BBM 413 Fundamentals of Image Processing

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Image Smoothing

Acknowledgement: The slides are mostly adapted from the course "A Gentle Introduction to Bilateral Filtering and its Applications" given by Sylvain Paris, Pierre Kornprobst, Jack Tumblin, and Frédo Durand (http://people.csail.mit.edu/sparis/bf_course/)

Review - Smoothing and Edge Detection

- While eliminating noise via smoothing, we also lose some of the (important) image details.
  - Fine details
  - Image edges
  - etc.
- What can we do to preserve such details?
  - Use edge information during denoising!
  - This requires a definition for image edges.

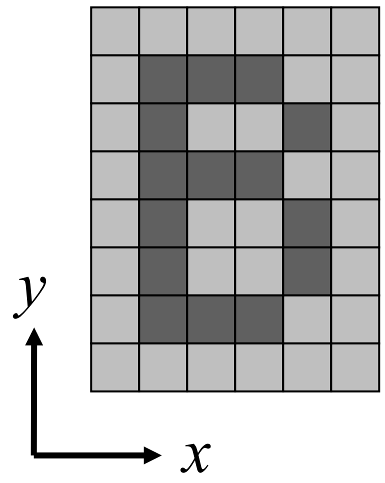
Chicken-and-egg dilemma!

- Edge preserving image smoothing

Today

- Bilateral filter (Tomasi et al., 1998)
- NL-means filter (Buades et al., 2005)
- Structure-texture decomposition via region covariances (Karacan et al. 2013)

Notation and Definitions



- Image = 2D array of pixels
- Pixel = intensity (scalar) or color (3D vector)
- $I_{\mathbf{p}}$ = value of image I at position: $\mathbf{p} = (\, p_x \,, p_y \,)$
- $F\,[\; I\;]$ = output of filter F applied to image I

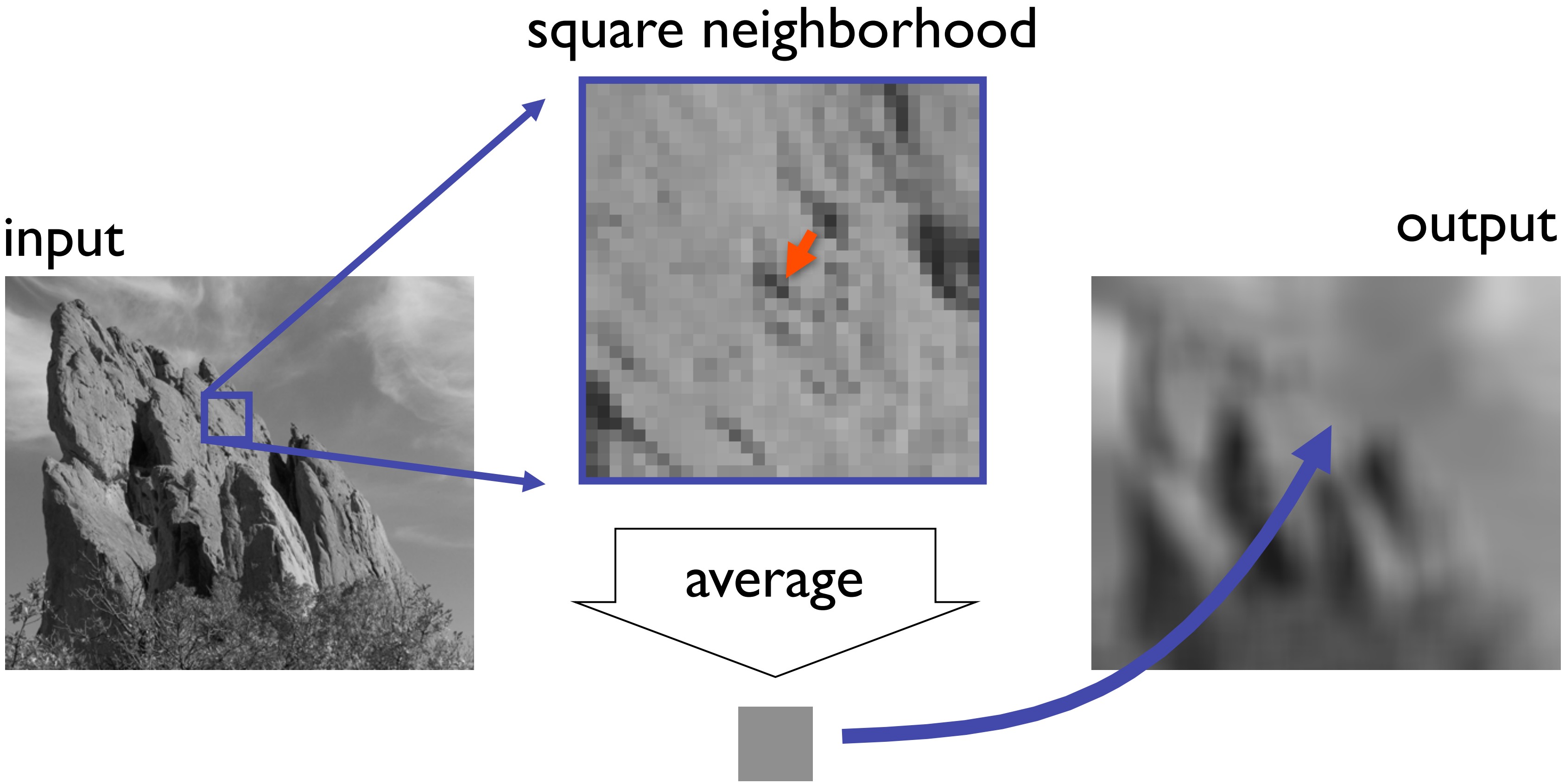
Strategy for Smoothing Images

- Images are not smooth because
  adjacent pixels are different.

- Smoothing = making adjacent pixels
  look more similar.

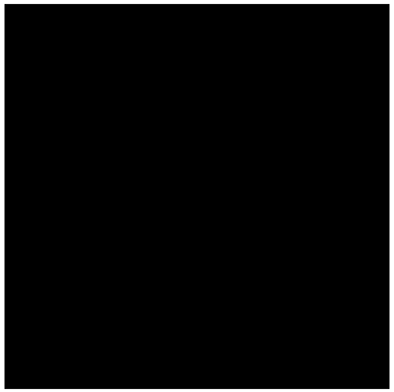
- Smoothing strategy
  pixel as average of its neighbors

Box Average



Equation of Box Average

$$\underbrace{BA[I]_{\mathbf{p}}}_{\text{result at pixel } \mathbf{p}} = \underbrace{\sum_{\mathbf{q}\in S}}_{\text{sum over all pixels } \mathbf{q}} \underbrace{B_\sigma(\mathbf{p}-\mathbf{q})}_{\text{normalized box function}} \underbrace{I_{\mathbf{q}}}_{\text{intensity at pixel } \mathbf{q}}$$



Square Box Generates Defects

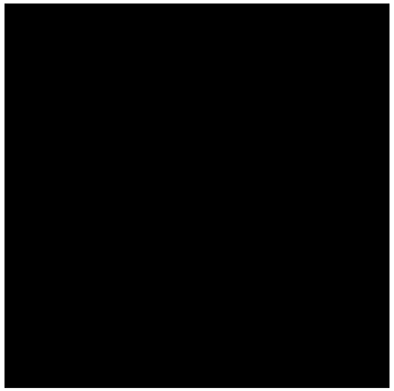
- Axis-aligned streaks
- Blocky results

output

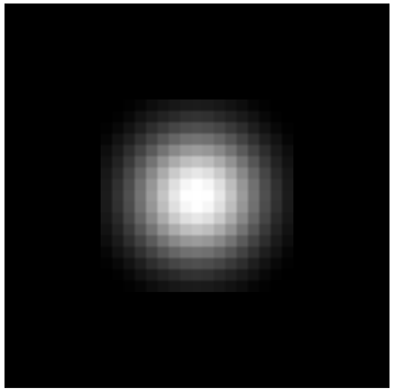
input

Strategy to Solve these Problems

- Use an isotropic (*i.e.* circular) window.
- Use a window with a smooth falloff.

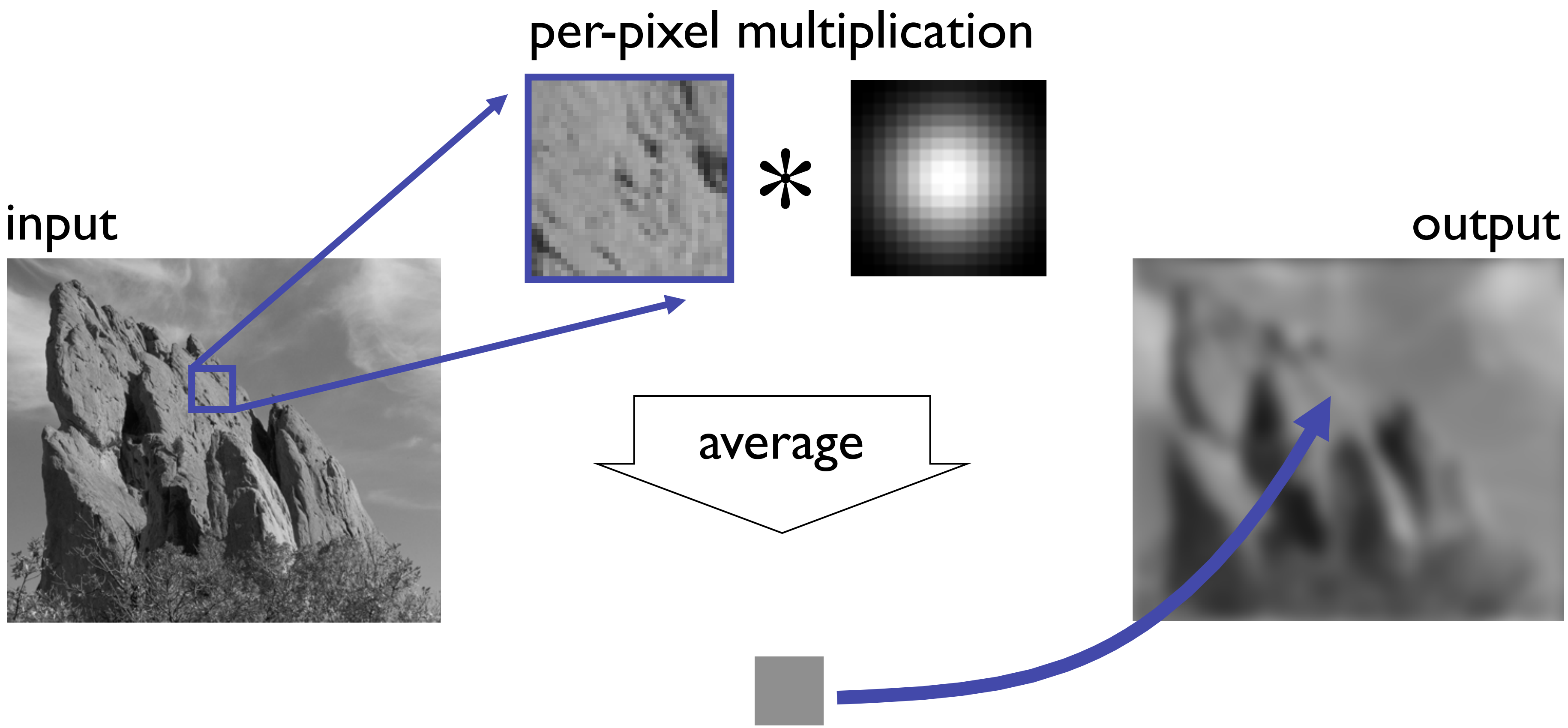


box window



Gaussian window

Gaussian Blur




input


box average

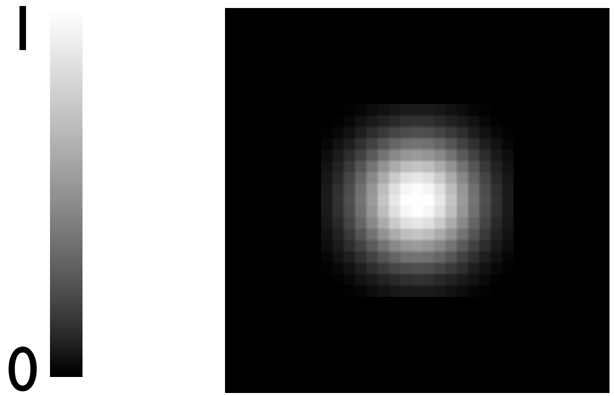

Gaussian blur

Equation of Gaussian Blur

Same idea: **weighted average of pixels**.

$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q}\in S} G_{\sigma}\left(\| \mathbf{p}-\mathbf{q} \|\right) I_{\mathbf{q}}$$

normalized
Gaussian function

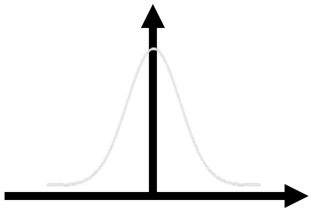


Spatial Parameter

input

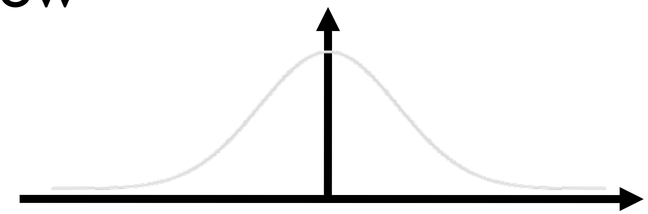
$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q}\in S} G_{\sigma}\left(\|\mathbf{p}-\mathbf{q}\|\right) I_{\mathbf{q}}$$

size of the window

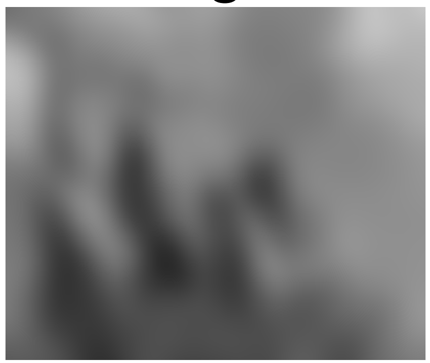


small *s*

limited smoothing



large *s*



strong smoothing

How to set *s*

- Depends on the application.

