BBM 413
Fundamentals of Image Processing

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Color and Point Operations
Today

• Perception of color and light
• Color spaces
• Point operations
• Histogram processing
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• Color spaces
• Point operations
• Histogram processing

Acknowledgement: This part is mostly adapted from the slides prepared by Steve Marschner, James Hays, Ali Farhadi and Anat Levin
What is color?

• Color is the result of interaction between physical light in the environment and our visual system

• Color is a psychological property of our visual experiences when we look at objects and lights, not a physical property of those objects or lights (S. Palmer, Vision Science: Photons to Phenomenology)
#thedress

• What is the color of the dress?
  • blue and black
  • white and gold
  • blue and brown

• What #thedress tell about our color perception?

http://nyti.ms/186m3wE
#thedress

- Let's take averages

http://nyti.ms/186m3wE
#thedress

• The dress in the photograph

http://nyti.ms/186m3wE
#thefdres

- Consider the dress is in shadow.

- Your brain remove the blue cast, and perceive it as white and gold.

http://nyti.ms/186m3wE
#thedress

- The dress in the photograph

http://nyti.ms/186m3wE
#thedress

• Consider the dress is in bright light.

• Your brain perceive the dress as a darker blue and black

http://nyti.ms/186m3wE
#thedress

- Answer:

- The dress is actually blue and black.

http://nyti.ms/186m3wE
Brightness perception

http://web.mit.edu/persci/people/adelson/illusions_demos.html
Brightness perception

http://web.mit.edu/persci/people/adelson/illusions_demos.html
Brightness perception

http://web.mit.edu/persci/people/adelson/illusions_demos.html
Brightness perception
Land’s Experiment (1959)

- Cover all patches except a blue rectangle
- Make it look gray by changing illumination
- Uncover the other patches

Color Constancy

We filter out illumination variations

Slide credit: S. Narasimhan
Land’s Experiment (1959)

- Cover all patches except a blue rectangle
- Make it look gray by changing illumination
- Uncover the other patches

**Color Constancy**

We filter out illumination variations
Color Constancy in Gold Fish

In David Ingle's experiment, a goldfish has been trained to swim to a patch of a given color for a reward—a piece of liver. It swims to the green patch regardless of the exact setting of the three projectors' intensities. The behavior is strikingly similar to the perceptual result in humans.

http://neuro.med.harvard.edu/site/dh/b45.htm

Slide credit: S. Narasimhan
Reading Assignment #2

• Watch Beau Lotto’s TED talk on “Optical illusions show how we see” [link available on course webpage]
Image Brightness (Intensity)

- Monochromatic Light: \((\lambda = \lambda_i)\)

\[
b'(x, y) = r'(x, y) e'(x, y) \quad q(\lambda_i) = 1
\]

NOTE: The analysis can be applied to COLORED LIGHT using FILTERS

**irradiance:** the power of electromagnetic radiation per unit area (radiative flux) incident on a surface.

**reflectance:** the fraction of incident electromagnetic power that is reflected at an interface

Slide credit: S. Narasimhan
Recovering Lightness

- Image Intensity: \( b'(x, y) = r'(x, y) e'(x, y) \)

Can we recover \( e' \) and \( r' \) from \( b' \) ?

- Retinex theory, Land and McCann, 1971

- use constraints (or priors) on shading and reflectance
- employ additional information (multiple images, depth maps, etc.)

Slide credit: S. Narasimhan
Color and light

- **Color of light** arriving at camera depends on
  - Spectral reflectance of the surface light is leaving
  - Spectral radiance of light falling on that patch

- **Color perceived** depends on
  - Physics of light
  - Visual system receptors
  - Brain processing, environment

- Color is a phenomenon of human perception; it is **not** a universal property of light

Slide credit: K. Grauman, S. Marschner
Color

White light: composed of about equal energy in all wavelengths of the visible spectrum

Newton 1665

4.1 NEWTON’S SUMMARY DRAWING of his experiments with light. Using a point source of light and a prism, Newton separated sunlight into its fundamental components. By reconverging the rays, he also showed that the decomposition is reversible.

From Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Slide credit: B. Freeman, A. Torralba, K. Grauman
Electromagnetic spectrum

- Light is electromagnetic radiation
  - exists as oscillations of different frequency (or, wavelength)

![Electromagnetic Spectrum Diagram]

**Human Luminance Sensitivity Function**

**Slide credit: A. Efros**
The Physics of light

Any source of light can be completely described physically by its spectrum: the amount of energy emitted (per time unit) at each wavelength 400 - 700 nm.

