BBM 413 Fundamentals of Image Processing

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Edge Preserving Image Smoothing

Acknowledgement: The slides are adapted from the course "A Gentle Introduction to Bilateral Filtering and its Applications" given by Sylvain Paris, Pierre Kornprobst, Jack Tumblin, and Frédo Durand (http://people.csail.mit.edu/sparis/bf_course/)

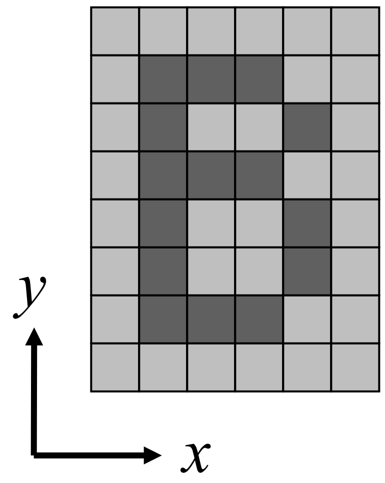
Review - Smoothing and Edge Detection

- While eliminating noise via smoothing, we also lose some of the (important) image details.
  - Fine details
  - Image edges
  - etc.
- What can we do to preserve such details?
  - Use edge information during denoising!
  - This requires a definition for image edges.

Chicken-and-egg dilemma!

- Edge preserving image smoothing

Notation and Definitions



- Image = 2D array of pixels
- Pixel = intensity (scalar) or color (3D vector)
- $I_{\mathbf{p}}$ = value of image I at position: $\mathbf{p} = (\, p_x \,, p_y \,)$
- $F\,[\,I\,]$ = output of filter F applied to image I

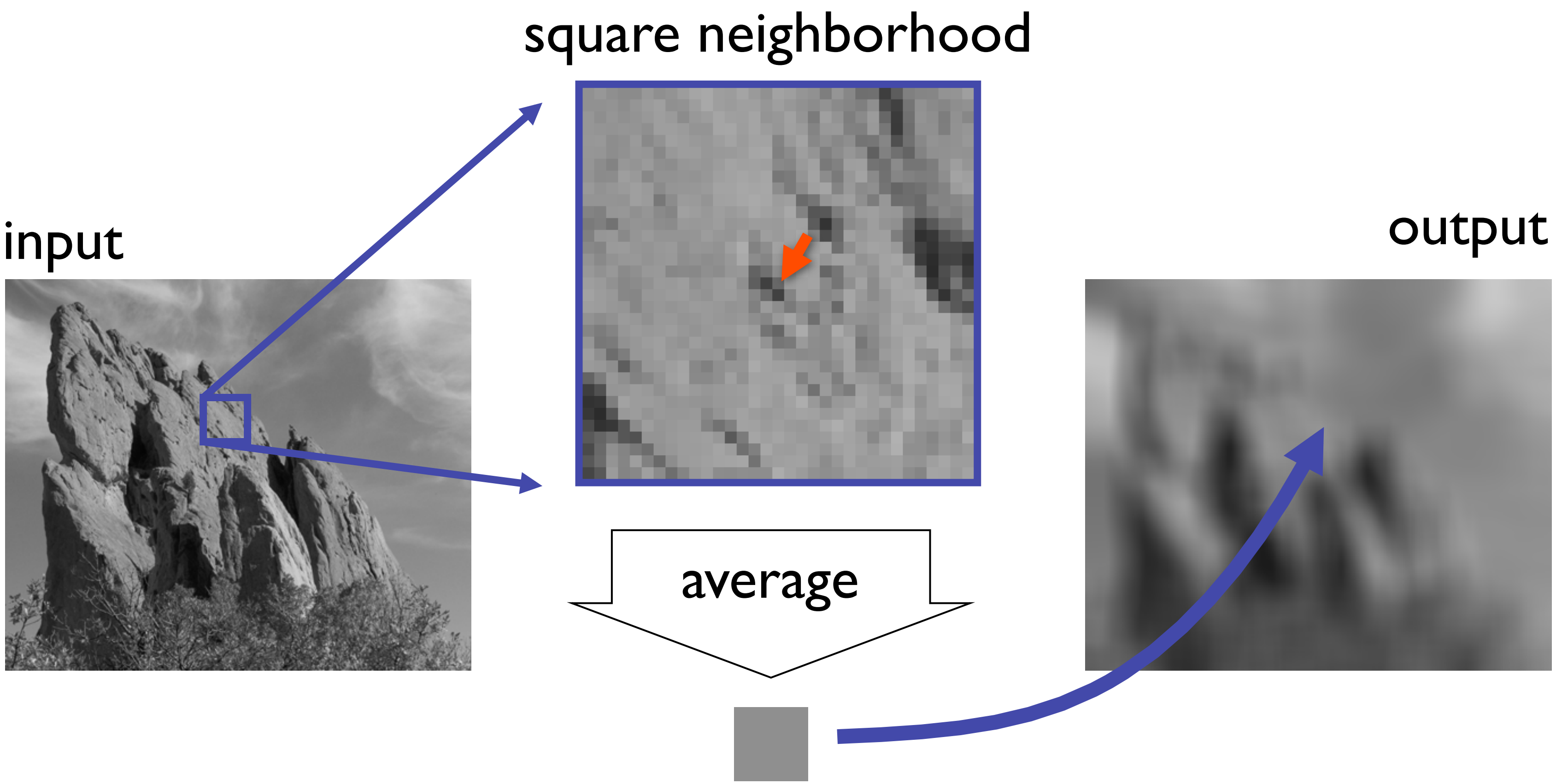
Strategy for Smoothing Images

- Images are not smooth because
  adjacent pixels are different.

- Smoothing = making adjacent pixels
  look more similar.

- Smoothing strategy
  pixel → average of its neighbors

Box Average



Equation of Box Average

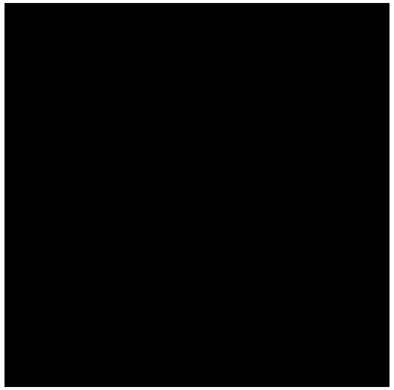
$$BA[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} B_{\sigma}(\mathbf{p} - \mathbf{q})\, I_{\mathbf{q}}$$

result at pixel **p**

sum over all pixels **q**

intensity at pixel **q**

normalized box function



Square Box Generates Defects

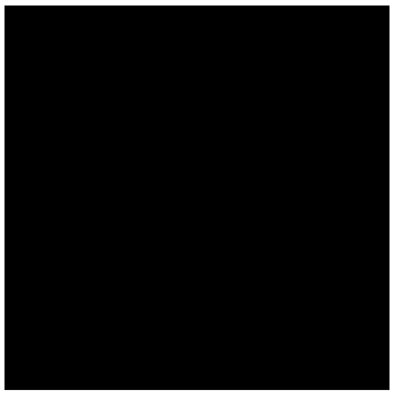
- Axis-aligned streaks
- Blocky results

output

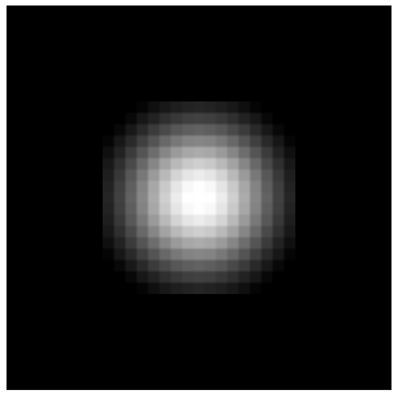
input

Strategy to Solve these Problems

- Use an isotropic (*i.e.* circular) window.
- Use a window with a smooth falloff.

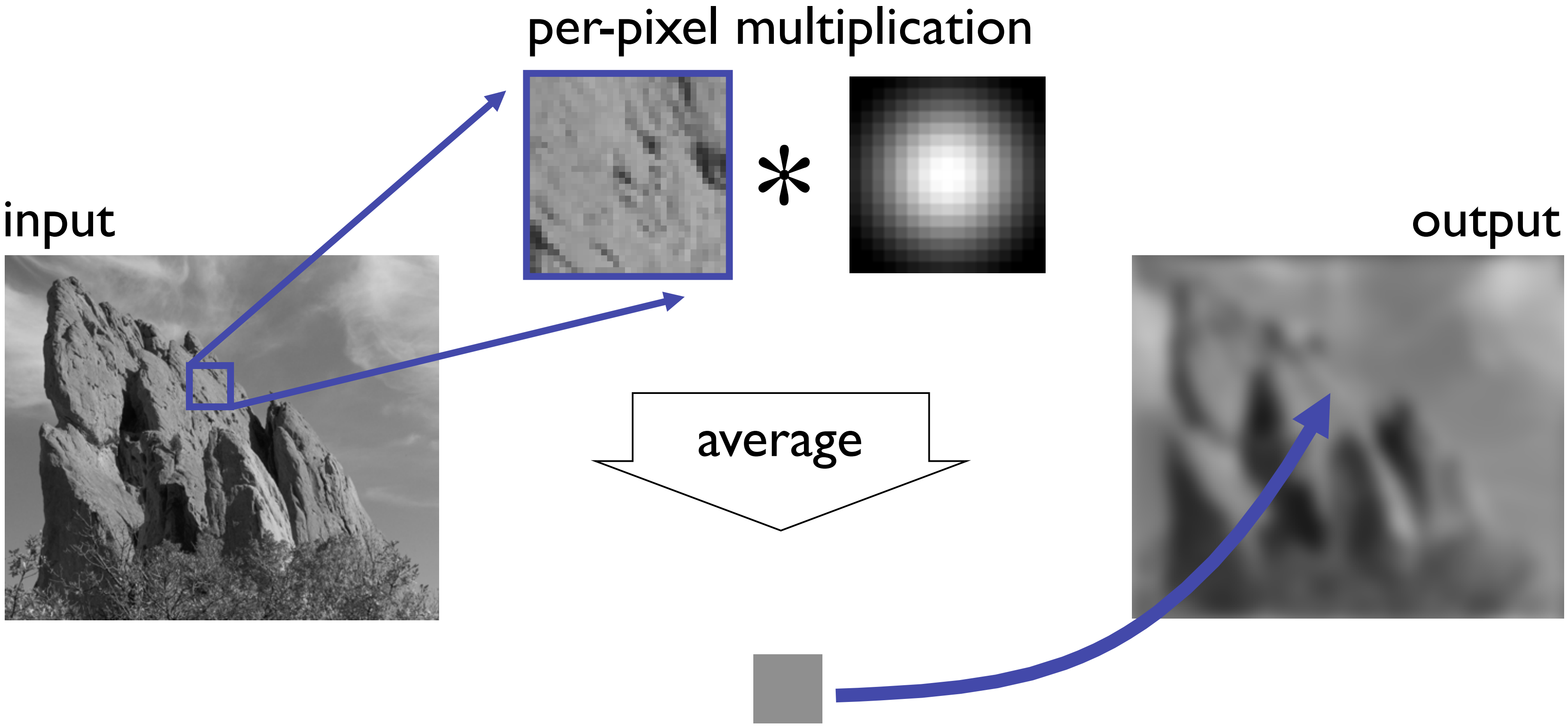


box window



Gaussian window

Gaussian Blur




input


box average

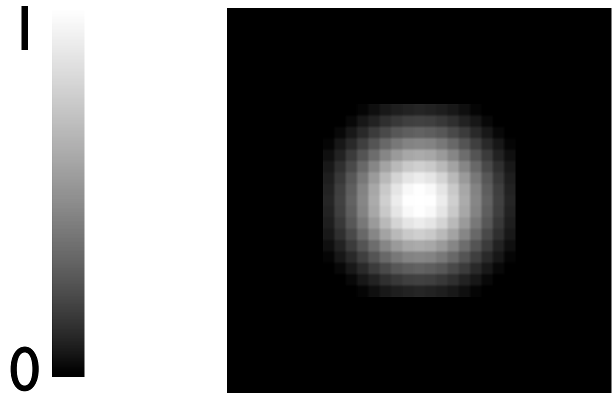

Gaussian blur

Equation of Gaussian Blur

Same idea: **weighted average of pixels**.

