### BBM 413 Fundamentals of Image Processing

Erkut Erdem
Dept. of Computer Engineering
Hacettepe University

Image Pyramids

### **Review - Fourier Transform**

We want to understand the frequency w of our signal. So, let's reparametrize the signal by w instead of x:

$$f(x)$$
 Fourier  $\longrightarrow$  F(w)

For every w from 0 to inf, F(w) holds the amplitude A and phase f of the corresponding sine  $A\sin(\omega x + \phi)$ 

• How can Fhold both? Complex number trick!

$$F(\omega) = R(\omega) + iI(\omega)$$

$$A = \pm \sqrt{R(\omega)^2 + I(\omega)^2} \qquad \phi = \tan^{-1} \frac{I(\omega)}{R(\omega)}$$

We can always go back:

$$F(w)$$
 Inverse Fourier  $f(x)$  Slide credit: A. Efros

# Review – Frequency Domain Techniques

- The name "filter" is borrowed from frequency domain processing
- Accept or reject certain frequency components
- Fourier (1807):
  Periodic functions
  could be represented
  as a weighted sum of
  sines and cosines

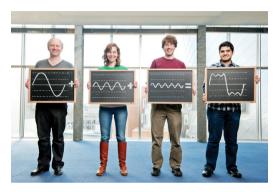


Image courtesy of Technology Review

### **Review - The Discrete Fourier transform**

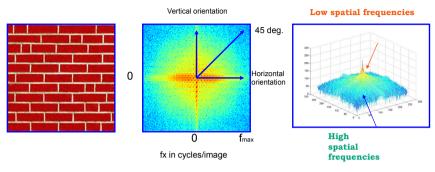
Forward transform

$$F[m,n] = \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} f[k,l] e^{-\pi i \left(\frac{km}{M} + \frac{\ln n}{N}\right)}$$

Inverse transform

$$f[k,l] = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} F[m,n] e^{+\pi l \left(\frac{km}{M} + \frac{\ln n}{N}\right)}$$

### **Review - The Discrete Fourier transform**



Log power spectrum

Slide credit: B. Freeman and A. Torralba

### **Review - The Convolution Theorem**

• The Fourier transform of the convolution of two functions is the product of their Fourier transforms

$$F[g * h] = F[g]F[h]$$

• The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms

$$F^{-1}[gh] = F^{-1}[g] * F^{-1}[h]$$

• Convolution in spatial domain is equivalent to multiplication in frequency domain!

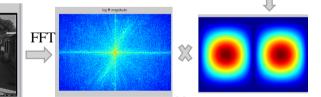
Slide credit: A. Efros

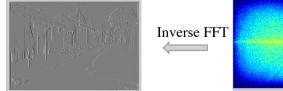
## Review - Filtering in frequency domain

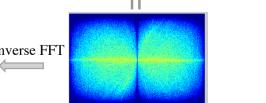


Slide credit: D. Hoiem

FFT







# Review - Low-pass, Band-pass, High-pass filters

low-pass:



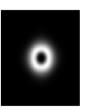




High-pass / band-pass:







Slide credit: A. Efros

### **Today - Image pyramids**

- Gaussian pyramid
- Laplacian pyramid
- Wavelet/QMF pyramid
- Steerable pyramid

Slide credit: B. Freeman and A. Torralba

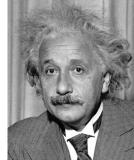
### **Matching with filters**

- Goal: find in image
- Method 0: filter the image with eye patch

$$h[m,n] = \sum_{k,l} g[k,l] f[m+k,n+l]$$

$$f = \text{image}$$

$$g = \text{filter}$$





response is stronger for higher intensity

What went wrong?

