Today
• Perception of color and light
• Color spaces
• Point operations
• Histogram processing

What is color?
• Color is the result of interaction between physical light in the environment and our visual system
• Color is a psychological property of our visual experiences when we look at objects and lights, not a physical property of those objects or lights (S. Palmer, Vision Science: Photons to Phenomenology)

Acknowledgement: This part is mostly adapted from the slides prepared by Steve Marschner, James Hays, Ali Farhadi and Anat Levin
#thedress

- What is the color of the dress?
  - blue and black
  - white and gold
  - blue and brown

- What #thedress tell about our color perception?

#thedress

- Let's take averages

<table>
<thead>
<tr>
<th>two pieces of the dress</th>
<th>averages</th>
<th>basic pattern</th>
</tr>
</thead>
</table>

#thedress

- The dress in the photograph

#thedress

- Consider the dress is in shadow.

- Your brain remove the blue cast, and perceive it as white and gold.
#thedress

• The dress in the photograph

[Image of the dress]

http://nyti.ms/186m3wE

#thedress

• Consider the dress is in bright light.

[Image of the dress in different lighting]

http://nyti.ms/186m3wE

• Your brain perceive the dress as a darker blue and black.

#thedress

• Answer:

• The dress is actually blue and black.

[Image of the dress]

http://nyti.ms/186m3wE

Brightness perception

[Image of a checkered surface with a green cylinder]

http://web.mit.edu/persci/people/adelson/illusions_demos.html

Edward Adelson
Brightness perception

Land’s Experiment (1959)

- Cover all patches except a blue rectangle
- Make it look gray by changing illumination
- Uncover the other patches

Color Constancy

We filter out illumination variations

Slide credit: S. Narasimhan
Land’s Experiment (1959)

- Cover all patches except a blue rectangle
- Make it look gray by changing illumination
- Uncover the other patches

We filter out illumination variations

Color Constancy

Color Constancy in Gold Fish

In David Ingle’s experiment, a goldfish has been trained to swim to a patch of a given color for a reward—a piece of liver. It swims to the green patch regardless of the exact setting of the three projectors’ intensities. The behavior is strikingly similar to the perceptual result in humans.

http://neuro.med.harvard.edu/site/dh/b45.htm

Reading Assignment #2

- Watch Beau Lotto’s TED talk on “Optical illusions show how we see” [link available on course webpage]
Recovering Lightness

- Image Intensity: $b'(x, y) = r'(x, y) e'(x, y)$

Can we recover $e'$ and $r'$ from $b'$?

- Retinex theory, Land and McCann, 1971

- Use constraints (or priors) on shading and reflectance
- Employ additional information (multiple images, depth maps, etc.)

An illposed problem!

Color and light

- **Color of light** arriving at camera depends on
  - Spectral reflectance of the surface light is leaving
  - Spectral radiance of light falling on that patch

- **Color perceived** depends on
  - Physics of light
  - Visual system receptors
  - Brain processing, environment

- Color is a phenomenon of human perception; it is not a universal property of light

Color

White light: composed of about equal energy in all wavelengths of the visible spectrum

Electromagnetic spectrum

- Light is electromagnetic radiation
  - exists as oscillations of different frequency (or, wavelength)
The Physics of light

Any source of light can be completely described physically by its spectrum: the amount of energy emitted (per time unit) at each wavelength 400 - 700 nm.

Relative spectral power

400 500 600 700
Wavelength (nm.)

Some examples of the reflectance spectra of surfaces

Red  Yellow  Blue  Purple
% Light Reflected

400 700 400 700 400 700 400 700
Wavelength (nm.)

Image formation

• What determines the brightness of an image pixel?
Color mixing
Cartoon spectra for color names:

- **Red**
  - 400
  - 500
  - 600
  - 700 nm

- **Green**
  - 400
  - 500
  - 600
  - 700 nm

- **Blue**
  - 400
  - 500
  - 600
  - 700 nm

- **Yellow**
  - 400
  - 500
  - 600
  - 700 nm

- **Cyan**
  - 400
  - 500
  - 600
  - 700 nm

- **Magenta**
  - 400
  - 500
  - 600
  - 700 nm

Credit: W. Freeman

Additive color mixing
Colors combine by *adding* color spectra

- **Red**
  - 400
  - 500
  - 600
  - 700 nm

- **Green**
  - 400
  - 500
  - 600
  - 700 nm

- **Yellow**
  - 400
  - 500
  - 600
  - 700 nm

Light adds to black.