Relative spectral power

Wavelength (nm.)

400 500 600 700

Slide credit: A. Efros
The Physics of light

Some examples of the spectra of light sources

A. Ruby Laser

B. Gallium Phosphide Crystal

C. Tungsten Lightbulb

D. Normal Daylight

Slide credit: A. Efros
The Physics of light

Some examples of the reflectance spectra of surfaces

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Red</th>
<th>Yellow</th>
<th>Blue</th>
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Image formation

• What determines the brightness of an image pixel?
Color mixing

Cartoon spectra for color names:

- Red
- Green
- Blue
- Cyan
- Magenta
- Yellow

400 nm - 500 nm - 600 nm - 700 nm

Credit: W. Freeman
Additive color mixing

Colors combine by adding color spectra

Light adds to black.
Examples of additive color systems

CRT phosphors
multiple projectors

Slide credit: K. Grauman

http://www.jegsworks.com
http://www.crtprojectors.co.uk/
Subtractive color mixing

Colors combine by *multiplying* color spectra.

Pigments *remove* color from incident light (white).

Credit: W. Freeman
Examples of subtractive color systems

- Printing on paper
- Crayons
- Photographic film
Interaction of light and surfaces

- Reflected color is the result of interaction of light source spectrum with surface reflectance
Reflection from colored surface

Slide credit: S. Marschner
The Eye

- **Iris** - colored annulus with radial muscles
- **Pupil** - the hole (aperture) whose size is controlled by the iris
- **Lens** - changes shape by using ciliary muscles (to focus on objects at different distances)
- **Retina** - photoreceptor cells

Slide credit: S. Seitz
The eye as a measurement device

- We can model the low-level behavior of the eye by thinking of it as a light-measuring machine
  - its optics are much like a camera
  - its detection mechanism is also much like a camera

- Light is measured by the photoreceptors in the retina
  - they respond to visible light
  - different types respond to different wavelengths

- The human eye is a camera!
Layers of the retina
Layers of the retina

- Cross-section of eye
- Cross section of retina
- Ganglion axons
- Ganglion cell layer
- Bipolar cell layer
- Receptor layer
- Pigmented epithelium


Slide credit: J. Hays
Receptors Density - Fovea

Slide credit: S. Ullman
Receptors Density - Fovea

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Slide credit: S. Ullman
Human Photoreceptors

3.4 THE SPATIAL MOSAIC OF THE HUMAN CONES. Cross sections of the human retina at the level of the inner segments showing (A) cones in the fovea, and (B) cones in the periphery. Note the size difference (scale bar = 10 μm), and that, as the separation between cones grows, the rod receptors fill in the spaces. (C) Cone density plotted as a function of distance from the center of the fovea for seven human retinas; cone density decreases with distance from the fovea. Source: Curcio et al., 1990.


Slide Credit: B. Freeman and A. Torralba
3.3 SPECTRAL SENSITIVITIES OF THE L-, M-, AND S-CONES in the human eye. The measurements are based on a light source at the cornea, so that the wavelength loss due to the cornea, lens, and other inert pigments of the eye plays a role in determining the sensitivity. Source: Stockman and MacLeod, 1993.


Slide Credit: B. Freeman and A. Torralba
Two types of light-sensitive receptors

**Cones**
- cone-shaped
- less sensitive
- operate in high light color vision

**Rods**
- rod-shaped
- highly sensitive
- operate at night gray-scale vision

Images by Shimon Ullman
Rods are responsible for intensity, cones for color perception

Rods and cones are non-uniformly distributed on the retina

- Fovea - Small region (1 or 2°) at the center of the visual field containing the highest density of cones (and no rods)
Physiology of Color Vision

Three kinds of cones:

- Ratio of L to M to S cones: approx. 10:5:1
- Almost no S cones in the center of the fovea

Slide credit: A. Efros
Color perception

Rods and cones act as filters on the spectrum
- To get the output of a filter, multiply its response curve by the spectrum, integrate over all wavelengths
  - Each cone yields one number

Q: How can we represent an entire spectrum with 3 numbers?
A: We can’t! Most of the information is lost.
  - As a result, two different spectra may appear indistinguishable

Slide credit: S. Seitz
Today

• Perception of color and light
• Color spaces
• Point operations
• Histogram processing

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Recall: Digital images

- **Sample** the 2D space on a regular grid
- **Quantize** each sample (round to nearest integer)

- Image thus represented as a matrix of integer values.