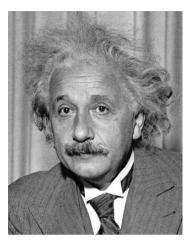
Input

Filtered Image

Slide: Hoiem

### **Template matching**

- Goal: find on in image
- Main challenge: What is a good similarity or distance measure between two patches?
  - Correlation
  - Zero-mean correlation
  - Sum Square Difference
  - Normalized Cross Correlation

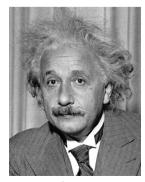


Slide: Hojem

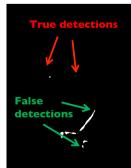
### **Matching with filters**

- Goal: find in image
- Method I: filter the image with zero-mean eye

$$h[m,n] = \sum_{k,l} (f[k,l] - \bar{f}) (g[m+k,n+l])$$
mean of f







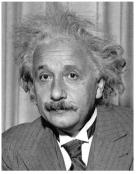
Input

Filtered Image (scaled) Thresholded Image

### **Matching with filters**

- Goal: find in image
- Method 2: SSD

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k,n+l])^2$$







Input

I-sqrt(SSD)

Thresholded Image

### **Matching with filters**

- Goal: find 🗑 in image
- Method 3: Normalized cross-correlation

mean template mean image patch

$$h[m,n] = \frac{\sum_{k,l} (g[k,l] - \bar{g})(f[m-k,n-l] - \bar{f}_{m,n})}{\left(\sum_{k,l} (g[k,l] - \bar{g})^2 \sum_{k,l} (f[m-k,n-l] - \bar{f}_{m,n})^2\right)^{0.5}}$$

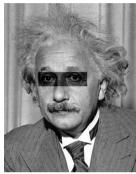
Matlab: normxcorr2 (template, im)

Slide: Hoiem

### **Matching with filters**

- Goal: find on in image
- Method 2: SSD

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k,n+l])^2$$



Input



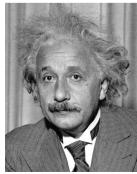
What's the potential downside of SSD?

SSD sensitive to average intensity

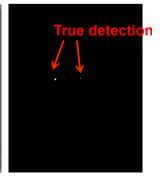
I - sqrt(SSD)

### **Matching with filters**

- Goal: find 🗑 in image
- Method 3: Normalized cross-correlation







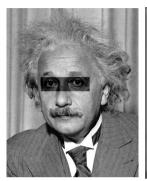
Slide: Hoiem

Input

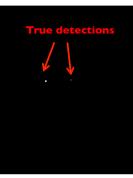
Normalized X-Correlation Thresholded Image

### **Matching with filters**

- Goal: find in image
- Method 3: Normalized cross-correlation







Slide: Hoiem

Input

Normalized X-Correlation Thresholded Image

### Q: What if we want to find larger or smaller eyes?

A: Image Pyramid

### Q: What is the best method to use?

### A: Depends

- SSD: faster, sensitive to overall intensity
- Normalized cross-correlation: slower, invariant to local average intensity and contrast

Slide: R. Pless



### **Image pyramids**

- · Gaussian pyramid
- · Laplacian pyramid
- Wavelet/QMF pyramid
- Steerable pyramid

Slide credit: B. Freeman and A. Torralba

### **Review of Sampling**



### **Image pyramids**

- · Gaussian pyramid
- · Laplacian pyramid
- Wavelet/QMF pyramid
- Steerable pyramid

Slide credit: B. Freeman and A. Torralba

### The Gaussian pyramid

- Smooth with Gaussians, because
  - A Gaussian\*Gaussian = another Gaussian
- Gaussians are low pass filters, so representation is redundant.
- Gaussian pyramid creates versions of the input image at multiple resolutions.
- This is useful for analysis across different spatial scales, but doesn't separate the image into different frequency bands.

Slide: Hoiem

Slide adapted from: B. Freeman and A. Torralba

### The computational advantage of pyramids

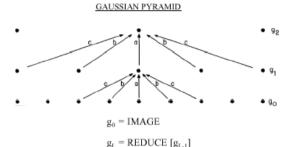
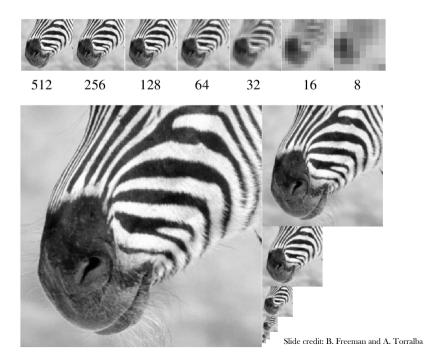


Fig 1. A one-dimensional graphic representation of the process which generates a Gaussian pyramid Each row of dots represents nodes within a level of the pyramid. The value of each node in the zero level is just the gray level of a corresponding image pixel. The value of each node in a high level is the weighted average of node values in the next lower level. Note that node spacing doubles from level to level, while the same weighting pattern or "generating kernel" is used to generate all levels.