Credit: W. Freeman

Subtractive color mixing
Colors combine by *multiplying* color spectra.

- **Cyan**
  - 400
  - 500
  - 600
  - 700 nm

- **Yellow**
  - 400
  - 500
  - 600
  - 700 nm

- **Green**
  - 400
  - 500
  - 600
  - 700 nm

Pigments remove color from incident light (white).

Credit: W. Freeman

Examples of additive color systems

- **CRT phosphors**
- **Multiple projectors**

Examples: [http://www.jegsworks.com](http://www.jegsworks.com)  [http://www.crtprojectors.co.uk/](http://www.crtprojectors.co.uk/)

Slide credit: K. Grauman
Examples of subtractive color systems

- Printing on paper
- Crayons
- Photographic film

Interaction of light and surfaces

- Reflected color is the result of interaction of light source spectrum with surface reflectance

Reflection from colored surface

The Eye

- Iris - colored annulus with radial muscles
- Pupil - the hole (aperture) whose size is controlled by the iris
- Lens - changes shape by using ciliary muscles (to focus on objects at different distances)
- Retina - photoreceptor cells
The eye as a measurement device

• We can model the low-level behavior of the eye by thinking of it as a light-measuring machine
  – its optics are much like a camera
  – its detection mechanism is also much like a camera

• Light is measured by the photoreceptors in the retina
  – they respond to visible light
  – different types respond to different wavelengths

• The human eye is a camera!

Layers of the retina

Receptors Density - Fovea
Receptors Density - Fovea

Human Photoreceptors

Human eye photoreceptor spectral sensitivities

Two types of light-sensitive receptors

Cones
- cone-shaped
- less sensitive
- operate in high light color vision

Rods
- rod-shaped
- highly sensitive
- operate at night gray-scale vision
Rods and cones

Rods are responsible for intensity, cones for color perception

Rods and cones are non-uniformly distributed on the retina

- Fovea - Small region (1 or 2') at the center of the visual field containing the highest density of cones (and no rods)

Color perception

Rods and cones act as filters on the spectrum

- To get the output of a filter, multiply its response curve by the spectrum, integrate over all wavelengths
  - Each cone yields one number

Q: How can we represent an entire spectrum with 3 numbers?
A: We can’t! Most of the information is lost.
  - As a result, two different spectra may appear indistinguishable

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Recall: Digital images

- **Sample** the 2D space on a regular grid
- **Quantize** each sample (round to nearest integer)
- Image thus represented as a matrix of integer values.

Digital color images

Color images, RGB color space

Images

- Images represented as a matrix
- Suppose we have a \( N \times M \) RGB image called "im"
  - \( \text{im}(1,1,1) \) = top-left pixel value in R-channel
  - \( \text{im}(y,x,b) \) = \( y \) pixels down, \( x \) pixels to right in the \( b \)th channel
  - \( \text{im}(N,M,3) \) = bottom-right pixel in B-channel

Color Images: Bayer Grid

- Estimate RGB at 'G' cells from neighboring values
Color spaces

• How can we represent color?

Slide credit: D. Hoiem

Color spaces: RGB

• Single wavelength primaries
• makes a particular monitor RGB standard
• Good for devices (e.g., phosphors for monitor), but not for perception

RGB color matching functions


Some drawbacks
• Strongly correlated channels
• Non-perceptual

Color spaces: RGB Default color space

0,1,0
0,0,1
1,0,0

Slide credit: D. Hoiem

Color spaces: CIE XYZ

• Standardized by CIE (*Commission Internationale de l’Eclairage*, the standards organization for color science)
• Based on three “imaginary” primaries X, Y, and Z
  – imaginary = only realizable by spectra that are negative at some wavelengths
  – separates out luminance: X, Z have zero luminance, so Y tells you the luminance by itself

CIE XYZ Color matching functions

Slide credit: D. Hoiem

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Slide credit: D. Hoiem

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Slide credit: K. Grauman, S. Marschner

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  - separates out luminance: X, Z have zero luminance, so Y tells you the luminance by itself

\[
\begin{bmatrix}
X \\
Y \\
Z \\
\end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix}
0.49 & 0.31 & 0.20 \\
0.17697 & 0.81240 & 0.01063 \\
0.00 & 0.01 & 0.99 \\
\end{bmatrix} \begin{bmatrix}
R \\
G \\
B \\
\end{bmatrix}
\]

*Slide credit: K. Grauman, S. Marschner*

**Perceptually organized color spaces**

- Artists often refer to colors as tints, shades, and tones of pure pigments
  - tint: mixture with white
  - shade: mixture with black
  - tones: mixture with black and white
  - gray: no color at all (aka. neutral)

- This seems intuitive
  - tints and shades are inherently related to the pure color
    - “same” color but lighter, darker, paler, etc.

