BBM 413 Fundamentals of Image Processing

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Point Operations
Histogram Processing

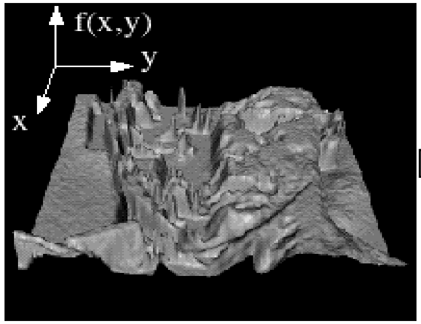
Today's topics

- Point operations
- Histogram processing

Digital images

- Sample the 2D space on a regular grid
- Quantize each sample (round to nearest integer)

- Image thus represented as a matrix of integer values.

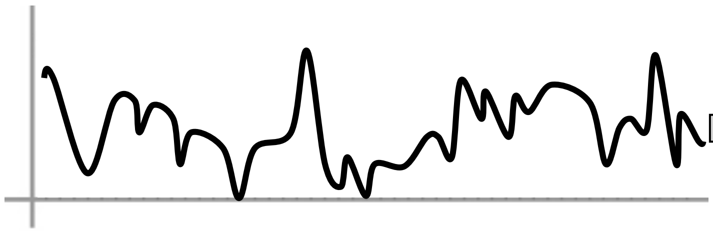


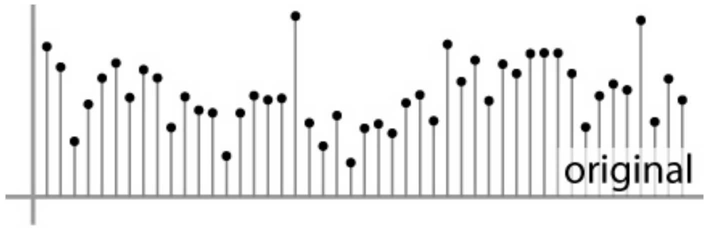
j

i

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

2D





1D

Slide credit: K. Grauman, S. Seitz

Image Transformations

- $g(x,y)=T[f(x,y)]$

$g(x,y)$: output image

$f(x,y)$: input image

T: transformation function

1. Point operations: operations on single pixels
2. Spatial filtering: operations considering pixel neighborhoods
3. Global methods: operations considering whole image

Point Operations

- Smallest possible neighborhood is of size 1x1
- Process each point independently of the others
- Output image *g* depends only on the value of *f* at a single point (x,y)
- Transformation function *T* remaps the sample's value:

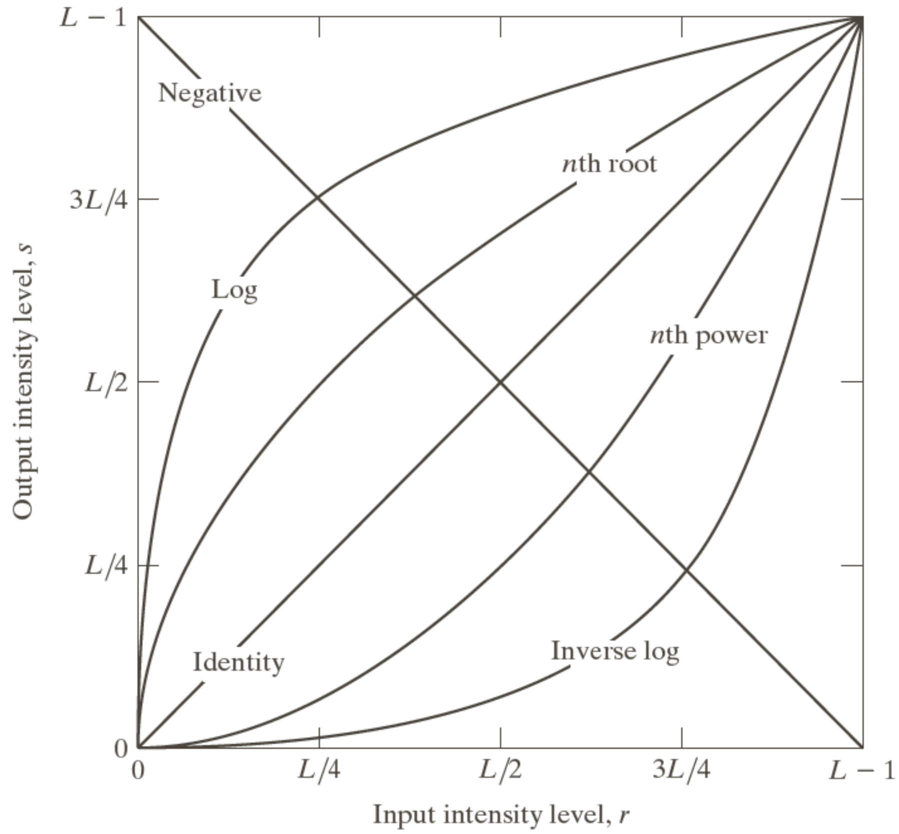
$$s = T(r)$$

where

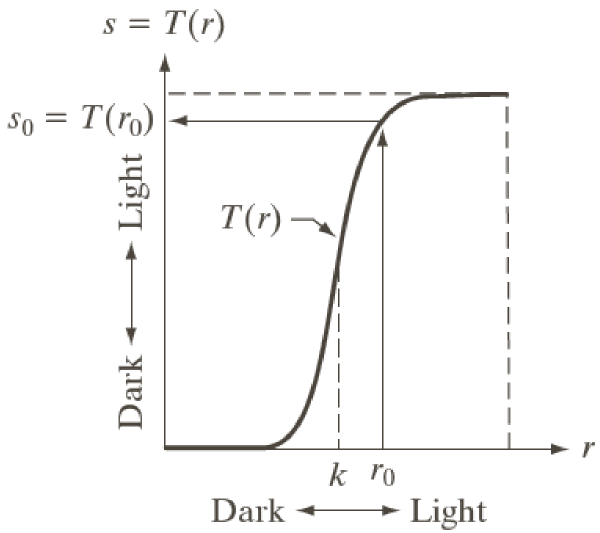
- *r* is the value at the point in question
- *s* is the new value in the processed result
- *T* is a *intensity transformation* function

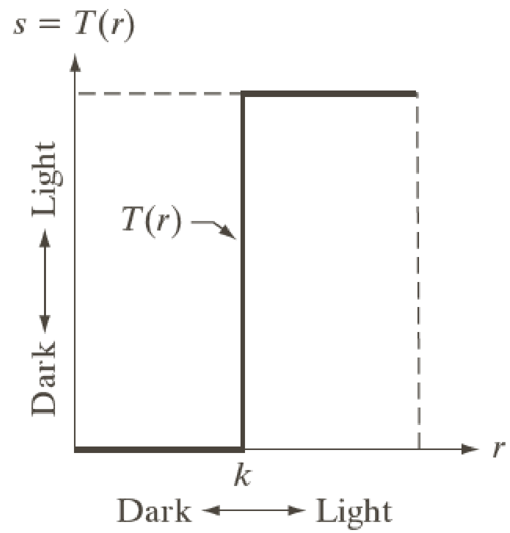
Sample intensity transformation functions

- Image negatives
- Log transformations
  - Compresses the dynamic range of images
- Power-law transformations
  - Gamma correction



Point Processing Examples



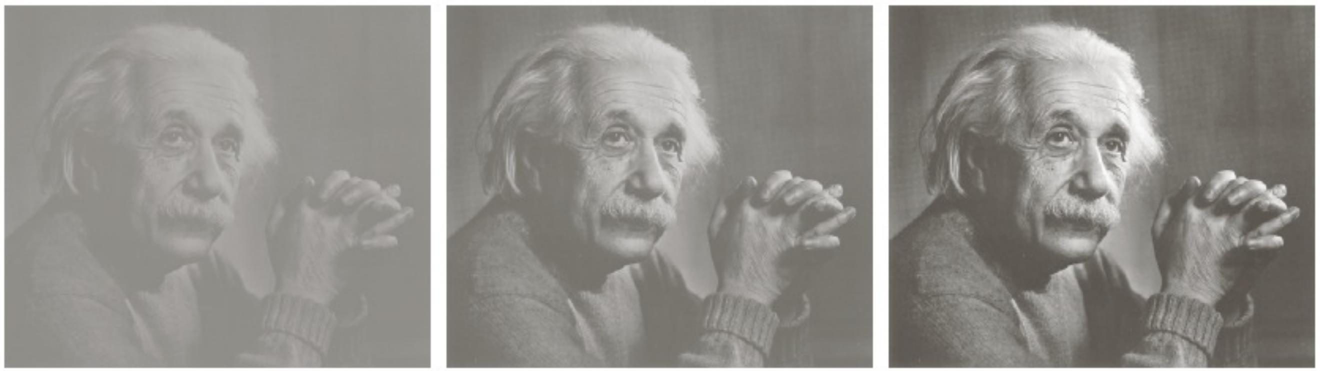


produces an image of higher contrast than the original by darkening the intensity levels below k and brightening intensities above k

produces a binary (two-intensity level) image

Dynamic range

- Dynamic range $R_d = I_{max} / I_{min}$, or $(I_{max} + k) / (I_{min} + k)$
  - determines the degree of image contrast that can be achieved
  - a major factor in image quality
- Ballpark values
  - Desktop display in typical conditions: 20:1
  - Photographic print: 30:1
  - High dynamic range display: 10,000:1