Slide credit: K. Grauman, S. Seitz
Color Images: Bayer Grid

• Estimate RGB at ‘G’ cells from neighboring values

http://www.cooldictionary.com/words/Bayer-filter.wikipedia

Slide credit: S. Seitz
Digital color images

Color images, RGB color space
Images

- Images represented as a matrix
- Suppose we have a NxM RGB image called “im”
  - \( \text{im}(1,1,1) \) = top-left pixel value in R-channel
  - \( \text{im}(y,x,b) \) = \( y \) pixels down, \( x \) pixels to right in the \( b^{th} \) channel
  - \( \text{im}(N,M,3) \) = bottom-right pixel in B-channel
Color spaces

• How can we represent color?
Color spaces: RGB

- Single wavelength primaries
- makes a particular monitor RGB standard
- Good for devices (e.g., phosphors for monitor), but not for perception

Slide credit: K. Grauman, S. Marschner
Some drawbacks

- Strongly correlated channels
- Non-perceptual
Color spaces: CIE XYZ

- Standardized by CIE (Commission Internationale de l’Eclairage, the standards organization for color science)
- Based on three “imaginary” primaries $X$, $Y$, and $Z$
  - imaginary = only realizable by spectra that are negative at some wavelengths
  - separates out luminance: $X$, $Z$ have zero luminance, so $Y$ tells you the luminance by itself

Slide credit: K. Grauman, S. Marschner
Color spaces: CIE XYZ

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\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix}
0.49 & 0.31 & 0.20 \\
0.17697 & 0.81240 & 0.01063 \\
0.00 & 0.01 & 0.99
\end{bmatrix} \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

Slide credit: K. Grauman, S. Marschner
Perceptually organized color spaces

• Artists often refer to colors as tints, shades, and tones of pure pigments
  – tint: mixture with white
  – shade: mixture with black
  – tones: mixture with black and white
  – gray: no color at all (aka. neutral)

• This seems intuitive
  – tints and shades are inherently related to the pure color
    • “same” color but lighter, darker, paler, etc.
Perceptual dimensions of color

- **Hue**
  - the “kind” of color, regardless of attributes
  - colorimetric correlate: dominant wavelength
  - artist’s correlate: the chosen pigment color

- **Saturation**
  - the “colorfulness”
  - colorimetric correlate: purity
  - artist’s correlate: fraction of paint from the colored tube

- **Lightness (or value)**
  - the overall amount of light
  - colorimetric correlate: luminance
  - artist’s correlate: tints are lighter, shades are darker
Color spaces: HSV

- **Hue, Saturation, Value**
- Nonlinear – reflects topology of colors by coding **hue** as an angle
- **Matlab**: `hsv2rgb`, `rgb2hsv`.

Image from mathworks.com

Slide credit: K. Grauman
Color spaces: HSV

- **Hue, Saturation, Value**
  
- Nonlinear – reflects topology of colors by coding **hue** as an angle
  
- **Matlab**: hsv2rgb, rgb2hsv.

\[
H = \begin{cases} 
\frac{G' - B'}{MAX - MIN} / 6, & \text{if } R' = MAX \\
2 + \frac{B' - R'}{MAX - MIN} / 6, & \text{if } G' = MAX \\
4 + \frac{R' - G'}{MAX - MIN} / 6, & \text{if } B' = MAX 
\end{cases} 
\]

\[
S = \frac{MAX - MIN}{MAX} \\
V = MAX
\]
Color spaces: HSV

Intuitive color space

Slide credit: D. Hoiem
Color spaces: YCbCr

Fast to compute, good for compression, used by TV

- Y = 0
- Y = 0.5
- Y = 1

Slide credit: D. Hoiem
Color spaces: YCbCr

Fast to compute, good for compression, used by TV

\[
\begin{bmatrix}
Y' \\
C_b \\
C_r
\end{bmatrix} = \begin{bmatrix}
0.299 & 0.587 & 0.114 \\
-0.168736 & -0.331264 & 0.5 \\
0.5 & -0.418688 & -0.081312
\end{bmatrix} \begin{bmatrix}
R' \\
G' \\
B'
\end{bmatrix} + \begin{bmatrix}
0 \\
128 \\
128
\end{bmatrix}
\]

Slide credit: D. Hoiem
Distances in color space

• Are distances between points in a color space perceptually meaningful?
Distances in color space

- Not necessarily: CIE XYZ is not a uniform color space, so magnitude of differences in coordinates are poor indicator of color “distance”.

