BBM 413 Fundamentals of Image Processing

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Image Pyramids

Review – Frequency Domain Techniques

- The name "filter" is borrowed from frequency domain processing
- Accept or reject certain frequency components
- Fourier (1807):
  Periodic functions could be represented as a weighted sum of sines and cosines

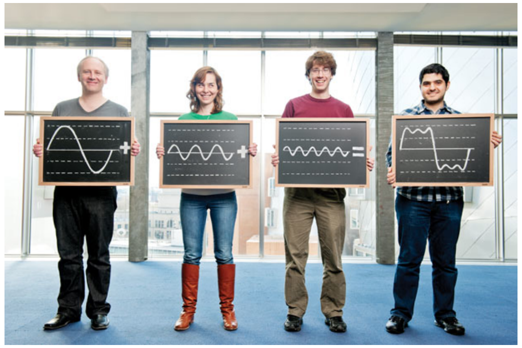


Image courtesy of Technology Review

Review - Fourier Transform

We want to understand the frequency *w* of our signal. So, let's reparametrize the signal by *w* instead of *x*:

f(x) → Fourier Transform → ***F(w)***

For every *w* from 0 to inf, ***F(w)*** holds the amplitude *A* and phase *f* of the corresponding sine $A\sin(\omega x+\phi)$

- How can *F* hold both? Complex number trick!

$$F(\omega)=R(\omega)+iI(\omega)$$

$$A=\pm\sqrt{R(\omega)^2+I(\omega)^2} \qquad \phi=\tan^{-1}\frac{I(\omega)}{R(\omega)}$$

We can always go back:

F(w) → Inverse Fourier Transform → ***f(x)***

Slide credit: A. Efros

Review - The Discrete Fourier transform

- Forward transform

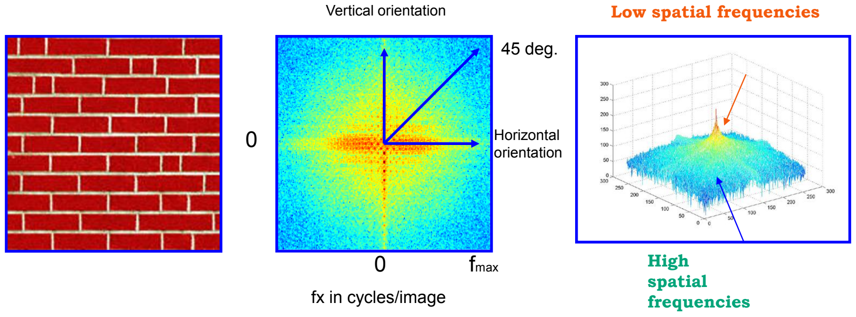
$$F[m,n]=\sum_{k=0}^{M-1}\sum_{l=0}^{N-1}f[k,l]e^{-\pi i\left(\frac{km}{M}+\frac{ln}{N}\right)}$$

- Inverse transform

$$f[k,l]=\frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1}F[m,n]e^{+\pi i\left(\frac{km}{M}+\frac{ln}{N}\right)}$$

Slide credit: B. Freeman and A. Torralba

Review - The Discrete Fourier transform



Log power spectrum

Slide credit: B. Freeman and A. Torralba

Review - The Convolution Theorem

- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

$$F[g * h] = F[g]F[h]$$

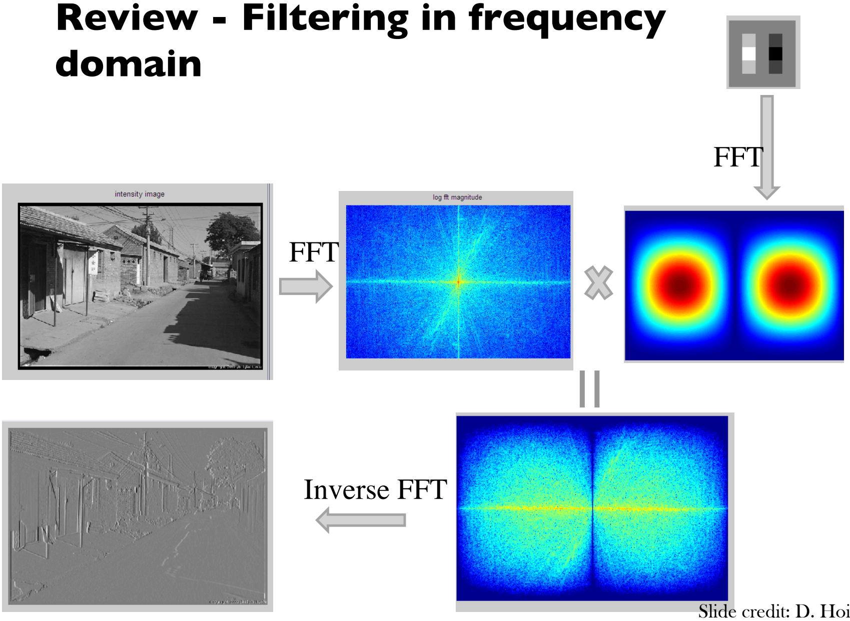
- The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms

$$F^{-1}[gh] = F^{-1}[g] * F^{-1}[h]$$

- **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

Slide credit: A. Efros

Review - Filtering in frequency domain

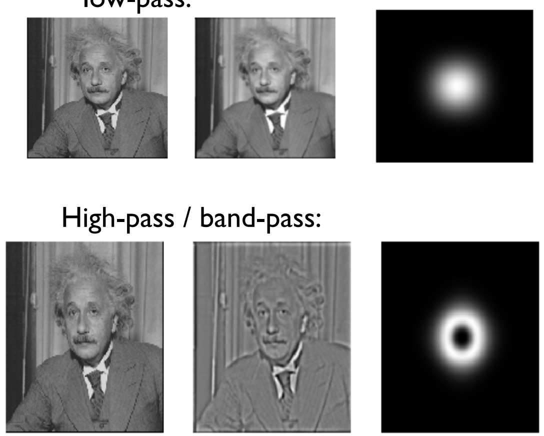


Slide credit: D. Hoiem

Review - Low-pass, Band-pass, High-pass filters

low-pass:

High-pass / band-pass:

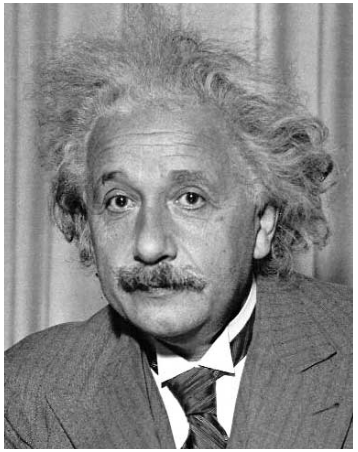


Slide credit: A. Efros

Template matching

- Goal: find  in image

- Main challenge: What is a good similarity or distance measure between two patches?
  - Correlation
  - Zero-mean correlation
  - Sum Square Difference
  - Normalized Cross Correlation