- Common strategy: proportional to image size
  - e.g. 2% of the image diagonal
  - property: independent of image resolution

Properties of Gaussian Blur

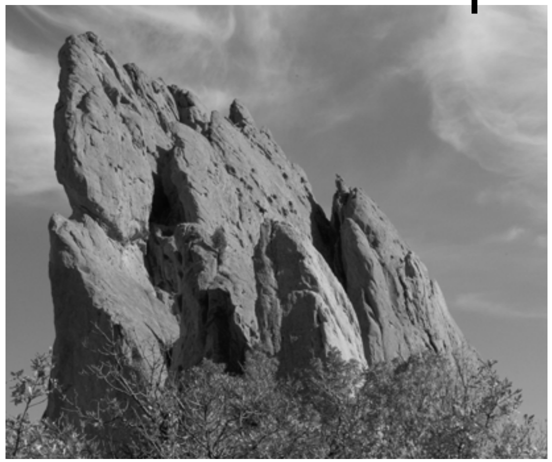
- Weights independent of spatial location
  - linear convolution
  - well-known operation
  - efficient computation (recursive algorithm, FFT…)

Properties of Gaussian Blur

- Does smooth images
- But smoothes too much:
  edges are blurred.
  - Only spatial distance matters
  - No edge term

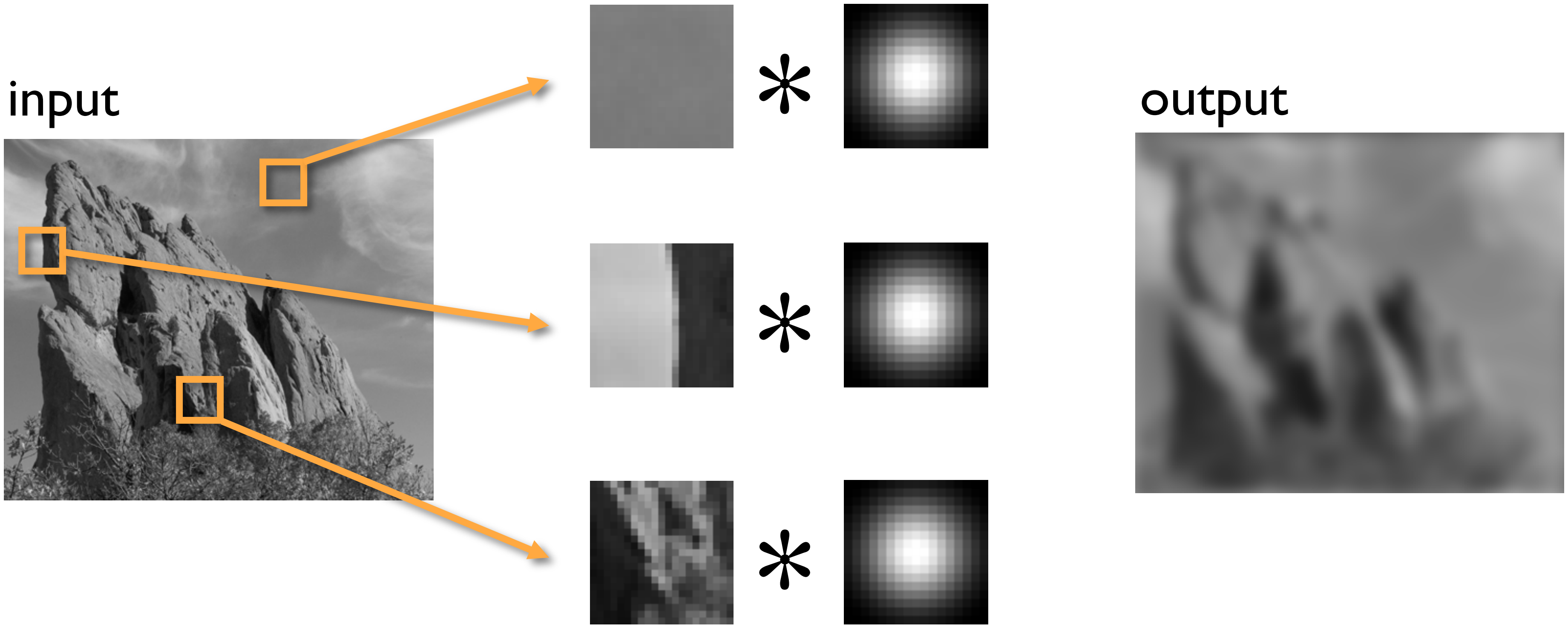
$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q}\in S} \underbrace{G_\sigma(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} I_{\mathbf{q}}$$

input



output

Blur Comes from Averaging across Edges

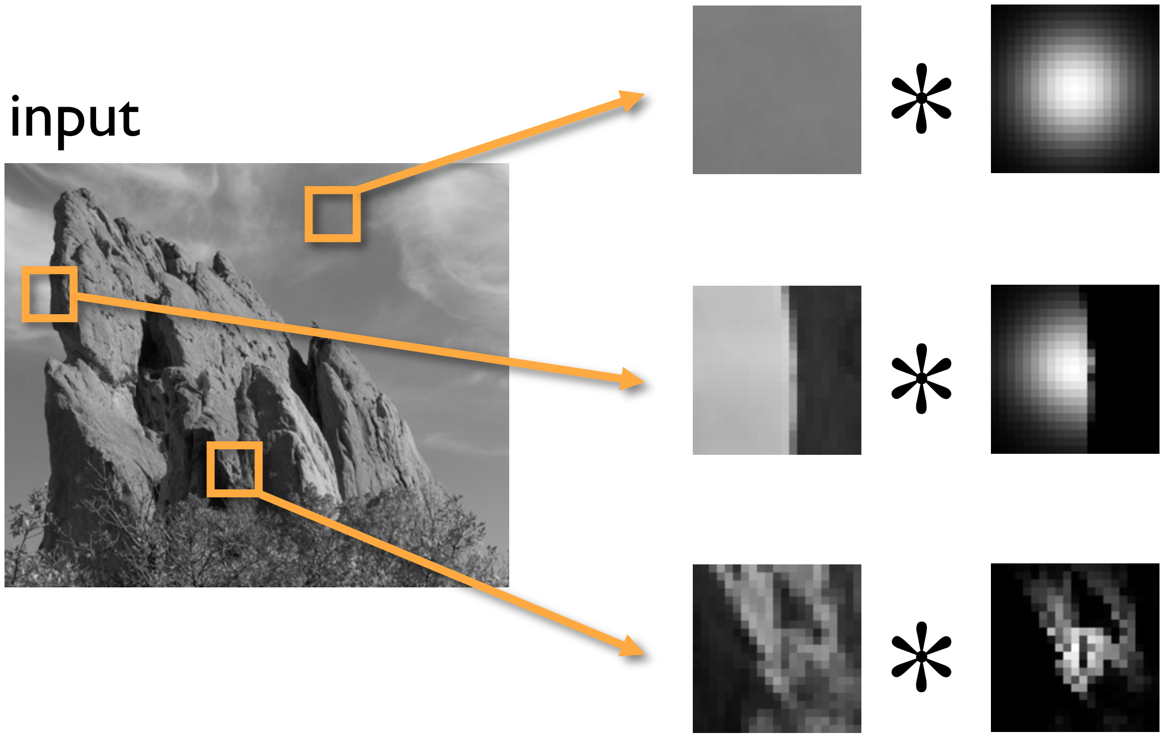


Same Gaussian kernel everywhere.

[Aurich 95, Smith 97, Tomasi 98]

Bilateral Filter
No Averaging across Edges

input



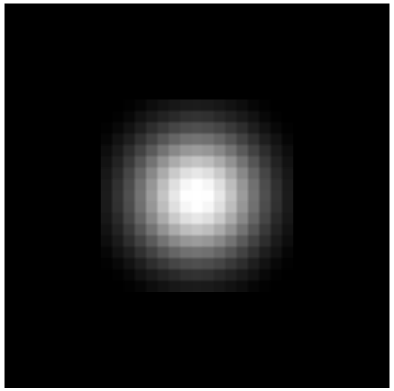
output

The kernel shape depends on the image content.

Bilateral Filter Definition: an Additional Edge Term

Same idea: **weighted average of pixels**.

$$BF[I]_{\mathbf{p}} = \underbrace{\overset{\text{new}}{\frac{1}{W_{\mathbf{p}}}}}_{\text{normalization factor}} \sum_{\mathbf{q}\in S} \underbrace{\overset{\text{not new}}{G_{\sigma_{\mathrm{s}}}\left(\| \mathbf{p}-\mathbf{q} \|\right)}}_{\textit{space}\text{ weight}} \; \underbrace{\overset{\text{new}}{G_{\sigma_{\mathrm{r}}}\left(| I_{\mathbf{p}} - I_{\mathbf{q}} |\right)}}_{\textit{range}\text{ weight}} \, I_{\mathbf{q}}$$



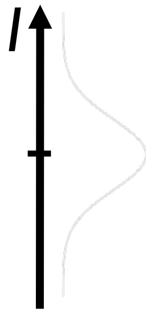
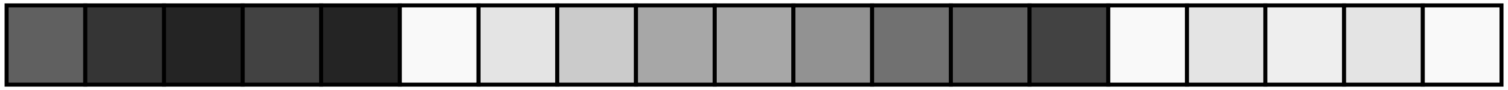
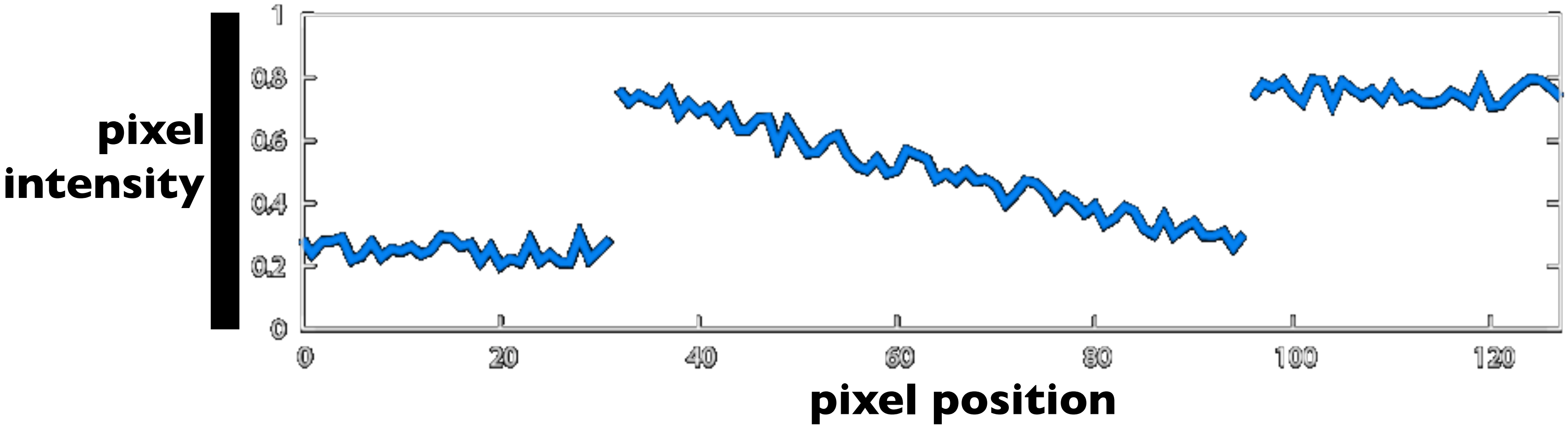


Illustration a 1D Image

- 1D image = line of pixels

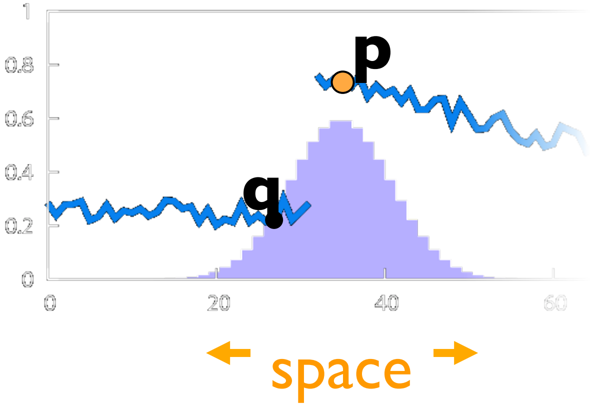


- Better visualized as a plot



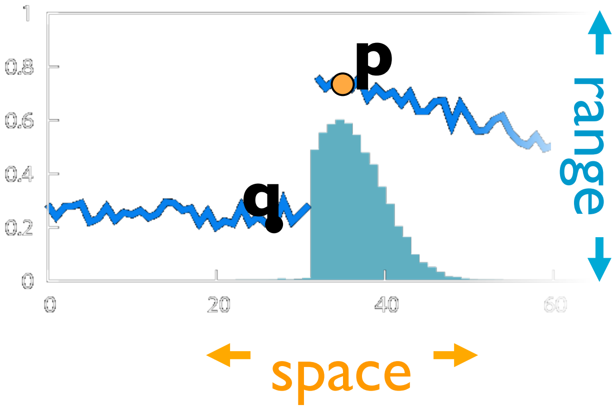
Gaussian Blur and Bilateral Filter

Gaussian blur



$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q}\in S} \underbrace{G_{\sigma}(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} I_{\mathbf{q}}$$

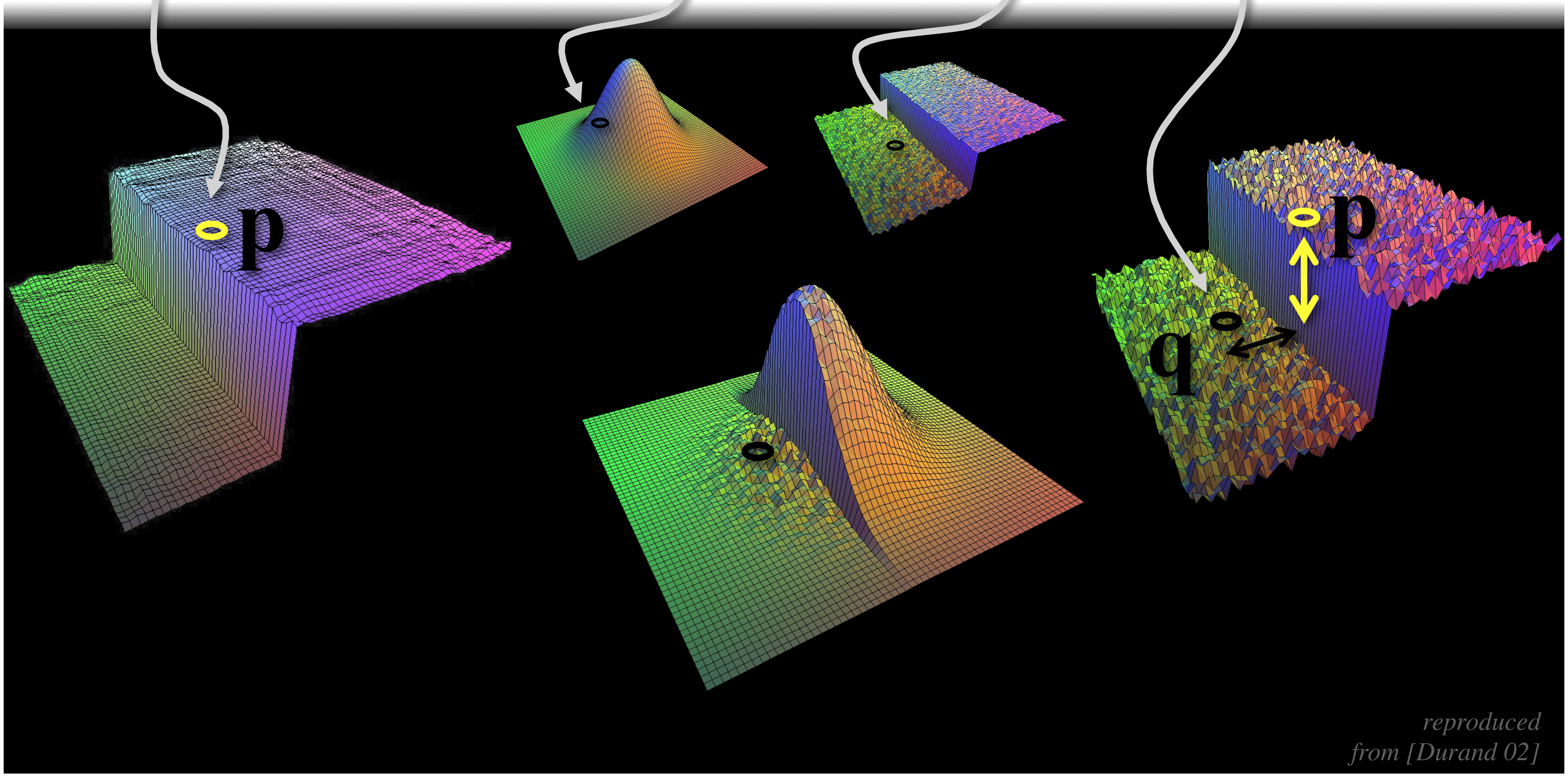
Bilateral filter
[Aurich 95, Smith 97, Tomasi 98]



$$BF[I]_{\mathbf{p}} = \underbrace{\frac{1}{W_{\mathbf{p}}}}_{\text{normalization}} \sum_{\mathbf{q}\in S} \underbrace{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} \underbrace{G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)}_{\text{range}} I_{\mathbf{q}}$$

