

BBM 413

**Fundamentals of
Image Processing**

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Point Operations
Histogram Processing

Today's topics

- Point operations
- Histogram processing

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- Point operations
- Histogram processing

Digital images

- Sample the 2D space on a regular grid
- Quantize each sample (round to nearest integer)
- Image thus represented as a matrix of integer values.

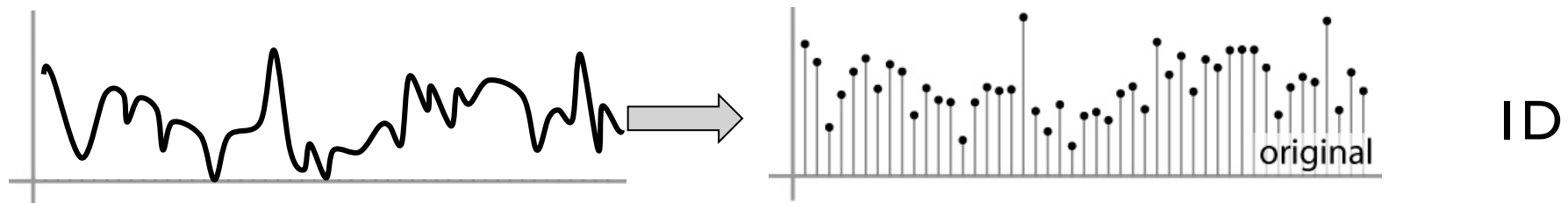
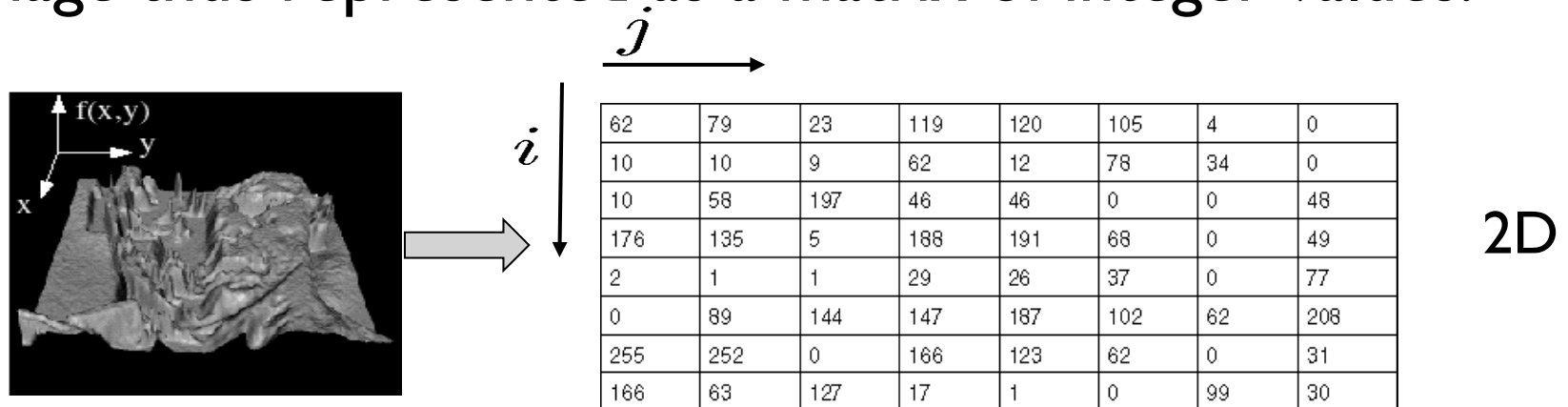


Image Transformations

- $g(x,y)=T[f(x,y)]$

$g(x,y)$: output image

$f(x,y)$: input image

T : transformation function

1. Point operations: operations on single pixels
2. Spatial filtering: operations considering pixel neighborhoods
3. Global methods: operations considering whole image

Point Operations

- Smallest possible neighborhood is of size 1×1
- Process each point independently of the others
- Output image g depends only on the value of f at a single point (x,y)
- Map each pixel's value to a new value
- Transformation function T remaps the sample's value:

$$s = T(r)$$

where

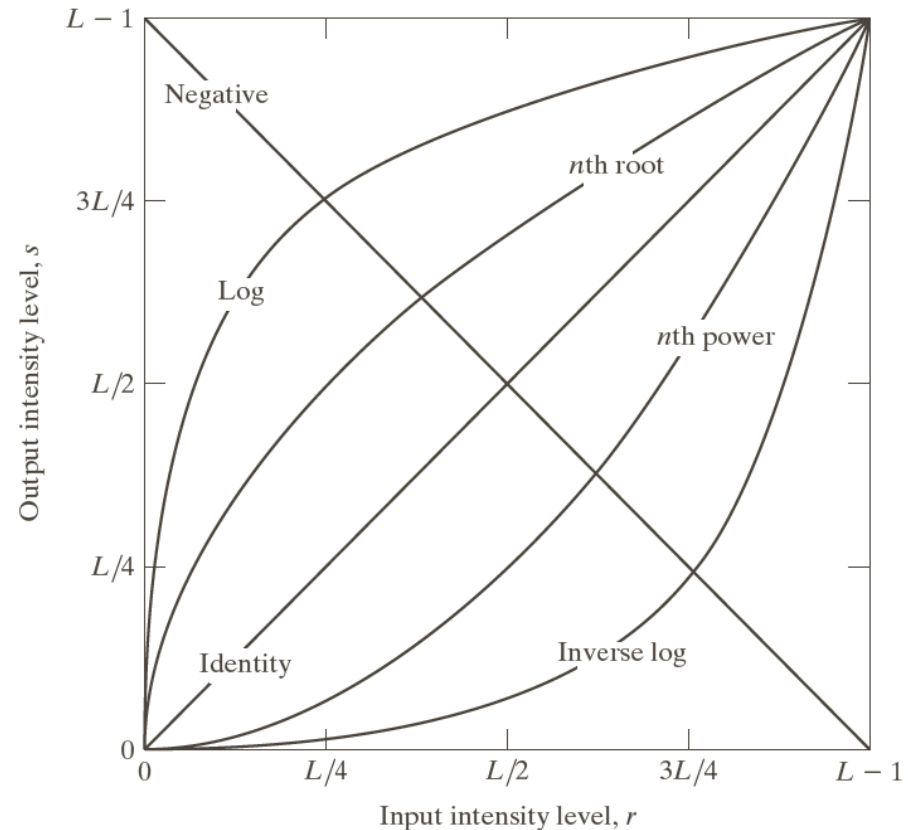
- r is the value at the point in question
- s is the new value in the processed result
- T is a *intensity transformation* function

Point operations

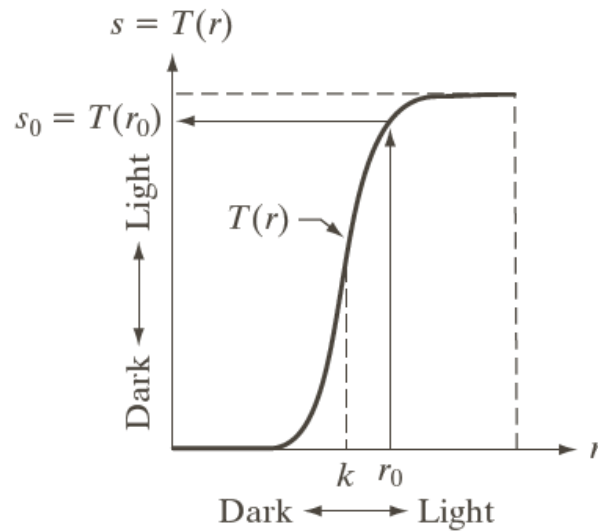
- Is mapping one color space to another (e.g. RGB2HSV) a point operation?
- Is image arithmetic a point operation?
- Is performing geometric transformations a point operation?
 - Rotation
 - Translation
 - Scale change
 - etc.