*Image from mathworks.com*  
*Slide credit: K. Grauman*

**Perceptual dimensions of color**

- **Hue**
  - the “kind” of color, regardless of attributes
  - colorimetric correlate: dominant wavelength
  - artist’s correlate: the chosen pigment color

- **Saturation**
  - the “colorfulness”
  - colorimetric correlate: purity
  - artist’s correlate: fraction of paint from the colored tube

- **Lightness (or value)**
  - the overall amount of light
  - colorimetric correlate: luminance
  - artist’s correlate: tints are lighter, shades are darker

*Slide credit: S. Marschner*

**Color spaces: HSV**

- **Hue, Saturation, Value**
- Nonlinear – reflects topology of colors by coding hue as an angle
- Matlab: hsv2rgb, rgb2hsv.
**Color spaces: HSV**

- **Hue, Saturation, Value**
- Nonlinear – reflects topology of colors by coding **hue** as an angle
- **Matlab**: hsv2rgb, rgb2hsv.

![Hue-Saturation-Value diagram](image1.png)

**Image from mathworks.com**

**Slide credit:** K. Grauman

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**Color spaces: YCbCr**

Fast to compute, good for compression, used by TV

- **Y**
  - (Cb=0.5, Cr=0.5)
  - Y=0: black, Y=1: white
- **Cb**
  - (Y=0, Cr=0.5)
  - Cb=0: blue, Cb=1: green
- **Cr**
  - (Y=0.5, Cb=0.5)
  - Cr=0: red, Cr=1: yellow

![YCbCr diagram](image2.png)

**Slide credit:** D. Hoiem
Distances in color space

- Are distances between points in a color space perceptually meaningful?

Slide credit: K. Grauman

Distances in color space

- Not necessarily: CIE XYZ is not a uniform color space, so magnitude of differences in coordinates are poor indicator of color “distance”.

McAdam ellipses:
Just noticeable differences in color

Slide credit: K. Grauman

Uniform color spaces

- Attempt to correct this limitation by remapping color space so that just noticeable differences are contained by circles → distances more perceptually meaningful.

- Examples:
  - CIE u’v’
  - CIE Lab

Slide credit: K. Grauman

Perceptually uniform spaces

- Two major spaces standardized by CIE
  - designed so that equal differences in coordinates produce equally visible differences in color
  - by remapping color space so that just noticeable differences are contained by circles → distances more perceptually meaningful.
  - LUV: earlier, simpler space; L*, u*, v*
  - LAB: more complex but more uniform: L*, a*, b*
  - both separate luminance from chromaticity
  - including a gamma-like nonlinear component is important

Slide credit: K. Grauman, S. Marschner
**Color spaces: L*a*b***

“Perceptually uniform” color space

- L: hue
- a: red-green axis
- b: blue-yellow axis

\[ L^* = 116 f \left( \frac{Y}{Y_n} \right) \]

\[ f(t) = \begin{cases} \frac{t^{1/3}}{\delta^2} + 2\delta/3 & t > \delta^2 \\ t/(3\delta^2) + 2\delta/3 & \text{else} \end{cases} \]

\[ a^* = 500 \left[ f \left( \frac{X}{X_n} \right) - f \left( \frac{Y}{Y_n} \right) \right] \]

\[ b^* = 200 \left[ f \left( \frac{Y}{Y_n} \right) - f \left( \frac{Z}{Z_n} \right) \right] \]

(X_n, Y_n, Z_n): measured white point

Slide credit: D. Hoiem

**Most information in intensity**

Only intensity shown – constant color

Slide credit: D. Hoiem

Slide credit: D. Hoiem

Original image
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Digital images

- **Sample** the 2D space on a regular grid
- **Quantize** each sample (round to nearest integer)
- Image thus represented as a matrix of integer values.