Bilateral Filter on a Height Field

$$\underbrace{BF[I]_{\mathbf{p}}}\ =\ \frac{1}{W_{\mathbf{p}}}\sum_{\mathbf{q}\in S}\underbrace{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)}\ \underbrace{G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)}\ I_{\mathbf{q}}$$



Space and Range Parameters

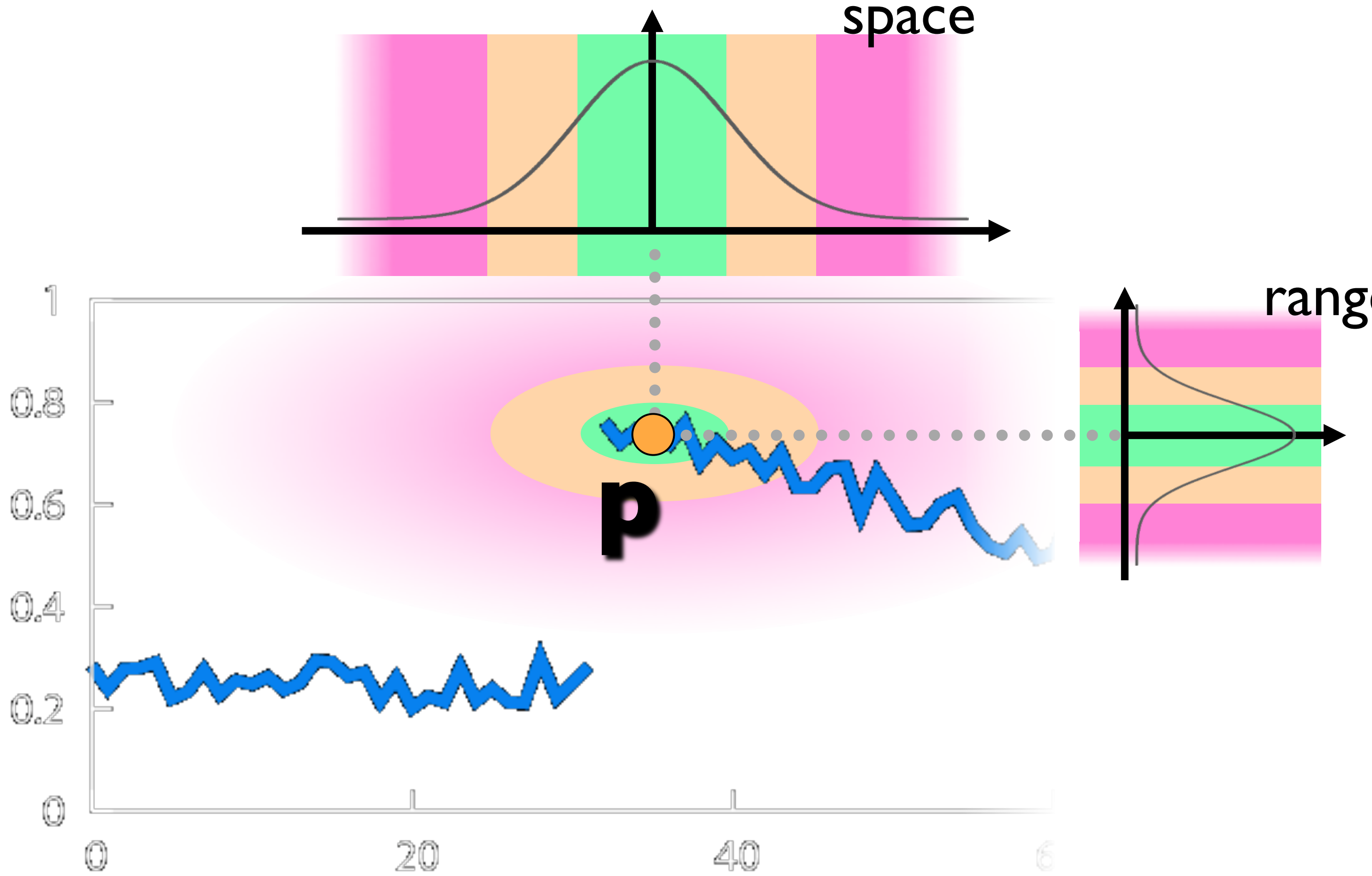
$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_{\mathrm{s}}}\left(\| \mathbf{p} - \mathbf{q} \|\right) G_{\sigma_{\mathrm{r}}}\left(| I_{\mathbf{p}} - I_{\mathbf{q}} |\right) I_{\mathbf{q}}$$

- space s_{s} : spatial extent of the kernel, size of the considered neighborhood.

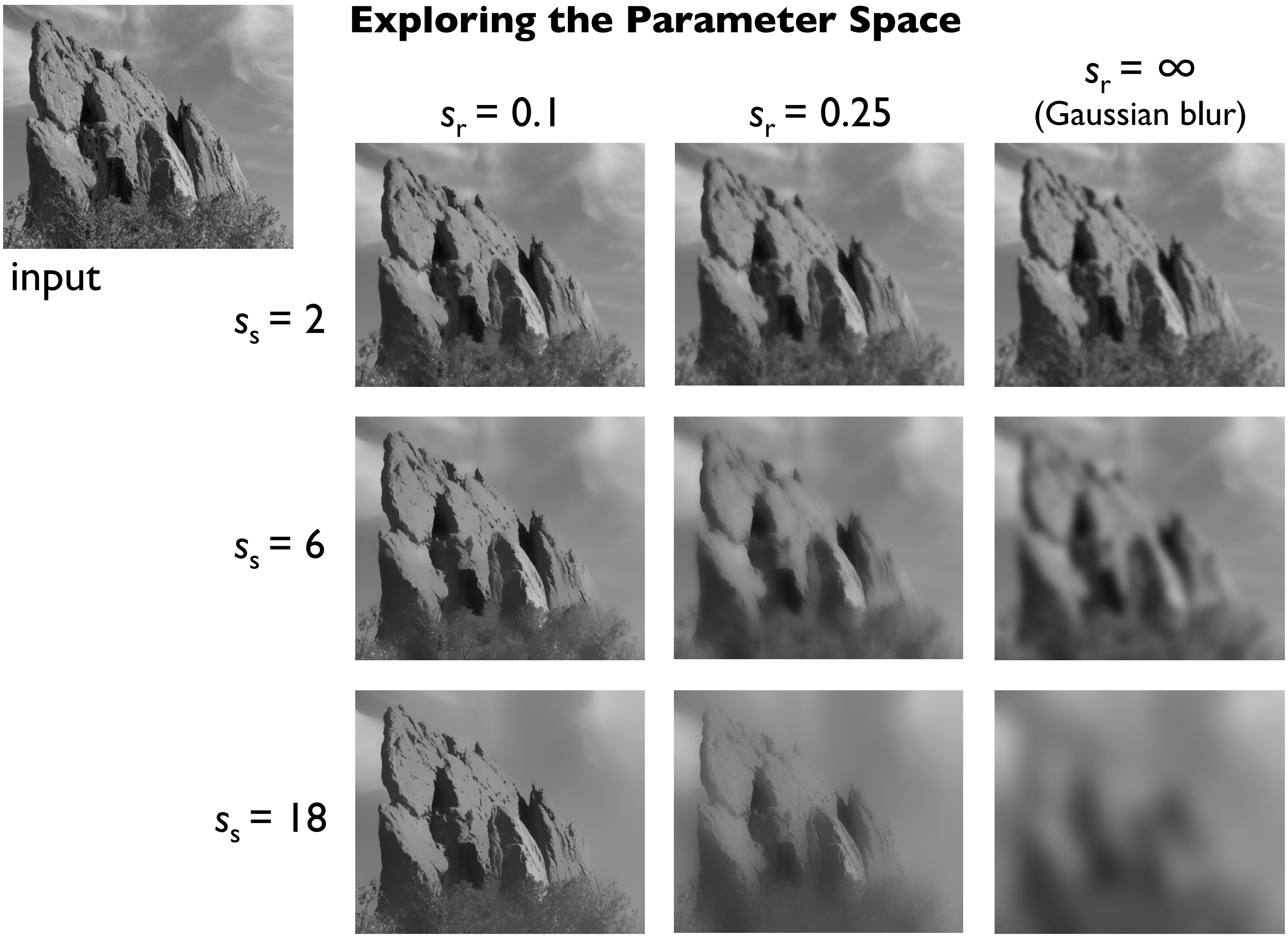
- range s_{r} : “minimum” amplitude of an edge

Influence of Pixels

Only pixels close in space and in range are considered.



Exploring the Parameter Space



input

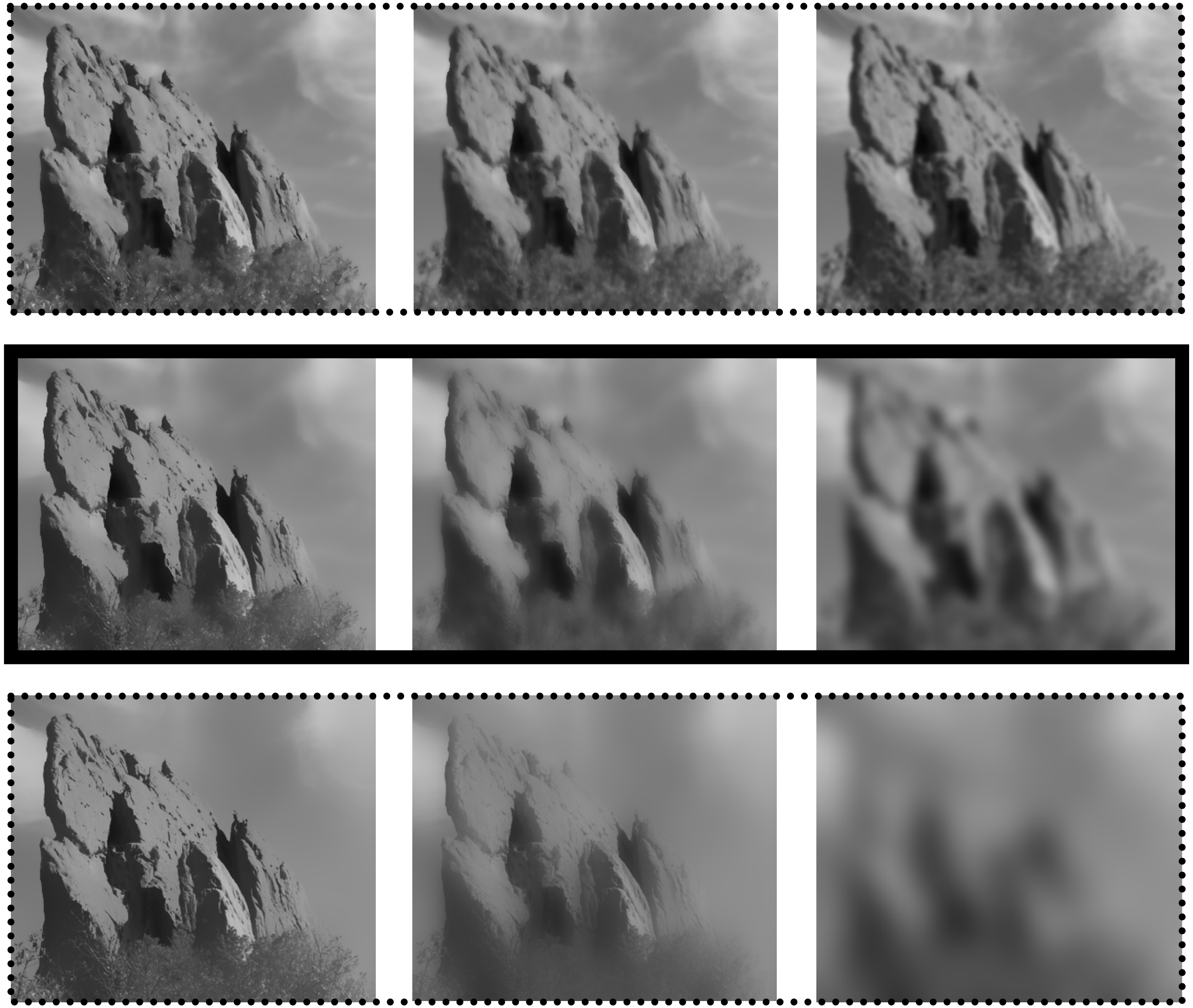
Varying the Range Parameter

$s_r = 0.1$ | $s_r = 0.25$ | $s_r = \infty$ (Gaussian blur)