$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_\sigma\left(\| \mathbf{p} - \mathbf{q} \|\right) I_{\mathbf{q}}$$

normalized
Gaussian function

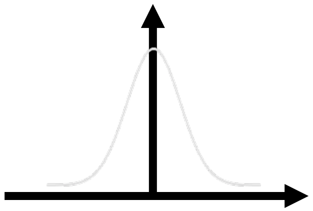


Spatial Parameter

input

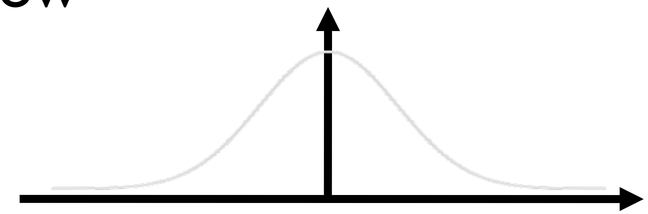
$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q}\in S} G_{\sigma}(\|\,\mathbf{p}-\mathbf{q}\,\|)\, I_{\mathbf{q}}$$

size of the window

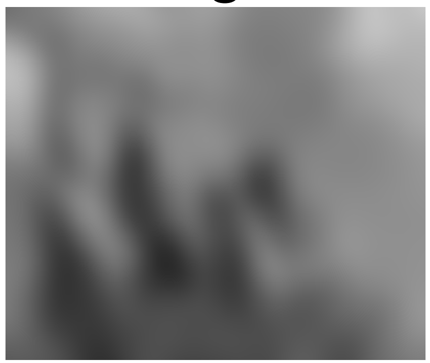


small *s*

limited smoothing



large *s*



strong smoothing

How to set σ

- Depends on the application.

- Common strategy: proportional to image size
  - e.g. 2% of the image diagonal
  - property: independent of image resolution

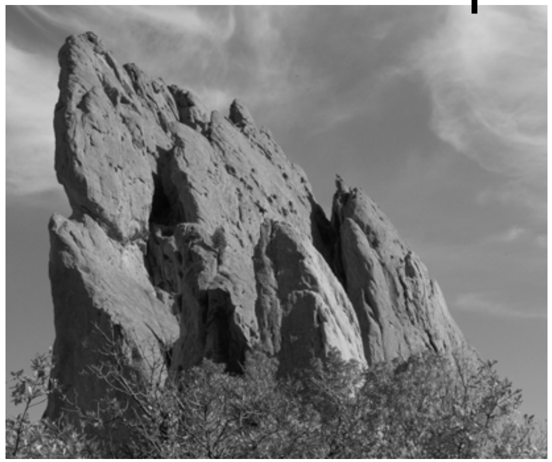
Properties of Gaussian Blur

- Weights independent of spatial location
  - linear convolution
  - well-known operation
  - efficient computation (recursive algorithm, FFT…)

Properties of Gaussian Blur

- Does smooth images
- But smoothes too much: **edges are blurred**.
  - Only spatial distance matters
  - No edge term

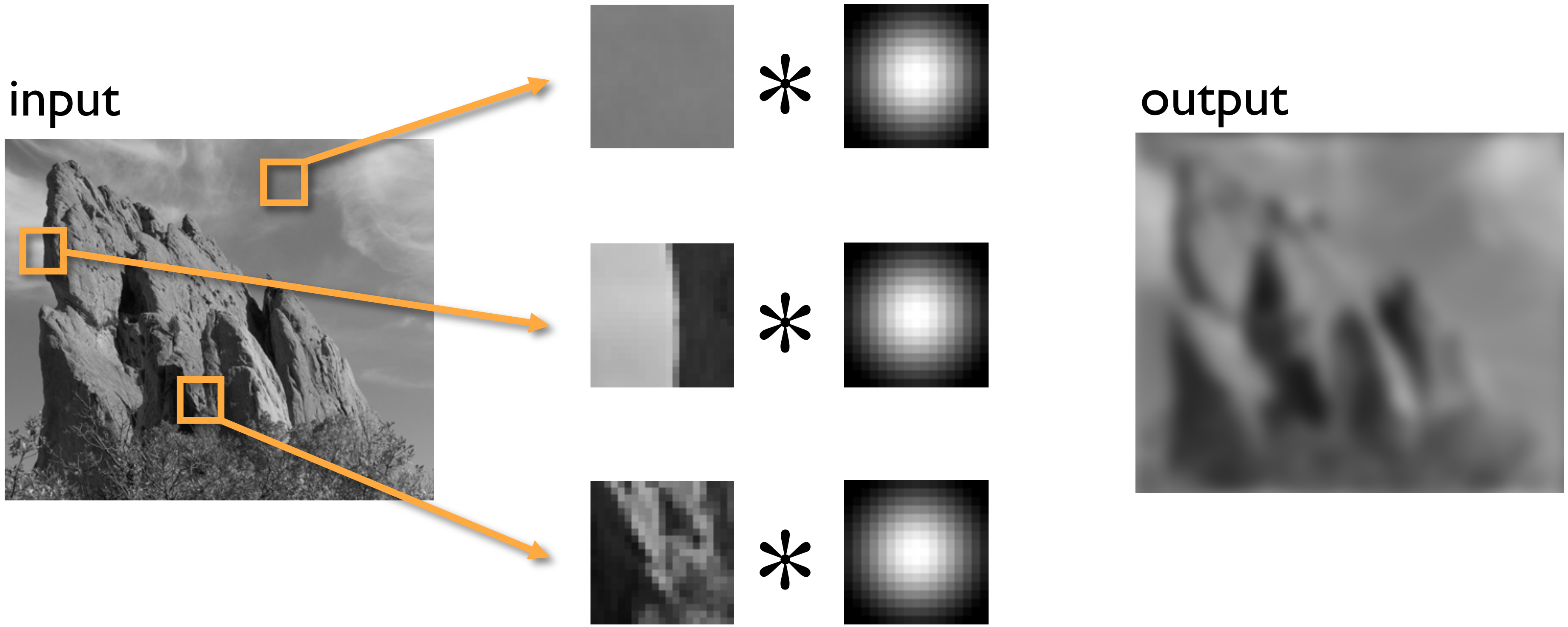
input



output

$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q}\in S} \underbrace{G_\sigma(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} I_{\mathbf{q}}$$

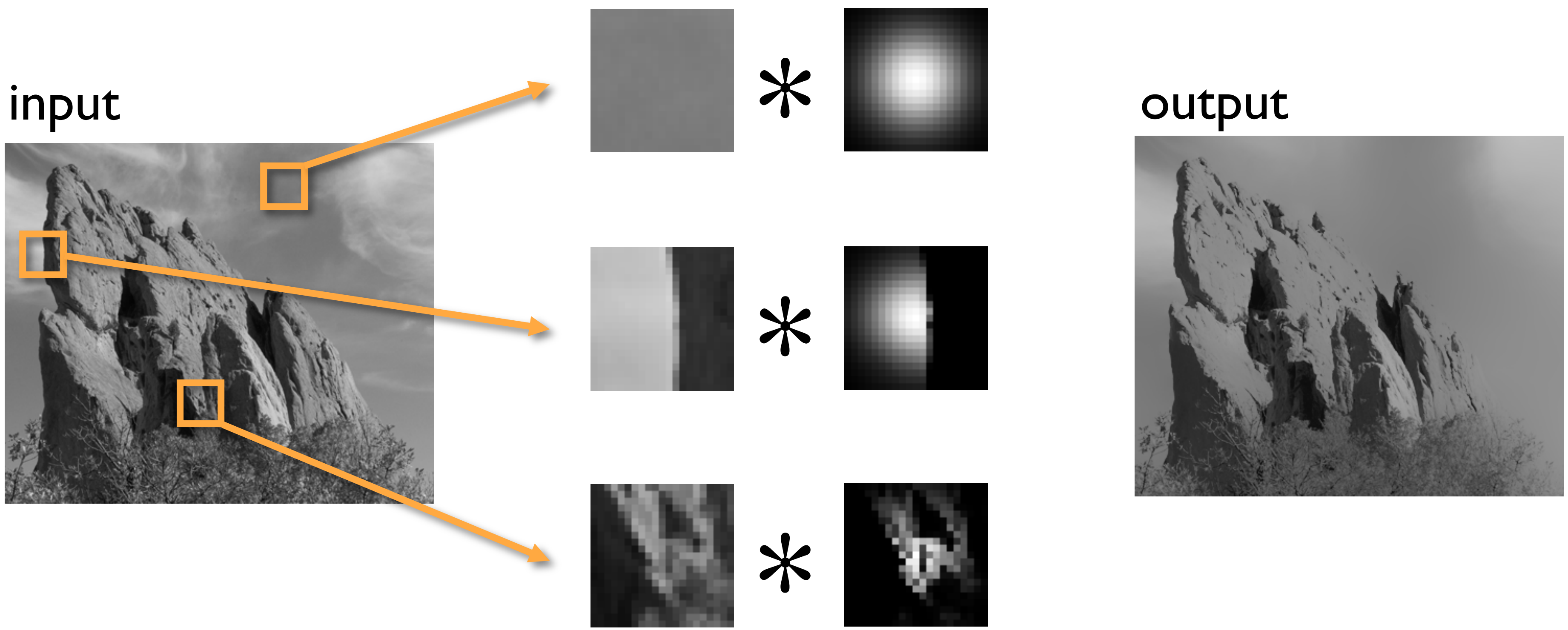
Blur Comes from Averaging across Edges



Same Gaussian kernel everywhere.

[Aurich 95, Smith 97, Tomasi 98]

Bilateral Filter
No Averaging across Edges



The kernel shape depends on the image content.

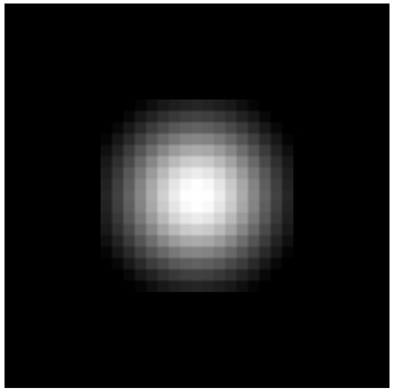
Bilateral Filter Definition: an Additional Edge Term

Same idea: **weighted average of pixels**.

$$BF[I]_{\mathbf{p}} = \overset{\text{new}}{\boxed{\frac{1}{W_{\mathbf{p}}}}} \sum_{\mathbf{q}\in S} \overset{\text{not new}}{\boxed{G_{\sigma_s}(\| \mathbf{p}-\mathbf{q} \|)}} \, \overset{\text{new}}{\boxed{G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |)}} \, I_{\mathbf{q}}$$

normalization factor

space weight



range weight

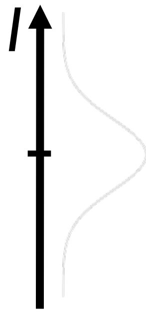
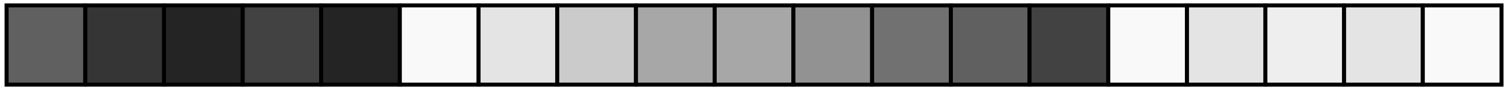
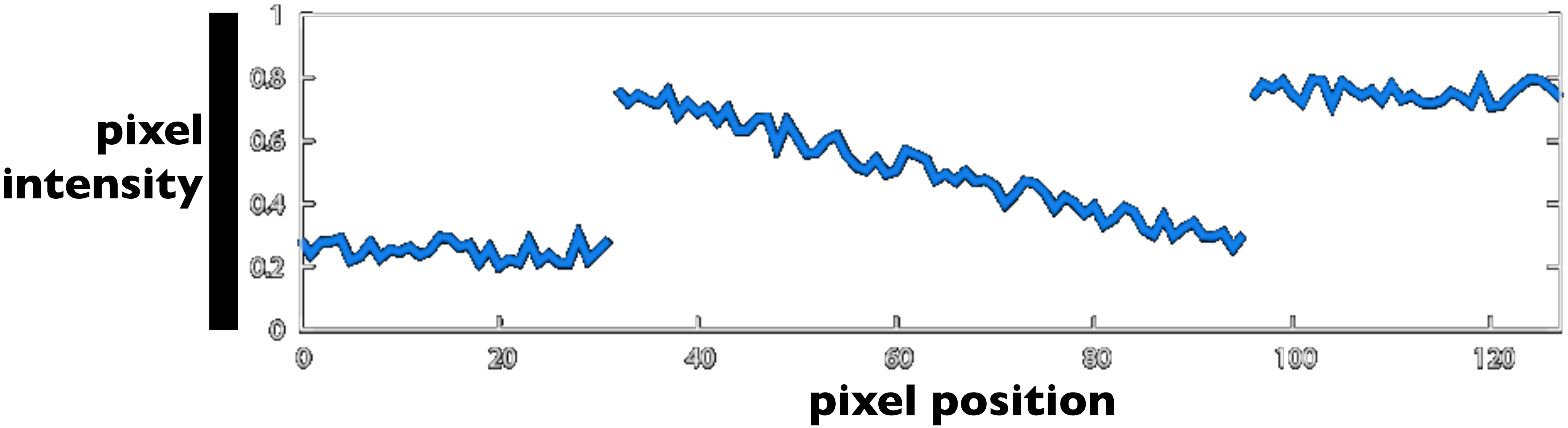