Sample intensity transformation functions

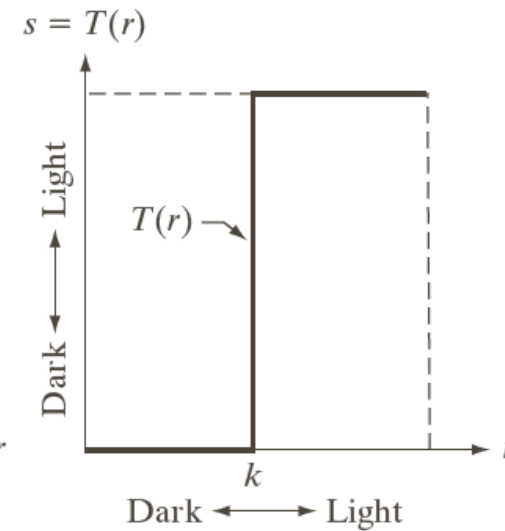
- Image negatives
- Log transformations
 - Compresses the dynamic range of images
- Power-law transformations
 - Gamma correction



Point Processing Examples



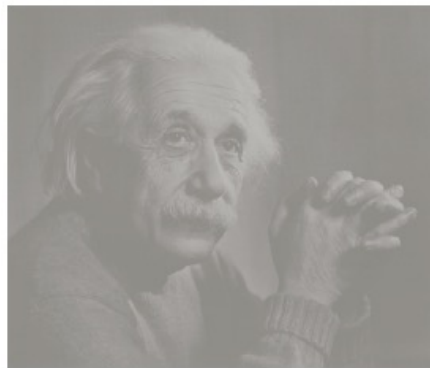
produces an image of higher contrast than the original by darkening the intensity levels below k and brightening intensities above k



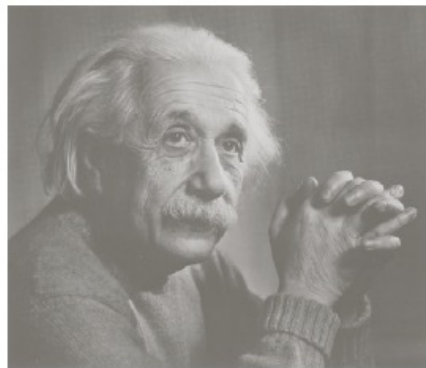
produces a binary (two-intensity level) image

Dynamic range

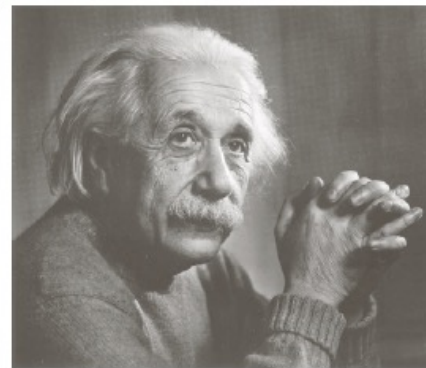
- Dynamic range $R_d = I_{\max} / I_{\min}$, or $(I_{\max} + k) / (I_{\min} + k)$
 - determines the degree of image contrast that can be achieved
 - a major factor in image quality
- Ballpark values
 - Desktop display in typical conditions: 20:1
 - Photographic print: 30:1
 - High dynamic range display: 10,000:1



low contrast



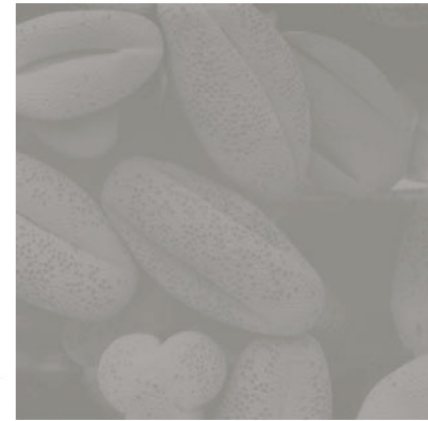
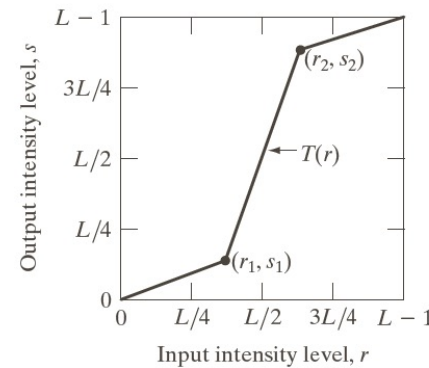
medium contrast



high contrast

Point Operations: Contrast stretching and Thresholding

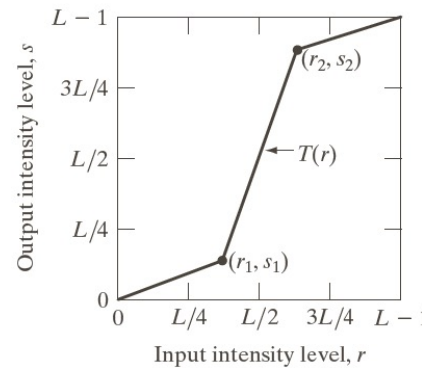
- Contrast stretching:
produces an image of
higher contrast than the
original



- Thresholding:
produces a binary
(two-intensity level) image

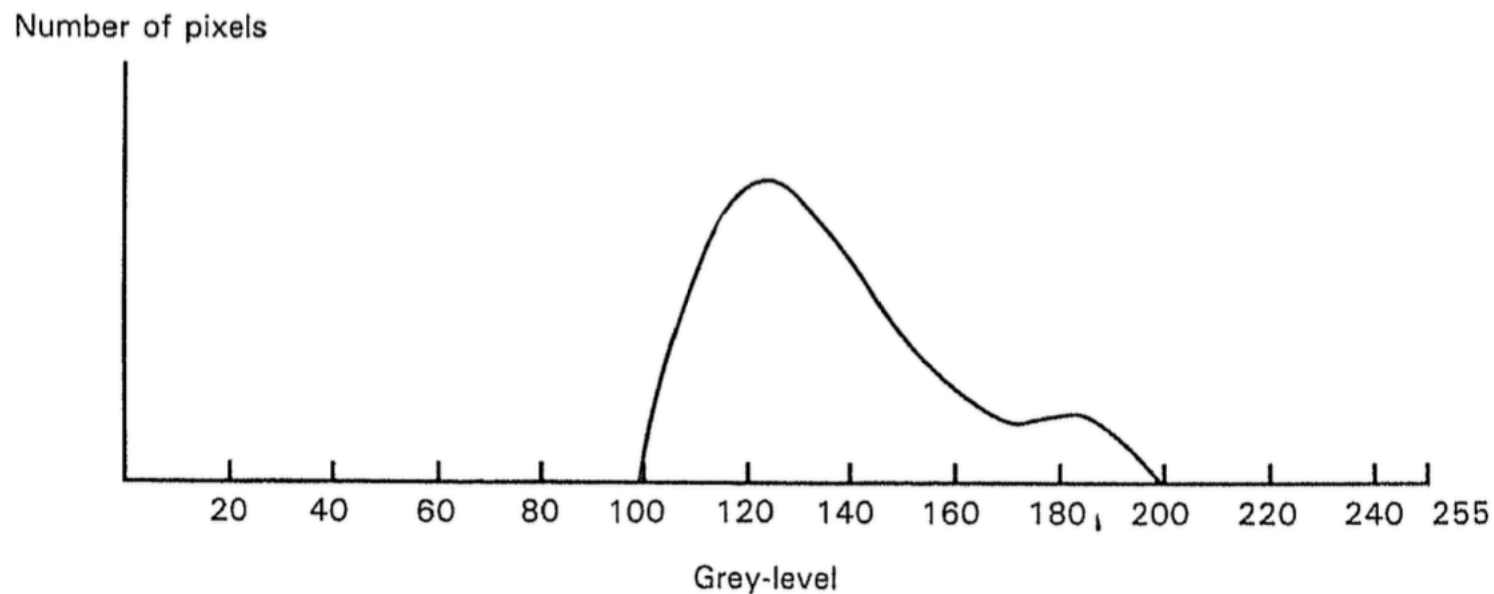
Point Operations: Contrast stretching and Thresholding

- Contrast stretching: produces an image of higher contrast than the original
- Thresholding: produces a binary (two-intensity level) image