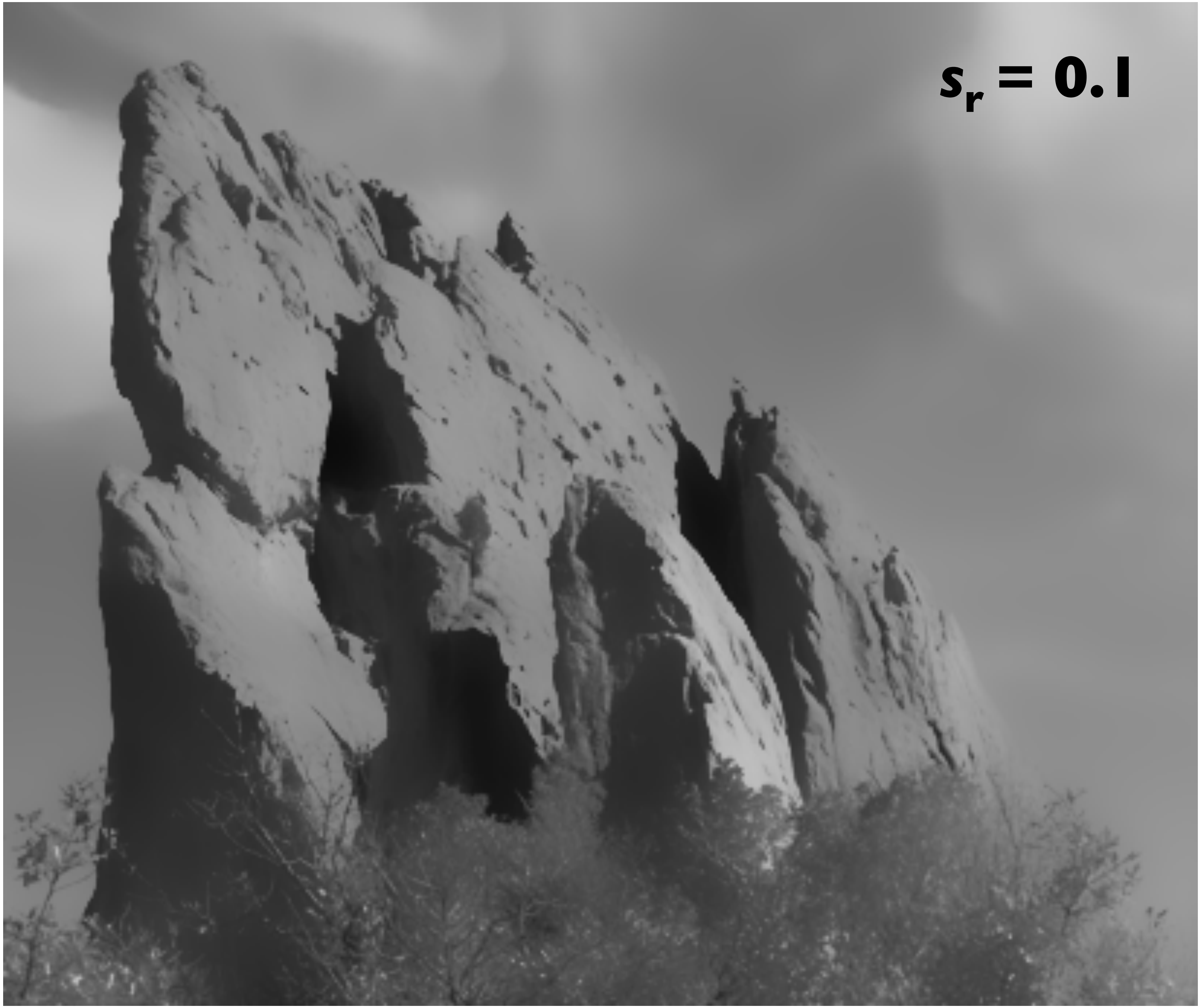
$s_s = 2$

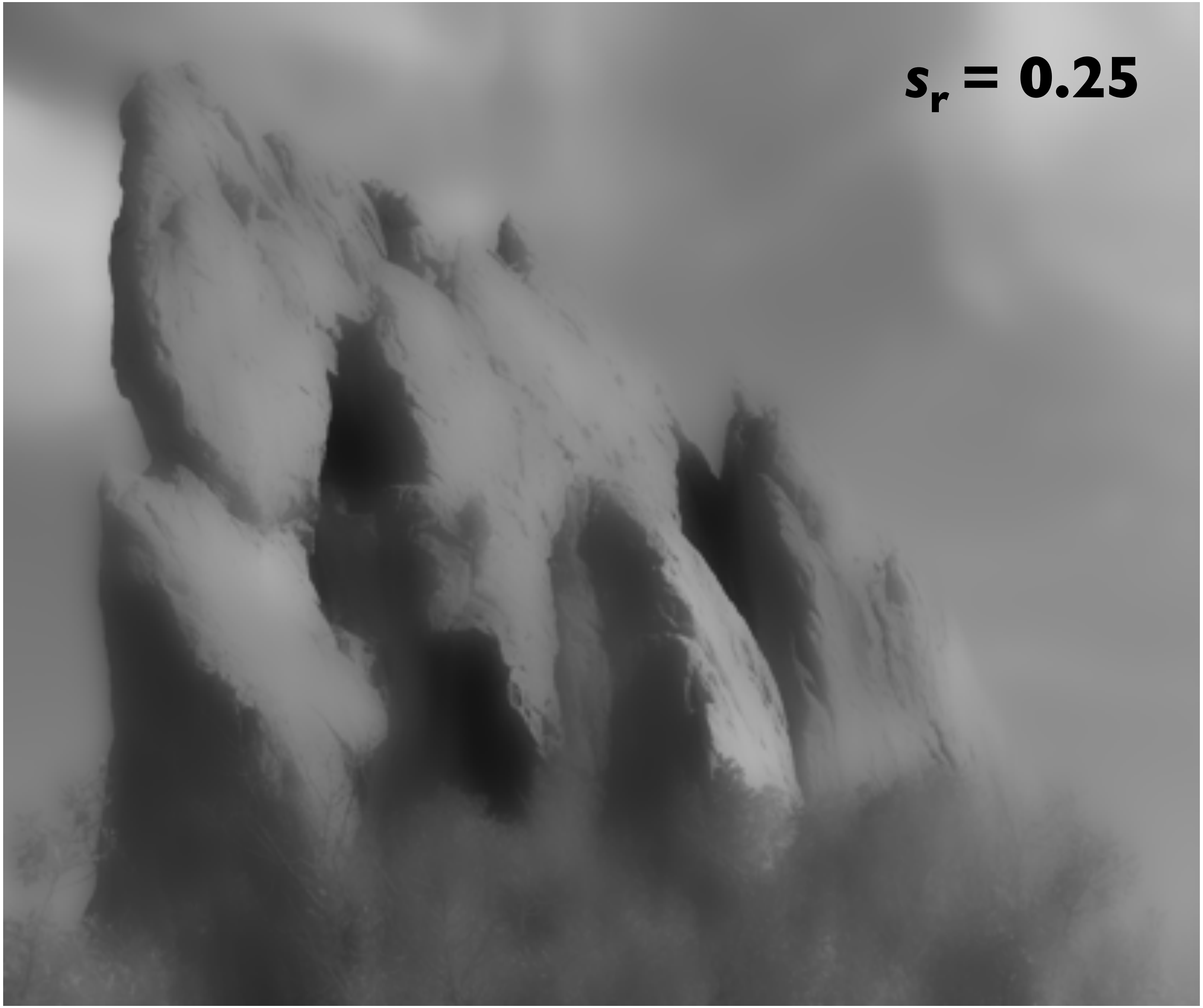
$s_s = 6$

$s_s = 18$




input

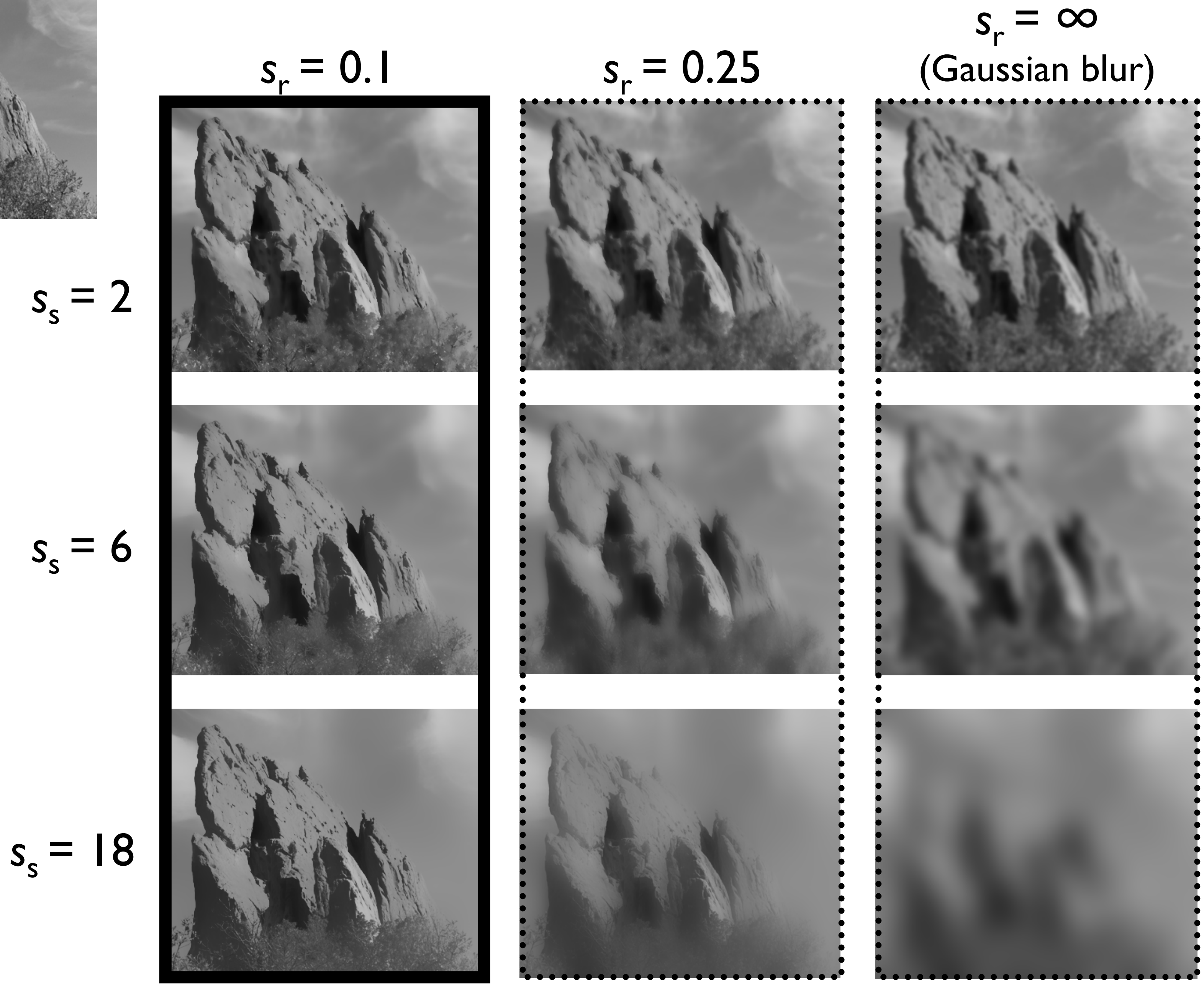

$s_r = 0.1$


$s_r = 0.25$


$s_r = \infty$
(Gaussian blur)

Varying the Space Parameter

input




input


$s_s = 2$


$s_s = 6$


$s_s = 18$

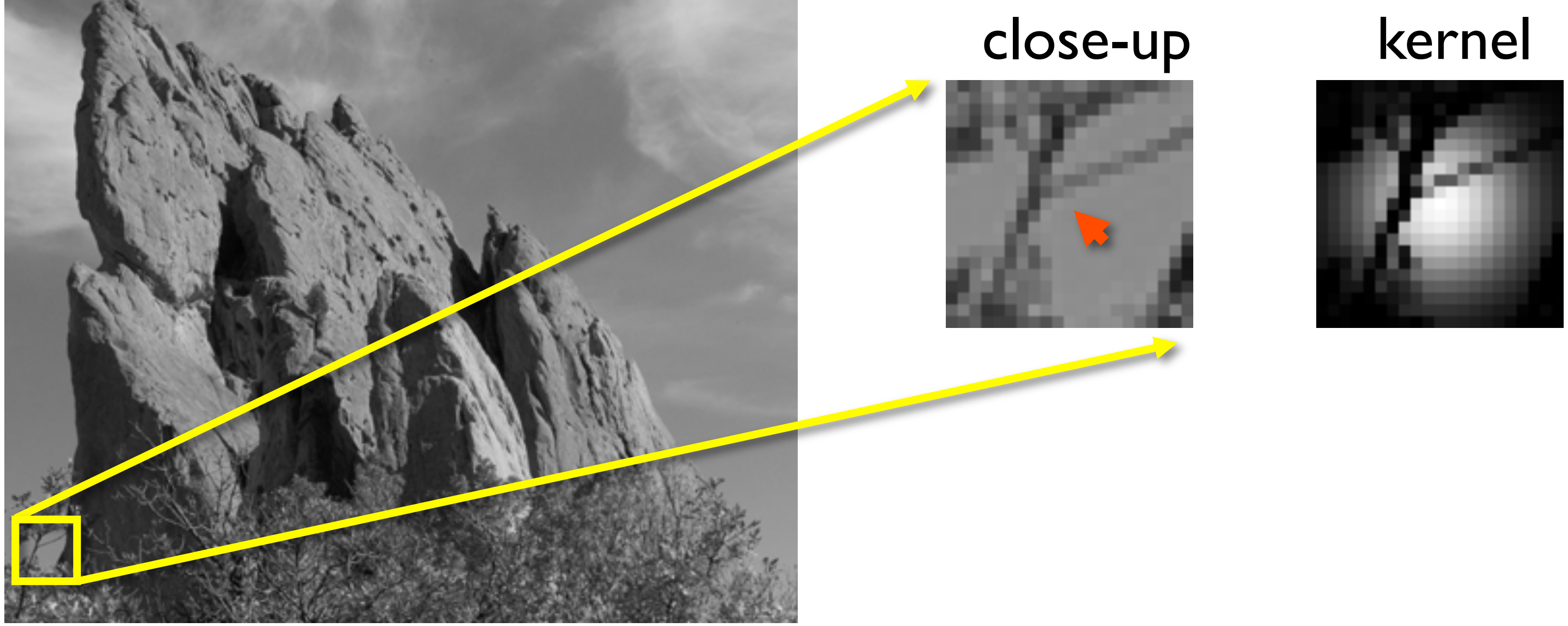
How to Set the Parameters

Depends on the application. For instance:

- space parameter: proportional to image size
  - e.g., 2% of image diagonal
- range parameter: proportional to edge amplitude
  - e.g., mean or median of image gradients
- independent of resolution and exposure

Bilateral Filter Crosses Thin Lines

- Bilateral filter averages across
  features thinner than $\sim 2s_s$
- Desirable for smoothing: more pixels = more robust
- Different from diffusion that stops at thin lines



Iterating the Bilateral Filter

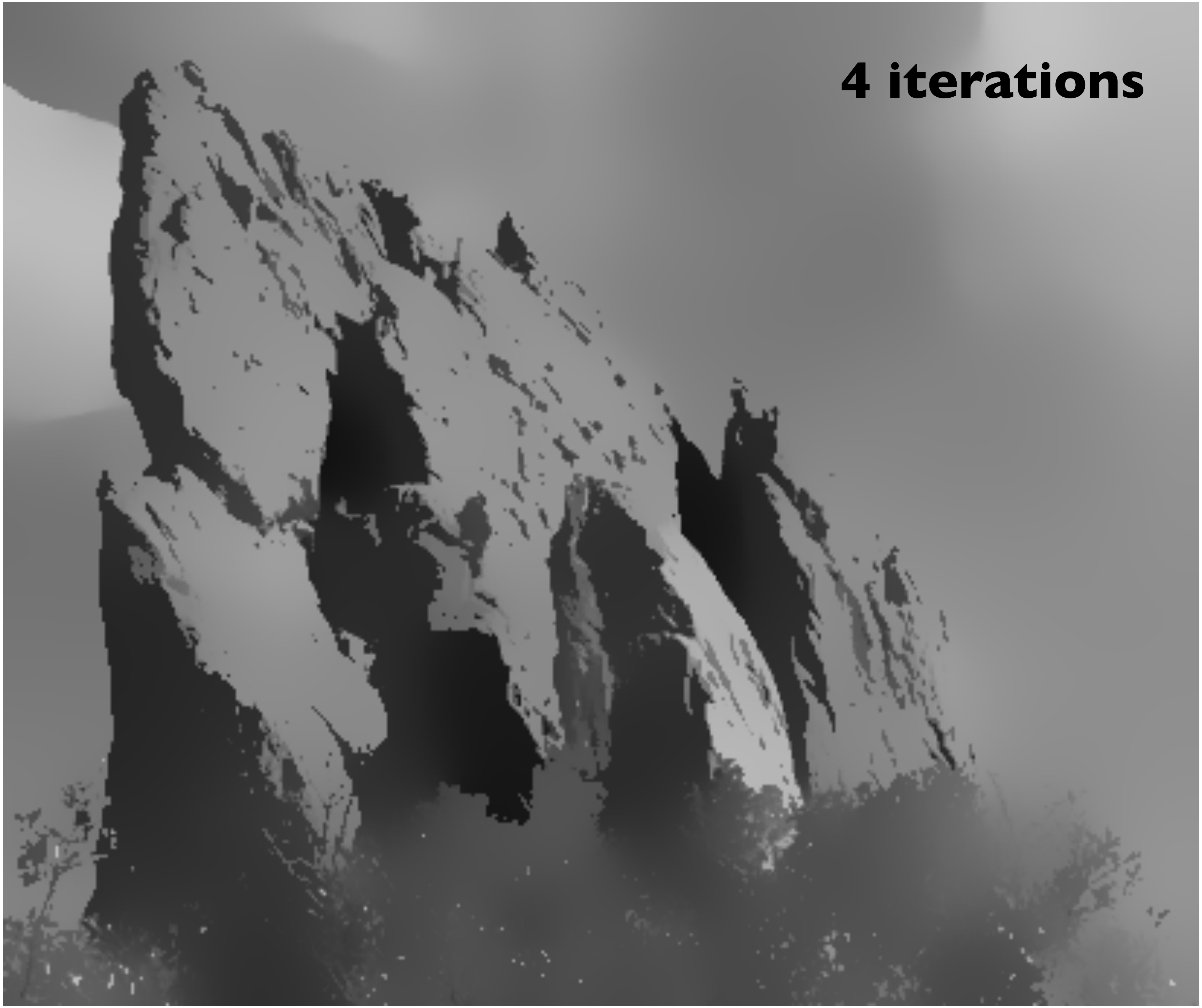
$$I_{(n+1)} = BF\left[I_{(n)} \right]$$

- Generate more piecewise-flat images
- Often not needed in computational photo.