[Burt and Adelson, 1983]

Slide credit: B. Freeman and A. Torralba



### The Gaussian Pyramid



#### **GAUSSIAN PYRAMID**







Fig. 4. First six levels of the Gaussian pyramid for the "Lady" image The original image, level 0, meusures 257 by 257 pixels and each higher level array is roughly half the dimensions of its predecessor. Thus, level 5 measures just 9 by 9 pixels.

[Burt and Adelson, 1983]

Slide credit: B. Freeman and A. Torralba

### Convolution and subsampling as a matrix multiply (ID case)

$$x_2 = G_1 x_1$$

$$G_1 =$$

 $\begin{smallmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{smallmatrix}$ 0 0 0 0 0 0 1 4 6 4 1 0 0 0 0 0 0  $0 \quad 0 \quad 1 \quad 4 \quad 6 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0$ 

Slide credit: B. Freeman and A. Torralba

(Normalization constant of 1/16 omitted for visual clarity.)

### **Next pyramid level**

$$x_3 = G_2 x_2$$

$$G_2 = \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0 \\ 0 \quad 0 \quad 1 \quad 4 \quad 6 \quad 4 \quad 1 \quad 0 \\ 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 4 \quad 6 \quad 4 \\ 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 4 \\ \end{array}$$

Slide credit: B. Freeman and A. Torralba

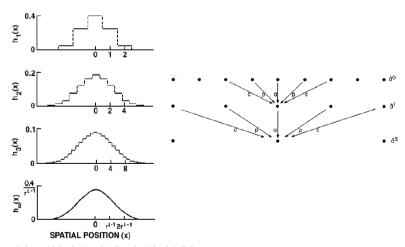


Fig. 2. The equivalent weighting functions h(x) for nodes in levels 1, 2, 3, and infinity of the Gaussian pyramid. Note that axis scales have been adjusted by factors of 2 to aid comparison Here the parameter a of the generating kernel is 0.4, and the resulting equivalent weighting functions closely resemble the Gaussian probability density functions.

Slide credit: B. Freeman and A. Torralba

# The combined effect of the two pyramid levels

$$x_3 = G_2 G_1 x_1$$

$$G_2G_1 =$$

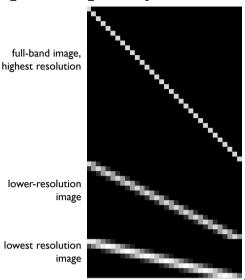
| 1 | 4 | 10 | 20 | 31 | 40 | 44 | 40 | 31 | 20 | 10 | 4  | 1   | 0  | 0  | 0  | 0   | 0  | 0 | 0 |
|---|---|----|----|----|----|----|----|----|----|----|----|-----|----|----|----|-----|----|---|---|
| 0 | 0 | 0  | 0  | 1  | 4  | 10 | 20 | 31 | 40 | 44 | 40 | 31  | 20 | 10 | 4  | 1   | 0  | 0 | 0 |
| 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 4  | 10 | 20 | 31  | 40 | 44 | 40 | 30  | 16 | 4 | 0 |
| 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | - 1 | 4  | 10 | 20 | 2.5 | 16 | 4 | 0 |

Slide credit: B. Freeman and A. Torralba

### Gaussian pyramids used for

- up- or down- sampling images.
- Multi-resolution image analysis
  - Look for an object over various spatial scales
  - Coarse-to-fine image processing: form blur estimate or the motion analysis on very low-resolution image, upsample and repeat. Often a successful strategy for avoiding local minima in complicated estimation tasks.

### ID Gaussian pyramid matrix, for [I 4 6 4 I] low-pass filter



Slide credit: B. Freeman and A. Torralba

# Template Matching with Image Pyramids

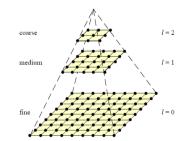
Input: Image, Template

- I. Match template at current scale
- 2. Downsample image
- 3. Repeat 1-2 until image is very small
- 4. Take responses above some threshold, perhaps with non-maxima suppression

Slide: Hoiem

### **Coarse-to-fine Image Registration**

- I. Compute Gaussian pyramid
- 2. Align with coarse pyramid
- 3. Successively align with finer pyramids
  - Search smaller range



Slide: Hoiem

Why is this faster?