McAdam ellipses:
Just noticeable differences in color
Uniform color spaces

- Attempt to correct this limitation by remapping color space so that just-noticeable differences are contained by circles \( \Rightarrow \) distances more perceptually meaningful.

- Examples:
  - CIE \( u'v' \)
  - CIE Lab

Slide credit: K. Grauman
Perceptually uniform spaces

- Two major spaces standardized by CIE
  - designed so that equal differences in coordinates produce equally visible differences in color
  - by remapping color space so that just-noticeable differences are contained by circles → distances more perceptually meaningful.
  - LUV: earlier, simpler space; \( L^*, u^*, v^* \)
  - LAB: more complex but more uniform: \( L^*, a^*, b^* \)
  - both separate luminance from chromaticity
  - including a gamma-like nonlinear component is important

Slide credit: K. Grauman, S. Marschner
Color spaces: L*a*b*

"Perceptually uniform"* color space

Slide credit: D. Hoiem
Color spaces: L*a*b*

“Perceptually uniform”* color space

\[ L^* = 116f \left( \frac{Y}{Y_n} \right) \]

\[ f(t) = \begin{cases} 
  t^{1/3} & t > \delta^3 \\
  t/(3\delta^2) + 2\delta/3 & \text{else,} 
\end{cases} \]

\[ a^* = 500 \left[ f \left( \frac{X}{X_n} \right) - f \left( \frac{Y}{Y_n} \right) \right] \]

\[ b^* = 200 \left[ f \left( \frac{Y}{Y_n} \right) - f \left( \frac{Z}{Z_n} \right) \right] \]

\((X_n, Y_n, Z_n)\): measured white point

\(L\) (a=0,b=0)

\(a\) (L-65,b=0)

\(b\) (L-65,a=0)

Slide credit: D. Hoiem
Most information in intensity

Only intensity shown – constant color

Slide credit: D. Hoiem
Most information in intensity

Original image

Slide credit: D. Hoiem
Today

- Perception of color and light
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- Point operations
- Histogram processing
Digital images

- **Sample** the 2D space on a regular grid
- **Quantize** each sample (round to nearest integer)

- Image thus represented as a matrix of integer values.

\[
\begin{array}{cccccccc}
62 & 79 & 23 & 119 & 120 & 105 & 4 & 0 \\
10 & 10 & 9 & 82 & 12 & 78 & 34 & 0 \\
10 & 58 & 197 & 46 & 46 & 0 & 0 & 48 \\
176 & 135 & 5 & 168 & 191 & 68 & 0 & 49 \\
2 & 1 & 1 & 29 & 26 & 37 & 0 & 77 \\
0 & 89 & 144 & 147 & 187 & 102 & 62 & 208 \\
255 & 252 & 0 & 166 & 123 & 62 & 0 & 31 \\
166 & 63 & 127 & 17 & 1 & 0 & 99 & 30 \\
\end{array}
\]

Slide credit: K. Grauman, S. Seitz
Image Transformations

- \( g(x,y) = T[f(x,y)] \)

\( g(x,y) \): output image  \( f(x,y) \): input image  \( M \): transformation function

1. Point operations: operations on single pixels
2. Spatial filtering: operations considering pixel neighborhoods
3. Global methods: operations considering whole image
Image Transformations

- \( g(x,y) = T[f(x,y)] \)

\( g(x,y) \): output image  \( f(x,y) \): input image  \( M \): transformation function

1. **Point operations:** operations on single pixels
2. **Spatial filtering:** operations considering pixel neighborhoods
3. **Global methods:** operations considering whole image

\[ g(x, y) = M(f(x, y)) \]
Image Transformations

- \( g(x,y) = M[f(x,y)] \)

\( g(x,y) \): output image  \( f(x,y) \): input image  \( M \): transformation function

1. Point operations: operations on single pixels
2. Spatial filtering: operations considering pixel neighborhoods
3. Global methods: operations considering whole image

\[
g(x, y) = M(\{ f(i, j) | (i, j) \in N(x, y) \})
\]
Point operations

- Smallest possible neighborhood is of size 1x1
- Process each point independently of the others
- Output image $g$ depends only on the value of $f$ at a single point $(x,y)$
- Map each pixel’s value to a new value
- Transformation function $T$ remaps the sample’s value:
  \[ s = T(r) \]

where
- $r$ is the value at the point in question
- $s$ is the new value in the processed result
- $T$ is a intensity transformation function
Point operations

• Is mapping one color space to another (e.g. RGB2HSV) a point operation?

• Is image arithmetic a point operation?

