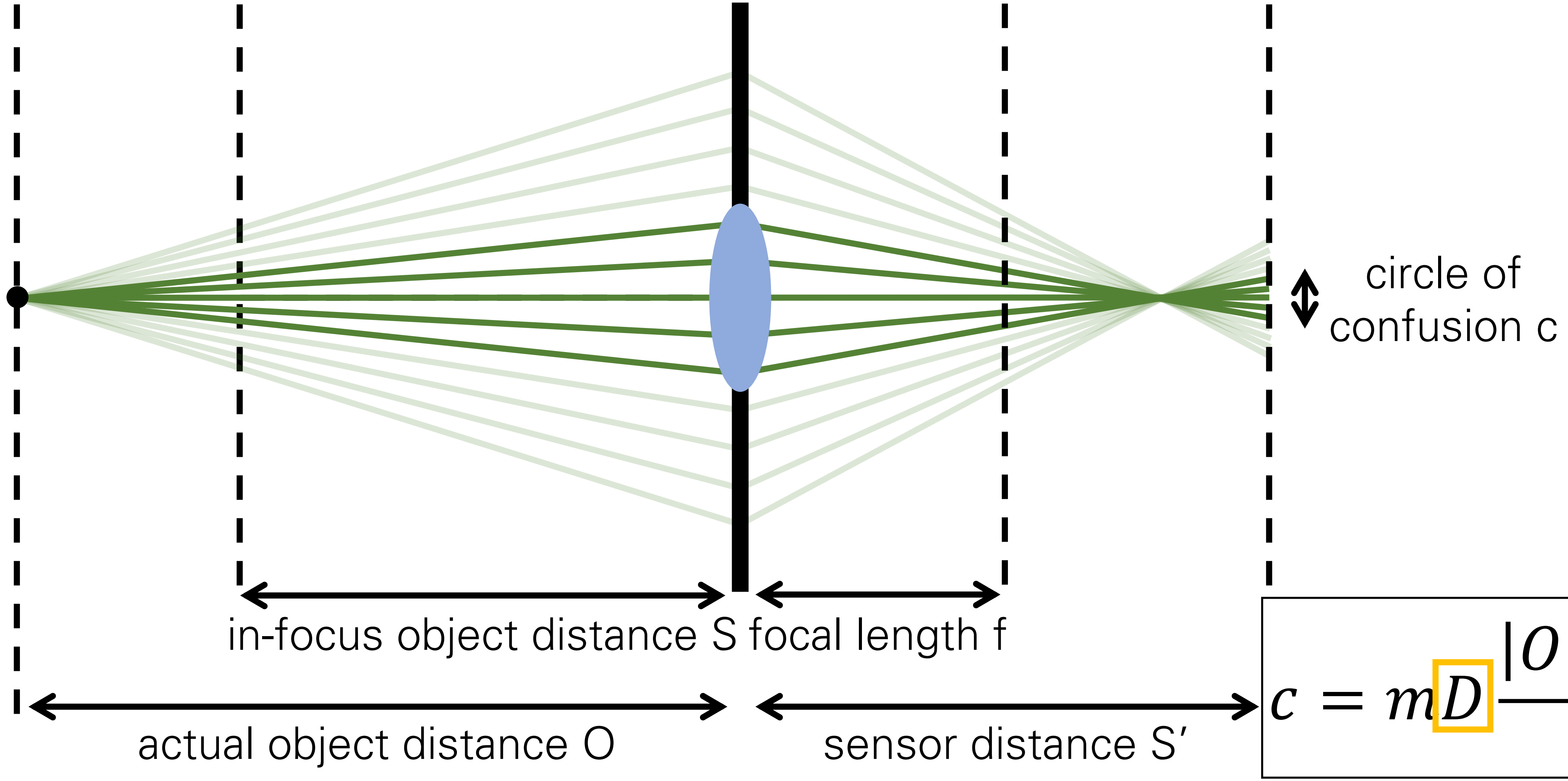
# Apertures are everywhere – Eclipse pinhole cameras!




[PetaPixel; https://petapixel.com/2012/05/21/crescent-shaped-projections-through-tree-leaves-during-the-solar-eclipse/
http://www.mreclipse.com/SEphoto/TSE2006/TSE2006galleryA.html]

# Defocus depends on aperture diameter

If small aperture sizes reduce defocus blur, should we always use the smallest aperture?

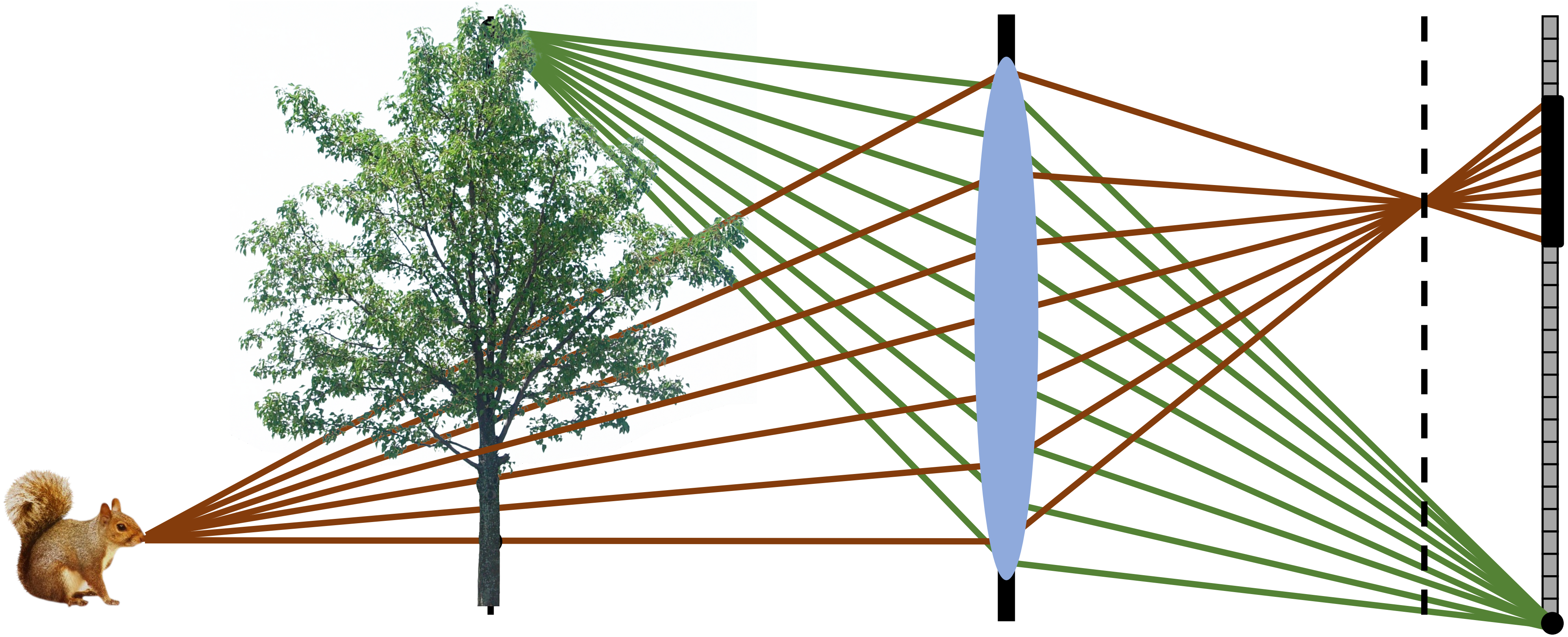



$$c = mD\frac{|O - S|}{O}$$

# Bokeh

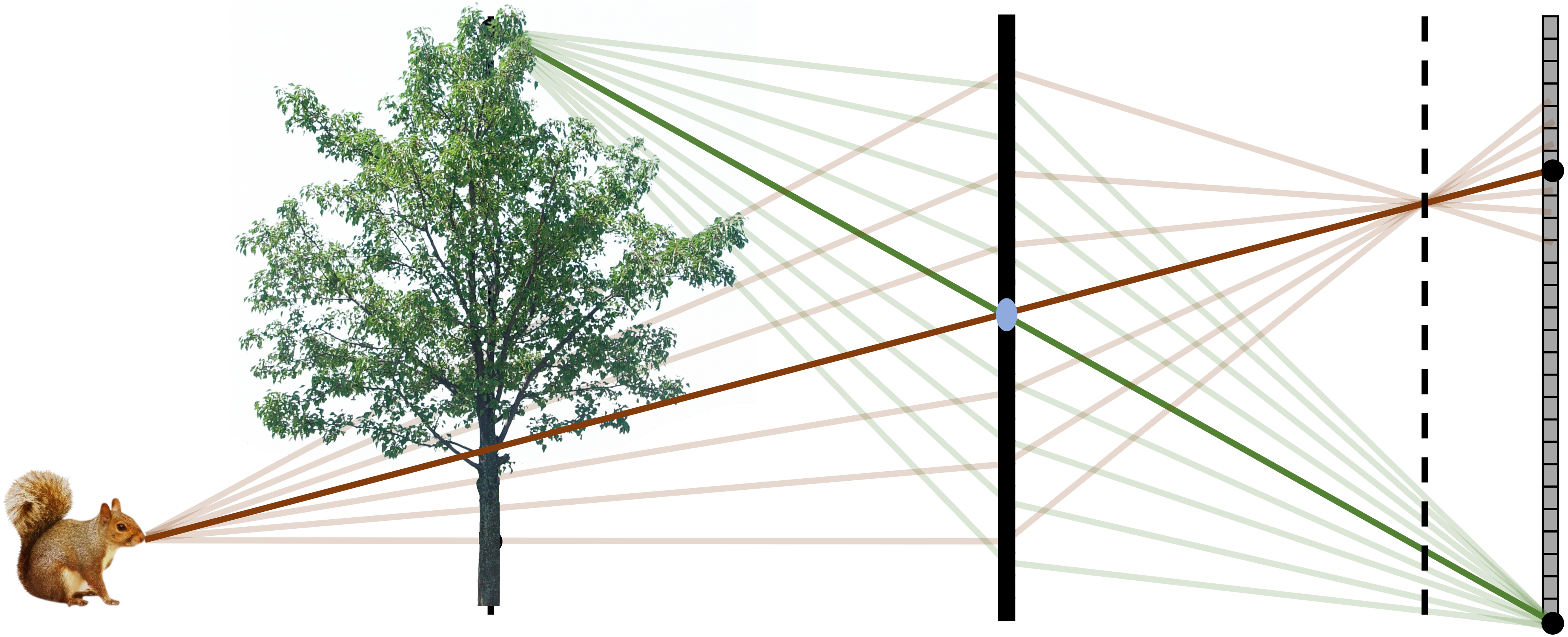
Sharp depth of field ("bokeh") is often desirable.

# What happens as the aperture keeps getting smaller?



# What happens as the aperture keeps getting smaller?

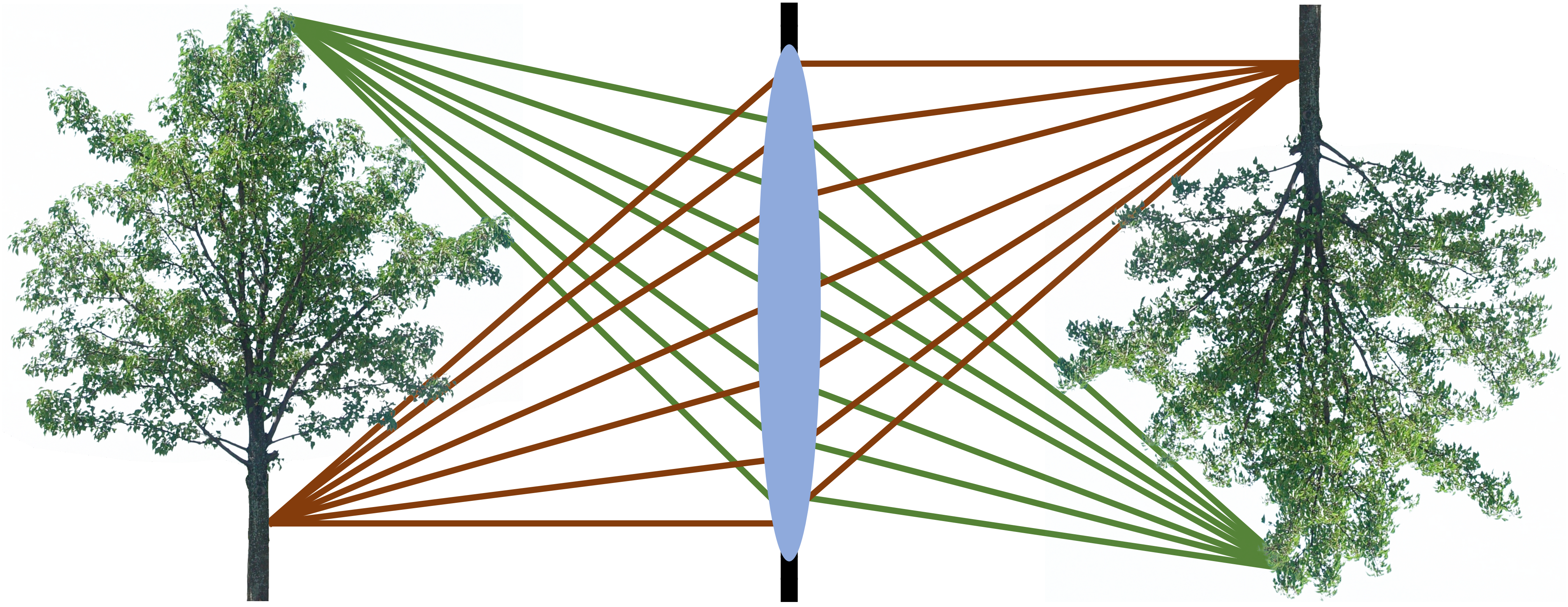
Lens becomes equivalent to a pinhole.



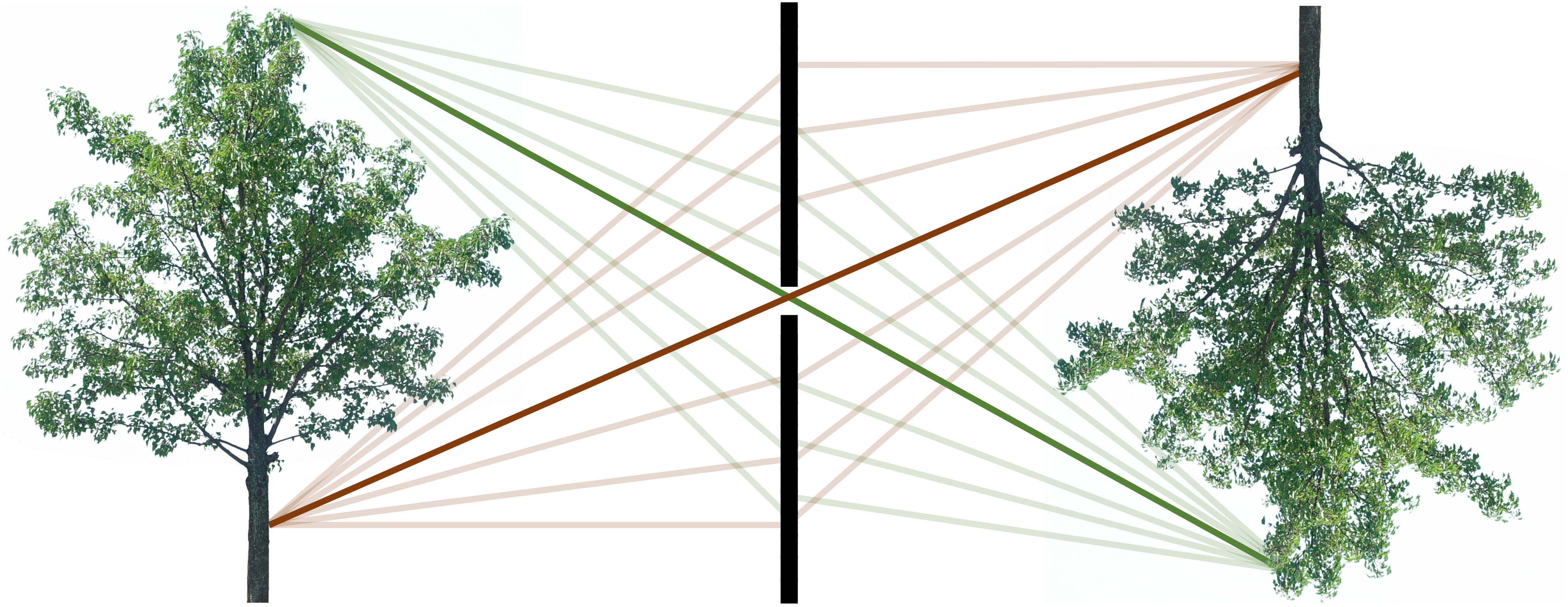

- No defocus, everything is sharp regardless of depth.
- Very little light, signal-to-noise ratio is just as bad as pinhole.

# Lens camera and pinhole camera

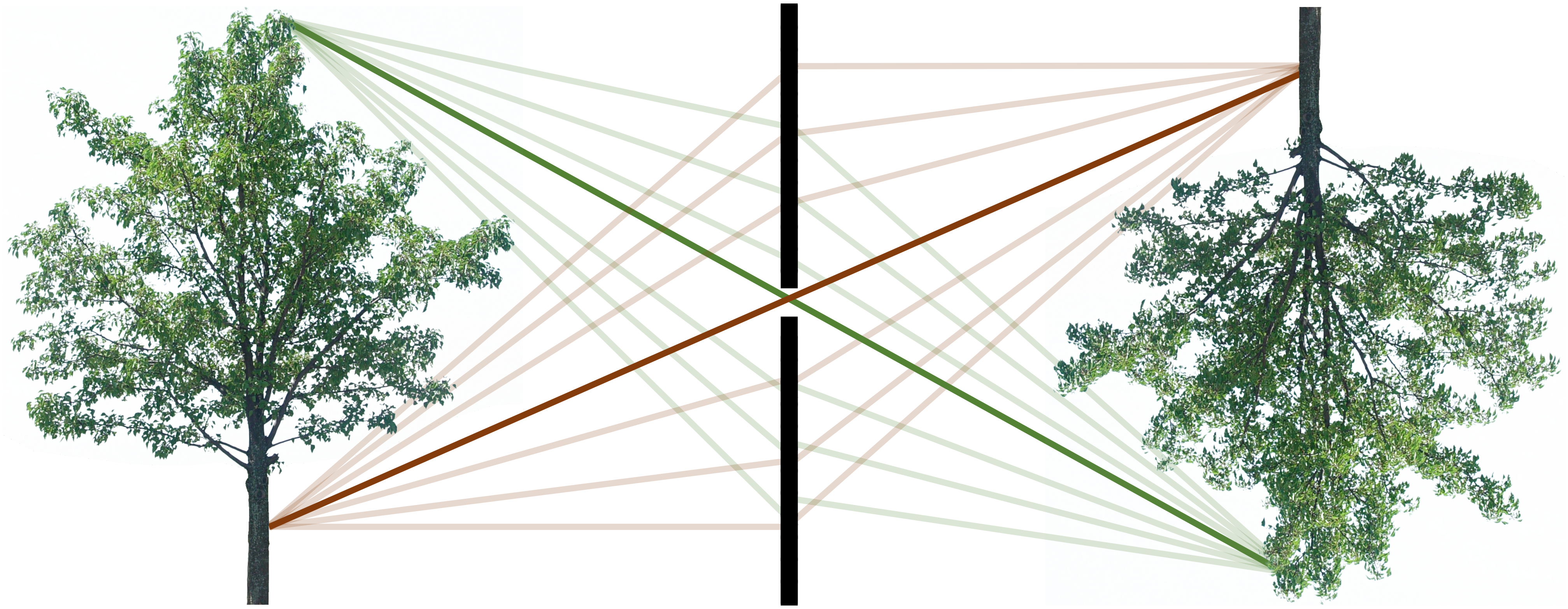
# The lens camera



# The pinhole camera

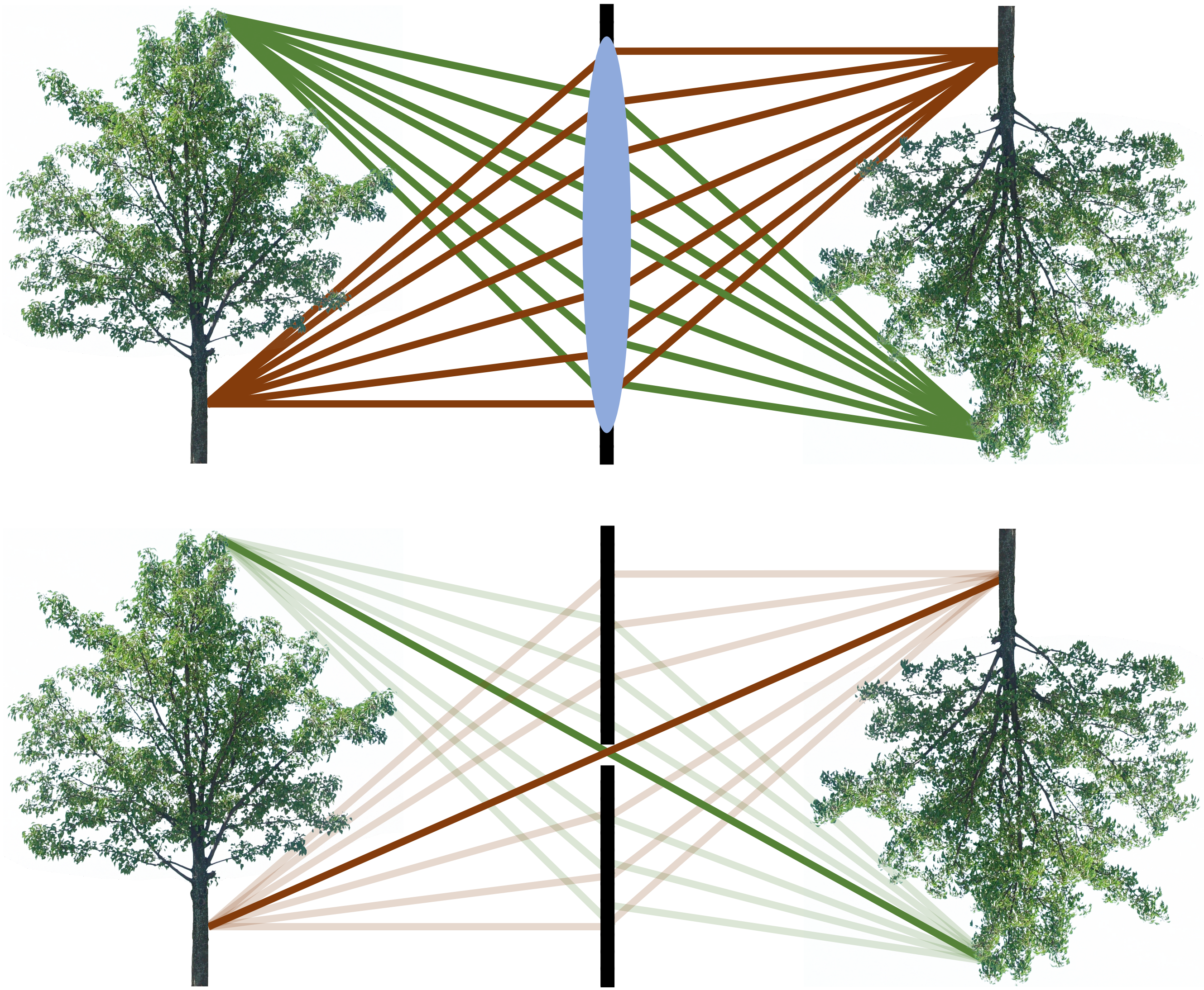


# The pinhole camera



Central rays propagate in the same way for both models!

# Describing both lens and pinhole cameras



We can derive properties and descriptions that hold for both camera models if:

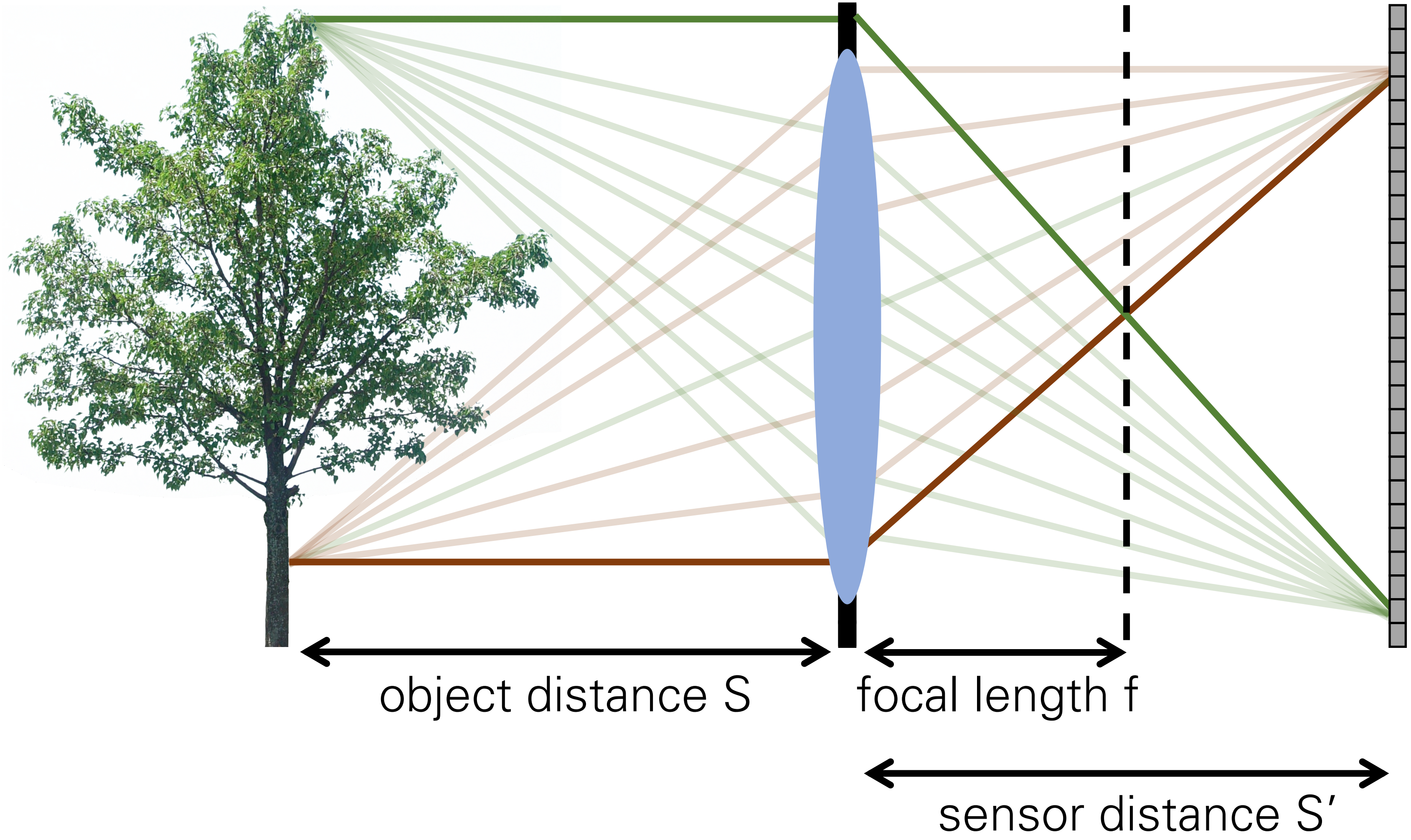
- We consider only central rays.
- We assume that everything of interest in the scene is within the depth of field.