Are we guaranteed to get the same result?

### **Image pyramids**

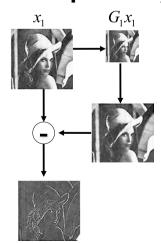
- Gaussian pyramid
- · Laplacian pyramid
- Wavelet/QMF pyramid
- Steerable pyramid

### The Laplacian Pyramid

- Synthesis
  - Compute the difference between upsampled Gaussian pyramid level and Gaussian pyramid level.
  - band pass filter each level represents spatial frequencies (largely) unrepresented at other level.
- Laplacian pyramid provides an extra level of analysis as compared to Gaussian pyramid by breaking the image into different isotropic spatial frequency bands.

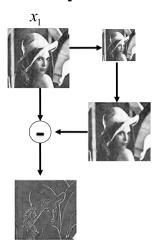
Slide adapted from: B. Freeman and A. Torralba

### The Laplacian Pyramid



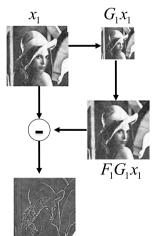
Slide credit: B. Freeman and A. Torralba

### The Laplacian Pyramid

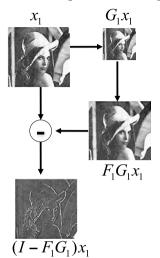


Slide credit: B. Freeman and A. Torralba

### **The Laplacian Pyramid**

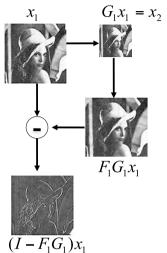


### The Laplacian Pyramid



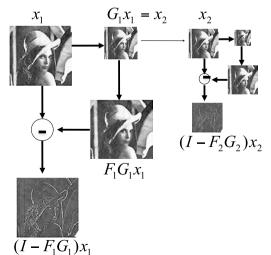
Slide credit: B. Freeman and A. Torralba

### **The Laplacian Pyramid**



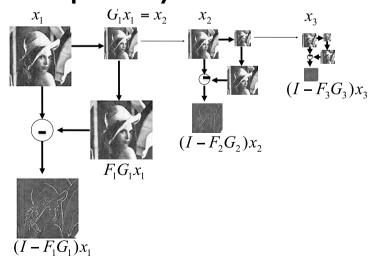
Slide credit: B. Freeman and A. Torralba

### The Laplacian Pyramid



Slide credit: B. Freeman and A. Torralba

### The Laplacian Pyramid



### **Upsampling**

$$y_2 = F_3 x_3$$

Insert zeros between pixels, then apply a low-pass filter, [I 4 6 4 I]

Slide credit: B. Freeman and A. Torralba

## Laplacian pyramid reconstruction algorithm: recover $x_1$ from $L_1$ , $L_2$ , $L_3$ and $x_4$

G# is the blur-and-downsample operator at pyramid level # F# is the blur-and-upsample operator at pyramid level #

Laplacian pyramid elements:

$$LI = (I - FI GI) \times I$$

$$L2 = (I - F2 G2) \times 2$$

$$L3 = (I - F3 G3) x3$$

$$x2 = GIxI$$

$$x3 = G2 x2$$

$$x4 = G3 x3$$

Reconstruction of original image  $(x \, I)$  from Laplacian pyramid elements:

$$x3 = L3 + F3 x4$$

$$x2 = L2 + F2 x3$$

$$xI = LI + FI \times 2$$

# Showing, at full resolution, the information captured at each level of a Gaussian (top) and Laplacian (bottom) pyramid.

