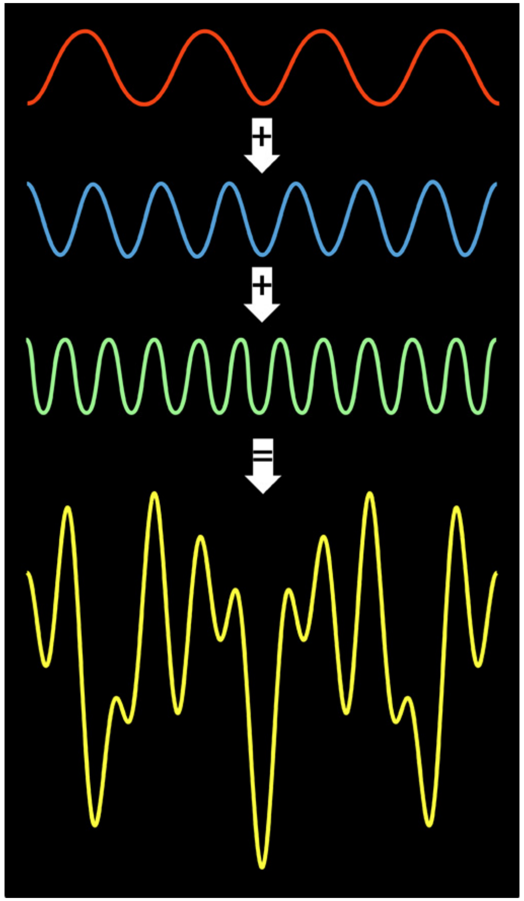
BBM 413 Fundamentals of Image Processing

Erkut Erdem
Dept. of Computer Engineering
Hacettepe University

Edge Detection
Hough Transform

Review – Signals and Images

- A signal is composed of low and high frequency components



low frequency components: smooth / piecewise smooth

Neighboring pixels have similar brightness values

You're within a region

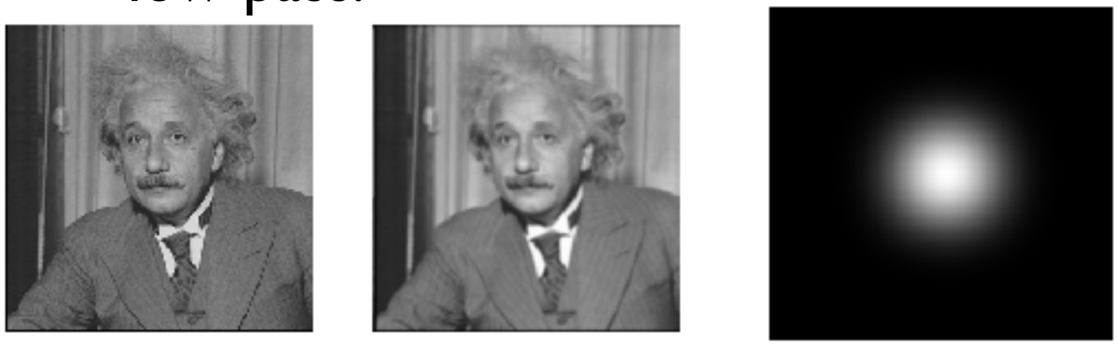
high frequency components: oscillatory

Neighboring pixels have different brightness values

You're either at the edges or noise points

Review - Low-pass, Band-pass, High-pass filters

low-pass:



High-pass / band-pass:

Slide credit: A. Efros

Today

- Edge detection
  - Difference filters
  - Laplacian of Gaussian
  - Canny edge detection
- Boundary detection
  - Hough transform

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  - Difference filters
  - Laplacian of Gaussian
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Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels

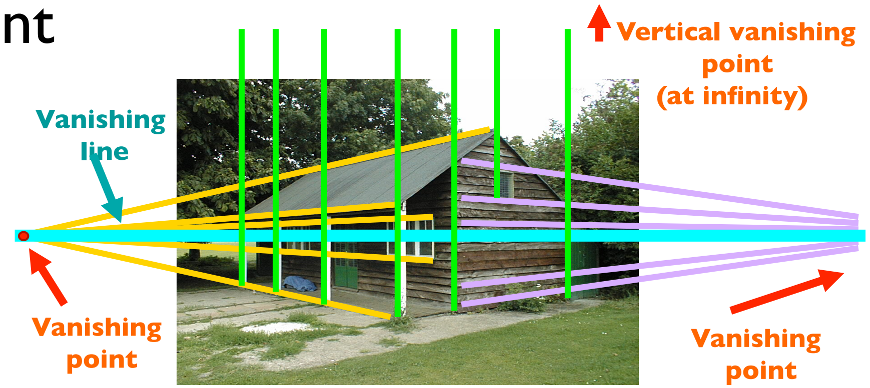
- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)

Slide credit: D. Lowe

Why do we care about edges?

- Extract information, recognize objects

- Recover geometry and viewpoint

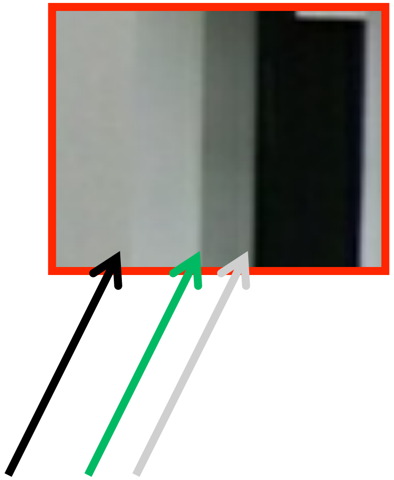


Source: J. Hays

Closeup of edges

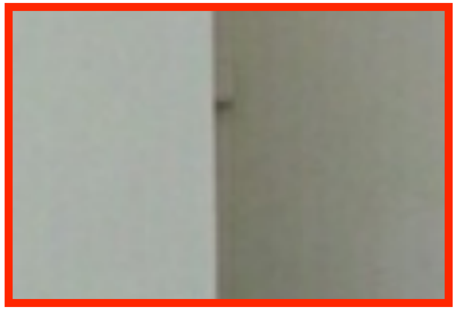
Slide credit: D. Hoiem

Closeup of edges



Slide credit: D. Hoiem

Closeup of edges



Slide credit: D. Hoiem

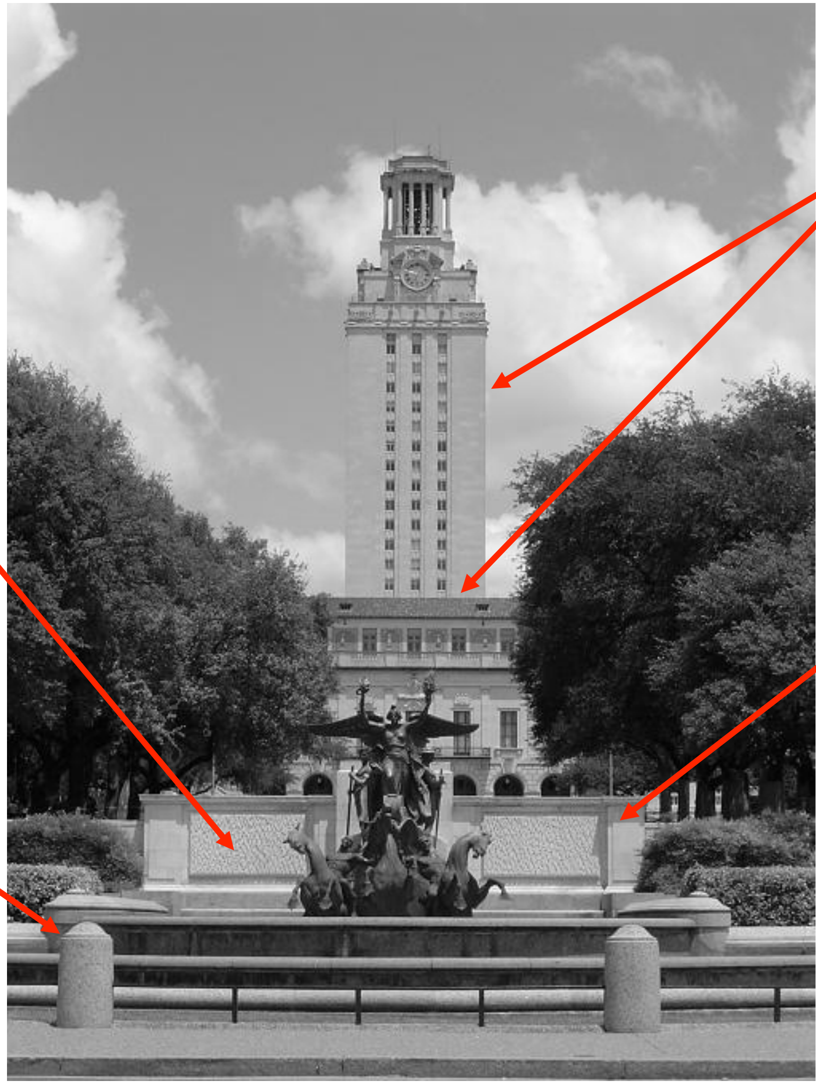
Closeup of edges

Slide credit: D. Hoiem

What causes an edge?

Reflectance change: appearance information, texture

Change in surface orientation: shape



Depth discontinuity: object boundary

Cast shadows

Slide credit: K. Grauman

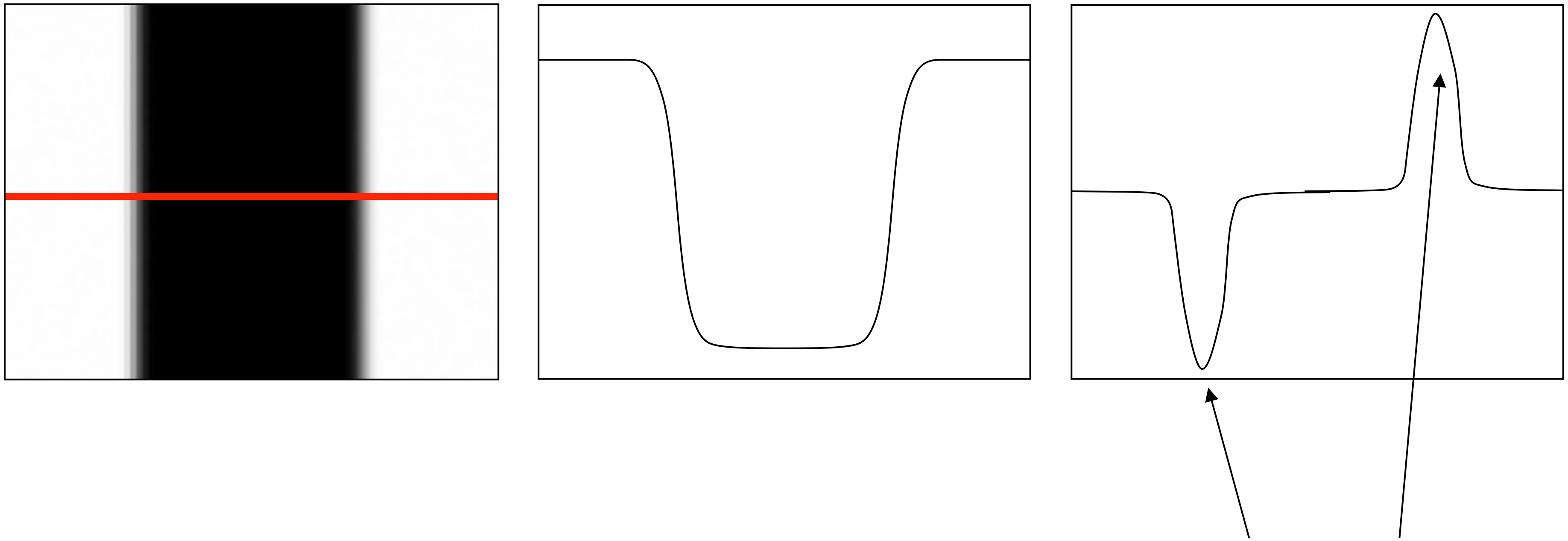
Characterizing edges

- An edge is a place of rapid change in the image intensity function

image

intensity function (along horizontal scanline)

first derivative



edges correspond to extrema of derivative

Slide credit: K. Grauman

Derivatives with convolution

For 2D function f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

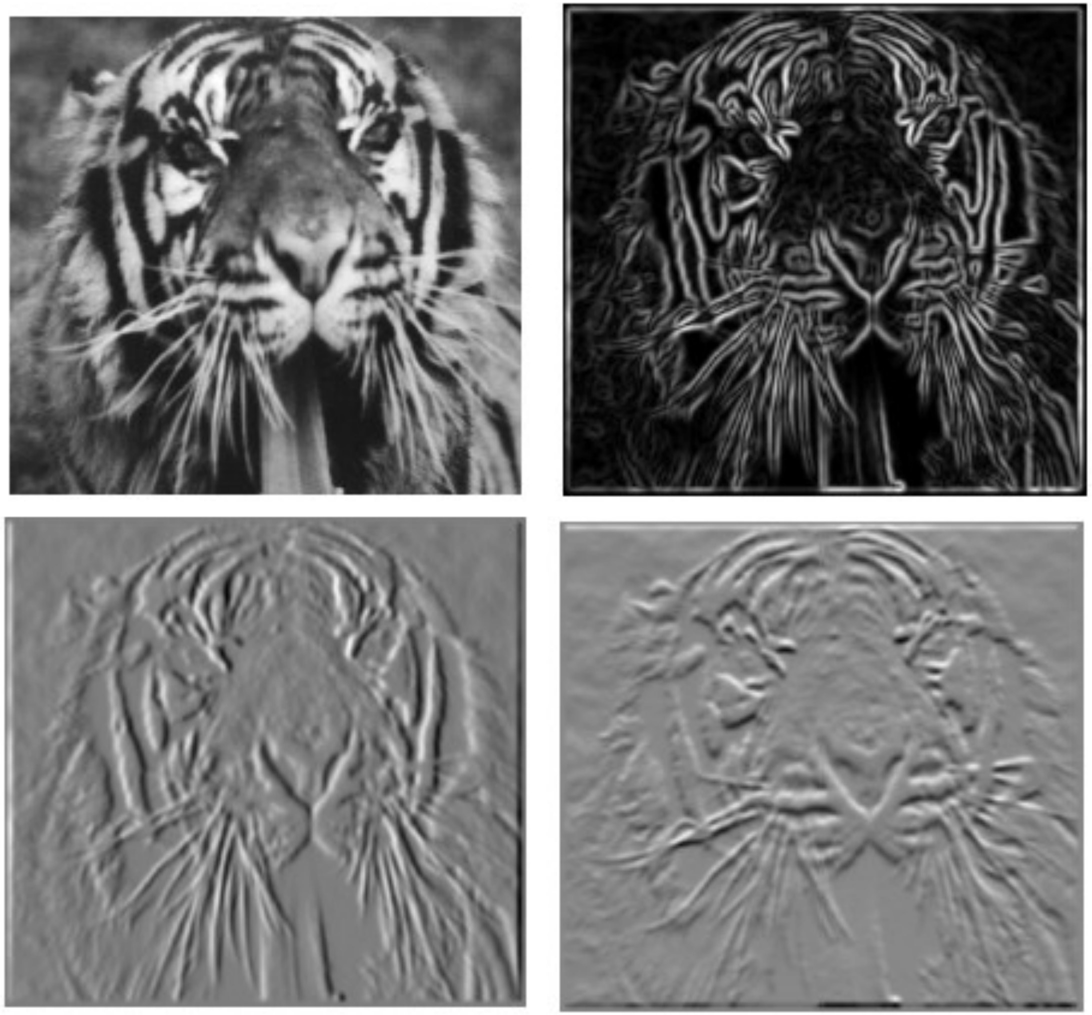
To implement above as convolution, what would be the associated filter?

Slide credit: K. Grauman

Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$

$$\frac{\partial f(x,y)}{\partial y}$$



-1	1

-1
1

Which shows changes with respect to x?

Slide credit: K. Grauman

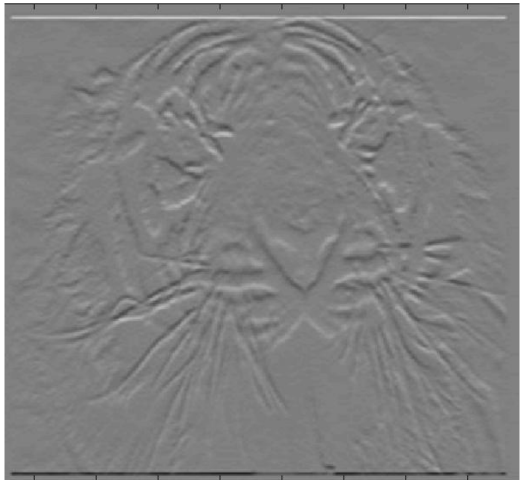
Assorted finite difference filters

Prewitt: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

Sobel: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$; $\boxed{M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}}$

Roberts: $M_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

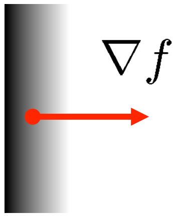
```
>> My = fspecial('sobel');
>> outim = imfilter(double(im), My);
>> imagesc(outim);
>> colormap gray;
```



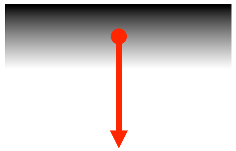
Slide credit: K. Grauman

Image gradient

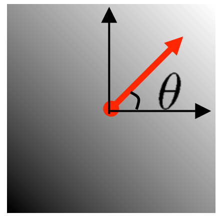
- The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