• Is performing geometric transformations a point operation?
  – Rotation
  – Translation
  – Scale change
  – etc.
Sample intensity transformation functions

- Image negatives
- Log transformations
  - Compresses the dynamic range of images
- Power-law transformations
  - Gamma correction
Point Processing Examples

produces an image of higher contrast than the original by darkening the intensity levels below $k$ and brightening intensities above $k$.

produces a binary (two-intensity level) image.
Image Mean

\[ I_{av} = \frac{\sum_{i} \sum_{j} I(i, j)}{\sum_{i} \sum_{j} 1} \]

\[ I_{NEW}(x,y) = I(x,y) - b \]
Image Mean

Changing the image mean
Image Negative

\[ M(v) = 255 - v \]
Dynamic range

- Dynamic range $R_d = \frac{l_{\text{max}}}{l_{\text{min}}}$, or $(l_{\text{max}} + k) / (l_{\text{min}} + k)$
  - determines the degree of image contrast that can be achieved
  - a major factor in image quality

- Ballpark values
  - Desktop display in typical conditions: 20:1
  - Photographic print: 30:1
  - High dynamic range display: 10,000:1
Point Operations: Contrast stretching and Thresholding

- **Contrast stretching:** produces an image of higher contrast than the original

- **Thresholding:** produces a binary (two-intensity level) image
Point Operations:
Contrast stretching and Thresholding

- **Contrast stretching:** produces an image of higher contrast than the original

- **Thresholding:** produces a binary (two-intensity level) image
Histogram

- Histogram: a discrete function $h(r)$ which counts the number of pixels in the image having intensity $r$
- If $h(r)$ is normalized, it measures the probability of occurrence of intensity level $r$ in an image
Point Operations

- What can you say about the image having the following histogram?

A low contrast image

- How we can process the image so that it has a better visual quality?
Point Operations

• How we can process the image so that it has a better visual quality?

• Answer is contrast stretching!
Point Operations

• Let us devise an appropriate point operation.

• Shift all values so that the observable pixel range starts at 0.
Point Operations

• Let us devise an appropriate point operation.

• Now, scale everything in the range 0-100 to 0-255.
Point Operations

- Let us devise an appropriate point operation.

- What is the corresponding transformation function?
  
  \[ T(r) = 2.55 \times (r-100) \]
Point Operations: Intensity-level Slicing

- highlights a certain range of intensities
Point Operations: Intensity-level Slicing

- highlights a certain range of intensities

**FIGURE 3.11** (a) This transformation highlights intensity range \([A, B]\) and reduces all other intensities to a lower level. (b) This transformation highlights range \([A, B]\) and preserves all other intensity levels.
Intensity encoding in images

- Recall that the pixel values determine how bright that pixel is.
- Bigger numbers are (usually) brighter
- *Transfer function*: function that maps input pixel value to luminance of displayed image
  \[ I = f(n) \quad f : [0, N] \rightarrow [I_{\text{min}}, I_{\text{max}}] \]
- What determines this function?
  - physical constraints of device or medium
  - desired visual characteristics

adapted from: S. Marschner
What this projector does?

- Something like this:

\[ I(n) \]

\[ \begin{align*}
  n &= 64 \\
  n &= 128 \\
  n &= 192 \\
  I = 0.25 & \quad I = 0.5 & \quad I = 0.75
\end{align*} \]

adapted from: S. Marschner
Constraints on transfer function

• Maximum displayable intensity, $I_{\text{max}}$
  – how much power can be channeled into a pixel?
    • LCD: backlight intensity, transmission efficiency ($<10\%$)
    • projector: lamp power, efficiency of imager and optics

• Minimum displayable intensity, $I_{\text{min}}$
  – light emitted by the display in its “off” state
    • e.g. stray electron flux in CRT, polarizer quality in LCD

• Viewing flare, $k$: light reflected by the display
  – very important factor determining image contrast in practice
    • 5% of $I_{\text{max}}$ is typical in a normal office environment [sRGB spec]
    • much effort to make very black CRT and LCD screens
    • all-black decor in movie theaters
Transfer function shape

- Desirable property: the change from one pixel value to the next highest pixel value should not produce a visible contrast
  - otherwise smooth areas of images will show visible bands

- What contrasts are visible?
  - rule of thumb: under good conditions we can notice a 2% change in intensity
  - therefore we generally need smaller quantization steps in the darker tones than in the lighter tones
  - most efficient quantization is logarithmic
How many levels are needed?