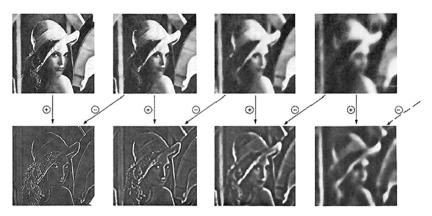
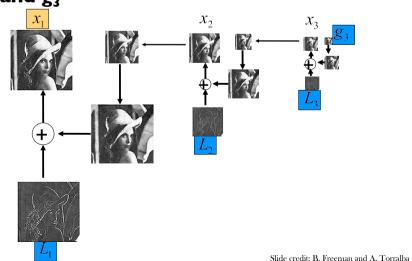
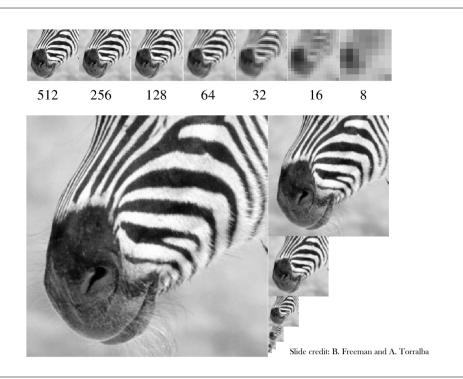


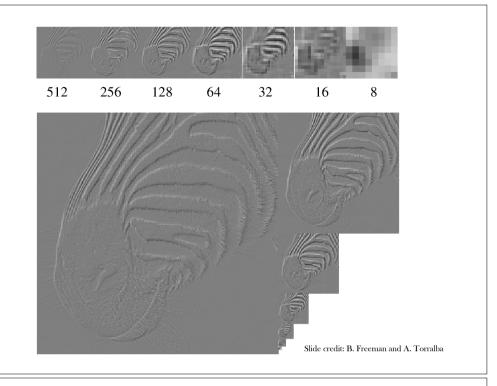
Fig. 5. First four levels of the Gaussian and Laplacian pyramid. Gaussian images, upper row, were obtained by exponding pyramid amoys (Fig. 4) through Gaussian interpolation. Each level of the Laplacian pyramid is the difference between the corresponding and next higher levels of the Gaussian pyramid.

Slide credit: B. Freeman and A. Torralba

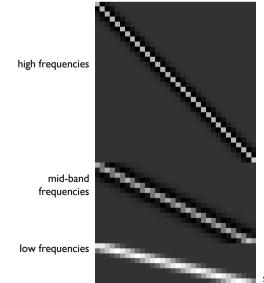
# Laplacian pyramid reconstruction algorithm: recover $x_1$ from $L_1$ , $L_2$ , $L_3$ and $g_3$







### ID Laplacian pyramid matrix, for [I 4 6 4 I] low-pass filter



Slide credit: B. Freeman and A. Torralba

### Laplacian pyramid applications

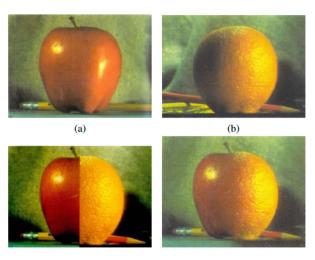
- Texture synthesis
- Image compression
- Noise removal

IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. COM-31, NO. 4, APRIL 1983

### The Laplacian Pyramid as a Compact Image Code

PETER J. BURT, MEMBER, IEEE, AND EDWARD H. ADELSON

### **Image blending**



Slide credit: B. Freeman and A. Torralba

### **Image blending**





- Build Laplacian pyramid for both images: LA, LB
- Build Gaussian pyramid for mask: G
- Build a combined Laplacian pyramid:
   L(j) = G(j) LA(j) + (I-G(j)) LB(j)
- · Collapse L to obtain the blended image



Slide credit: B. Freeman and A. Torralba

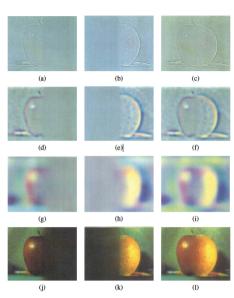


Figure 3.42 Laplacian pyramid blending details (Burt and Adelson 1983b) ⊚ 1983 ACM. The first three rows show the high, medium, and low frequency parts of the Laplacian pyramid (taken from levels 0, 2, and 4). The left and middle columns show the original apple and orange images weighted by the smooth interpolation functions, while the right column shows the averaged contributions.