input


1 iteration


2 iterations


4 iterations

Bilateral Filtering Color Images

For gray-level images

intensity difference

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) \, G_{\sigma_r}\left(\boxed{| I_{\mathbf{p}} - I_{\mathbf{q}} |} \right) \boxed{I_{\mathbf{q}}}$$

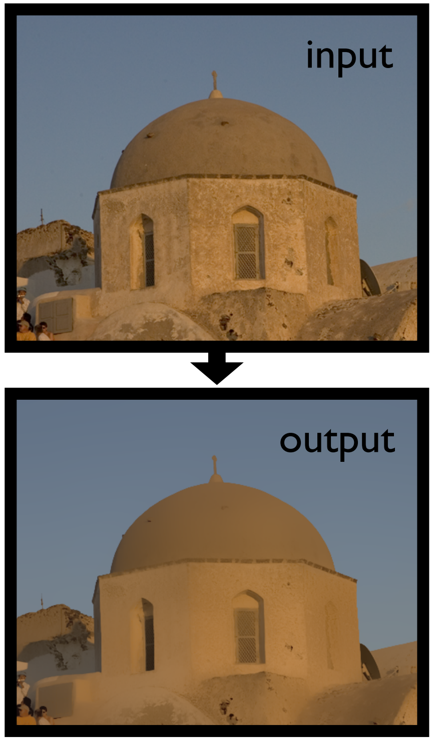
scalar

For color images

color difference

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) \, G_{\sigma_r}\left(\boxed{\| \mathbf{C}_{\mathbf{p}} - \mathbf{C}_{\mathbf{q}} \|} \right) \boxed{\mathbf{C}_{\mathbf{q}}}$$

3D vector
(RGB, Lab)



Hard to Compute

- Nonlinear

$$BF[I]_{\mathbf{p}} = \boxed{\frac{1}{W_{\mathbf{p}}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) \boxed{G_{\sigma_r}\left(| I_{\mathbf{p}} - I_{\mathbf{q}} | \right)} I_{\mathbf{q}}$$

- Complex, spatially varying kernels
  - Cannot be precomputed, no FFT…

- Brute-force implementation is slow > 10min

Basic denoising

Noisy input

Bilateral filter 7x7 window

Basic denoising

Bilateral filter

Median 3x3

Basic denoising

Bilateral filter

Median 5x5

Basic denoising

Bilateral filter

Bilateral filter – lower sigma

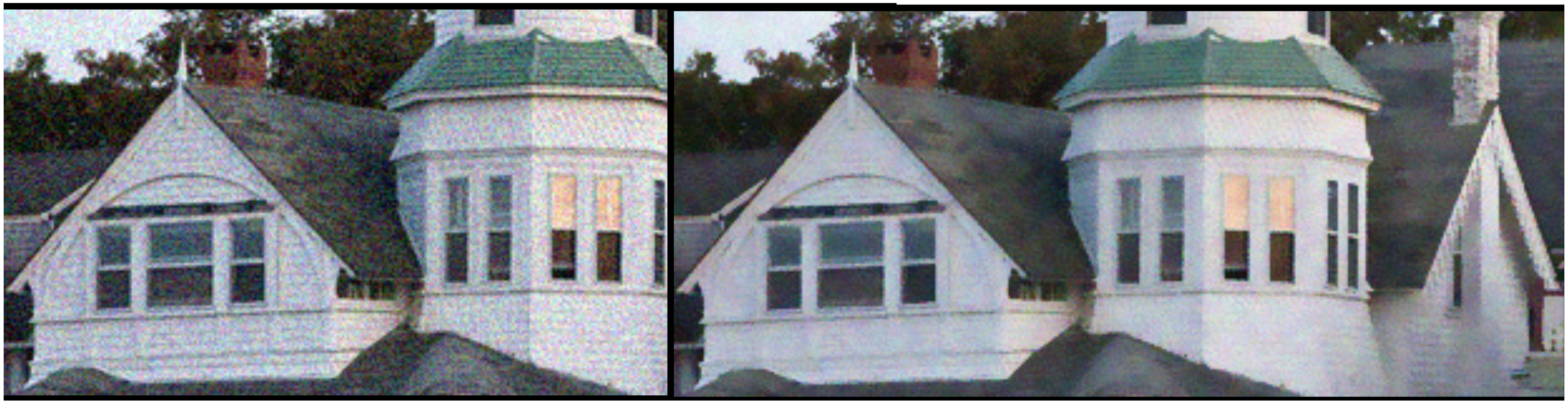
Basic denoising

Bilateral filter

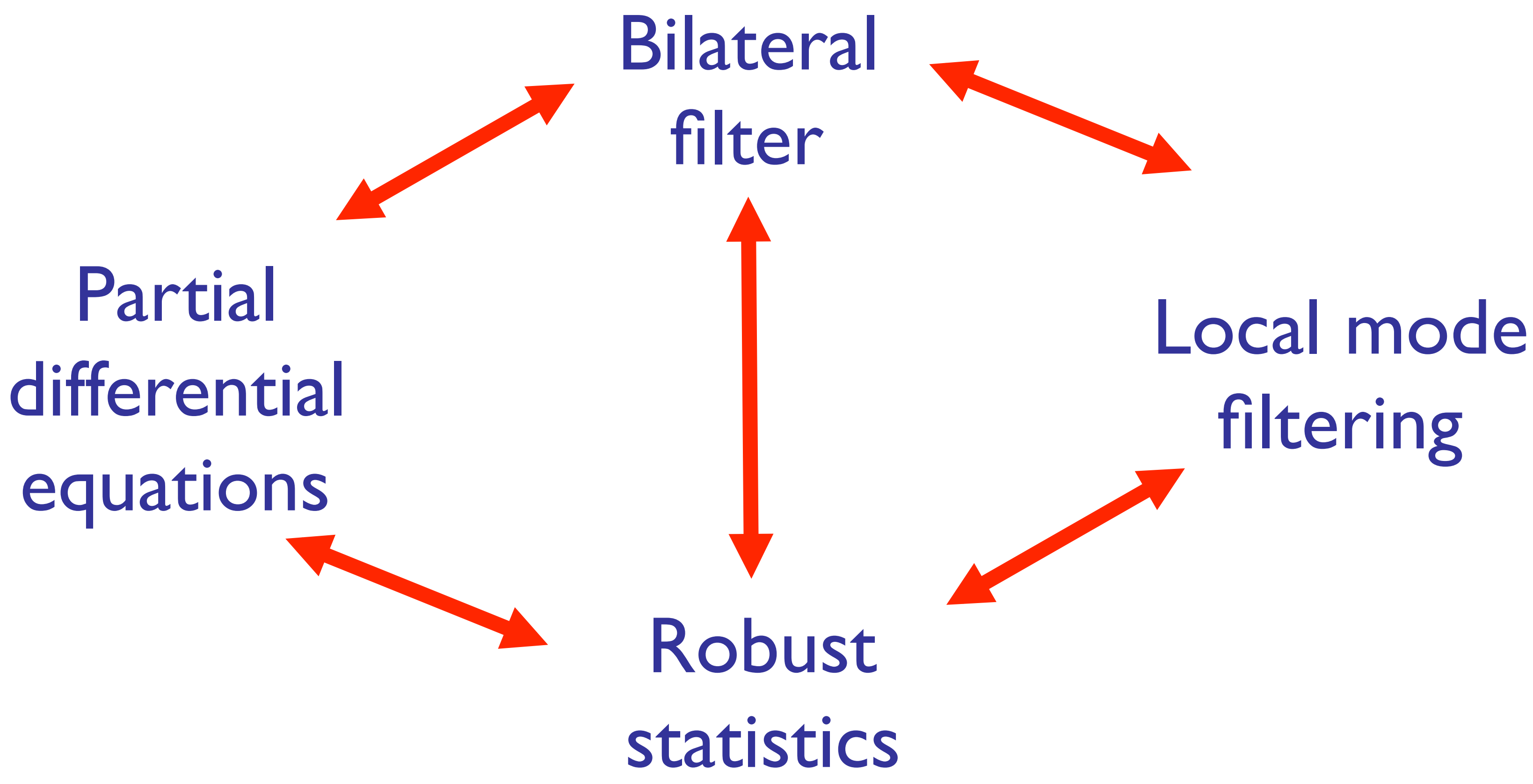
Bilateral filter – higher sigma

Denoising

- Small spatial sigma (e.g. 7x7 window)
- Adapt range sigma to noise level
- Maybe not best denoising method, but best simplicity/quality tradeoff
  - No need for acceleration (small kernel)
  - But the denoising feature in e.g. Photoshop is better



Goal: Understand how does bilateral filter relates with other methods



more in BIL717 Image Processing graduate course..

New Idea:
NL-Means Filter (Buades 2005)

- Same goals: 'Smooth within Similar Regions'

- **KEY INSIGHT**: Generalize, extend'Similarity'
  - **Bilateral:**
    Averages neighbors with **similar intensities**;

  - **NL-Means:**
    Averages neighbors with **similar neighborhoods!**

NL-Means Method: Buades (2005)

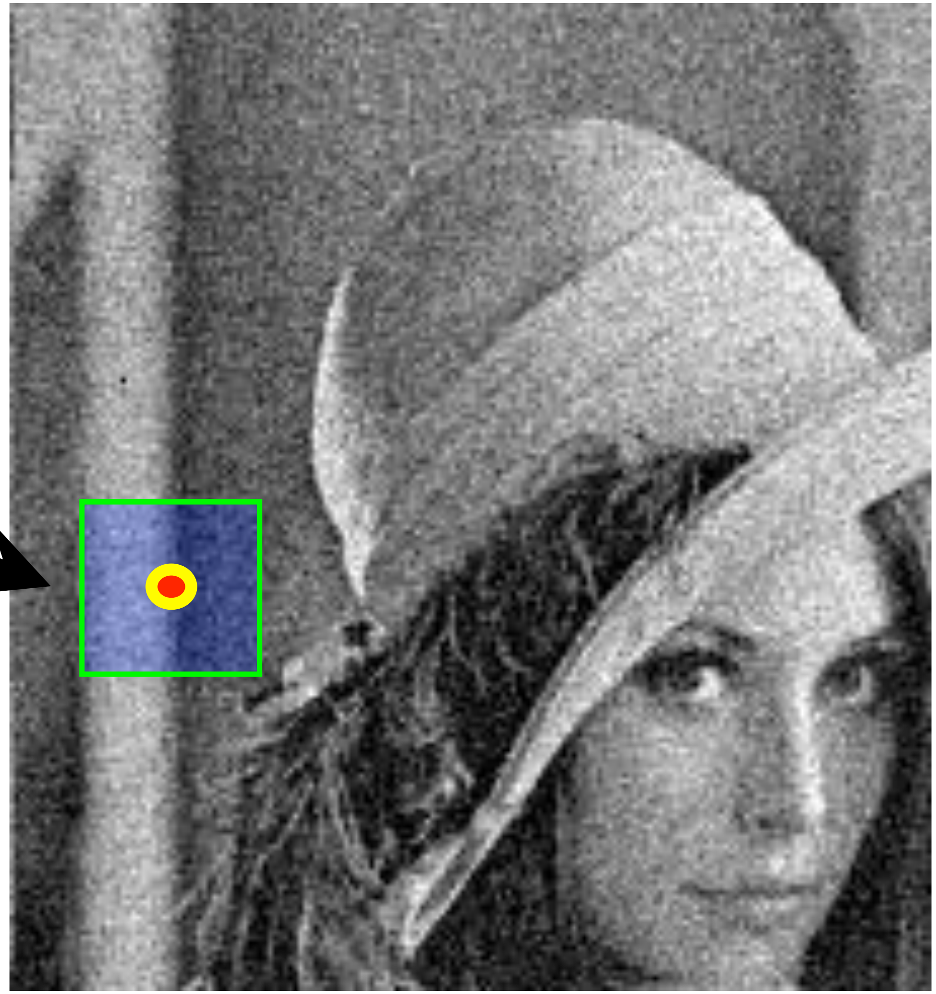
- For each and every pixel **p:**

NL-Means Method: Buades (2005)

- For each and every pixel **p:**
  - Define a small, simple fixed size neighborhood;

NL-Means Method: Buades (2005)

$$\mathbf{V_p} = \begin{bmatrix} 0.74 \\ 0.32 \\ 0.41 \\ 0.55 \\ \cdots \\ \cdots \end{bmatrix}$$
$\cdots$



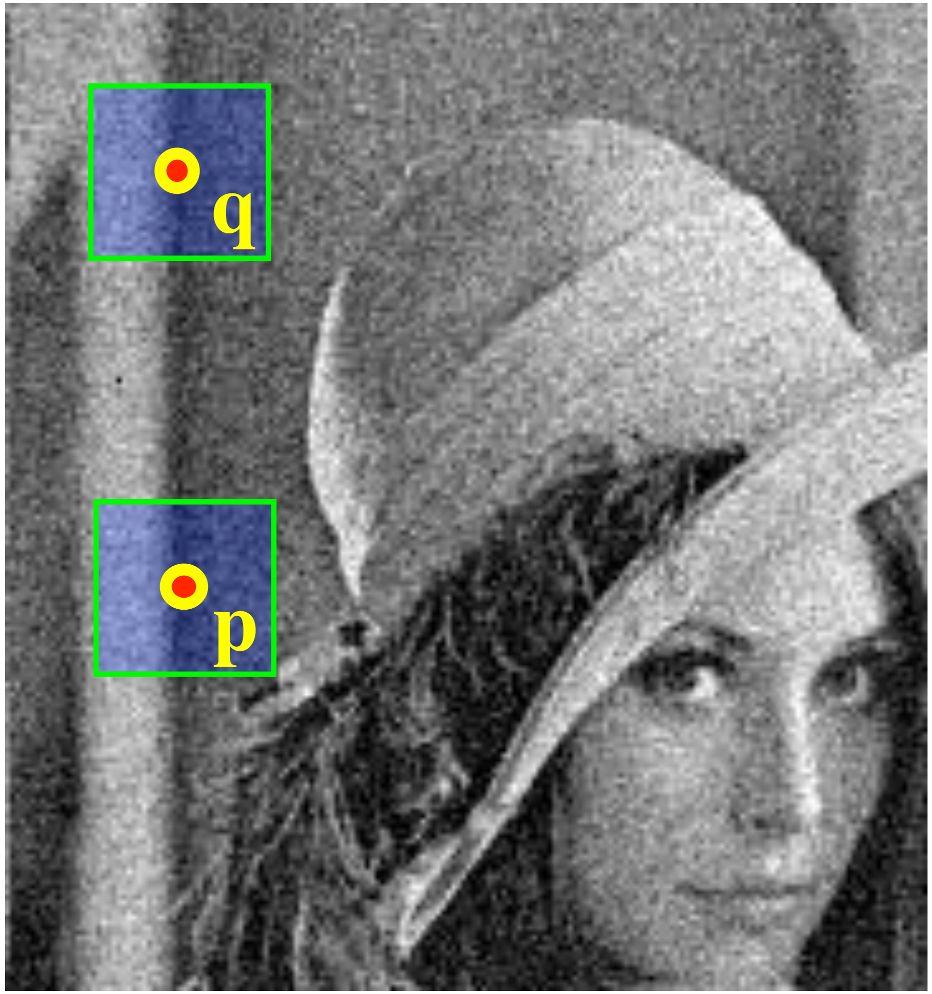
- For each and every pixel **p**:
  - Define a small, simple fixed size neighborhood;
  - Define vector $\mathbf{V_p}$: a list of neighboring pixel values.