- Depends on dynamic range
  - 2% steps are most efficient:
    \[
    0 \mapsto I_{\min}; 
    1 \mapsto 1.02I_{\min}; 
    2 \mapsto (1.02)^2I_{\min}; \ldots
    \]
  - \[\log 1.02 \text{ is about } 1/120, \text{ so } 120 \text{ steps per decade of dynamic range}\]
    - 240 for desktop display
    - 360 to print to film
    - 480 to drive HDR display

- If we want to use linear quantization (equal steps)
  - one step must be < 2% (1/50) of \(I_{\min}\)
  - need to get from \(\sim 0\) to \(I_{\min} \cdot R_{d}\) so need about 50 \(R_{d}\) levels
    - 1500 for a print; 5000 for desktop display; 500,000 for HDR display

- Moral: 8 bits is just barely enough for low-end applications
  - but only if we are careful about quantization

Slide credit: S. Marschner
Intensity quantization in practice

• Option 1: linear quantization \( I(n) = (n/N) I_{\text{max}} \)
  – pro: simple, convenient, amenable to arithmetic
  – con: requires more steps (wastes memory)
  – need 12 bits for any useful purpose; more than 16 for HDR

• Option 2: power-law quantization \( I(n) = (n/N)^\gamma I_{\text{max}} \)
  – pro: fairly simple, approximates ideal exponential quantization
  – con: need to linearize before doing pixel arithmetic
  – con: need to agree on exponent
  – 8 bits are OK for many applications; 12 for more critical ones

• Option 2: floating-point quantization \( I(x) = (x/w) I_{\text{max}} \)
  – pro: close to exponential; no parameters; amenable to arithmetic
  – con: definitely takes more than 8 bits
  – 16-bit “half precision” format is becoming popular

Slide credit: S. Marschner
Why gamma?

- Power-law quantization, or *gamma correction* is most popular
- Original reason: CRTs are like that
  - intensity on screen is proportional to (roughly) voltage\(^2\)
- Continuing reason: inertia + memory savings
  - inertia: gamma correction is close enough to logarithmic that there’s no sense in changing
  - memory: gamma correction makes 8 bits per pixel an acceptable option
Gamma quantization

\[ I(n) = (n/N)^\gamma I_{\text{max}} \]

- Close enough to ideal perceptually uniform exponential

Slide credit: S. Marschner
Gamma correction

- Sometimes (often, in graphics) we have computed intensities $a$ that we want to display linearly

- In the case of an ideal monitor with zero black level,
  $$I(n) = (n/N)^\gamma$$
  (where $N = 2^n - 1$ in $n$ bits). Solving for $n$:
  $$n = N a^{\frac{1}{\gamma}}$$

- This is the “gamma correction” recipe that has to be applied when computed values are converted to 8 bits for output
  - failing to do this (implicitly assuming gamma = 1) results in dark, oversaturated images
Gamma correction

corrected for $\gamma$ lower than display

OK

corrected for $\gamma$ higher than display

Slide credit: S. Marschner
Instagram Filters

• How do they make those Instagram filters?

“It's really a combination of a bunch of different methods. In some cases we draw on top of images, in others we do pixel math. It really depends on the effect we're going for.” --- Kevin Systrom, co-founder of Instagram

Source: C. Dyer
Example Instagram Steps

1. Perform an independent RGB color point transformation on the original image to increase contrast or make a color cast.
Example Instagram Steps

2. Overlay a circle background image to create a vignette effect
Example Instagram Steps

3. Overlay a background image as decorative grain

Source: C. Dyer
Example Instagram Steps

4. Add a border or frame
Result

Javascript library for creating Instagram-like effects, see: http://alexmic.net/filtrr/

Source: C. Dyer
Today

• Perception of color and light
• Color spaces
• Point operations
• Histogram processing
Histogram

- Histogram: a discrete function $h(r)$ which counts the number of pixels in the image having intensity $r$
- If $h(r)$ is normalized, it measures the probability of occurrence of intensity level $r$ in an image

What histograms say about images?

A descriptor for visual information

What they don’t?
- No spatial information
Histogram

Normalized Histogram

Cumulative Histogram
Images and histograms

- How do histograms change when
  - we adjust brightness? shifts the histogram horizontally
  - we adjust contrast? stretches or shrinks the histogram horizontally
Image Representations: Histograms

Global histogram

- Represent distribution of features
  - Color, texture, depth, …
Image Representations: Histograms

Histogram: Probability or count of data in each bin

Joint histogram
- Requires lots of data
- Loss of resolution to avoid empty bins

Marginal histogram
- Requires independent features
- More data/bin than joint histogram

Image credit: D. Kauchak
Histograms: Implementation issues

- Quantization
  - Grids: fast but applicable only with few dimensions

- Matching
  - Histogram intersection or Euclidean may be faster
  - Chi-squared often works better
  - Earth mover’s distance is good for when nearby bins represent similar values

Slide credit: J. Hays
What kind of things do we compute histograms of?