Slide credit: B. Freeman & A. Torralba

### **Eulerian Video Magnification**

Video

Szeliski, Computer Vision, 2010



### **Image pyramids**

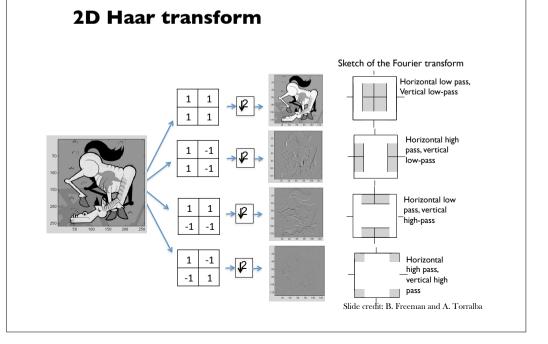
- Gaussian pyramid
- · Laplacian pyramid
- Wavelet/QMF pyramid
- Steerable pyramid

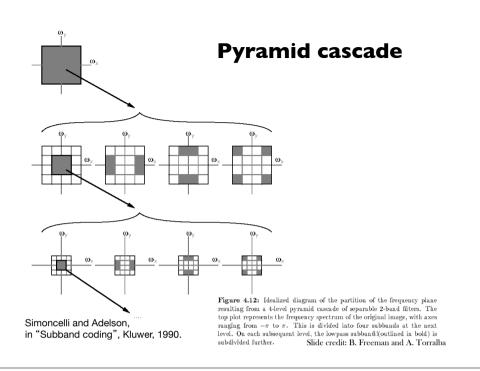
Slide credit: B. Freeman and A. Torralba

### Wavelet/QMF pyramid

- · Subband coding
- Wavelet or QMF (quadrature mirror filter) pyramid provides some splitting of the spatial frequency bands according to orientation (although in a somewhat limited way).
- Image is decomposed into a set of band-limited components (subbands).
- Original image can be reconstructed without error by reassemblying these subbands.

# Basic elements: 1</td





### Image pyramids

- Gaussian pyramid
- Laplacian pyramid
- Wavelet/QMF pyramid
- Steerable pyramid

### Wavelet/QMF representation



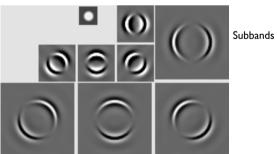
Same number of pixels!

Slide credit: B. Freeman and A. Torralba

### **Steerable Pyramid**

Low pass residual

2 Level decomposition of white circle example:



• The Steerable pyramid provides a clean separation of the image into different scales and orientations.

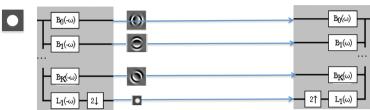
Images from: http://www.cis.upenn.edu/~eero/steerpyr.html

Slide credit: B. Freeman and A. Torralba

### **Steerable Pyramid**

We may combine Steerability with Pyramids to get a Steerable Laplacian Pyramid as shown below.

#### **Decomposition** Reconstruction



Images from: http://www.cis.upenn.edu/~eero/steerpyr.html

Slide credit: B. Freeman and A. Torralba

### **Steerable Pyramid**

But we need to get rid of the corner regions before starting the recursive circular filtering

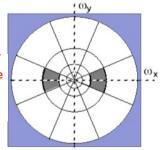


Figure 1. Idealized illustration of the spectral decomposition performed by a steerable pyramid with k = 4. Frequency axes range from  $-\pi$  to  $\pi$ . The basis functions are related by translations, dilations and rotations (except for the initial highpass subband and the final lowpass subband). For example, the shaded region Simoncelli and Freeman, corresponds to the spectral support of a single (vertically-oriented) subband. Slide credit: B. Freeman and A. Torralba

ICIP 1995

### **Steerable Pyramid**

We may combine Steerability with Pyramids to get a Steerable Laplacian Pyramid as shown below

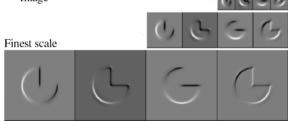
# **Decomposition** Reconstruction

Images from: http://www.cis.upenn.edu/~eero/steerpyr.html

Slide credit: B. Freeman and A. Torralba



Filter Kernels



Reprinted from "Shiftable MultiScale Transforms," by Simoncelli et al., IEEE Transactions on Information Theory, 1992, copyright 1992, IEEE

There is also a high pass residual...