# Important difference: focal length

In a pinhole camera, focal length is distance between aperture and sensor

# Important difference: focal length

In a lens camera, focal length is distance where parallel rays intersect



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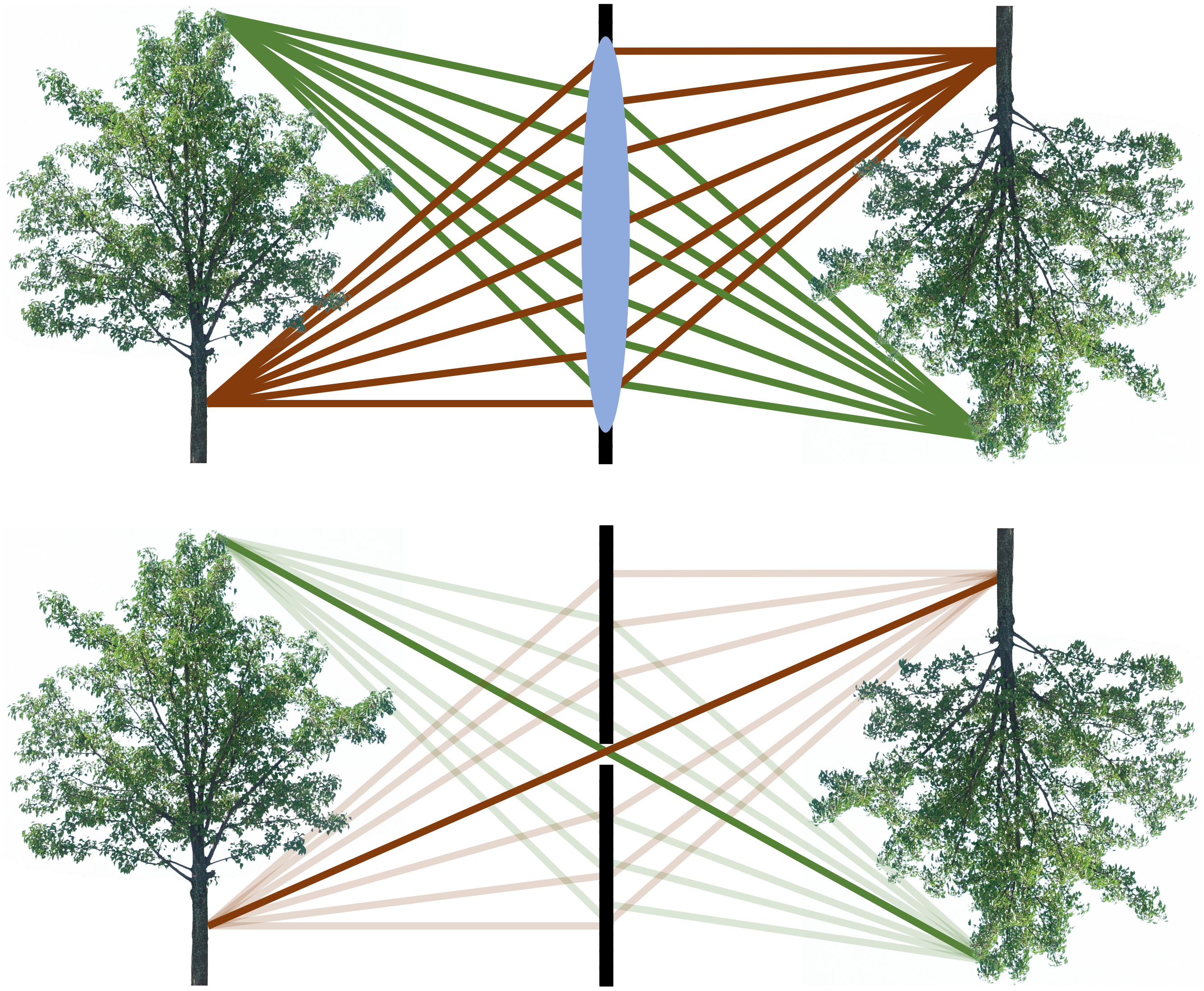
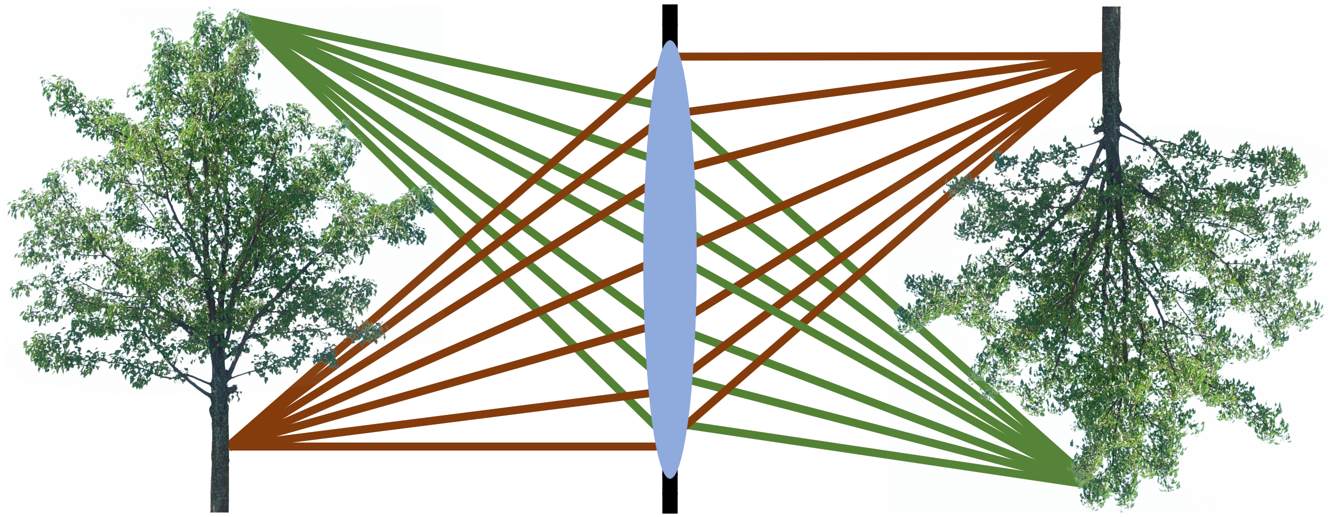


We can derive properties and descriptions that hold for both camera models if:

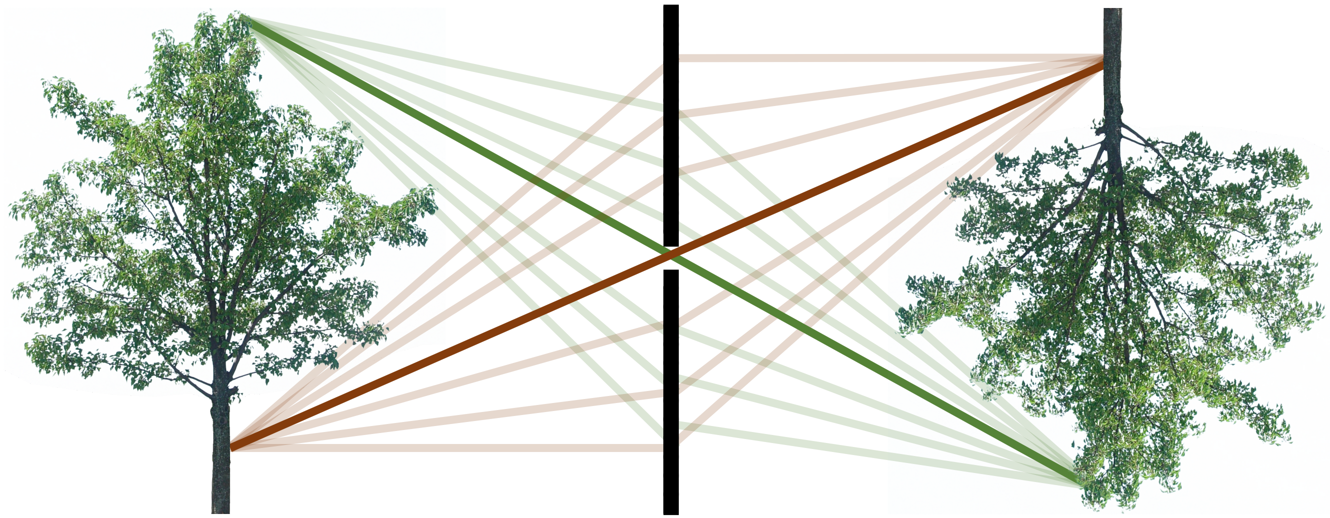
- We consider only central rays.
- We assume everything of interest in the scene is within the depth of field.
- We assume that the focus distance of the lens camera is equal to the focal length of the pinhole camera.

# Effect of aperture size on lens and pinhole cameras



Doubling the aperture diameter:

- Increases light throughput by four times.
- Increases circle of confusion for out-of-focus plane by two times.
- Decreases depth of field by two times.



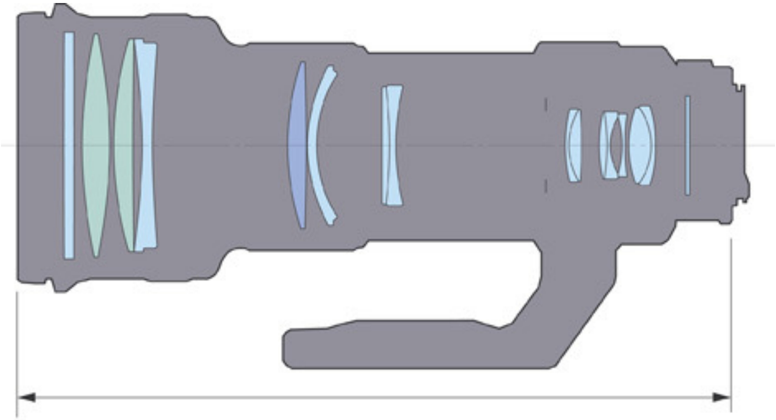
Doubling the aperture diameter:

- Increases light throughput by four times.
- Increases circle of confusion for all planes by two times.

# Thin lenses are fiction!

# Thin lenses are a fiction

The thin lens model assumes that the lens has no thickness, but this is rarely true...

To make real lenses behave like ideal thin lenses, we have to use combinations of multiple lens elements (compound lenses).

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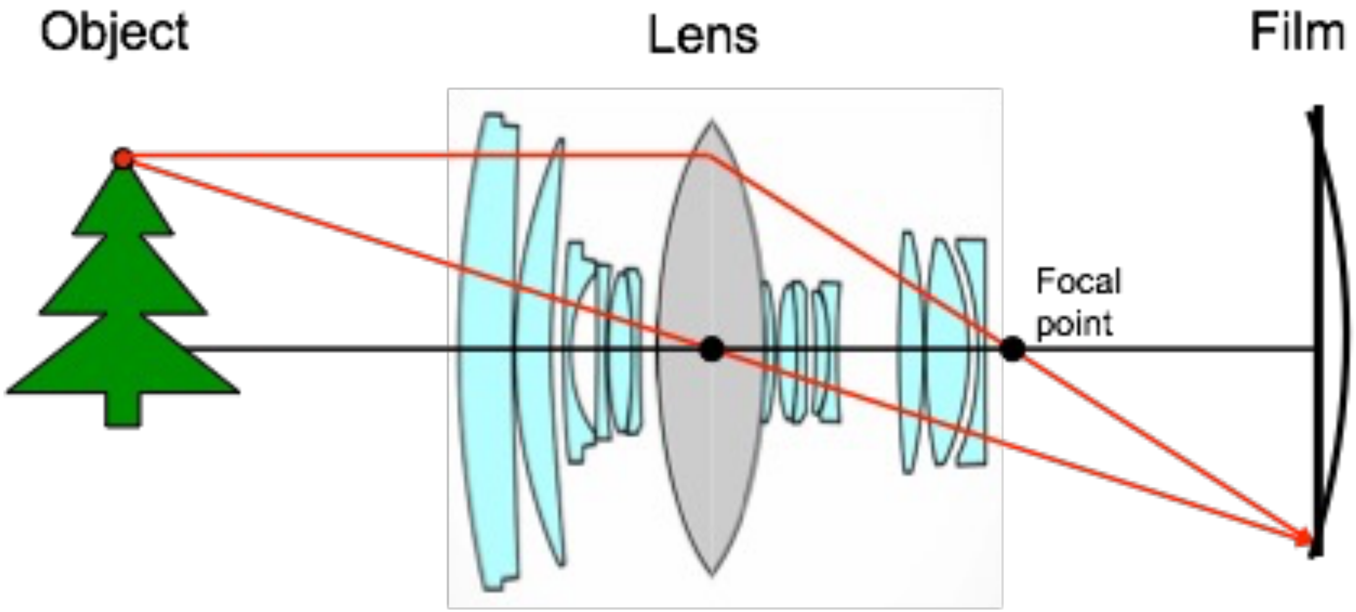
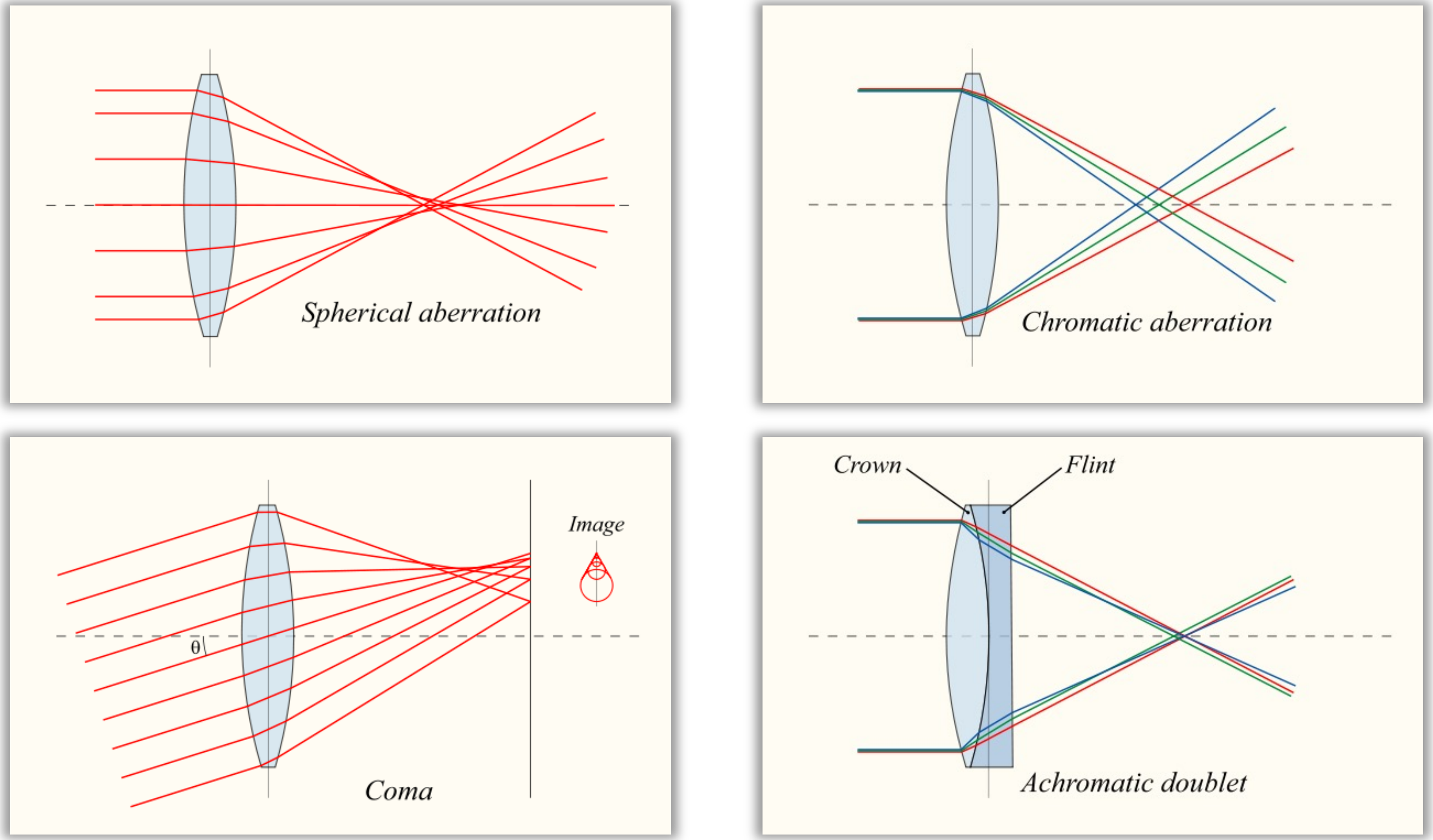


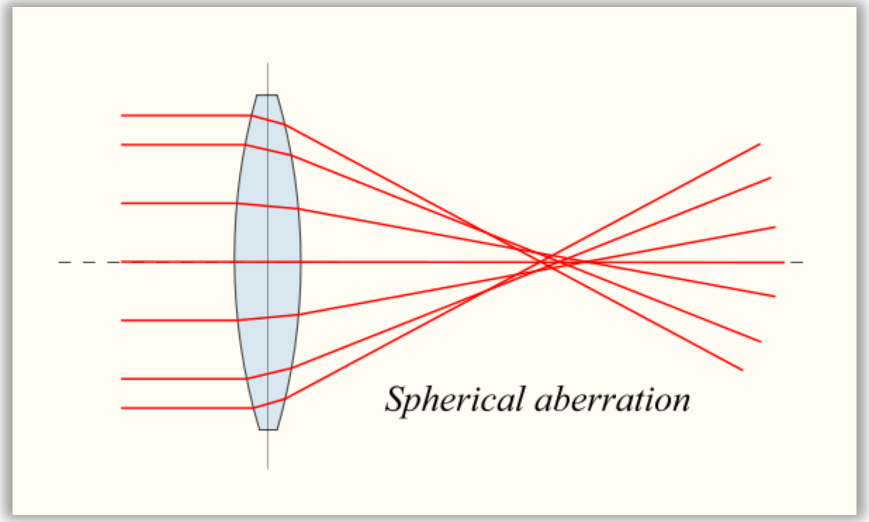

Even though we have multiple lenses, the entire optical system can be (paraxially) described using a single thin lens of some equivalent focal length and aperture number.

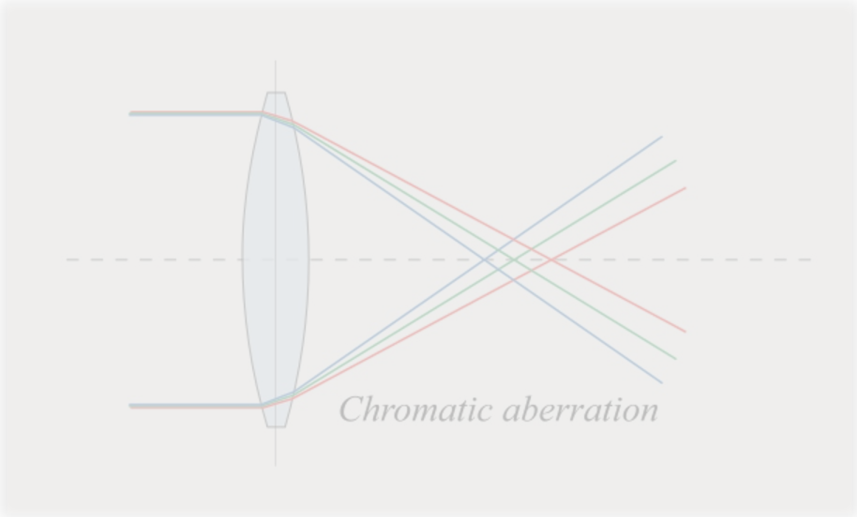
To make real lenses behave like ideal thin lenses, we have to use combinations of multiple lens elements (compound lenses).

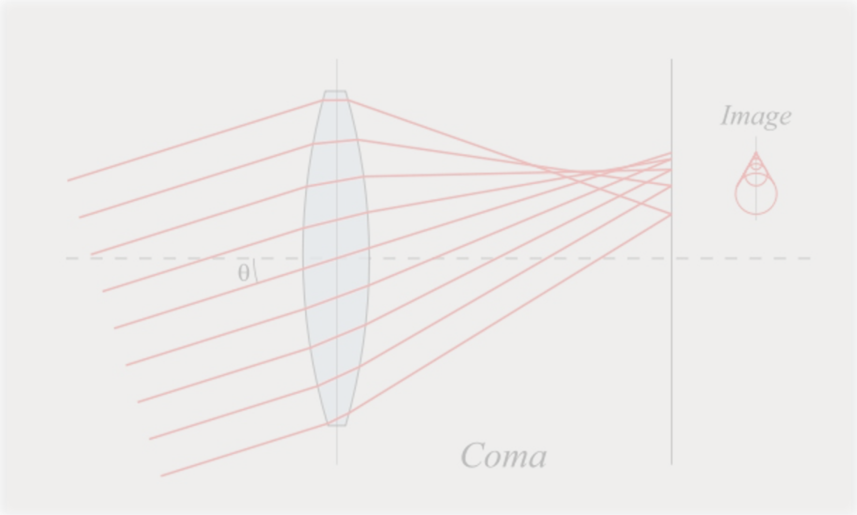
# Lenses - Aberrations

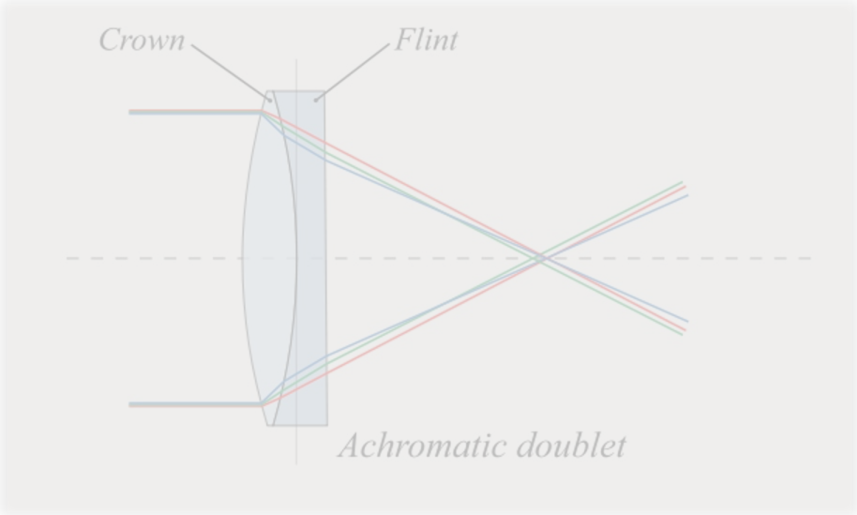


# Lenses - Aberrations



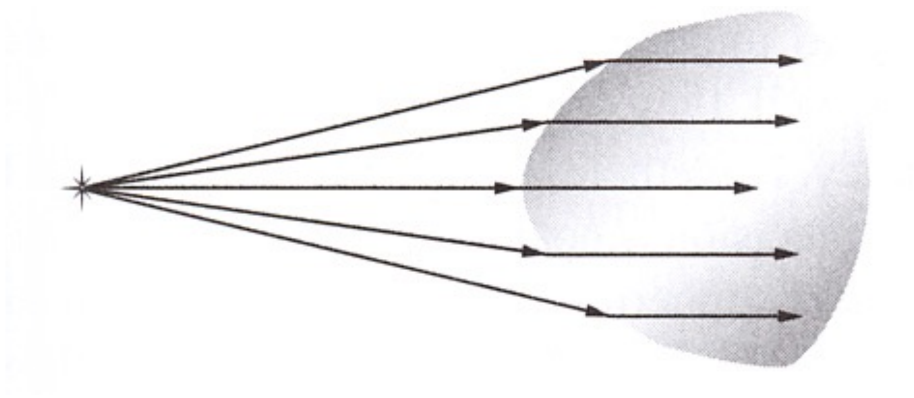



# Refraction at interfaces of complicated shapes

What shape should an interface have to make parallel rays converge to a point?



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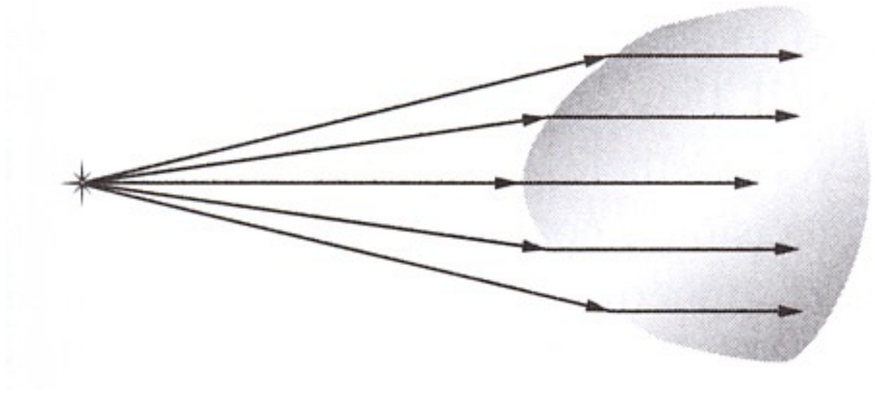
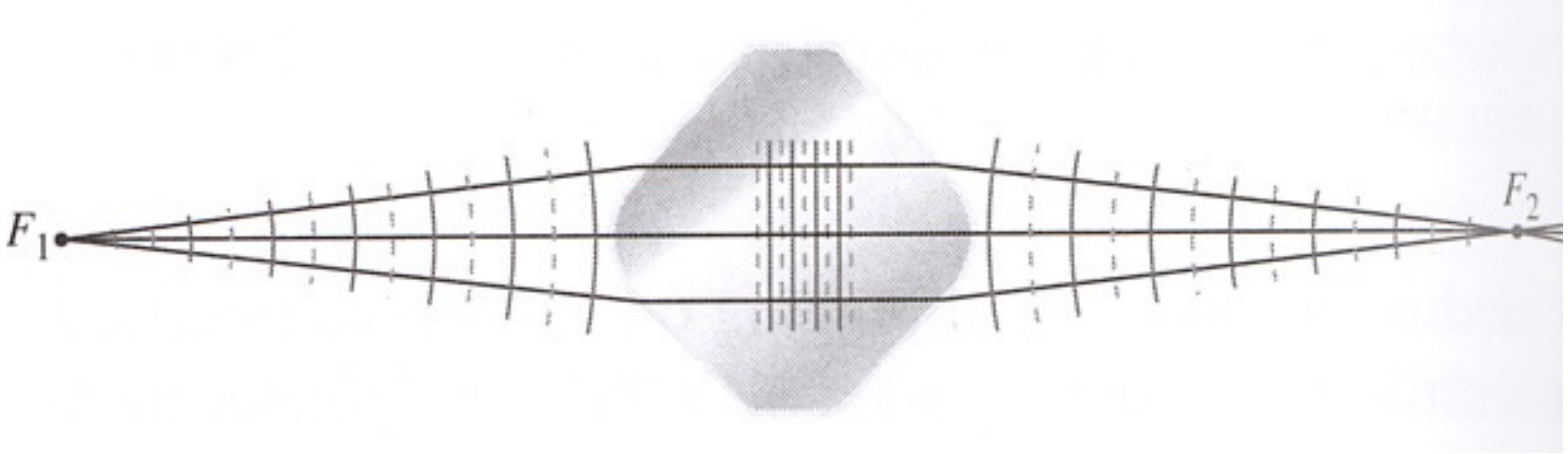


Single hyperbolic interface:
point to parallel rays

Double hyperbolic interface:
point to point rays

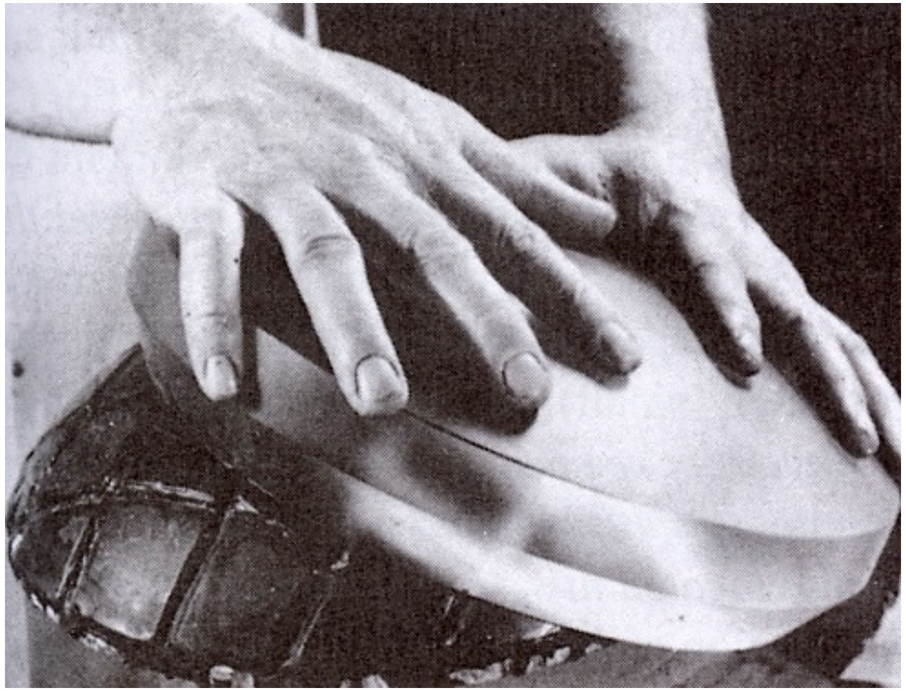


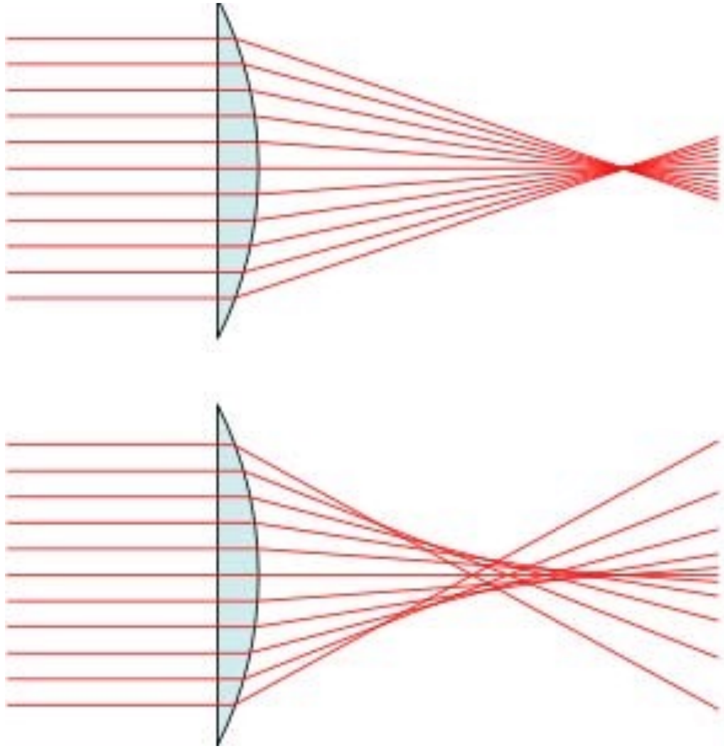

What shape should an interface have to make parallel rays converge to a point?

# Spherical lenses

In practice, lenses are often made to have spherical interfaces for ease of fabrication.

- Two roughly fitting curved surfaces ground together will eventually become spherical.





