

# BBM444

## FUNDAMENTALS OF COMPUTATIONAL PHOTOGRAPHY

Lecture #05 – Image Filtering

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HACETTEPE  
UNIVERSITY  
COMPUTER  
VISION LAB

# Today's Lecture

- Gaussian filtering
- Sharpening
- Bilateral filter
- Non-local means filter
- RegCov smoothing
- Rolling guidance filter

**Disclaimer:** The material and slides for this lecture were borrowed from

- Ioannis Gkioulekas' 15-463/15-663/15-862 "Computational Photography" class
- Wojciech Jarosz's CS 89.15/189.5 "Computational Aspects of Digital Photography" class
- Steve Marschner's CS6640 "Computational Photography" class
- Jiaya Jia's slides on Rolling guidance filter

# Filtering

- The name “filter” is borrowed from frequency domain processing
- Accept or reject certain frequency components
- Fourier (1807):  
Periodic functions could be represented as a weighted sum of sines and cosines

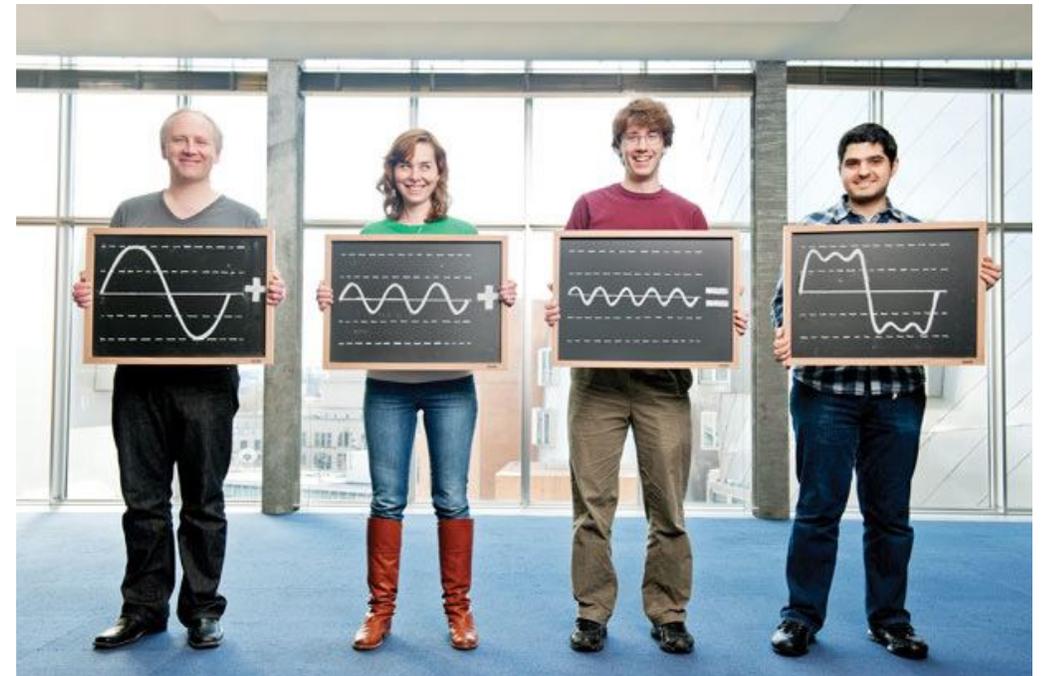
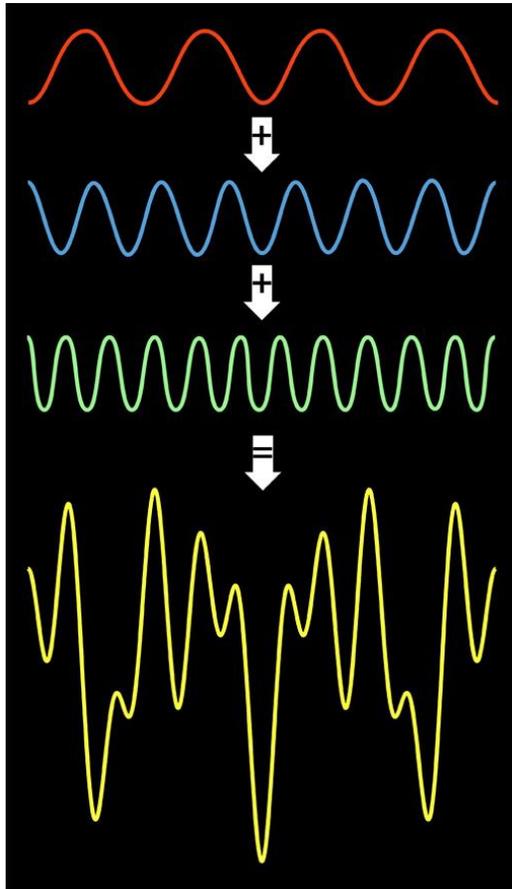