- Color
  - L*a*b* color space
  - HSV color space
- Texture (filter banks over regions – later on)

Slide credit: J. Hays
What kind of things do we compute histograms of?

- Histograms of oriented gradients (later on)

SIFT – Lowe IJCV 2004
Examples

- The image histogram does not fully represent the image

Slide credit: Y. Hel-Or
Examples

Original image

Decreasing contrast

Increasing average
Image Statistics

- The image mean: \[ E\{I\} = \frac{1}{N} \sum_{i,j} I(i, j) = \frac{1}{N} \sum_k k H(k) = \sum_k k P(k) \]

- Generally: \[ E\{g(k)\} = \sum_k g(k) P(k) \]

- The image s.t.d.: \[ \sigma(I) = \sqrt{E\{(I - E\{I\})^2\}} = \sqrt{E(I^2) - E^2(I)} \]

where \( E\{I^2\} = \sum_k k^2 P(k) \)
Image Entropy

\[ \text{Entropy}(I) = - \sum_k P(k) \log P(k) \]

- The image entropy specifies the uncertainty in the image values.
- Measures the averaged amount of information required to encode the image values.

Slide credit: Y. Hel-Or
Image Entropy

• An infrequent event provides more information than a frequent event

• Entropy is a measure of histogram dispersion

entropy=7.4635

entropy=0
Adaptive Histogram

• In many cases histograms are needed for local areas in an image

• Examples:
  – Pattern detection
  – adaptive enhancement
  – adaptive thresholding
  – tracking

Slide credit: Y. Hel-Or
Histogram Usage

- Digitizing parameters
- Measuring image properties:
  - Average
  - Variance
  - Entropy
  - Contrast
  - Area (for a given gray-level range)
- Threshold selection
- Image distance

- Image Enhancement
  - Histogram equalization
  - Histogram stretching
  - Histogram matching
Example: Auto-Focus

- In some optical equipment (e.g. slide projectors) inappropriate lens position creates a blurred (“out-of-focus”) image
- We would like to automatically adjust the lens
- How can we measure the amount of blurring?
Example: Auto-Focus

- Image mean is not affected by blurring
- Image s.t.d. (entropy) is decreased by blurring
- **Algorithm**: Adjust lens according the changes in the histogram s.t.d.
Recall: Thresholding

\[ k_{\text{new}} \]

\[ F(k) \]

\[ k_{\text{old}} \]

Threshold value

Slide credit: Y. Hel-Or
Threshold Selection

Original Image

Binary Image

Threshold too low

Threshold too high
Segmentation using Thresholding

Original

Histogram

Threshold = 50

Threshold = 75

Slide credit: Y. Hel-Or
Segmentation using Thresholding

Original

Threshold = 21

Histogram

Slide credit: Y. Hel-Or
Adaptive Thresholding

- Thresholding is space variant.
- How can we choose the local threshold values?
**Histogram based image distance**

- **Problem**: Given two images A and B whose (normalized) histogram are $P_A$ and $P_B$, define the distance $D(A,B)$ between the images.

- **Example Usage:**
  - Tracking
  - Image retrieval
  - Registration
  - Detection
  - Many more ...

Porikli 05
Option 1: Minkowski Distance

\[ D_p(A, B) = \left[ \sum_k |P_A(k) - P_B(k)|^p \right]^{1/p} \]

- **Problem**: distance may not reflect the perceived dissimilarity:
Option 2: Kullback-Leibler (KL) Distance

\[ D_{KL}(A \parallel B) = - \sum_k P_A(k) \log \frac{P_A(k)}{P_B(k)} \]

- Measures the amount of added information needed to encode image A based on the histogram of image B.
- Non-symmetric: \( D_{KL}(A,B) \neq D_{KL}(B,A) \)
- Suffers from the same drawback of the Minkowski distance.
Option 3: The Earth Mover Distance (EMD)

• Suggested by Rubner & Tomasi 98

• Defines as the minimum amount of “work” needed to transform histogram $H_A$ towards $H_B$

• The term $d_{ij}$ defines the “ground distance” between gray-levels $i$ and $j$.