Point Operations

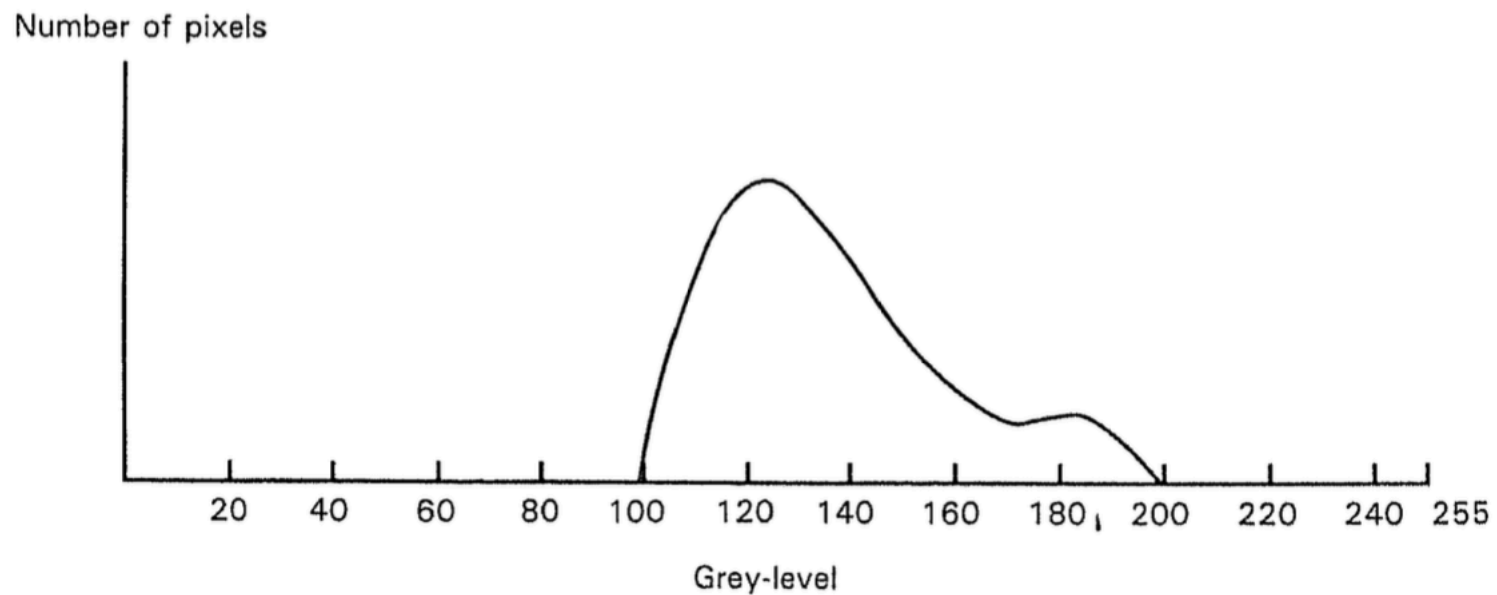
- What can you say about the image having the following histogram?



- A low contrast image
- How we can process the image so that it has a better visual quality?

Point Operations

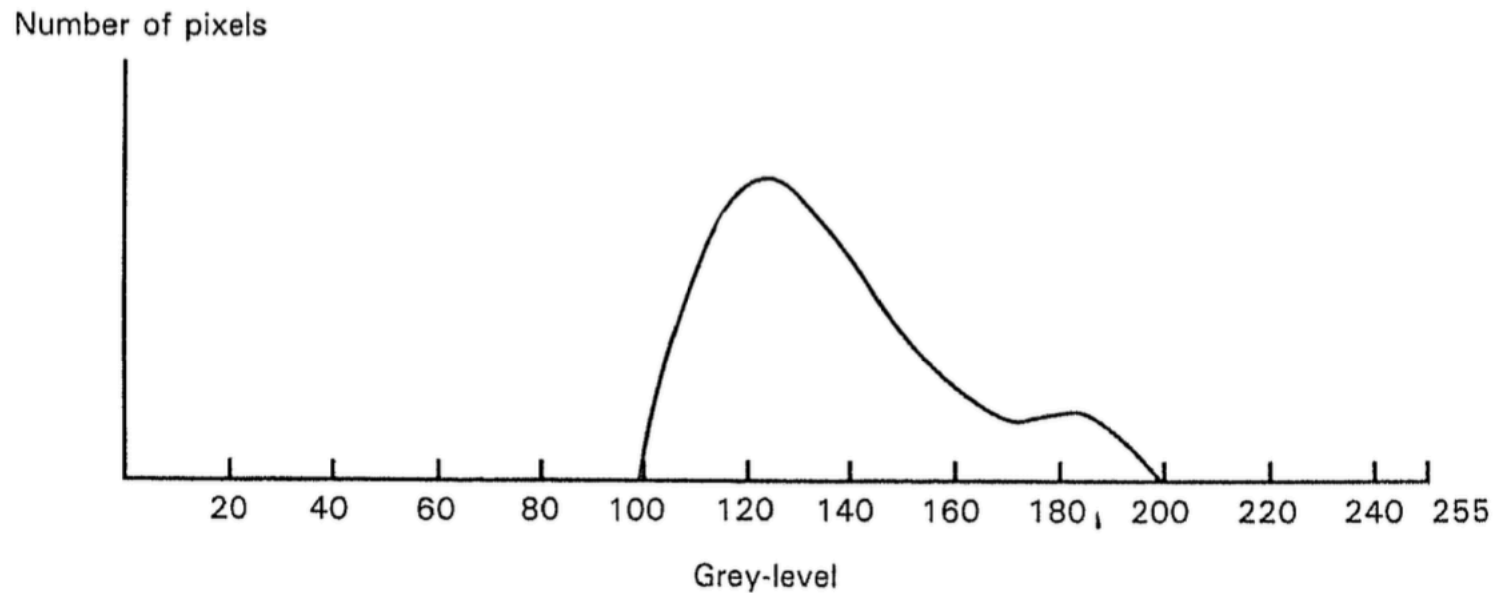
- How we can process the image so that it has a better visual quality?



- Answer is contrast stretching!

Point Operations

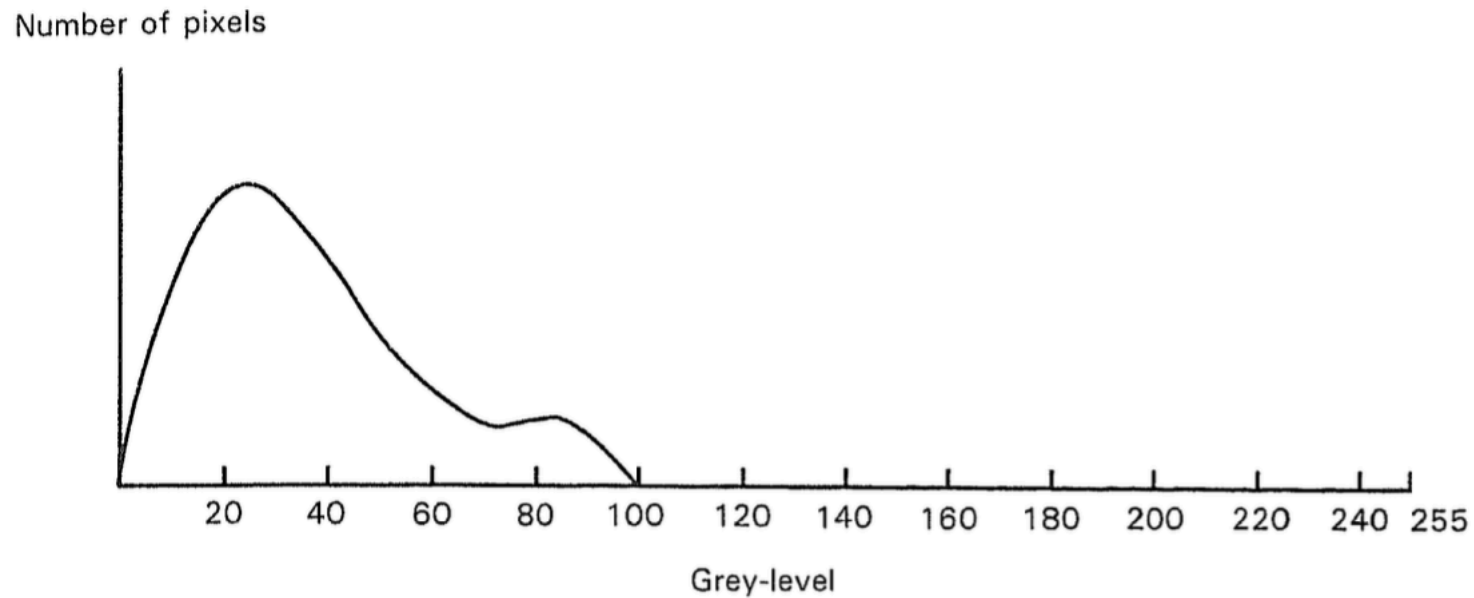
- Let us devise an appropriate point operation.