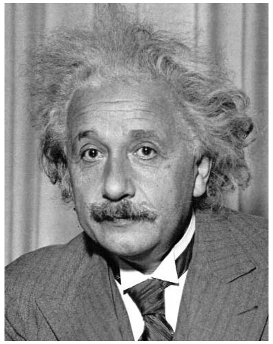
Slide: Hoiem

Matching with filters

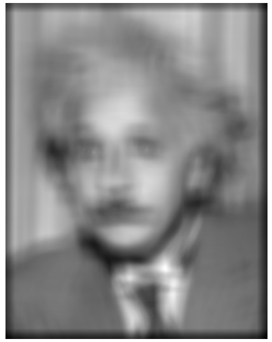
- Goal: find  in image
- Method 0: filter the image with eye patch

$$h[m,n]=\sum_{k,l} g[k,l]\, f[m+k,n+l]$$

f = image
g = filter



Input



Filtered Image

What went wrong?

response is stronger for higher intensity

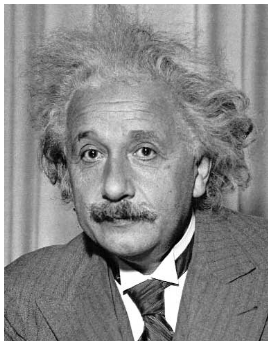
Slide: Hoiem

Matching with filters

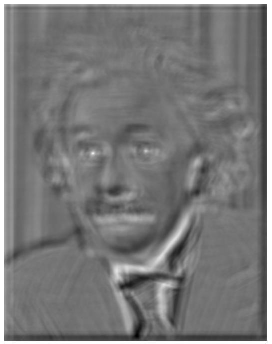
- Goal: find  in image
- Method 1: filter the image with zero-mean eye

$$h[m,n]=\sum_{k,l}(f[k,l]-\bar{f})\,(g[m+k,n+l])$$

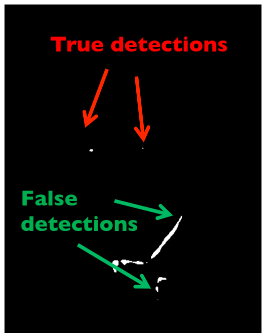
mean of f



Input



Filtered Image (scaled)



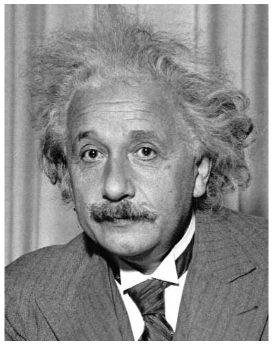
Thresholded Image

Slide: Hoiem

Matching with filters

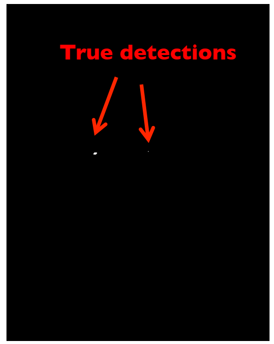
- Goal: find  in image
- Method 2: SSD

$$h[m,n]=\sum_{k,l}(g[k,l]-f[m+k,n+l])^2$$



Input

1- sqrt(SSD)



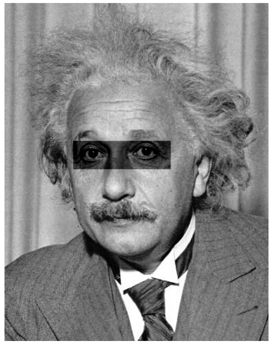
Thresholded Image

Slide: Hoiem

Matching with filters

- Goal: find  in image
- Method 2: SSD

$$h[m,n]=\sum_{k,l}(g[k,l]-f[m+k,n+l])^2$$



Input

1- sqrt(SSD)

What's the potential downside of SSD?

SSD sensitive to average intensity

Slide: Hoiem

Matching with filters

- Goal: find  in image
- Method 3: Normalized cross-correlation

mean template　　mean image patch

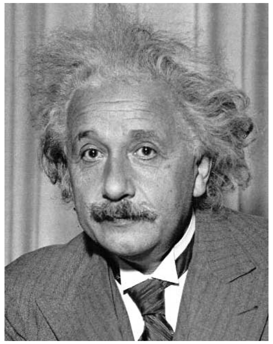
$$h[m,n]=\frac{\sum_{k,l}(g[k,l]-\bar{g})(f[m-k,n-l]-\bar{f}_{m,n})}{\left(\sum_{k,l}(g[k,l]-\bar{g})^2\sum_{k,l}(f[m-k,n-l]-\bar{f}_{m,n})^2\right)^{0.5}}$$

Matlab: `normxcorr2(template, im)`

Slide: Hoiem

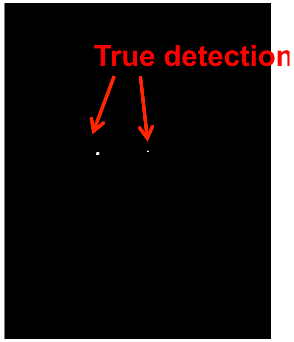
Matching with filters

- Goal: find  in image
- Method 3: Normalized cross-correlation



Input

Normalized X-Correlation

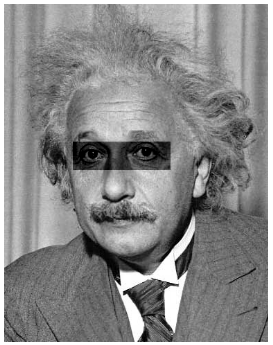


Thresholded Image

Slide: Hoiem

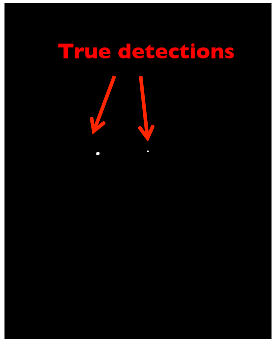
Matching with filters

- Goal: find  in image
- Method 3: Normalized cross-correlation



Input

Normalized X-Correlation



Thresholded Image

Slide: Hoiem

Q: What is the best method to use?

A: Depends

- SSD: faster, sensitive to overall intensity
- Normalized cross-correlation: slower, invariant to local average intensity and contrast

Slide: R. Pless

Q: What if we want to find larger or smaller eyes?