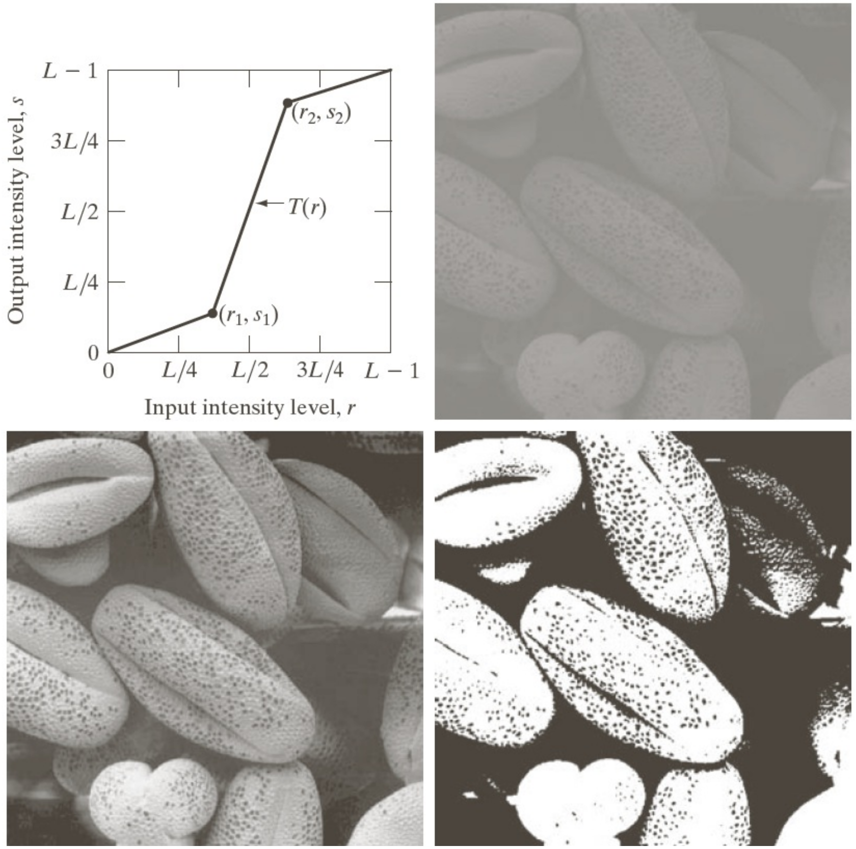
low contrast medium contrast high contrast

Slide credit: S. Marschner

Point Operations: Contrast stretching and Thresholding

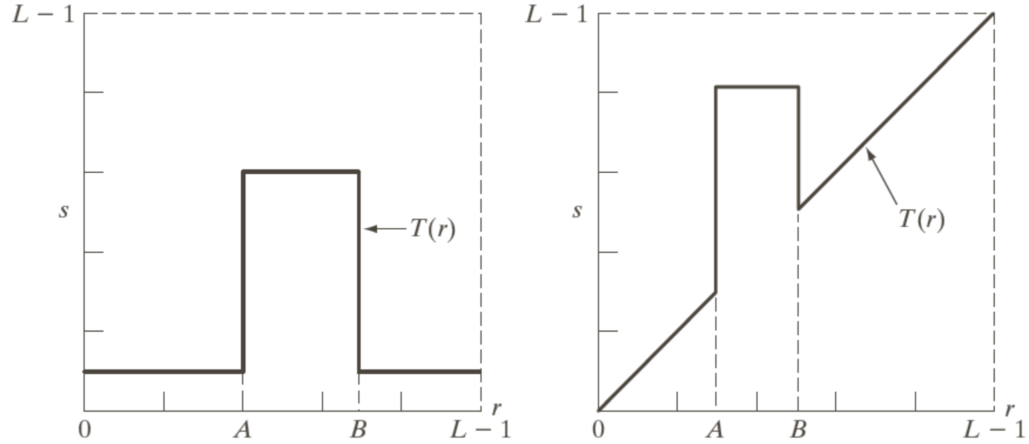
- Contrast stretching: produces an image of higher contrast than the original

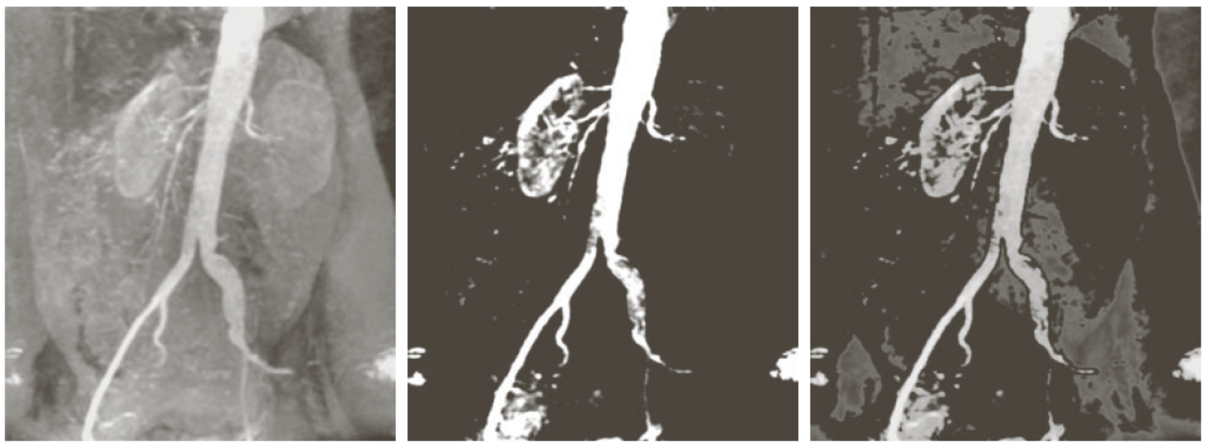
- Thresholding: produces a binary (two-intensity level) image



Point Operations: Intensity-level Slicing

- highlights a certain range of intensities





Intensity encoding in images

- Recall that the pixel values determine how bright that pixel is.
- Bigger numbers are (usually) brighter
- *Transfer function*: function that maps input pixel value to luminance of displayed image