- Shift all values so that the observable pixel range starts at 0.

Point Operations

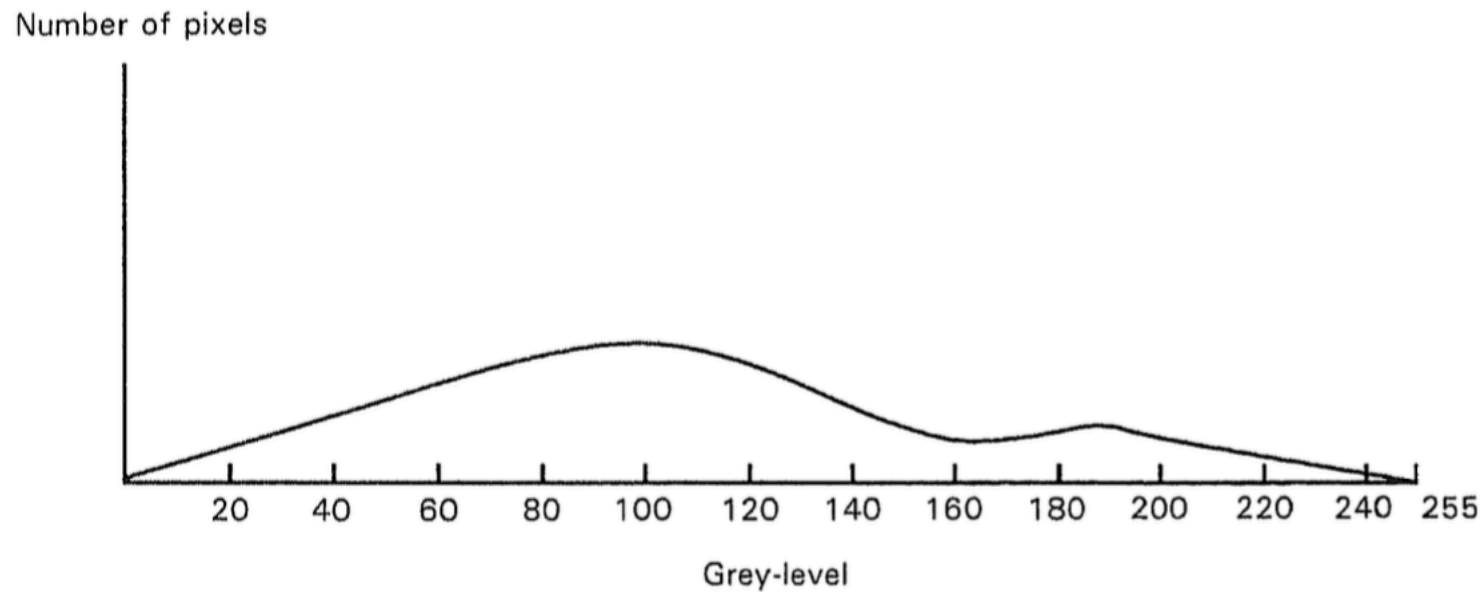
- Let us devise an appropriate point operation.



- Now, scale everything in the range 0-100 to 0-255.

Point Operations

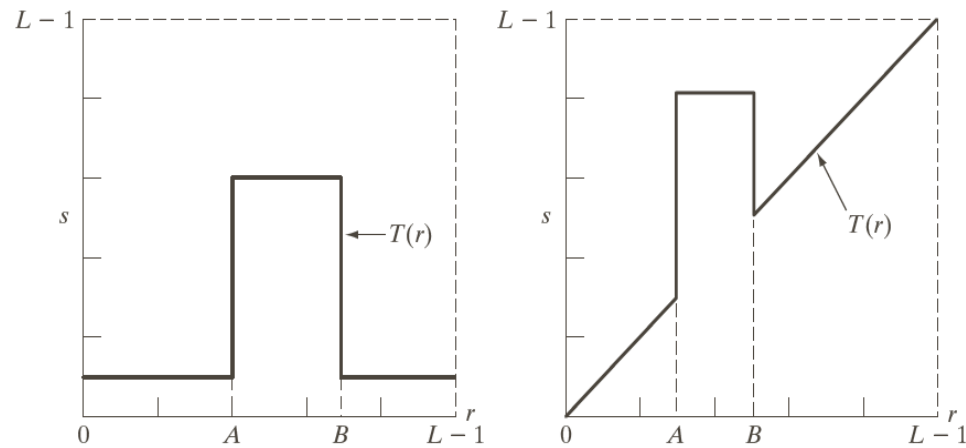
- Let us devise an appropriate point operation.



- What is the corresponding transformation function?
- $T(r) = 2.55*(r-100)$

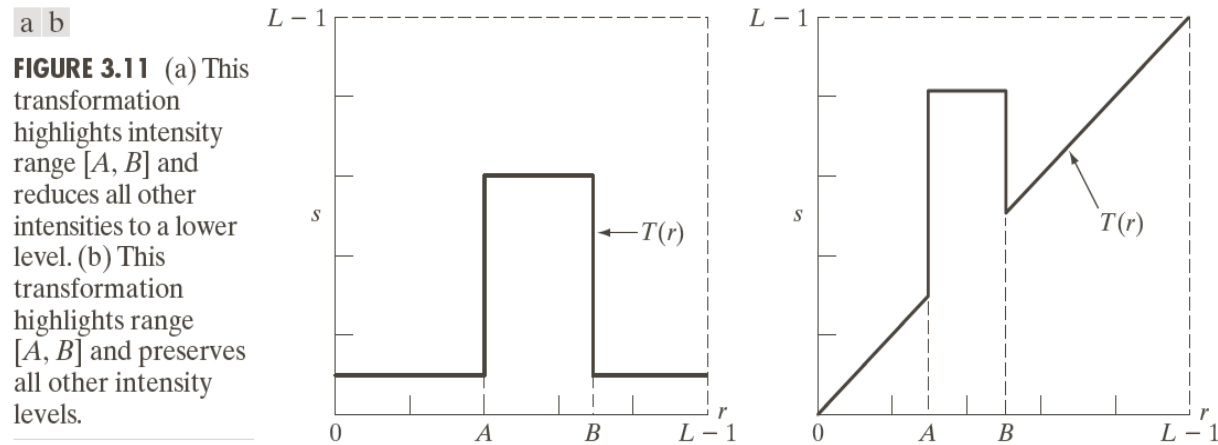
Point Operations: Intensity-level Slicing