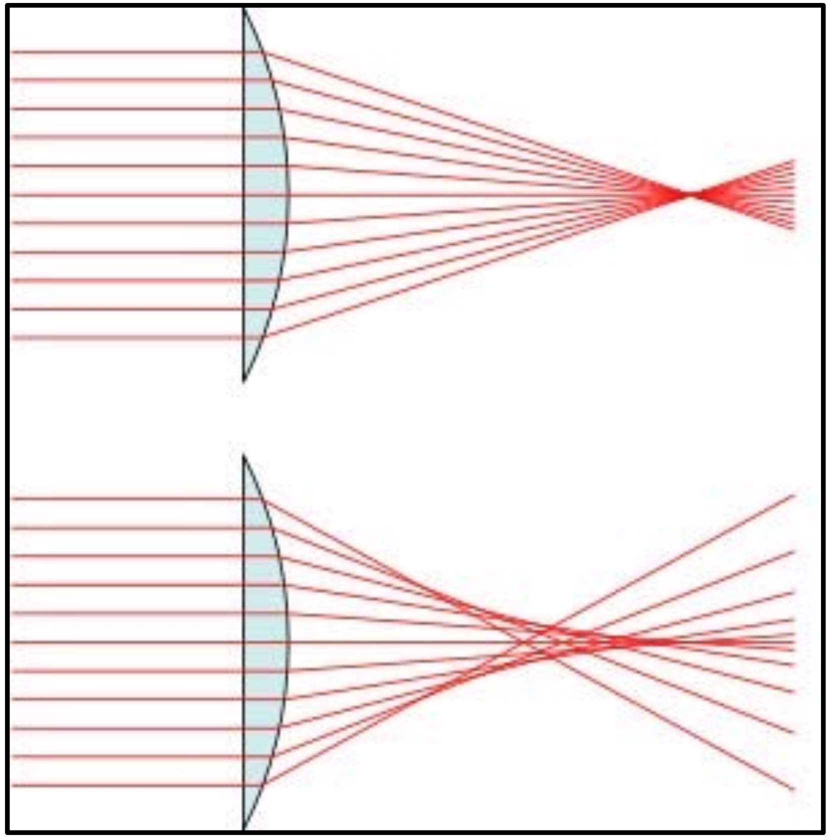
Spherical lenses don't bring parallel rays to a point.

- This is called spherical aberration.
- Approximately axial (i.e., paraxial) rays behave better.

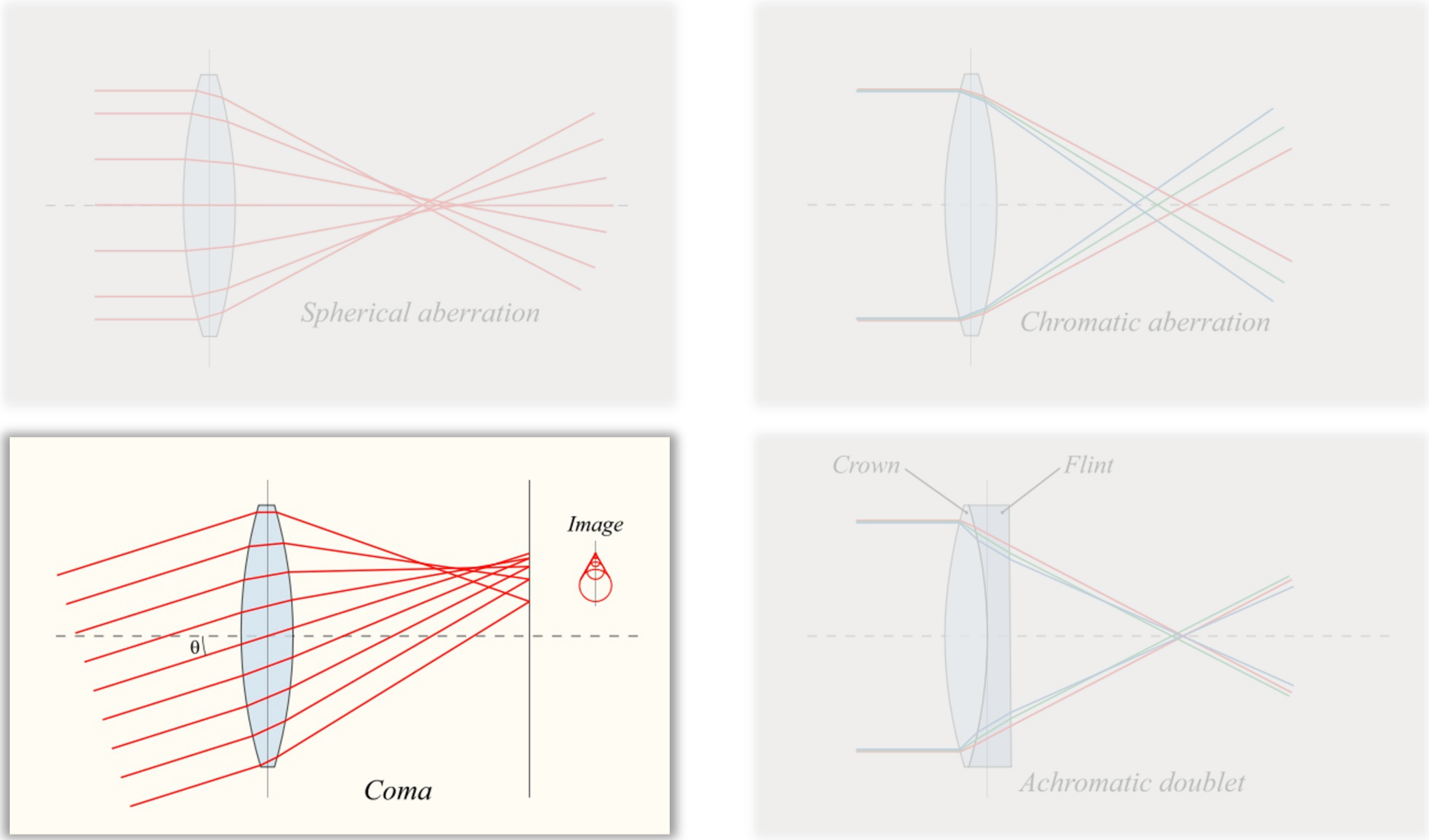
# Aberrations

Deviations from ideal thin lens behavior (e.g., imperfect focus).

- Example: spherical aberration.



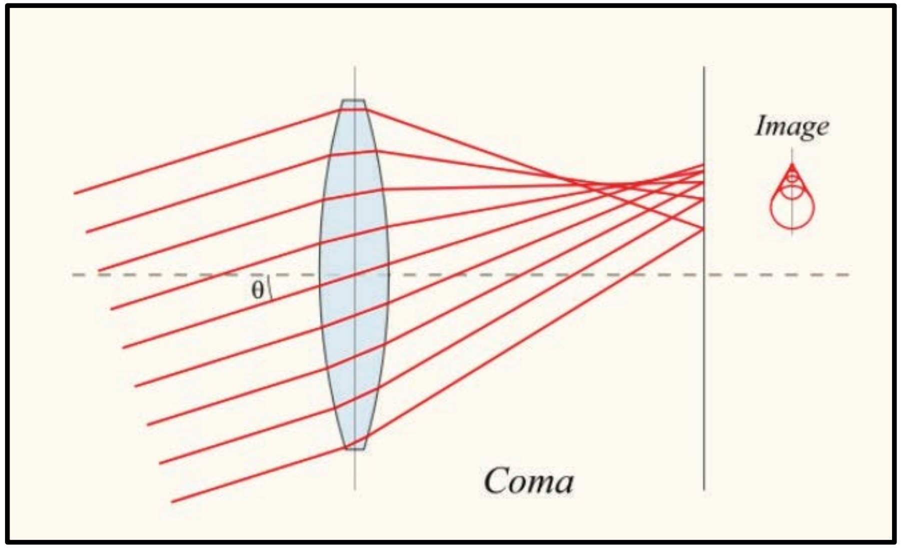
# Lenses - Aberrations



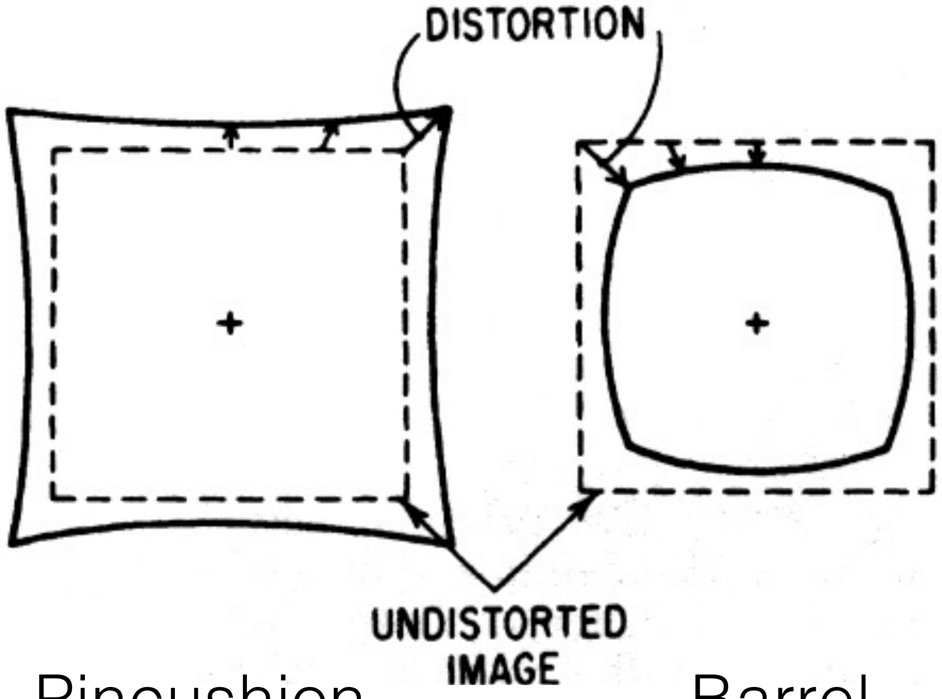
# Oblique aberrations

These appear only as we move further from the center of the field of view.

- Contrast with spherical and chromatic, which appear everywhere.
- Many other examples (astigmatism, field curvature, etc.).




Coma



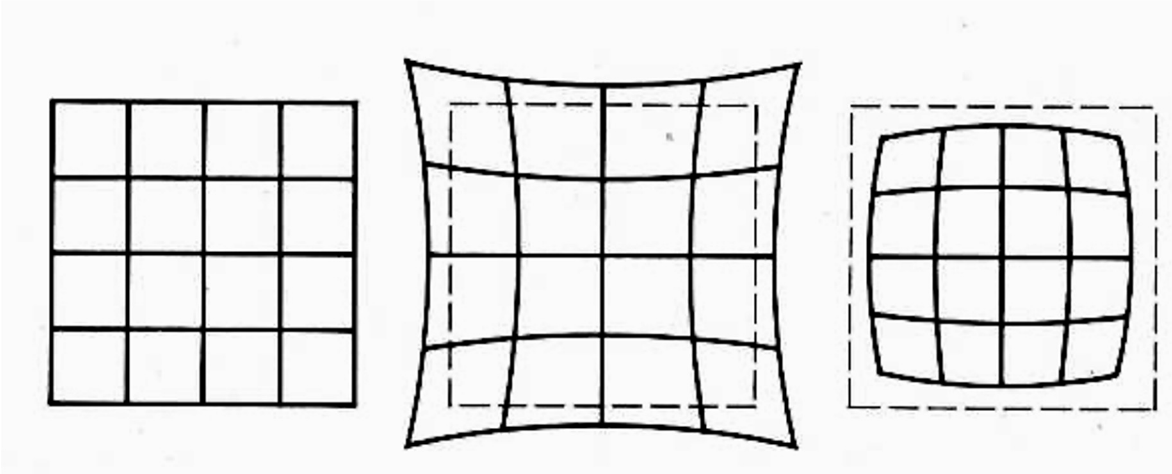

Pincushion Barrel

# Radial Distortion

Straight lines curve around the image center

# Radial Distortion

# Radial Distortion



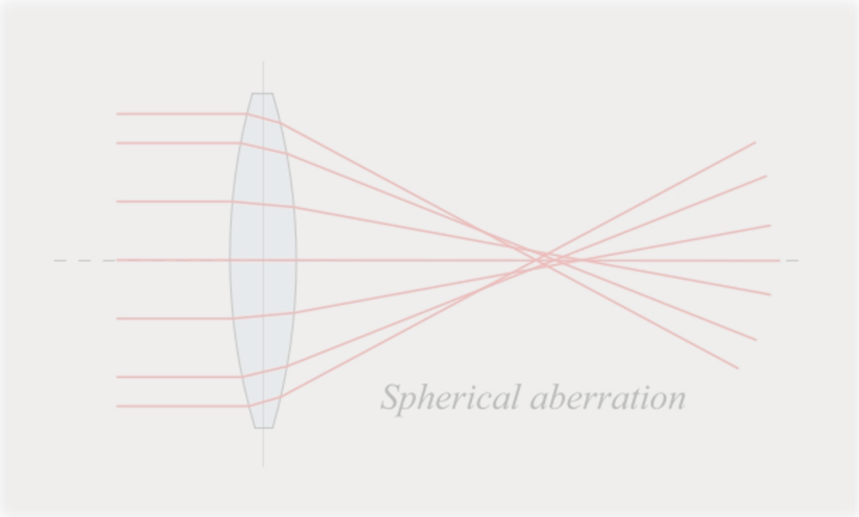
No distortion  Pin cushion  Barrel

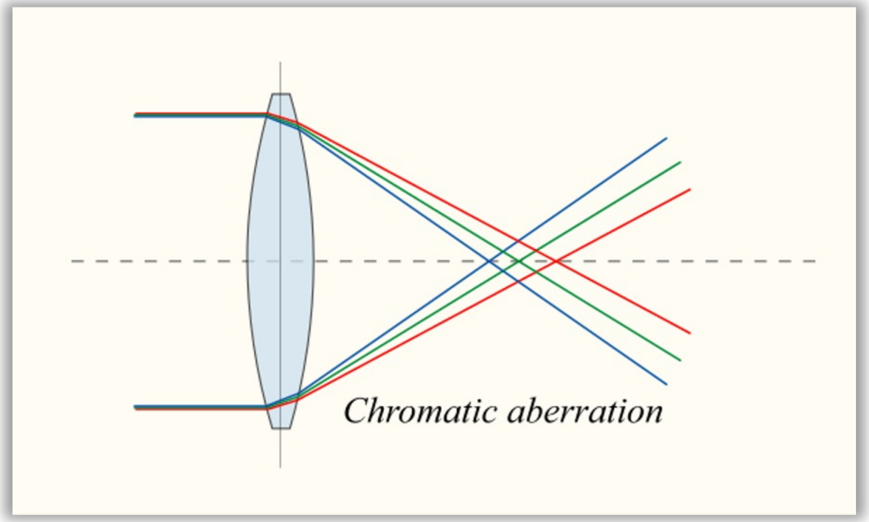
- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens

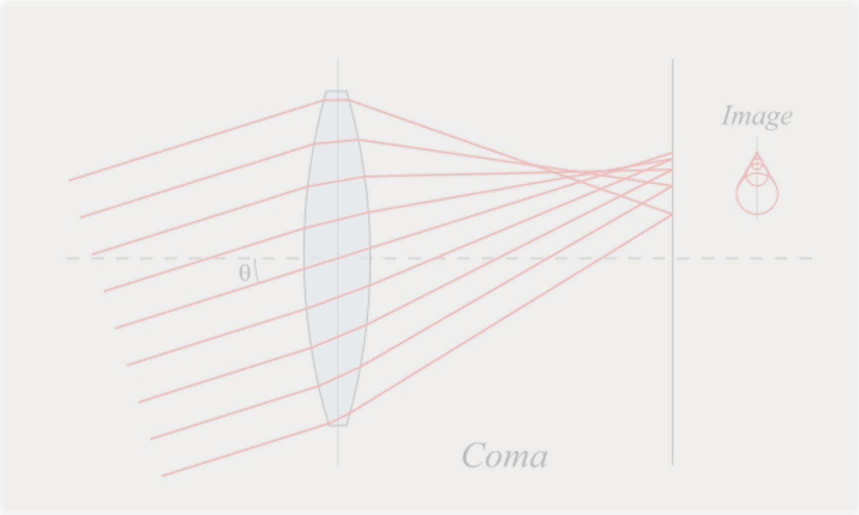
Corrected Barrel Distortion

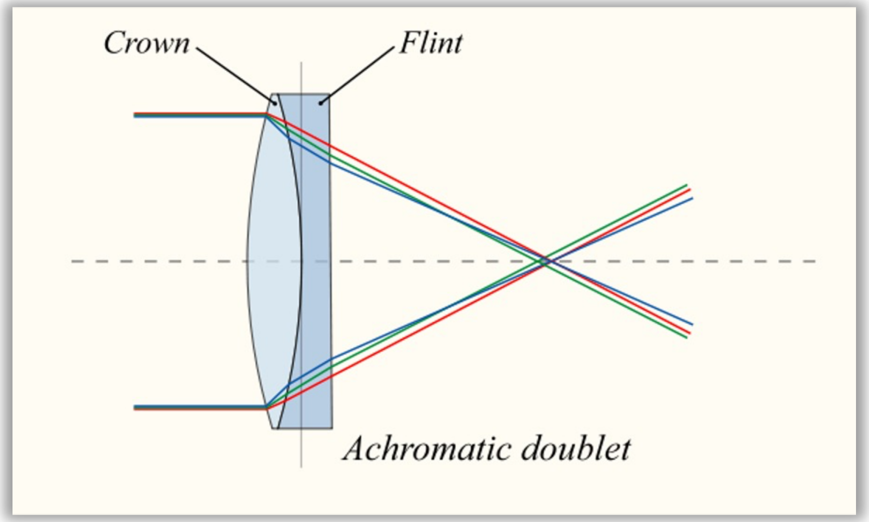
Image from Martin Habbecke

# Lenses - Aberrations



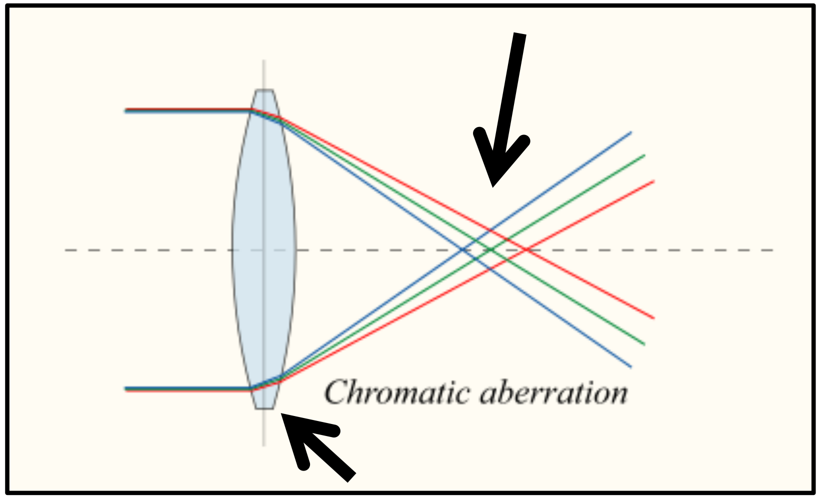



# Aberrations

Deviations from ideal thin lens behavior (e.g., imperfect focus).

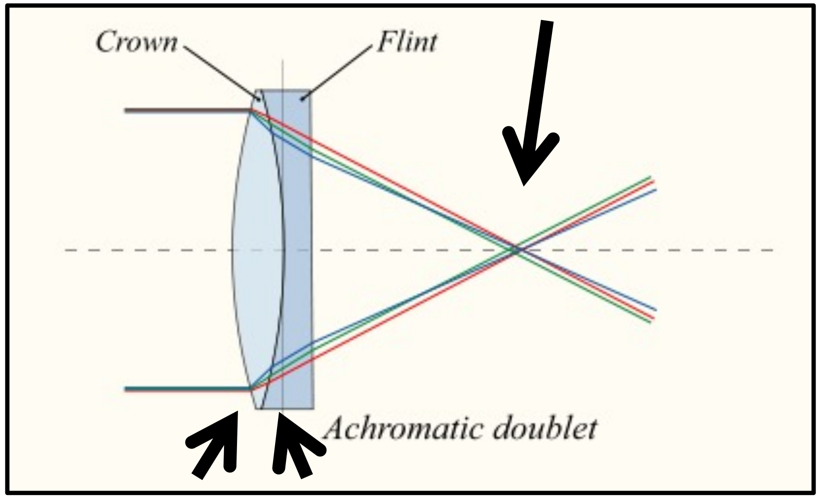
- Example: chromatic aberration.

focal length shifts with wavelength




glass has dispersion (refractive index changes with wavelength)

one lens cancels out dispersion of other



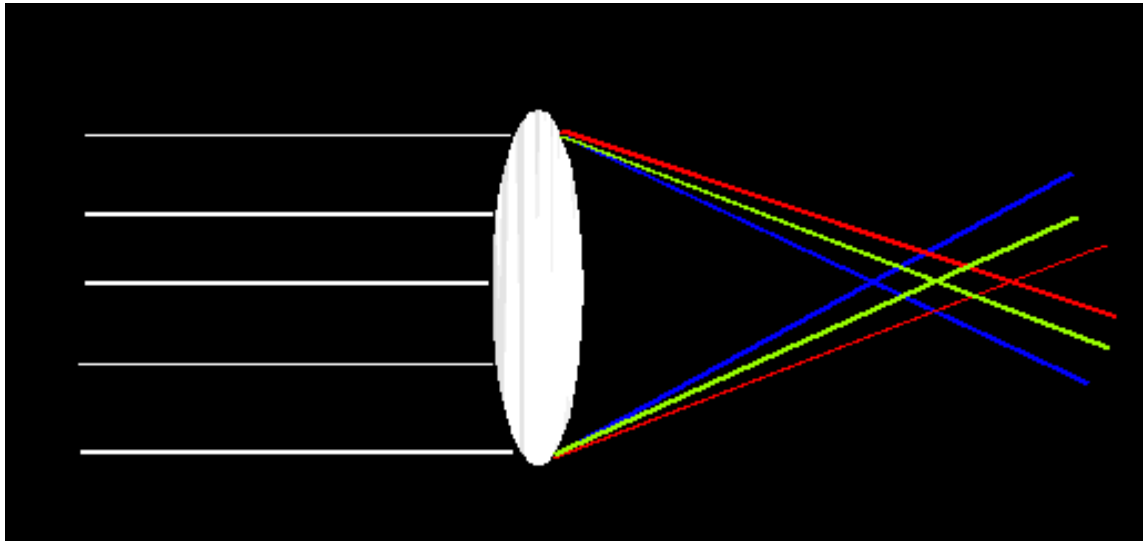

glasses of different refractive index

Using a doublet (two-element compound lens), we can reduce chromatic aberration.

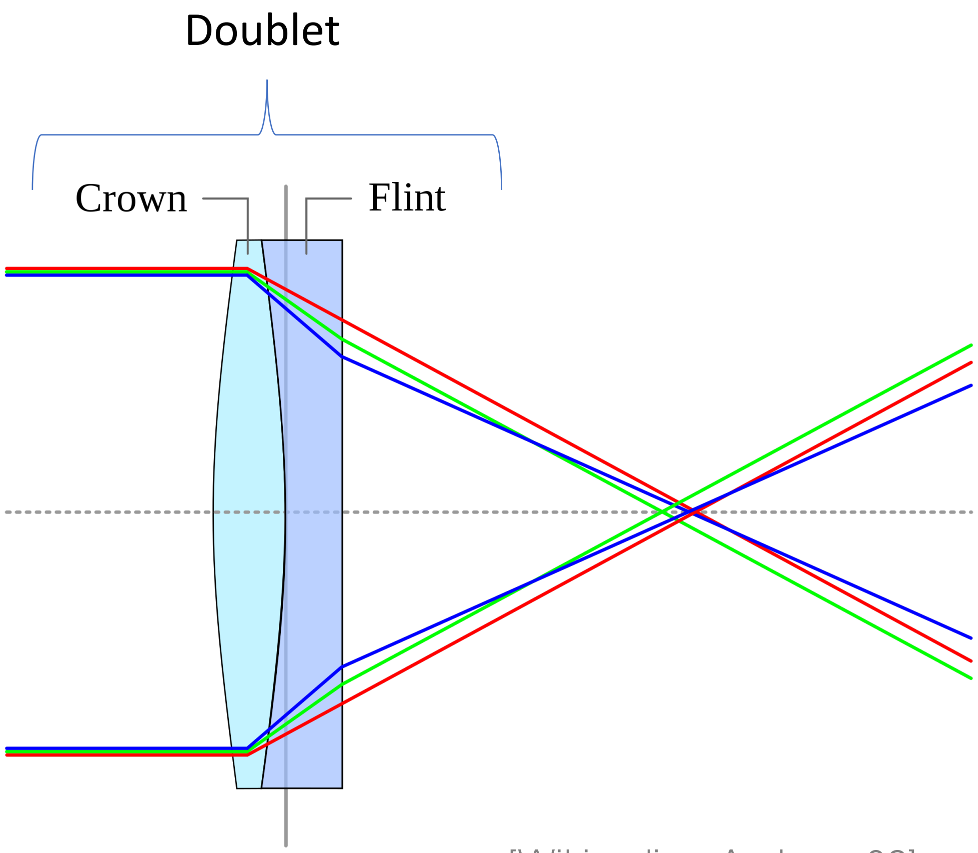
# Chromatic aberration examples

# Chromatic Aberration

- Dispersion: wavelength λ dependent refractive index
  - E.g., a prism spreads white light into rainbow
- Modifies ray-bending and lens focal length: f(λ)
- Causes color fringes near edges of image



- Corrections: add ‘doublet’ lens of flint glass




[Wikipedia – Andreas 06]

# Vignetting

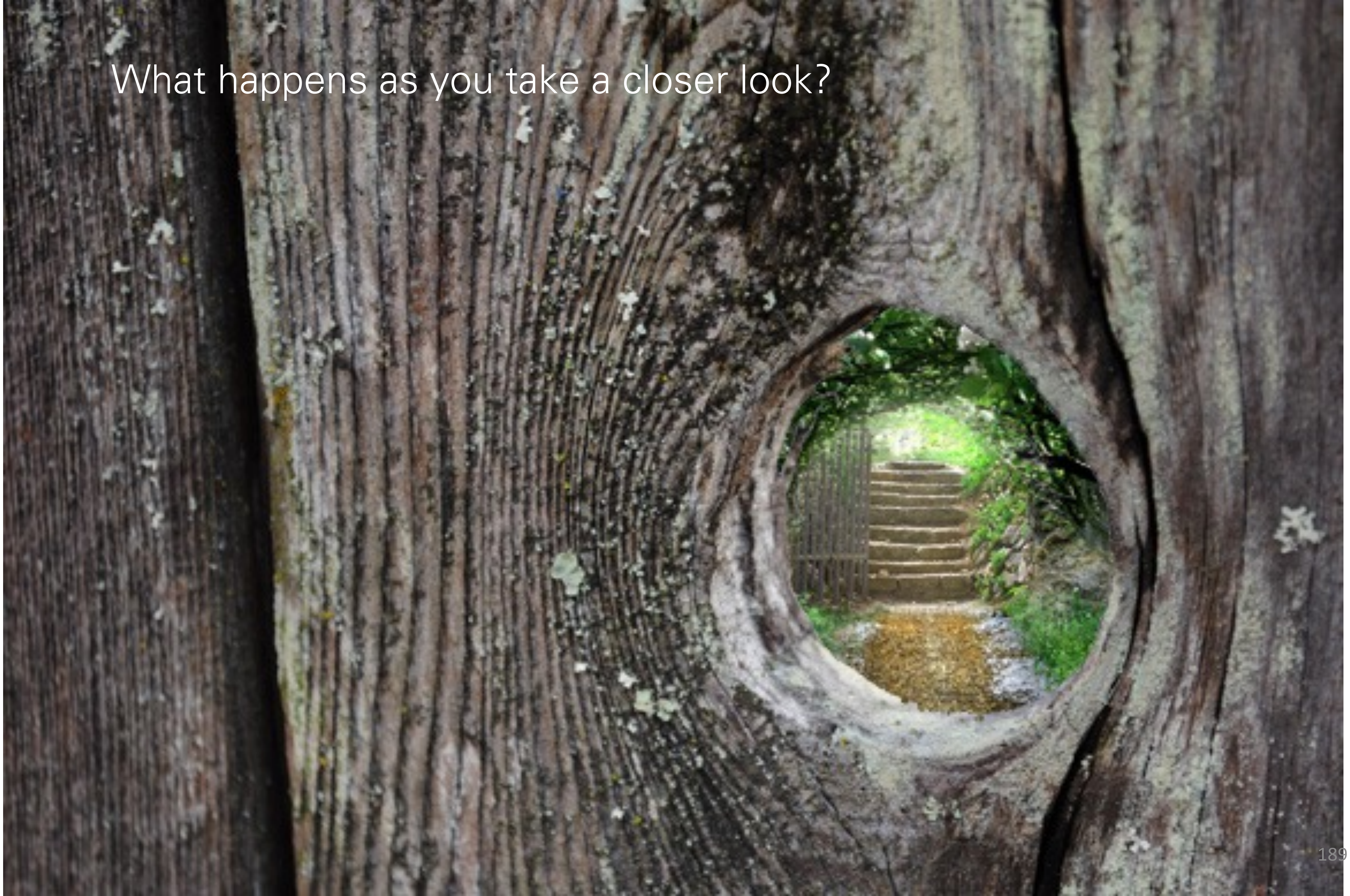
- Optical system occludes rays entering at obtuse angles