Illustration a 1D Image

- 1D image = line of pixels

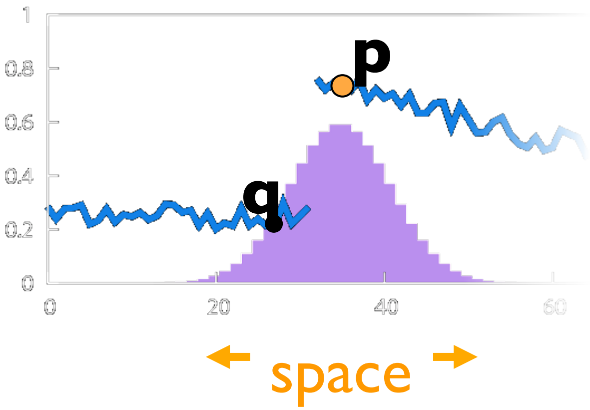


- Better visualized as a plot



Gaussian Blur and Bilateral Filter

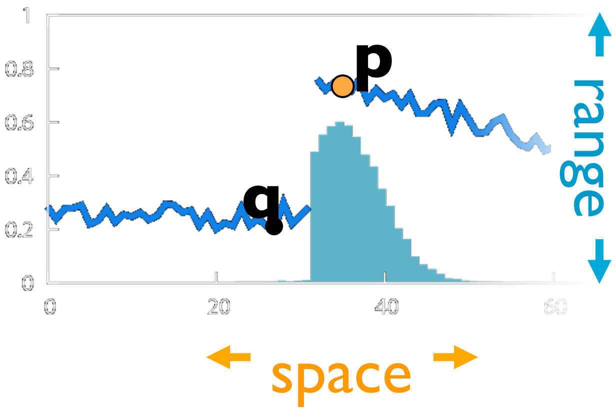
Gaussian blur



$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q}\in S} \underbrace{G_{\sigma}(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} I_{\mathbf{q}}$$

Bilateral filter

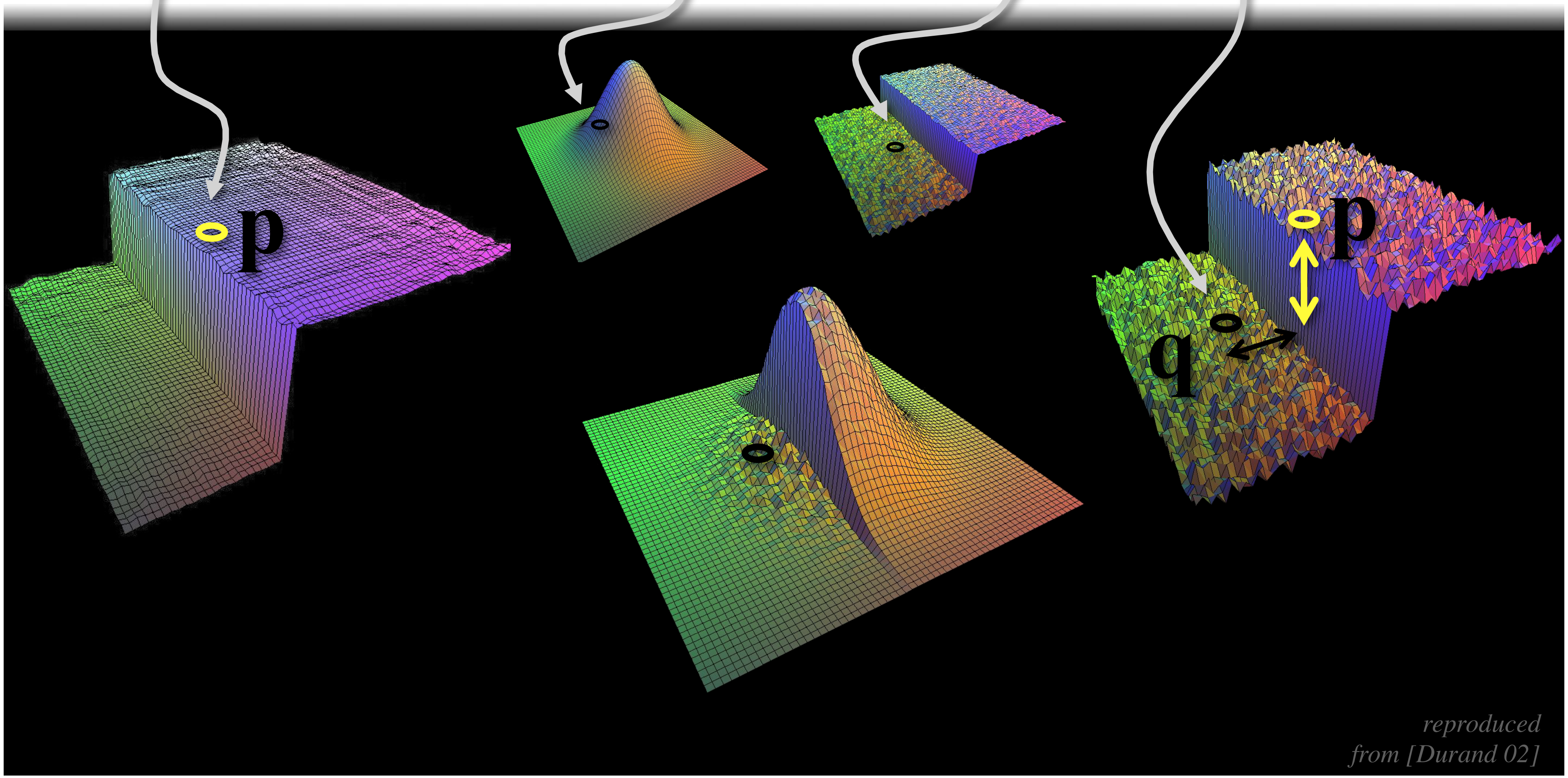
[Aurich 95, Smith 97, Tomasi 98]



$$BF[I]_{\mathbf{p}} = \underbrace{\frac{1}{W_{\mathbf{p}}}}_{\text{normalization}} \sum_{\mathbf{q}\in S} \underbrace{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} \underbrace{G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)}_{\text{range}} I_{\mathbf{q}}$$

Bilateral Filter on a Height Field

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q}\in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\; G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)\; I_{\mathbf{q}}$$



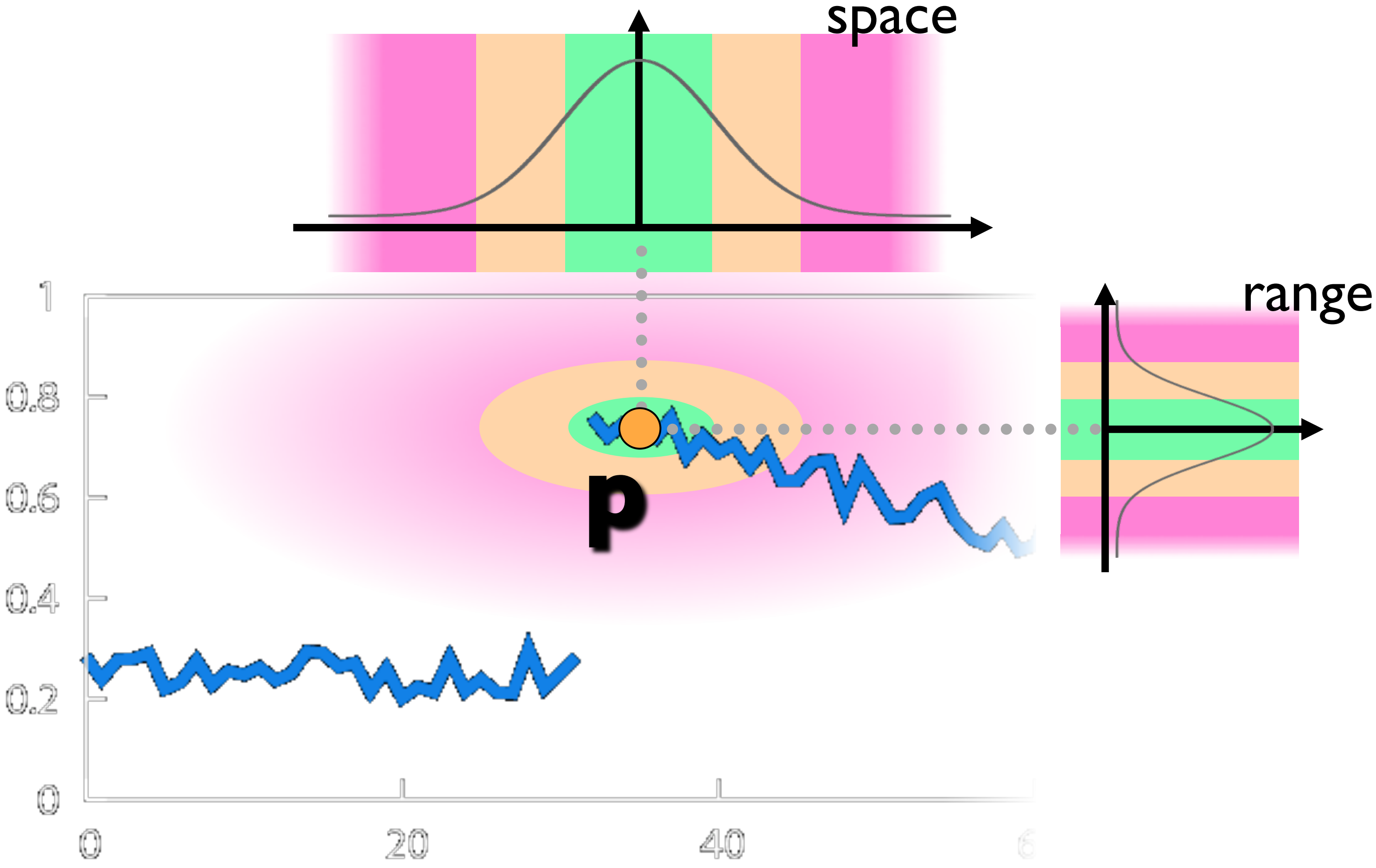
Space and Range Parameters

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_{\mathrm{s}}}\left(\| \mathbf{p} - \mathbf{q} \|\right) G_{\sigma_{\mathrm{r}}}\left(| I_{\mathbf{p}} - I_{\mathbf{q}} |\right) I_{\mathbf{q}}$$

- space σ_{s} : spatial extent of the kernel, size of the considered neighborhood.
- range σ_{r} : "minimum" amplitude of an edge

Influence of Pixels

Only pixels close in space and in range are considered.