- 

$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

The gradient direction is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

The *edge strength* is given by the gradient magnitude

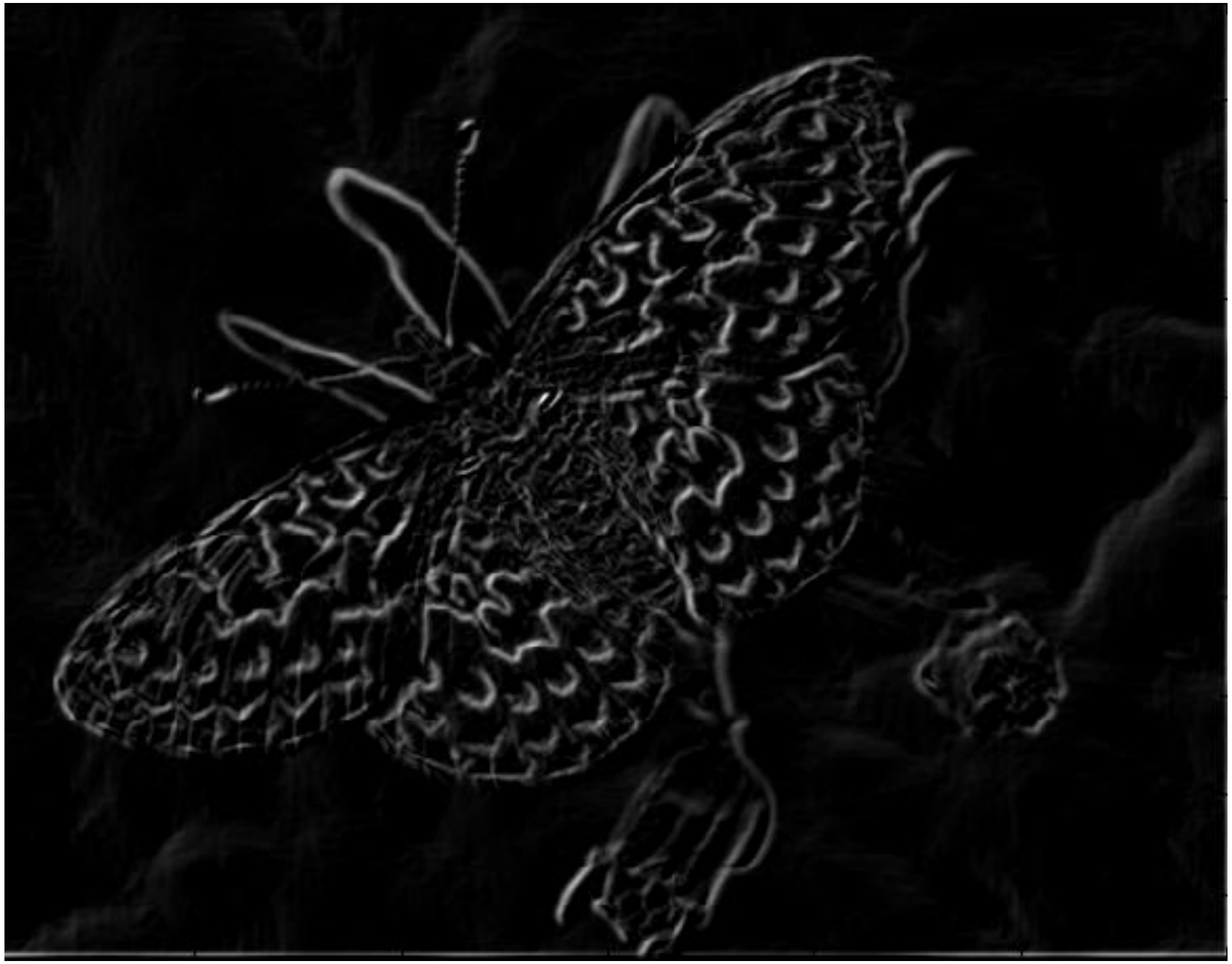
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Slide credit: S. Seitz

Original Image

Slide credit: K. Grauman

Gradient magnitude image



Slide credit: K. Grauman

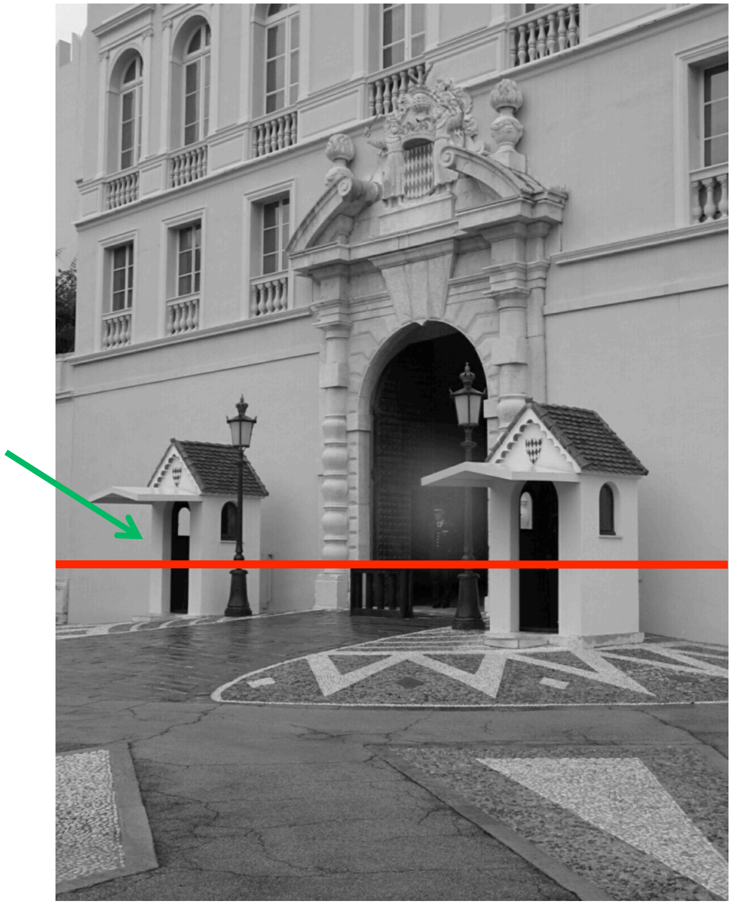
Thresholding gradient with a lower threshold

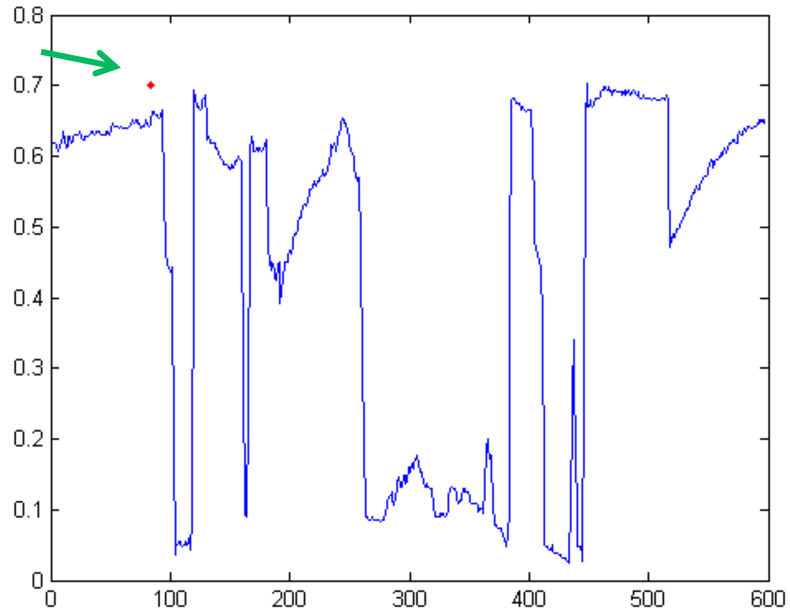
Slide credit: K. Grauman

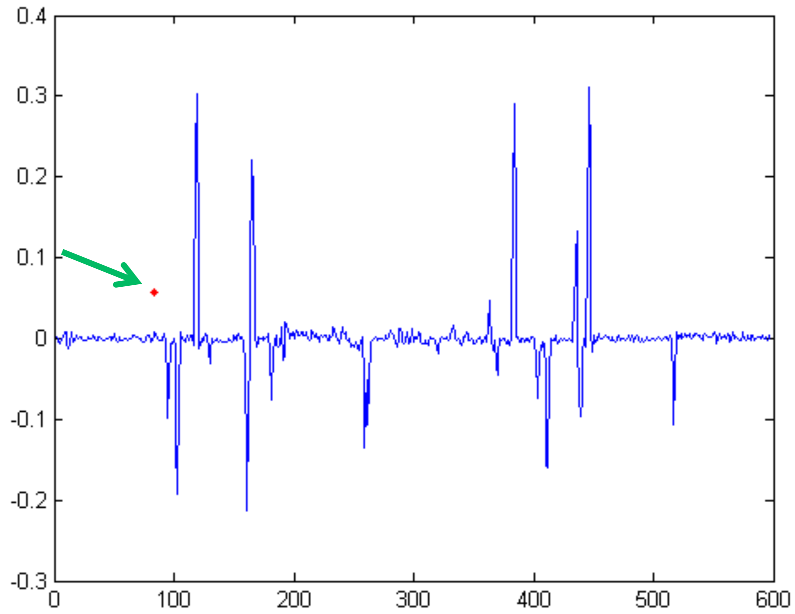
Thresholding gradient with a higher threshold

Slide credit: K. Grauman

Intensity profile

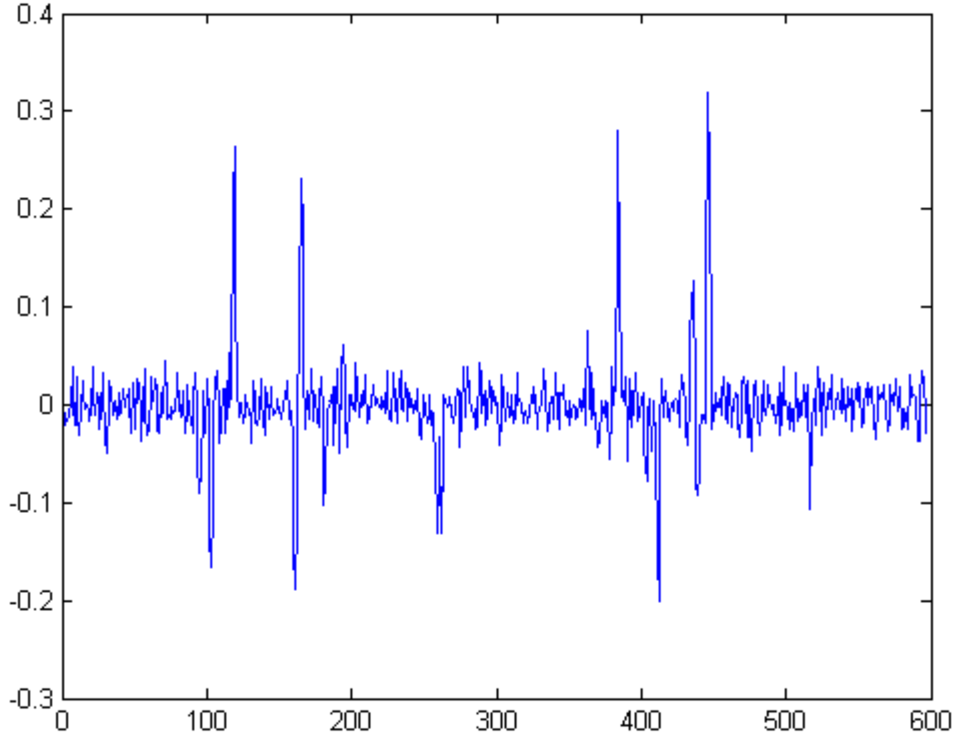






Slide credit: D. Hoiem

With a little Gaussian noise



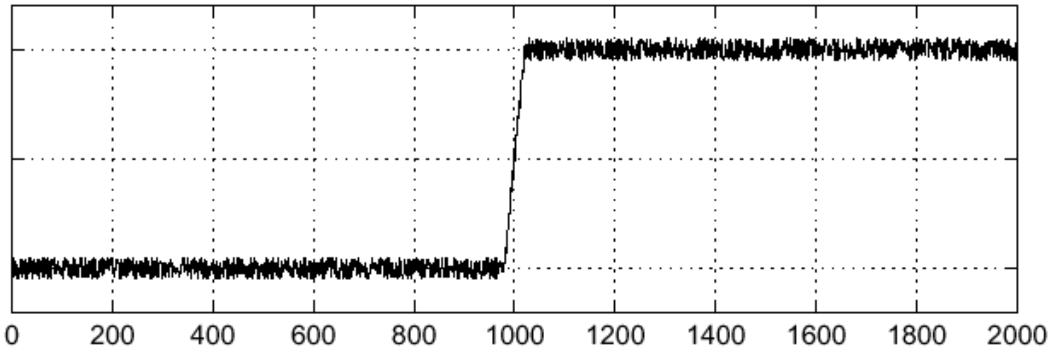
Gradient

Slide credit: D. Hoiem

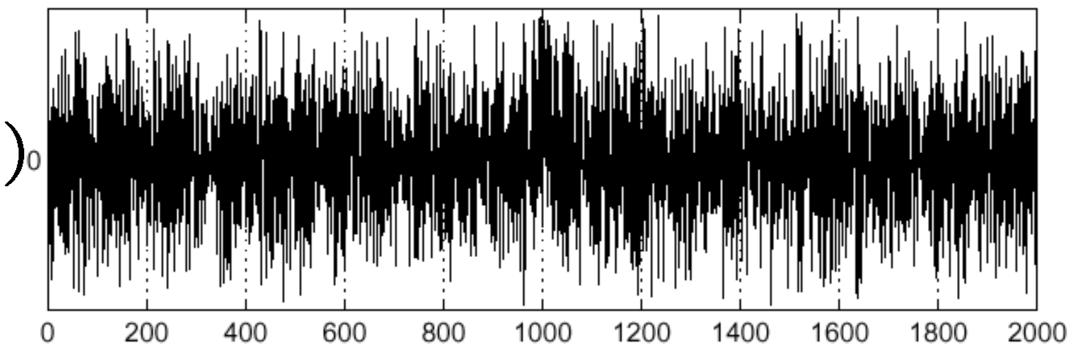
Effects of noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal

$f(x)$



$\frac{d}{dx}f(x)$



Where is the edge?

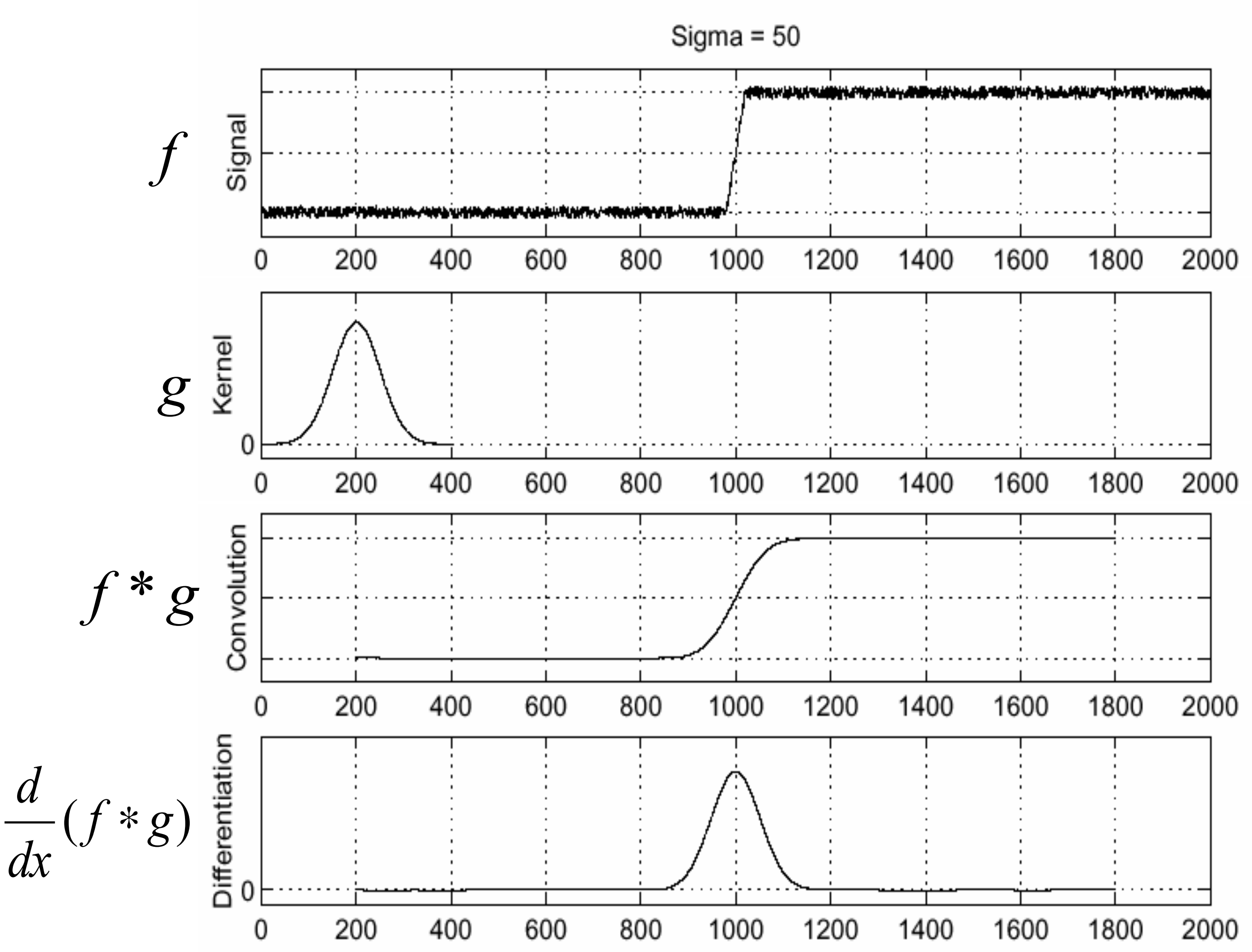
Slide credit: S. Seitz

Effects of noise

- Difference filters respond strongly to noise
  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response
- What can we do about it?

Slide credit: D. Forsyth

Solution: smooth first

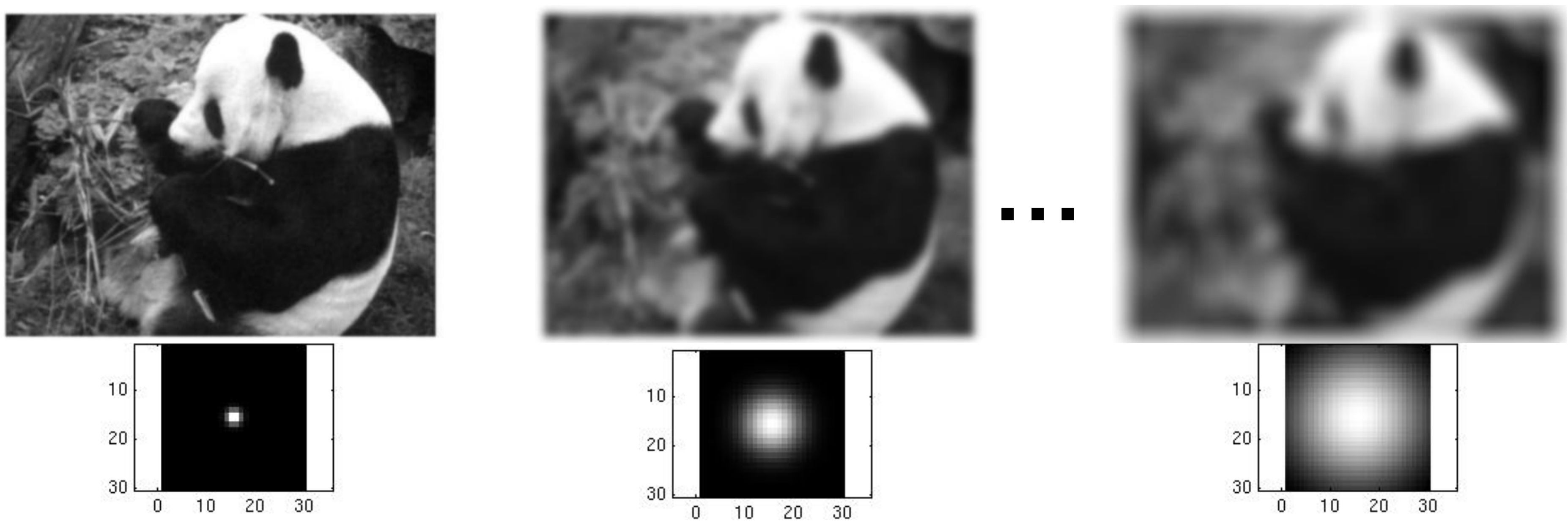


- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Slide credit: S. Seitz

Smoothing with a Gaussian

Recall: parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.