A: Image Pyramid

Slide: R. Pless

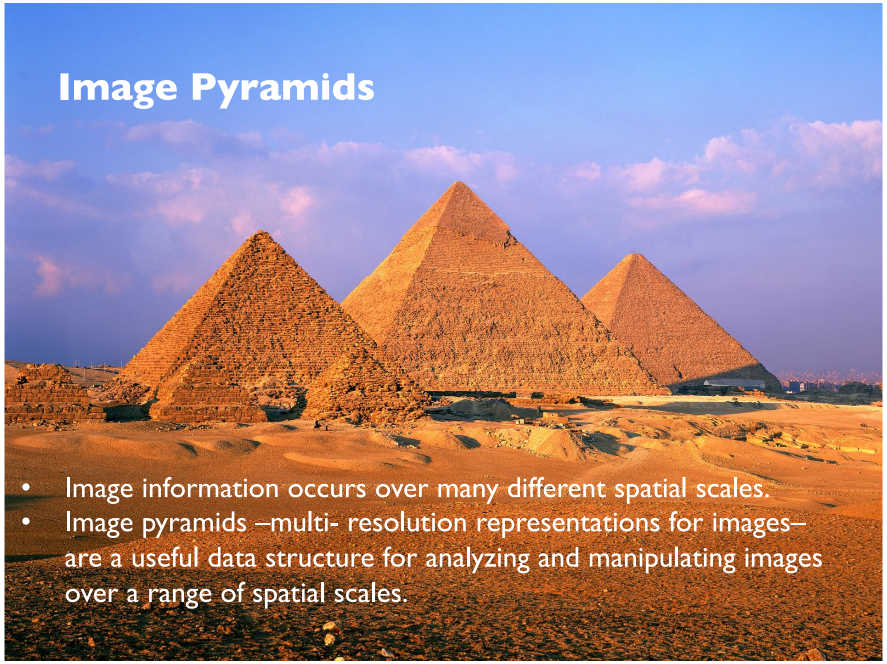


Image pyramids

Image information occurs at all spatial scales

- Gaussian pyramid
- Laplacian pyramid
- Wavelet/QMF pyramid
- Steerable pyramid

Slide credit: B. Freeman and A. Torralba

Image pyramids

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Review of Sampling

Slide: Hoiem

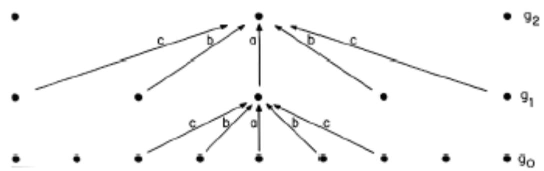
The Gaussian pyramid

- Smooth with Gaussians, because
  - A Gaussian*Gaussian = another Gaussian
- Gaussians are low pass filters, so representation is redundant.
- Gaussian pyramid creates versions of the input image at multiple resolutions.
- This is useful for analysis across different spatial scales, but doesn't separate the image into different frequency bands.

Slide adapted from: B. Freeman and A. Torralba

The computational advantage of pyramids

GAUSSIAN PYRAMID



g_0 = IMAGE

g_L = REDUCE [g_{L-1}]

Fig 1. A one-dimensional graphic representation of the process which generates a Gaussian pyramid Each row of dots represents nodes within a level of the pyramid. The value of each node in the zero level is just the gray level of a corresponding image pixel. The value of each node in a high level is the weighted average of node values in the next lower level. Note that node spacing doubles from level to level, while the same weighting pattern or "generating kernel" is used to generate all levels.

[Burt and Adelson, 1983]

Slide credit: B. Freeman and A. Torralba

The Gaussian Pyramid

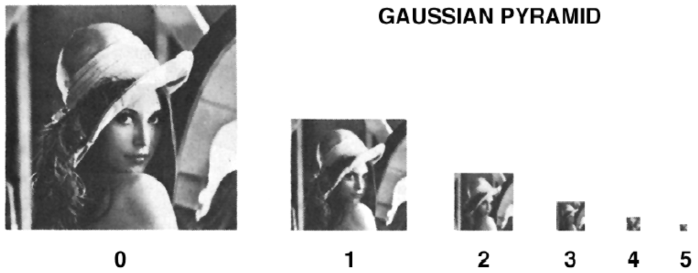
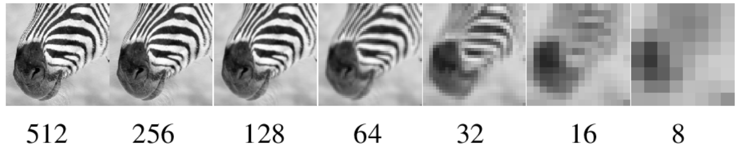


Fig. 4. First six levels of the Gaussian pyramid for the "Lady" image The original image, level 0, meusures 257 by 257 pixels and each higher level array is roughly half the dimensdons of its predecessor. Thus, level 5 measures just 9 by 9 pixels.

[Burt and Adelson, 1983]

Slide credit: B. Freeman and A. Torralba



Slide credit: B. Freeman and A. Torralba

Convolution and subsampling as a matrix multiply (1D case)

$$x_2 = G_1 x_1$$

$G_1 =$

$$\begin{matrix}
1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0
\end{matrix}$$

Slide credit: B. Freeman and A. Torralba

(Normalization constant of 1/16 omitted for visual clarity.)

Next pyramid level

$$x_3 = G_2 x_2$$

$G_2 =$

$$\begin{matrix}
1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 4
\end{matrix}$$

Slide credit: B. Freeman and A. Torralba

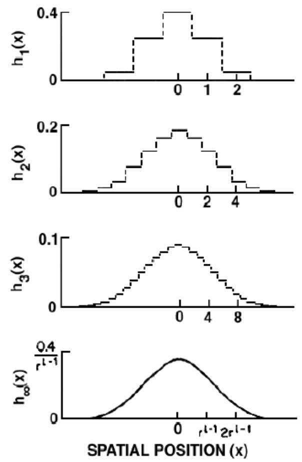
The combined effect of the two pyramid levels

$$x_3 = G_2 G_1 x_1$$

$G_2 G_1 =$

1	4	10	20	31	40	44	40	31	20	10	4	1	0	0	0	0	0	0	0
0	0	0	0	1	4	10	20	31	40	44	40	31	20	10	4	1	0	0	0
0	0	0	0	0	0	0	0	1	4	10	20	31	40	44	40	30	16	4	0
0	0	0	0	0	0	0	0	0	0	0	0	1	4	10	20	25	16	4	0

Slide credit: B. Freeman and A. Torralba



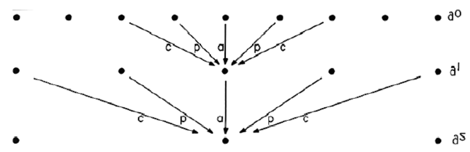


Fig. 2. The equivalent weighting functions $h_l(x)$ for nodes in levels 1, 2, 3, and infinity of the Gaussian pyramid. Note that axis scales have been adjusted by factors of 2 to aid comparison Here the parameter a of the generating kernel is 0.4, and the resulting equivalent weighting functions closely resemble the Gaussian probability density functions.

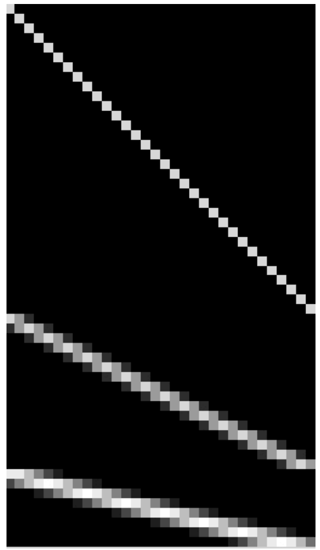
Slide credit: B. Freeman and A. Torralba

Gaussian pyramids used for

- up- or down- sampling images.
- Multi-resolution image analysis
  - Look for an object over various spatial scales
  - Coarse-to-fine image processing: form blur estimate or the motion analysis on very low-resolution image, upsample and repeat. Often a successful strategy for avoiding local minima in complicated estimation tasks.