input

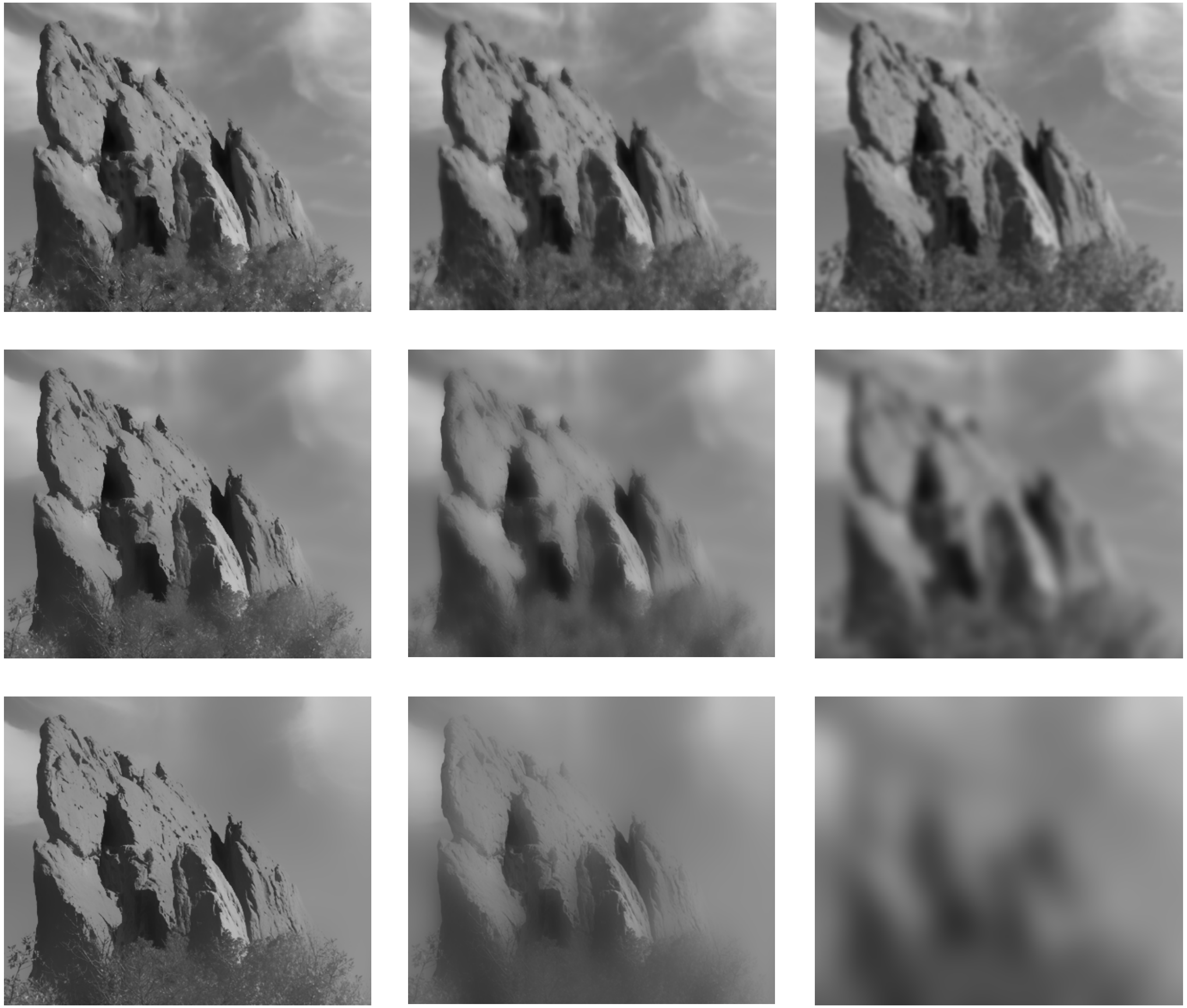
Exploring the Parameter Space

$s_r = 0.1$ | $s_r = 0.25$ | $s_r = \infty$ (Gaussian blur)

$s_s = 2$

$s_s = 6$

$s_s = 18$



input

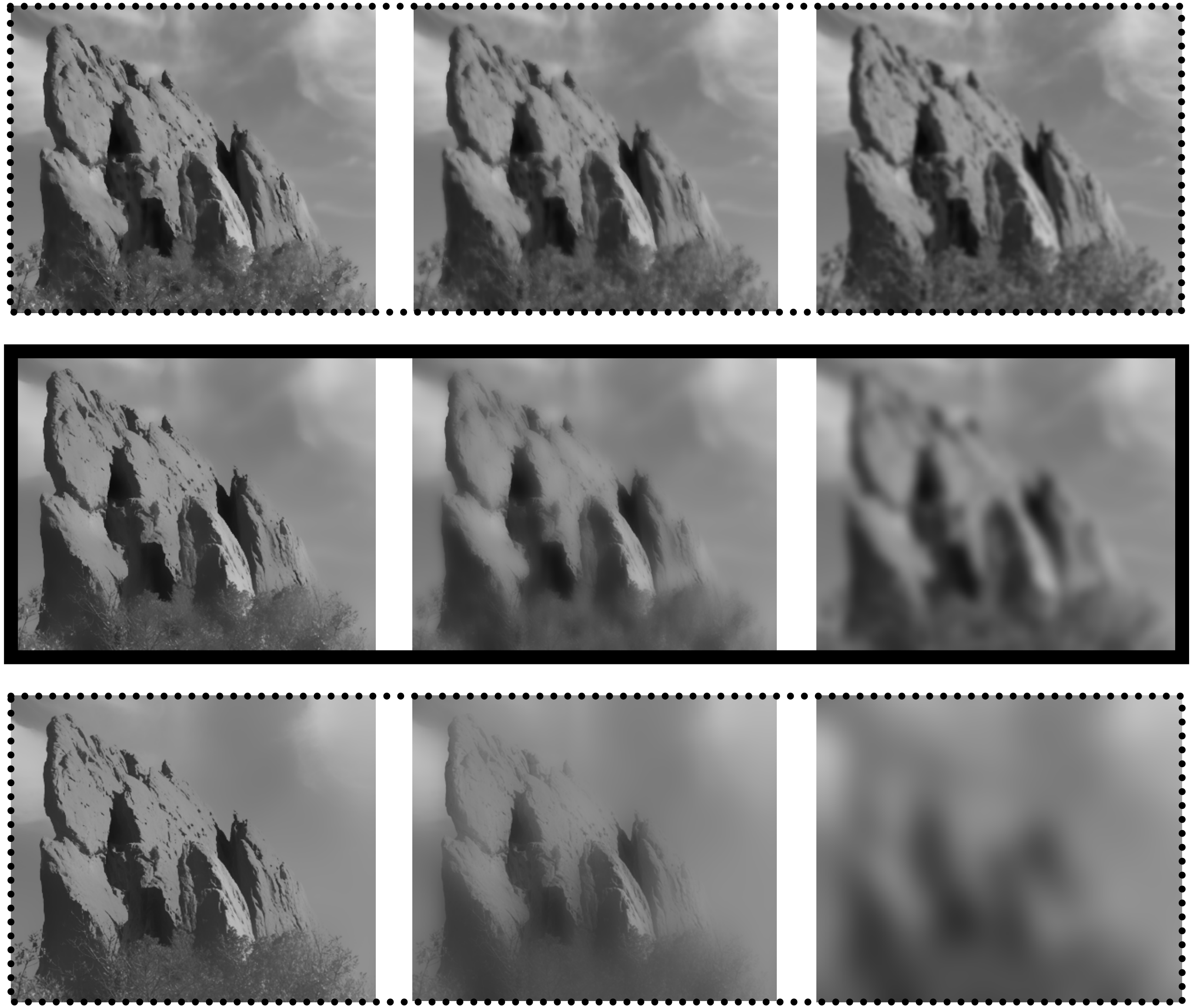
Varying the Range Parameter

$\sigma_r = 0.1$ | $\sigma_r = 0.25$ | $\sigma_r = \infty$ (Gaussian blur)