- Causes darkening at edges

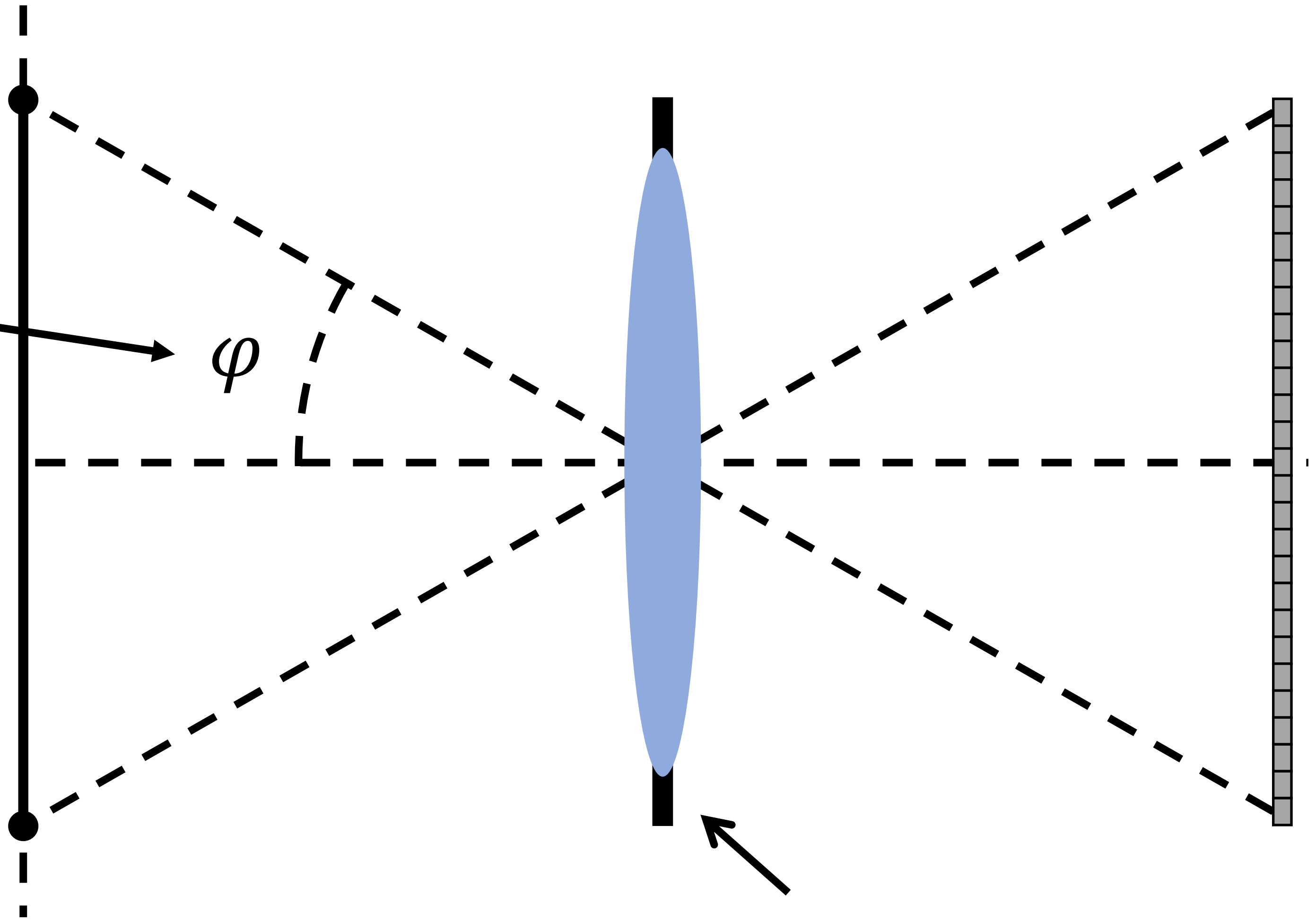
# Field of view

# Field of view

- Determines how much of scene is in frame
- Traditionally specified by focal length
  - but interpreting this number requires considering the “format,” or size of the film or sensor
- After decades of 35mm, that format is stuck in our heads
  - fields of view are usually discussed using the numbers that would be written on a lens for the 35mm format
- Changing FOV while keeping the camera fixed
  - strictly “crops” the image: relationships between objects are fixed corresponds to turning the zoom control on a modern camera


What happens as you take a closer look?
189

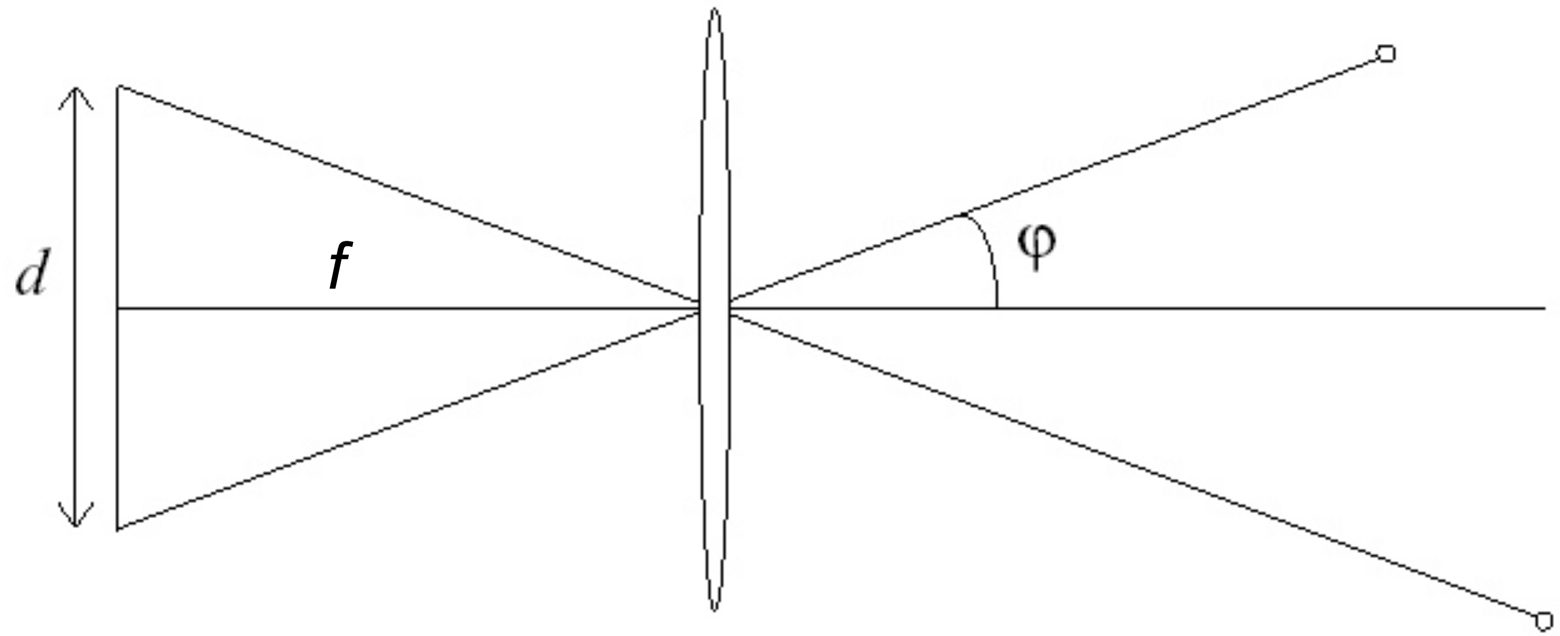
# Field of view




also described using angle of view

Note: here I drew a lens, but I could have just as well drawn a pinhole

# Field of view depends of focal length



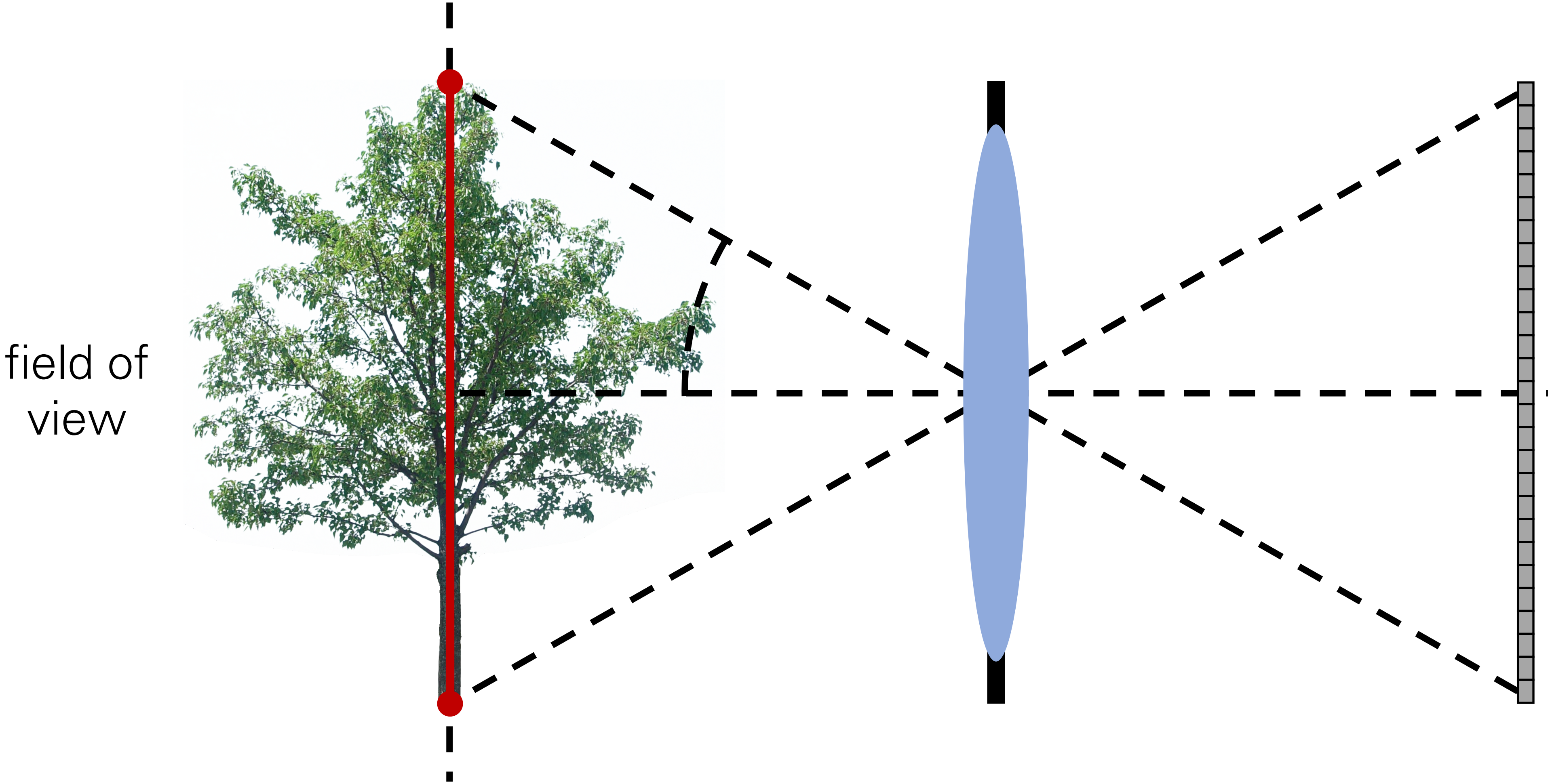

Size of field of view governed by size of the camera retina:

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Smaller field of view = larger focal length

# Field of view

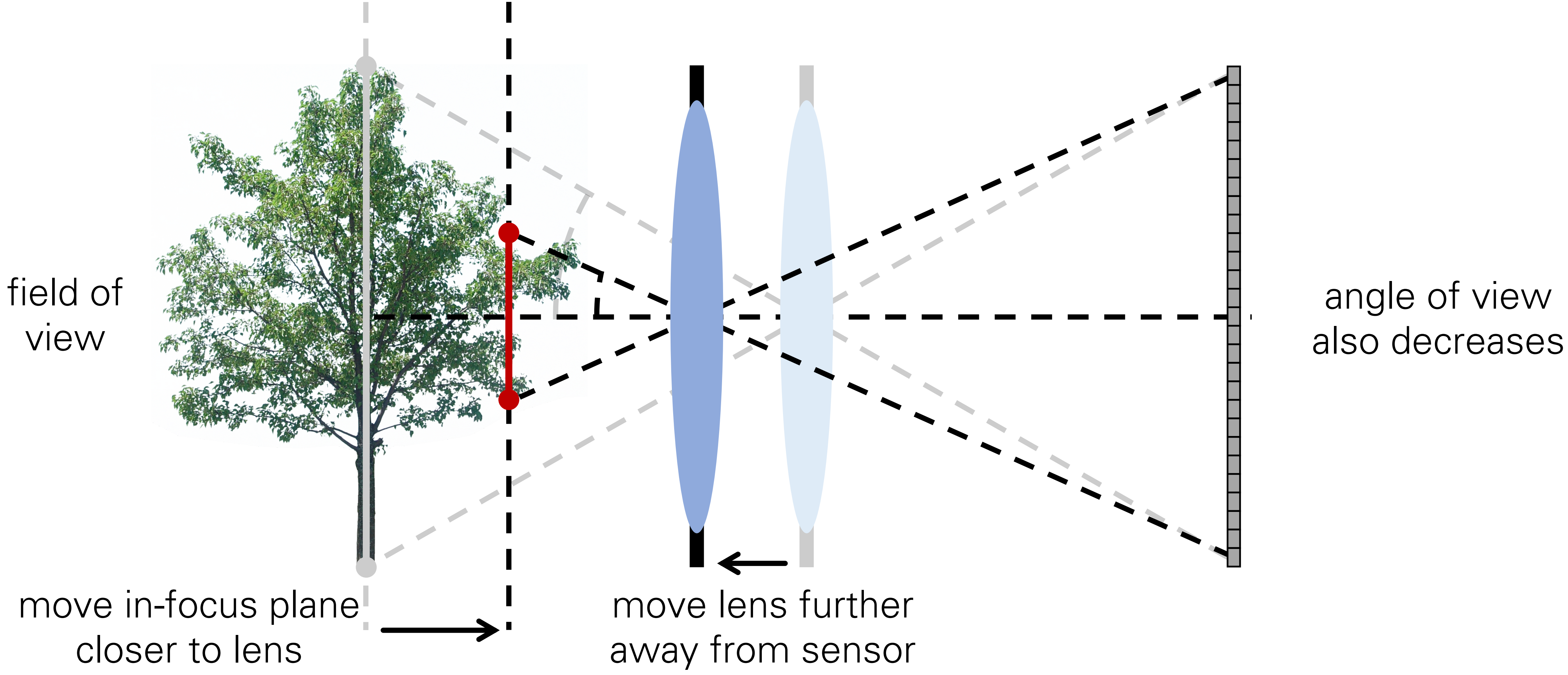
The part of the in-focus plane that gets mapped on the sensor.




- What happens to field of view as we focus closer?

# Field of view

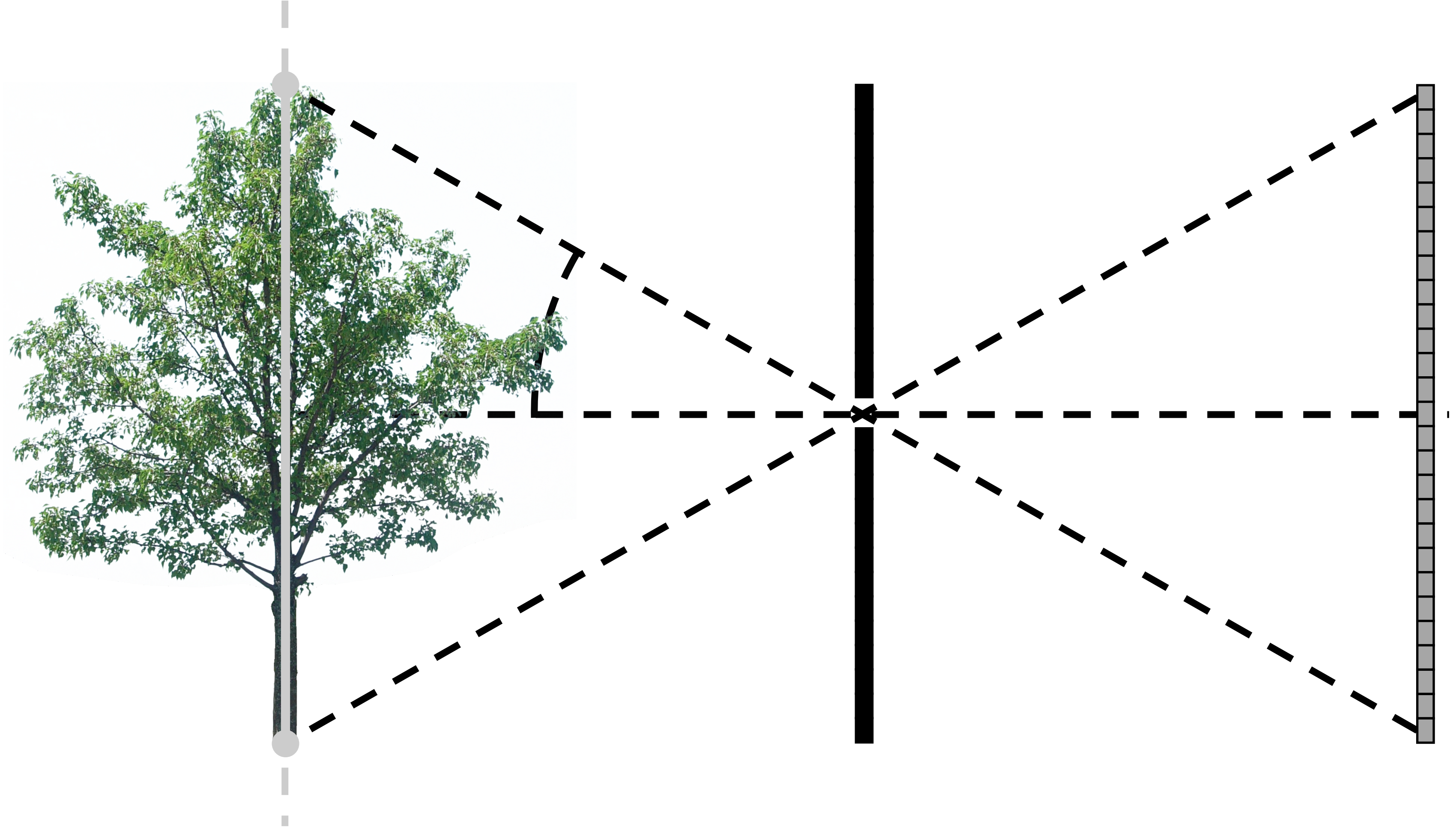
The part of the in-focus plane that gets mapped on the sensor.




- What happens to field of view as we focus closer? → It becomes smaller.

# Comparison with pinhole camera

field of
view

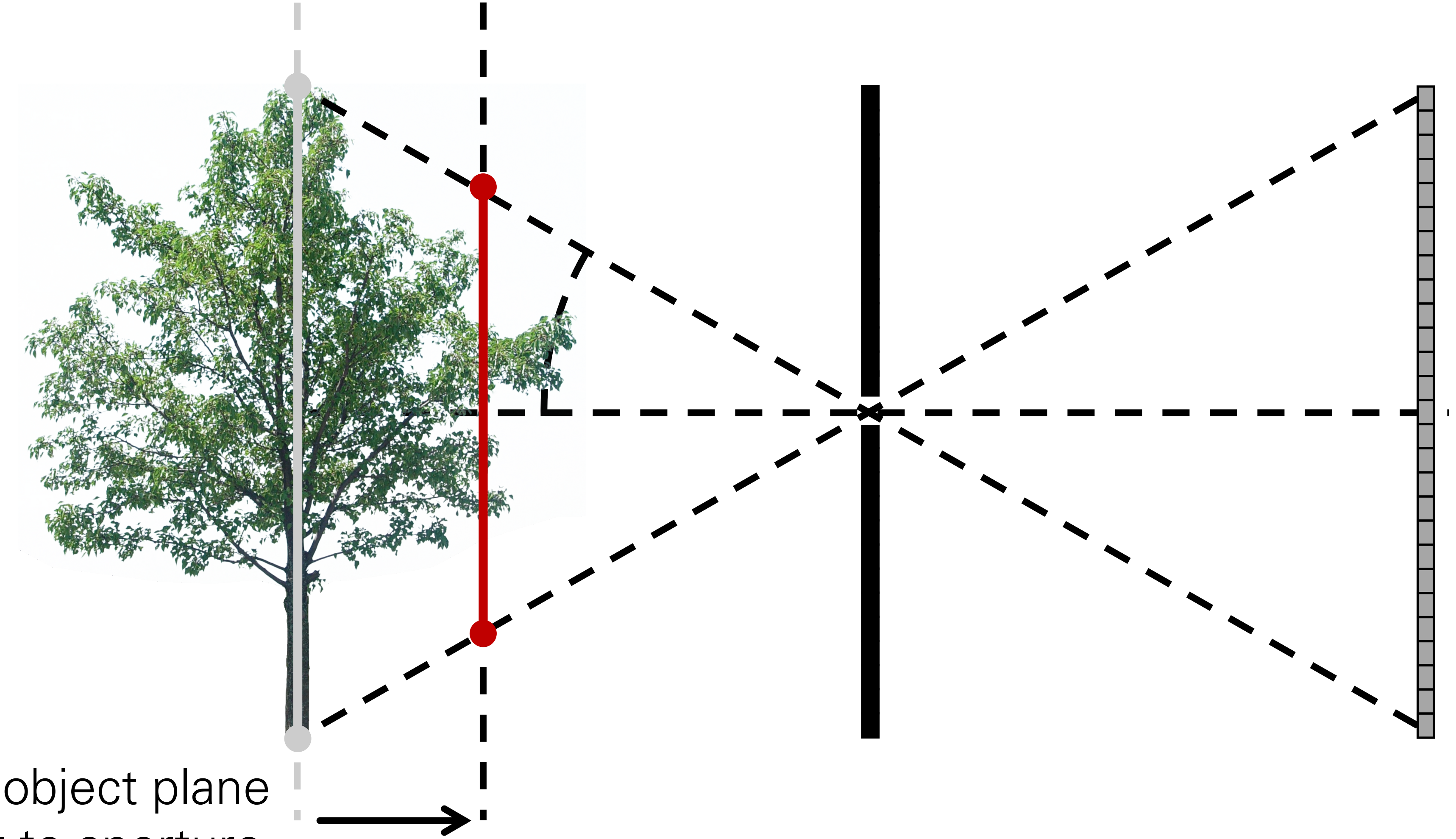


- What happens to field of view as we move closer?

# Comparison with pinhole camera

No need to refocus: we can move object closer without changing aperture-sensor distance.

field of view



angle of view does not change

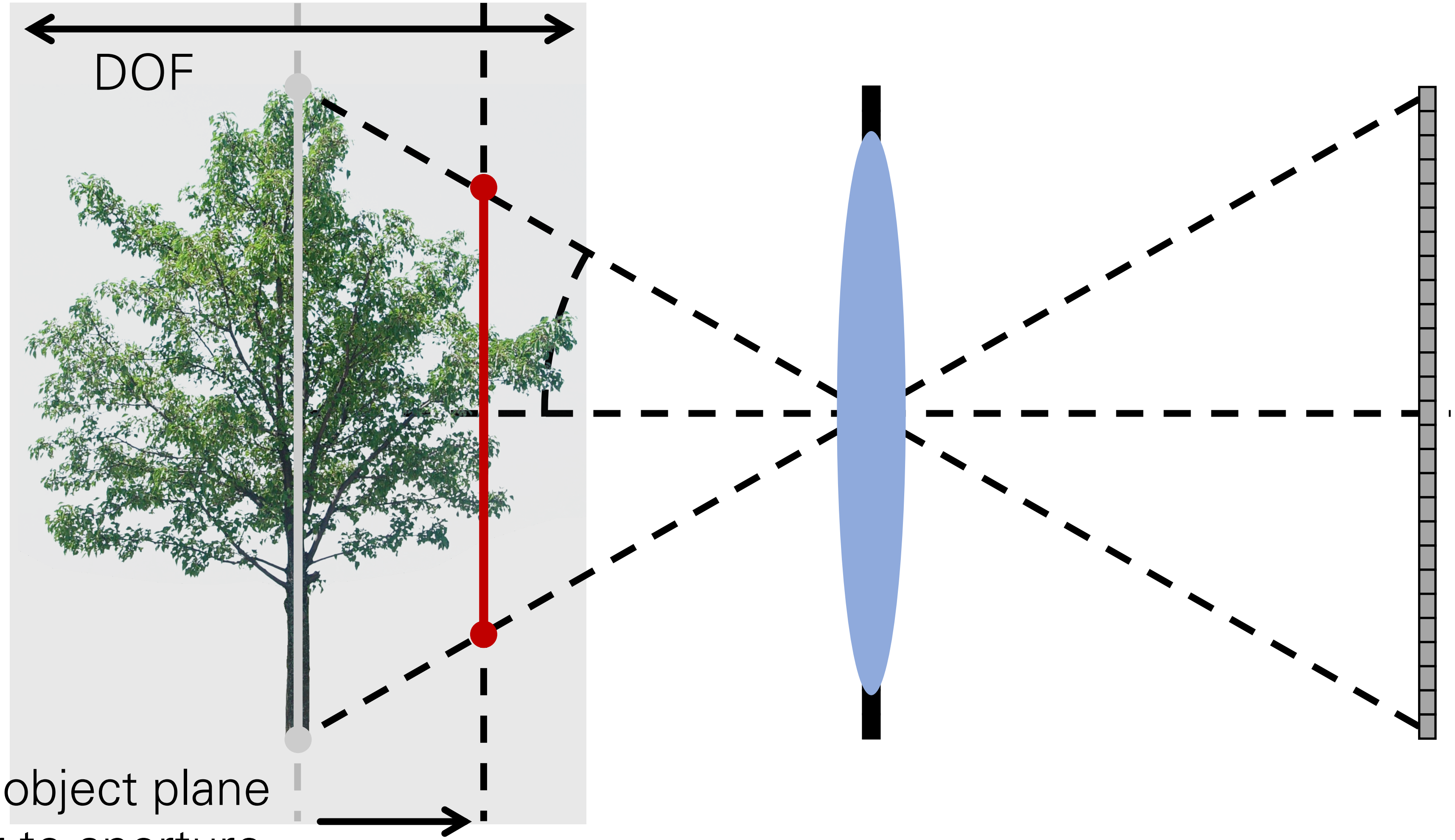
move object plane closer to aperture

- What happens to field of view as we move closer? → It becomes smaller, but amount differs.