Slide credit: B. Freeman and A. Torralba

1D Gaussian pyramid matrix, for [1 4 6 4 1] low-pass filter



Slide credit: B. Freeman and A. Torralba

Template Matching with Image Pyramids

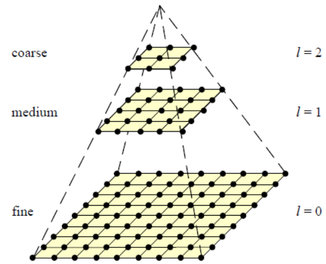
Input: Image, Template

1. Match template at current scale
2. Downsample image
3. Repeat 1-2 until image is very small
4. Take responses above some threshold, perhaps with non-maxima suppression

Slide: Hoiem

Coarse-to-fine Image Registration

1. Compute Gaussian pyramid
2. Align with coarse pyramid
3. Successively align with finer pyramids
   - Search smaller range



Why is this faster?

Are we guaranteed to get the same result?

Slide: Hoiem

Image pyramids

Image information occurs at all spatial scales

- Gaussian pyramid
- Laplacian pyramid
- Wavelet/QMF pyramid
- Steerable pyramid

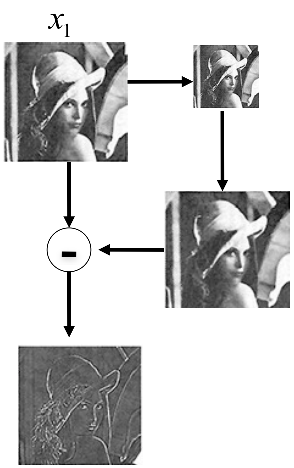
Slide credit: B. Freeman and A. Torralba

The Laplacian Pyramid

- Synthesis
  - Compute the difference between upsampled Gaussian pyramid level and Gaussian pyramid level.
  - band pass filter - each level represents spatial frequencies (largely) unrepresented at other level.
- Laplacian pyramid provides an extra level of analysis as compared to Gaussian pyramid by breaking the image into different isotropic spatial frequency bands.

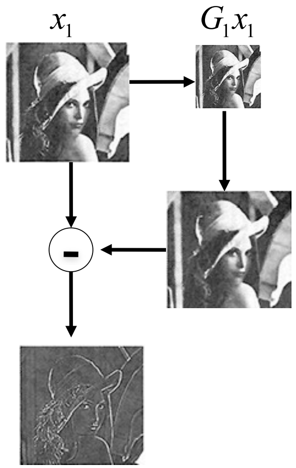
Slide adapted from: B. Freeman and A. Torralba

The Laplacian Pyramid



Slide credit: B. Freeman and A. Torralba

The Laplacian Pyramid



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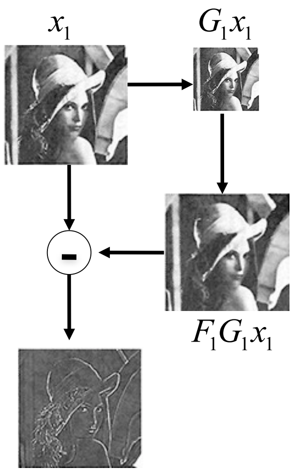
The Laplacian Pyramid



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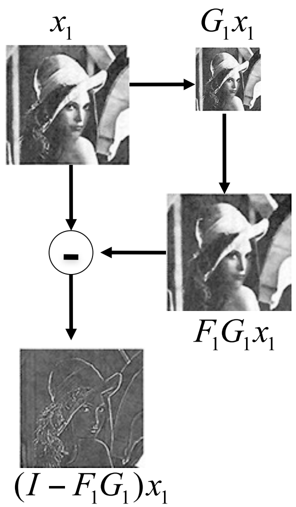
The Laplacian Pyramid



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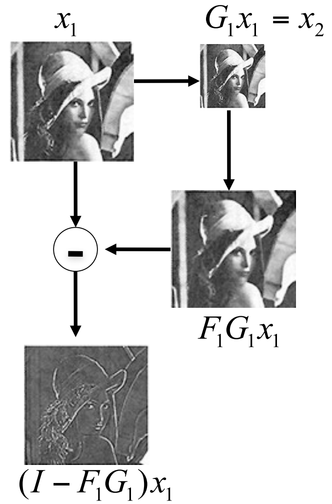
The Laplacian Pyramid



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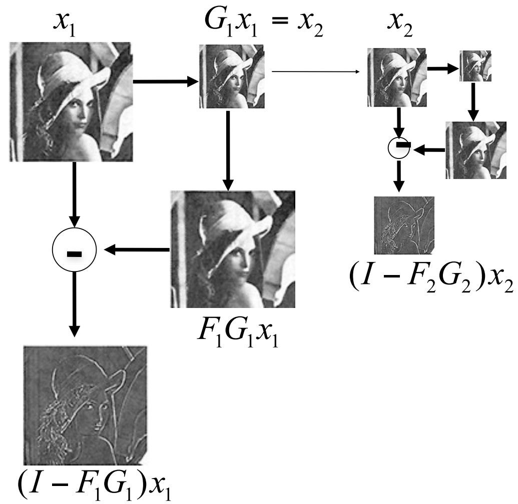

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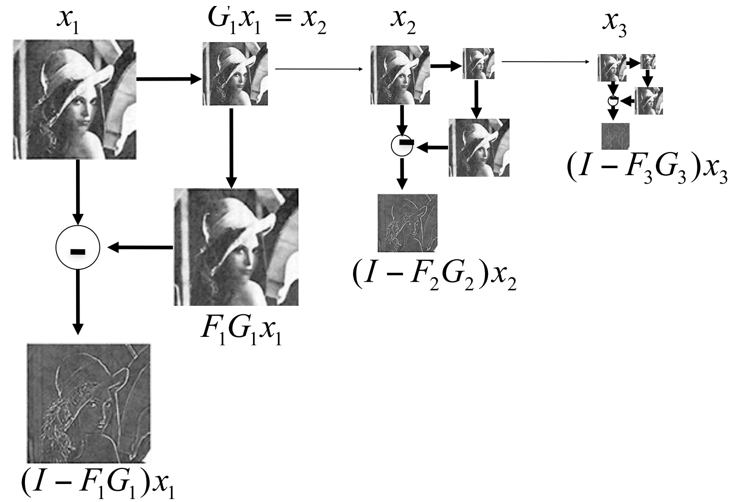
The Laplacian Pyramid



Slide credit: B. Freeman and A. Torralba

Upsampling

$$y_2 = F_3 x_3$$

Insert zeros between pixels, then apply a low-pass filter, [1 4 6 4 1]

$$F_3 = \begin{matrix} 6 & 1 & 0 & 0 \\ 4 & 4 & 0 & 0 \\ 1 & 6 & 1 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & 1 & 6 & 1 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 4 \end{matrix}$$

Slide credit: B. Freeman and A. Torralba

Showing, at full resolution, the information captured at each level of a Gaussian (top) and Laplacian (bottom) pyramid.