$\sigma_s = 2$

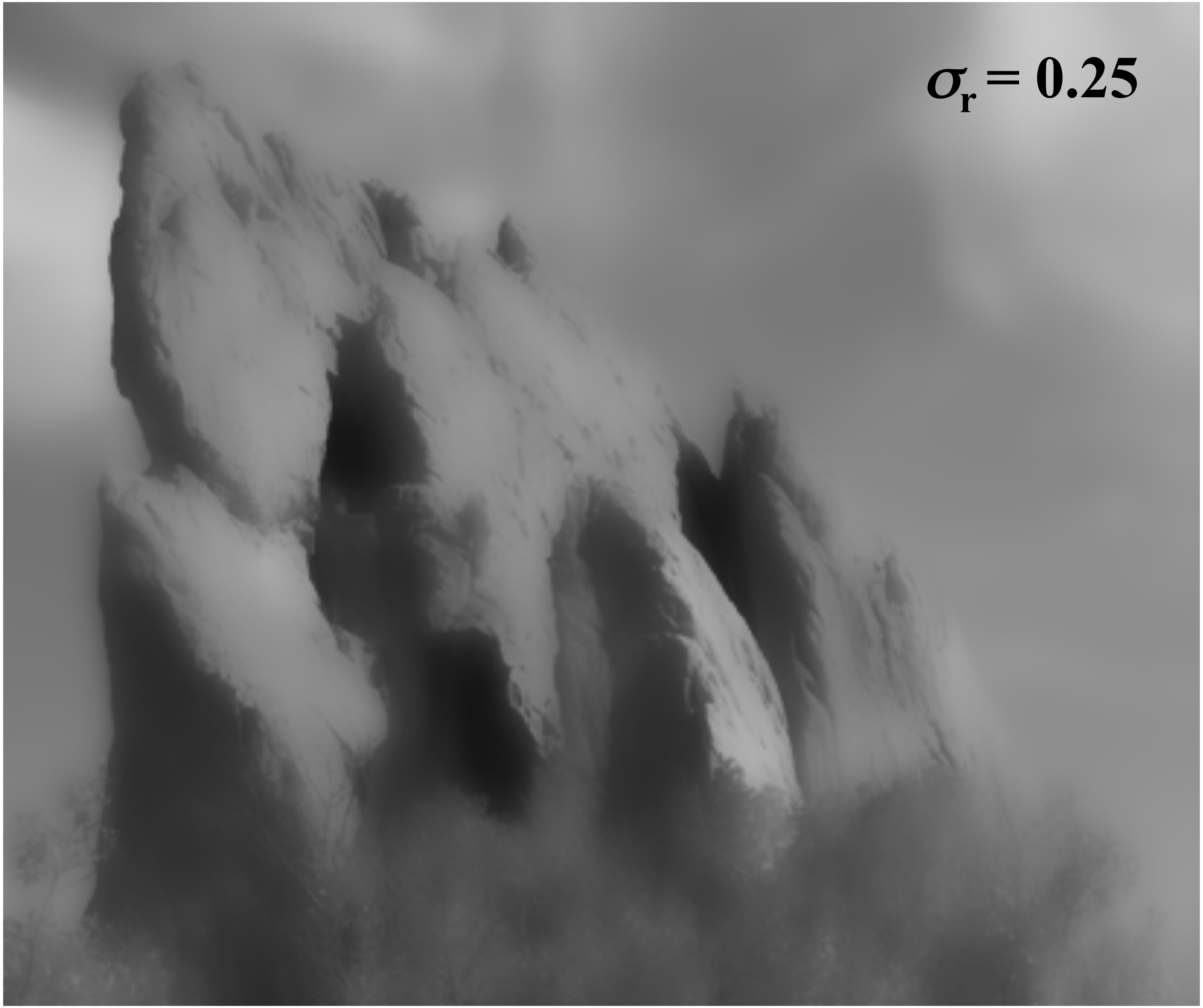
$\sigma_s = 6$

$\sigma_s = 18$



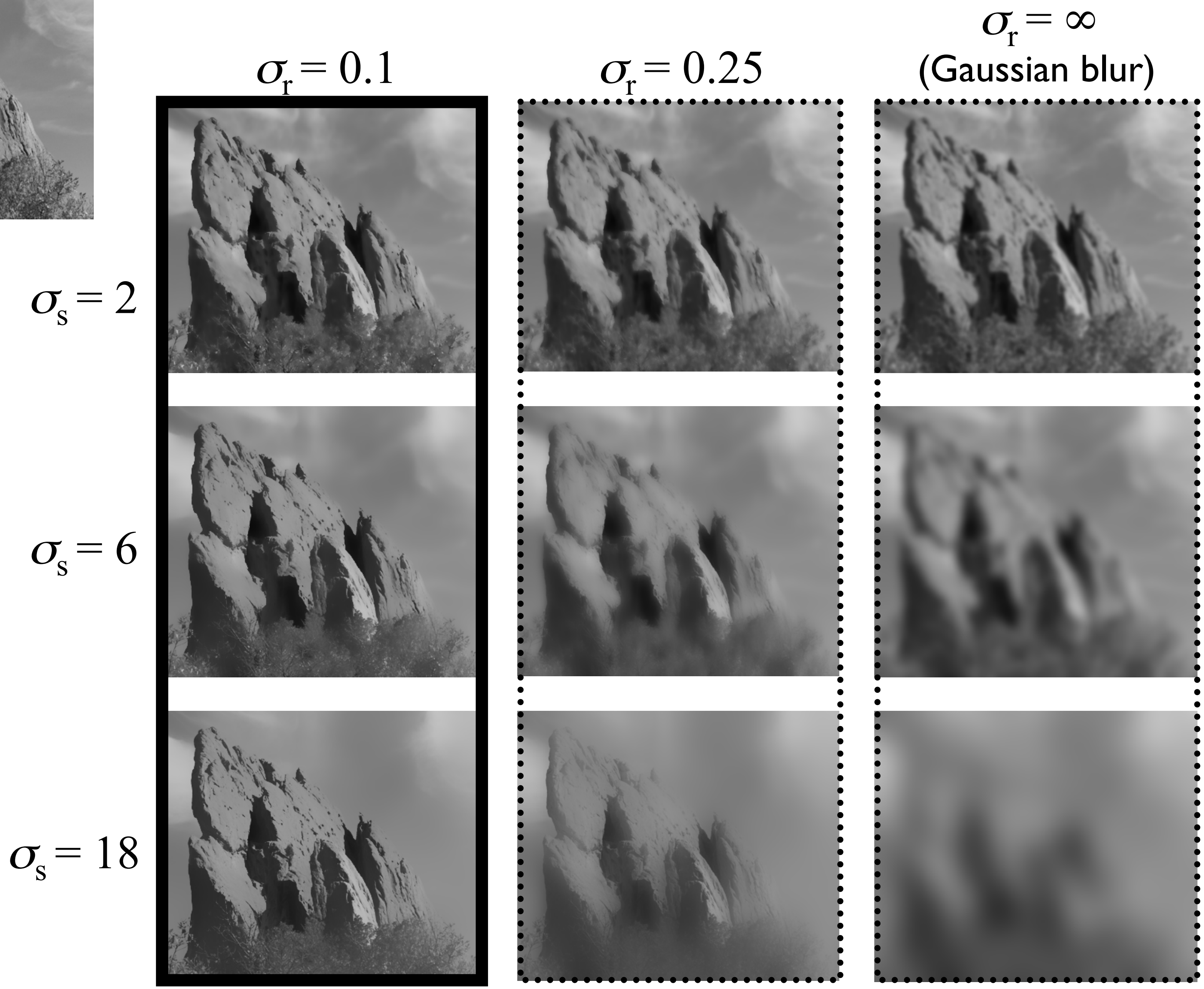

input


$\sigma_r = 0.1$


$\sigma_r = 0.25$


$\sigma_r = \infty$
(Gaussian blur)

Varying the Space Parameter




input


$\sigma_s = 2$


$\sigma_s = 6$


$\sigma_s = 18$

How to Set the Parameters

Depends on the application. For instance:

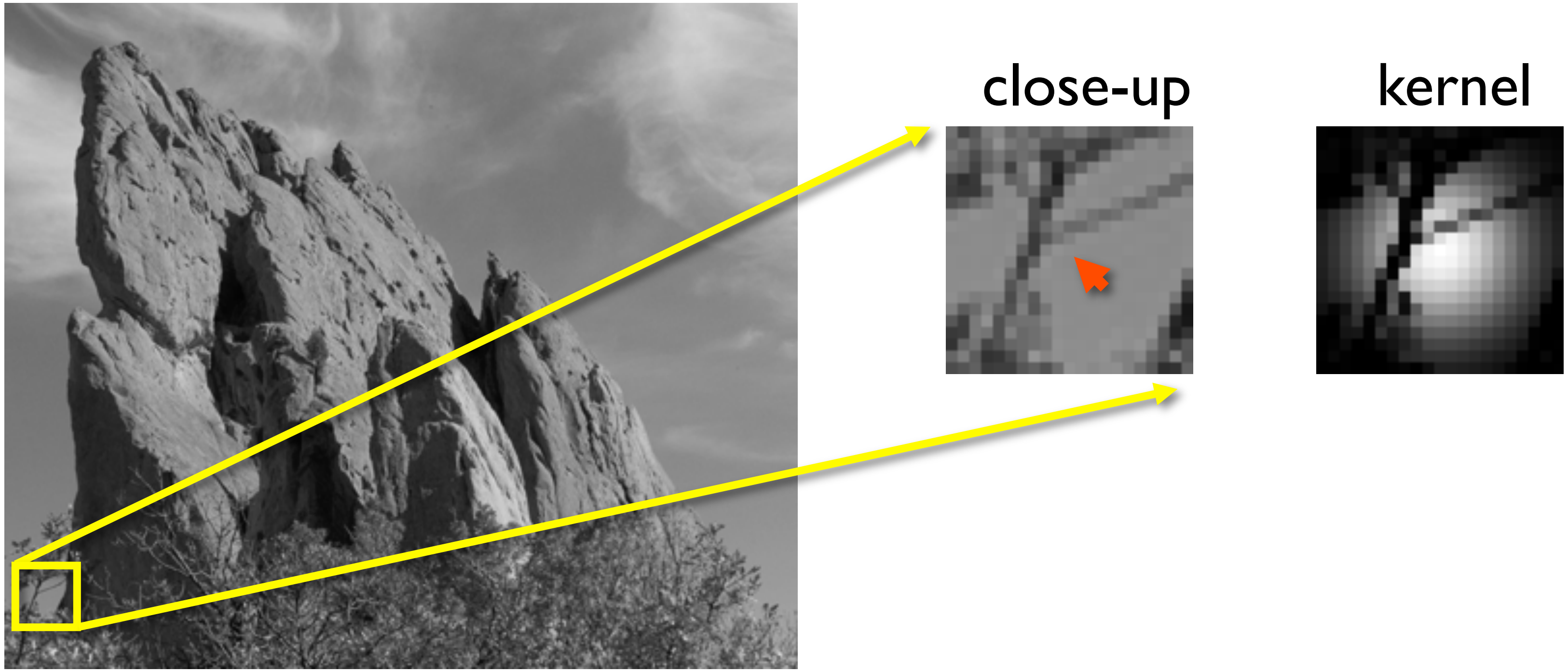
- space parameter: proportional to image size
  - e.g., 2% of image diagonal

- range parameter: proportional to edge amplitude
  - e.g., mean or median of image gradients

- independent of resolution and exposure

Bilateral Filter Crosses Thin Lines

- Bilateral filter averages across features thinner than ~2s_s
- Desirable for smoothing: more pixels = more robust
- Different from diffusion that stops at thin lines



Iterating the Bilateral Filter

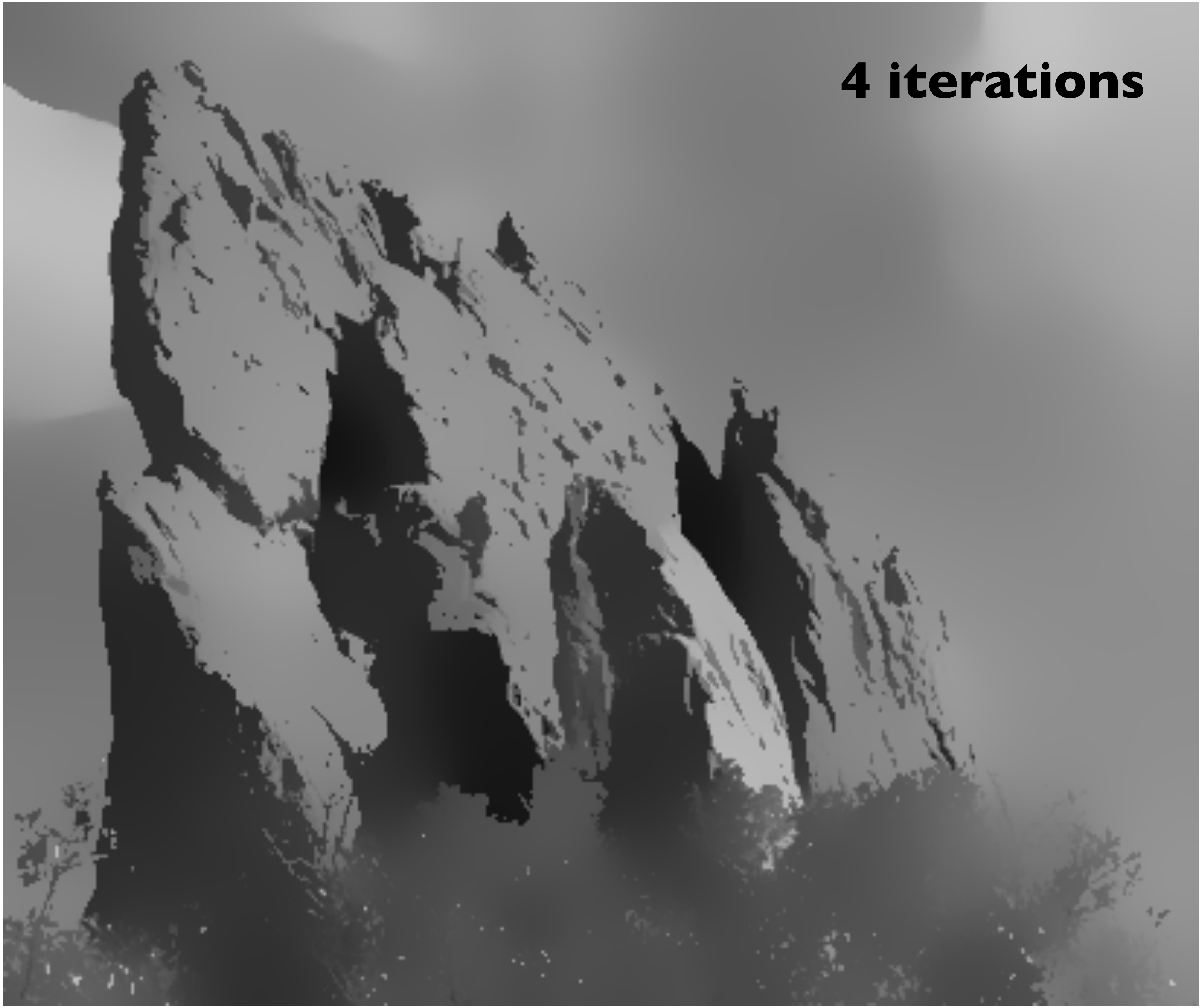
$$I_{(n+1)} = BF\left[I_{(n)}\right]$$

- Generate more piecewise-flat images
- Often not needed in computational photo.


input


1 iteration


2 iterations


4 iterations

Bilateral Filtering Color Images

For gray-level images

intensity difference

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}\left(\boxed{|I_{\mathbf{p}} - I_{\mathbf{q}}|}\right) \boxed{I_{\mathbf{q}}}$$

scalar

For color images

color difference

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}\left(\boxed{\|\mathbf{C}_{\mathbf{p}} - \mathbf{C}_{\mathbf{q}}\|}\right) \boxed{\mathbf{C}_{\mathbf{q}}}$$

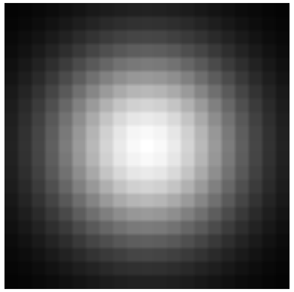
3D vector
(RGB, Lab)

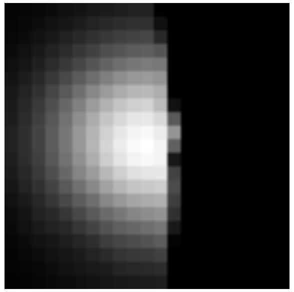
Hard to Compute

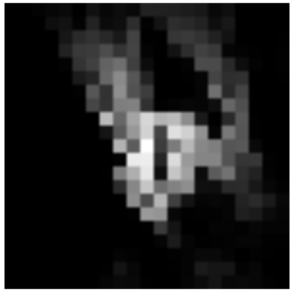
- Nonlinear

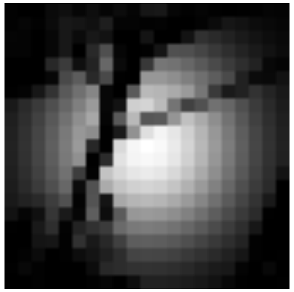
$$BF[I]_{\mathbf{p}} = \boxed{\frac{1}{W_{\mathbf{p}}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) \boxed{G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |)} I_{\mathbf{q}}$$