# Comparison with pinhole camera

No need to refocus: we can move object closer without changing aperture-sensor distance.

field of view

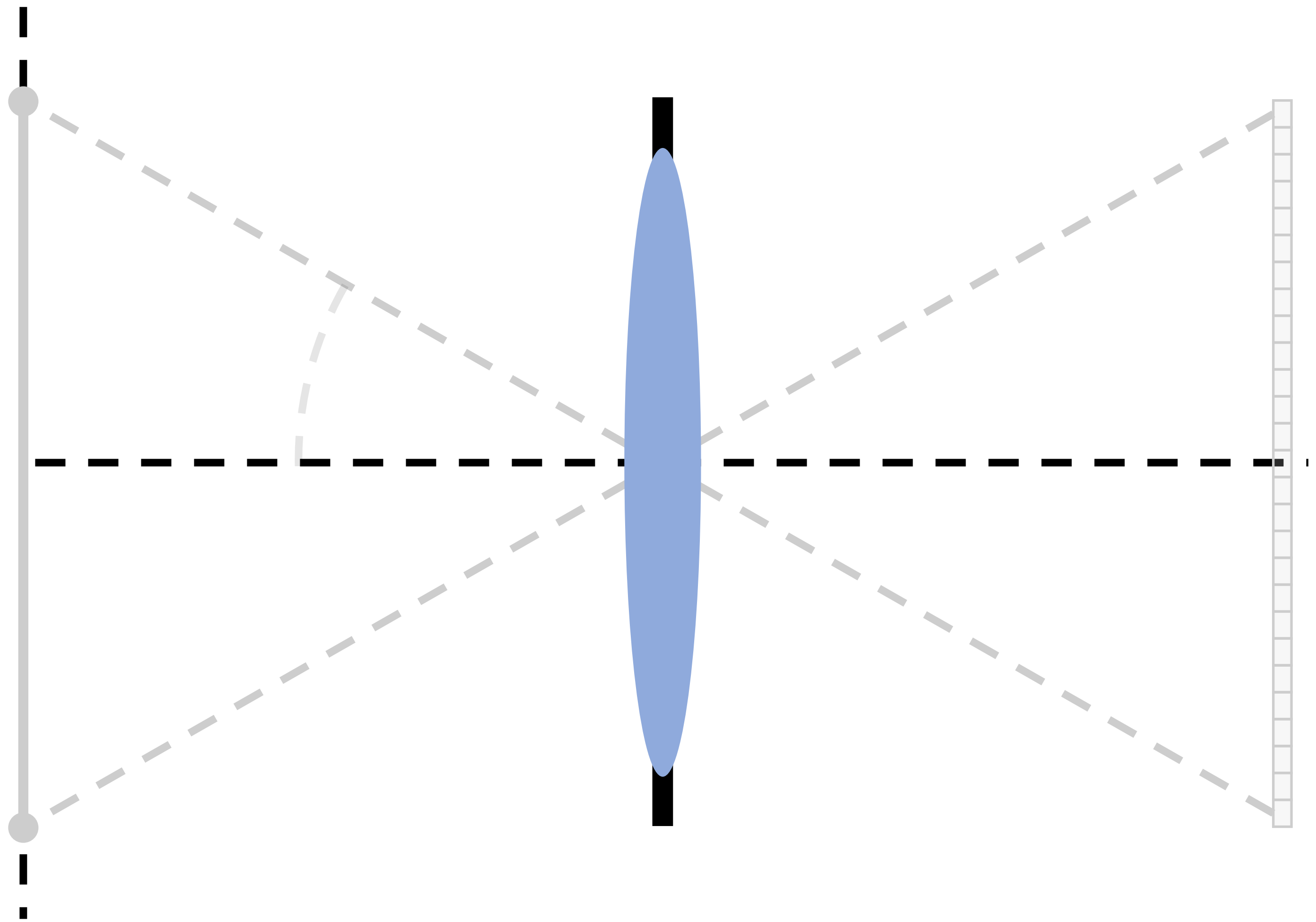



angle of view does not change

move object plane closer to aperture

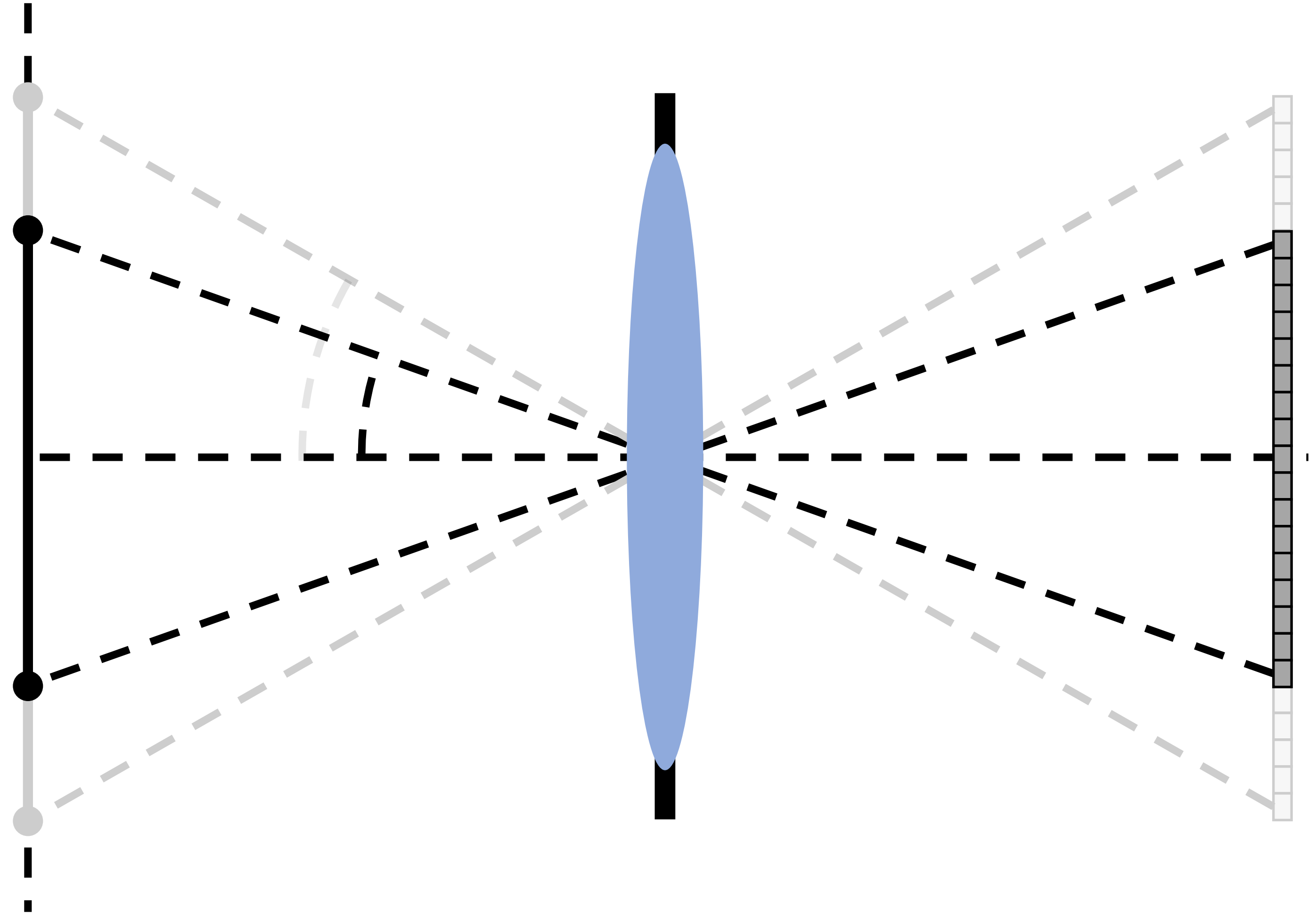
This can be done with a lens only if depth of field is large enough. Then the two behave the same.

# Field of view also depends on sensor size



- What happens to field of view when we reduce sensor size?

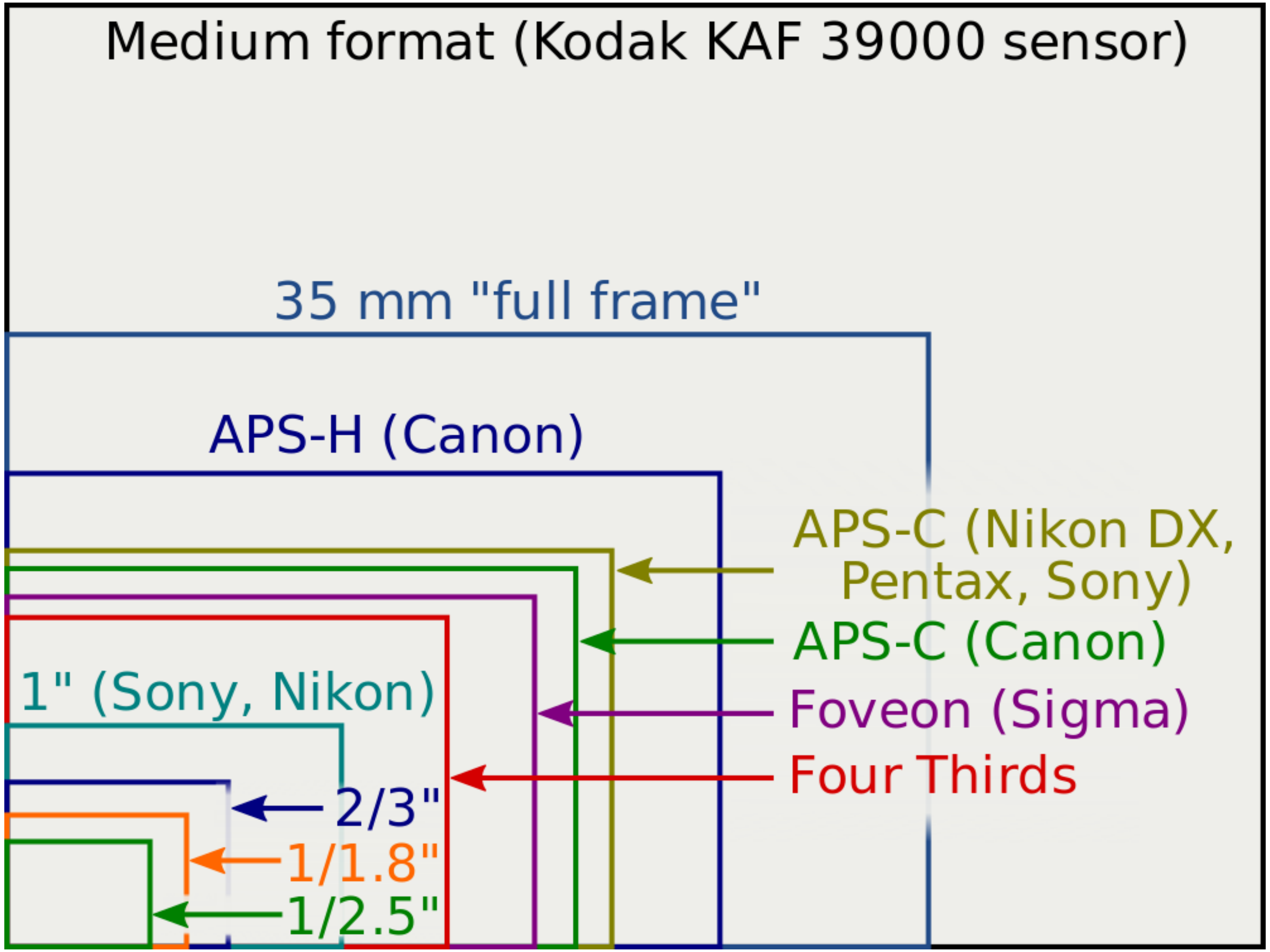
# Field of view also depends on sensor size



Lens and pinhole cameras behave the same in this case.

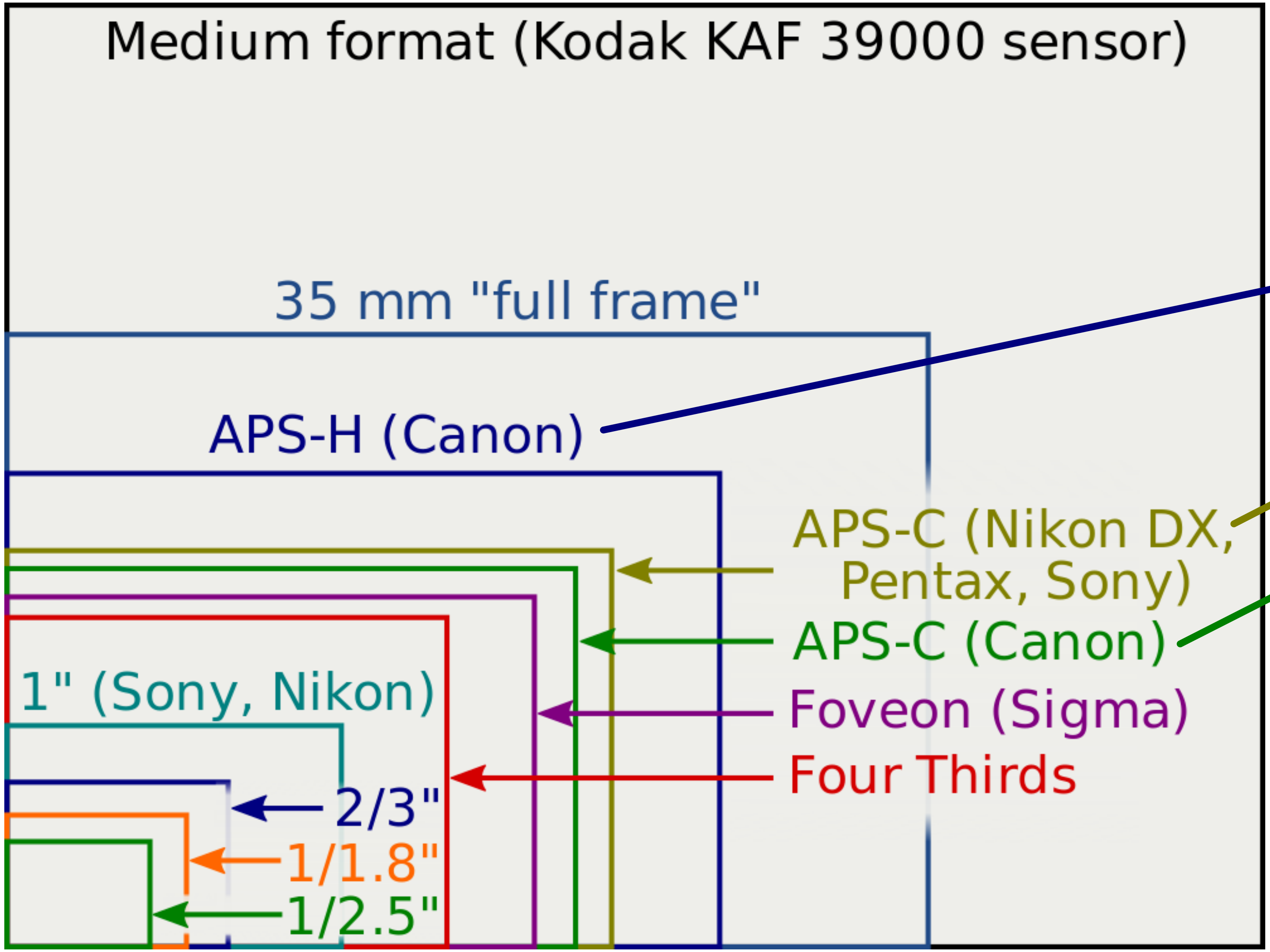
- What happens to field of view when we reduce sensor size? → It decreases.

# Field of view also depends on sensor size




- "Full frame" corresponds to standard film size.
- Digital sensors come in smaller formats due to manufacturing limitations (now mostly overcome).
- Lenses are often described in terms of field of view on film instead of focal length.
- These descriptions are invalid when not using full-frame sensor.

# Crop factor

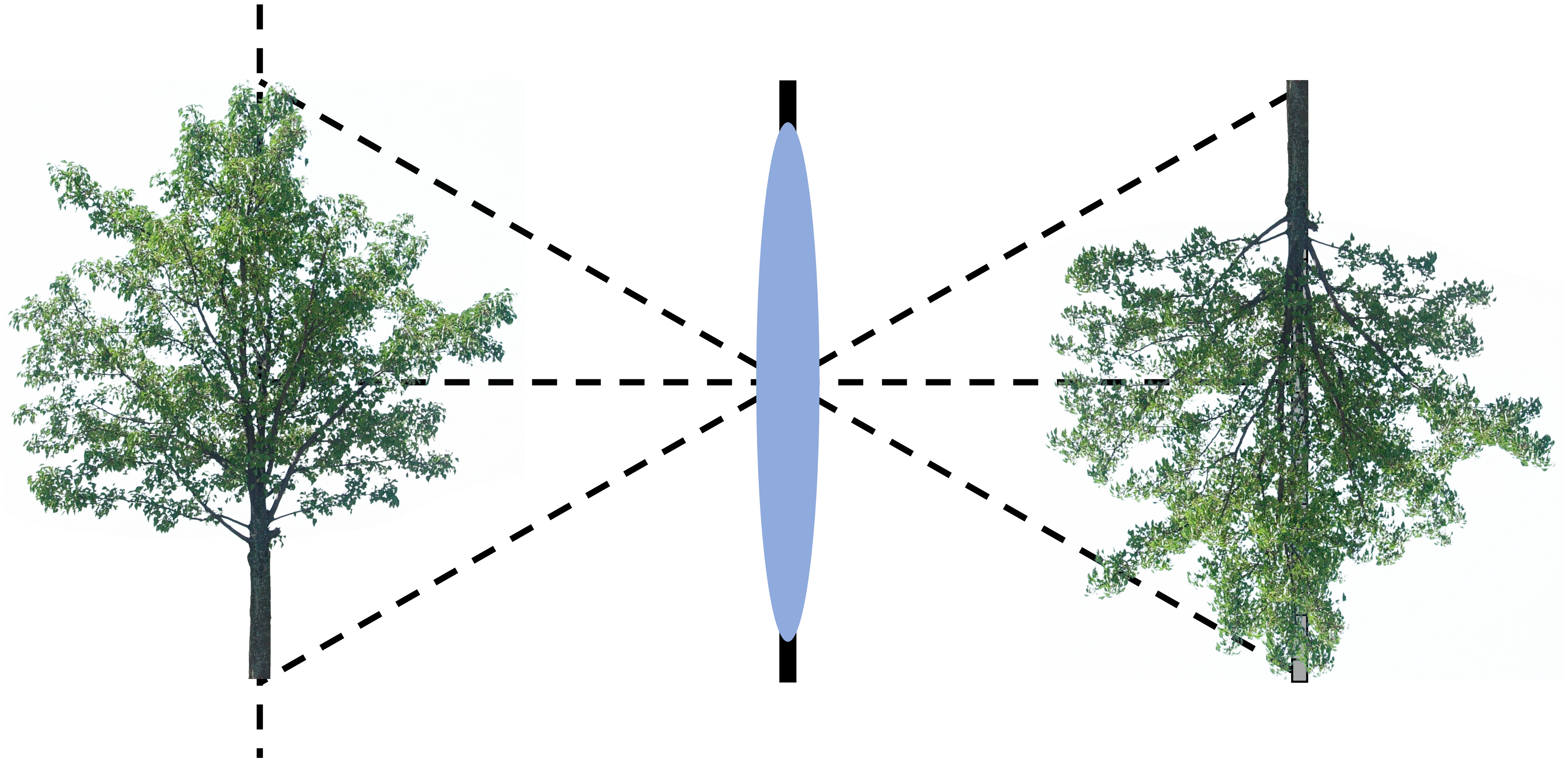




How much field of view is cropped when using a sensor smaller than full frame.

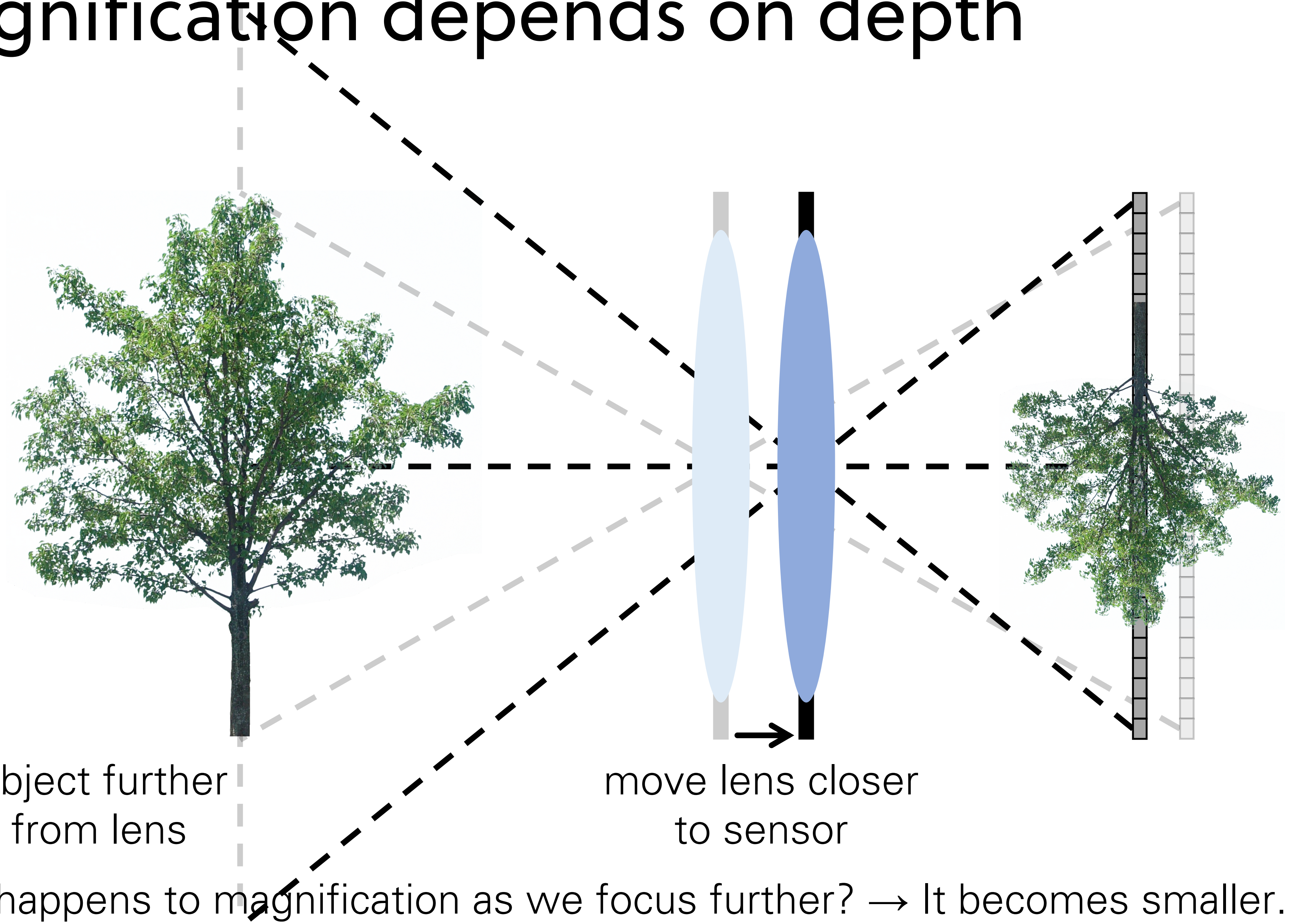
# Magnification and perspective

# Magnification depends on depth



- What happens to magnification as we focus further away?

# Magnification depends on depth



move object further away from lens

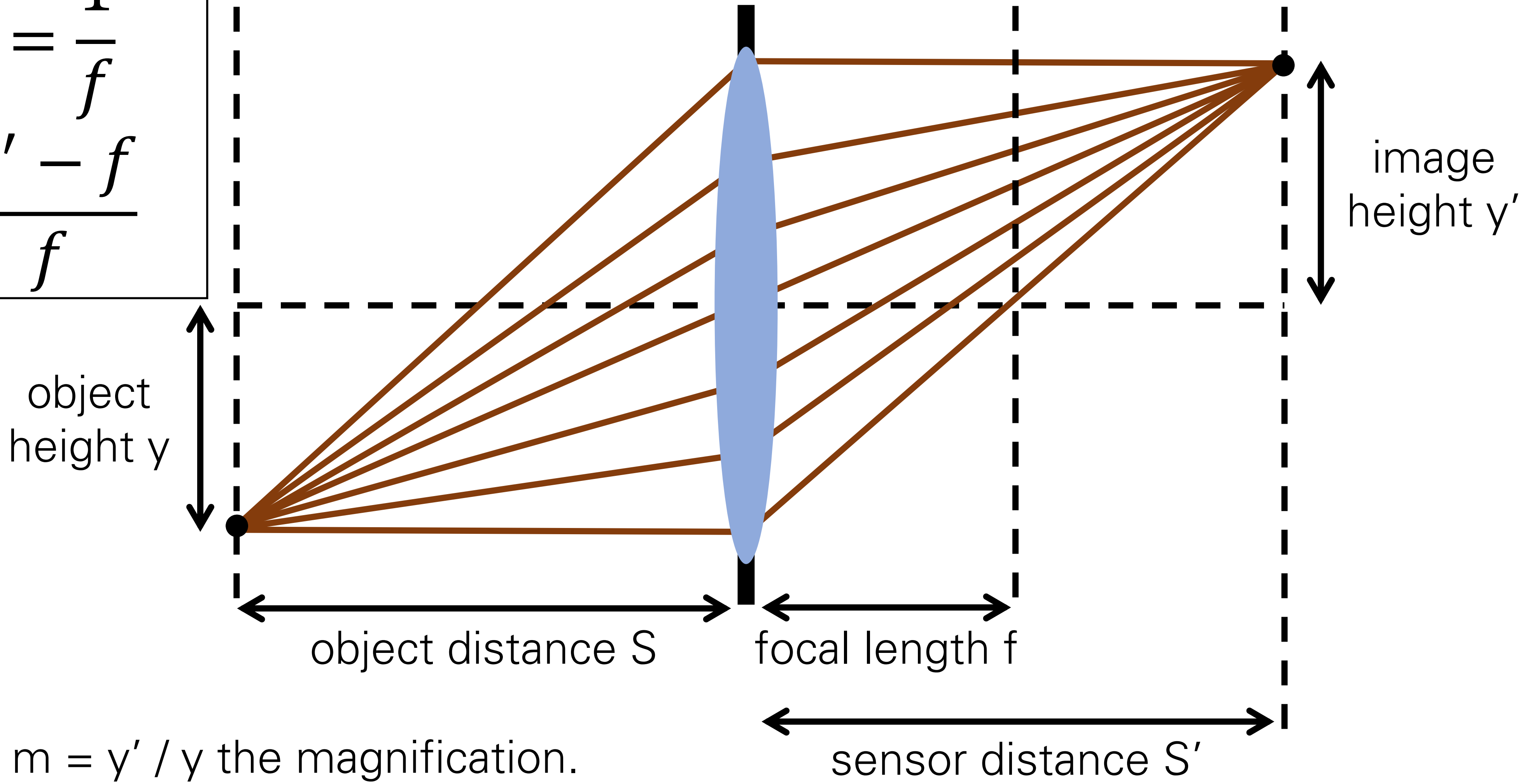
move lens closer to sensor

- What happens to magnification as we focus further? → It becomes smaller.

# Magnification depends on depth

$$\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$

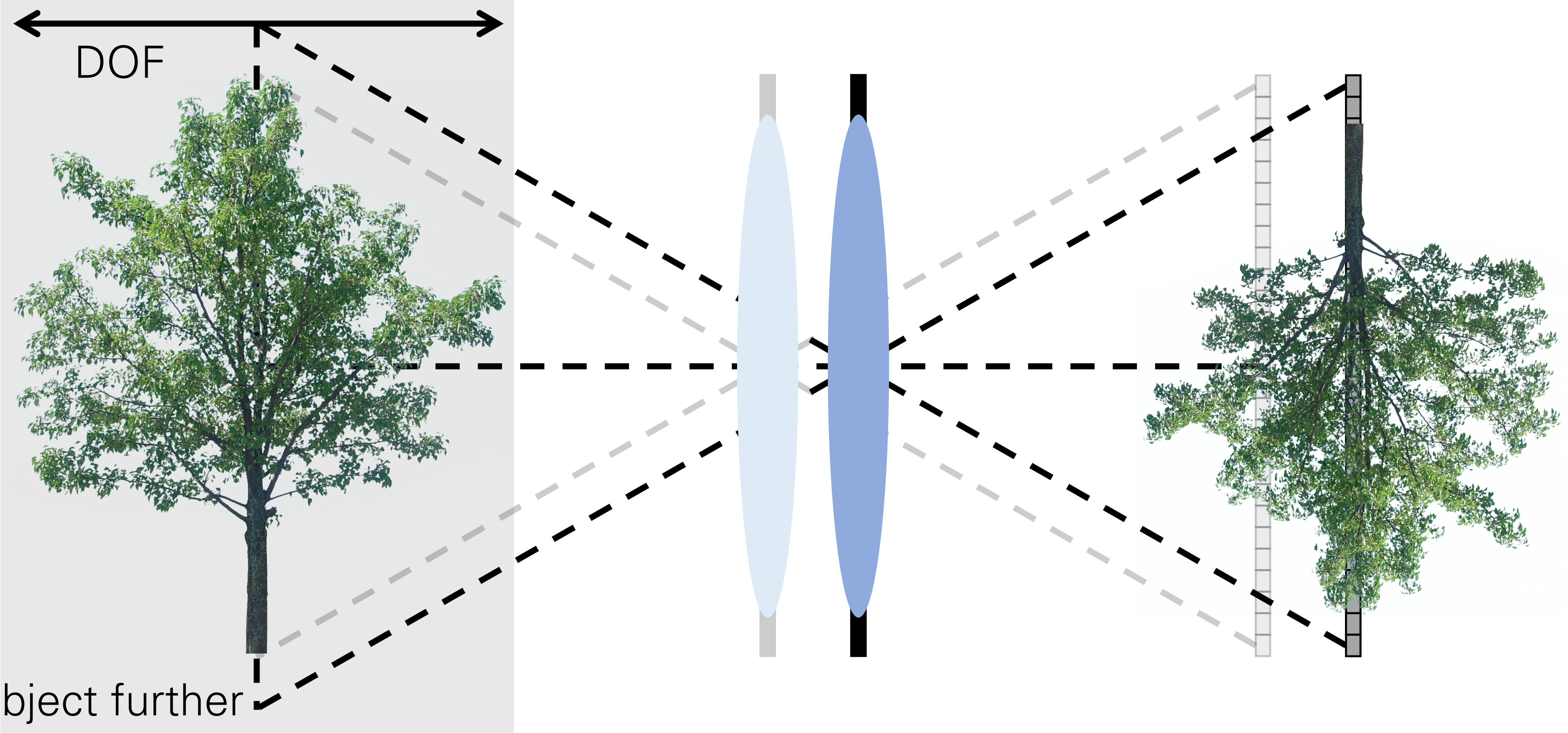
$$m = \frac{S' - f}{f}$$




- We call m = y' / y the magnification.

# Comparison with pinhole camera

No need to refocus: we can move object further without changing aperture-sensor distance.