- highlights a certain range of intensities



Point Operations: Intensity-level Slicing

- highlights a certain range of intensities



Intensity encoding in images

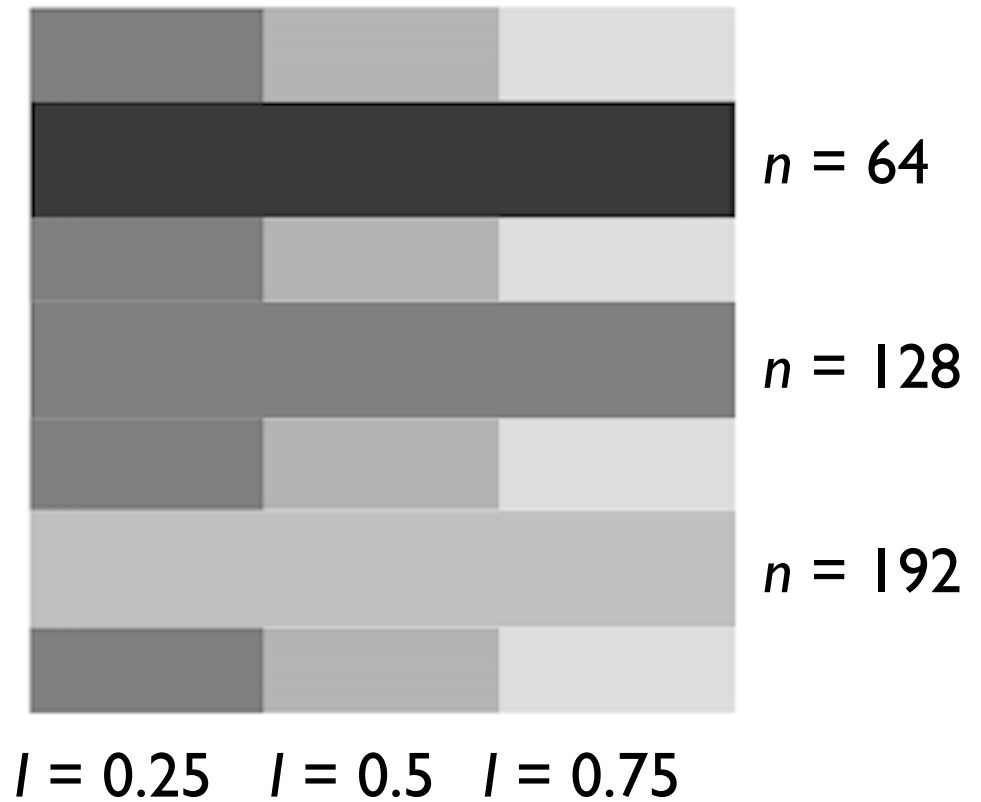
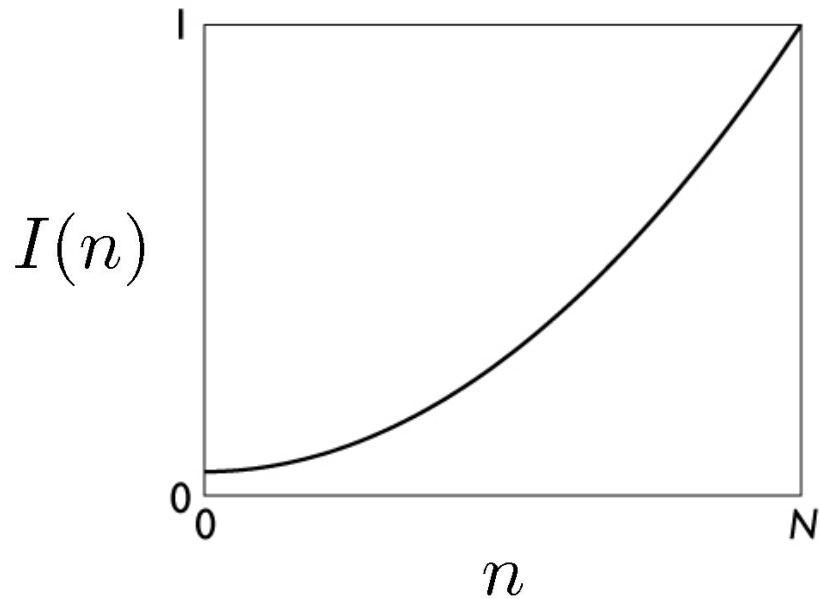
- Recall that the pixel values determine how bright that pixel is.
- Bigger numbers are (usually) brighter
- *Transfer function*: function that maps input pixel value to luminance of displayed image

$$I = f(n) \quad f : [0, N] \rightarrow [I_{\min}, I_{\max}]$$

- What determines this function?
 - physical constraints of device or medium
 - desired visual characteristics

What this projector does?

- Something like this:

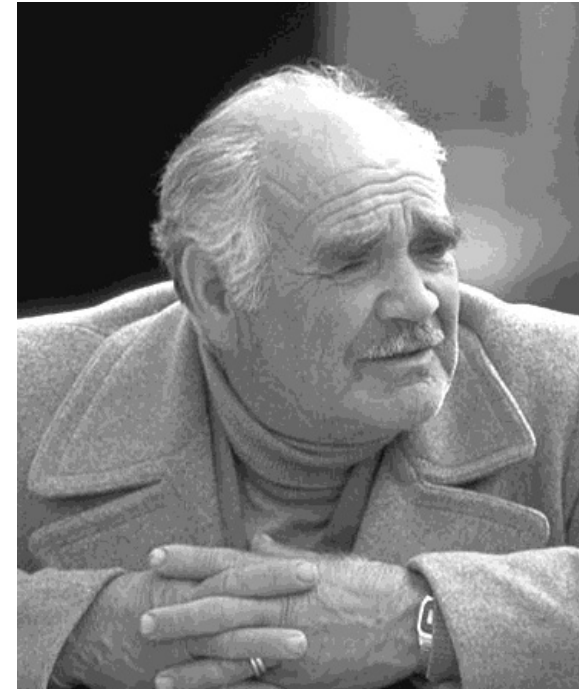


Constraints on transfer function

- Maximum displayable intensity, I_{\max}
 - how much power can be channeled into a pixel?
 - LCD: backlight intensity, transmission efficiency (<10%)
 - projector: lamp power, efficiency of imager and optics
- Minimum displayable intensity, I_{\min}
 - light emitted by the display in its “off” state
 - e.g. stray electron flux in CRT, polarizer quality in LCD
- Viewing flare, k : light reflected by the display
 - very important factor determining image contrast in practice
 - 5% of I_{\max} is typical in a normal office environment [sRGB spec]
 - much effort to make very black CRT and LCD screens
 - all-black decor in movie theaters

Transfer function shape

- Desirable property: the change from one pixel value to the next highest pixel value should not produce a visible contrast
 - otherwise smooth areas of images will show visible bands
- What contrasts are visible?
 - rule of thumb: under good conditions we can notice a 2% change in intensity
 - therefore we generally need smaller quantization steps in the darker tones than in the lighter tones
 - most efficient quantization is logarithmic



an image with severe *banding*

[Philip Greenspun]

How many levels are needed?

- Depends on dynamic range
 - 2% steps are most efficient:
 - $0 \mapsto I_{\min}; 1 \mapsto 1.02I_{\min}; 2 \mapsto (1.02)^2 I_{\min}; \dots$
 - $\log 1.02$ is about $1/120$, so 120 steps per decade of dynamic range
 - 240 for desktop display
 - 360 to print to film
 - 480 to drive HDR display
 - If we want to use linear quantization (equal steps)
 - one step must be $< 2\%$ ($1/50$) of I_{\min}
 - need to get from ~ 0 to $I_{\min} \cdot R_d$ so need about $50 R_d$ levels
 - 1500 for a print; 5000 for desktop display; 500,000 for HDR display
 - Moral: 8 bits is just barely enough for low-end applications
 - but only if we are careful about quantization