Slide credit: K. Grauman

Effect of σ on derivatives

σ = 1 pixel

σ = 3 pixels

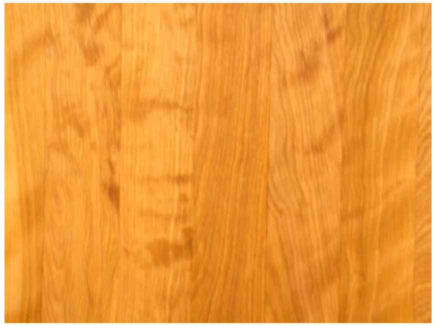
The apparent structures differ depending on Gaussian's scale parameter.

Larger values: larger scale edges detected
Smaller values: finer features detected

Slide credit: K. Grauman

So, what scale to choose?

It depends what we're looking for.



Slide credit: K. Grauman

Smoothing and Edge Detection

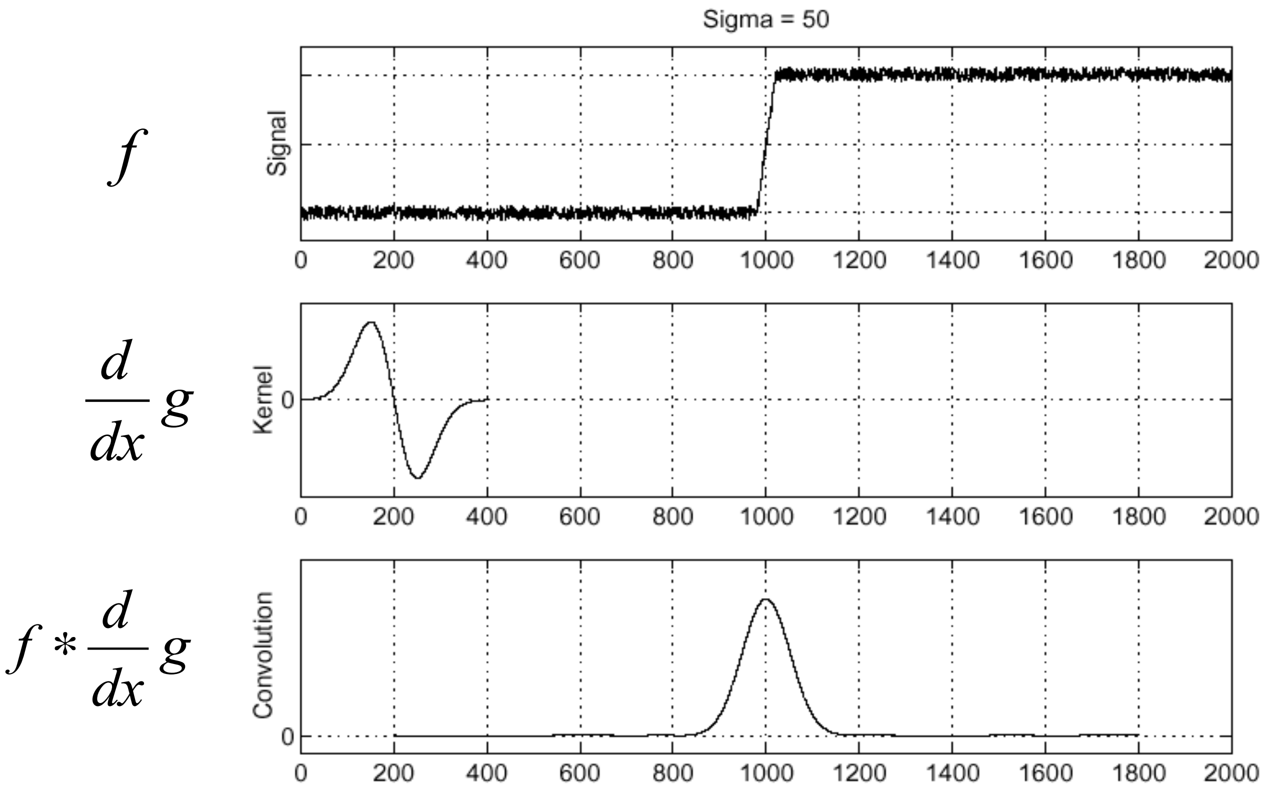
- While eliminating noise via smoothing, we also lose some of the (important) image details.
  - Fine details
  - Image edges
  - etc.
- What can we do to preserve such details?
  - Use edge information during denoising!
  - This requires a definition for image edges.

Chicken-and-egg dilemma!

- Edge preserving image smoothing (Next week's topic!)

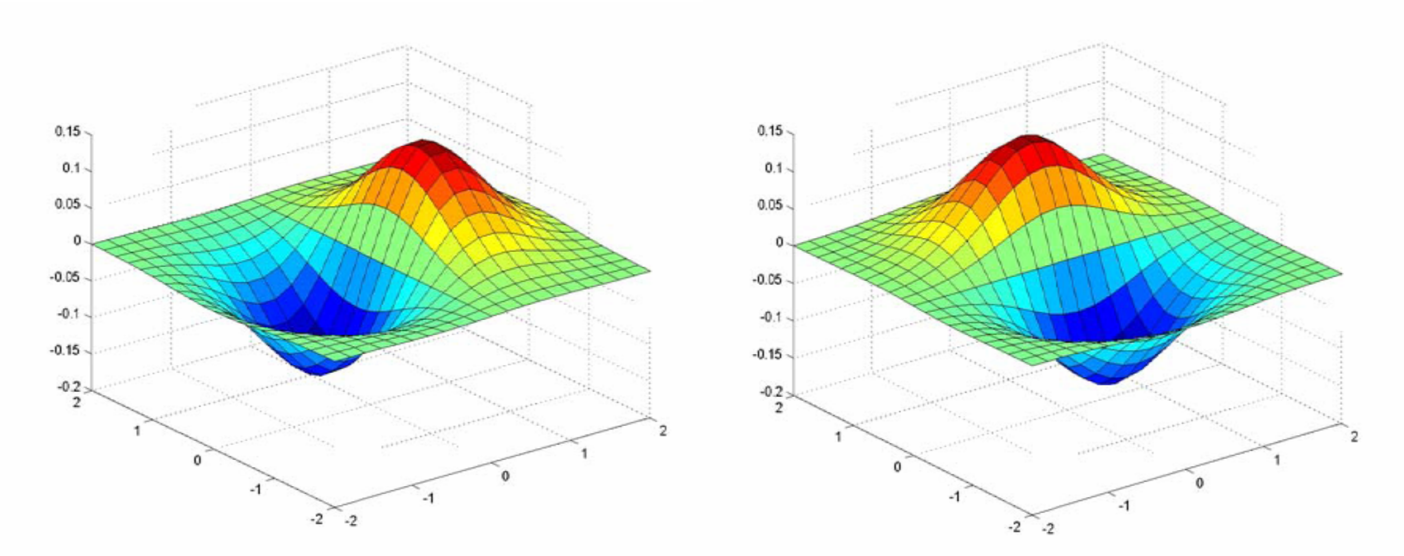
Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative:
- This saves us one operation: $\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$



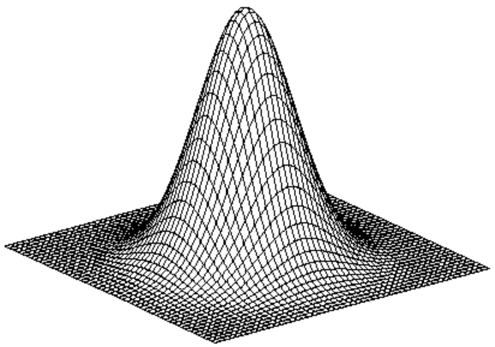
Slide credit: S. Seitz

Derivative of Gaussian filter

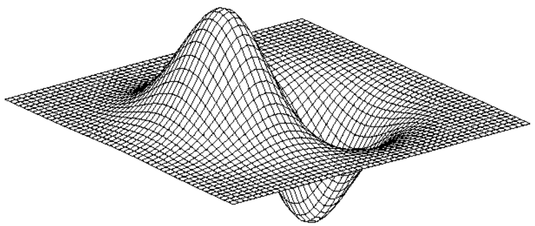


x-direction

y-direction

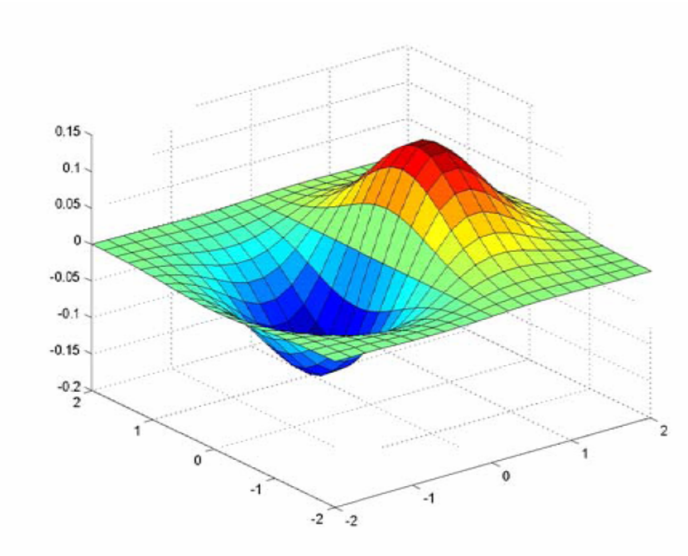


* [1 -1] =

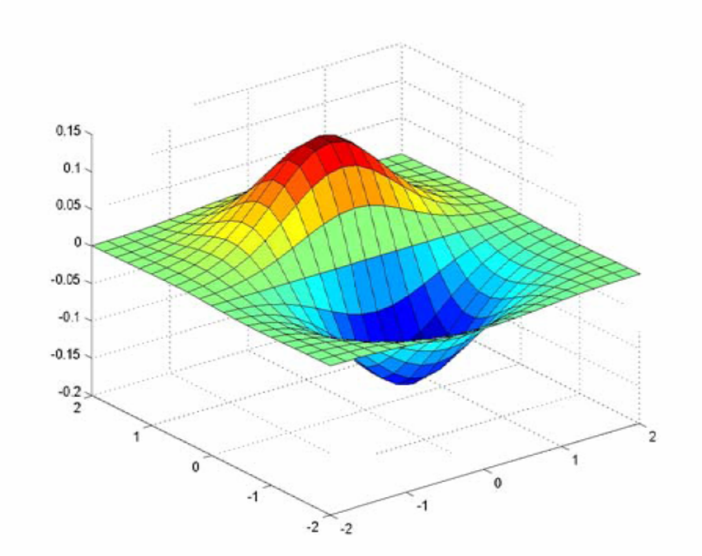


Slide credit: S. Lazebnik

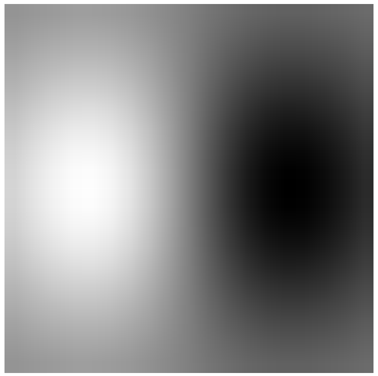
Derivative of Gaussian filter

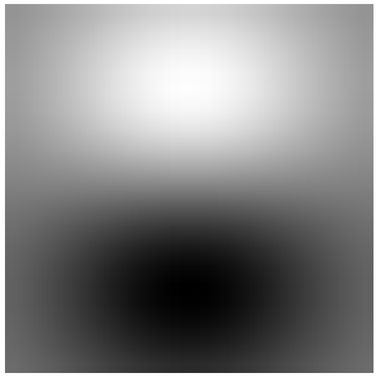


x-direction



y-direction

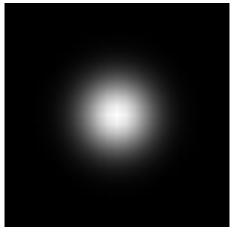




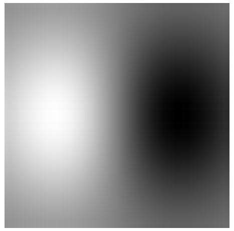
- Which one finds horizontal/vertical edges?

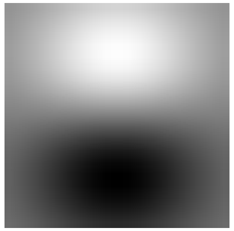
Slide credit: S. Lazebnik

Smoothing vs. derivative filters



- Smoothing filters
  - Gaussian: remove "high-frequency" components; "low-pass" filter
  - Can the values of a smoothing filter be negative?
  - What should the values sum to?
    - **One:** constant regions are not affected by the filter





- Derivative filters
  - Derivatives of Gaussian
  - Can the values of a derivative filter be negative?
  - What should the values sum to?
    - **Zero:** no response in constant regions
  - High absolute value at points of high contrast

Slide credit: S. Lazebnik

Reading Assignment #4

PROCEEDINGS OF THE ROYAL SOCIETY B | BIOLOGICAL SCIENCES

Theory of Edge Detection

D. Marr and E. Hildreth

Proc. R. Soc. Lond. B 1980 **207**, 187-217

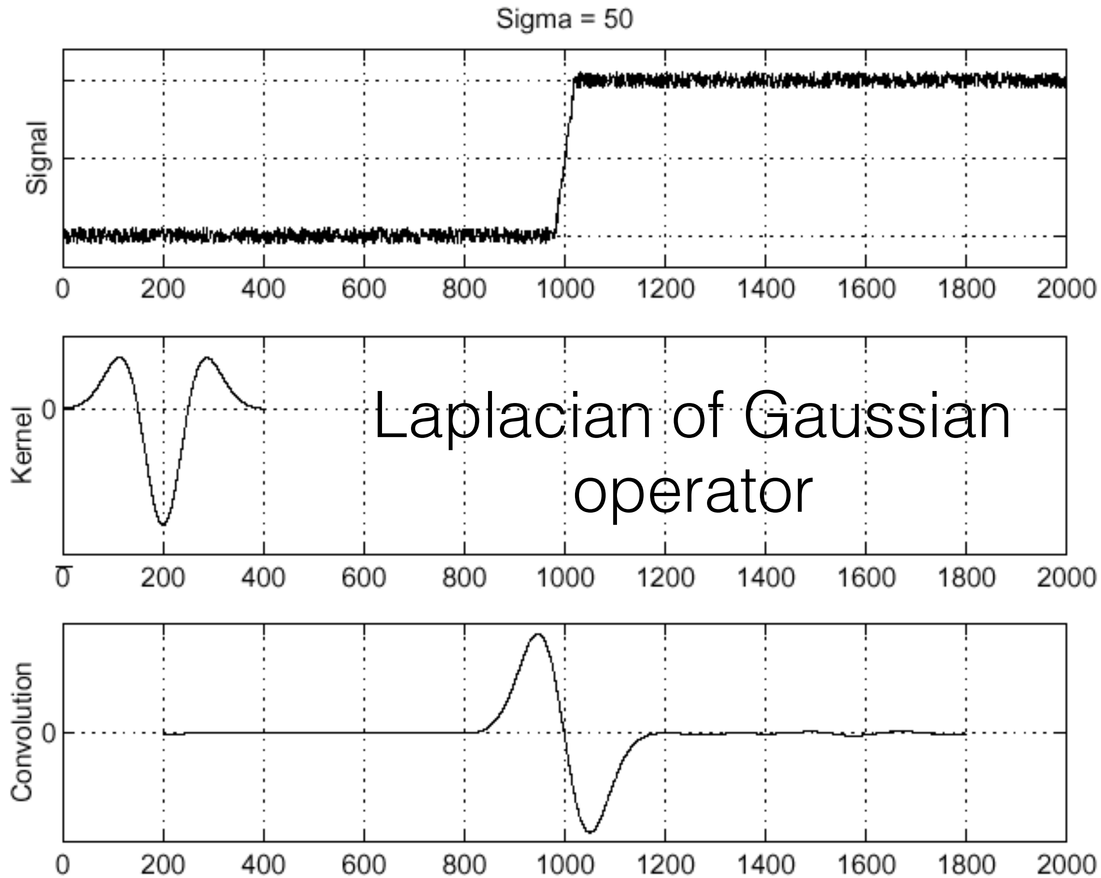
- One of the 60 seminal articles appeared in the journal Philosophical Transactions, which is made available online due to the celebration of 350th birthday of the Royal Society in 2010. [http://trailblazing.royalsociety.org]
- Due on 21^{st} of December

Laplacian of Gaussian

Consider $\frac{\partial^2}{\partial x^2}(h \star f)$

$\frac{\partial^2}{\partial x^2}h$

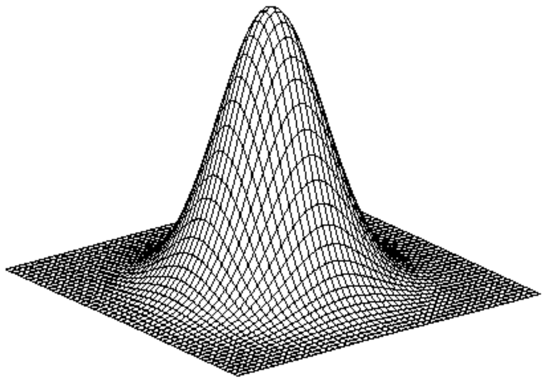
$(\frac{\partial^2}{\partial x^2}h) \star f$



Where is the edge? Zero-crossings of bottom graph

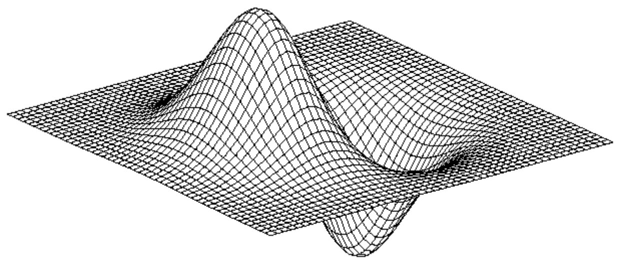
Slide credit: K. Grauman

2D edge detection filters



Gaussian

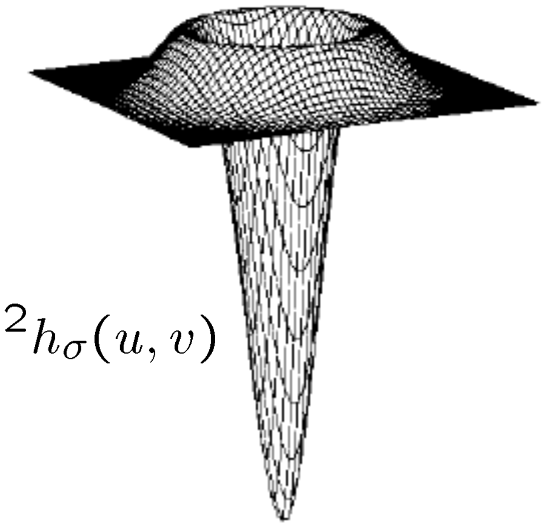
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