Image Transformations

- \( g(x,y) = T[f(x,y)] \)

\( g(x,y) \): output image \( f(x,y) \): input image \( M \): transformation function

1. Point operations: operations on single pixels
2. Spatial filtering: operations considering pixel neighborhoods
3. Global methods: operations considering whole image
**Image Transformations**

- \( g(x,y) = M[f(x,y)] \)

\( g(x,y) \): output image  \( f(x,y) \): input image  \( M \): transformation function

1. Point operations: operations on single pixels
2. Spatial filtering: operations considering pixel neighborhoods
3. Global methods: operations considering whole image

\[ g(x,y) = M(f(i,j)(i,j) \in N(x,y)) \]

**Point operations**

- Smallest possible neighborhood is of size 1x1
- Process each point independently of the others
- Output image \( g \) depends only on the value of \( f \) at a single point \((x,y)\)
- Map each pixel's value to a new value
- Transformation function \( T \) remaps the sample's value:
  \[ s = T(r) \]
  where
  - \( r \) is the value at the point in question
  - \( s \) is the new value in the processed result
  - \( T \) is an intensity transformation function

**Sample intensity transformation functions**

- Image negatives
- Log transformations
  - Compresses the dynamic range of images
- Power-law transformations
  - Gamma correction

- Is mapping one color space to another (e.g. RGB2HSV) a point operation?
- Is image arithmetic a point operation?
- Is performing geometric transformations a point operation?
  - Rotation
  - Translation
  - Scale change
  - etc.
**Point Processing Examples**

Produces an image of higher contrast than the original by darkening the intensity levels below \( k \) and brightening intensities above \( k \).

**Image Mean**

\[
I_{av} = \frac{\sum \sum I(i,j)}{\sum \sum 1}
\]

**Image Negative**

**Changing the image mean**
Dynamic range

• Dynamic range $R_d = \frac{l_{\text{max}}}{l_{\text{min}}}$, or $(l_{\text{max}} + k) / (l_{\text{min}} + k)$
  – determines the degree of image contrast that can be achieved
  – a major factor in image quality

• Ballpark values
  – Desktop display in typical conditions: 20:1
  – Photographic print: 30:1
  – High dynamic range display: 10,000:1

Point Operations: Contrast stretching and Thresholding

• Contrast stretching: produces an image of higher contrast than the original

• Thresholding: produces a binary (two-intensity level) image

Histogram

• Histogram: a discrete function $h(r)$ which counts the number of pixels in the image having intensity $r$

• If $h(r)$ is normalized, it measures the probability of occurrence of intensity level $r$ in an image
Point Operations

- What can you say about the image having the following histogram?

![Histogram]

- A low contrast image
- How we can process the image so that it has a better visual quality?

Answer is contrast stretching!

Point Operations

- Let us devise an appropriate point operation.

![Histogram]

- Shift all values so that the observable pixel range starts at 0.

Point Operations

- How we can process the image so that it has a better visual quality?

![Histogram]

- Answer is contrast stretching!

Point Operations

- Let us devise an appropriate point operation.

![Histogram]

- Now, scale everything in the range 0-100 to 0-255.
**Point Operations**

- Let us devise an appropriate point operation.
- What is the corresponding transformation function?
  \[ T(r) = 2.55^{(r-100)} \]

**Point Operations: Intensity-level Slicing**

- highlights a certain range of intensities

**Intensity encoding in images**

- Recall that the pixel values determine how bright that pixel is.
- Bigger numbers are (usually) brighter
- *Transfer function*: function that maps input pixel value to luminance of displayed image
  \[ I = f(n) \quad f : [0, N] \rightarrow [I_{\text{min}}, I_{\text{max}}] \]
- What determines this function?
  - physical constraints of device or medium
  - desired visual characteristics
**What this projector does?**

- Something like this:

  ![Graph](image)

  - $I(n) = 64$
  - $I(n) = 128$
  - $I(n) = 192$
  - $I = 0.25$
  - $I = 0.5$
  - $I = 0.75$

adapted from: S. Marschner

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**Constraints on transfer function**

- Maximum displayable intensity, $I_{\text{max}}$
  - how much power can be channeled into a pixel?
    - LCD: backlight intensity, transmission efficiency (<10%)
    - projector: lamp power, efficiency of imager and optics