Image courtesy of Technology Review

# Signals

- A signal is composed of low and high frequency components



low frequency components: smooth / piecewise smooth

Neighboring pixels have similar brightness values

You're within a region

high frequency components: oscillatory

Neighboring pixels have different brightness values

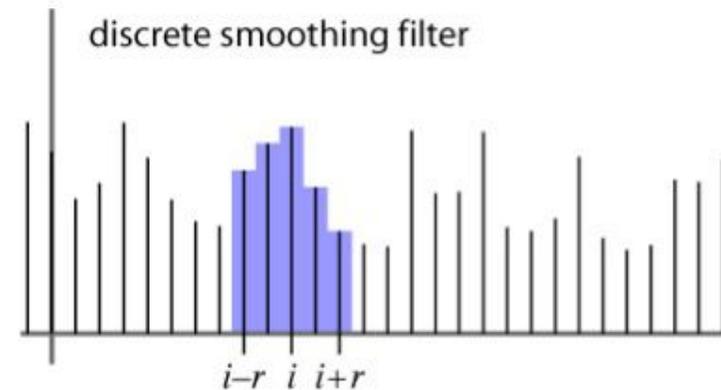
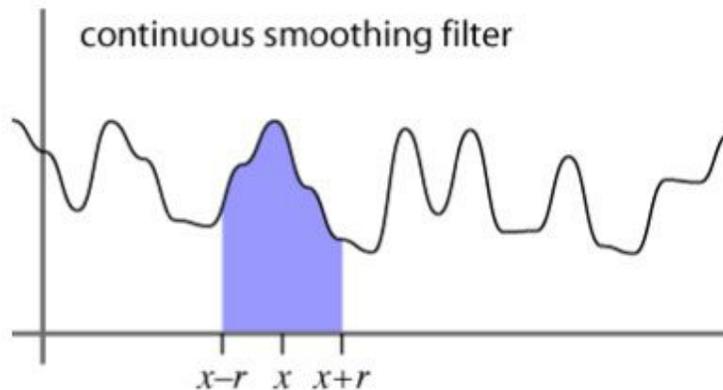
You're either at the edges or noise points

# Image Filtering

- Idea: Use the information coming from the neighboring pixels for processing
- Design a transformation function of the local neighborhood at each pixel in the image
  - Function specified by a “filter” or mask saying how to combine values from neighbors.
- Various uses of filtering:
  - Enhance an image (denoise, resize, etc)
  - Extract information (texture, edges, etc)
  - Detect patterns (template matching)

# Filtering

- Processing done on a function
  - can be executed in continuous form (e.g. analog circuit)
  - but can also be executed using sampled representation
- Simple example: smoothing by averaging
- Can be modeled mathematically by convolution



# Discrete convolution

- Simple averaging:

$$b_{\text{smooth}}[i] = \frac{1}{2r + 1} \sum_{j=i-r}^{i+r} b[j]$$

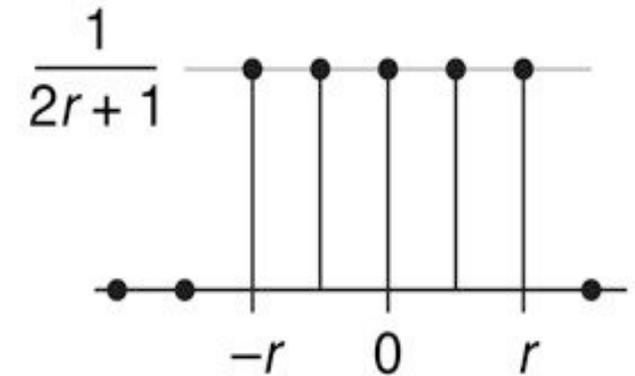
- every sample gets the same weight
- Convolution: same idea but with weighted average

$$(a \star b)[i] = \sum_j a[j]b[i - j]$$

- each sample gets its own weight (normally zero far away)
- This is all convolution is: it is a moving weighted average