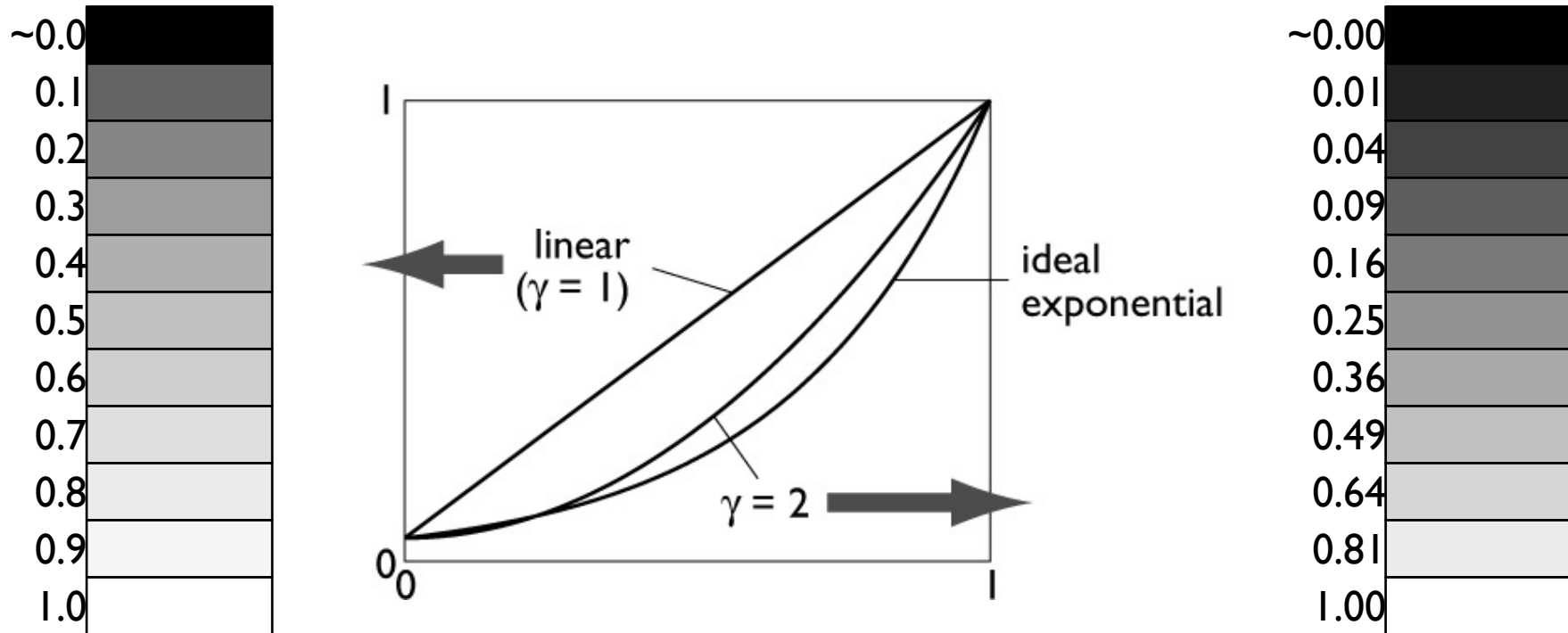
Intensity quantization in practice

- Option 1: linear quantization $I(n) = (n/N) I_{\max}$
 - pro: simple, convenient, amenable to arithmetic
 - con: requires more steps (wastes memory)
 - need 12 bits for any useful purpose; more than 16 for HDR
- Option 2: power-law quantization $I(n) = (n/N)^\gamma I_{\max}$
 - pro: fairly simple, approximates ideal exponential quantization
 - con: need to linearize before doing pixel arithmetic
 - con: need to agree on exponent
 - 8 bits are OK for many applications; 12 for more critical ones
- Option 2: floating-point quantization $I(x) = (x/w) I_{\max}$
 - pro: close to exponential; no parameters; amenable to arithmetic
 - con: definitely takes more than 8 bits
 - 16-bit “half precision” format is becoming popular

Why gamma?

- Power-law quantization, or *gamma correction* is most popular
- Original reason: CRTs are like that
 - intensity on screen is proportional to (roughly) voltage^2
- Continuing reason: inertia + memory savings
 - inertia: gamma correction is close enough to logarithmic that there's no sense in changing
 - memory: gamma correction makes 8 bits per pixel an acceptable option

Gamma quantization



- Close enough to ideal perceptually uniform exponential

Gamma correction

- Sometimes (often, in graphics) we have computed intensities a that we want to display linearly

- In the case of an ideal monitor with zero black level,

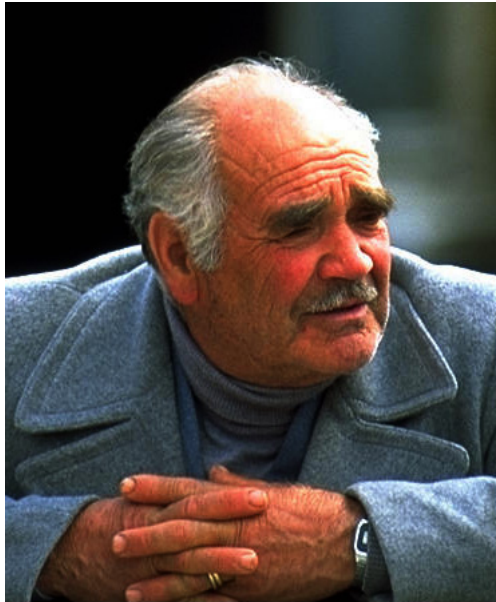
$$I(n) = (n/N)^\gamma$$

(where $N = 2^n - 1$ in n bits). Solving for n :

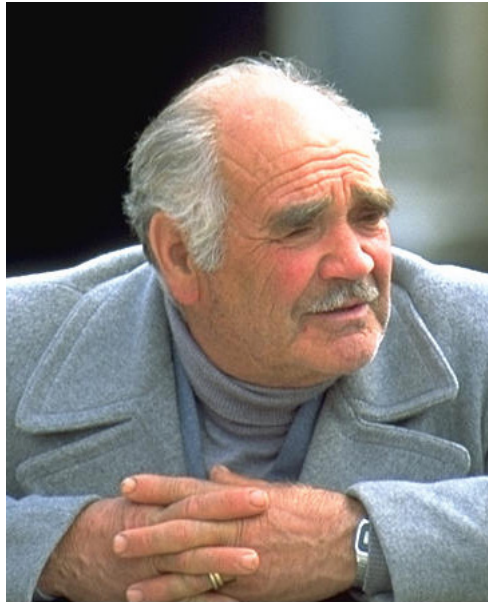
$$n = Na^{\frac{1}{\gamma}}$$

- This is the “gamma correction” recipe that has to be applied when computed values are converted to 8 bits for output
 - failing to do this (implicitly assuming gamma = 1) results in dark, oversaturated images

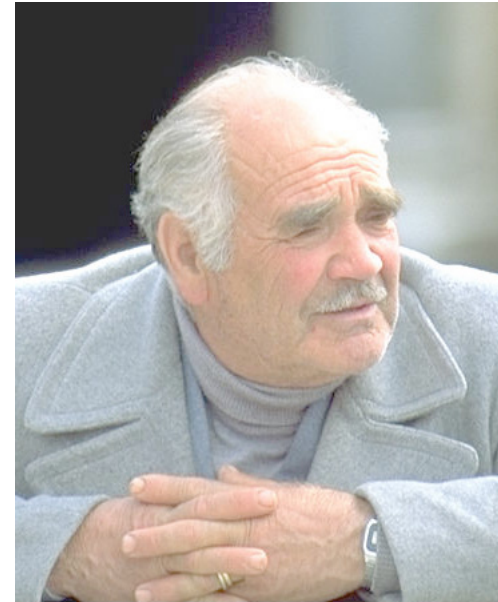
Gamma correction



corrected for
 γ lower than
display



OK



corrected for
 γ higher than
display

[Philip Greenspun]

Instagram Filters

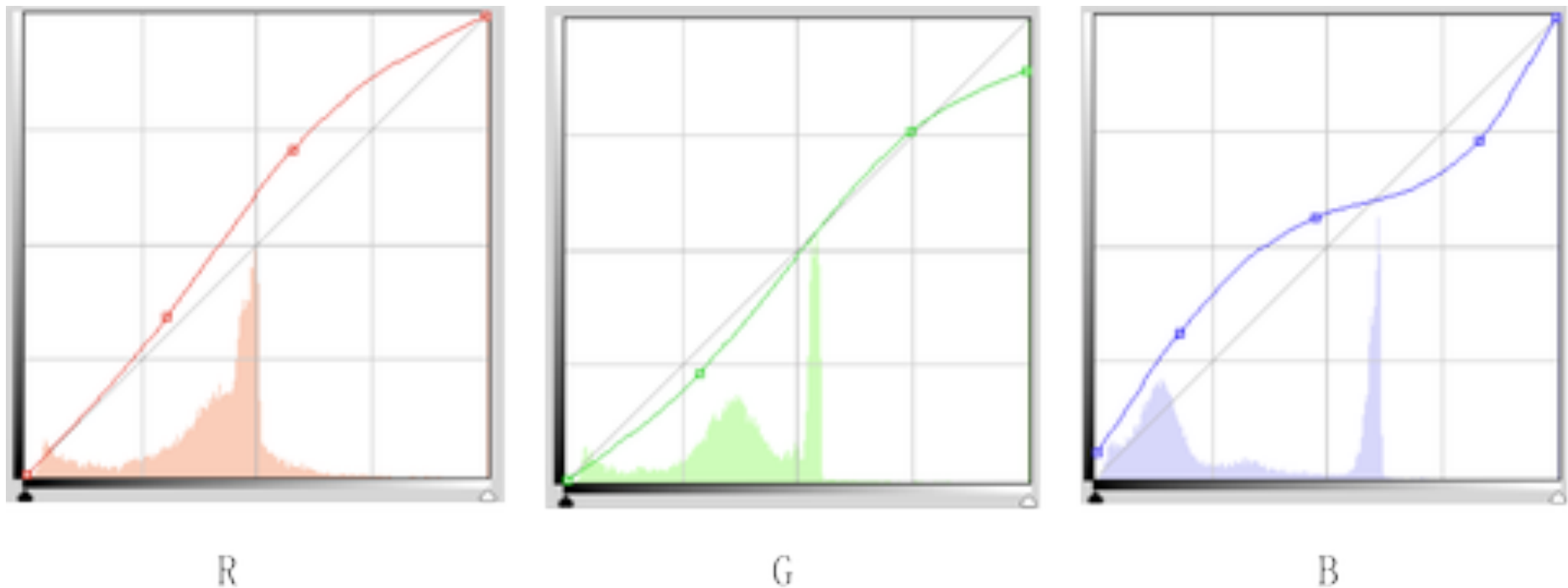
- How do they make those Instagram filters?