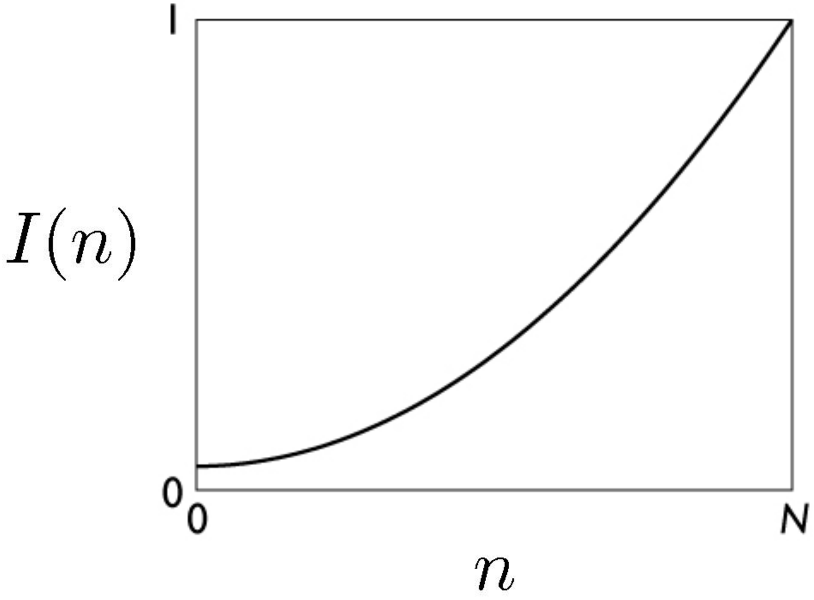
$$I = f(n) \quad f : [0, N] \to [I_{\min}, I_{\max}]$$

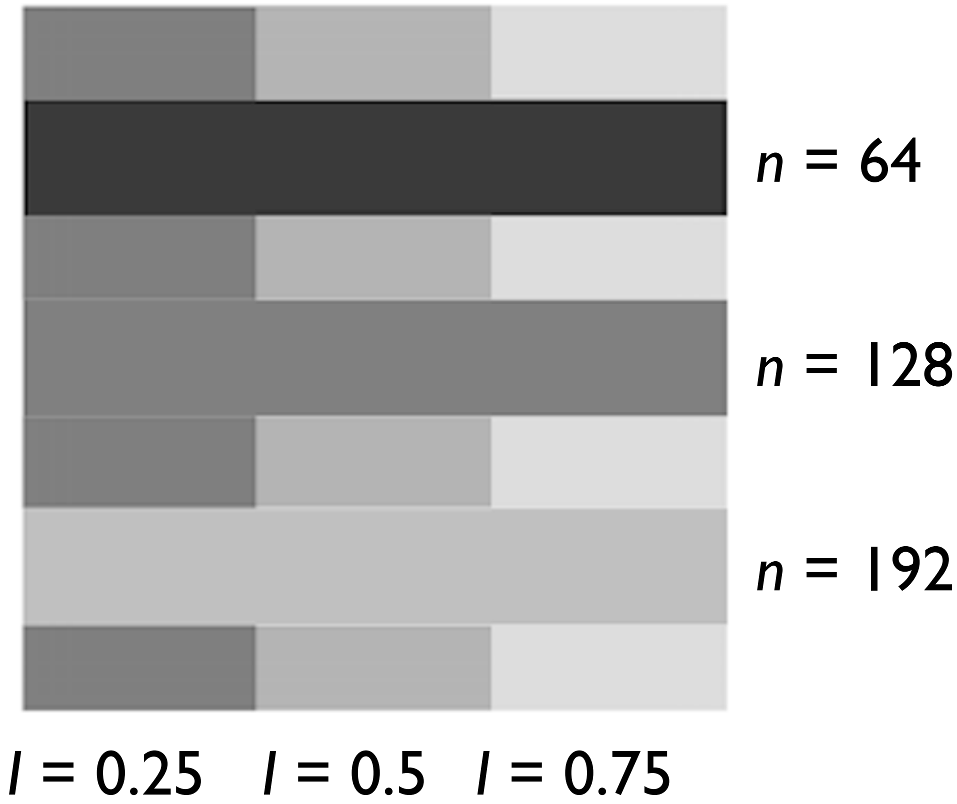
- What determines this function?
  - physical constraints of device or medium
  - desired visual characteristics

adapted from: S. Marschner

What this projector does?

- Something like this:





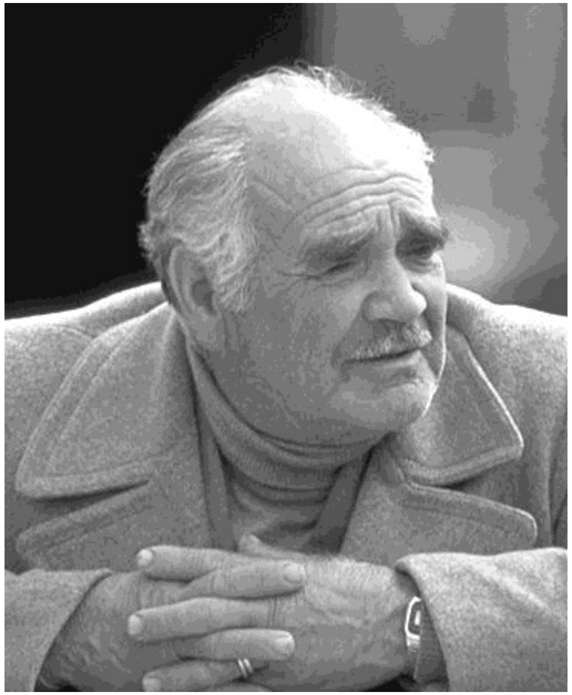
adapted from: S. Marschner

Constraints on transfer function

- Maximum displayable intensity, I_{max}
  - how much power can be channeled into a pixel?
    - LCD: backlight intensity, transmission efficiency (<10%)
    - projector: lamp power, efficiency of imager and optics
- Minimum displayable intensity, I_{min}
  - light emitted by the display in its "off" state
    - *e.g.* stray electron flux in CRT, polarizer quality in LCD
- Viewing flare, k: light reflected by the display
  - very important factor determining image contrast in practice
    - 5% of I_{max} is typical in a normal office environment [sRGB spec]
    - much effort to make very black CRT and LCD screens
    - all-black decor in movie theaters

Transfer function shape

- Desirable property: the change from one pixel value to the next highest pixel value should not produce a visible contrast
  - otherwise smooth areas of images will show visible bands
- What contrasts are visible?
  - rule of thumb: under good conditions we can notice a 2% change in intensity
  - therefore we generally need smaller quantization steps in the darker tones than in the lighter tones
  - most efficient quantization is logarithmic



an image with severe *banding*

[Philip Greenspun]

Slide credit: S. Marschner

How many levels are needed?

- Depends on dynamic range
  - 2% steps are most efficient:

$$0 \mapsto I_{\min}; 1 \mapsto 1.02 I_{\min}; 2 \mapsto (1.02)^2 I_{\min}; \ldots$$

  - log 1.02 is about 1/120, so 120 steps per decade of dynamic range
    - 240 for desktop display
    - 360 to print to film
    - 480 to drive HDR display
- If we want to use linear quantization (equal steps)
  - one step must be < 2% (1/50) of $I_{\min}$
  - need to get from ~0 to $I_{\min} \cdot R_d$ so need about 50 R_d levels
    - 1500 for a print; 5000 for desktop display; 500,000 for HDR display
- Moral: 8 bits is just barely enough for low-end applications
  - but only if we are careful about quantization

Slide credit: S. Marschner

Intensity quantization in practice

- Option 1: linear quantization $I(n) = (n/N)\, I_{\max}$
  - pro: simple, convenient, amenable to arithmetic
  - con: requires more steps (wastes memory)
  - need 12 bits for any useful purpose; more than 16 for HDR
- Option 2: power-law quantization $I(n) = (n/N)^{\gamma}\, I_{\max}$
  - pro: fairly simple, approximates ideal exponential quantization
  - con: need to linearize before doing pixel arithmetic
  - con: need to agree on exponent
  - 8 bits are OK for many applications; 12 for more critical ones
- Option 2: floating-point quantization $I(x) = (x/w)\, I_{\max}$
  - pro: close to exponential; no parameters; amenable to arithmetic
  - con: definitely takes more than 8 bits
  - 16–bit "half precision" format is becoming popular