- Complex, spatially varying kernels
  - Cannot be precomputed, no FFT…









- Brute-force implementation is slow > 10min

Basic denoising

Noisy input

Bilateral filter 7x7 window

Basic denoising

Bilateral filter

Median 3x3

Basic denoising

Bilateral filter

Median 5x5

Basic denoising

Bilateral filter

Bilateral filter – lower sigma

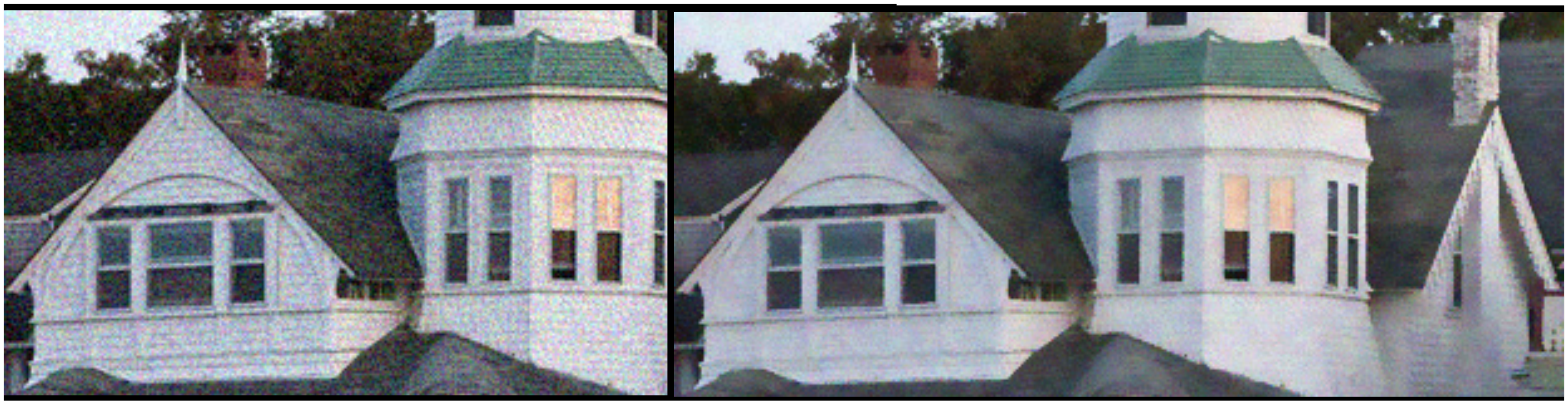
Basic denoising

Bilateral filter

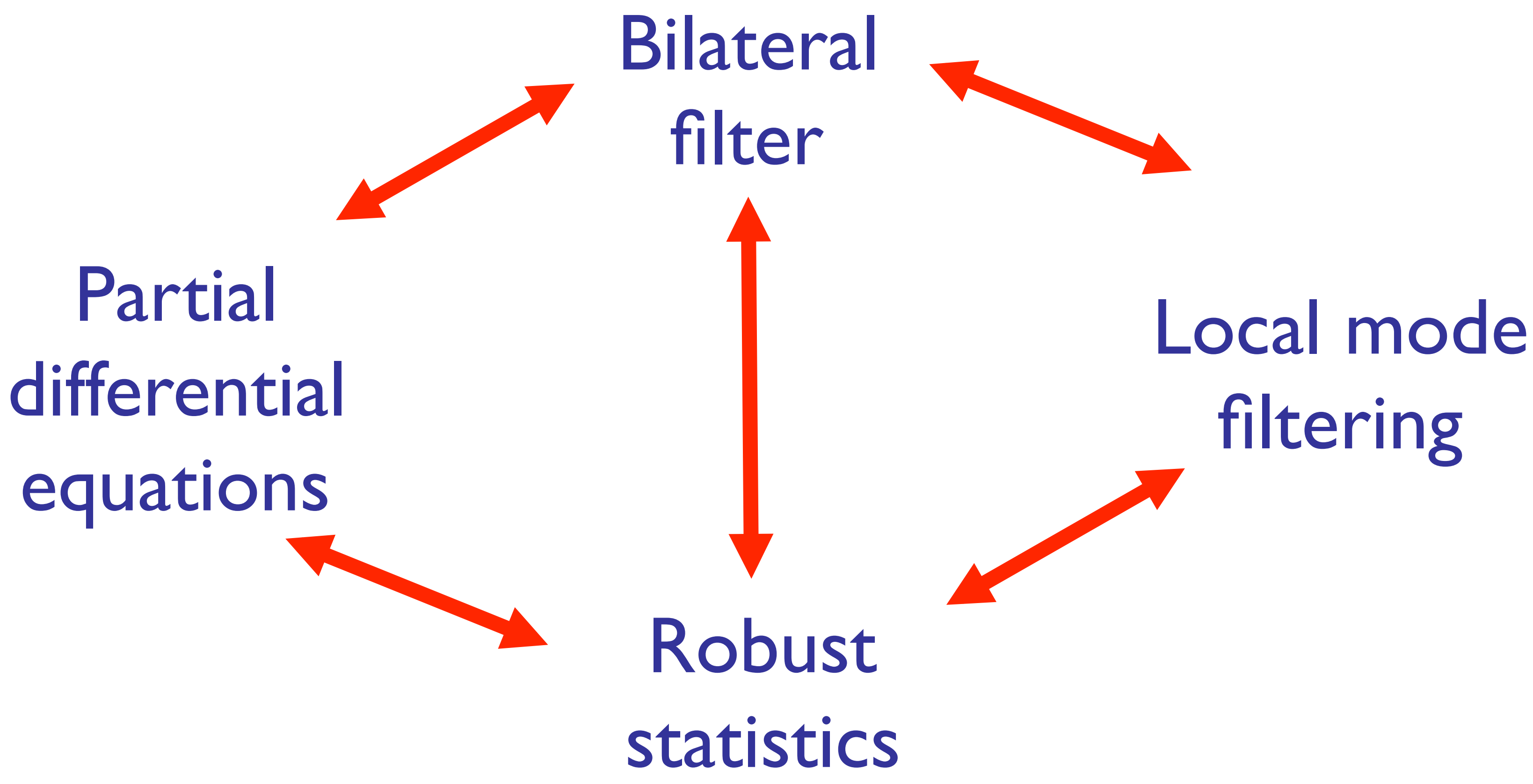
Bilateral filter – higher sigma

Denoising

- Small spatial sigma (e.g. 7x7 window)
- Adapt range sigma to noise level
- Maybe not best denoising method, but best simplicity/quality tradeoff
  - No need for acceleration (small kernel)
  - But the denoising feature in e.g. Photoshop is better



Goal: Understand how does bilateral filter relates with other methods



New Idea:
NL-Means Filter (Buades 2005)

- Same goals: 'Smooth within Similar Regions'

- **KEY INSIGHT**: Generalize, extend'Similarity'
  - **Bilateral:**
    Averages neighbors with **similar intensities**;

  - **NL-Means:**
    Averages neighbors with **similar neighborhoods!**

NL-Means Method: Buades (2005)

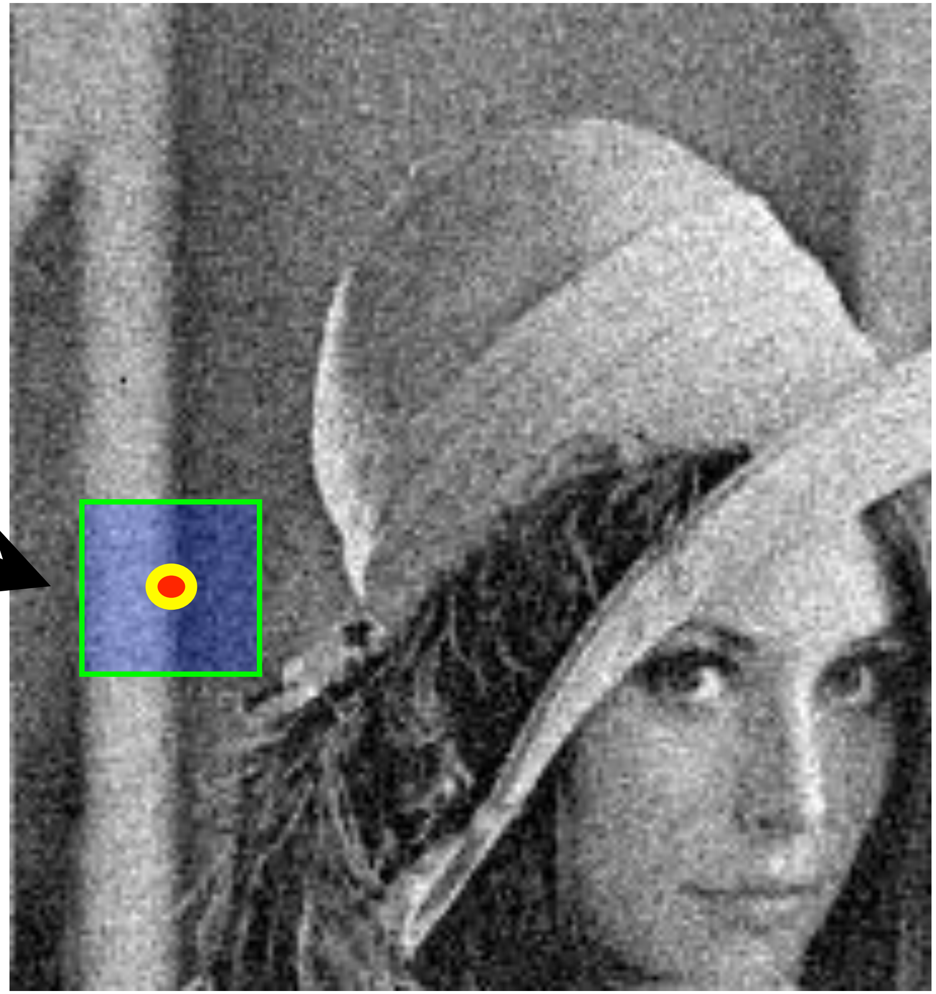
- For each and every pixel **p:**

NL-Means Method: Buades (2005)

- For each and every pixel **p:**
  - Define a small, simple fixed size neighborhood;

NL-Means Method: Buades (2005)

$$\mathbf{V_p} = \begin{bmatrix} 0.74 \\ 0.32 \\ 0.41 \\ 0.55 \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}$$



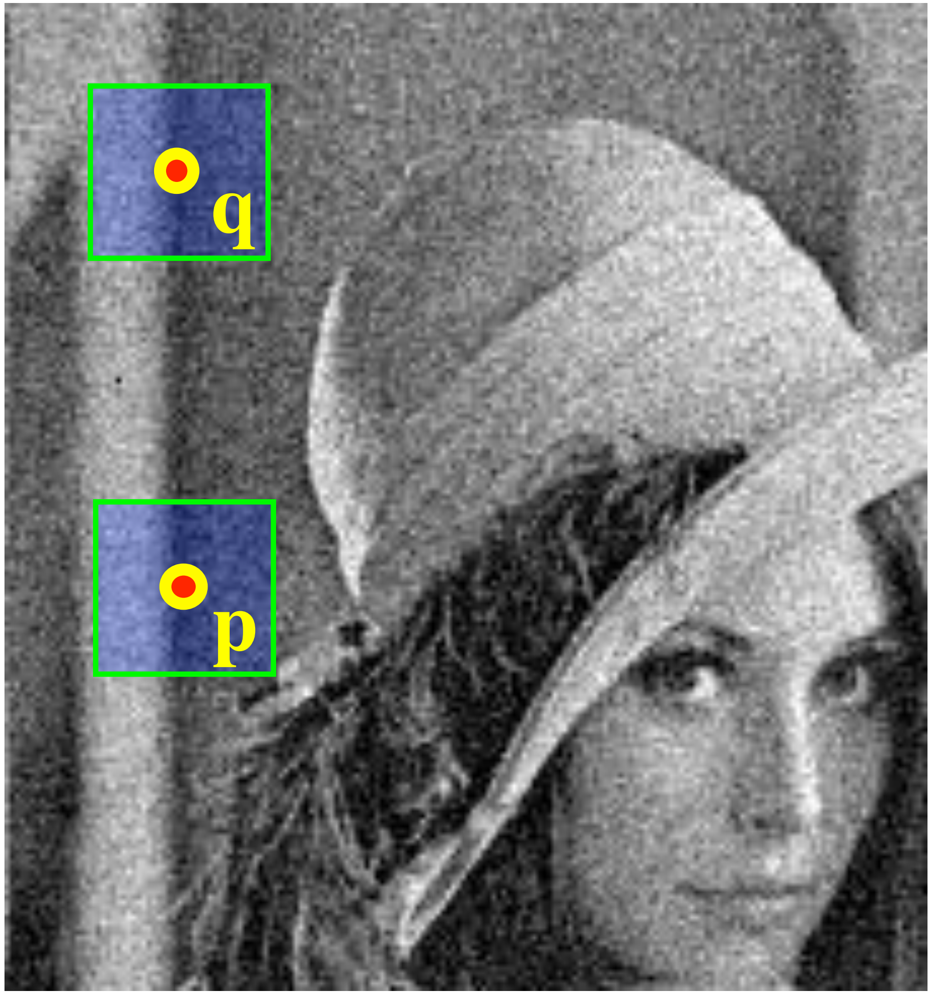
- For each and every pixel **p**:
  - Define a small, simple fixed size neighborhood;
  - Define vector $\mathbf{V_p}$: a list of neighboring pixel values.