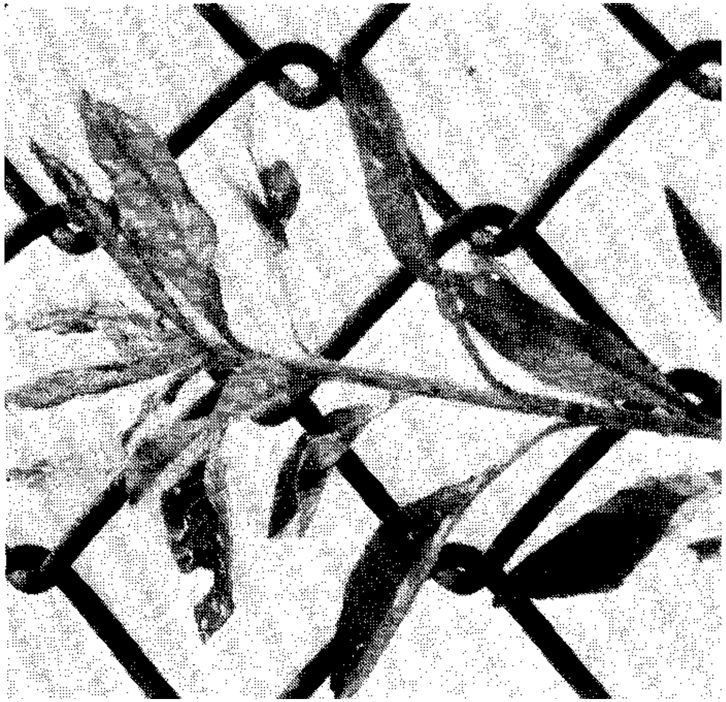
$$\nabla^2 h_\sigma(u,v)$$

- The Laplacian operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Slide credit: K. Grauman

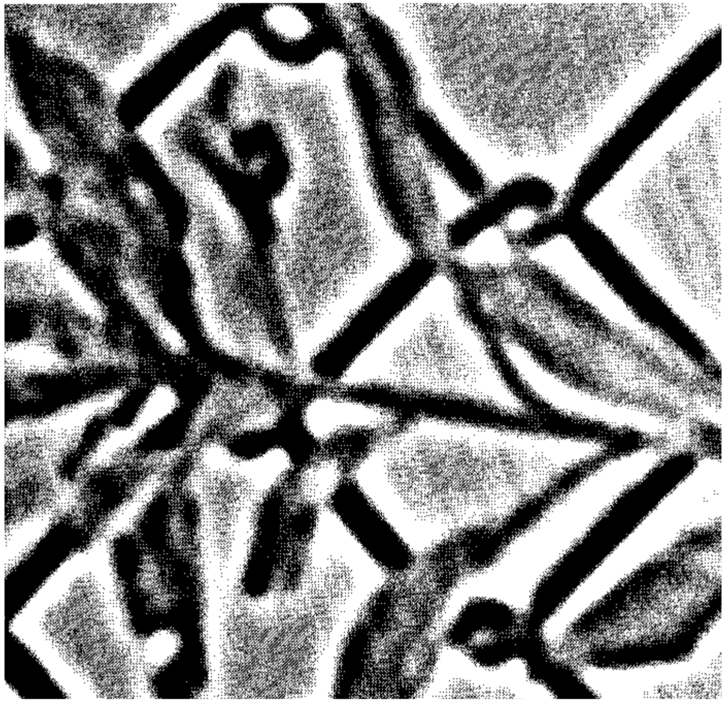
Laplacian of Gaussian



original image

Source: D. Marr and E. Hildreth (1980)

Laplacian of Gaussian



convolution with
$$\nabla^2 h_\sigma(u,v)$$

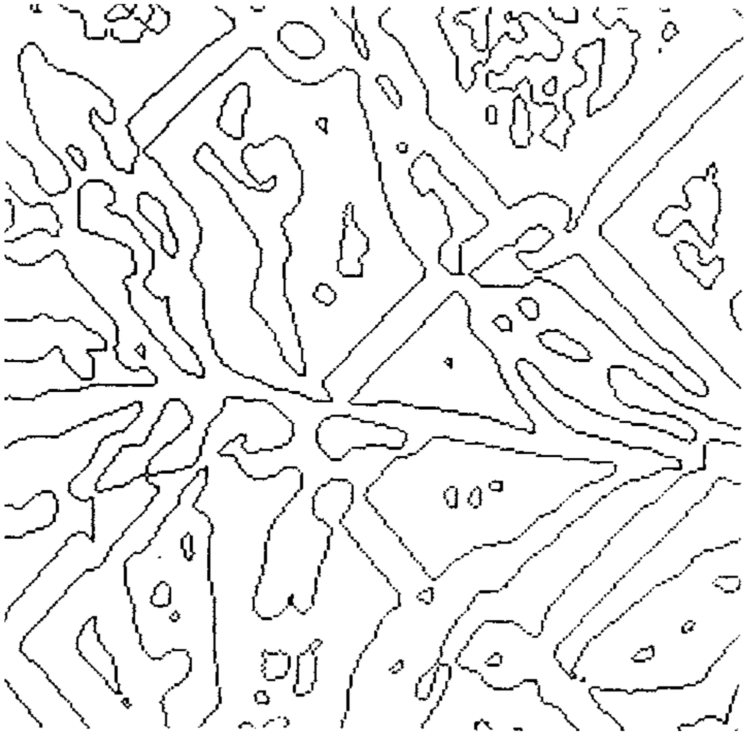
Source: D. Marr and E. Hildreth (1980)

Laplacian of Gaussian

convolution with
$$\nabla^2 h_\sigma(u, v)$$
(pos. values – white, neg. values – black)

Source: D. Marr and E. Hildreth (1980)

Laplacian of Gaussian



zero-crossings

Source: D. Marr and E. Hildreth (1980)

Designing an edge detector

- Criteria for a good edge detector:
  - **Good detection:** the optimal detector should find all real edges, ignoring noise or other artifacts
  - **Good localization**
    - the edges detected must be as close as possible to the true edges
    - the detector must return one point only for each true edge point
- Cues of edge detection
  - Differences in color, intensity, or texture across the boundary
  - Continuity and closure
  - High-level knowledge

Slide credit: L. Fei-Fei

The Canny edge detector

original image (Lena)

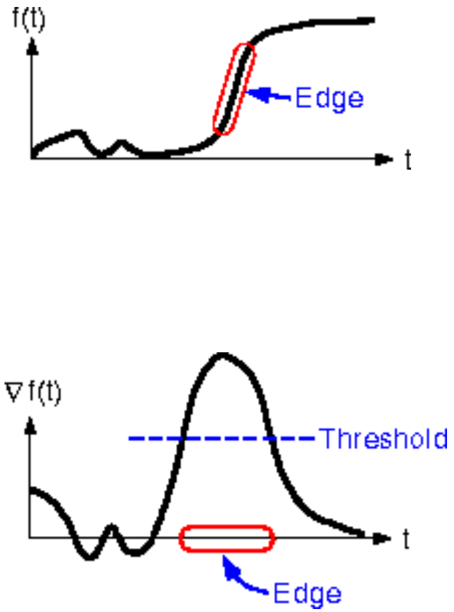
Slide credit: K. Grauman

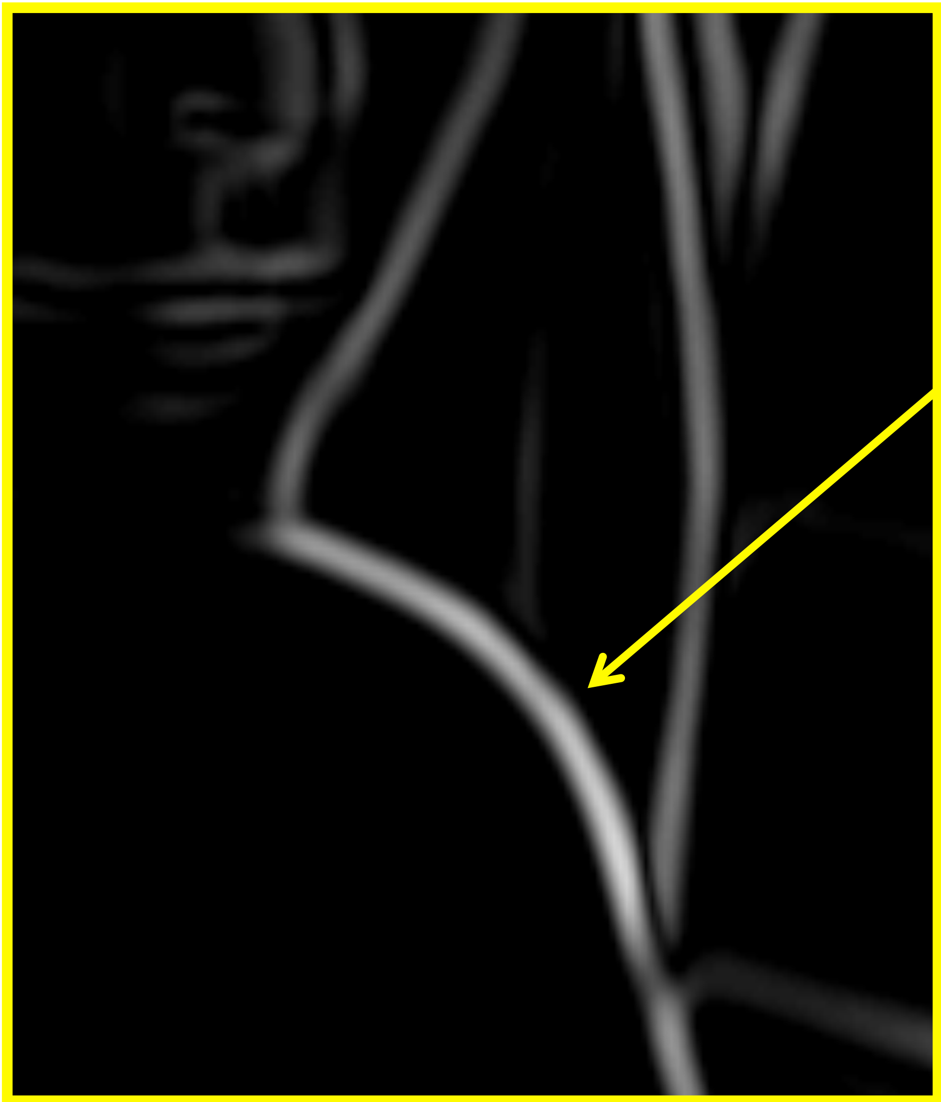
The Canny edge detector

thresholding

Slide credit: K. Grauman

The Canny edge detector

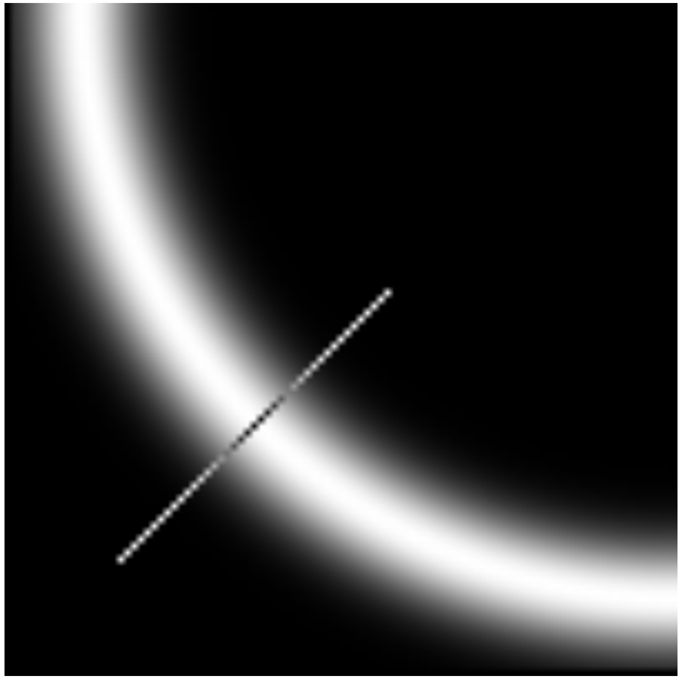


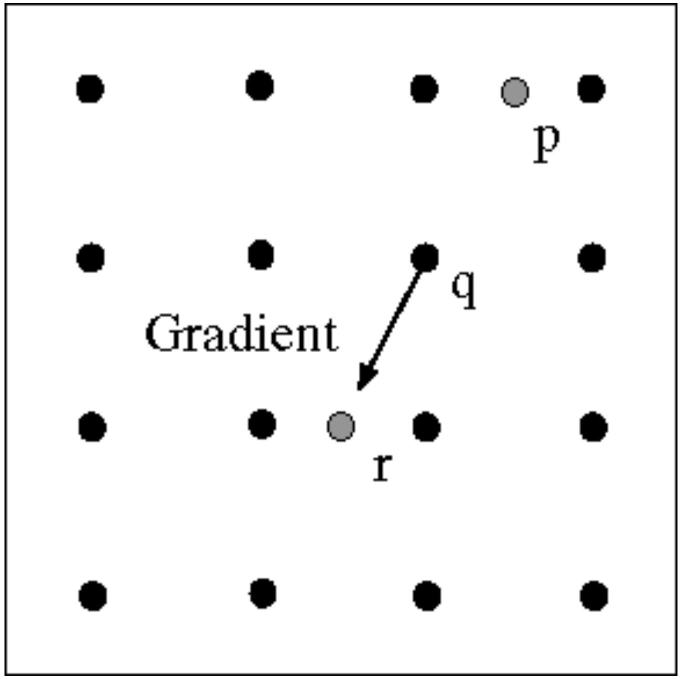


How to turn these thick regions of the gradient into curves?

Slide credit: K. Grauman

Non-maximum suppression





Check if pixel is local maximum along gradient direction, select single max across width of the edge

– requires checking interpolated pixels p and r

Slide credit: K. Grauman

The Canny Edge Detector

Problem: pixels along this edge didn't survive the thresholding

thinning
(non-maximum suppression)

Slide credit: K. Grauman

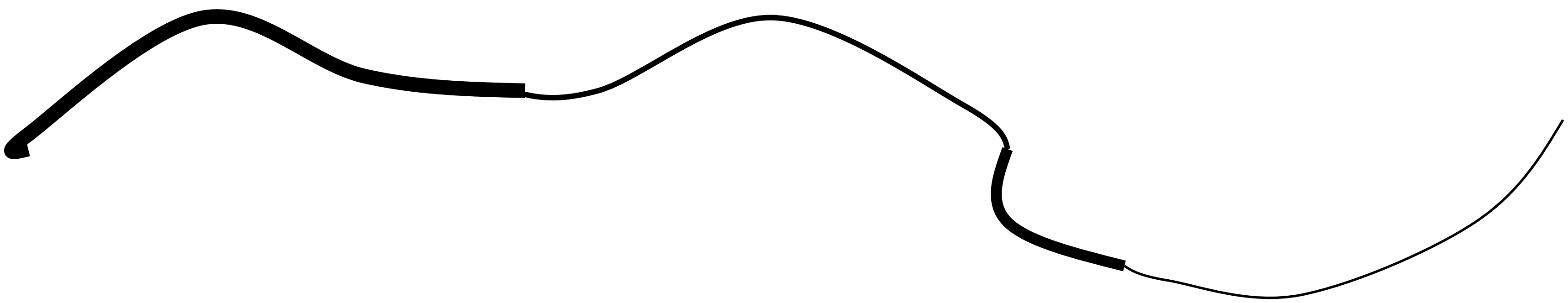
Hysteresis thresholding

- Threshold at low/high levels to get weak/strong edge pixels
- Do connected components, starting from strong edge pixels

Slide credit: J. Hays

Hysteresis thresholding

- Check that maximum value of gradient value is sufficiently large
  - drop-outs? use **hysteresis**
    - use a high threshold to start edge curves and a low threshold to continue them.