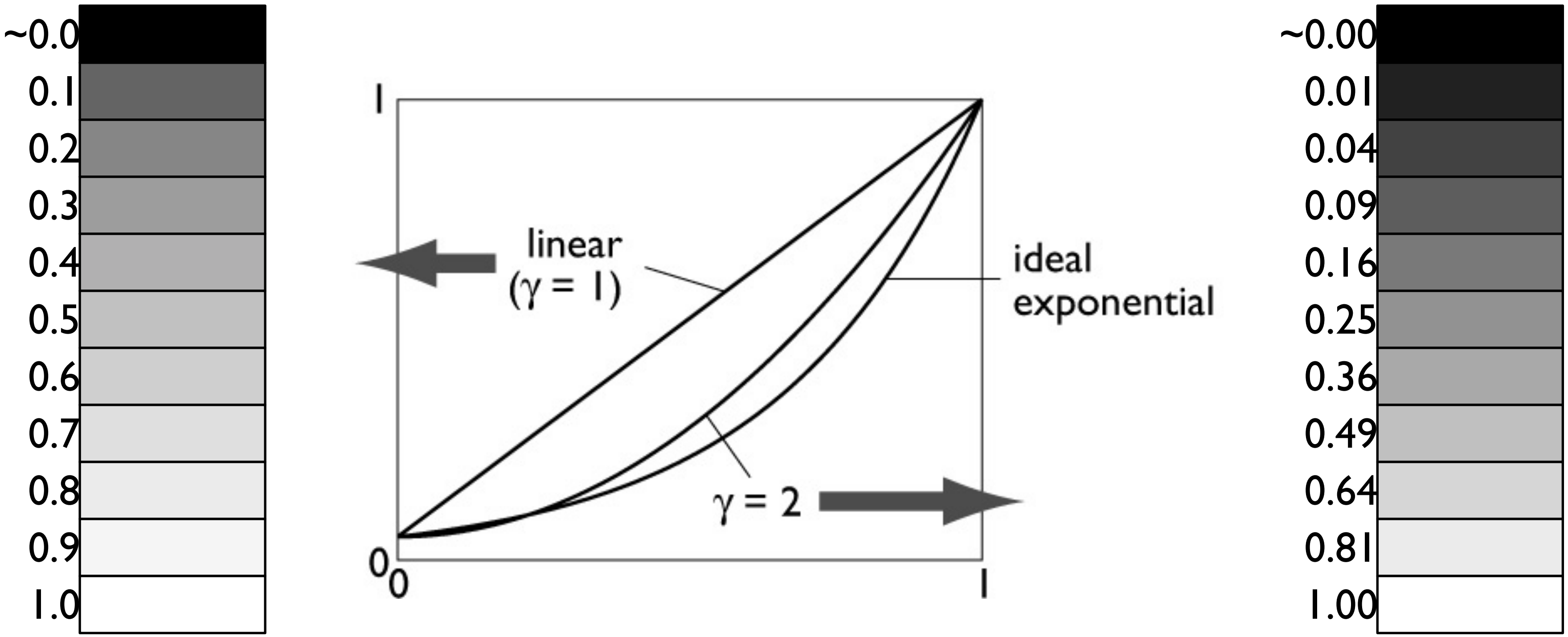
Slide credit: S. Marschner

Why gamma?

- Power-law quantization, or *gamma correction* is most popular
- Original reason: CRTs are like that
  - intensity on screen is proportional to (roughly) voltage^2
- Continuing reason: inertia + memory savings
  - inertia: gamma correction is close enough to logarithmic that there's no sense in changing
  - memory: gamma correction makes 8 bits per pixel an acceptable option

Slide credit: S. Marschner

Gamma quantization



- Close enough to ideal perceptually uniform exponential

Slide credit: S. Marschner

Gamma correction

- Sometimes (often, in graphics) we have computed intensities *a* that we want to display linearly
- In the case of an ideal monitor with zero black level,

  $$I(n) = (n/N)^{\gamma}$$

  (where N = 2^n – 1 in *n* bits). Solving for *n*:

  $$n = Na^{\frac{1}{\gamma}}$$

- This is the "gamma correction" recipe that has to be applied when computed values are converted to 8 bits for output
  - failing to do this (implicitly assuming gamma = 1) results in dark, oversaturated images

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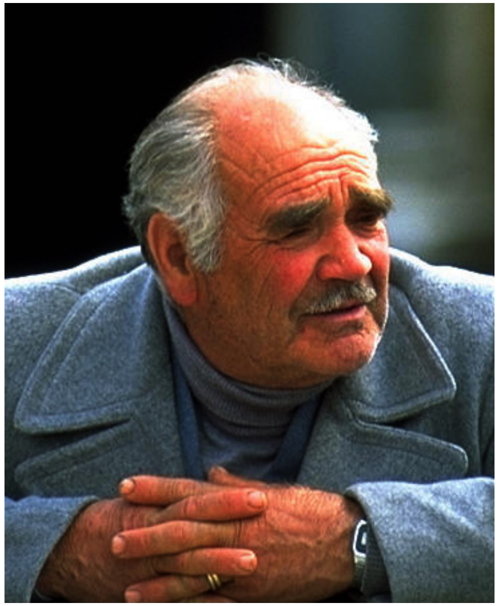
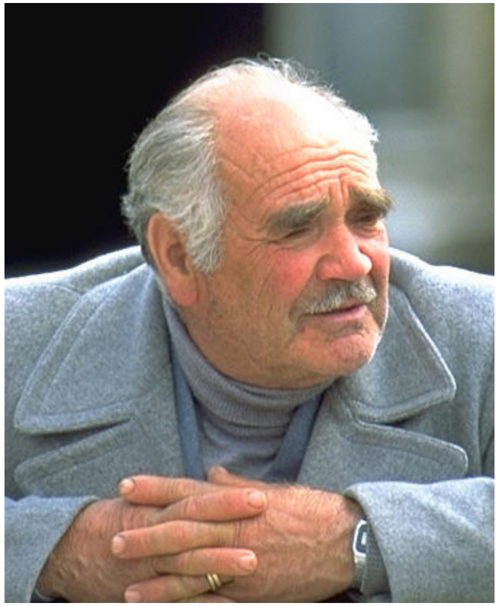
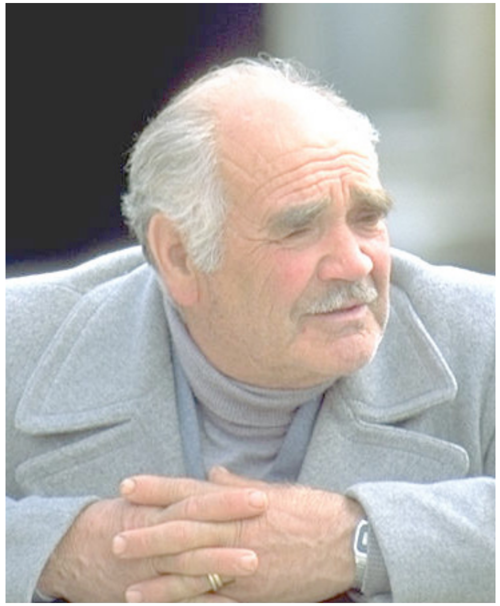
Gamma correction



corrected for γ lower than display



OK



corrected for γ higher than display

[Philip Greenspun]