### **Phase-based Video Magnification**

Video



### **Image pyramids**

Gaussian



Progressively blurred and subsampled versions of the image. Adds scale invariance to fixed-size algorithms.

- Laplacian
- Wavelet/QMF
- Steerable pyramid

Slide credit: B. Freeman and A. Torralba

### **Image pyramids**

- Gaussian
- Laplacian
- Wavelet/QMF
- · Steerable pyramid

Slide credit: B. Freeman and A. Torralba

### **Image pyramids**

• Gaussian



Progressively blurred and subsampled versions of the image. Adds scale invariance to fixed-size algorithms.

• Laplacian



Shows the information added in Gaussian pyramid at each spatial scale. Useful for noise reduction & coding.

- Wavelet/QMF
- · Steerable pyramid

### **Image pyramids**

Gaussian



Progressively blurred and subsampled versions of the image. Adds scale invariance to fixed-size algorithms.

Laplacian



Shows the information added in Gaussian pyramid at each spatial scale. Useful for noise reduction & coding.

Wavelet/OMF



Bandpassed representation, complete, but with aliasing and some non-oriented subbands.

Steerable pyramid

Slide credit: B. Freeman and A. Torralba

### Schematic pictures of each matrix transform

Shown for I-d images

The matrices for 2-d images are the same idea, but more complicated, to account for vertical, as well as horizontal, neighbor relationships.

### transformed image $\vec{F} = U\vec{f}$ Vectorized image Fourier transform, or Wavelet transform, or Steerable pyramid transform

Slide credit: B. Freeman and A. Torralba

### **Image pyramids**

Gaussian



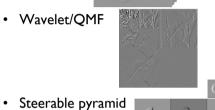
Progressively blurred and subsampled versions of the image. Adds scale invariance to fixed-size algorithms.

Laplacian



Shows the information added in Gaussian pyramid at each spatial scale. Useful for noise reduction & coding.

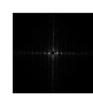
Wavelet/OMF



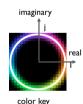
Bandpassed representation, complete, but with aliasing and some non-oriented subbands.

> Shows components at each scale and orientation separately. Nonaliased subbands. Good for texture and feature analysis. But overcomplete and with HF residual.