Slide credit: S. Seitz

Hysteresis thresholding

original image

high threshold
(strong edges)

low threshold
(weak edges)

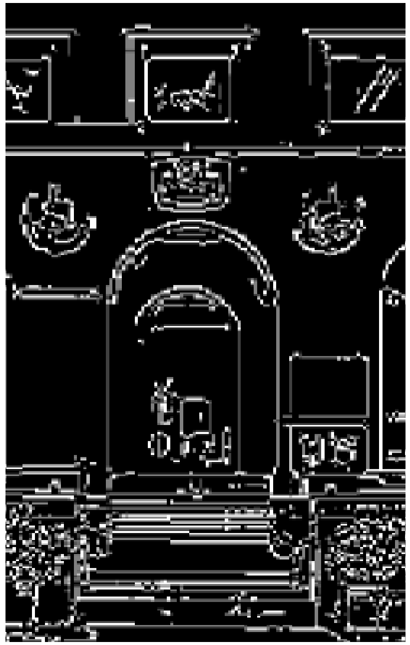
hysteresis threshold

Slide credit: L. Fei-Fei

Hysteresis thresholding

high threshold
(strong edges)

low threshold
(weak edges)



hysteresis threshold

Slide credit: L. Fei-Fei

Recap: Canny edge detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. **Non-maximum suppression**:
   - Thin wide "ridges" down to single pixel width
4. **Linking and thresholding** (**hysteresis**):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB: `edge(image, 'canny');`

Slide credit: D. Lowe, L. Fei-Fei

Effect of σ (Gaussian kernel spread/size)

original | Canny with $\sigma = 1$ | Canny with $\sigma = 2$

The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

Slide credit: S. Seitz

Low-level edges vs. perceived contours

Background

Texture

Shadows

Slide credit: K. Grauman

Edge detection is just the beginning…

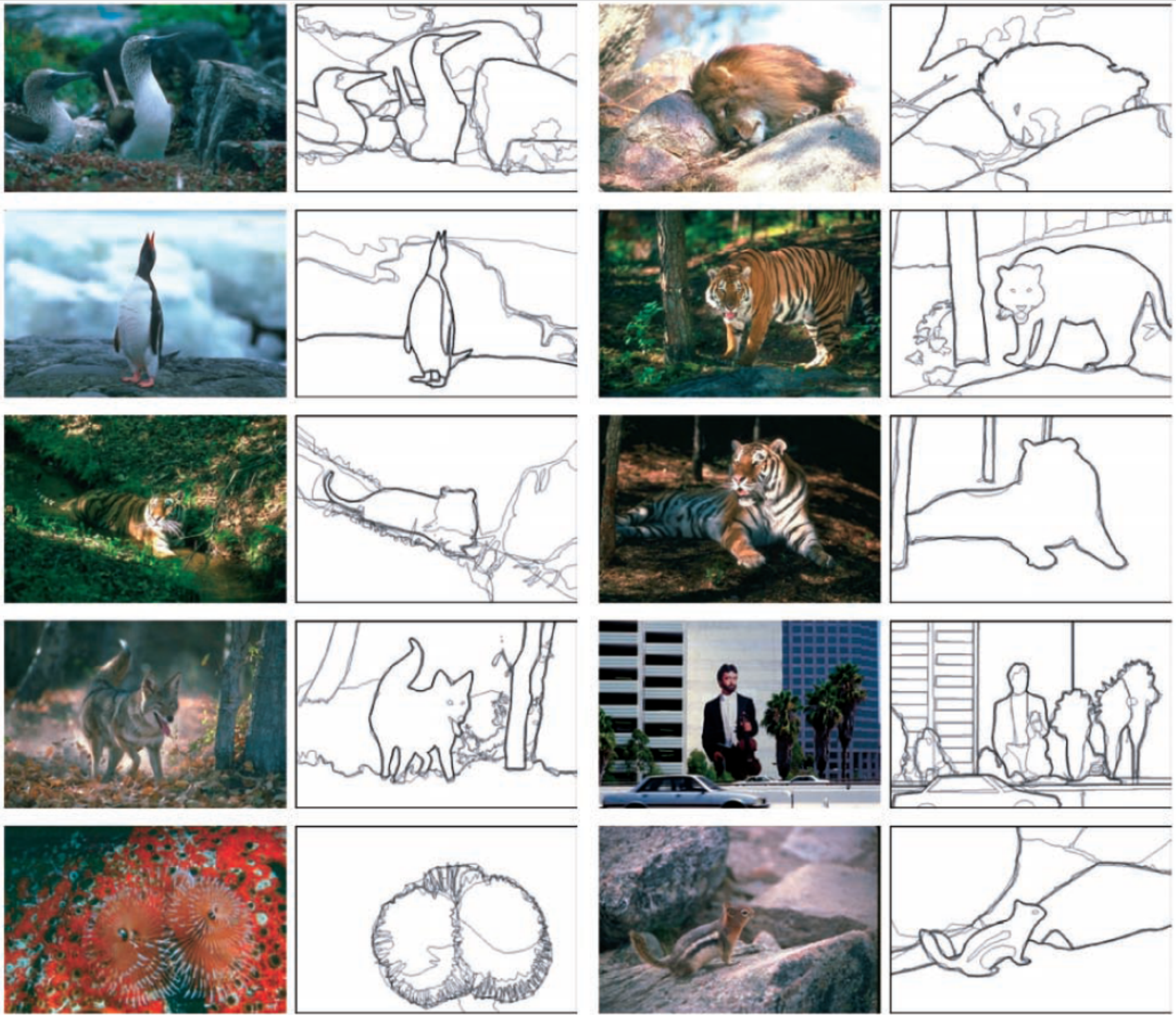
image | human segmentation | gradient magnitude

- Berkeley segmentation database: http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/

Source: S. Lazebnik

Learn from humans which combination of features is most indicative of a "good" contour?

[D. Martin et al. PAMI 2004]



Human-marked segment boundaries

Slide credit: K. Grauman

Today

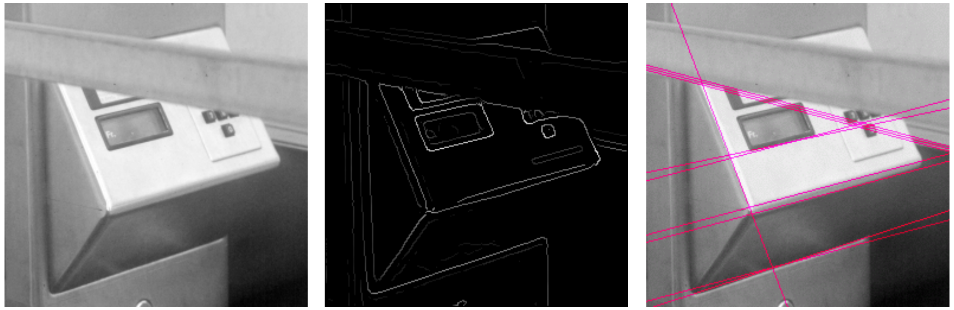
- Edge detection
  - Difference filters
  - Laplacian of Gaussian
  - Canny edge detection
- Boundary detection
  - Hough transform

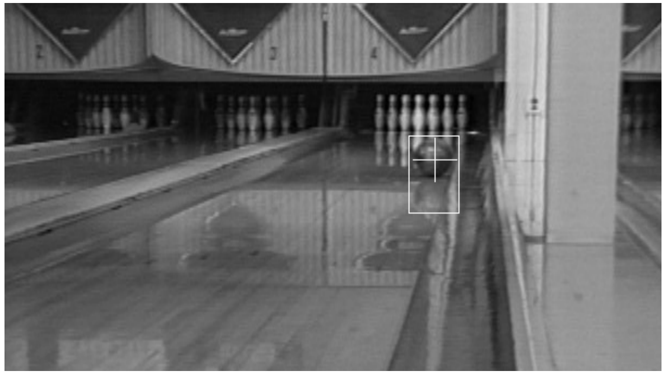
Edges vs. Boundaries

- Edges
  - abrupt changes in the intensity
  - discontinuities in intensity values
  - a local entity
- Edge detection may result in
  - Breaks in the edges due to non-uniform illumination
  - Spurious edges
- Boundaries
  - related to regions
  - a global entity
  - assemble of meaningful edge points
- Boundary detection requires grouping or fitting

Fitting

- Want to associate a model with observed features





[Fig from Marszalek & Schmid, 2007]

For example, the model could be a line, a circle, or an arbitrary shape.

Slide credit: K. Grauman

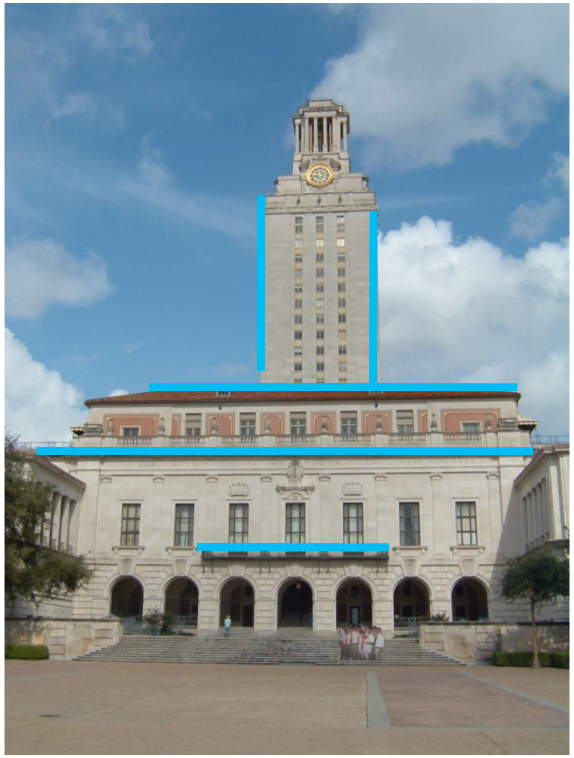
Fitting: Main idea

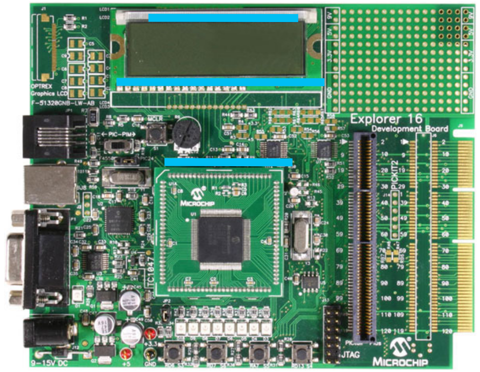
- Choose a parametric model to represent a set of features
- Membership criterion is not local
  - Can't tell whether a point belongs to a given model just by looking at that point
- Three main questions:
  - What model represents this set of features best?
  - Which of several model instances gets which feature?
  - How many model instances are there?
- Computational complexity is important
  - It is infeasible to examine every possible set of parameters and every possible combination of features

Slide credit: L. Lazebnik

Example: Line fitting

- Why fit lines?
  - Many objects characterized by presence of straight lines

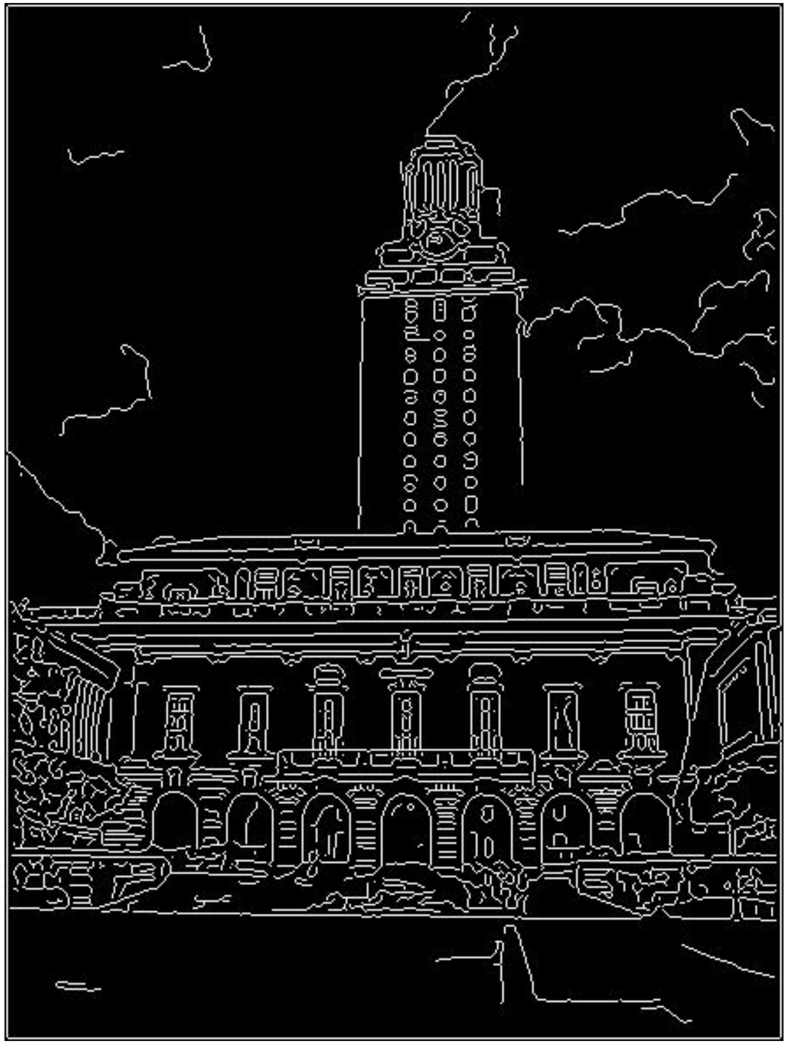


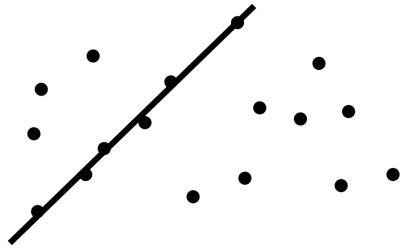


Wait, why aren't we done just by running edge detection?

Slide credit: K. Grauman

Difficulty of line fitting





- **Extra** edge points (clutter), multiple models:
  - which points go with which line, if any?
- Only some parts of each line detected, and some parts are **missing:**
  - how to find a line that bridges missing evidence?
- **Noise** in measured edge points, orientations:
  - how to detect true underlying parameters?

Slide credit: K. Grauman

Voting

- It's not feasible to check all combinations of features by fitting a model to each possible subset.

- **Voting** is a general technique where we let the features *vote for all models that are compatible with it.*

  – Cycle through features, cast votes for model parameters.

  – Look for model parameters that receive a lot of votes.

- Noise & clutter features will cast votes too, *but* typically their votes should be inconsistent with the majority of "good" features.

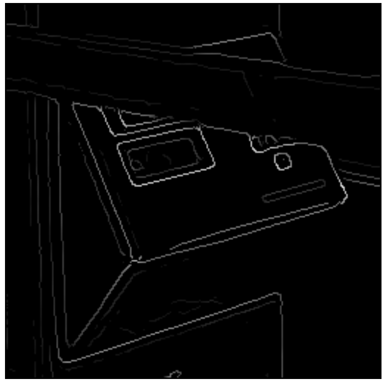
Slide credit: K. Grauman

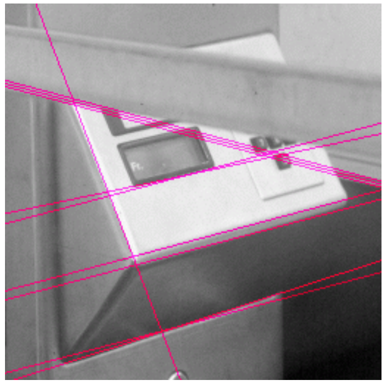
Fitting lines: Hough transform

- Given points that belong to a line, what is the line?
- How many lines are there?
- Which points belong to which lines?

- **Hough Transform** is a voting technique that can be used to answer all of these questions.

  Main idea:
  1. Record vote for each possible line on which each edge point lies.
  2. Look for lines that get many votes.





Slide credit: K. Grauman

Finding lines in an image: Hough space

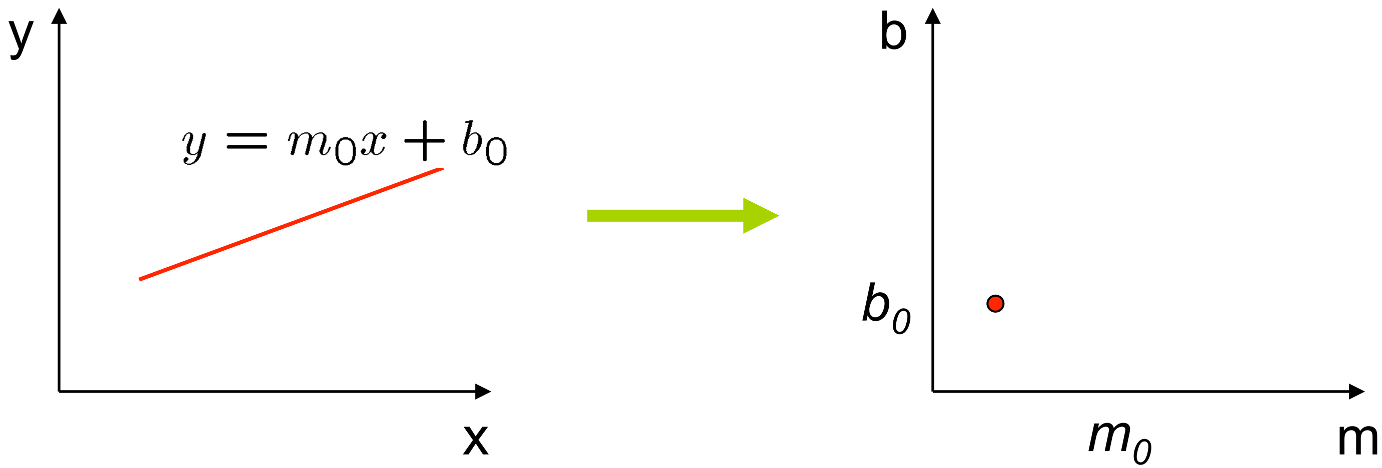


image space

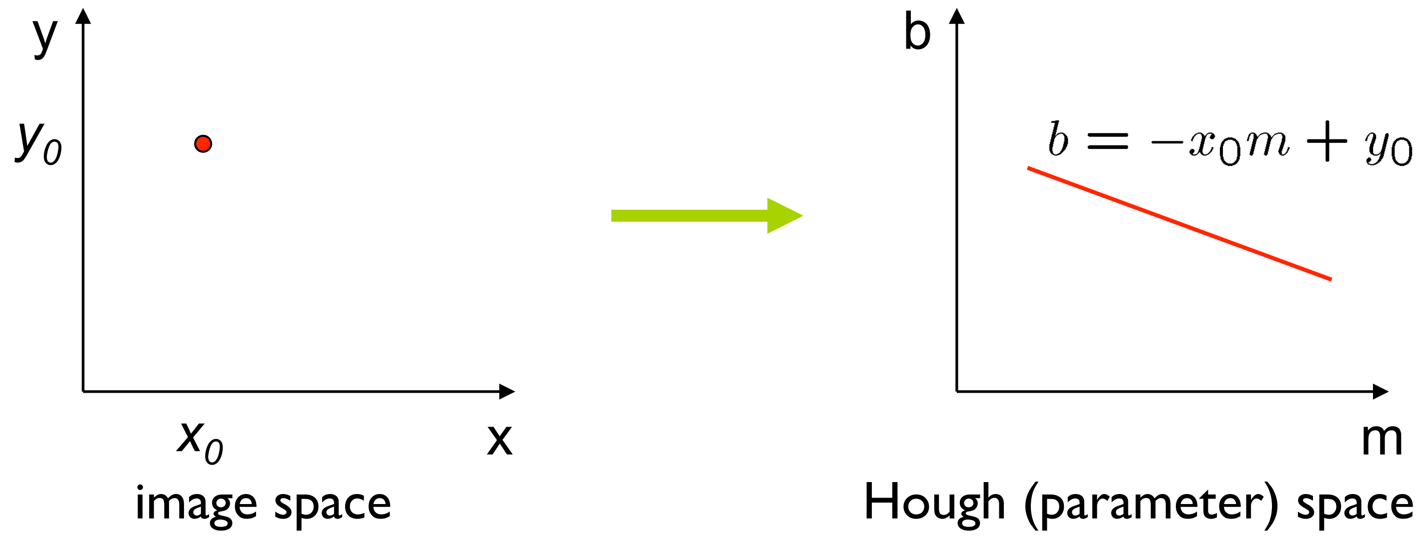
Hough (parameter) space

Connection between image (x,y) and Hough (m,b) spaces

- A line in the image corresponds to a point in Hough space
- To go from image space to Hough space:
    - given a set of points (x,y), find all (m,b) such that y = mx + b

Slide credit: S. Seitz

Finding lines in an image: Hough space

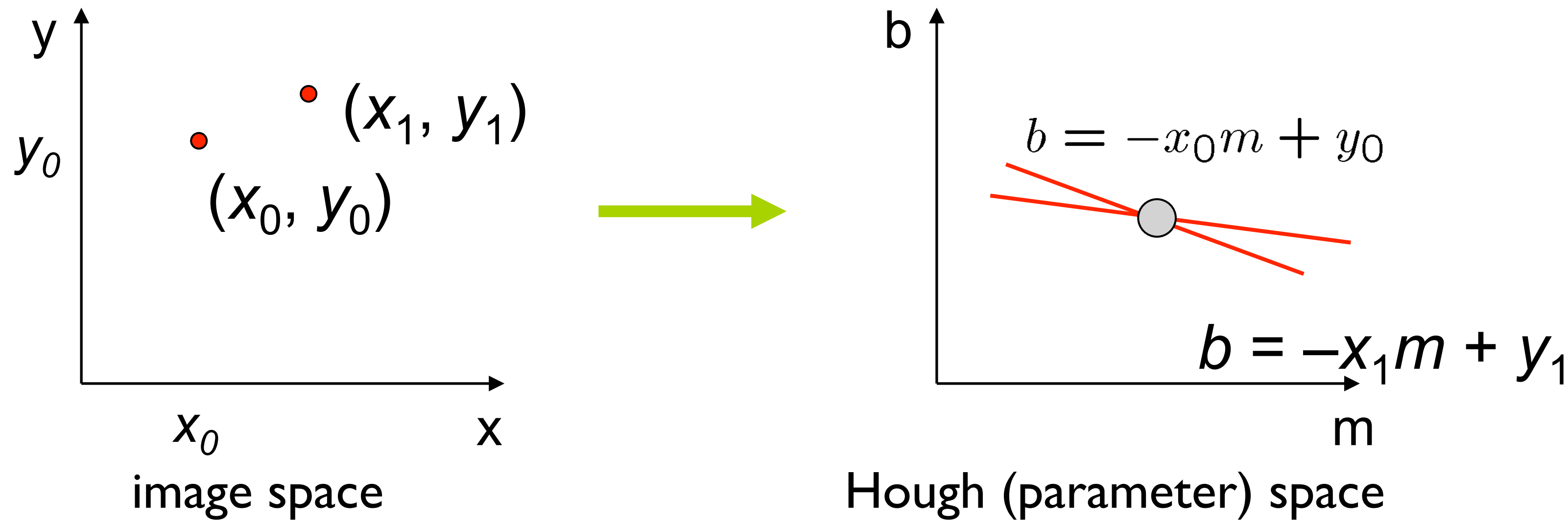


Connection between image (x,y) and Hough (m,b) spaces

- A line in the image corresponds to a point in Hough space
- To go from image space to Hough space:
  - given a set of points (x,y), find all (m,b) such that y = mx + b
- What does a point (x_0, y_0) in the image space map to?
  - Answer: the solutions of $b = -x_0 m + y_0$
  - this is a line in Hough space

Slide credit: S. Seitz

Finding lines in an image: Hough space



What are the line parameters for the line that contains both (x_0, y_0) and (x_1, y_1)?