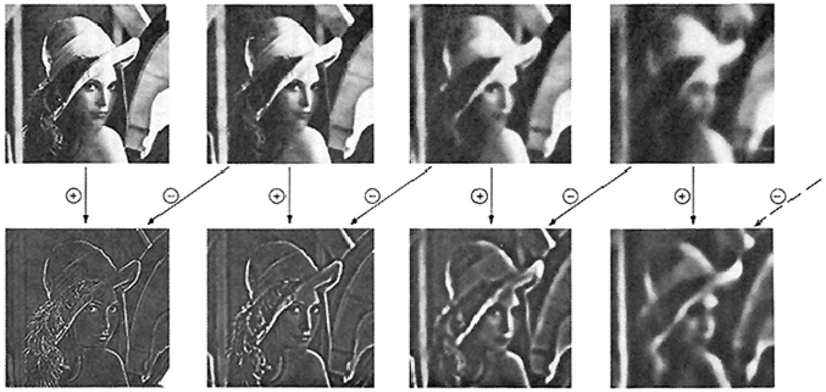


Fig 5. First four levels of the Gaussian and Laplacian pyramid. Gaussian images, upper row, were obtained by expanding pyramid arrays (Fig. 4) through Gaussian interpolation. Each level of the Laplacian pyramid is the difference between the corresponding and next higher levels of the Gaussian pyramid.

Slide credit: B. Freeman and A. Torralba

Laplacian pyramid reconstruction algorithm: recover x_1 from L_1, L_2, L_3 and x_4

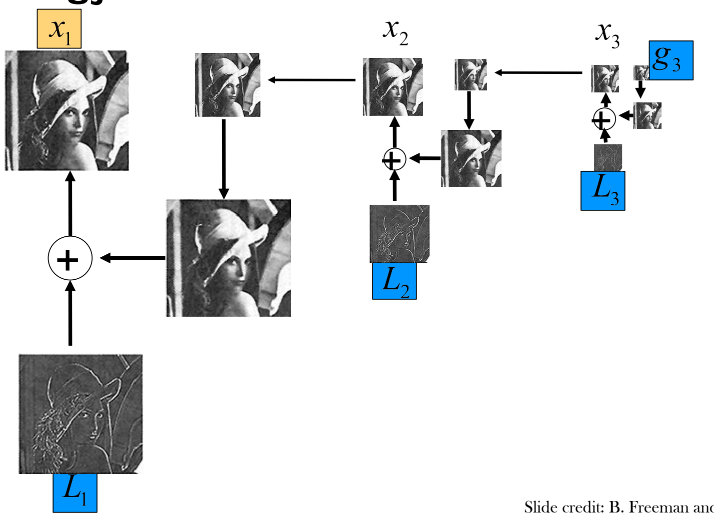
G# is the blur-and-downsample operator at pyramid level #
F# is the blur-and-upsample operator at pyramid level #

Laplacian pyramid elements:
L1 = (I – F1 G1) x1
L2 = (I – F2 G2) x2
L3 = (I – F3 G3) x3
x2 = G1 x1
x3 = G2 x2
x4 = G3 x3

Reconstruction of original image (x1) from Laplacian pyramid elements:
x3 = L3 + F3 x4
x2 = L2 + F2 x3
x1 = L1 + F1 x2

Slide credit: B. Freeman and A. Torralba

Laplacian pyramid reconstruction algorithm: recover x_1 from L_1, L_2, L_3 and g_3



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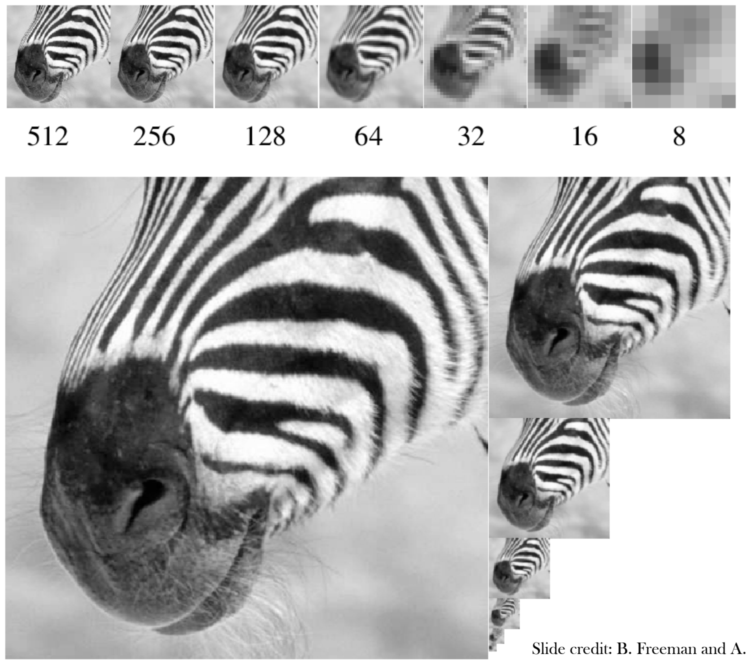

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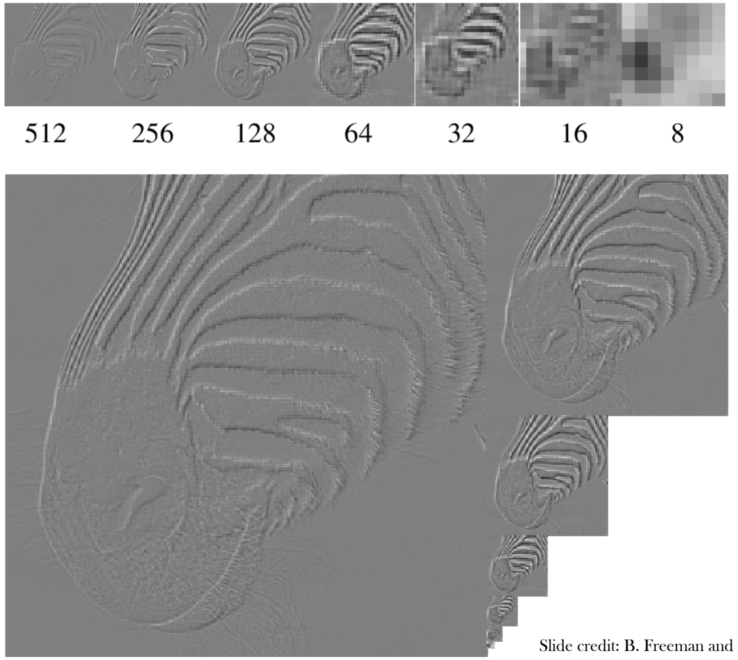
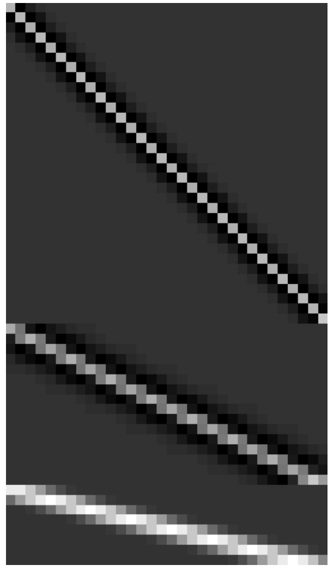