> > Slide credit: B. Freeman and A. Torralba

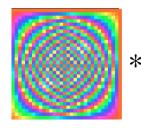


### Fourier transform



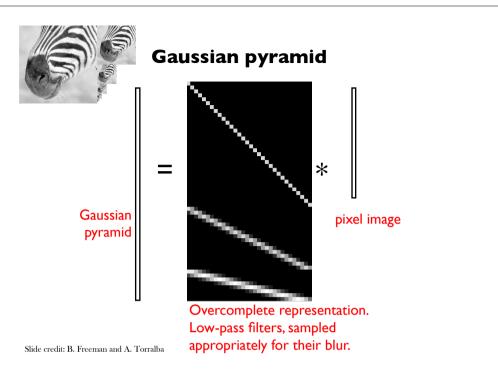
**Fourier** 

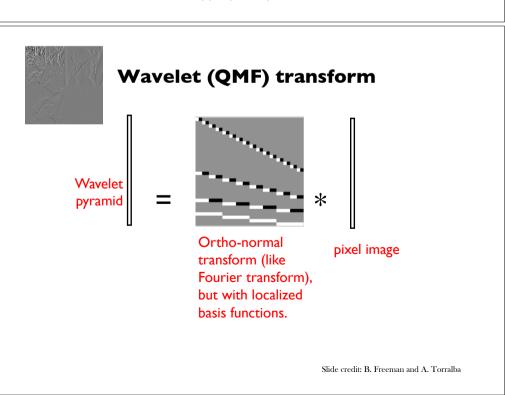
transform

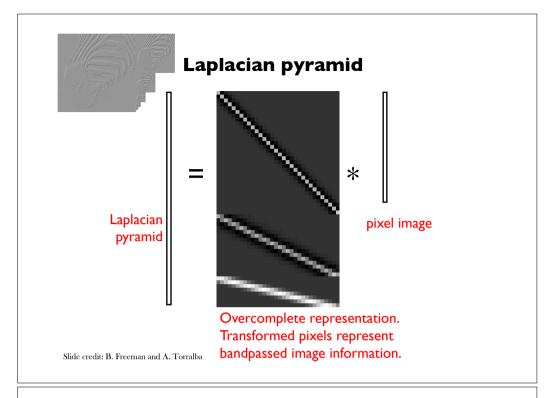


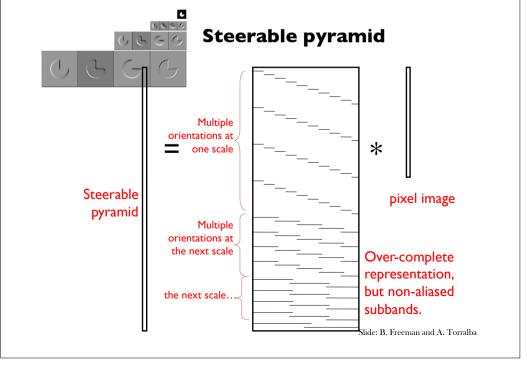
Fourier bases are global: each transform coefficient depends on all pixel locations.

pixel domain image









### Why use image pyramids?

- Handle real-world size variations with a constant-size vision algorithm.
- Remove noise
- Analyze texture
- Recognize objects
- · Label image features
- Image priors can be specified naturally in terms of wavelet pyramids.

Slide credit: B. Freeman and A. Torralba

# Slide credit; J. Hays

 A. Oliva, A. Torralba, P.G. Schyns (2006). Hybrid Images. ACM Transactions on Graphics, ACM SIGGRAPH, 25-3, 527-530.

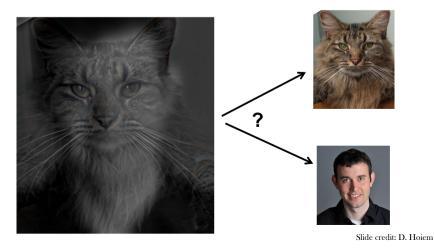
Reading Assignment #3 - Hybrid Images

• Due on 27th of November





Why do we get different, distance-dependent interpretations of hybrid images?

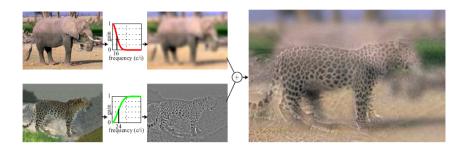


Salvador Dali invented Hybrid Images?

#### Salvador Dali

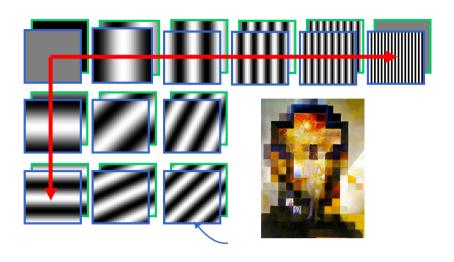
"Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln", 1976

### **Hybrid Images**



Slide credit: J. Hays

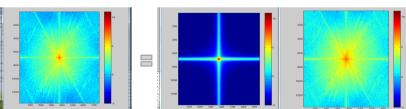
### Fourier bases



Slide credit: M. H. Yang

### **Hybrid Image in FFT**

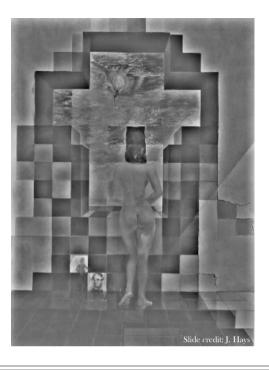
Hybrid Image Low-passed Image ♣ High-passed Image



Slide credit: J. Hays



**Salvador Dali**"Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln", 1976



## **Salvador Dali**"Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln", 1976

### **Announcements**

- There will be no classes next week!
- You will take your midterm exams on 20<sup>th</sup> of November.
- The exam will cover all the topics we covered in the class and the reading materials distributed.

### **Summary – Image pyramids**

- · Gaussian pyramid
- · Laplacian pyramid
- Wavelet/QMF pyramid
- Steerable pyramid

Slide credit: B. Freeman and A. Torralba

### After midterm exam

Edge detection