- It is the intersection of the lines $b = -x_0 m + y_0$ and $b = -x_1 m + y_1$

Slide credit: K. Grauman

Finding lines in an image: Hough space

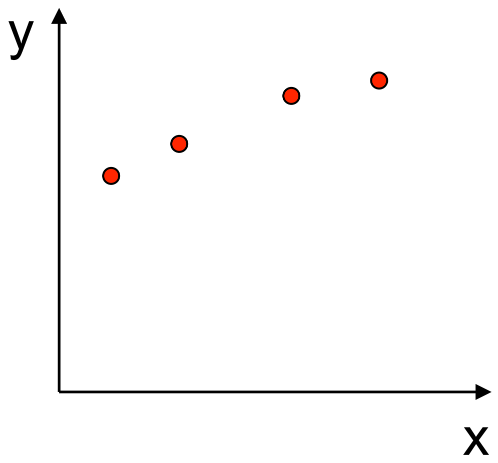
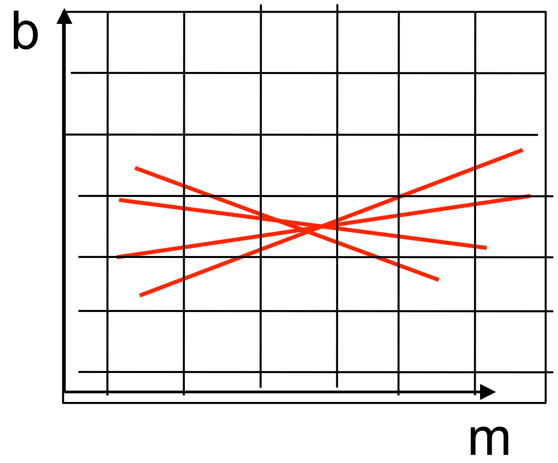


image space



Hough (parameter) space

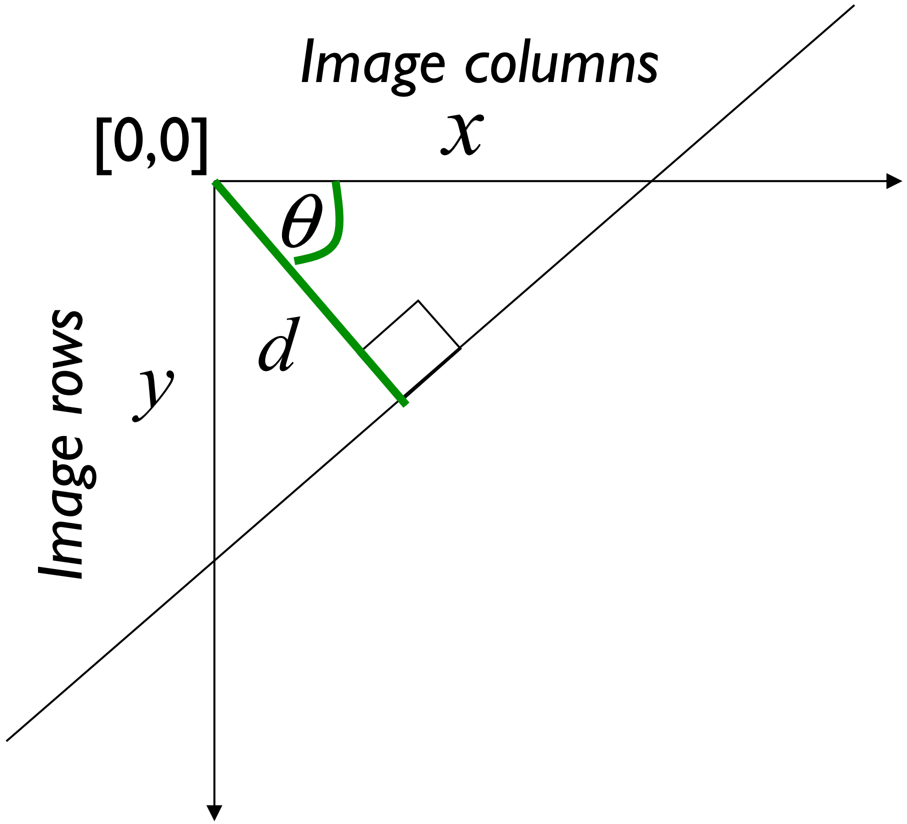
How can we use this to find the most likely parameters (m,b) for the most prominent line in the image space?

- Let each edge point in image space *vote* for a set of possible parameters in Hough space
- Accumulate votes in discrete set of bins; parameters with the most votes indicate line in image space.

Slide credit: K. Grauman

Polar representation for lines

Issues with usual (*m,b*) parameter space: can take on infinite values, undefined for vertical lines.



d : perpendicular distance from line to origin

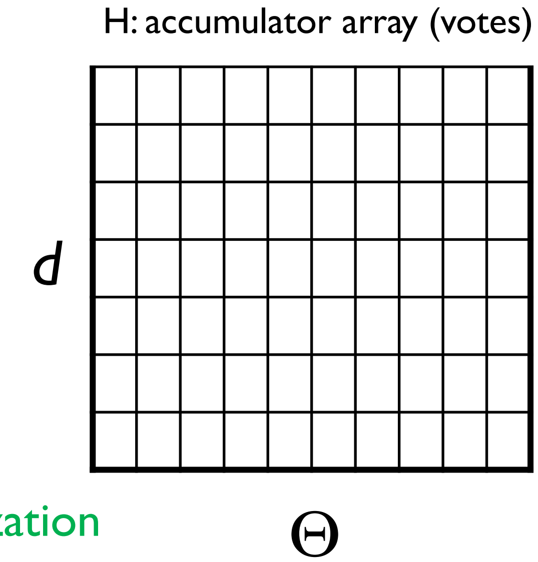
θ : angle the perpendicular makes with the x-axis

$$x\cos\theta - y\sin\theta = d$$

Point in image space → sinusoid segment in Hough space

Slide credit: K. Grauman

Hough transform algorithm



Using the polar parameterization:

$$x\cos\theta - y\sin\theta = d$$

Basic Hough transform algorithm

1. Initialize H[d, Θ]=0
2. for each edge point I[x,y] in the image

   for Θ = [Θ_{min} to Θ_{max}] // some quantization

   $$d = x\cos\theta - y\sin\theta$$

   H[d, Θ] += 1
3. Find the value(s) of (d, Θ) where H[d, Θ] is maximum
4. The detected line in the image is given by $d = x\cos\theta - y\sin\theta$

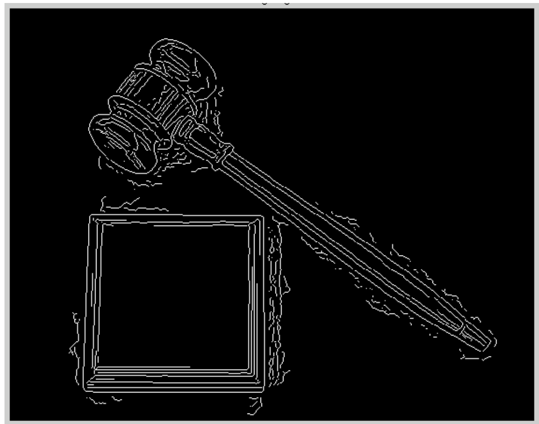
Time complexity (in terms of number of votes per pt)?

Slide credit: S. Seitz

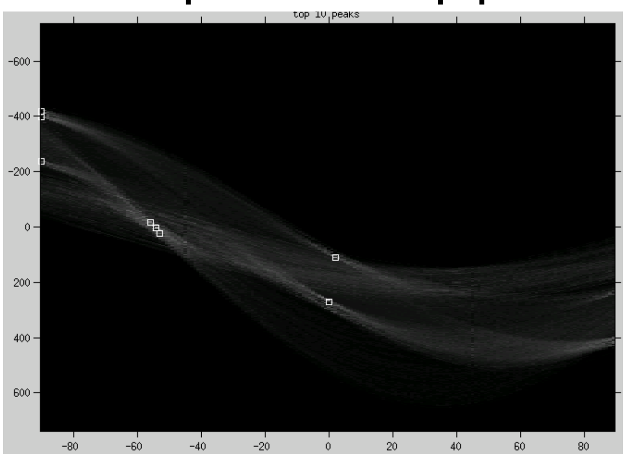
Hough transform algorithm

Original image

Canny edges

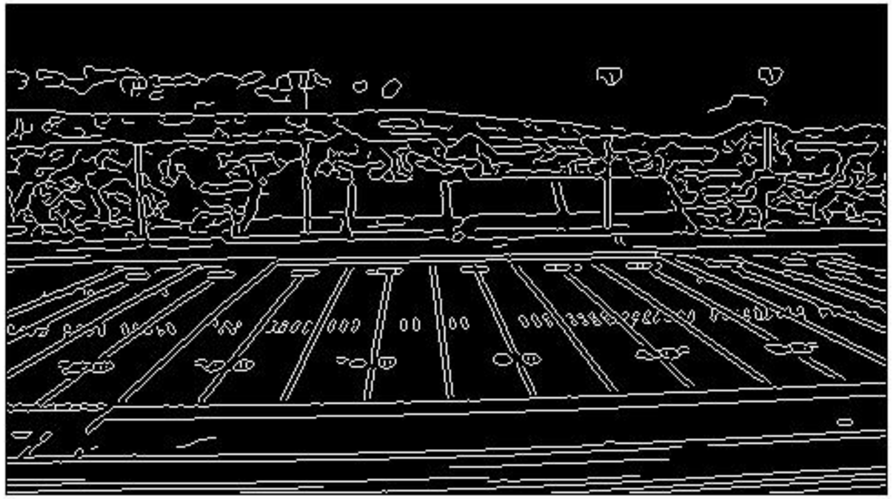


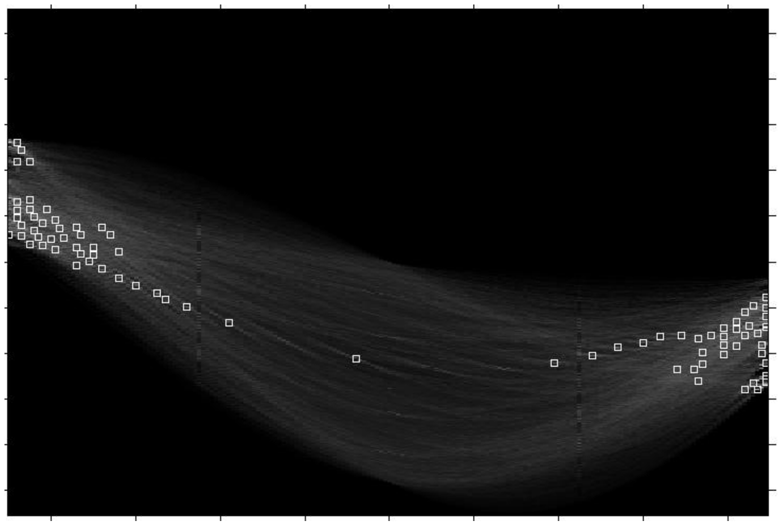
Vote space and top peaks

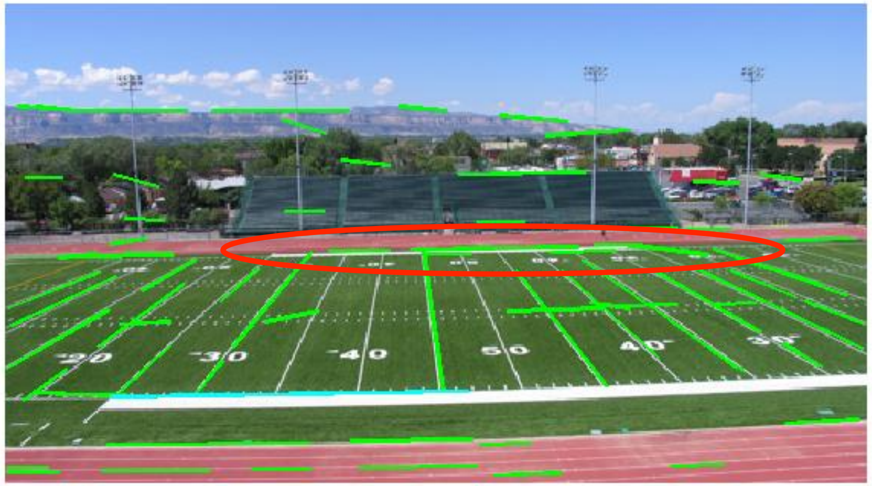


Slide credit: K. Grauman

Hough transform algorithm







Showing longest segments found

Slide credit: K. Grauman

Impact of noise on Hough

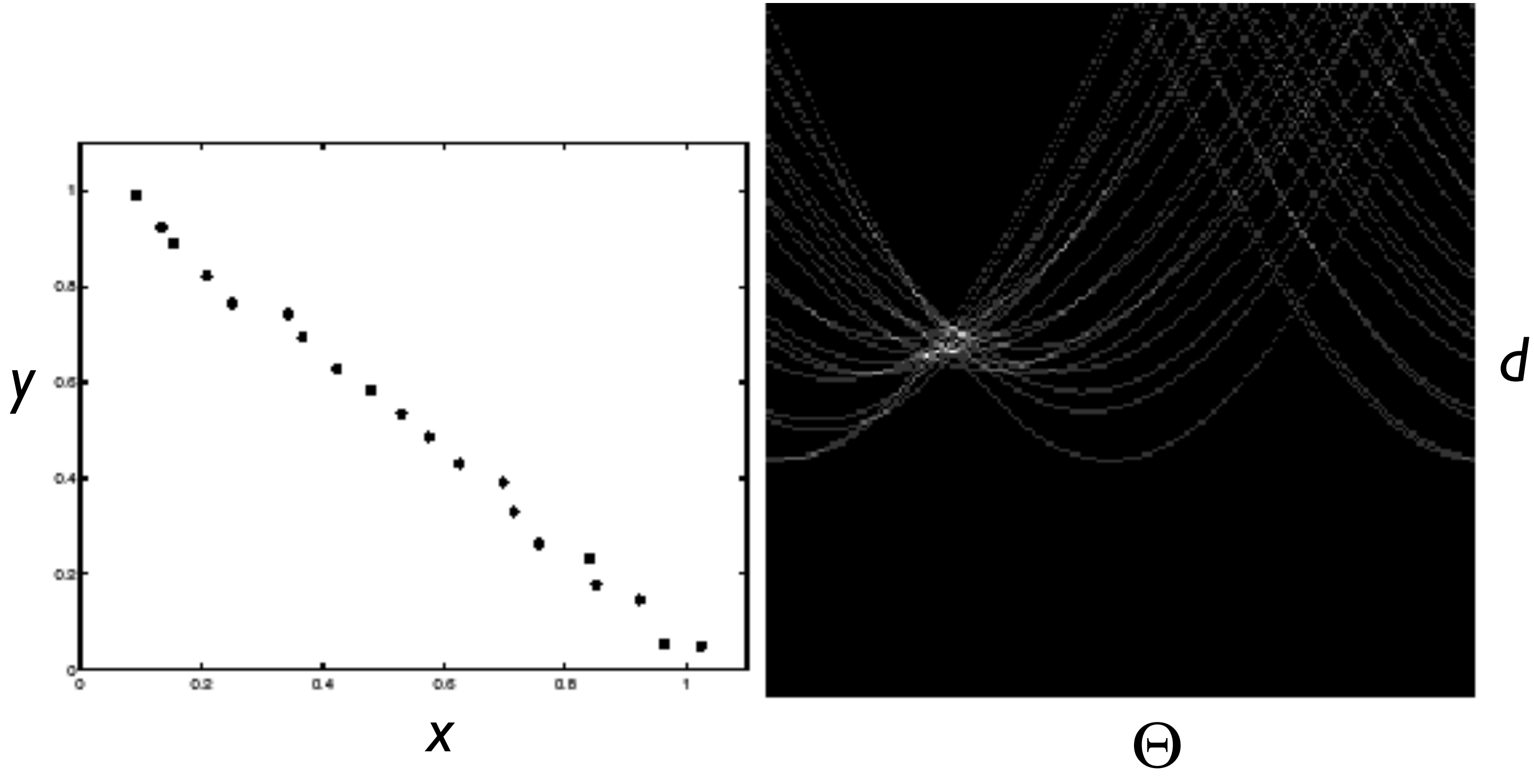


Image space
edge coordinates

Votes

What difficulty does this present for an implementation?