NL-Means Method: Buades (2005)

'Similar' pixels **p, q**

→ **SMALL** vector distance;

$$\| \mathbf{V}_p - \mathbf{V}_q \|^2$$

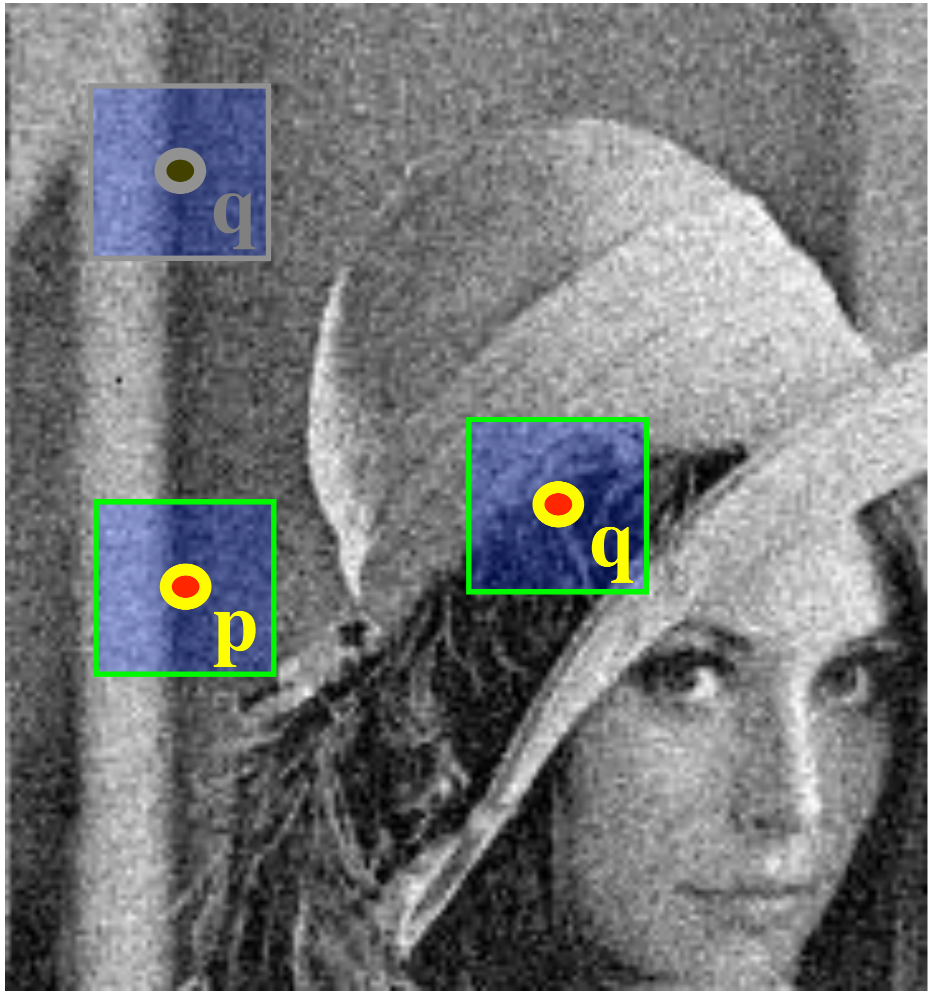


NL-Means Method: Buades (2005)

'Dissimilar' pixels **p, q**

→ **LARGE**
vector distance;

$$\| \mathbf{V_p} - \mathbf{V_q} \|^2$$



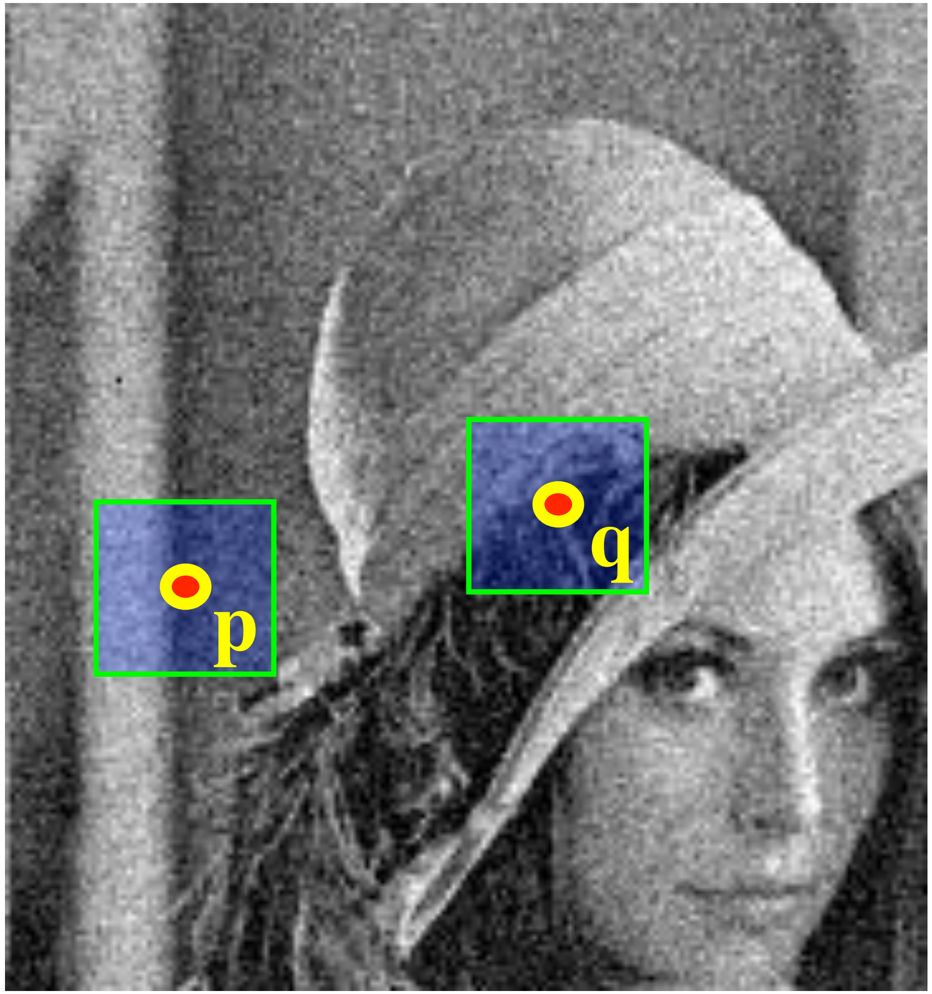
NL-Means Method: Buades (2005)

'Dissimilar' pixels **p, q**

→ **LARGE** vector distance;

$$\boxed{\| \mathbf{V}_p - \mathbf{V}_q \|^2}$$

Filter with this!



NL-Means Method: Buades (2005)

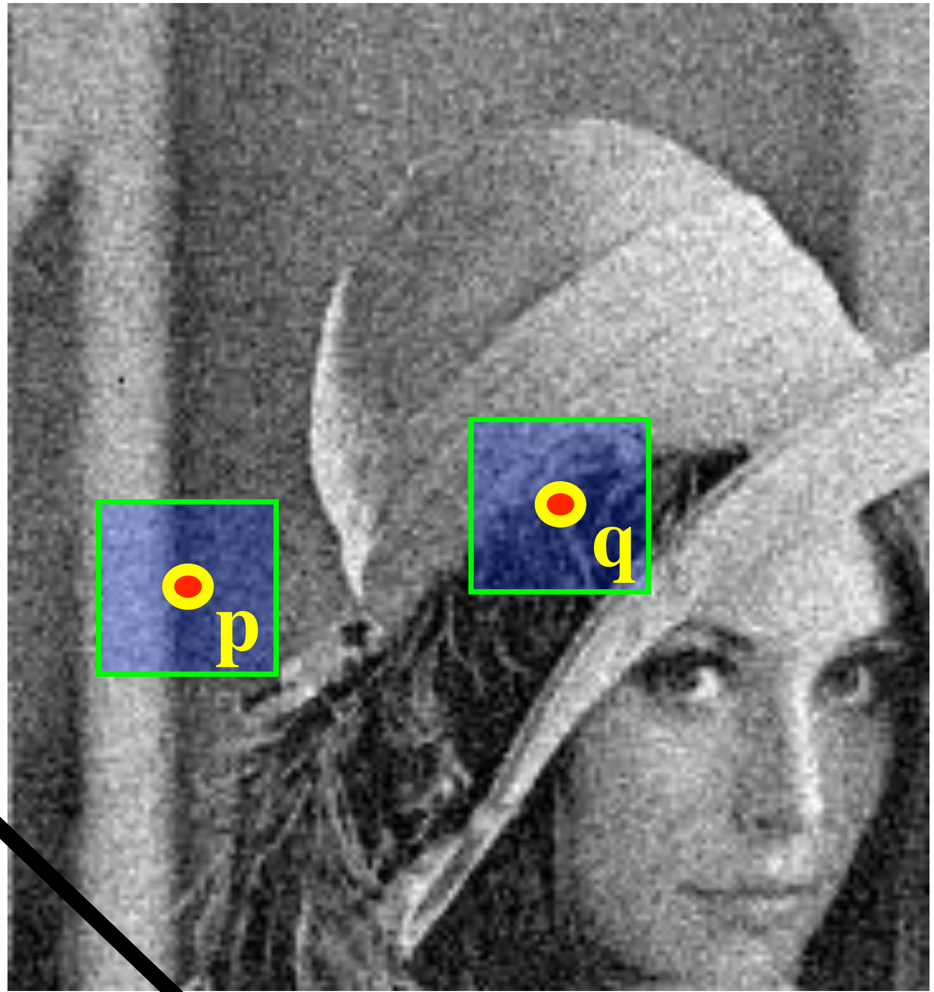
p, q neighbors define

a vector distance;

Filter with this:

$$\boxed{\| \mathbf{V_p} - \mathbf{V_q} \|^2}$$

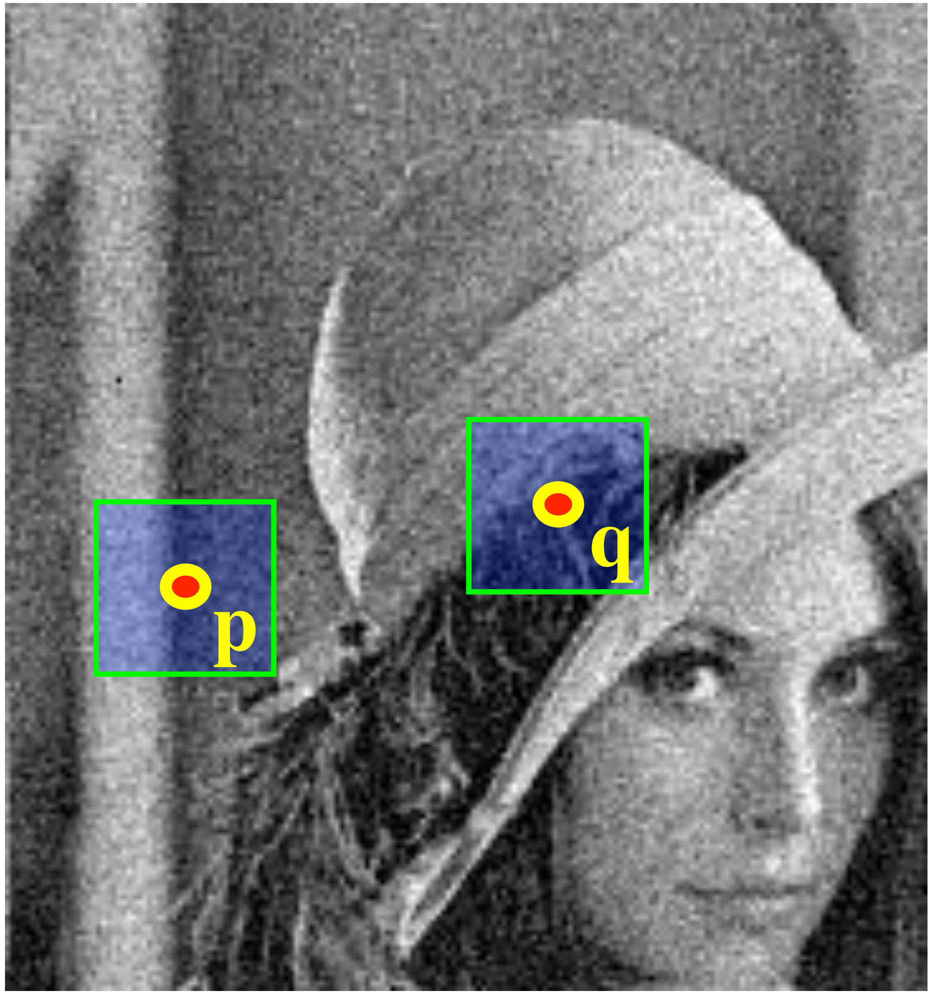
No spatial term!



$$NLMF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} \cancel{G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|)}\, \boxed{G_{\sigma_r}\left(\boxed{\| \vec{V}_{\mathbf{p}} - \vec{V}_{\mathbf{q}} \|^2}\right)} I_{\mathbf{q}}$$

NL-Means Method: Buades (2005)

pixels **p, q** neighbors
Set a vector distance;



$$\| \mathbf{V_p} - \mathbf{V_q} \|^2$$

Vector Distance to p sets weight for each pixel q

$$NLMF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_r}\left(\| \vec{V}_{\mathbf{p}} - \vec{V}_{\mathbf{q}} \|^2 \right) I_{\mathbf{q}}$$

NL-Means Filter (Buades 2005)

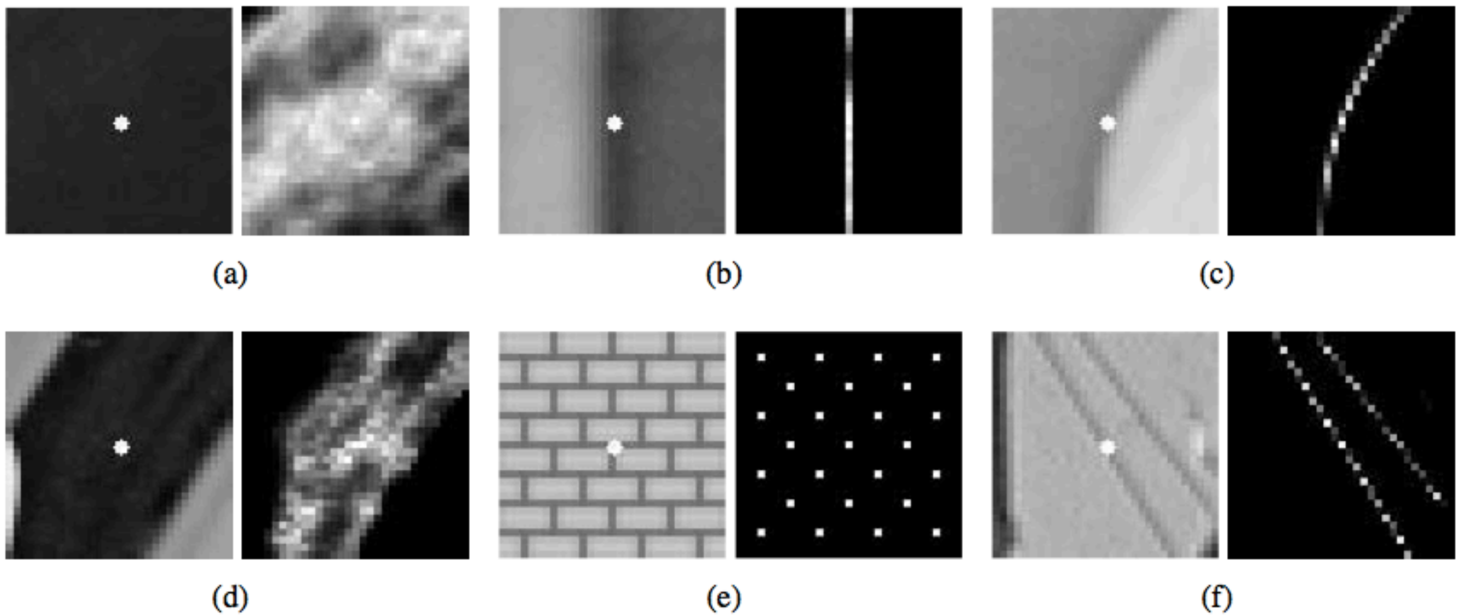


Figure 2. Display of the NL-means weight distribution used to estimate the central pixel of every image. The weights go from 1(white) to zero(black).