- Minimum displayable intensity, $I_{\text{min}}$
  - light emitted by the display in its “off” state
    - e.g. stray electron flux in CRT, polarizer quality in LCD

- Viewing flare, $k$: light reflected by the display
  - very important factor determining image contrast in practice
    - 5% of $I_{\text{max}}$ is typical in a normal office environment [sRGB spec]
    - much effort to make very black CRT and LCD screens
    - all-black decor in movie theaters

---

**Transfer function shape**

- Desirable property: the change from one pixel value to the next highest pixel value should not produce a visible contrast
  - otherwise smooth areas of images will show visible bands

- What contrasts are visible?
  - rule of thumb: under good conditions we can notice a 2% change in intensity
  - therefore we generally need smaller quantization steps in the darker tones than in the lighter tones
  - most efficient quantization is logarithmic

![Image with severe banding](image)

---

**How many levels are needed?**

- Depends on dynamic range
  - 2% steps are most efficient:
    - $0 \mapsto I_{\text{min}}; 1 \mapsto 1.02I_{\text{min}}; 2 \mapsto (1.02)^2 I_{\text{min}}; \ldots$
    - log 1.02 is about 1/120, so 120 steps per decade of dynamic range
      - 240 for desktop display
      - 360 to print to film
      - 480 to drive HDR display

- If we want to use linear quantization (equal steps)
  - one step must be < 2% (1/50) of $I_{\text{min}}$
  - need to get from ~0 to $I_{\text{min}}$, $R_d$ so need about $50R_d$ levels
    - 1500 for a print; 5000 for desktop display; 500,000 for HDR display

- Moral: 8 bits is just barely enough for low-end applications
  - but only if we are careful about quantization
**Intensity quantization in practice**

- **Option 1: linear quantization** $I(n) = (n/N) I_{\text{max}}$
  - pro: simple, convenient, amenable to arithmetic
  - con: requires more steps (wastes memory)
  - need 12 bits for any useful purpose; more than 16 for HDR
- **Option 2: power-law quantization** $I(n) = (n/N)^\gamma I_{\text{max}}$
  - pro: fairly simple, approximates ideal exponential quantization
  - con: need to linearize before doing pixel arithmetic
  - con: need to agree on exponent
  - 8 bits are OK for many applications; 12 for more critical ones
- **Option 2: floating-point quantization** $I(x) = (x/w) I_{\text{max}}$
  - pro: close to exponential; no parameters; amenable to arithmetic
  - con: definitely takes more than 8 bits
  - 16-bit “half precision” format is becoming popular

**Why gamma?**

- Power-law quantization, or *gamma correction* is most popular
- Original reason: CRTs are like that
  - intensity on screen is proportional to (roughly) voltage$^2$
- Continuing reason: inertia + memory savings
  - inertia: gamma correction is close enough to logarithmic that there’s no sense in changing
  - memory: gamma correction makes 8 bits per pixel an acceptable option

**Gamma quantization**

![Gamma quantization diagram]

- Close enough to ideal perceptually uniform exponential

**Gamma correction**

- Sometimes (often, in graphics) we have computed intensities $a$ that we want to display linearly
- In the case of an ideal monitor with zero black level, $I(n) = (n/N)^\gamma$
  (where $N = 2^n - 1$ in $n$ bits). Solving for $n$:
  $n = N a^{\frac{1}{\gamma}}$
- This is the “gamma correction” recipe that has to be applied when computed values are converted to 8 bits for output
  - failing to do this (implicitly assuming gamma = 1) results in dark, oversaturated images
**Gamma correction**

Corrected for $\gamma$ lower than display

**OK**

Corrected for $\gamma$ higher than display

Slide credit: S. Marschner

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**Instagram Filters**

- How do they make those Instagram filters?