Slide credit: K. Grauman

Impact of noise on Hough

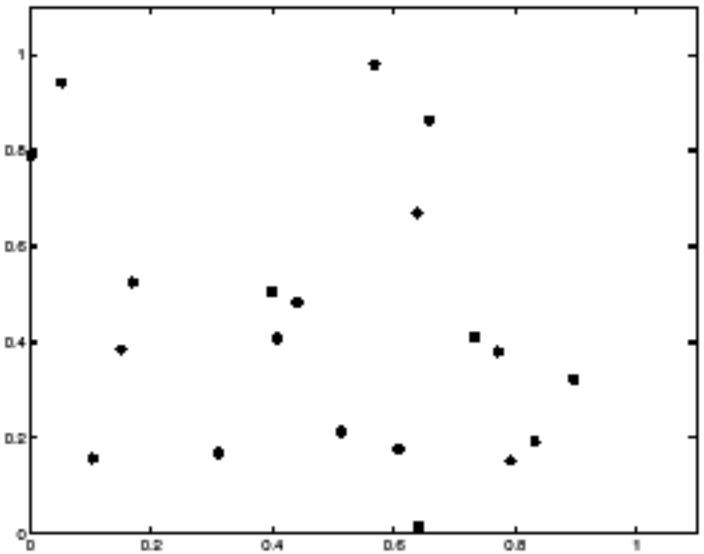
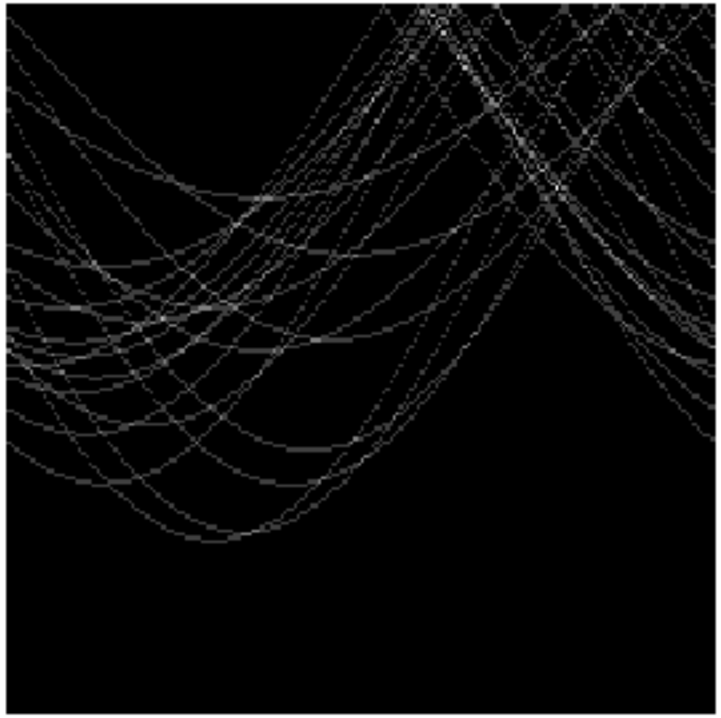


Image space
edge coordinates



Votes

Here, everything appears to be "noise", or random edge points, but we still see peaks in the vote space.

Slide credit: K. Grauman

Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For a fixed radius r, unknown gradient direction

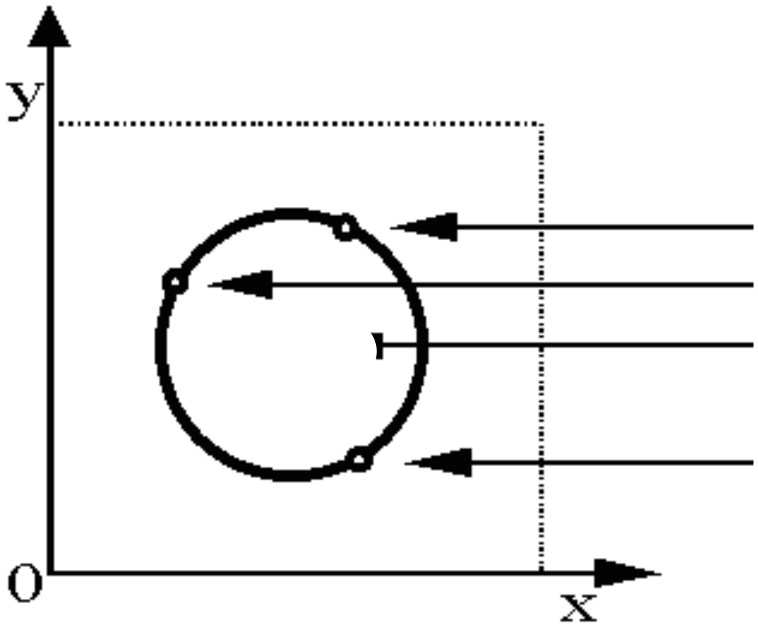
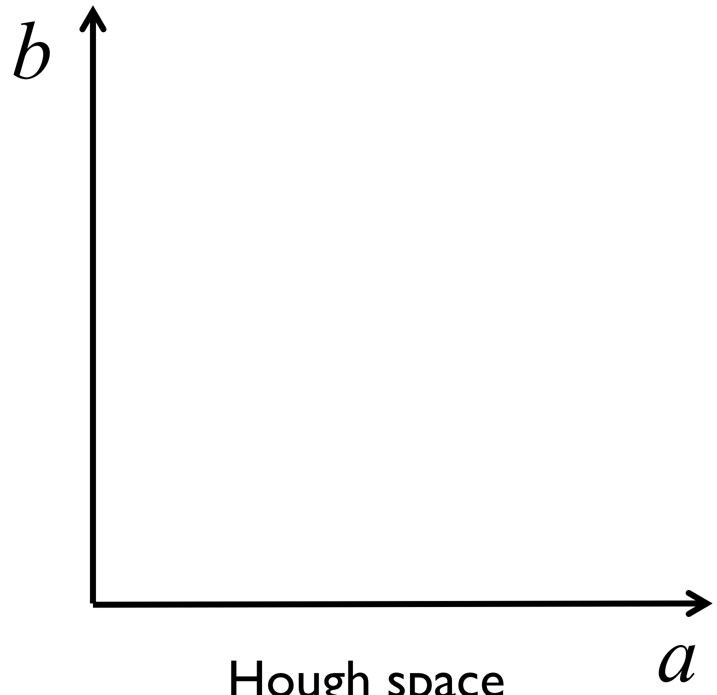


Image space



Hough space

Slide credit: K. Grauman

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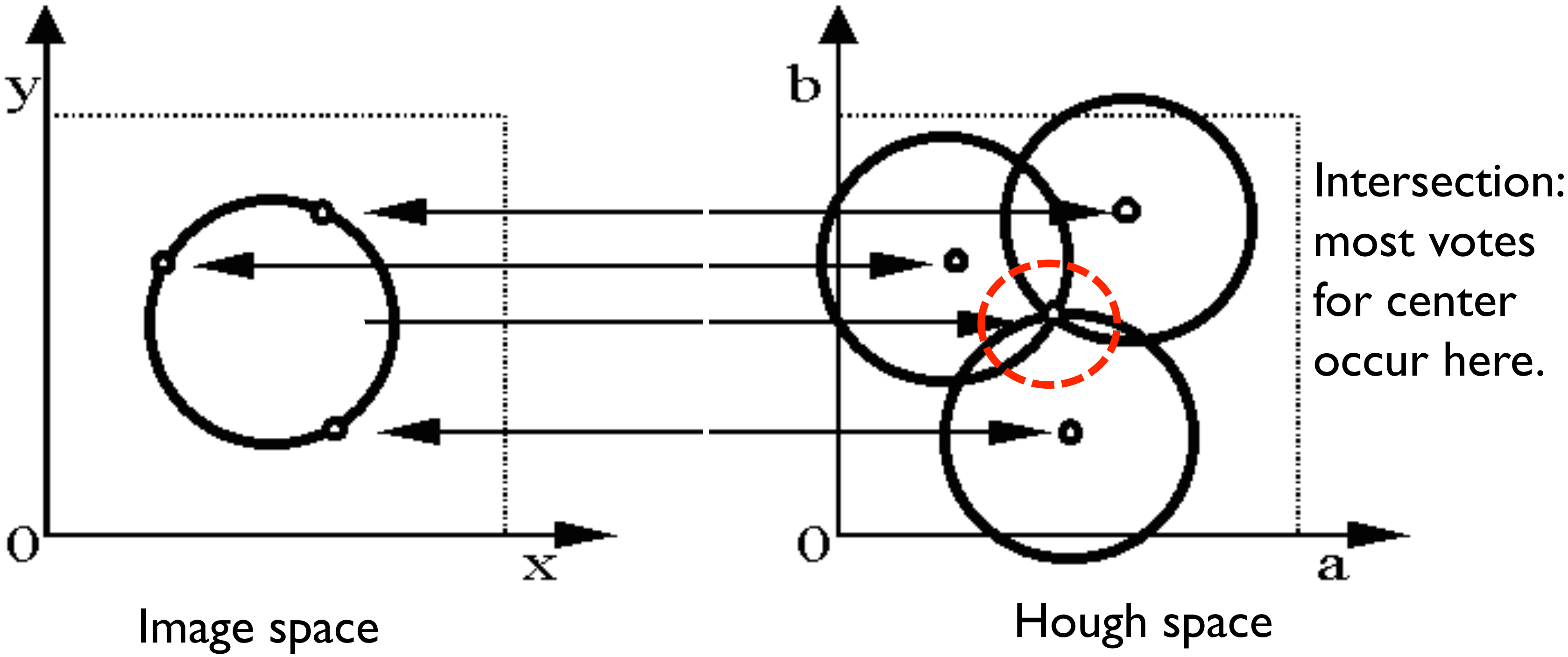

Slide credit: K. Grauman

Hough transform for circles

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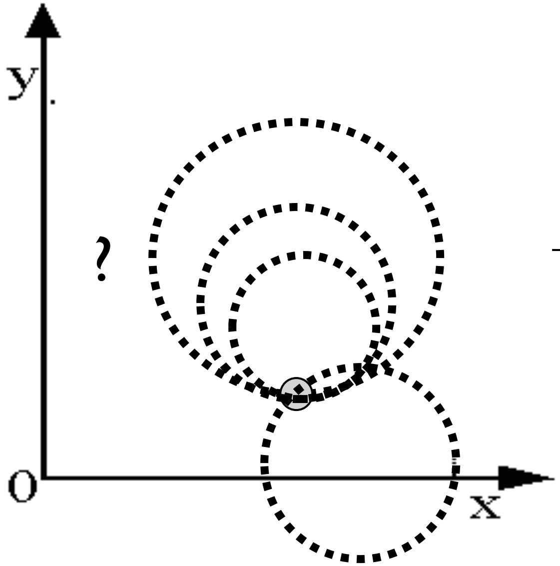
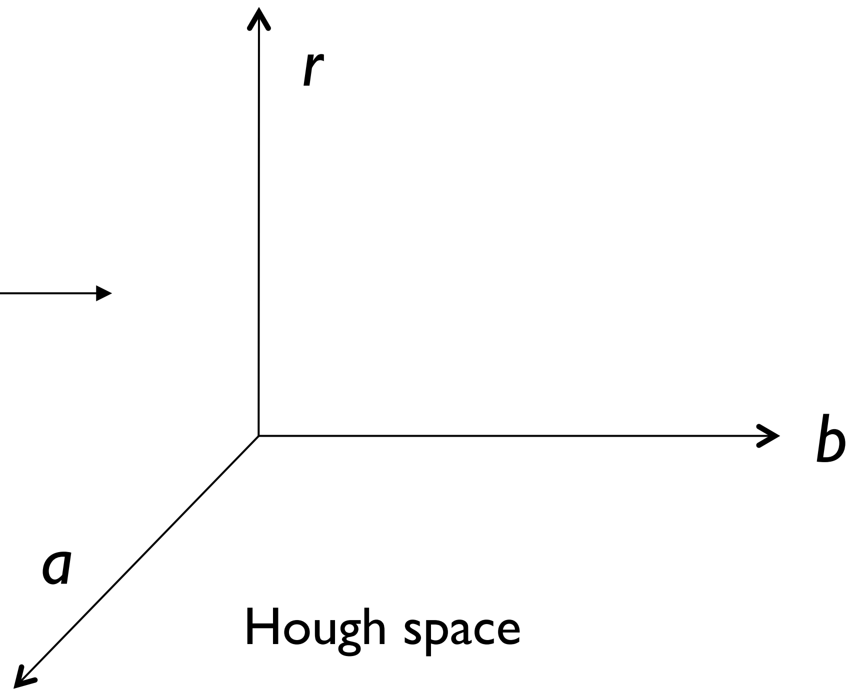


Image space



Hough space

Slide credit: K. Grauman

Hough transform for circles

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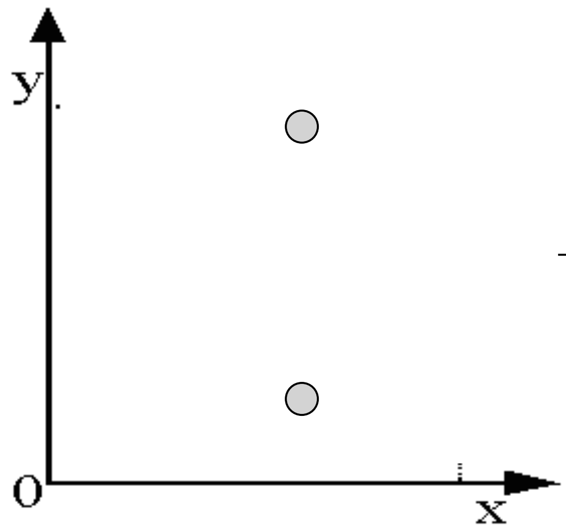
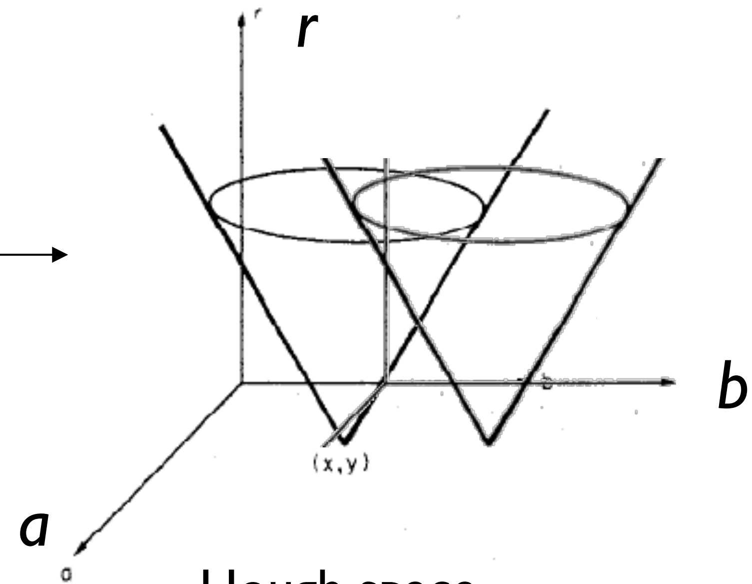


Image space



Hough space

Slide credit: K. Grauman

Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For an unknown radius r, **known** gradient direction

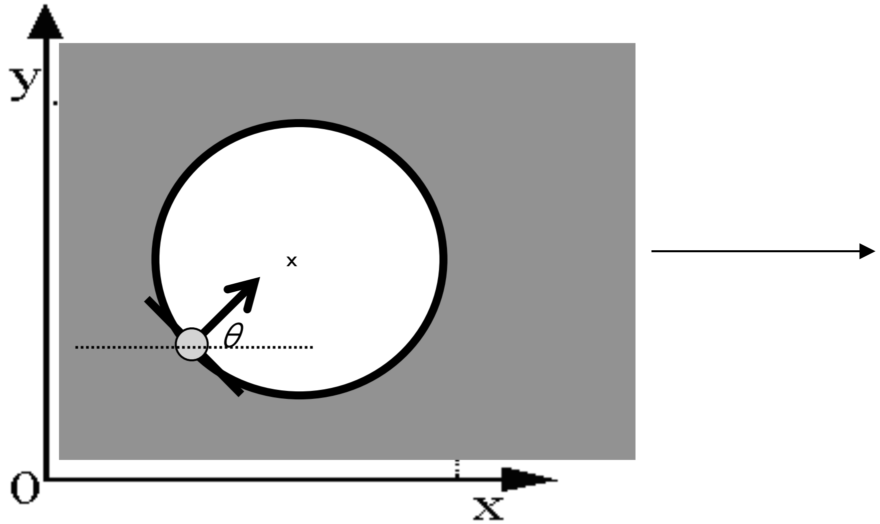
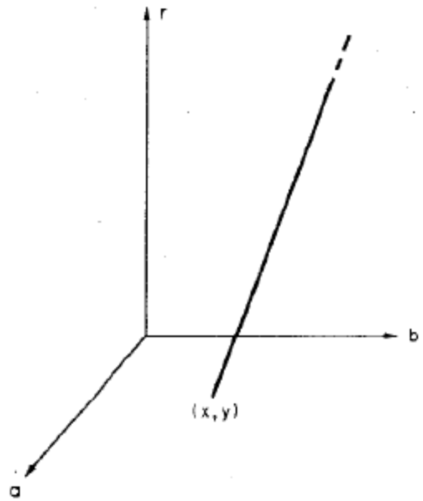


Image space



Hough space

Slide credit: K. Grauman

Hough transform for circles

For every edge pixel (x,y) :

  For each possible radius value r:

    For each possible gradient direction θ:

      // or use estimated gradient at (x,y)

$$a = x - r\cos(\theta) \quad \text{// column}$$

$$b = y + r\sin(\theta) \quad \text{// row}$$

$$H[a,b,r] \mathrel{+}= 1$$

  end

end

Time complexity per edgel?