• The term $F = \{f_{ij}\}$ is an admissible flow from $H_A(i)$ to $H_B(j)$

Slide credit: Y. Hel-Or
Option 3: The Earth Mover Distance (EMD)
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Option 3: The Earth Mover Distance (EMD)

(amount moved)

Slide credit: P. Barnum
Option 3: The Earth Mover Distance (EMD)

work = (amount moved) \times (distance moved)
Option 3: The Earth Mover Distance (EMD)

\[ D_{EMD}(A, B) = \min_F \sum_i \sum_j f_{ij} \cdot d_{ij} \]

\[ \text{s.t. } f_{ij} \geq 0 ; \quad P_B(k) = \sum_i f_{ik} ; \quad P_A(k) \geq \sum_i f_{ki} \]

- Constraints:
  - Move earth only from A to B
  - After move \( P_A \) will be equal to \( P_B \)
  - Cannot send more “earth” than there is

- Can be solved using Linear Programming
- Can be applied in high dim. histograms (color).
Special case: EMD in 1D

- Define $C_A$ and $C_B$ as the cumulative histograms of image A and B respectively:

$$D_{EMD}(A, B) = \sum_{k} |C_A(k) - C_B(k)|$$
Histogram equalization

• A good quality image has a nearly uniform distribution of intensity levels. Why?

• Every intensity level is equally likely to occur in an image

• *Histogram equalization:* Transform an image so that it has a uniform distribution
  – create a lookup table defining the transformation
Histogram as a probability density function

- Recall that a normalized histogram measures the probability of occurrence of an intensity level $r$ in an image.
- We can normalize a histogram by dividing the intensity counts by the area.

$$p(r) = \frac{h(r)}{\text{Area}}$$
Histogram equalization: Continuous domain

- Define a transformation function of the form

\[ s = T(r) = (L - 1) \int_0^r p(w) dw \]

where
- \( r \) is the input intensity level
- \( s \) is the output intensity level
- \( p \) is the normalized histogram of the input signal
- \( L \) is the desired number of intensity levels

(Continuous) output signal has a uniform distribution!
Histogram equalization: 
Discrete domain

- Define the following transformation function for an MxN image

\[ s_k = T(r_k) = (L - 1) \sum_{j=0}^{k} \frac{n_j}{MN} = \frac{(L - 1)}{MN} \sum_{j=0}^{k} n_j \]

for \( k = 0, \ldots, L - 1 \)

where
- \( r_k \) is the input intensity level
- \( s_k \) is the output intensity level
- \( n_j \) is the number of pixels having intensity value \( j \) in the input image
- \( L \) is the number of intensity levels

(Discrete) output signal has a nearly uniform distribution!
Histogram equalization

- Define: $C_b(v) = v \times \frac{(# \text{ pixels})}{# \text{ grayValues}}$

- Assign: $v_b = C_b^{-1}(C_a(v_a)) = M(v_a)$
Histogram equalization

Original

Goal

$P_{\text{old}}$

$P_{\text{new}}$

Slide credit: Y. Hel-Or
Histogram equalization

Original

Goal

Result

0 1 2 3 4 5 6 7 8 9 10 11
0 0 0 1 1 1 3 5 8 9 11

old
new

Slide credit: Y. Hel-Or
Histogram equalization examples

Slide credit: Y. Hel-Or
Histogram equalization examples

Original

Equalized
Histogram equalization examples

Original

Equalized

Slide credit: Y. Hel-Or
Histogram equalization examples

Slide credit: C. Dyer
Histogram equalization examples

(1) (2) (3) (4)
Histogram Specification

• Given an input image $f$ and a specific histogram $p_2(r)$, transform the image so that it has the specified histogram.

• How to perform histogram specification?
  • Histogram equalization produces a (nearly) uniform output histogram.
  • Use histogram equalization as an intermediate step.

Image credit: Y. Hel-Or
Histogram Specification

1. Equalize the histogram of the input image

\[ T_1(r) = (L - l) \int_0^r p_1(w) \, dw \]

2. Histogram equalize the desired output histogram

\[ T_2(r) = (L - l) \int_0^r p_2(w) \, dw \]

3. Histogram specification can be carried out by the following point operation:

\[ s = T(r) = T_2^{-1}(T_1(r)) \]
Histogram Specification

- In cases where corresponding colors between images are not “consistent”, this mapping may fail:


Slide credit: Y. Hel-Or
Histogram Specification: Discussion

- Histogram matching produces the optimal **monotonic** mapping so that the resulting histogram will be as **close** as possible to the target histogram.

- This does not necessarily imply similar images.
Next week

• Spatial filtering