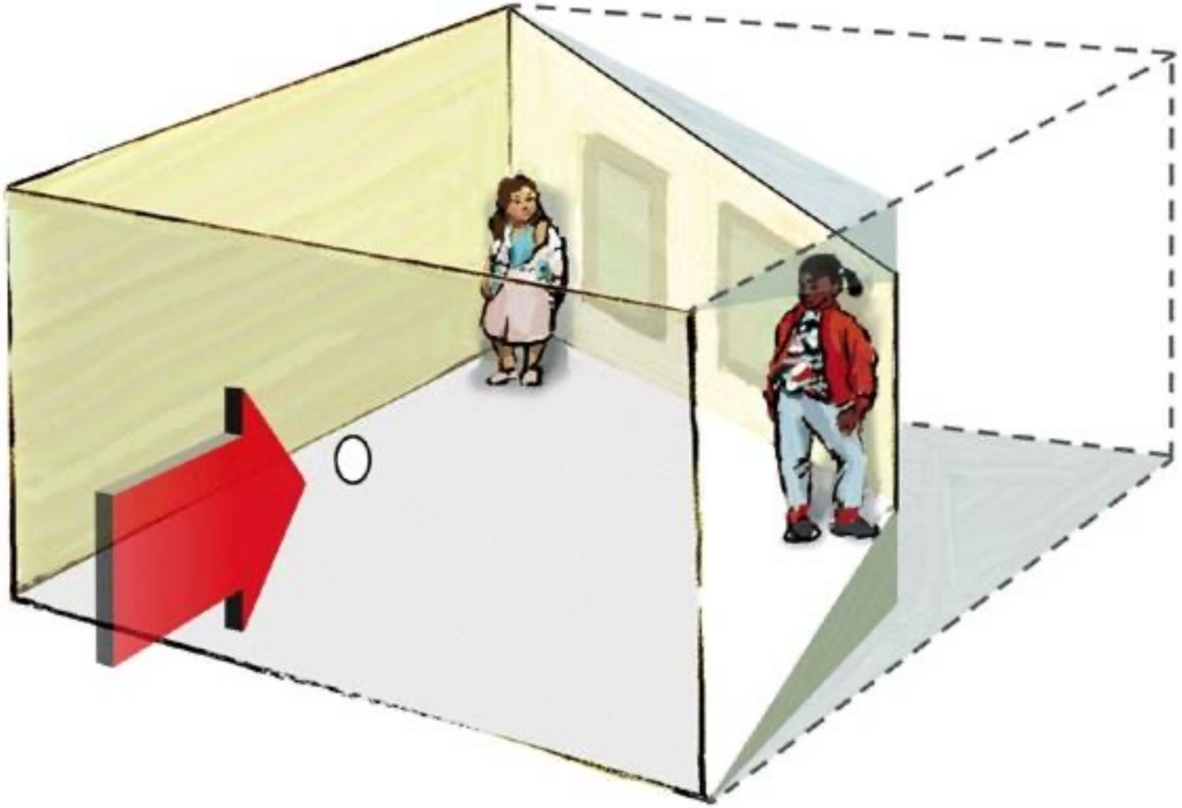

move object further
away from aperture

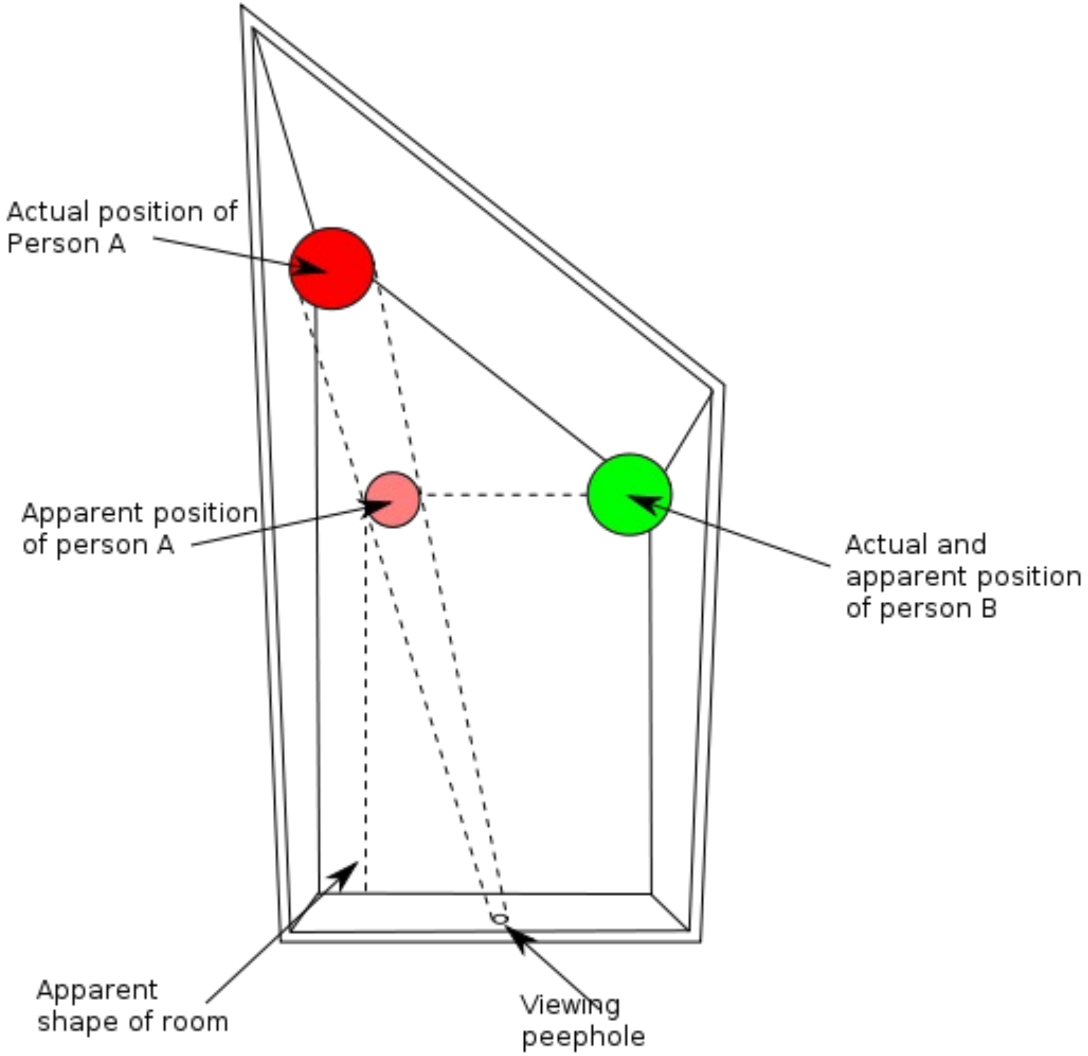
This can be done with a lens only if depth of field is large enough. Then the two behave the same.

# Forced perspective

# The Ames room illusion

# The Ames room illusion





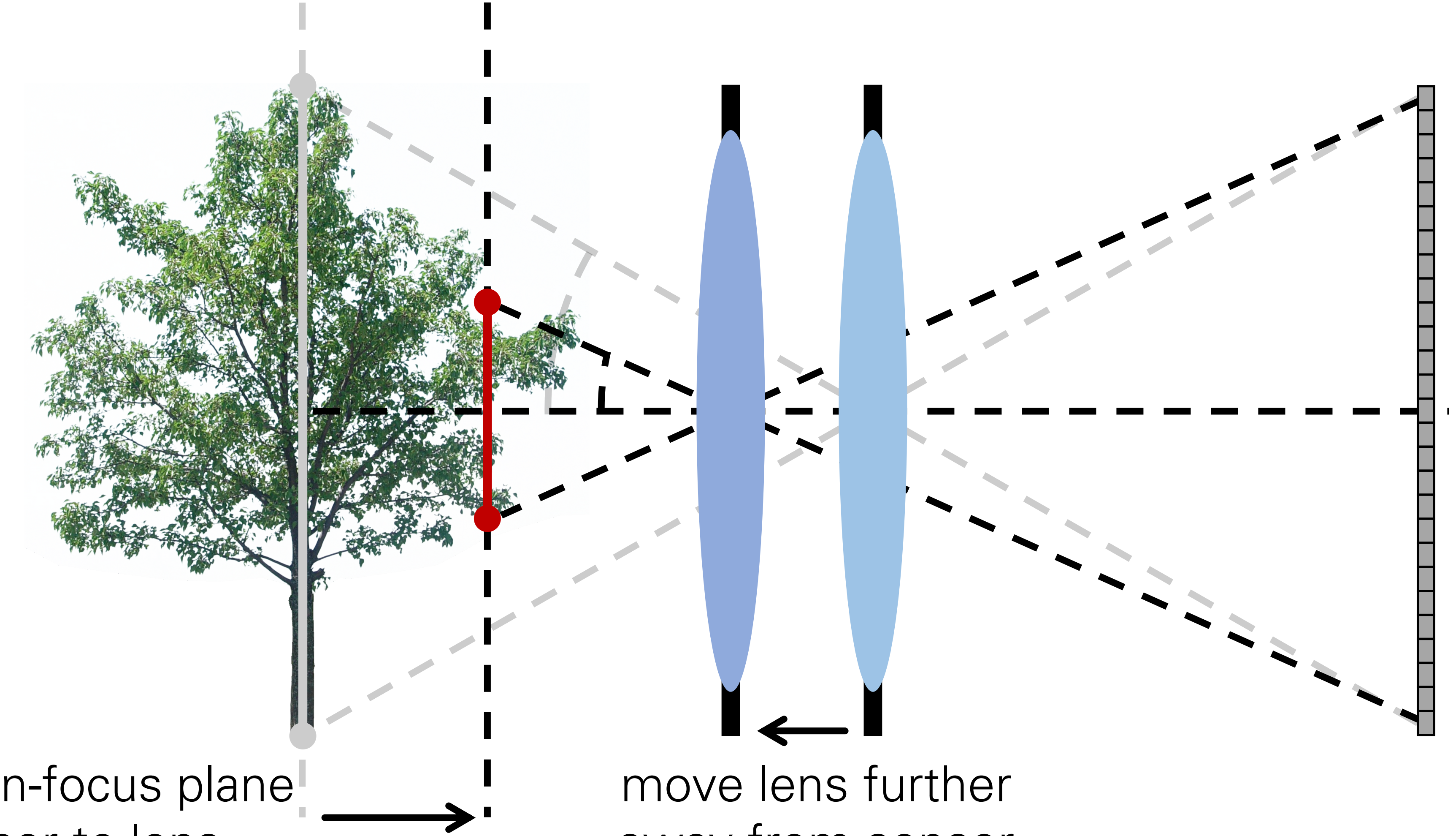
# The arrow illusion

Prof. Kokichi Sugihara has many other amazing illusions involving perspective distortion, check them out on YouTube or on his website:
http://www.isc.meiji.ac.jp/~kokichis/

# Zooming

# Field of view

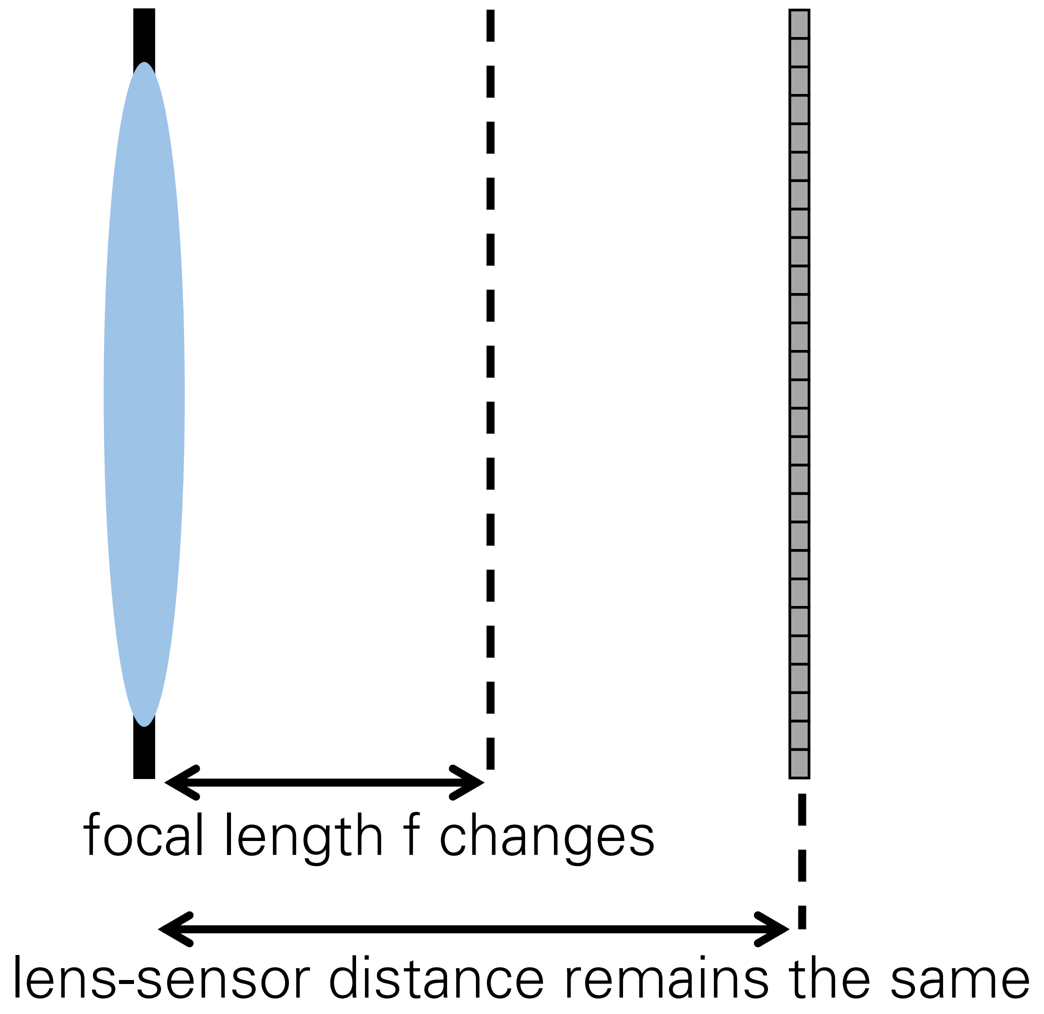
The part of the in-focus plane that gets mapped on the sensor.



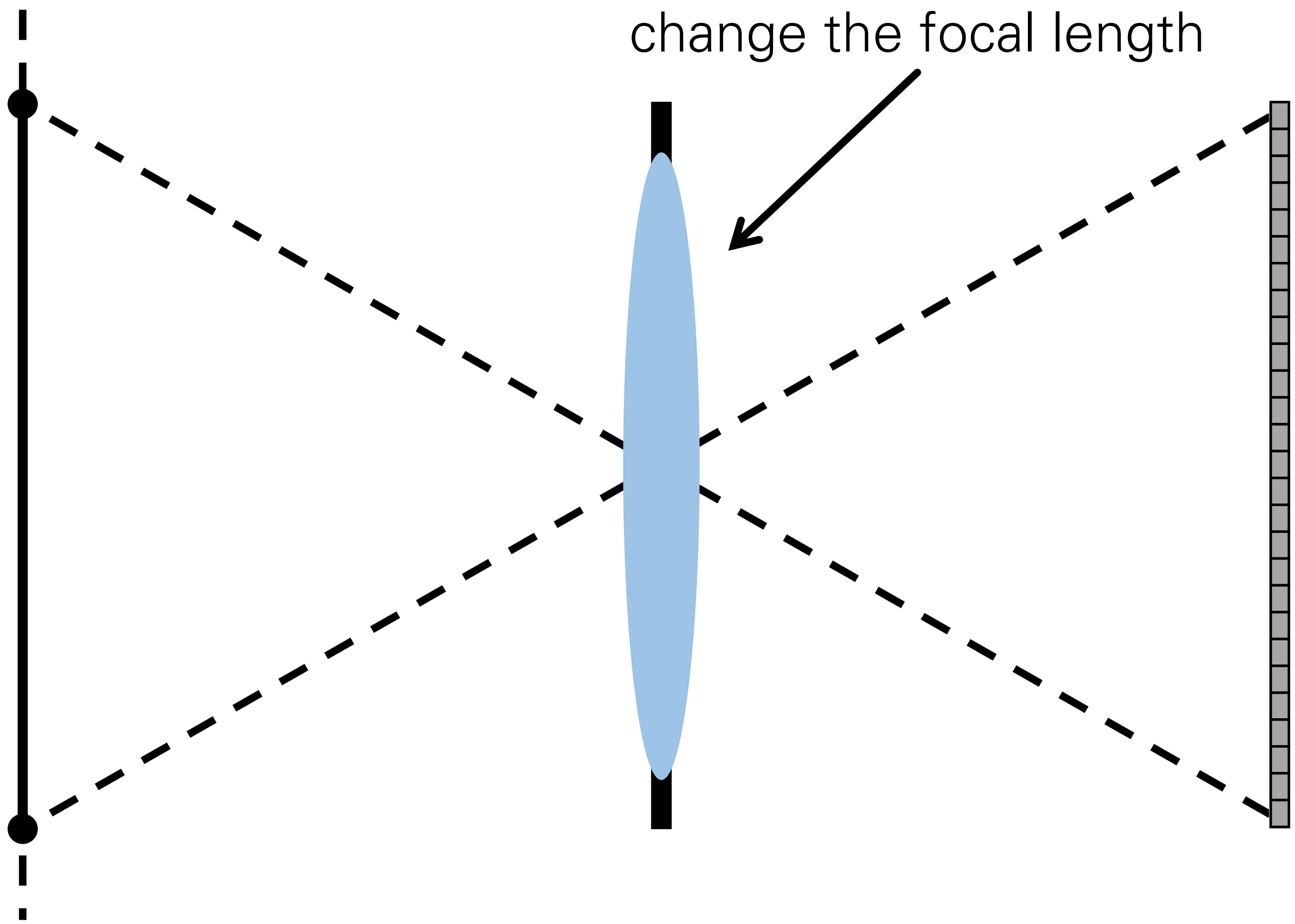

- What happens to field of view as we focus closer? → It becomes smaller.

# Zooming means changing the focal length

Very different process from refocusing

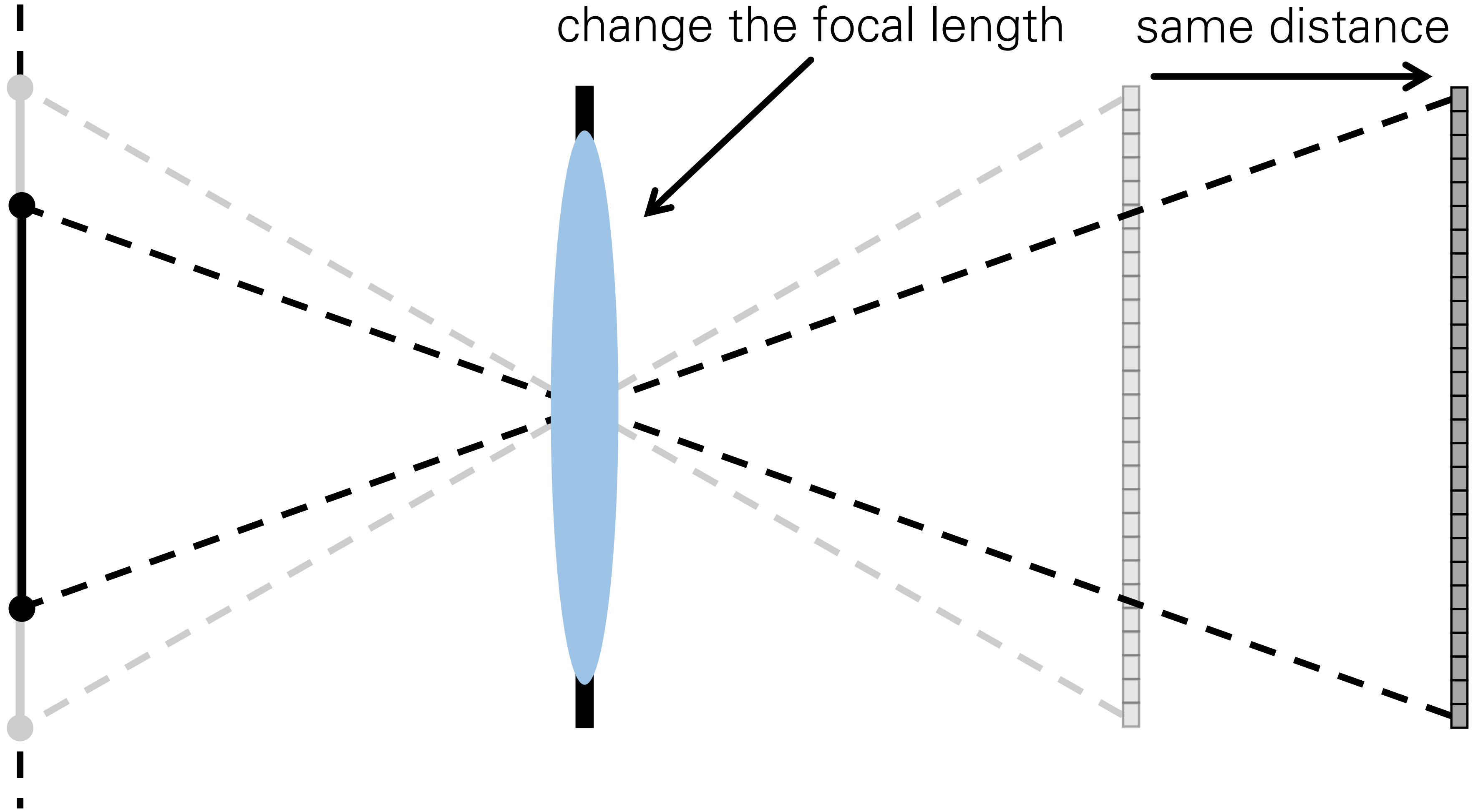


# Zooming and field of view



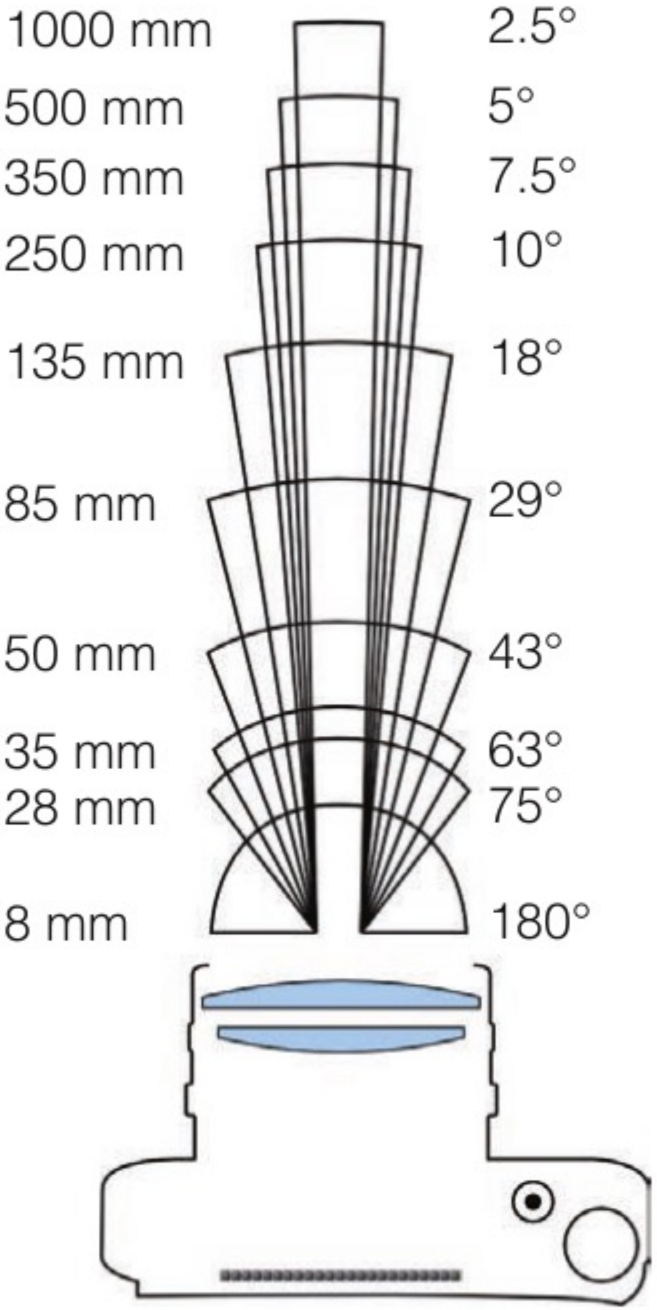

- What happens to field of view when we focus closer? → It decreases.
- What happens to field of view when we increase <u>lens</u> focal length?

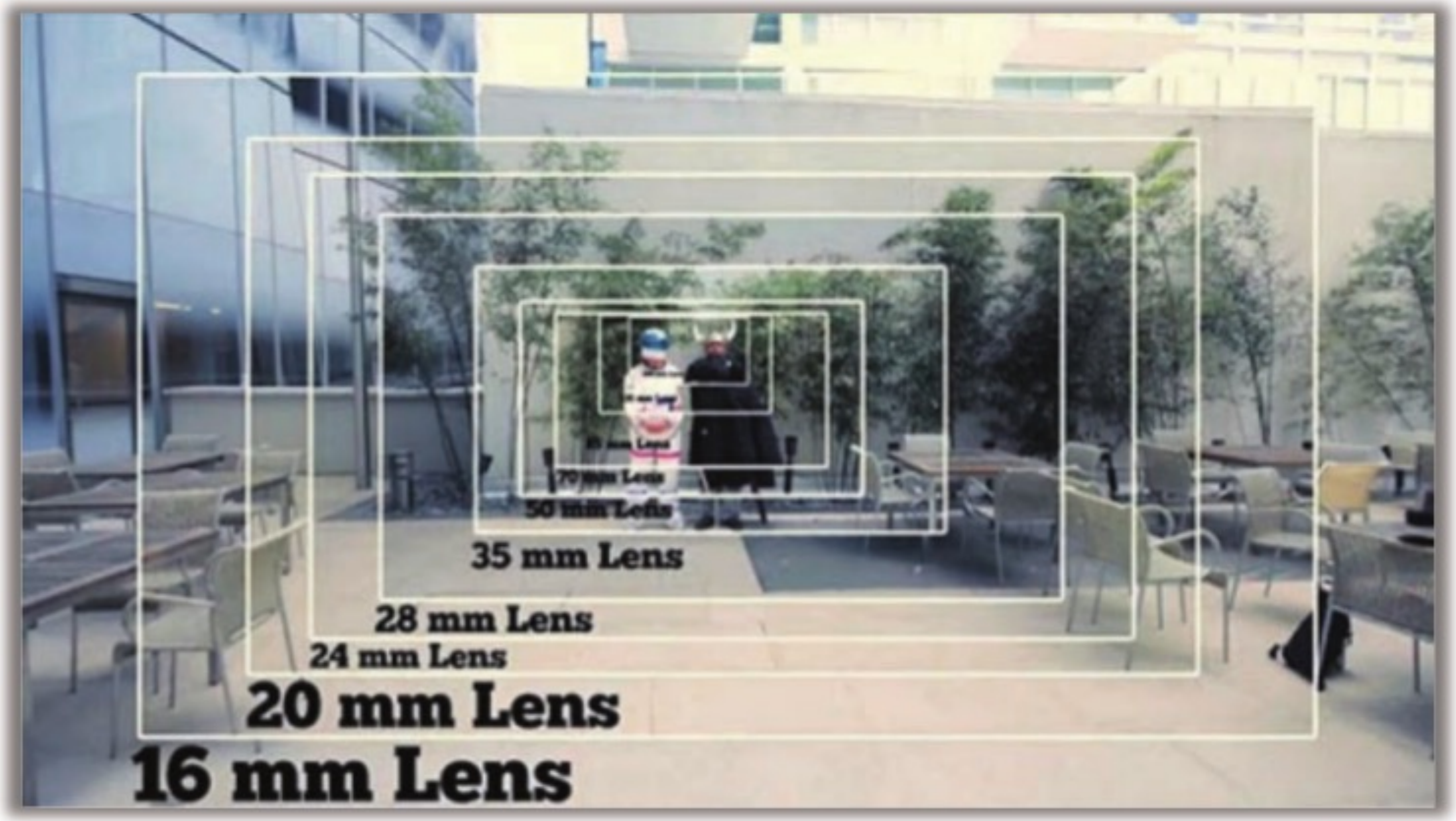
# Zooming and field of view




- When we increase lens focal length, field of view decreases (we “zoom in”).