Slide credit: B. Freeman and A. Torralba

1D Laplacian pyramid matrix, for [1 4 6 4 1] low-pass filter



Slide credit: B. Freeman and A. Torralba

Laplacian pyramid applications

- Texture synthesis
- Image compression
- Noise removal

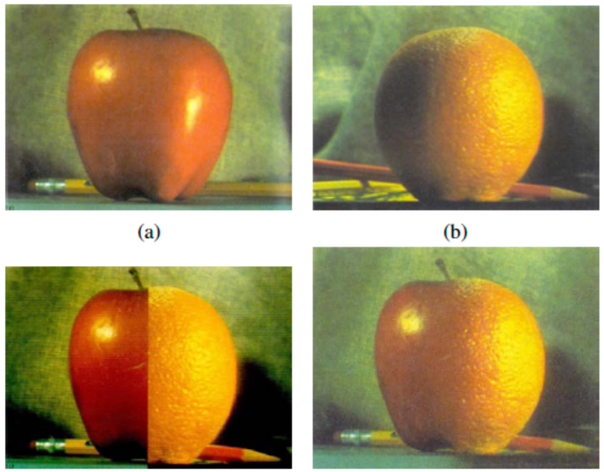
IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. COM-31, NO. 4, APRIL 1983

The Laplacian Pyramid as a Compact Image Code

PETER J. BURT, MEMBER, IEEE, AND EDWARD H. ADELSON

Slide credit: B. Freeman and A. Torralba

Image blending



Slide credit: B. Freeman and A. Torralba

Szeliski, Computer Vision, 2010

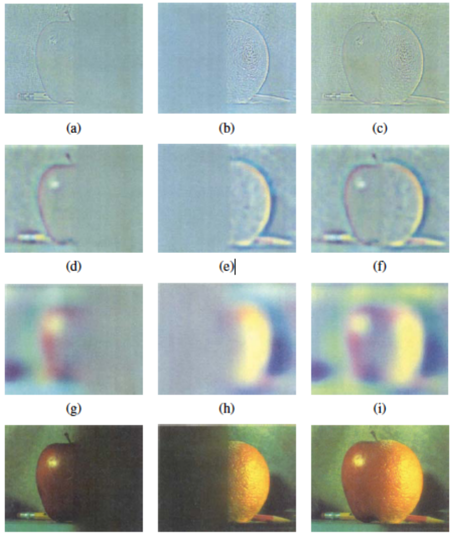
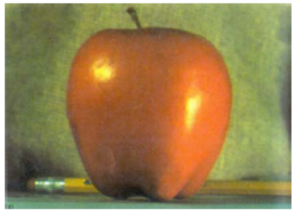
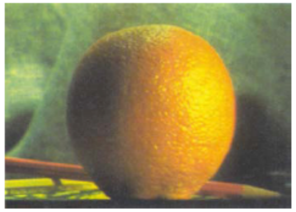


Figure 3.42 Laplacian pyramid blending details (Burt and Adelson 1983b) © 1983 ACM. The first three rows show the high, medium, and low frequency parts of the Laplacian pyramid (taken from levels 0, 2, and 4). The left and middle columns show the original apple and orange images weighted by the smooth interpolation functions, while the right column shows the averaged contributions.

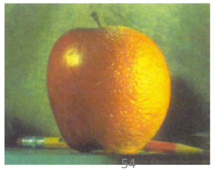
Slide credit: B. Freeman & A. Torralba

Image blending





- Build Laplacian pyramid for both images: LA, LB
- Build Gaussian pyramid for mask: G
- Build a combined Laplacian pyramid: L(j) = G(j) LA(j) + (1-G(j)) LB(j)
- Collapse L to obtain the blended image



Slide credit: B. Freeman and A. Torralba

Image pyramids

Image information occurs at all spatial scales

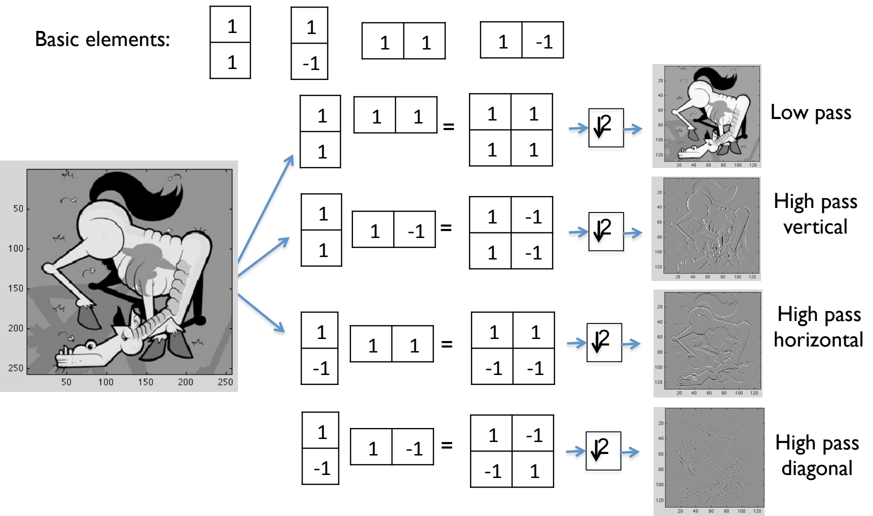
- Gaussian pyramid
- Laplacian pyramid
- Wavelet/QMF pyramid
- Steerable pyramid

Slide credit: B. Freeman and A. Torralba

Wavelet/QMF pyramid

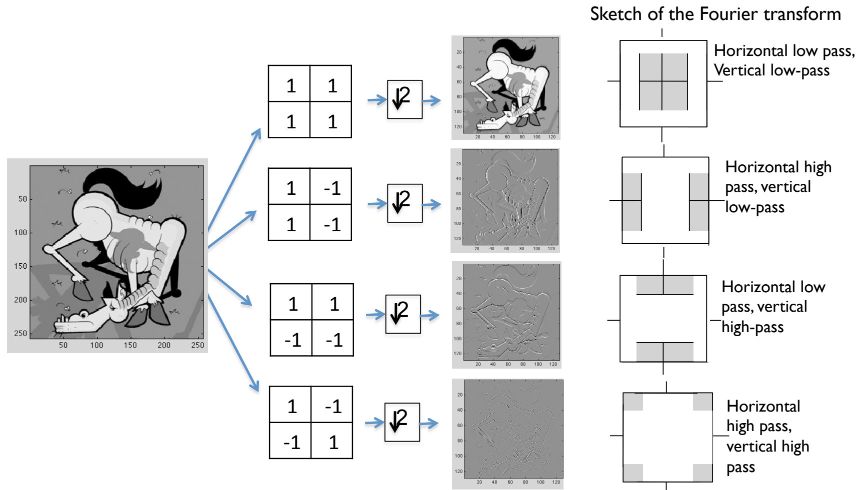
- Subband coding
- Wavelet or QMF (quadrature mirror filter) pyramid provides some splitting of the spatial frequency bands according to orientation (although in a somewhat limited way).
- Image is decomposed into a set of band-limited components (subbands).
- Original image can be reconstructed without error by reassembling these subbands.

2D Haar transform



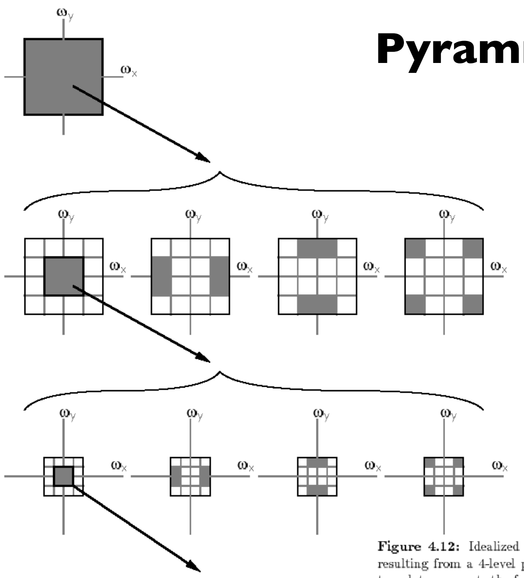
Slide credit: B. Freeman and A. Torralba

2D Haar transform



Slide credit: B. Freeman and A. Torralba

Pyramid cascade



Simoncelli and Adelson, in "Subband coding", Kluwer, 1990.