“It's really a combination of a bunch of different methods. In some cases we draw on top of images, in others we do pixel math. It really depends on the effect we're going for.” --- Kevin Systrom, co-founder of Instagram

Example Instagram Steps

1. Perform an independent RGB color point transformation on the original image to increase contrast or make a color cast



Example Instagram Steps

2. Overlay a circle background image to create a vignette effect



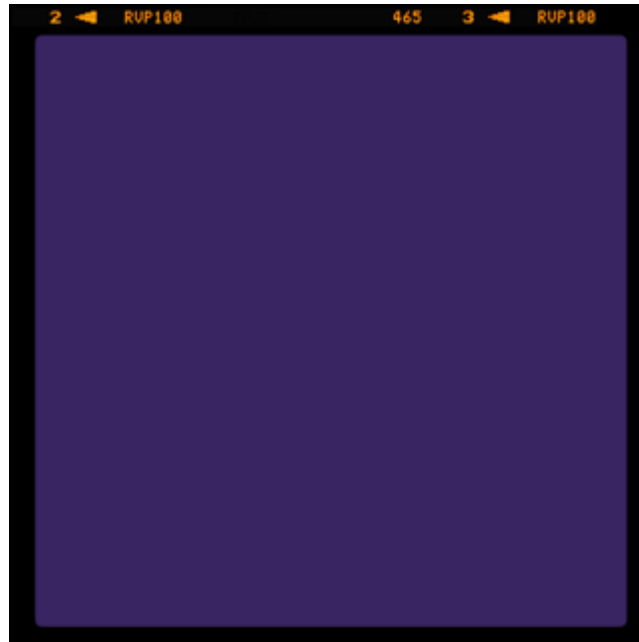
Example Instagram Steps

3. Overlay a background image as decorative grain



Example Instagram Steps

4. Add a border or frame



Result



Javascript library for creating Instagram-like effects, see:

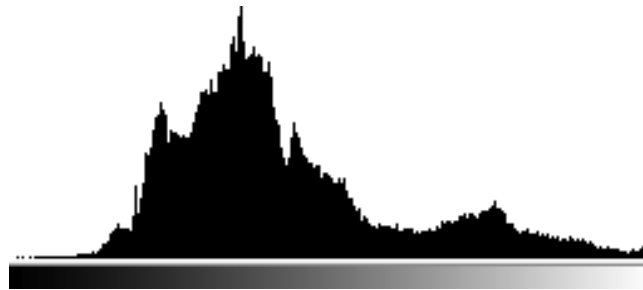
<http://alexmic.net/filtrr/>

Today's topics

- Point operations
- Histogram processing

Histogram

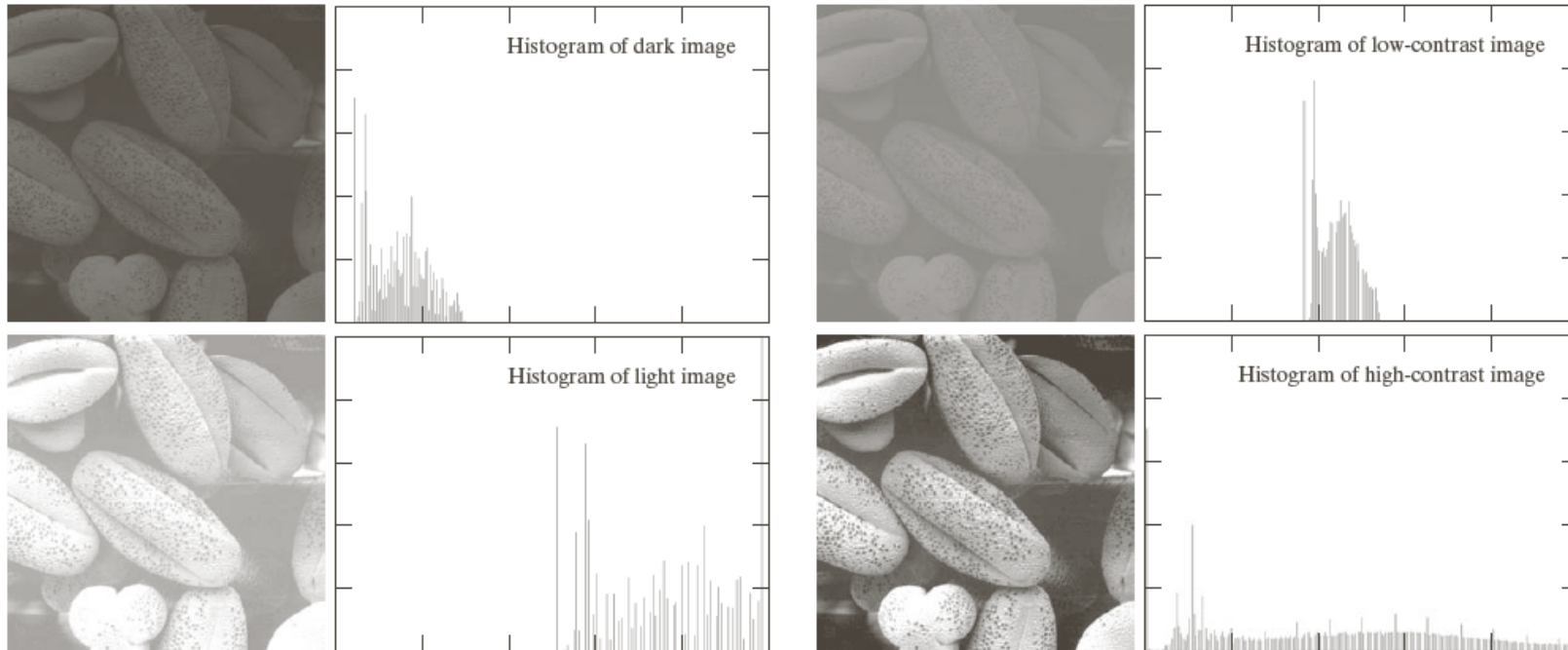
- Histogram: a discrete function $h(r)$ which counts the number of pixels in the image having intensity r
- If $h(r)$ is normalized, it measures the probability of occurrence of intensity level r in an image