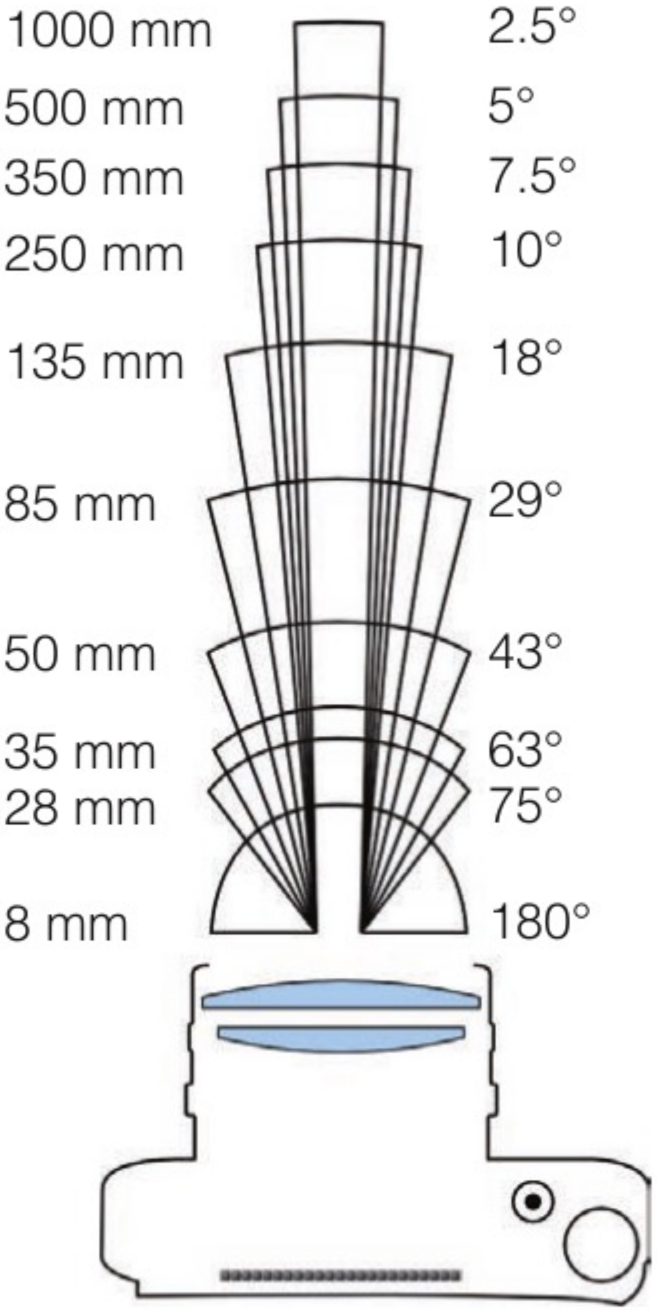
# Field of view



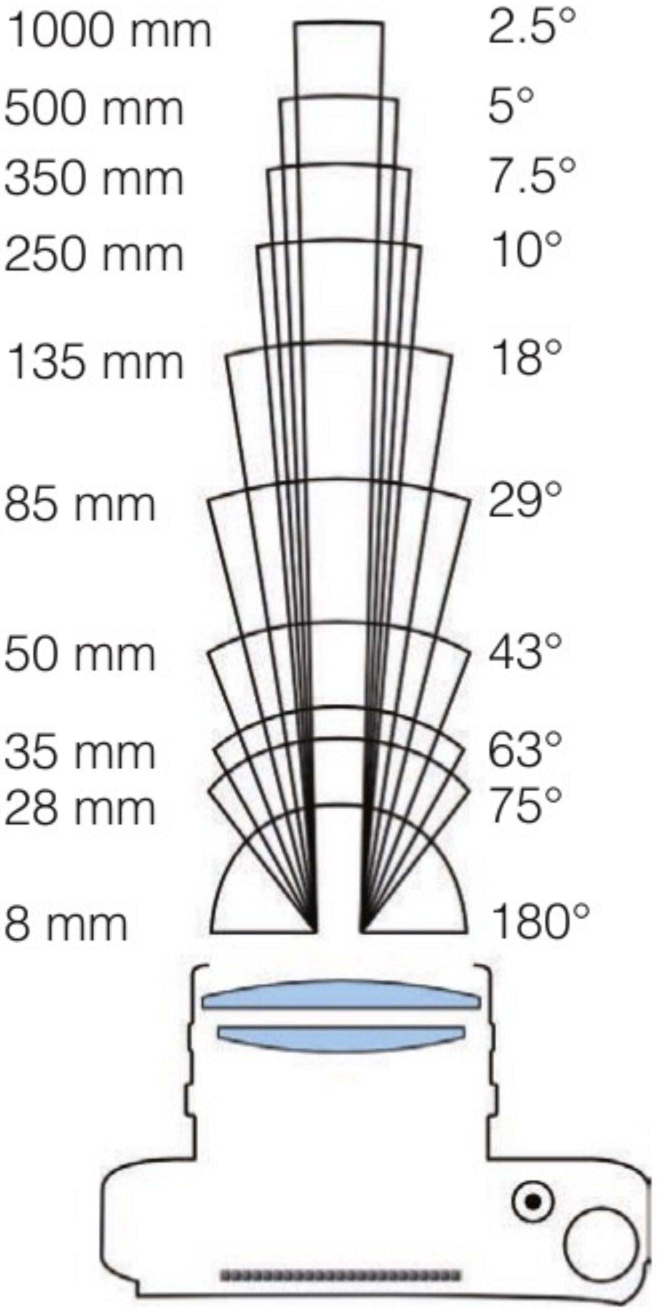



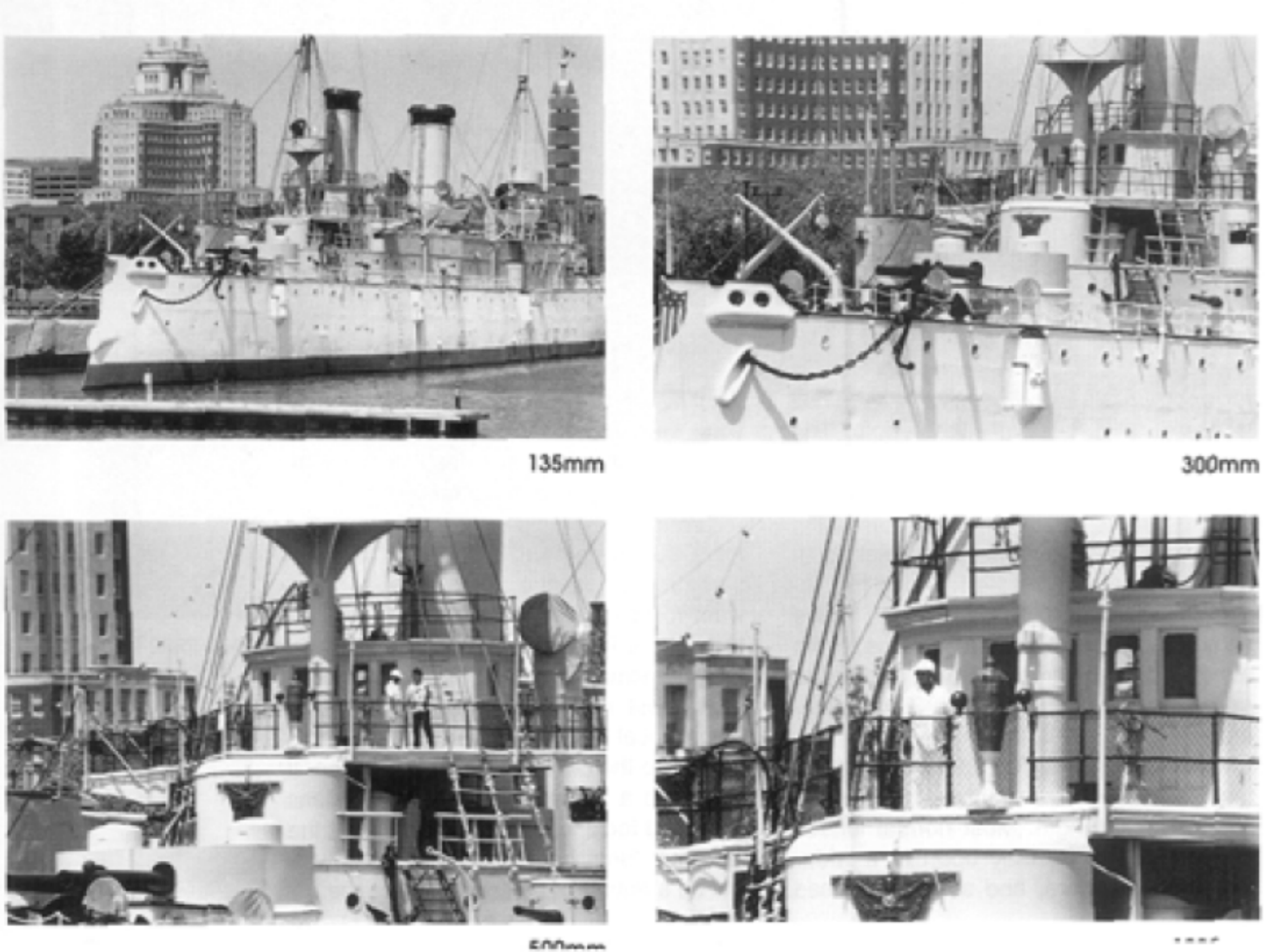

Andrew McWilliams

# Field of view



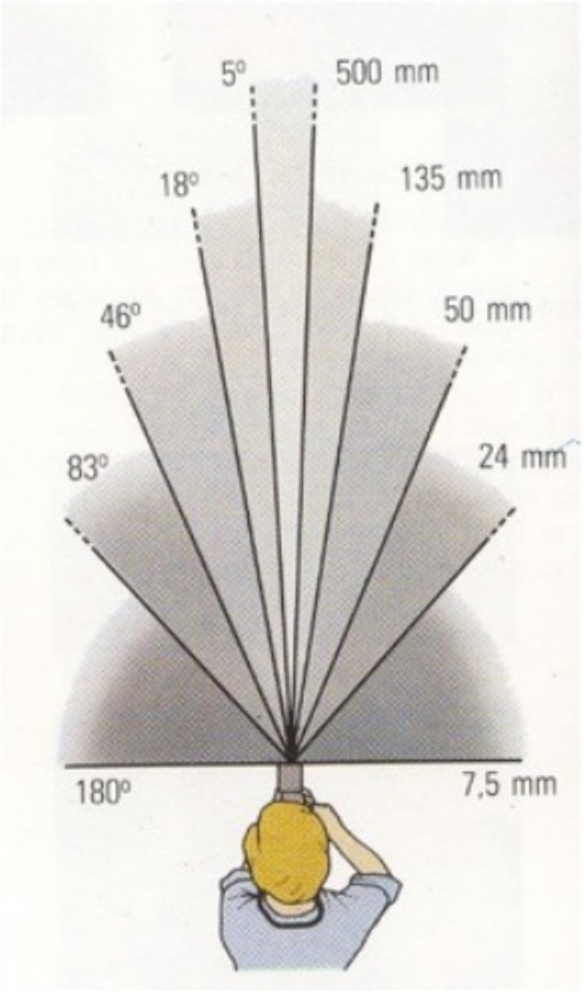

# Field of view



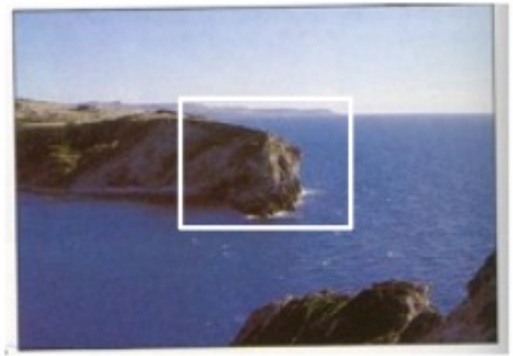
# Field of view

Increasing the <u>lens</u> focal length is similar to cropping




f = 25 mm

f = 50 mm



f = 135 mm

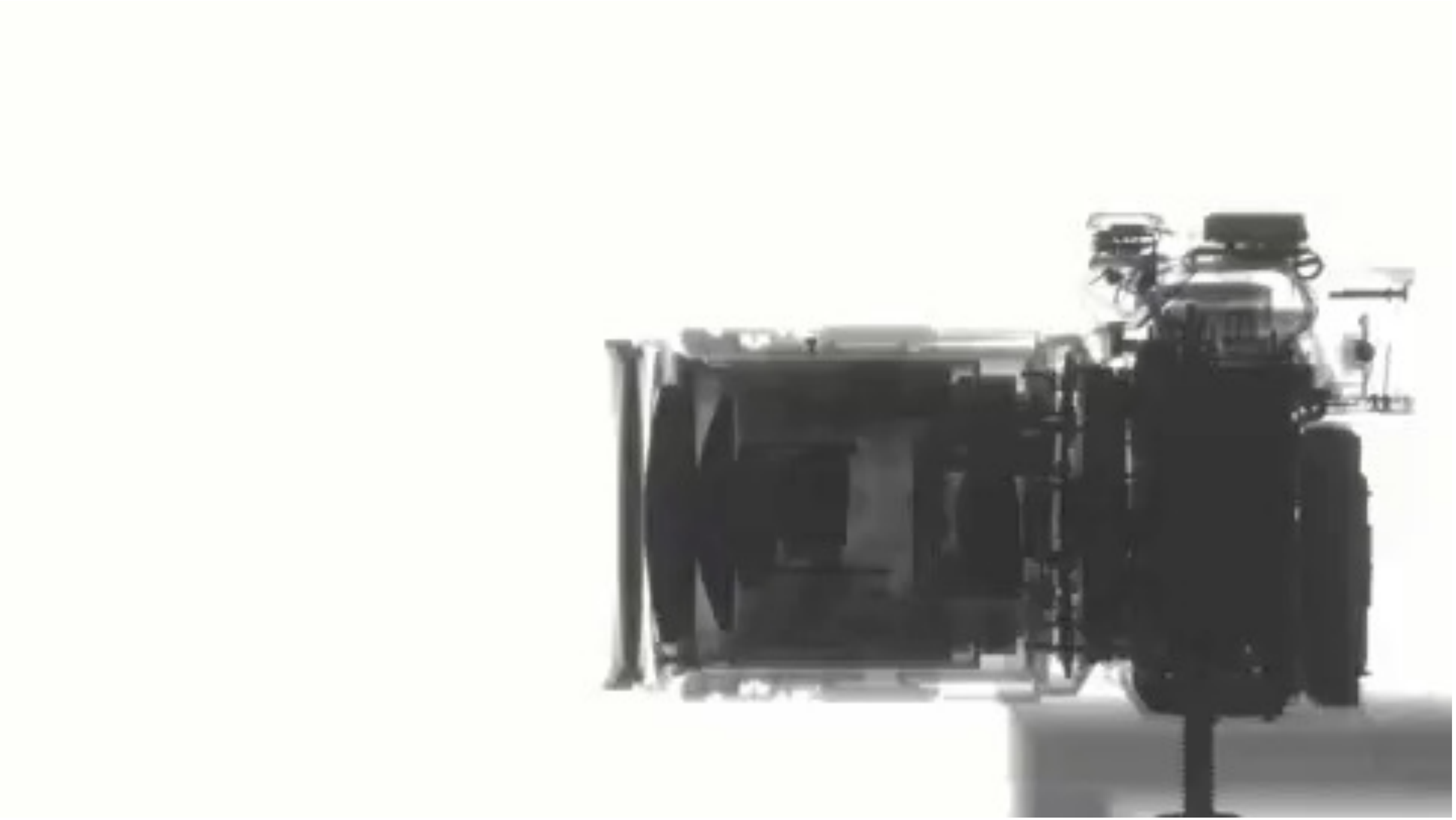
Is this effect identical to cropping?

# The lens on your camera

Focus ring: controls distance of lens from sensor


Zoom ring: controls focal length of lens

# The lens on your camera



Credit: P. Milanfar

# Focusing versus zooming

When you turn the focus ring to bring lens further-away from the sensor:
1. The in-focus distance decreases (you need to get closer to object).
2. The field of view decreases (you see a smaller part of the object).
3. The magnification increases (same part of the object is bigger on sensor).

When you turn the zoom ring to decrease the focal length of the lens:
1. The in-focus distance increases (you need to move away from the object).
2. The field of view increases (you see a larger part of the object).
3. The magnification decreases (same part of the object is smaller on sensor).

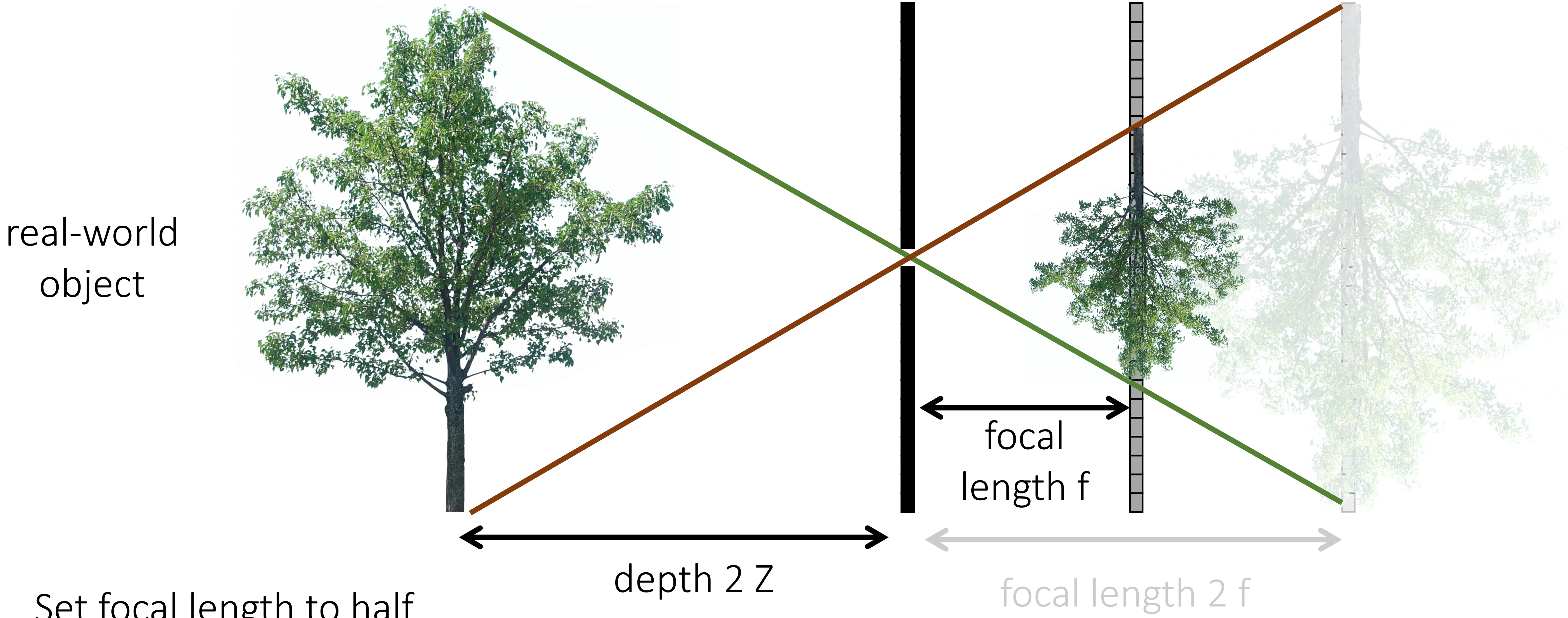
# Focusing versus zooming

When you turn the focus ring to bring lens further-away from the sensor:
1. The in-focus distance decreases (you need to get closer to object).
2. The field of view decreases (you see a smaller part of the object).
3. The magnification increases (same part of the object is bigger on sensor).

When you turn the zoom ring to decrease the focal length of the lens:
1. The in-focus distance increases (you need to move away from the object).
2. The field of view increases (you see a larger part of the object).
3. The magnification decreases (same part of the object is smaller on sensor).

We can use both focus and zoom to cancel out their effects.

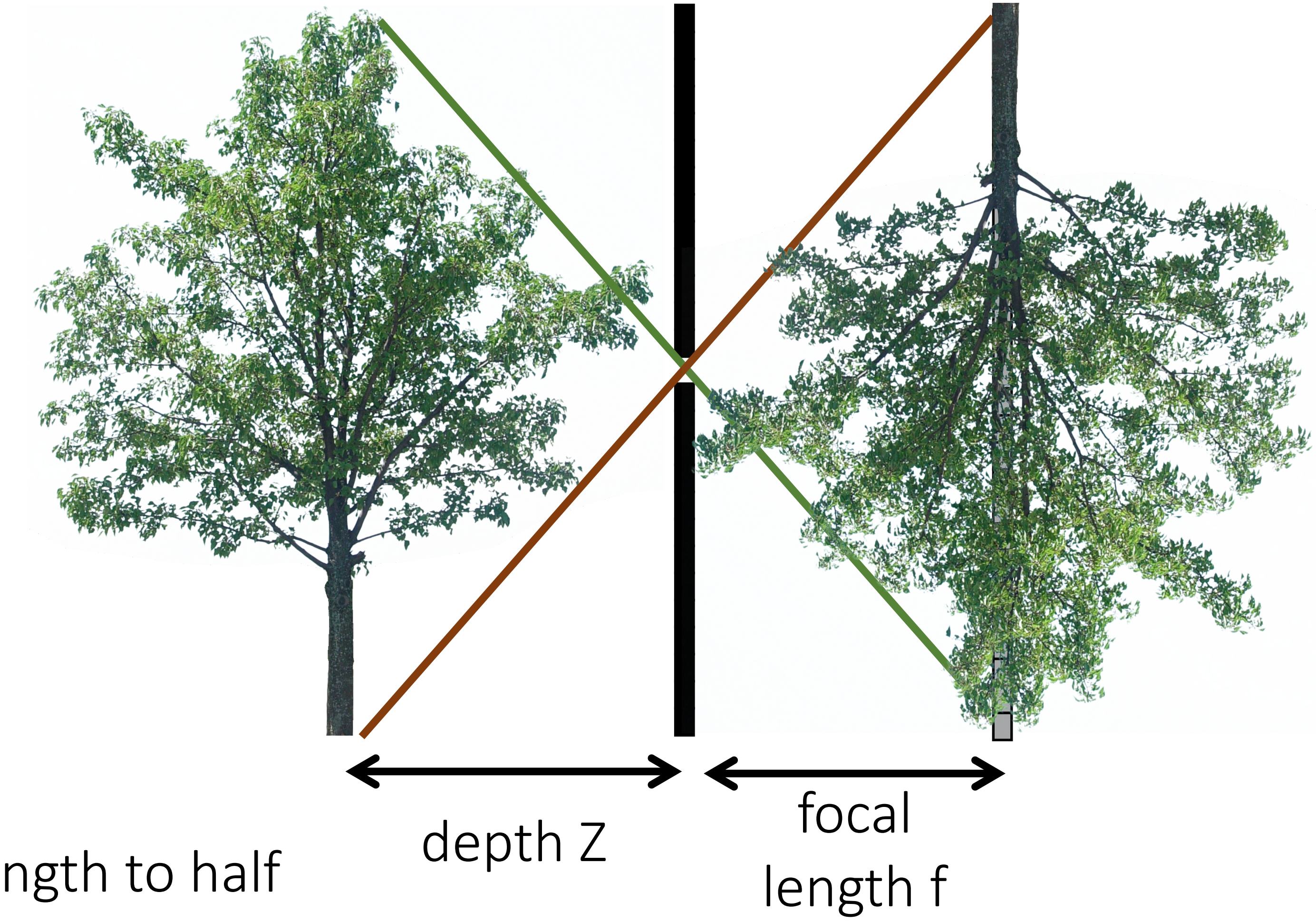
# What if…




1. Set focal length to half

# What if…

real-world
object



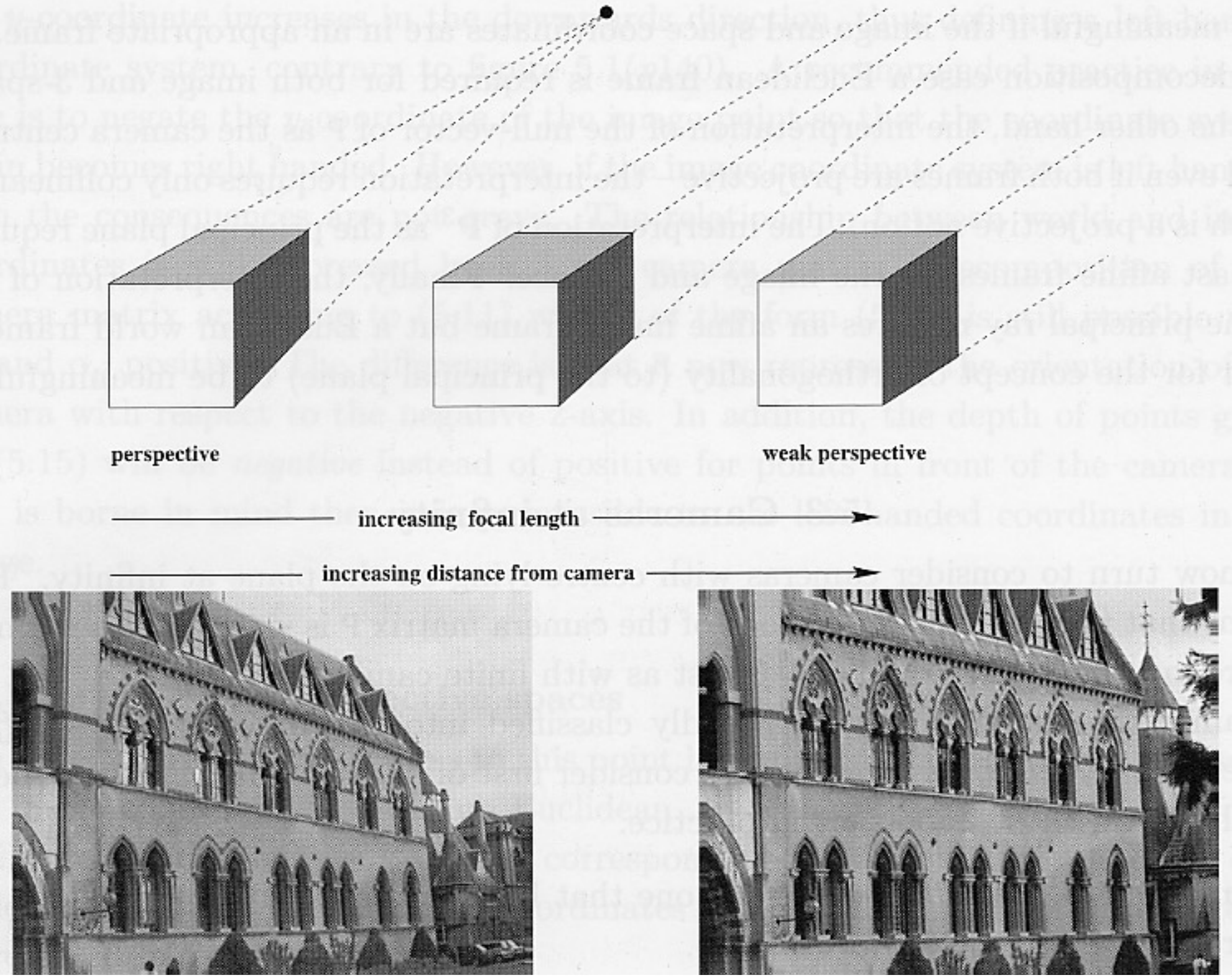

Is this the same image as the one we had at focal length 2f and distance 2Z?

1. Set focal length to half
2. Set depth to half

Similar construction can be done with lenses, after taking care of refocusing.

# Perspective distortion

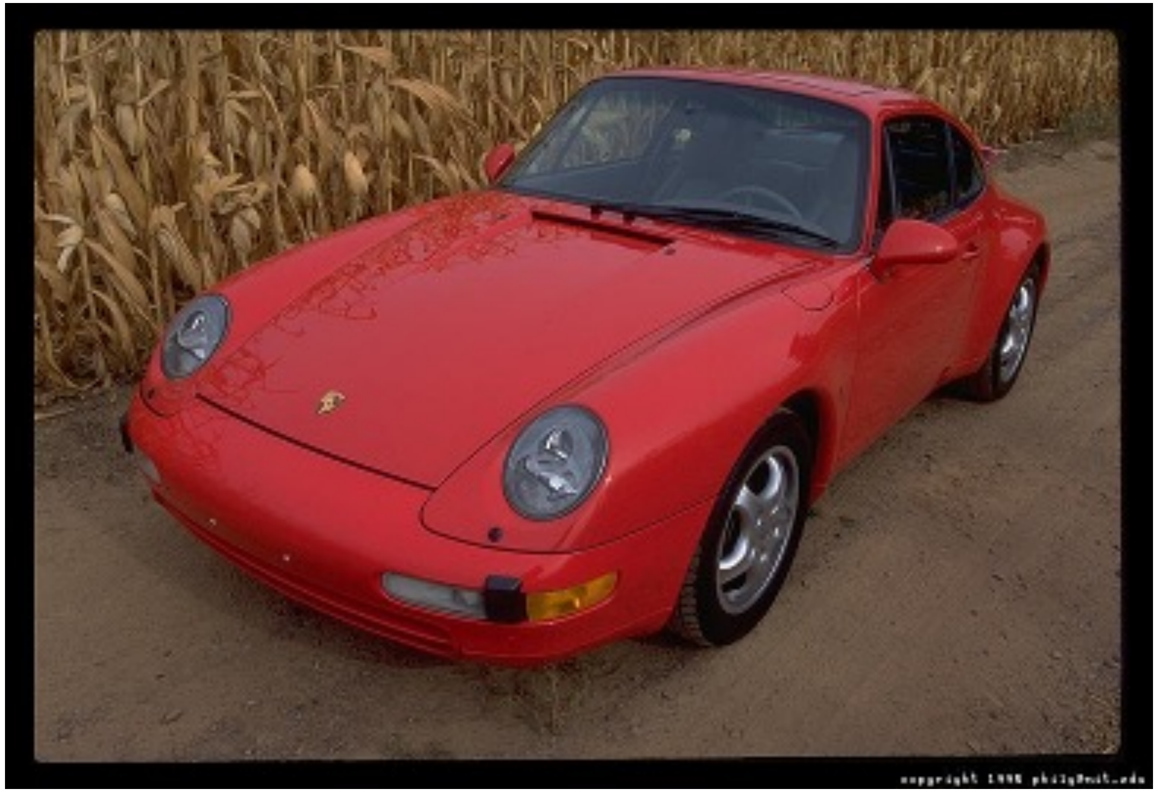
- Field of view affects perspective effects!