“It’s really a combination of a bunch of different methods. In some cases we draw on top of images, in others we do pixel math. It really depends on the effect we’re going for.” --- Kevin Systrom, co-founder of Instagram

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**Example Instagram Steps**

1. Perform an independent RGB color point transformation on the original image to increase contrast or make a color cast

2. Overlay a circle background image to create a vignette effect
Example Instagram Steps

3. Overlay a background image as decorative grain

Source: C. Dyer

4. Add a border or frame

Source: C. Dyer

Result

Javascript library for creating Instagram-like effects, see: http://alexmic.net/filrr/

Source: C. Dyer

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Histogram

- Histogram: a discrete function $h(r)$ which counts the number of pixels in the image having intensity $r$
- If $h(r)$ is normalized, it measures the probability of occurrence of intensity level $r$ in an image

- What histograms say about images? A descriptor for visual information
- What they don’t?
  - No spatial information

Images and histograms

- How do histograms change when
  - we adjust brightness? shifts the histogram horizontally
  - we adjust contrast? stretches or shrinks the histogram horizontally

Image Representations: Histograms

- Represent distribution of features
  - Color, texture, depth, …
**Histograms: Implementation issues**

- **Quantization**
  - Grids: fast but applicable only with few dimensions

- **Matching**
  - Histogram intersection or Euclidean may be faster
  - Chi-squared often works better
  - Earth mover’s distance is good for when nearby bins represent similar values

**What kind of things do we compute histograms of?**

- **Color**
  - L*a*b* color space
  - HSV color space
  - Texture (filter banks over regions – later on)

- **Histograms of oriented gradients (later on)**
• The image histogram does not fully represent the image

Image Statistics
• The image mean: \[ E[I] = \frac{1}{N} \sum_{i,j} I(i,j) = \frac{1}{N} \sum_k k H(k) = \sum_k k P(k) \]
• Generally: \[ E[g(k)] = \sum_k g(k) P(k) \]
• The image s.t.d.: \[ \sigma(I) = \sqrt{E[I - E[I]^2]} = \sqrt{E[I^2] - E^2(I)} \]
where \[ E[I^2] = \sum_k k^2 P(k) \]

Image Entropy
\[ Entropy(I) = -\sum P(k) \log P(k) \]
• The image entropy specifies the uncertainty in the image values.
• Measures the averaged amount of information required to encode the image values.
**Image Entropy**

- An infrequent event provides more information than a frequent event
- Entropy is a measure of histogram dispersion

Entropy=$7.4635$

Entropy=0

---

**Adaptive Histogram**

- In many cases histograms are needed for local areas in an image
- Examples:
  - Pattern detection
  - adaptive enhancement
  - adaptive thresholding
  - tracking

---

**Histogram Usage**

- Digitizing parameters
- Measuring image properties:
  - Average
  - Variance
  - Entropy
  - Contrast
  - Area (for a given gray-level range)
- Threshold selection
- Image distance

---

**Example: Auto-Focus**

- In some optical equipment (e.g. slide projectors) inappropriate lens position creates a blurred (“out-of-focus”) image
- We would like to automatically adjust the lens
- How can we measure the amount of blurring?
Example: Auto-Focus

- Image mean is not affected by blurring
- Image s.t.d. (entropy) is decreased by blurring
- Algorithm: Adjust lens according to changes in the histogram s.t.d.

Recall: Thresholding

Threshold Selection

Original Image

Binary Image

Threshold too low

Threshold too high

Segmentation using Thresholding

Original

Histogram

Threshold = 50

Threshold = 75
Segmentation using Thresholding

Original

Histogram

Threshold = 21

Slide credit: Y. Hel-Or

Adaptive Thresholding

• Thresholding is space variant.
• How can we choose the local threshold values?

Slide credit: Y. Hel-Or

Histogram based image distance

• **Problem:** Given two images A and B whose (normalized) histogram are $P_A$ and $P_B$, define the distance $D(A,B)$ between the images.

• Example Usage:
  - Tracking
  - Image retrieval
  - Registration
  - Detection
  - Many more ...

Porikli 05

Slide credit: Y. Hel-Or

Option 1: Minkowski Distance

$$D_p(A,B) = \left[ \sum_k |P_A(k) - P_B(k)|^p \right]^{1/p}$$

• **Problem:** distance may not reflect the perceived dissimilarity:

Slide credit: Y. Hel-Or
Option 2: Kullback-Leibler (KL) Distance

\[ D_{KL}(A \parallel B) = -\sum_k P_A(k) \log \frac{P_A(k)}{P_B(k)} \]

- Measures the amount of added information needed to encode image A based on the histogram of image B.
- Non-symmetric: \( D_{KL}(A,B) \neq D_{KL}(B,A) \)
- Suffers from the same drawback of the Minkowski distance.