- What histograms say about images?
- What they don't?
 - No spatial information

A descriptor for visual information

Images and histograms

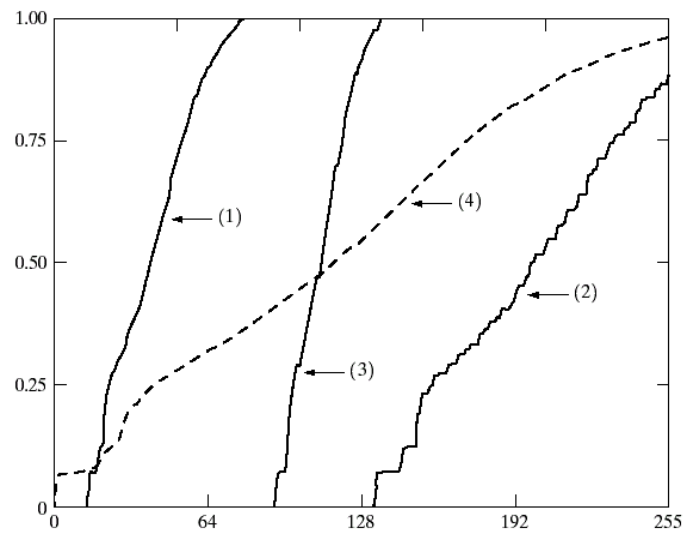
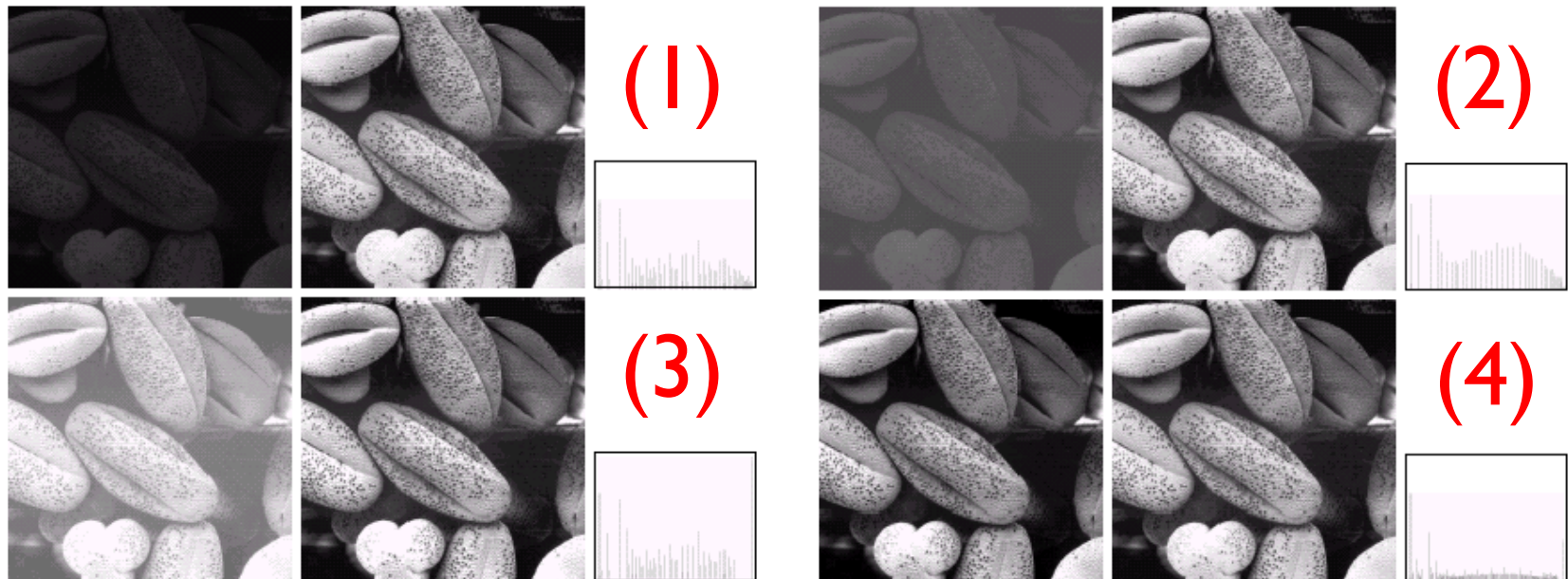


- How do histograms change when
 - we adjust brightness? **shifts the histogram horizontally**
 - we adjust contrast? **stretches or shrinks the histogram horizontally**

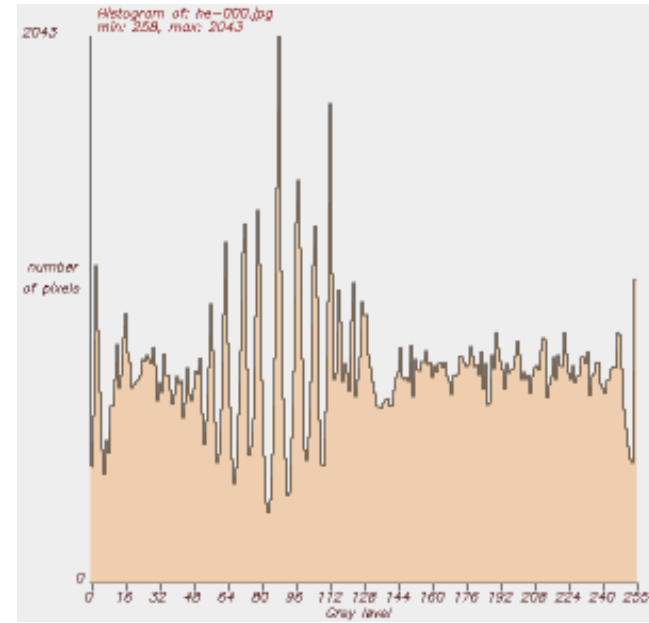
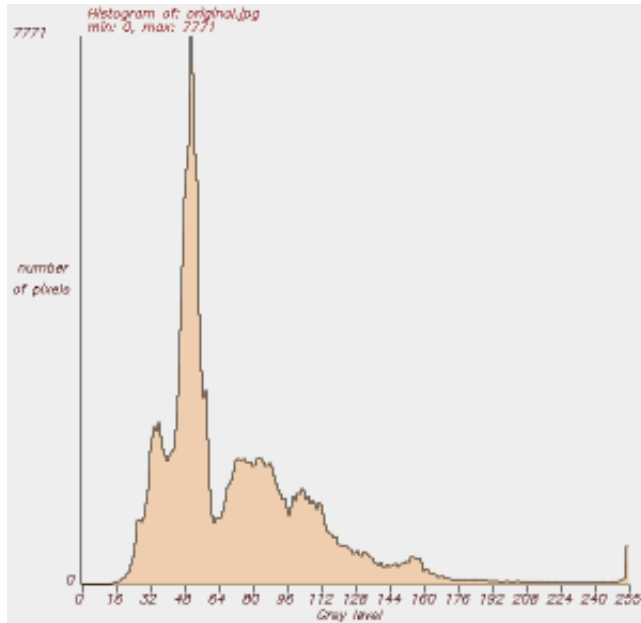
Histogram equalization

- A good quality image has a nearly uniform distribution of intensity levels. Why?
- Every intensity level is equally likely to occur in an image
- *Histogram equalization*: Transform an image so that it has a uniform distribution
 - create a lookup table defining the transformation

Histogram equalization examples



Histogram Equalization



Histogram as a probability density function

- Recall that a normalized histogram measures the probability of occurrence of an intensity level r in an image
- We can normalize a histogram by dividing the intensity counts by the area

$$p(r) = \frac{h(r)}{\text{Area}}$$

Histogram equalization: Continuous domain

- Define a transformation function of the form

$$s = T(r) = (L - 1) \underbrace{\int_0^r p(w) dw}_{\text{cumulative distribution function}}$$

where

- r is the input intensity level
- s is the output intensity level
- p is the normalized histogram of the input signal
- L is the desired number of intensity levels

(Continuous) output signal has a uniform distribution!

Histogram equalization: Discrete domain

- Define the following transformation function for an MxN image

$$s_k = T(r_k) = (L-1) \sum_{j=0}^k \frac{n_j}{MN} = \frac{(L-1)}{MN} \sum_{j=0}^k n_j$$

for $k = 0, \dots, L-1$

where

- r_k is the input intensity level
- s_k is the output intensity level
- n_j is the number of pixels having intensity value j in the input image
- L is the number of intensity levels

(Discrete) output signal has a nearly uniform distribution!

Histogram Specification

- Given an input image f and a specific histogram $p_2(r)$, transform the image so that it has the specified histogram
- How to perform histogram specification?
- Histogram equalization produces a (nearly) uniform output histogram
- Use histogram equalization as an intermediate step

Histogram Specification

1. Equalize the histogram of the input image

$$T_1(r) = (L - 1) \int_0^r p_1(w) dw$$

2. Histogram equalize the desired output histogram

$$T_2(r) = (L - 1) \int_0^r p_2(w) dw$$

3. Histogram specification can be carried out by the following point operation:

$$s = T(r) = T_2^{-1}(T_1(r))$$

Next week

- Spatial filtering