Figure 4.12: Idealized diagram of the partition of the frequency plane resulting from a 4-level pyramid cascade of separable 2-band filters. The top plot represents the frequency spectrum of the original image, with axes ranging from $-\pi$ to π. This is divided into four subbands at the next level. On each subsequent level, the lowpass subband (outlined in bold) is subdivided further.

Slide credit: B. Freeman and A. Torralba

Wavelet/QMF representation

Same number of pixels!

Slide credit: B. Freeman and A. Torralba

Image pyramids

Image information occurs at all spatial scales

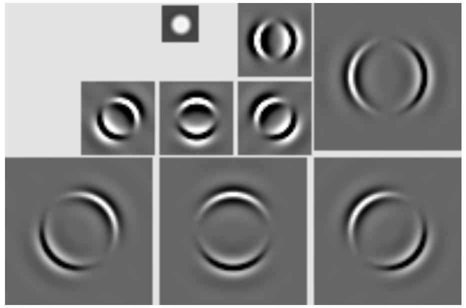
- Gaussian pyramid
- Laplacian pyramid
- Wavelet/QMF pyramid
- Steerable pyramid

Slide credit: B. Freeman and A. Torralba

Steerable Pyramid

2 Level decomposition of white circle example:

Low pass residual



Subbands

- The Steerable pyramid provides a clean separation of the image into different scales and orientations.

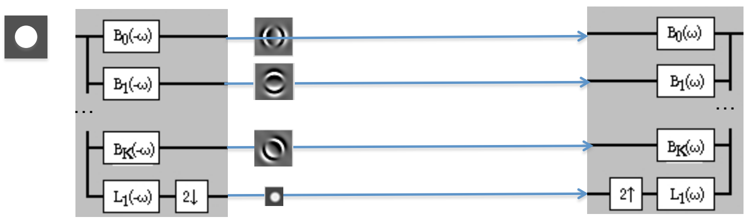
Images from: http://www.cis.upenn.edu/~eero/steerpyr.html

Slide credit: B. Freeman and A. Torralba

Steerable Pyramid

We may combine Steerability with Pyramids to get a Steerable Laplacian Pyramid as shown below.

Decomposition **Reconstruction**



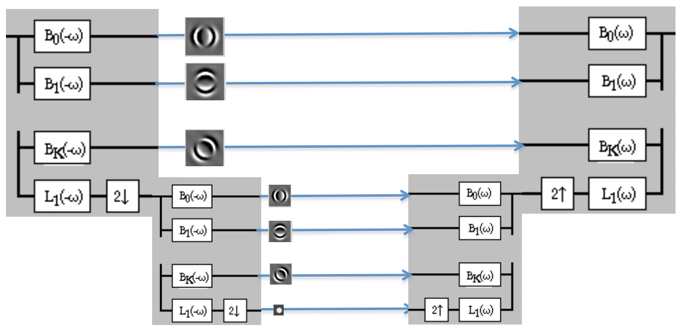
Images from: http://www.cis.upenn.edu/~eero/steerpyr.html

Slide credit: B. Freeman and A. Torralba

Steerable Pyramid

We may combine Steerability with Pyramids to get a Steerable Laplacian Pyramid as shown below

Decomposition **Reconstruction**



Images from: http://www.cis.upenn.edu/~eero/steerpyr.html

Slide credit: B. Freeman and A. Torralba

Steerable Pyramid

But we need to get rid of the corner regions before starting the recursive circular filtering

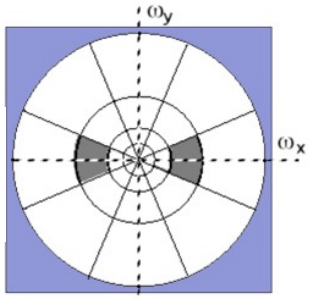
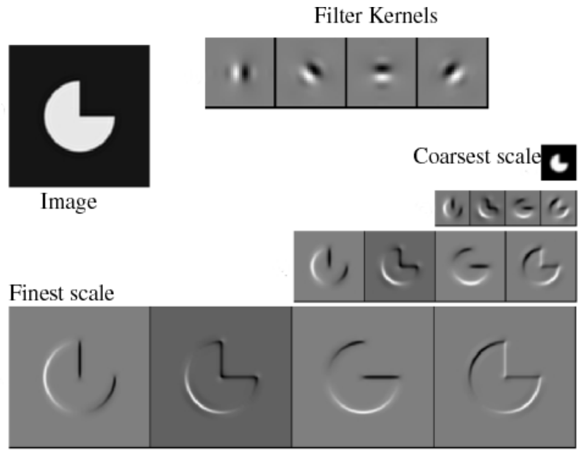


Figure 1. Idealized illustration of the spectral decomposition performed by a steerable pyramid with $k = 4$. Frequency axes range from $-\pi$ to π. The basis functions are related by translations, dilations and *rotations* (except for the initial highpass subband and the final lowpass subband). For example, the shaded region corresponds to the spectral support of a single (vertically-oriented) subband.

Simoncelli and Freeman, ICIP 1995

Slide credit: B. Freeman and A. Torralba



Reprinted from "Shiftable MultiScale Transforms," by Simoncelli et al., IEEE Transactions on Information Theory, 1992, copyright 1992, IEEE

There is also a high pass residual…

Slide credit: B. Freeman and A. Torralba

Image pyramids

- Gaussian
- Laplacian
- Wavelet/QMF
- Steerable pyramid

Slide credit: B. Freeman and A. Torralba

Image pyramids

- Gaussian

Progressively blurred and subsampled versions of the image. Adds scale invariance to fixed-size algorithms.

- Laplacian
- Wavelet/QMF
- Steerable pyramid

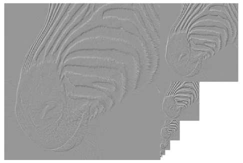
Slide credit: B. Freeman and A. Torralba

Image pyramids

- Gaussian

Progressively blurred and subsampled versions of the image. Adds scale invariance to fixed-size algorithms.

- Laplacian



Shows the information added in Gaussian pyramid at each spatial scale. Useful for noise reduction & coding.

- Wavelet/QMF
- Steerable pyramid

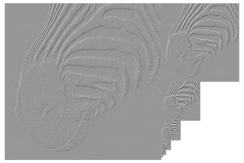
Slide credit: B. Freeman and A. Torralba

Image pyramids

- Gaussian

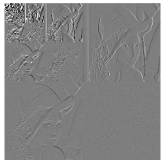
Progressively blurred and subsampled versions of the image. Adds scale invariance to fixed-size algorithms.

- Laplacian



Shows the information added in Gaussian pyramid at each spatial scale. Useful for noise reduction & coding.