From Zisserman & Hartley

# Perspective distortion

Large FOV, small f
Camera close to car



Small FOV, large f
Camera far from the car

# Perspective distortion

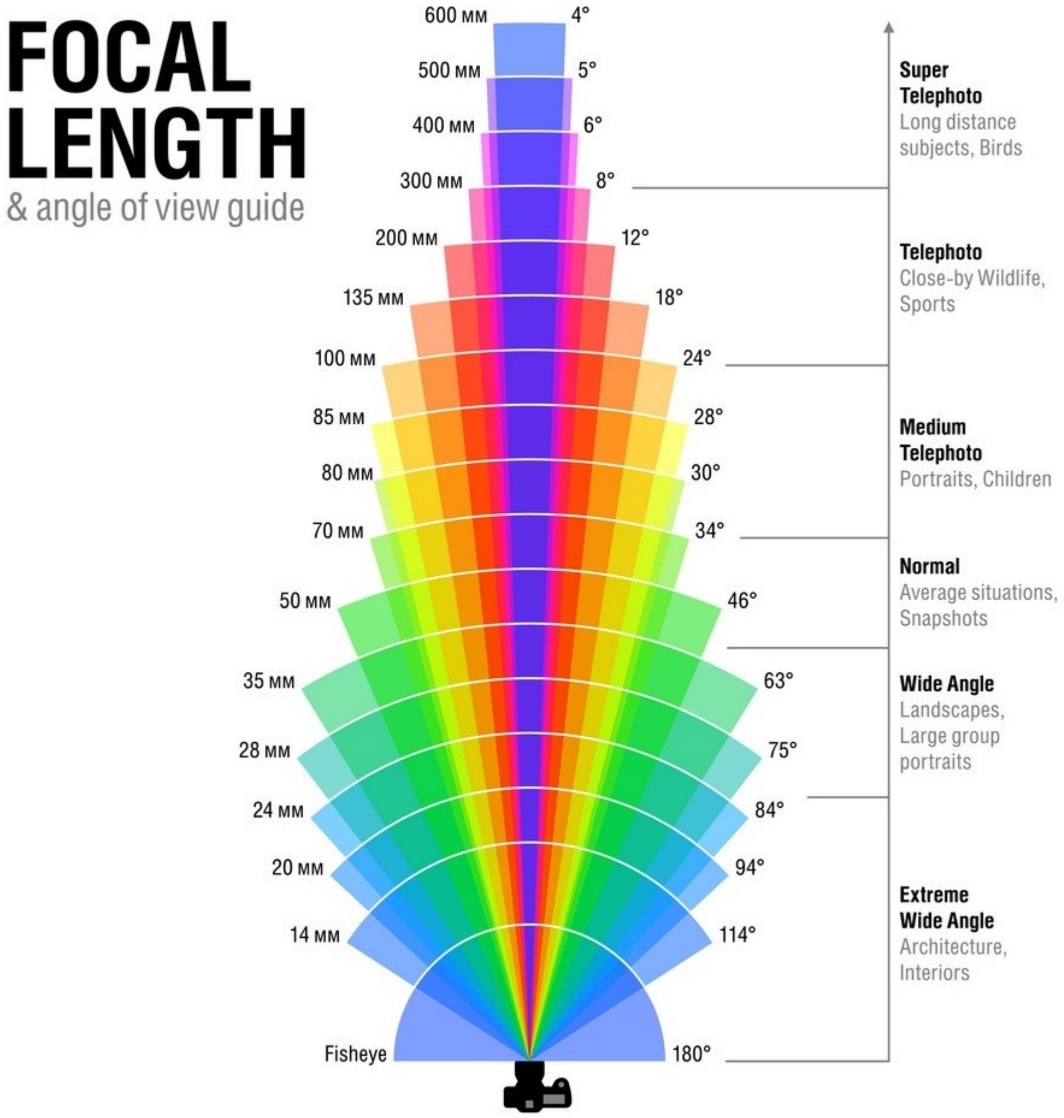
long focal length

mid focal length

short focal length

# Perspective distortion

# Perspective distortion





Credit: P. Milanfar

# What is the best focal length for portraits?

That’s like asking which is better, vi or emacs…

long focal length

mid focal length

short focal length

# Vertigo effect

Named after Alfred Hitchcock's movie
- also known as "dolly zoom"

# Vertigo effect

How would you create this effect?

# Long focal length

When the focal length is long,
the field of view becomes very small
and the resulting image appears
more flat.

Credit: P. Milanfar

# Long focal length

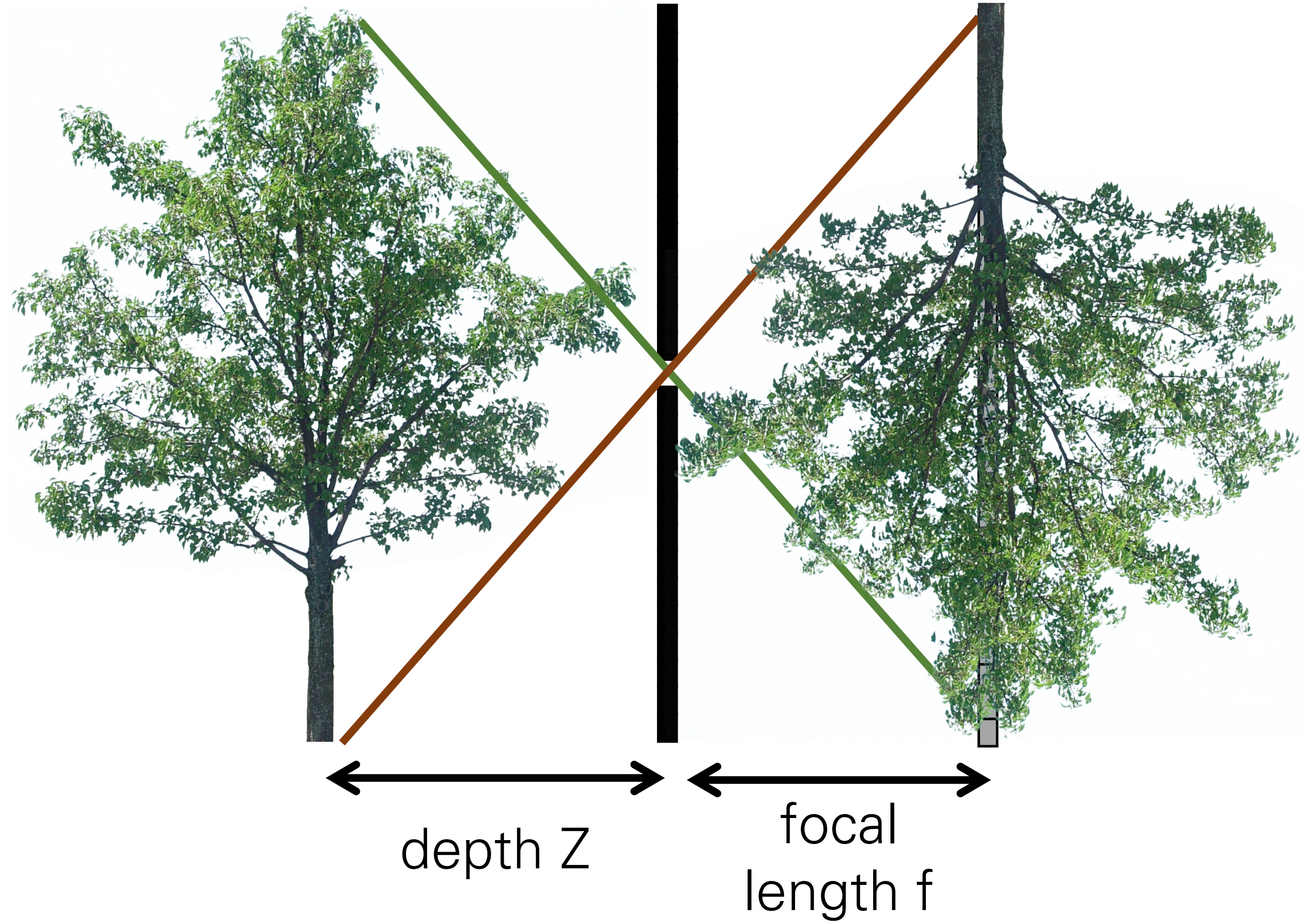
Here's another example:
Empire State building and the Statue of Liberty are about 4.5 miles apart, and the former is 5x taller.

Credit: P. Milanfar

# Orthographic camera and telecentric lenses

# What if...

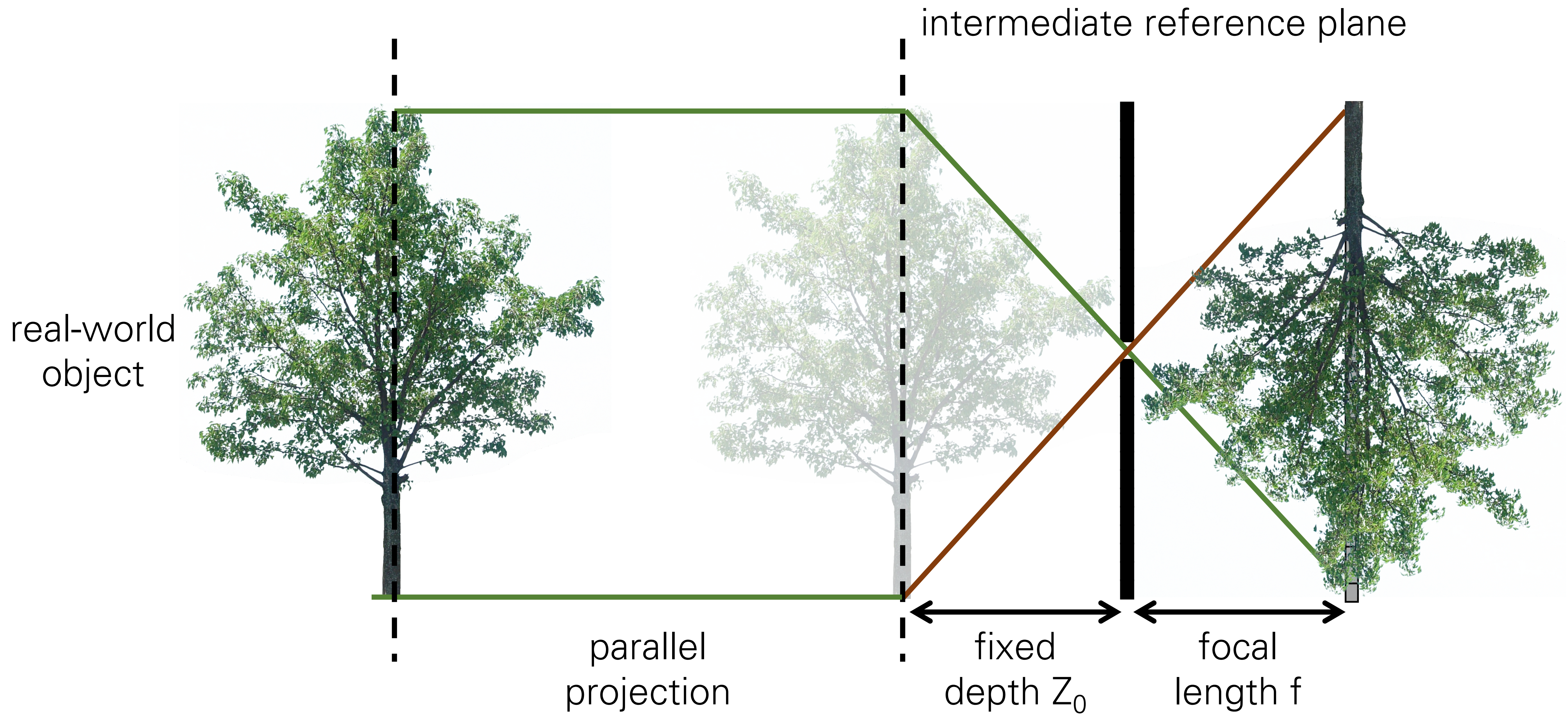
real-world object




Continue increasing Z and f while maintaining same magnification?

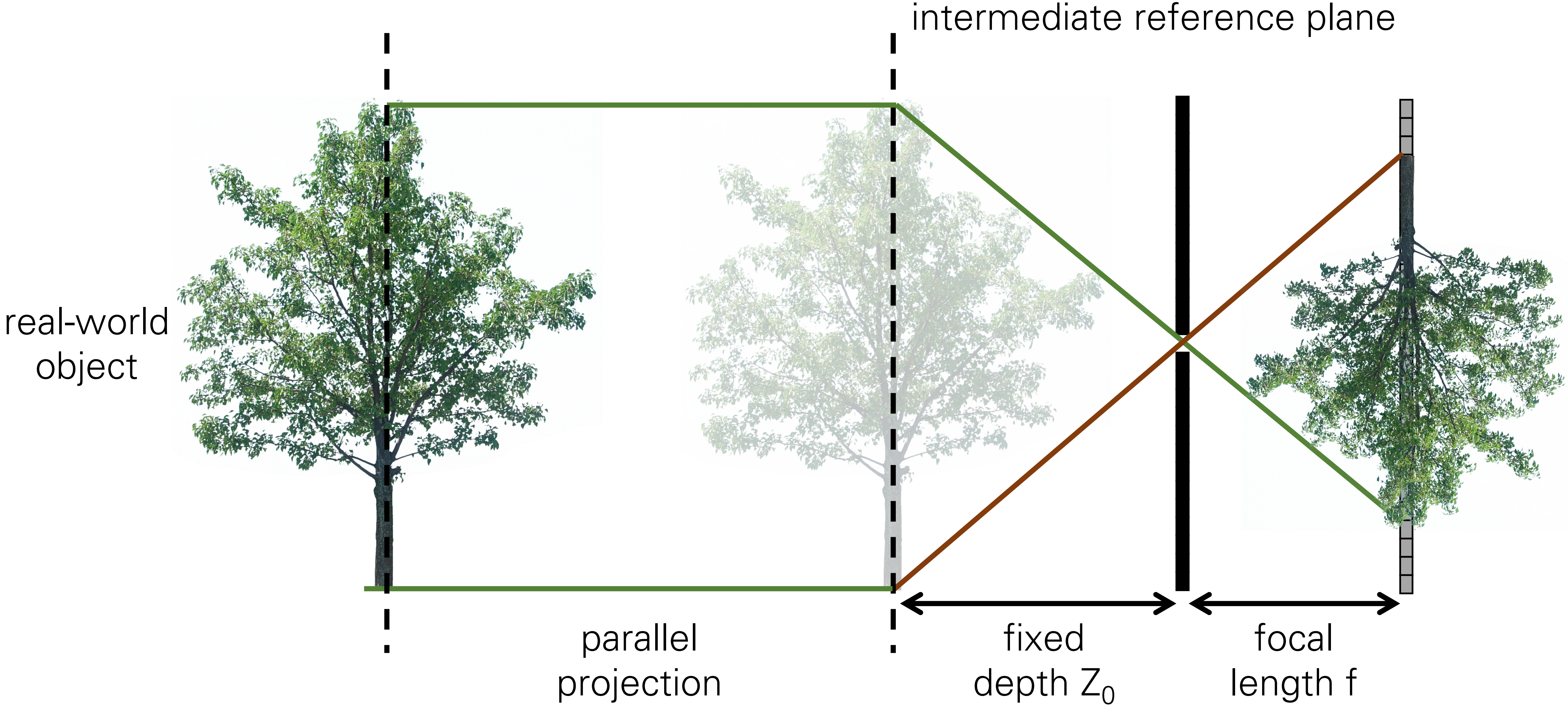
$$f \to \infty \text{ and } \frac{f}{Z} = \text{constant}$$

# Orthographic camera




Depth-independent magnification m = 1 (real-life size).

# Weak-perspective camera

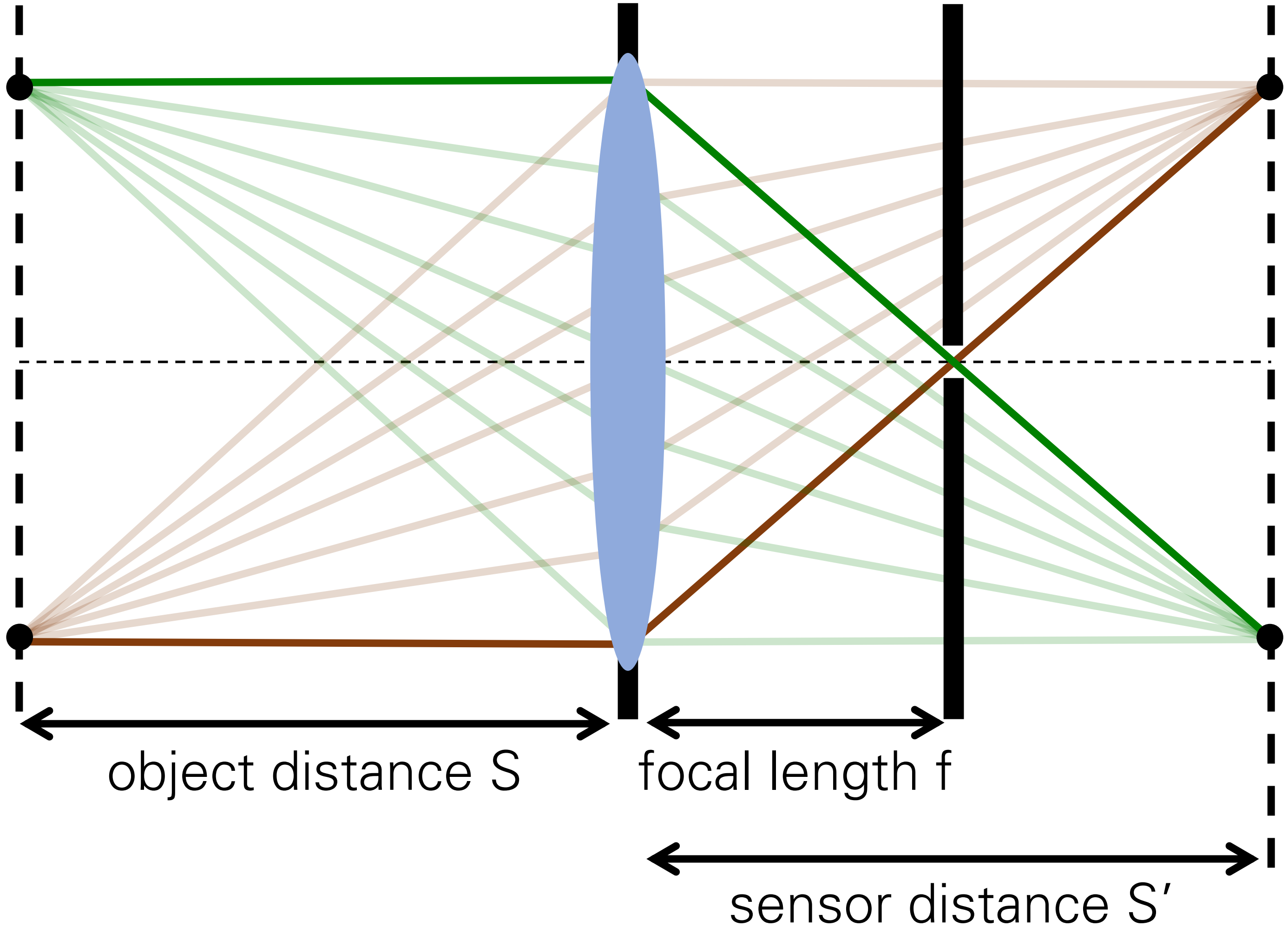



Depth-independent magnification $m = f / Z_0$.

# How can we implement such a camera with lenses?

# Telecentric lens

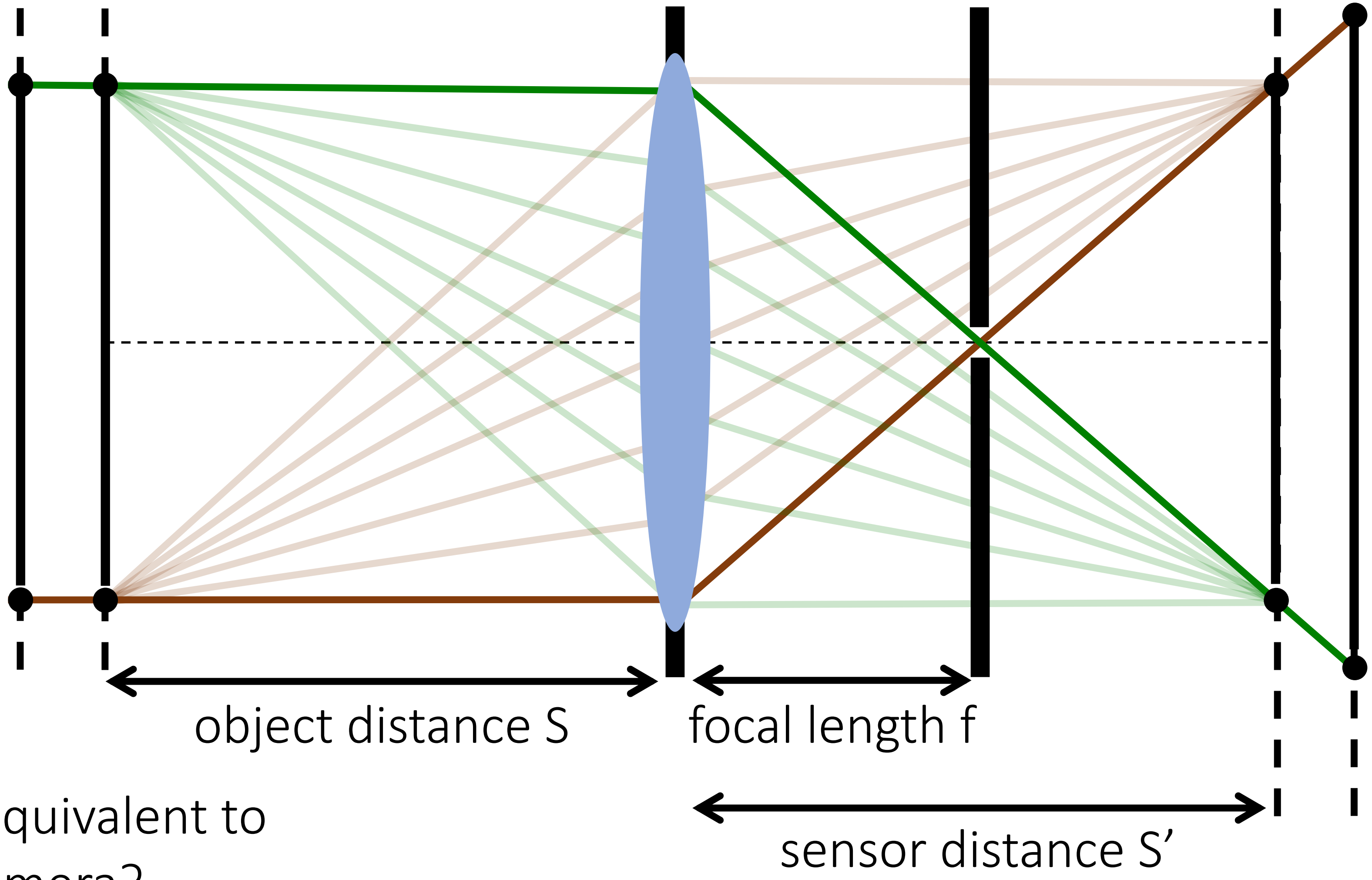
Place a pinhole at focal length, so that only rays parallel to primary ray pass through.



# Telecentric lens

Place a pinhole at focal length, so that only rays parallel to primary ray pass through.

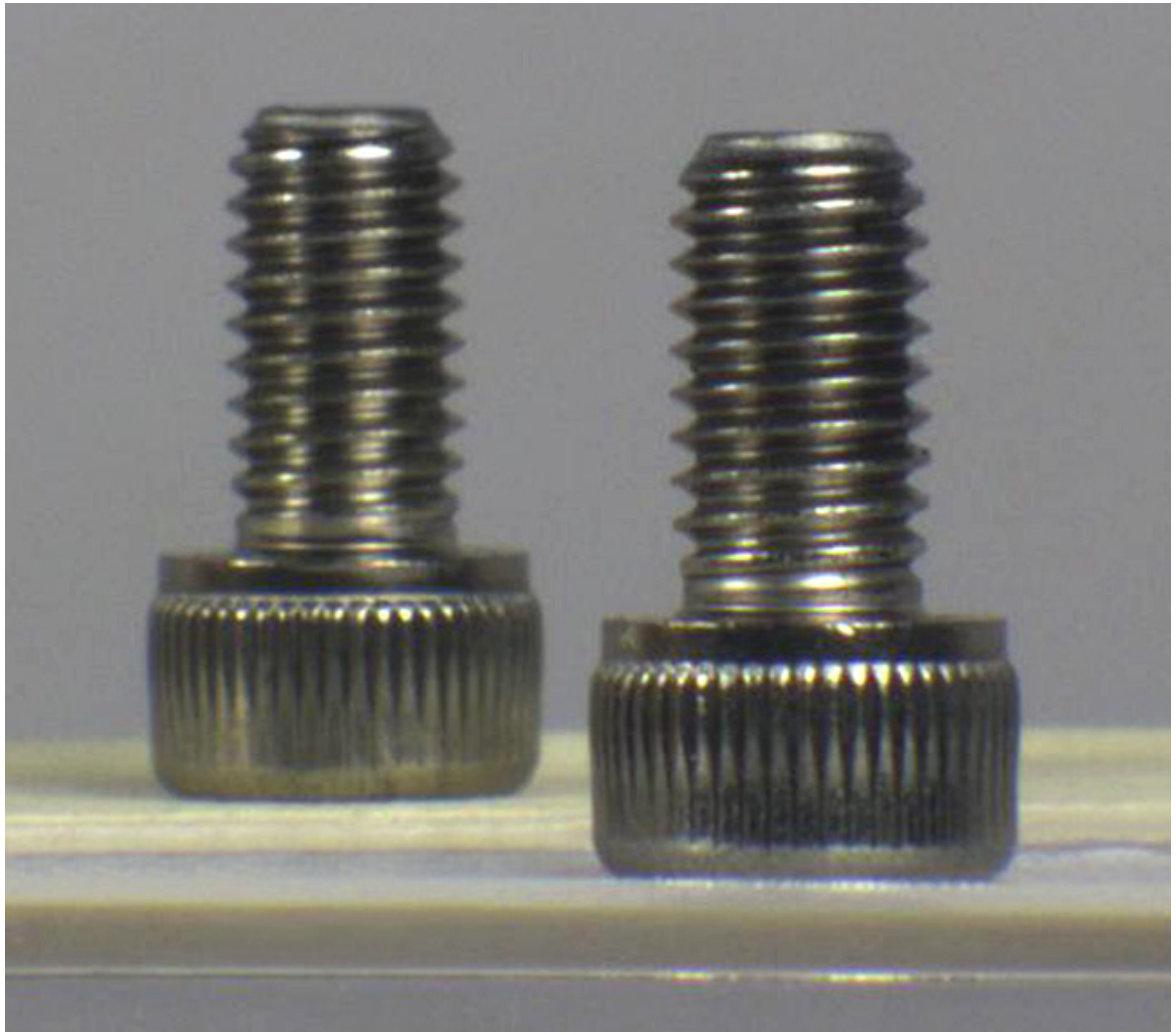
Magnification independent of object depth.



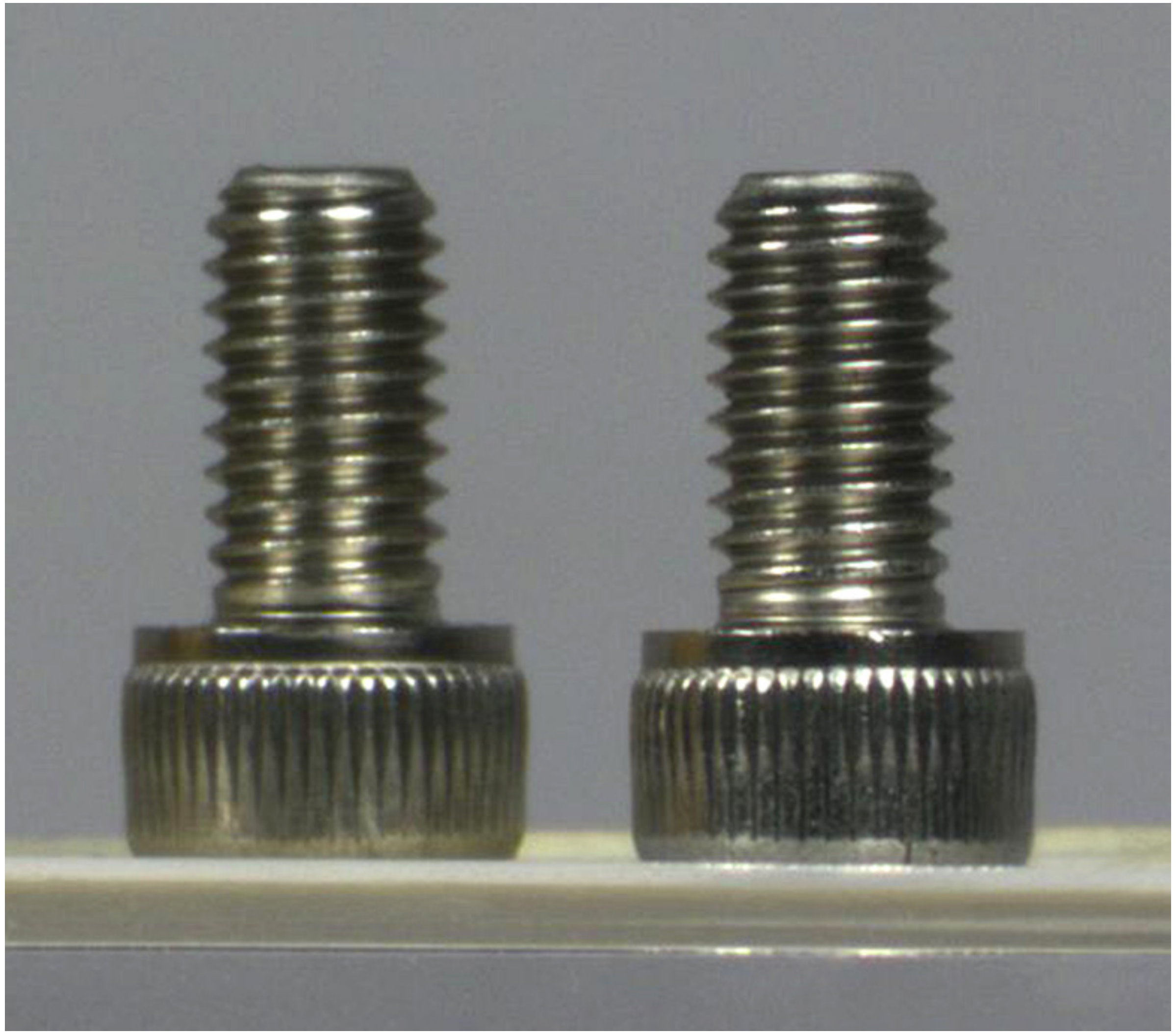

Magnification depends only on sensor-lens distance S’.

When is this lens equivalent to an orthographic camera?

# Regular vs telecentric lens



regular lens



telecentric lens

# Next Lecture:
Noise and Color