NL-Means Filter (Buades 2005)

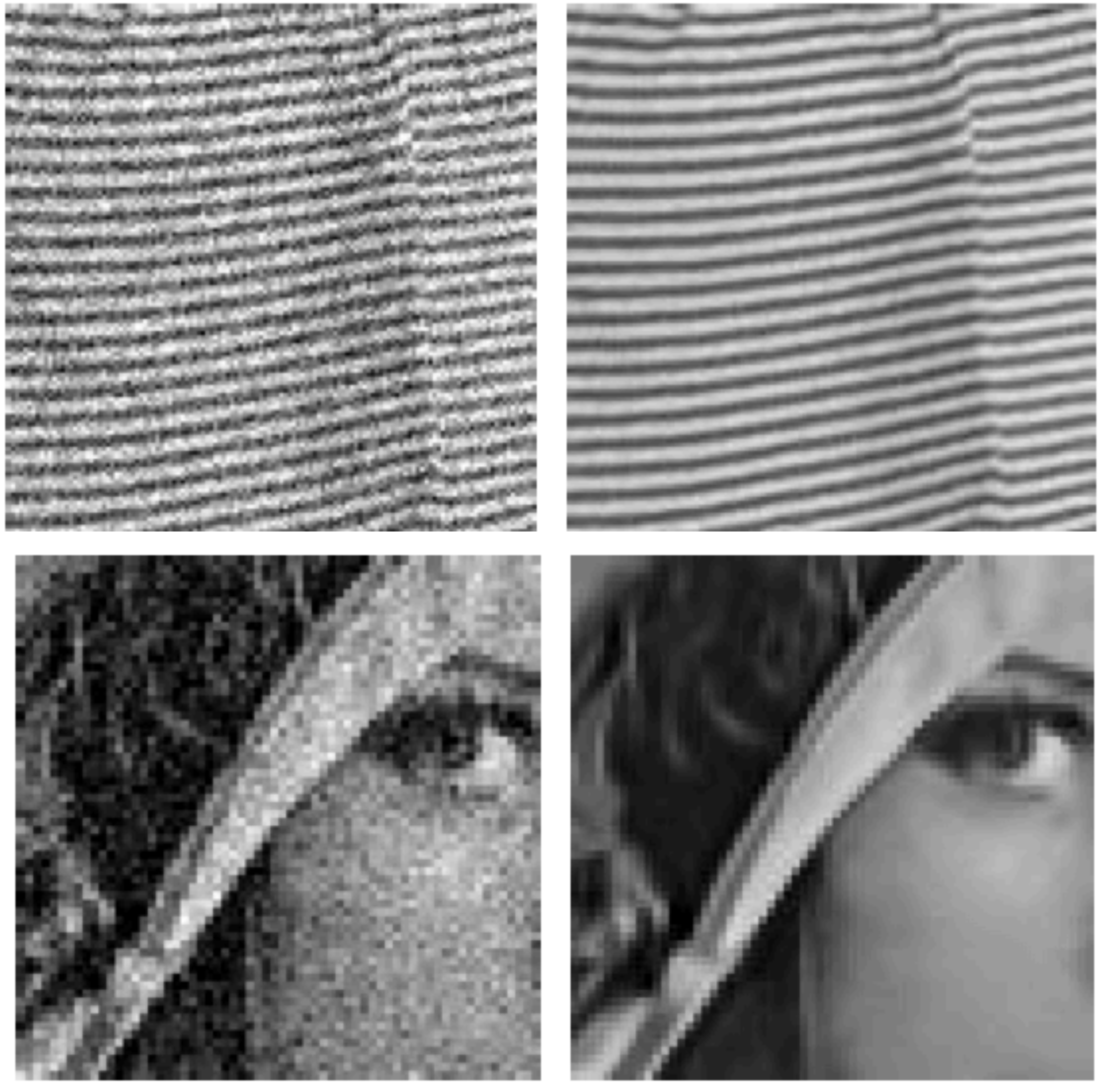


FIG. 9. *NL-means denoising experiment with a natural image. Left: Noisy image with standard deviation* 20. *Right: Restored image.*

NL-Means Filter (Buades 2005)

- Noisy source image:

NL-Means Filter (Buades 2005)

- Gaussian Filter

Low noise,

Low detail

NL-Means Filter (Buades 2005)

- Anisotropic Diffusion

(Note 'stairsteps': ~ piecewise constant)

NL-Means Filter (Buades 2005)

- Bilateral Filter

(better, but similar 'stairsteps':

NL-Means Filter (Buades 2005)

- NL-Means:

Sharp,

Low noise,

Few artifacts.

NL-Means Filter (Buades 2005)

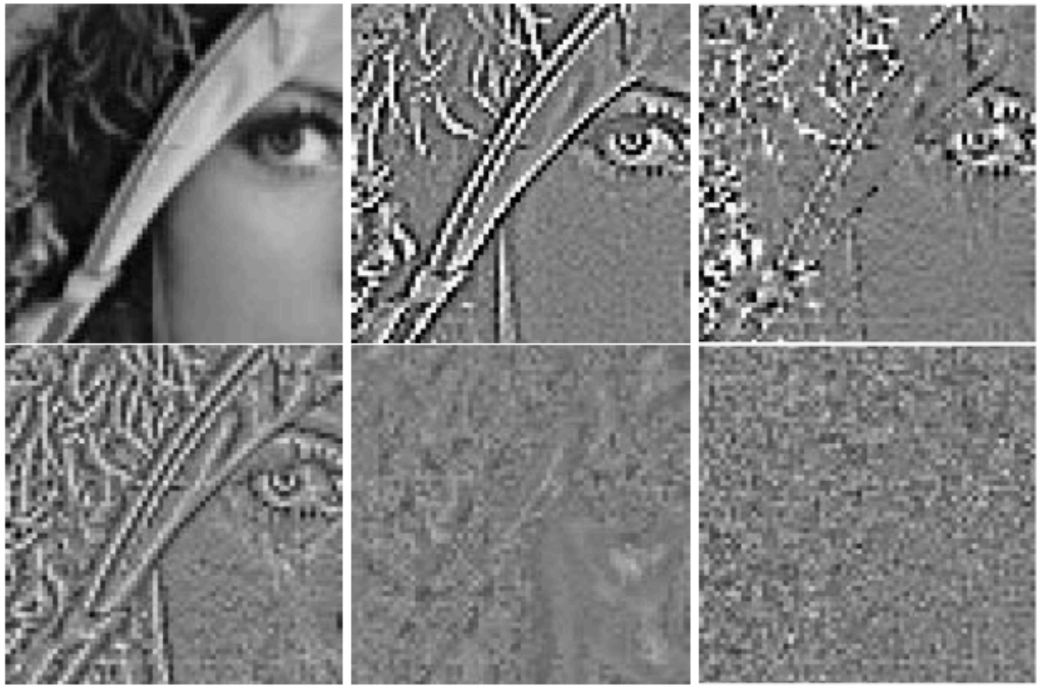


Figure 4. Method noise experience on a natural image. Displaying of the image difference $u - D_h(u)$. From left to right and from top to bottom: original image, Gauss filtering, anisotropic filtering, Total variation minimization, Neighborhood filtering and NL-means algorithm. The visual experiments corroborate the formulas of section 2.

NL-Means Filter (Buades 2005)

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

original

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

noisy

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

denoised

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

original

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

noisy

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

NL-Means Filter (Buades 2005)

denoised

http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/

Structure-Texture Decomposition
Karacan et al., SIGGRAPH Asia 2013

Input Image

Structure-Texture Decomposition
Karacan et al., SIGGRAPH Asia 2013

Structure Component

Structure-Texture Decomposition
Karacan et al., SIGGRAPH Asia 2013

Texture Component

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Karacan et al., SIGGRAPH Asia 2013

Input Image

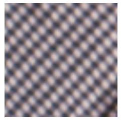
Structure

Texture

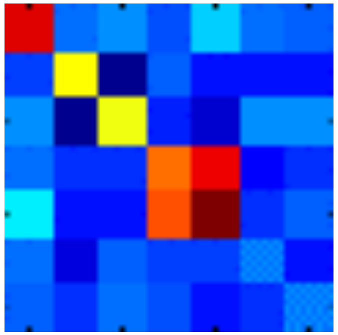
Region Covariances as Region Descriptors

Tuzel et al., ECCV 2006

$$F(x,y) = \phi(I, x, y)$$



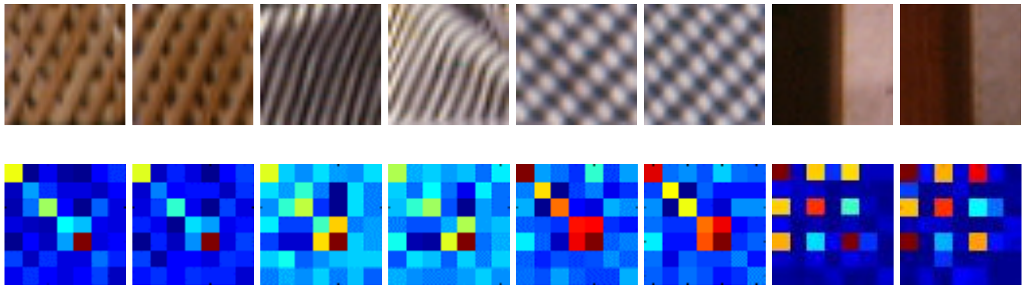
$$F(x,y) = \left[I(x,y) \quad \left|\frac{\partial I}{\partial x}\right| \quad \left|\frac{\partial I}{\partial y}\right| \quad \left|\frac{\partial^2 I}{\partial x^2}\right| \quad \left|\frac{\partial^2 I}{\partial y^2}\right| \quad x \quad y \right]^T$$



$$\mathbf{C}_R = \frac{1}{n-1}\sum_{i=0}^{n}(\mathbf{z}_k - \mu)(\mathbf{z}_k - \mu)^T$$

Main motivation

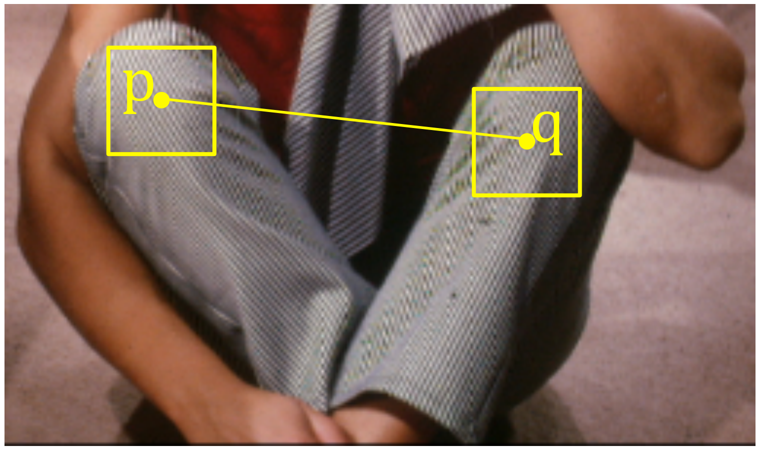
- Region covariances well capture local structure and texture information.
- Similar regions have similar statistics.



Formulation

$$I = S + T$$

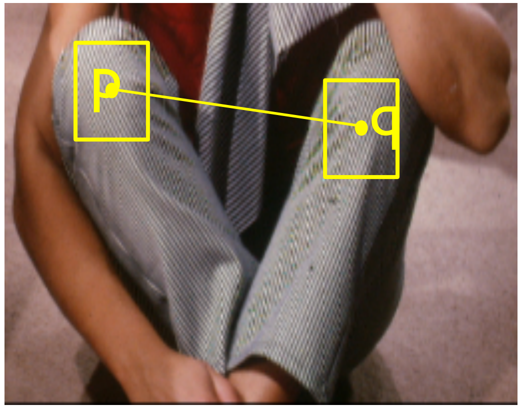
$$S(\mathbf{p}) = \frac{1}{Z_{\mathbf{p}}} \sum_{\mathbf{q} \in N(\mathbf{p},r)} w_{\mathbf{pq}} I(\mathbf{q})$$



- Structure-texture decomposition via smoothing
- Smoothing as weighted averaging
- Different kernels (w_{pq}) result in different types of filters.
- Two novel patch-based kernels for structure-texture decomposition

Model I

- Covariance matrices do not live on Euclidean space.
- Hong et al.,CVPR'09 suggested a way to transform covariance matrices into Euclidean Space.
- Every covariance matrix has a unique Cholesky decomposition



$C = LL^T$ Cholesky Decomposition

$S = \{s_i\}$ Sigma Points

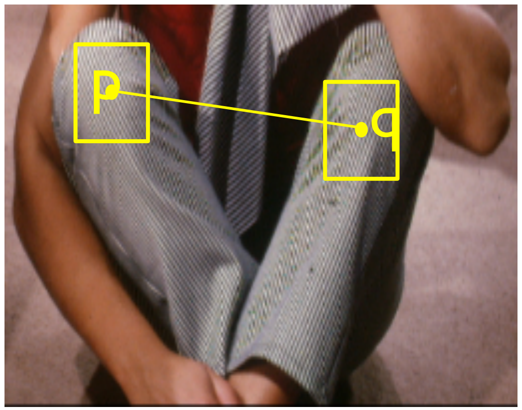
$$\mathbf{s}_i = \begin{cases} \alpha\sqrt{d}\mathbf{L}_i & \text{if } 1 \leq i \leq d \\ -\alpha\sqrt{d}\mathbf{L}_i & \text{if } d+1 \leq i \leq 2d \end{cases}$$

- First order statistics can be easily incorporated to the formulation.