Slide credit: S. Marschner

Today's topics

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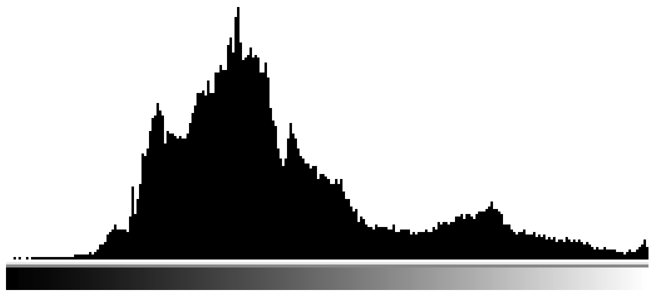
- Histogram processing

Histogram

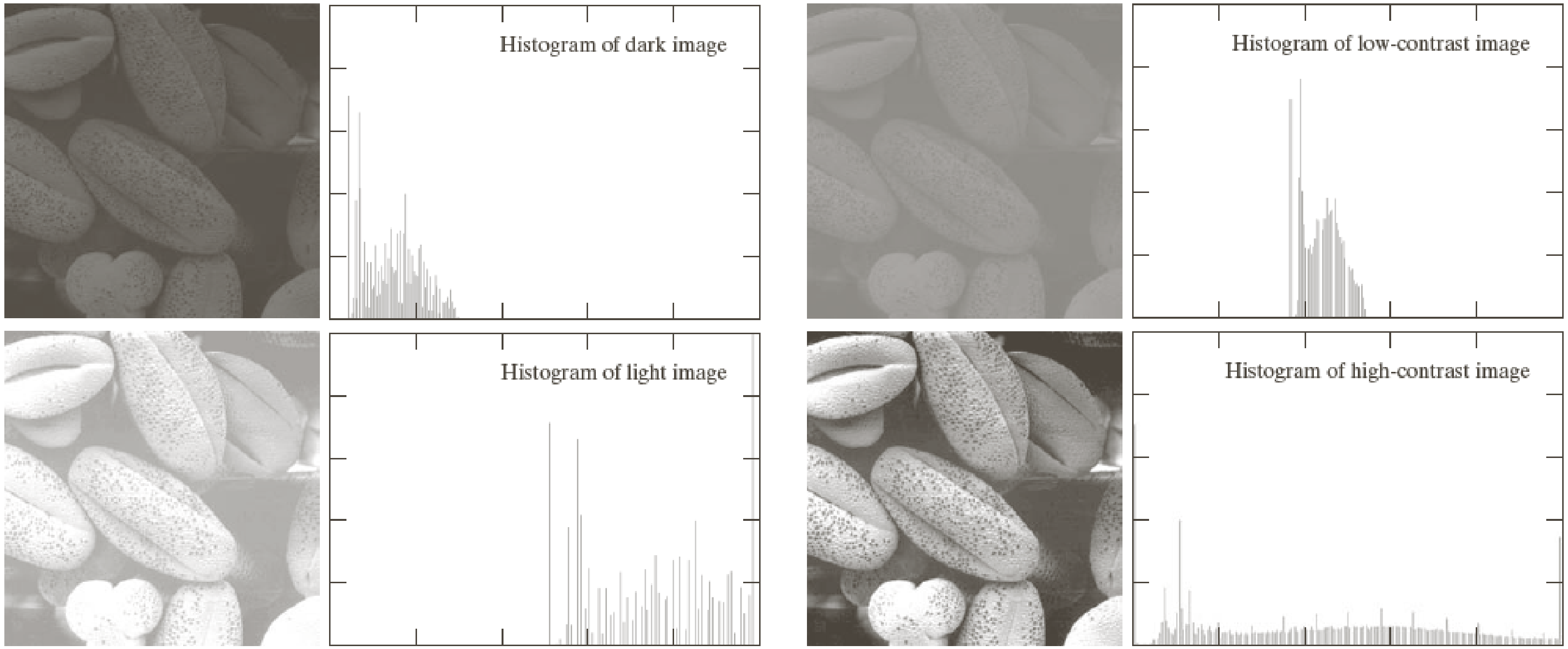
- Histogram: a discrete function *h(r)* which counts the number of pixels in the image having intensity *r*
- If *h(r)* is normalized, it measures the probability of occurrence of intensity level *r* in an image



- What histograms say about images?
- What they don't?
  - No spatial information