Slide credit: Y. Hel-Or

Option 3: The Earth Mover Distance (EMD)

- Suggested by Rubner & Tomasi 98
- Defines as the minimum amount of “work” needed to transform histogram \( H_A \) towards \( H_B \)
- The term \( d_{ij} \) defines the “ground distance” between gray-levels i and j.
- The term \( F=\{f_{ij}\} \) is an admissible flow from \( H_A(i) \) to \( H_B(j) \)

Slide credit: Y. Hel-Or

From: Pete Barnum
Option 3: The Earth Mover Distance (EMD)

\[
\text{work} = \text{(amount moved)} \times \text{(distance moved)}
\]

\[
\begin{align*}
D_{\text{EMD}}(A, B) &= \min_F \sum_i \sum_j f_{ij} \cdot d_{ij} \\
\text{s.t. } f_{ij} &\geq 0 \quad ; \quad P_A(k) = \sum_i f_{ik} \quad ; \quad P_B(k) \geq \sum_i f_{ki}
\end{align*}
\]

- Constraints:
  - Move earth only from A to B
  - After move \( P_A \) will be equal to \( P_B \)
  - Cannot send more “earth” than there is
- Can be solved using Linear Programming
- Can be applied in high dim. histograms (color).
Special case: EMD in 1D

- Define $C_A$ and $C_B$ as the cumulative histograms of image A and B respectively:

$$D_{EMD}(A, B) = \sum_k |C_A(k) - C_B(k)|$$

Histogram equalization

- A good quality image has a nearly uniform distribution of intensity levels. Why?
- Every intensity level is equally likely to occur in an image

- Histogram equalization: Transform an image so that it has a uniform distribution
  - create a lookup table defining the transformation

Histogram as a probability density function

- Recall that a normalized histogram measures the probability of occurrence of an intensity level $r$ in an image
- We can normalize a histogram by dividing the intensity counts by the area

$$p(r) = \frac{h(r)}{\text{Area}}$$

Histogram equalization: Continuous domain

- Define a transformation function of the form

$$s = T(r) = (L - 1) \int_0^r p(w) dw$$

where
- $r$ is the input intensity level
- $s$ is the output intensity level
- $p$ is the normalized histogram of the input signal
- $L$ is the desired number of intensity levels

(Continuous) output signal has a uniform distribution!
Histogram equalization: Discrete domain

- Define the following transformation function for an MxN image

\[ s_k = T(r_k) = (L-1) \sum_{j=0}^{k} \frac{n_j}{MN} = \frac{(L-1)}{MN} \sum_{j=0}^{k} n_j \]

for \( k = 0, \ldots, L - 1 \)

where

- \( r_k \) is the input intensity level
- \( s_k \) is the output intensity level
- \( n_j \) is the number of pixels having intensity value \( j \) in the input image
- \( L \) is the number of intensity levels

(Discrete) output signal has a nearly uniform distribution!

- Define: \( C_b(v) = v \ast \frac{\text{(# pixels)}}{\text{#grayValues}} \)

- Assign: \( v_b = C_b^{-1}(C_a(v_a)) = M(v_a) \)
Histogram equalization examples

Original

Equalized

Slide credit: Y. Hel-Or

Histogram equalization examples

Original

Equalized

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Equalized

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Histogram equalization examples

Original

Equalized

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Histogram equalization examples

Histogram Specification

1. Equalize the histogram of the input image
   \[ T_1(r) = (L - 1) \int_0^r p_1(w) \, dw \]
2. Histogram equalize the desired output histogram
   \[ T_2(r) = (L - 1) \int_0^r p_2(w) \, dw \]
3. Histogram specification can be carried out by the following point operation:
   \[ s = T(r) = T_2^{-1}(T_1(r)) \]

Histogram equalization: Examples

- Given an input image \( f \) and a specific histogram \( p_2(r) \), transform the image so that it has the specified histogram

- How to perform histogram specification?
- Histogram equalization produces a (nearly) uniform output histogram
- Use histogram equalization as an intermediate step

Histogram Specification

- In cases where corresponding colors between images are not “consistent”, this mapping may fail:

Histogram Specification: Discussion

- Histogram matching produces the optimal **monotonic** mapping so that the resulting histogram will be as **close** as possible to the target histogram.
- This does not necessarily imply similar images.

Next week

- Spatial filtering