NL-Means Method: Buades (2005)

'Similar' pixels **p, q**

→ **SMALL** vector distance;

$$\| V_p - V_q \|^2$$

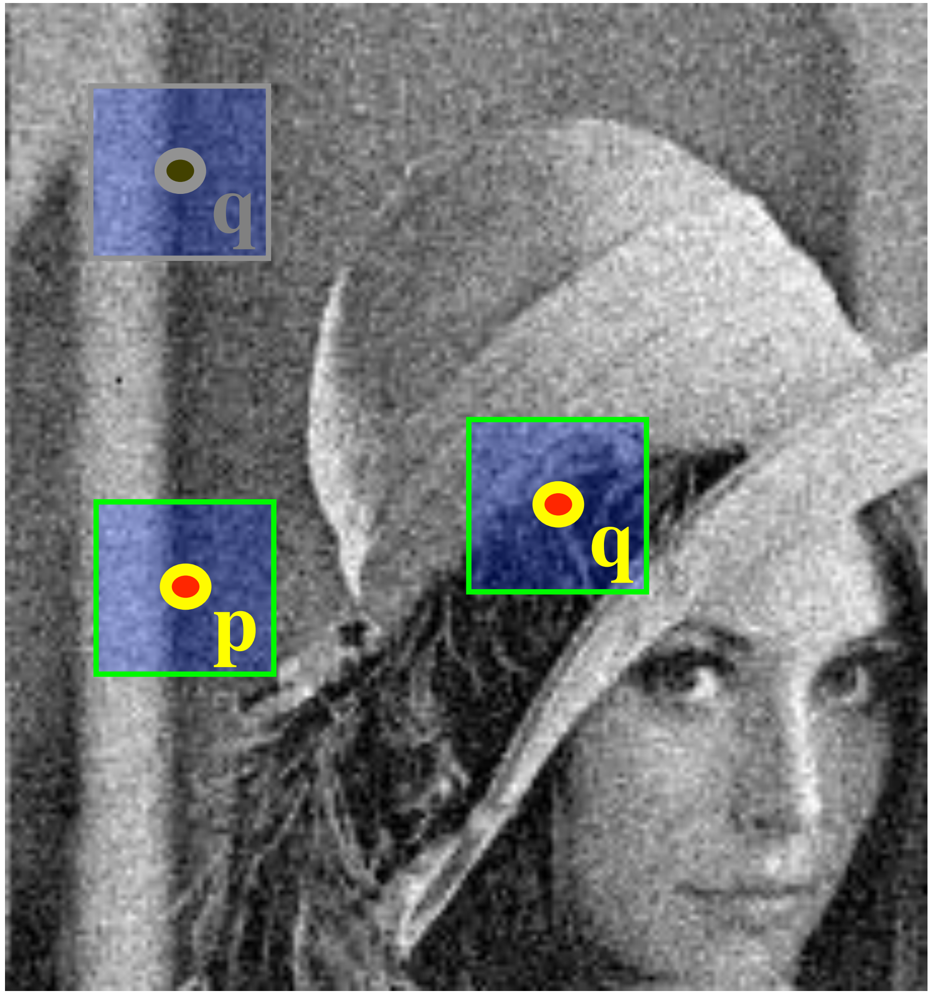


NL-Means Method: Buades (2005)

'Dissimilar' pixels **p, q**

→ **LARGE**
vector distance;

$$\| \mathbf{V_p} - \mathbf{V_q} \|^2$$



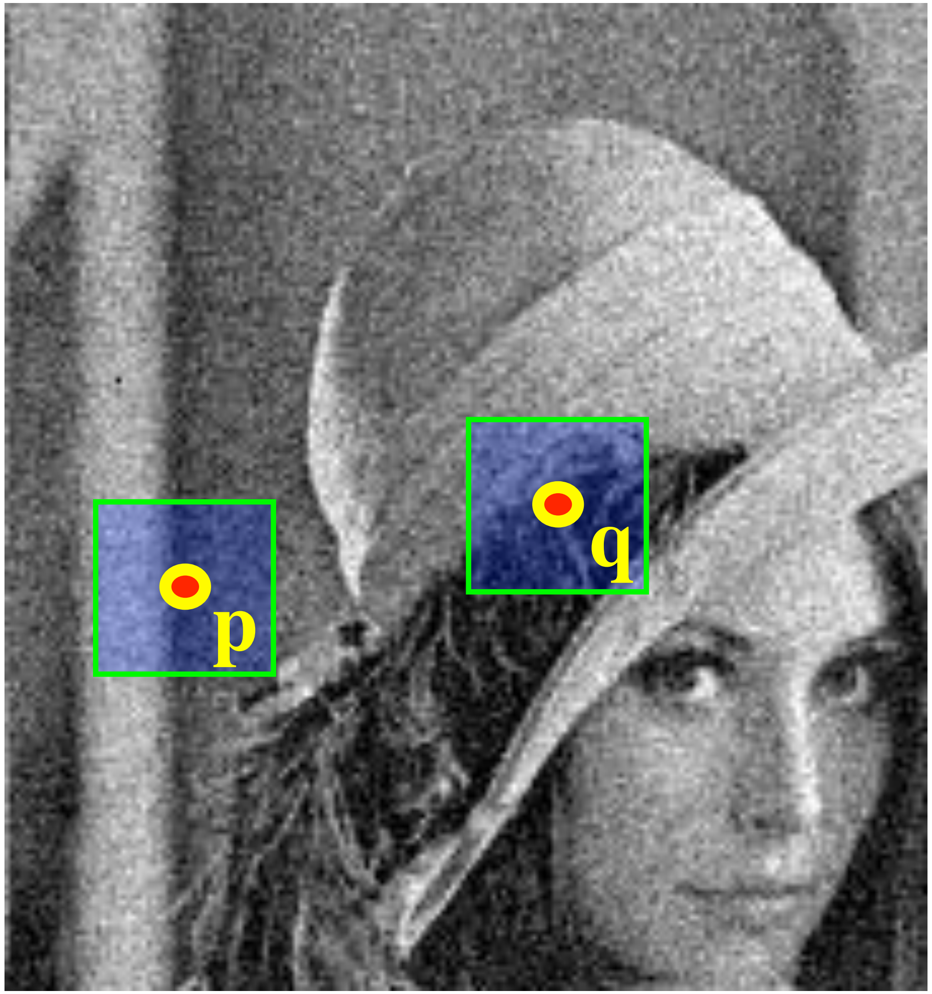
NL-Means Method: Buades (2005)

'Dissimilar' pixels **p, q**

→ **LARGE** vector distance;

$$\boxed{\|V_p - V_q\|^2}$$

Filter with this!



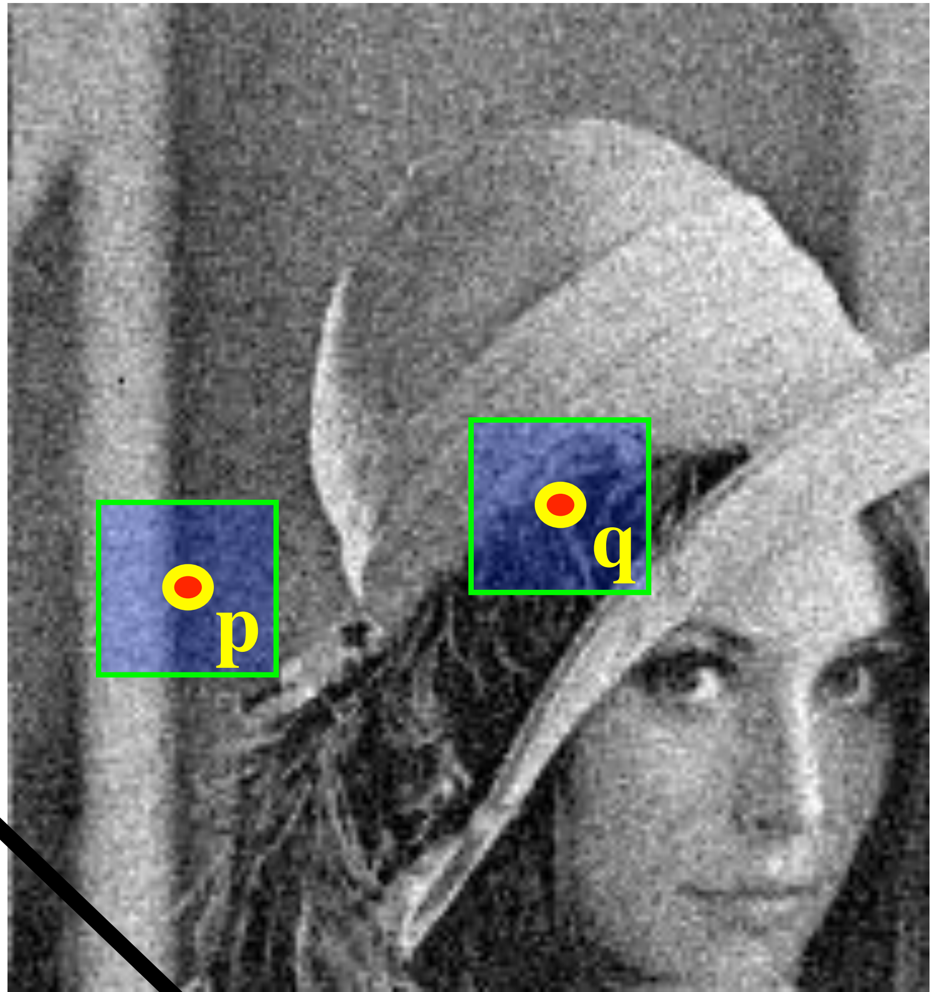
NL-Means Method: Buades (2005)

p, q neighbors define

a vector distance;

Filter with this:

$$\| \mathbf{V_p} - \mathbf{V_q} \|^2$$



No spatial term!