A descriptor for visual information

Images and histograms



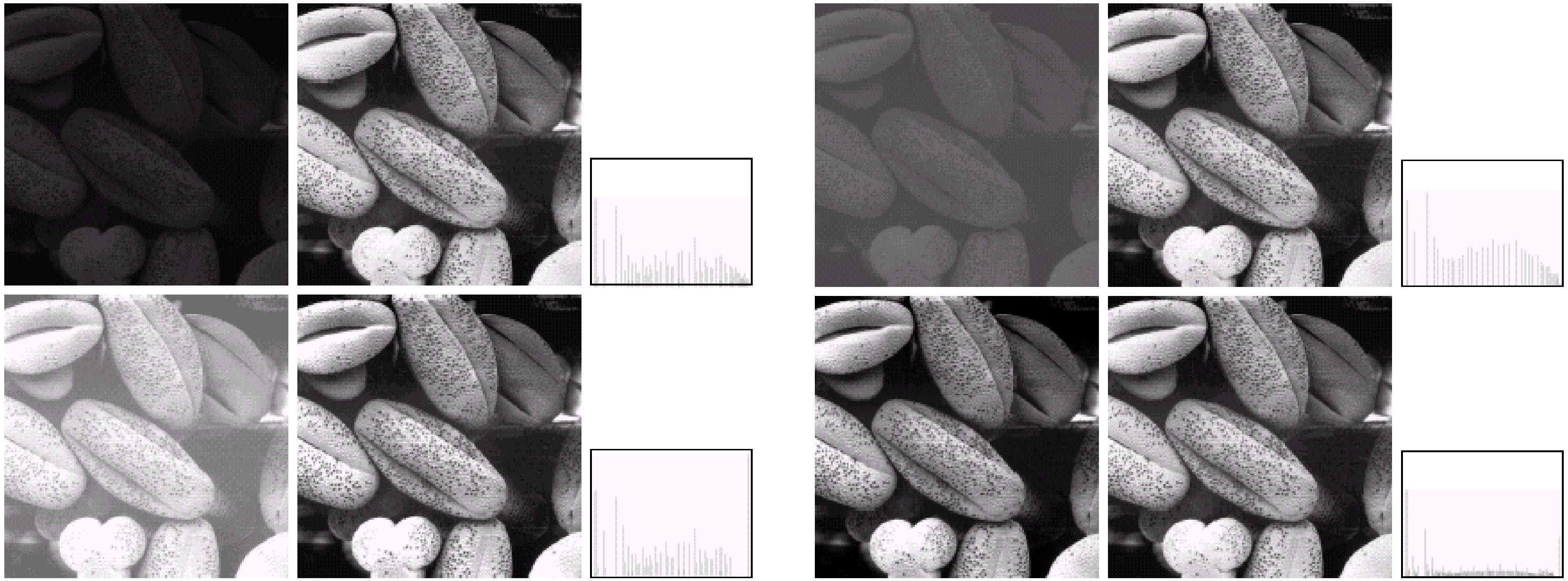
- How do histograms change when
  - we adjust brightnesss? shifts the histogram horizontally
  - we adjust constrast? stretches or shrinks the histogram horizontally

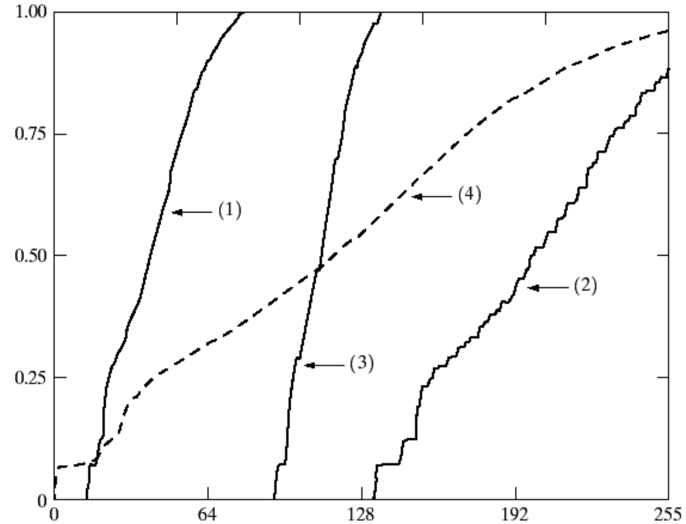
Histogram equalization

- A good quality image has a nearly uniform distribution of intensity levels
- Every intensity level is equally likely to occur in an image

- *Histogram equalization:* Transform an image so that it has a uniform distribution

Histogram equalization examples





Histogram as a probability density function

- Recall that a normalized histogram measures the probability of occurrence of an intensity level *r* in an image
- We can normalize a histogram by dividing the intensity counts by the area

$$p(r) = \frac{h(r)}{Area}$$

Histogram equalization: Continuous domain

- Define a transformation function of the form

$$s = T(r) = (L-1) \underbrace{\int_0^r p(w)\,dw}_{\text{cumulative distribution function}}$$

where

- r is the input intensity level
- s is the output intensity level
- p is the normalized histogram of the input signal
- L is the desired number of intensity levels

(Continuous) output signal has a uniform distribution!

Histogram equalization: Discrete domain

- Define the following transformation function for an MxN image

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k}\frac{n_j}{MN} = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

$$\text{for} \quad k = 0,\ldots,L-1$$

where

- r_k is the input intensity level
- s_k is the output intensity level
- n_j is the number of pixels having intensity value j in the input image
- L is the number of intensity levels

(Discrete) output signal has a nearly uniform distribution!

Histogram Specification

- Given an input image f and a specific histogram $p_2(r)$, transform the image so that it has the specified histogram
- How to perform histogram specification?
- Histogram equalization produces a (nearly) uniform output histogram
- Use histogram equalization as an intermediate step

Histogram Specification

1. Equalize the histogram of the input image

$$T_1(r) = (L-1)\int_0^r p_1(w)\,dw$$

2. Histogram equalize the desired output histogram

$$T_2(r) = (L-1)\int_0^r p_2(w)\,dw$$

3. Histogram specification can be carried out by the following point operation:

$$s = T(r) = T_2^{-1}(T_1(r))$$