- Wavelet/QMF



Bandpassed representation, complete, but with aliasing and some non-oriented subbands.

- Steerable pyramid

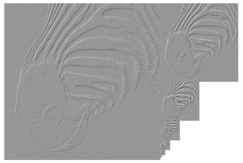
Slide credit: B. Freeman and A. Torralba

Image pyramids

- Gaussian

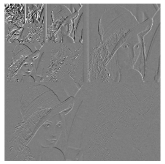
Progressively blurred and subsampled versions of the image. Adds scale invariance to fixed-size algorithms.

- Laplacian



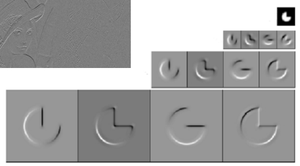
Shows the information added in Gaussian pyramid at each spatial scale. Useful for noise reduction & coding.

- Wavelet/QMF



Bandpassed representation, complete, but with aliasing and some non-oriented subbands.

- Steerable pyramid



Shows components at each scale and orientation separately. Non-aliased subbands. Good for texture and feature analysis. But overcomplete and with HF residual.

Slide credit: B. Freeman and A. Torralba

Schematic pictures of each matrix transform

Shown for 1-d images

The matrices for 2-d images are the same idea, but more complicated, to account for vertical, as well as horizontal, neighbor relationships.

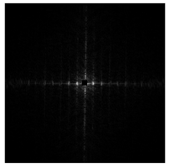
transformed image

$$\vec{F} = U\vec{f}$$

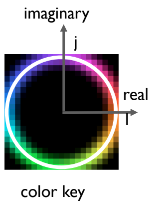
Vectorized image

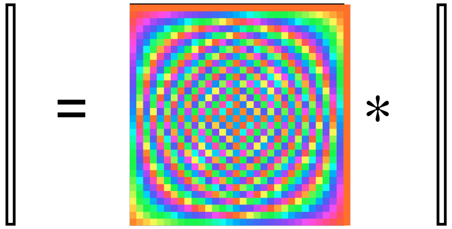
Fourier transform, or
Wavelet transform, or
Steerable pyramid transform

Slide credit: B. Freeman and A. Torralba



Fourier transform





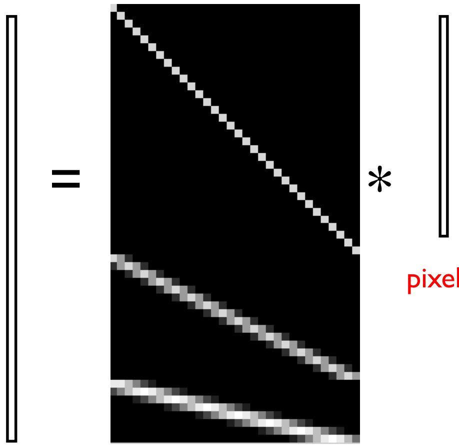
Fourier transform

Fourier bases are global: each transform coefficient depends on all pixel locations.

pixel domain image

Slide credit: B. Freeman and A. Torralba

Gaussian pyramid

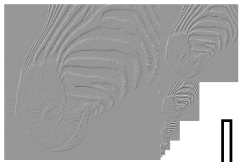


Gaussian pyramid

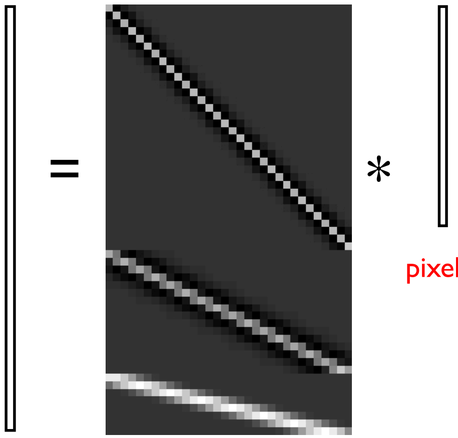
pixel image

Overcomplete representation. Low-pass filters, sampled appropriately for their blur.

Slide credit: B. Freeman and A. Torralba



Laplacian pyramid



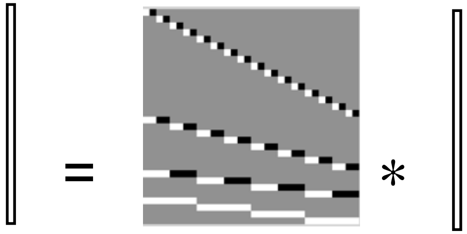
Laplacian pyramid

pixel image

Overcomplete representation. Transformed pixels represent bandpassed image information.

Slide credit: B. Freeman and A. Torralba

Wavelet (QMF) transform



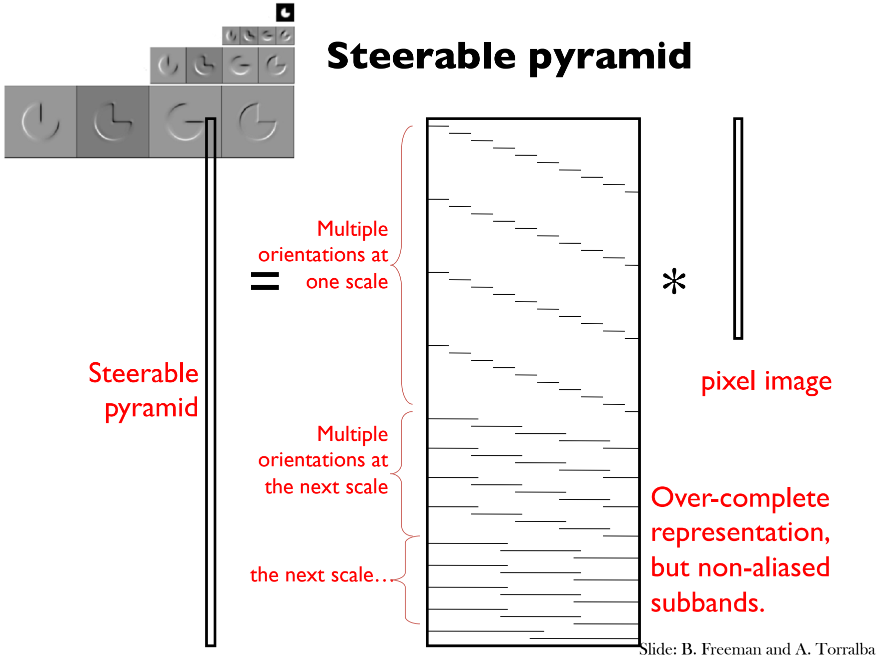
Wavelet pyramid

Ortho-normal transform (like Fourier transform), but with localized basis functions.

pixel image

Slide credit: B. Freeman and A. Torralba

Steerable pyramid



Why use image pyramids?

- Handle real-world size variations with a constant-size vision algorithm.
- Remove noise
- Analyze texture
- Recognize objects
- Label image features
- Image priors can be specified naturally in terms of wavelet pyramids.

Slide credit: B. Freeman and A. Torralba

Reading Assignment #3 – Hybrid Images

- A. Oliva, A. Torralba, P.G. Schyns (2006). Hybrid Images. ACM Transactions on Graphics, ACM SIGGRAPH, 25-3, 527-530.
- Due on 20th of December