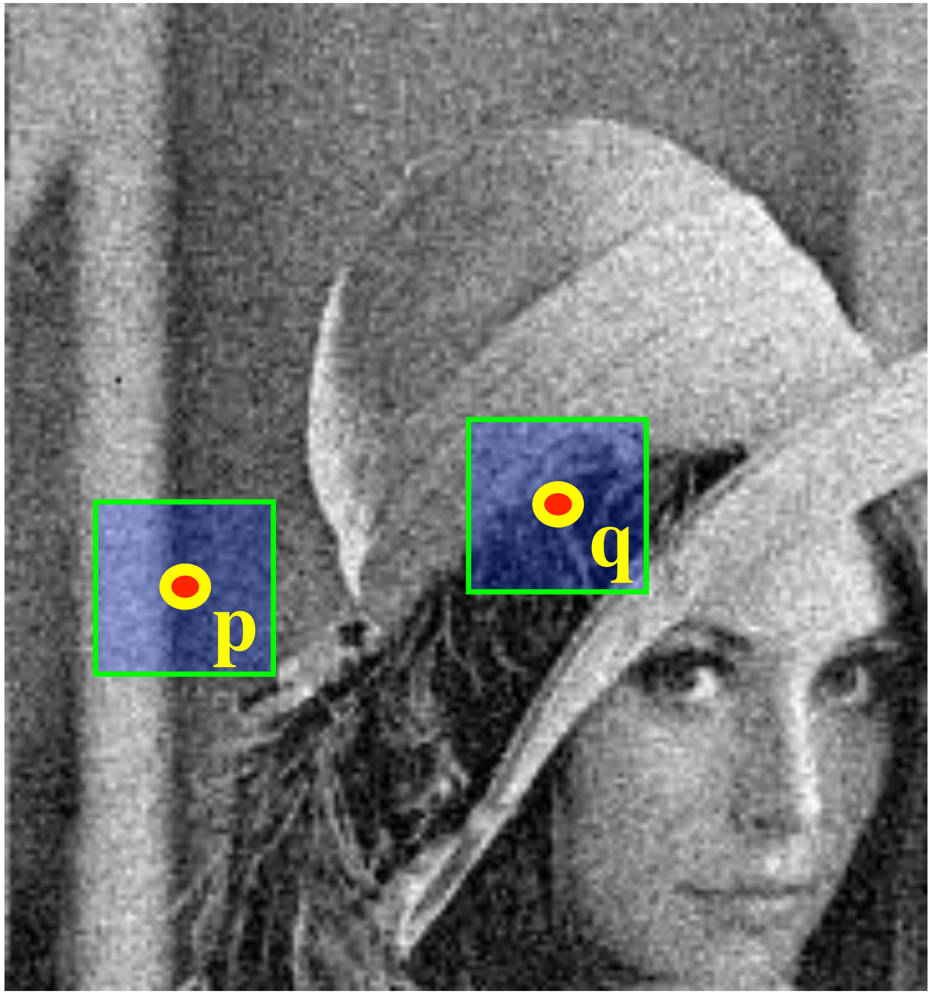
$$NLMF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} \cancel{G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|)} \, G_{\sigma_r}\left(\| \vec{V}_{\mathbf{p}} - \vec{V}_{\mathbf{q}} \|^2 \right) I_{\mathbf{q}}$$

NL-Means Method: Buades (2005)

pixels **p, q** neighbors
Set a vector distance;

$$\boxed{\| \mathbf{V}_{\mathbf{p}} - \mathbf{V}_{\mathbf{q}} \|^2}$$

Vector Distance to p sets weight for each pixel q



$$NLMF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_r}\left(\| \vec{V}_{\mathbf{p}} - \vec{V}_{\mathbf{q}} \|^2 \right) I_{\mathbf{q}}$$

NL-Means Filter (Buades 2005)

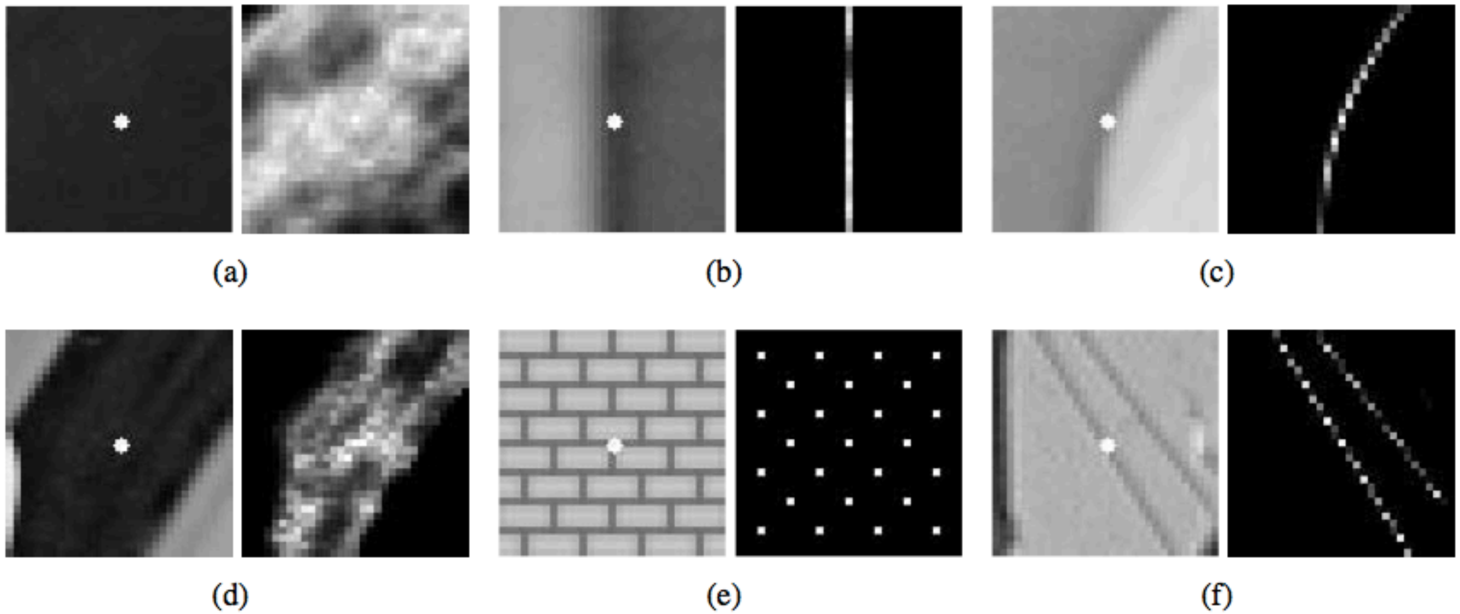


Figure 2. Display of the NL-means weight distribution used to estimate the central pixel of every image. The weights go from 1(white) to zero(black).

NL-Means Filter (Buades 2005)

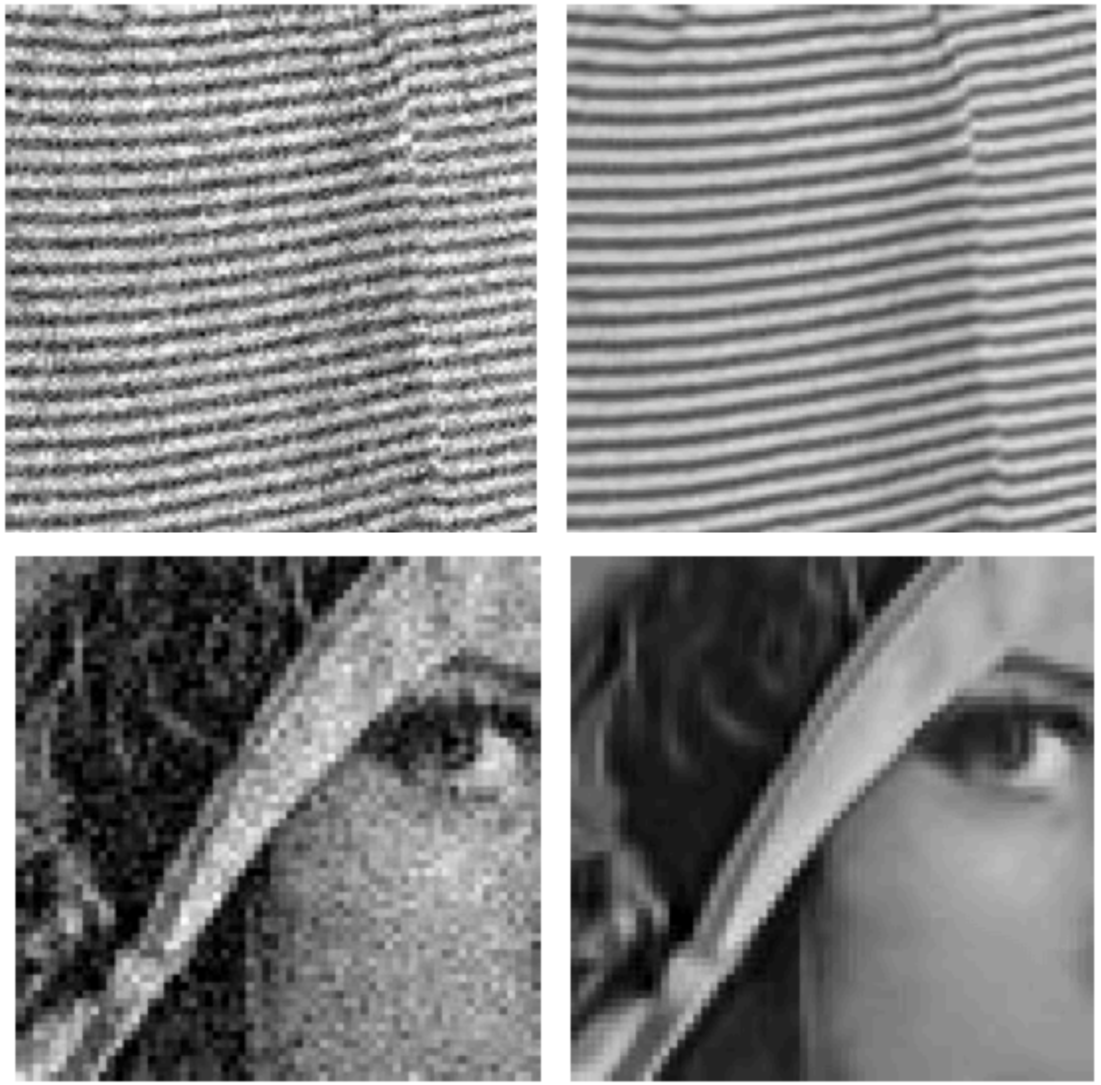


FIG. 9. *NL-means denoising experiment with a natural image. Left: Noisy image with standard deviation* 20. *Right: Restored image.*

NL-Means Filter (Buades 2005)

- Noisy source image:

NL-Means Filter (Buades 2005)

- Gaussian Filter

Low noise,

Low detail

NL-Means Filter (Buades 2005)

- Anisotropic Diffusion

(Note 'stairsteps': ~ piecewise constant)

NL-Means Filter (Buades 2005)

- Bilateral Filter

(better, but similar 'stairsteps':

NL-Means Filter (Buades 2005)

- NL-Means:

Sharp,

Low noise,

Few artifacts.

NL-Means Filter (Buades 2005)

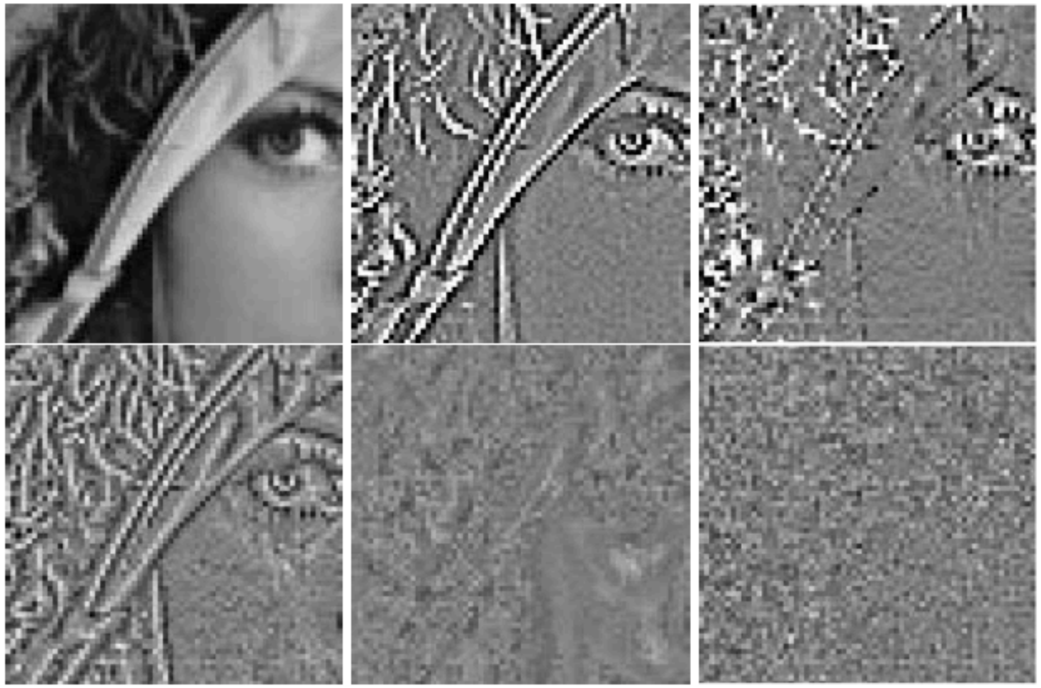


Figure 4. Method noise experience on a natural image. Displaying of the image difference $u - D_h(u)$. From left to right and from top to bottom: original image, Gauss filtering, anisotropic filtering, Total variation minimization, Neighborhood filtering and NL-means algorithm. The visual experiments corroborate the formulas of section 2.

NL-Means Filter (Buades 2005)

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

original

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

noisy

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

denoised

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

original

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

noisy

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

denoised

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/