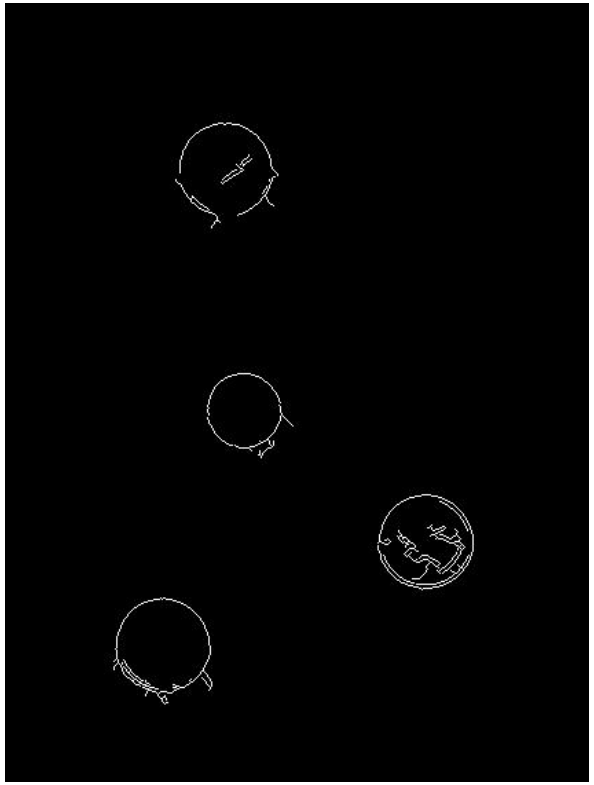
Check out online demo : http://www.markschulze.net/java/hough/

Slide credit: K. Grauman

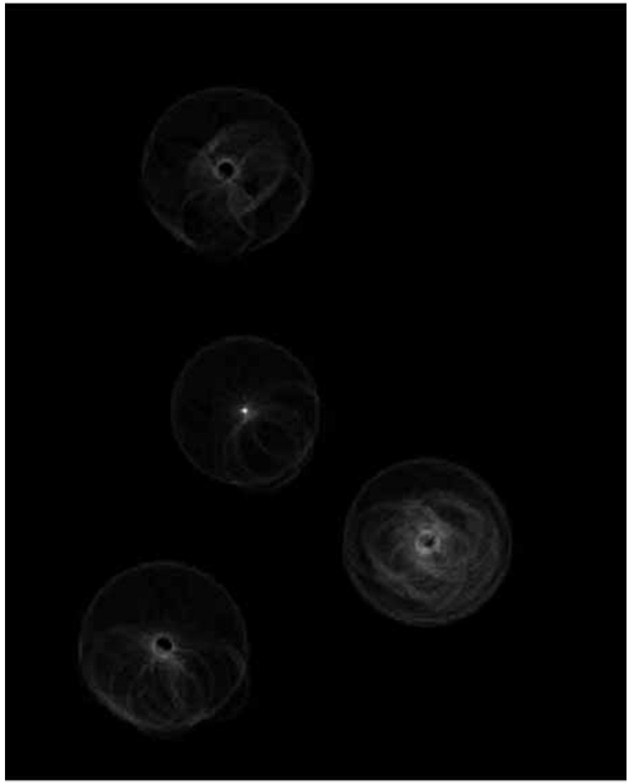
Example: detecting circles with Hough

Original

Edges



Votes: Penny



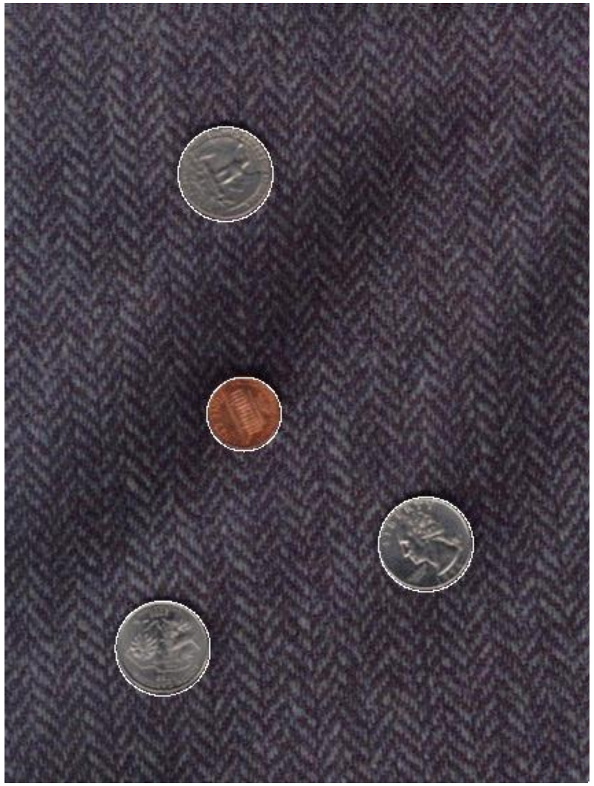
Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).

Slide credit: K. Grauman

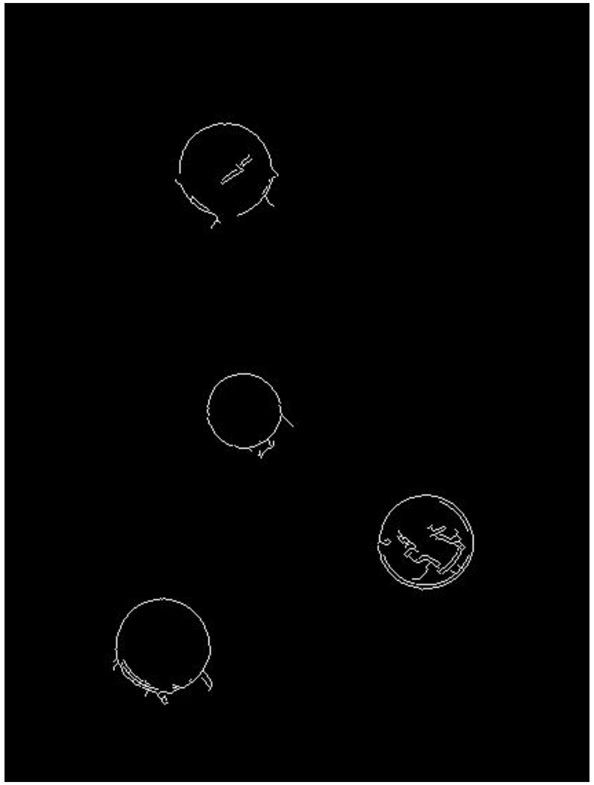
Example: detecting circles with Hough

Original

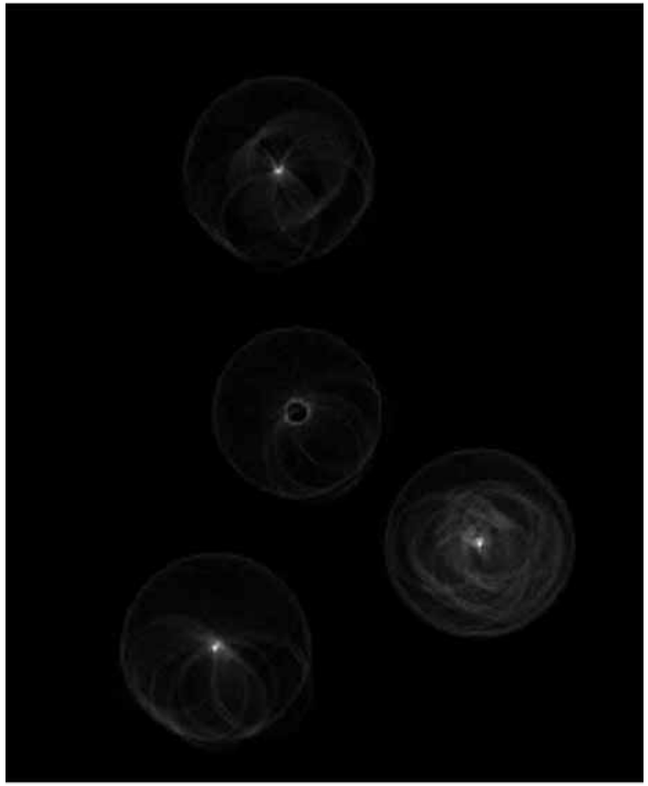
Combined detections



Edges



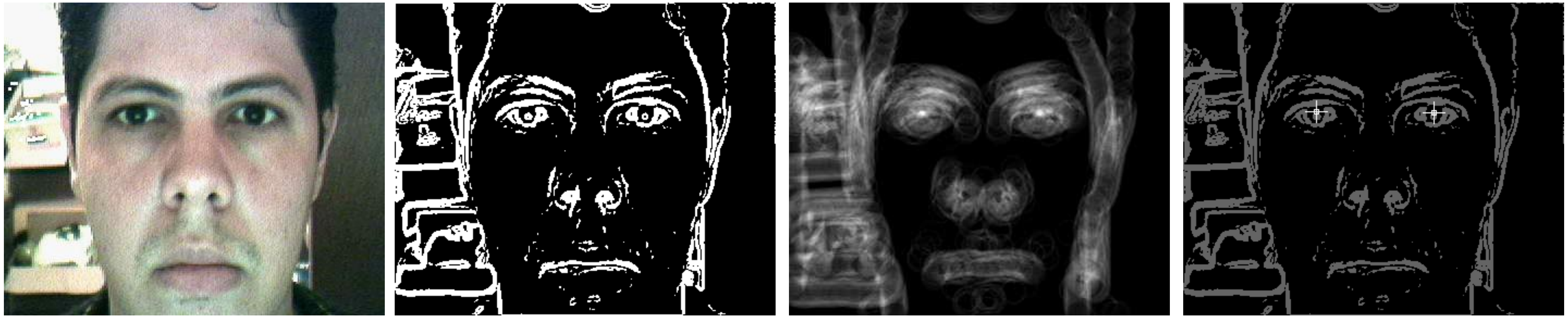
Votes: Quarter



Coin finding sample images from: Vivek Kwatra

Slide credit: K. Grauman

Example: iris detection



Gradient+threshold | Hough space (fixed radius) | Max detections

Hemerson Pistori and Eduardo Rocha Costa http://rsbweb.nih.gov/ij/plugins/hough-circles.html

Slide credit: K. Grauman

Voting: practical tips

- Minimize irrelevant tokens first
- Choose a good grid / discretization

Too fine ? Too coarse

- Vote for neighbors, also (smoothing in accumulator array)
- Use direction of edge to reduce parameters by 1
- To read back which points voted for "winning" peaks, keep tags on the votes.

Slide credit: K. Grauman

Hough transform: pros and cons

Pros

- All points are processed independently, so can cope with occlusion, gaps
- Some robustness to noise: noise points unlikely to contribute *consistently* to any single bin
- Can detect multiple instances of a model in a single pass

Cons

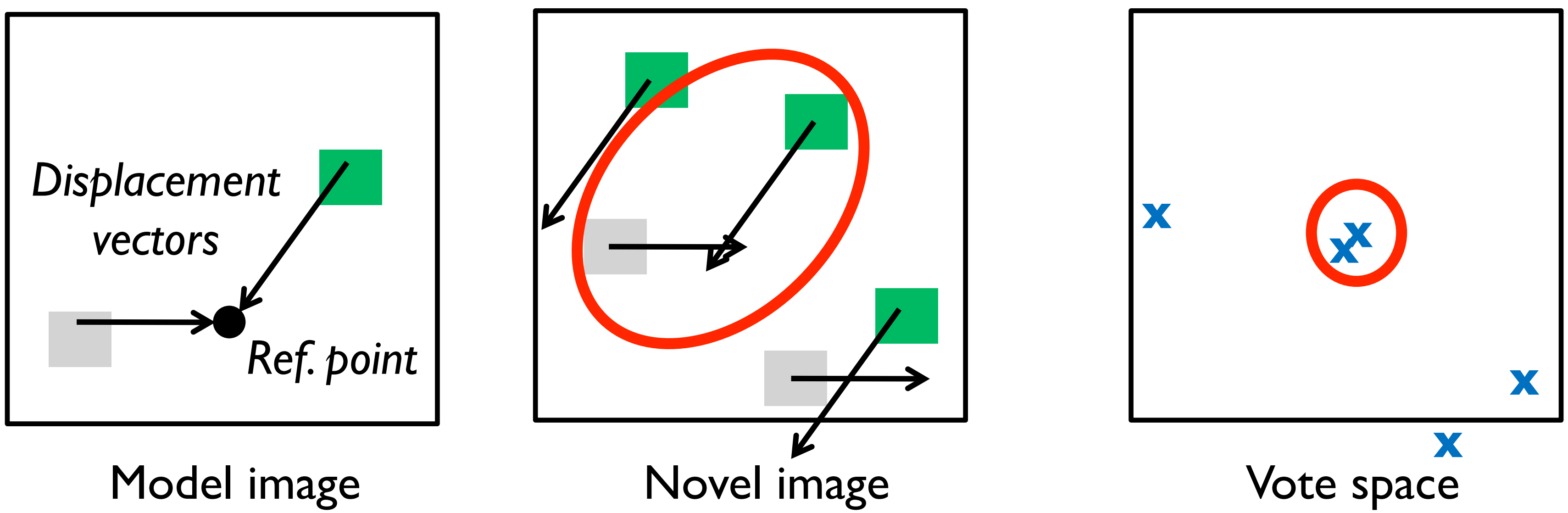
- Complexity of search time increases exponentially with the number of model parameters
- Non-target shapes can produce spurious peaks in parameter space
- Quantization: can be tricky to pick a good grid size

Slide credit: K. Grauman

Generalized Hough Transform

- What if we want to detect arbitrary shapes?

Intuition:

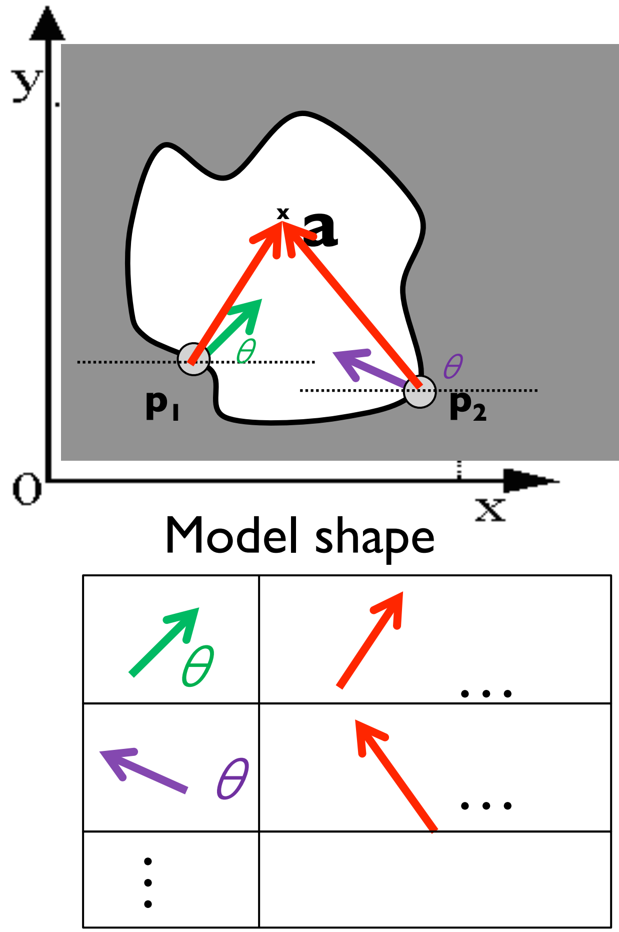


Now suppose those colors encode gradient directions…

Slide credit: K. Grauman

Generalized Hough Transform

- Define a model shape by its boundary points and a reference point.



Offline procedure:

At each boundary point, compute displacement vector: $\mathbf{r} = \mathbf{a} - \mathbf{p_i}$.

Store these vectors in a table indexed by gradient orientation θ.

[Dana H. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, 1980]

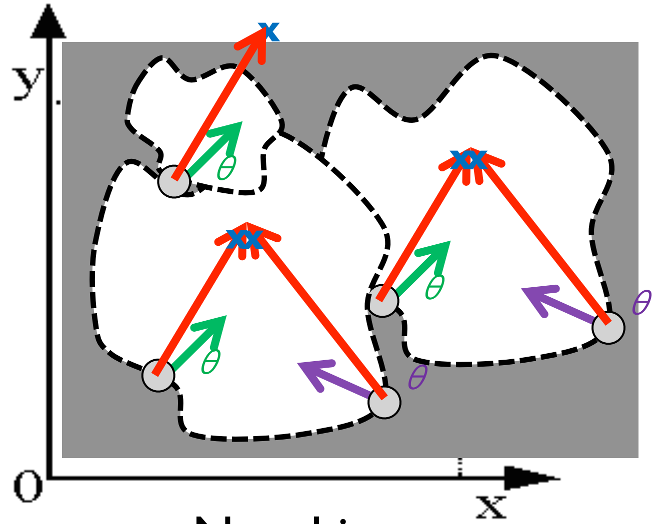
Slide credit: K. Grauman

Generalized Hough Transform

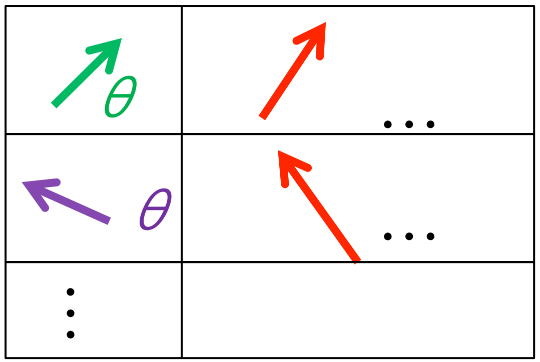
Detection procedure:

For each edge point:

- Use its gradient orientation θ to index into stored table
- Use retrieved **r** vectors to vote for reference point



Novel image

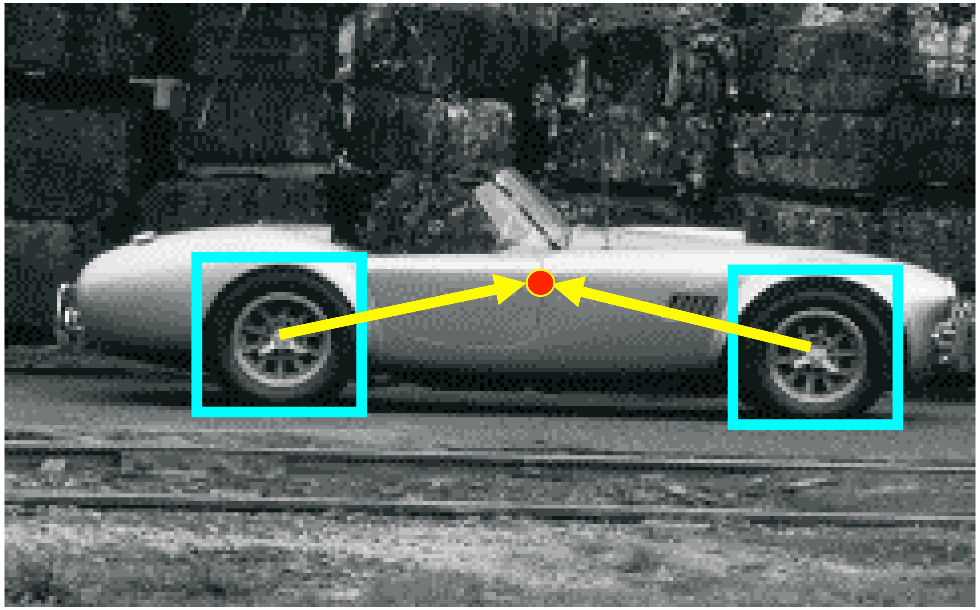


Assuming translation is the only transformation here, i.e., orientation and scale are fixed.

Slide credit: K. Grauman

Generalized Hough for object detection

- Instead of indexing displacements by gradient orientation, index by matched local patterns.



training image

"visual codeword" with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele,
Combined Object Categorization and Segmentation with an Implicit Shape Model,
ECCV Workshop on Statistical Learning in Computer Vision 2004

Slide credit: S. Lazebnik

Generalized Hough for object detection

- Instead of indexing displacements by gradient orientation, index by “visual codeword”

test image

B. Leibe, A. Leonardis, and B. Schiele,
[Combined Object Categorization and Segmentation with an Implicit Shape Model](),
ECCV Workshop on Statistical Learning in Computer Vision 2004

Slide credit: S. Lazebnik

Summary

- Edge detection
  - Difference filters
  - Laplacian of Gaussian
  - Canny edge detection
- Boundary detection
  - Hough transform

Next week

- Image segmentation