Model I

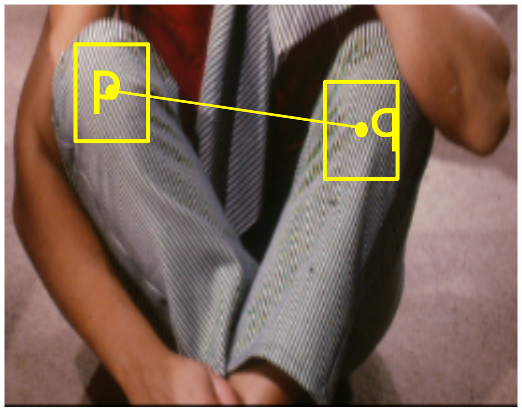
$$\Psi(\mathbf{C}) = (\mu, \mathbf{s}_1, \ldots, \mathbf{s}_d, \mathbf{s}_{d+1}, \ldots, \mathbf{s}_{2d})^T$$ Final representation

$$w_{\mathbf{pq}} \propto \exp\left(-\frac{\|\Psi(\mathbf{C_p}) - \Psi(\mathbf{C_q})\|^2}{2\sigma^2}\right)$$ Resulting kernel



Model 2

- An alternative way is to use statistical measures.
- A Mahalanobis-like distance measure to compare to image patches



$$d(\mathbf{p},\mathbf{q}) = \sqrt{(\mu_{\mathbf{p}} - \mu_{\mathbf{q}})\mathbf{C}^{-1}(\mu_{\mathbf{p}} - \mu_{\mathbf{q}})^T}$$

$$\mathbf{C} = \mathbf{C}_{\mathbf{p}} + \mathbf{C}_{\mathbf{q}}$$

Resulting kernel

$$w_{\mathbf{pq}} \propto \exp\left(-\frac{d(\mathbf{p},\mathbf{q})^2}{2\sigma^2}\right)$$

Illustrative Example

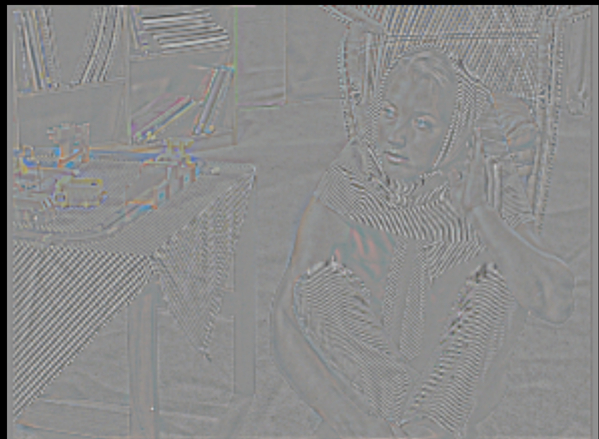
Model1

σ=0.1 k=9

Model2

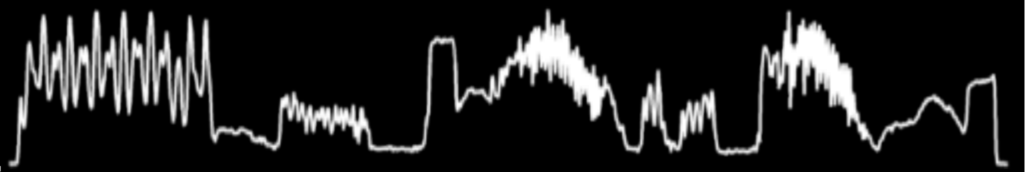
σ=0.3 k=9

Input

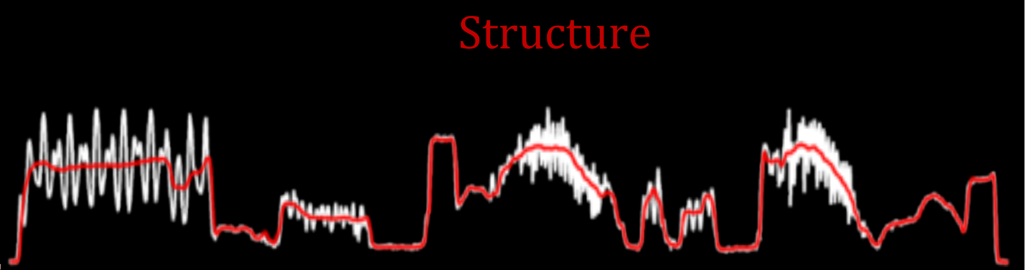


Illustrative Example

Input

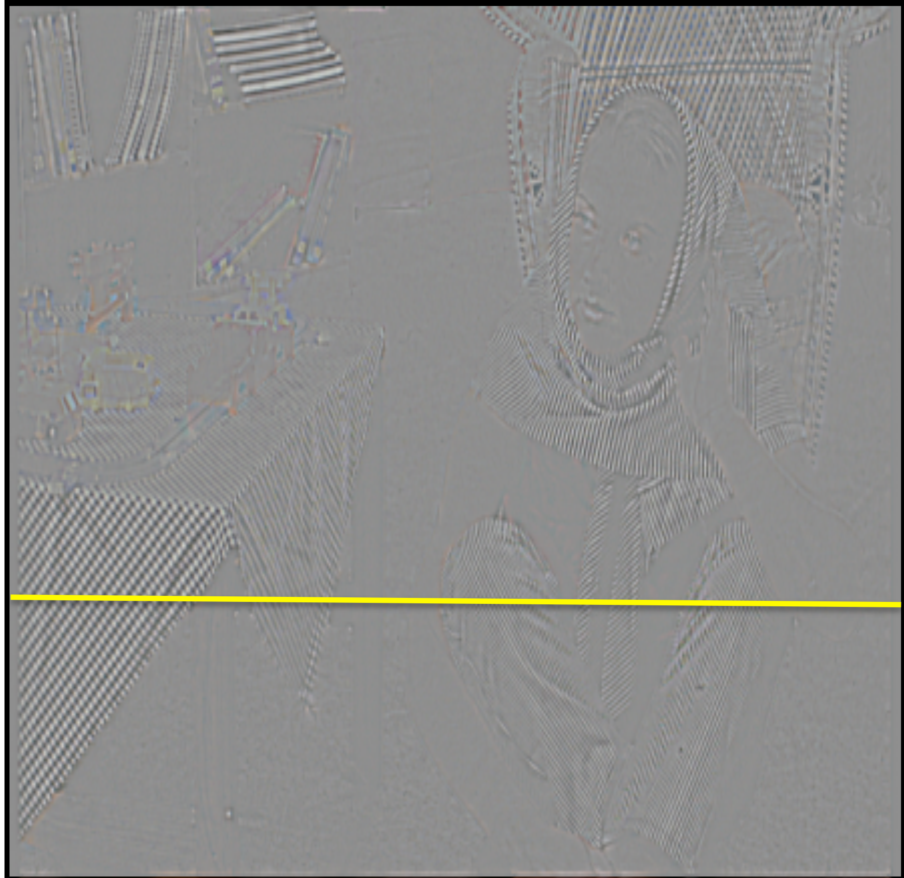


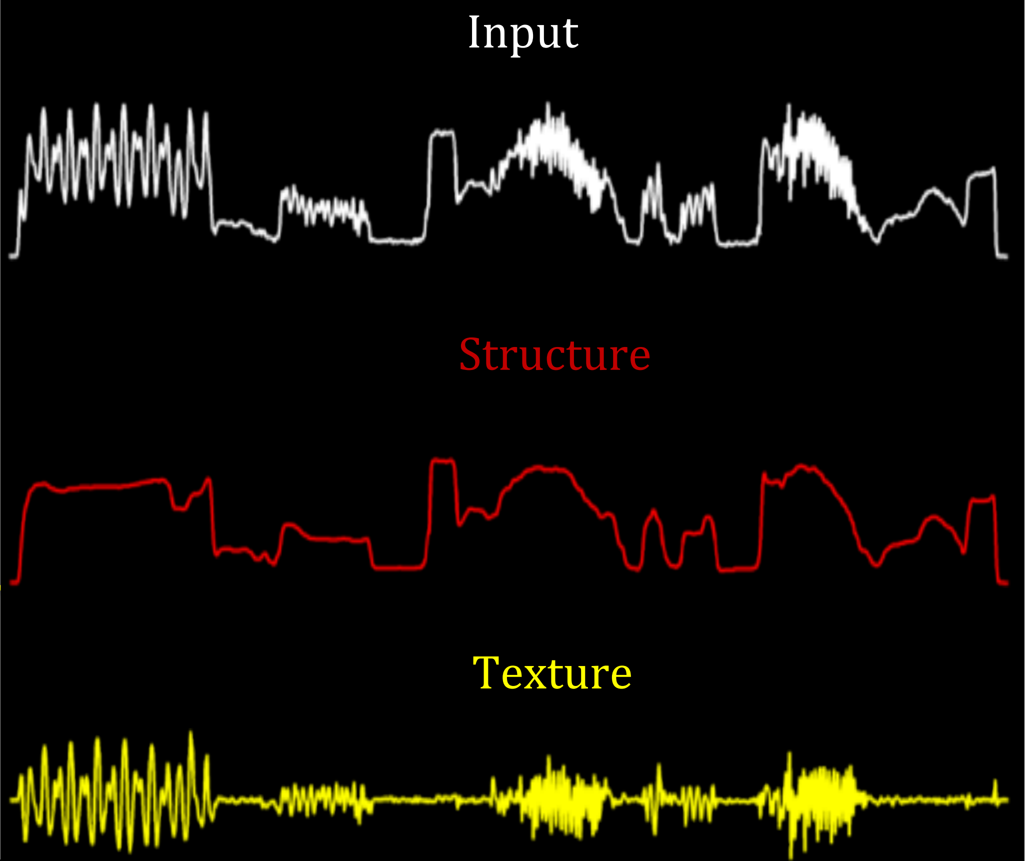
Illustrative Example



Model2 Structure

Illustrative Example



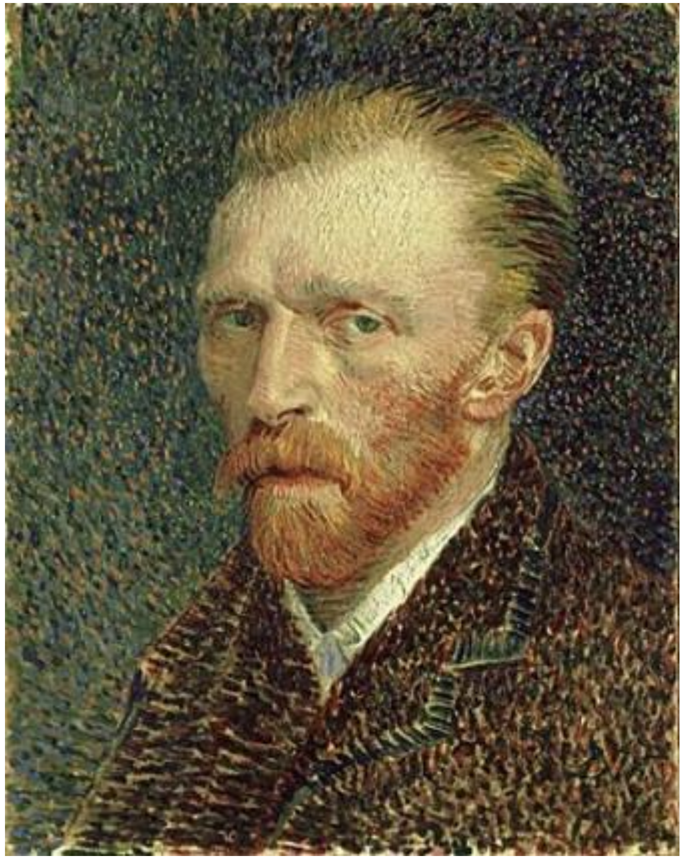


Model2 Texture

Multiscale Decomposition

$$I(\mathbf{p}) = \sum_{i=0}^{n} T_i(\mathbf{p}) + S_n(\mathbf{p})$$

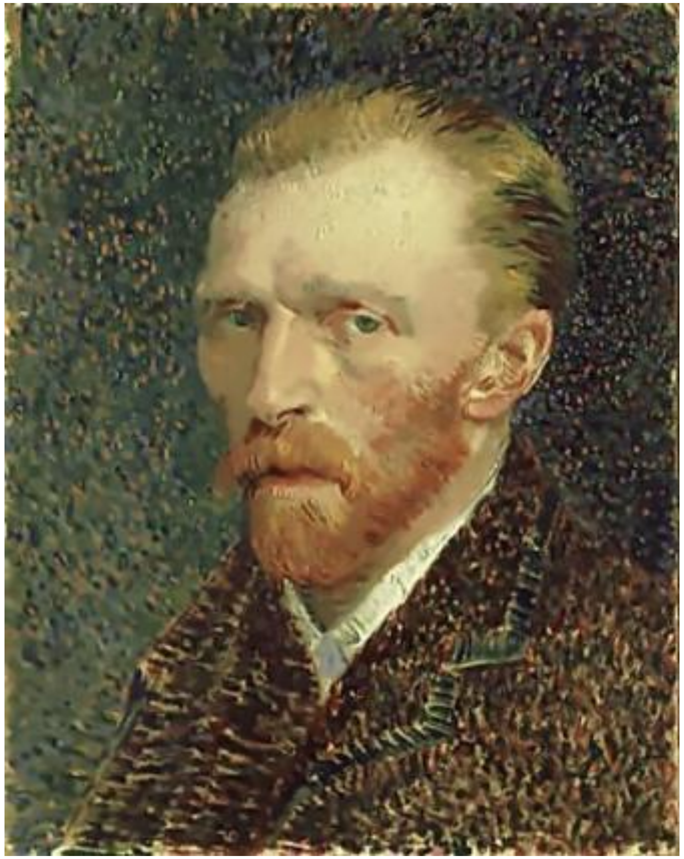
Input



Multiscale Decomposition

$$I(\mathbf{p}) = \sum_{i=0}^{n} T_i(\mathbf{p}) + S_n(\mathbf{p})$$

$S_1(k = 5)$



Multiscale Decomposition

$$I(\mathbf{p}) = \sum_{i=0}^{n} T_i(\mathbf{p}) + S_n(\mathbf{p})$$

$S_2(k = 7)$

Multiscale Decomposition

$$I(\mathbf{p}) = \sum_{i=0}^{n} T_i(\mathbf{p}) + S_n(\mathbf{p})$$

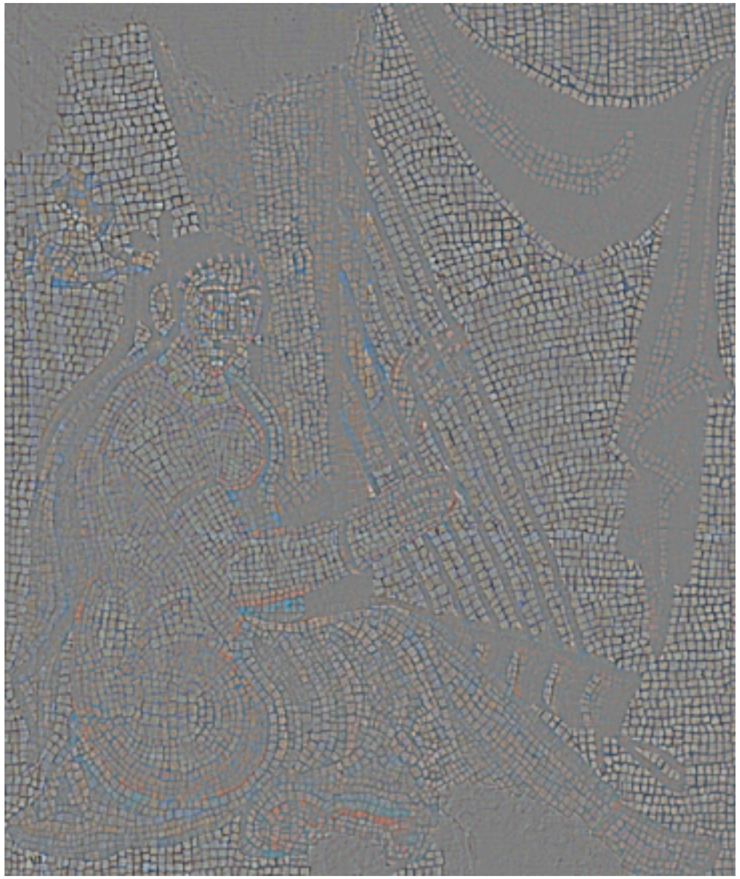
$S_3(k = 9)$

Model 2 + Model 1

Model2 Structure

Model 2 + Model 1

Model2 Texture



Model 2 + Model 1

Model2+Model1

Model 2 + Model 1

Input

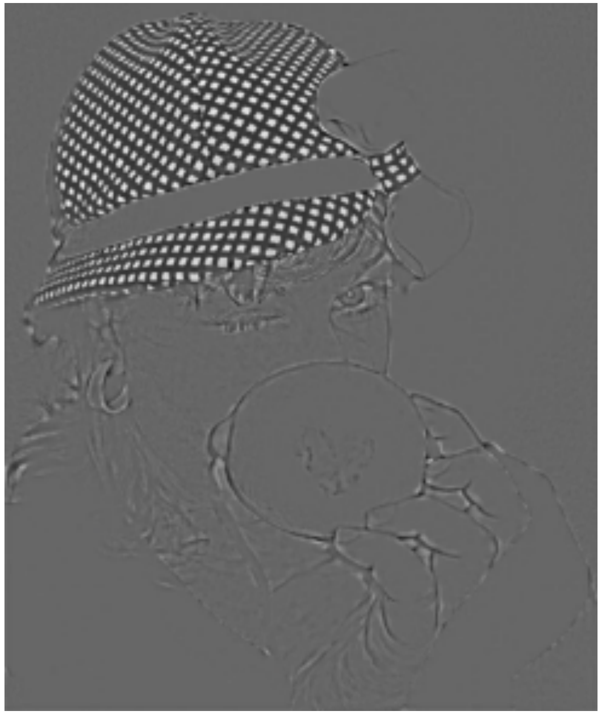
Model2

Model2+Model1

Model 2 + Model 1

Input

Model2 Texture



Model2+Model1