# Filters

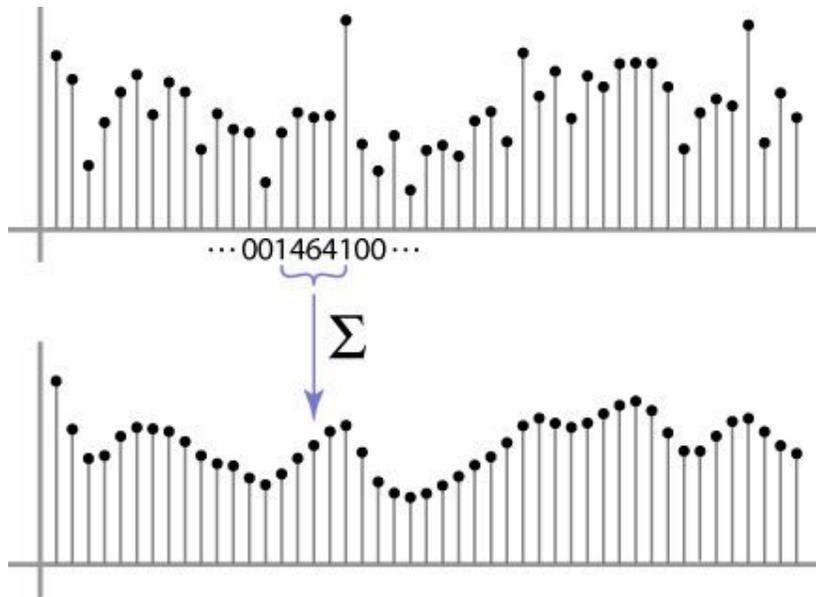
- Sequence of weights  $a[j]$  is called a filter
- Filter is nonzero over its region of support
  - usually centered on zero: support radius  $r$
- Filter is normalized so that it sums to 1.0
  - this makes for a weighted average, not just any old weighted sum
- Most filters are symmetric about 0
  - since for images we usually want to treat left and right the same



a box filter

# Convolution and filtering

- Convolution applies with any sequence of weights
- Example: bell curve (gaussian-like) [..., 1, 4, 6, 4, 1, ...]/16



# Discrete filtering in 2D

- Same equation, one more index

$$(a \star b)[i, j] = \sum_{i', j'} a[i', j'] b[i - i', j - j']$$

- now the filter is a rectangle you slide around over a grid of numbers
- Usefulness of associativity
- often apply several filters one after another:  $((a * b_1) * b_2) * b_3$
- this is equivalent to applying one filter:  $a * (b_1 * b_2 * b_3)$

# Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0									

# Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10							

# Moving Average In 2D

$$F[x, y]$$

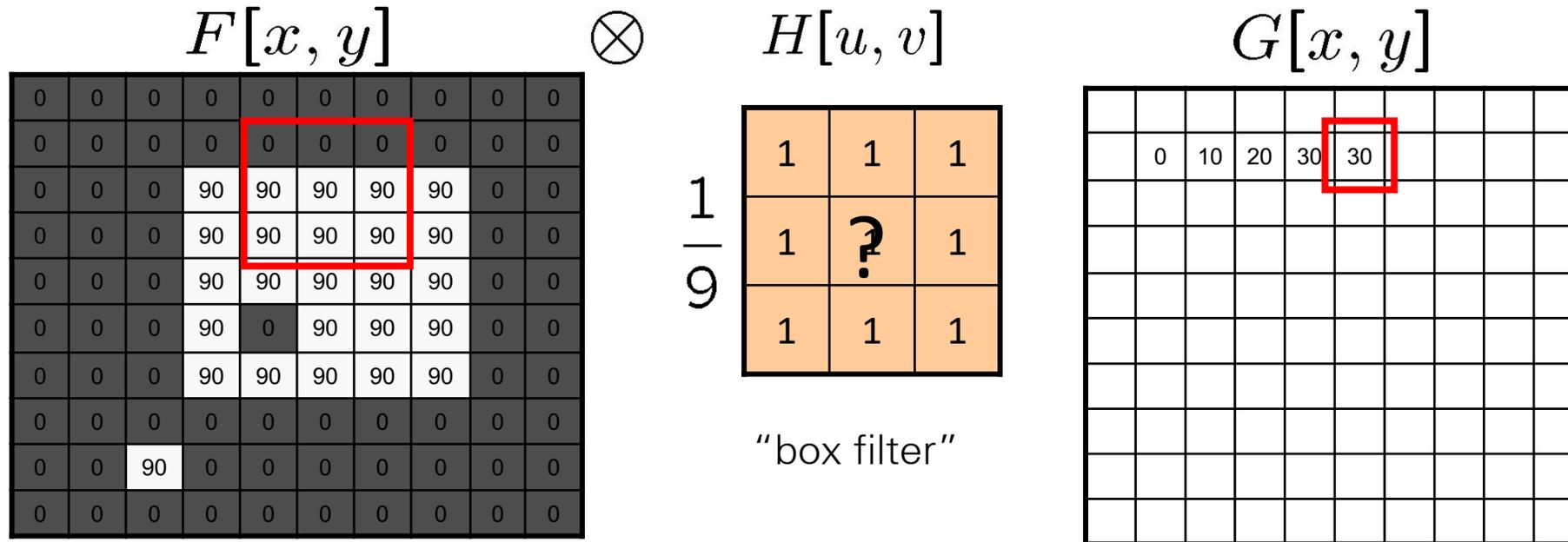
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G[x, y]$$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

# Averaging Filter

- What values belong in the kernel  $H$  for the moving average example?



$$G = H \otimes F$$

# Smoothing by averaging



depicts box filter:  
white = high value, black = low value



original

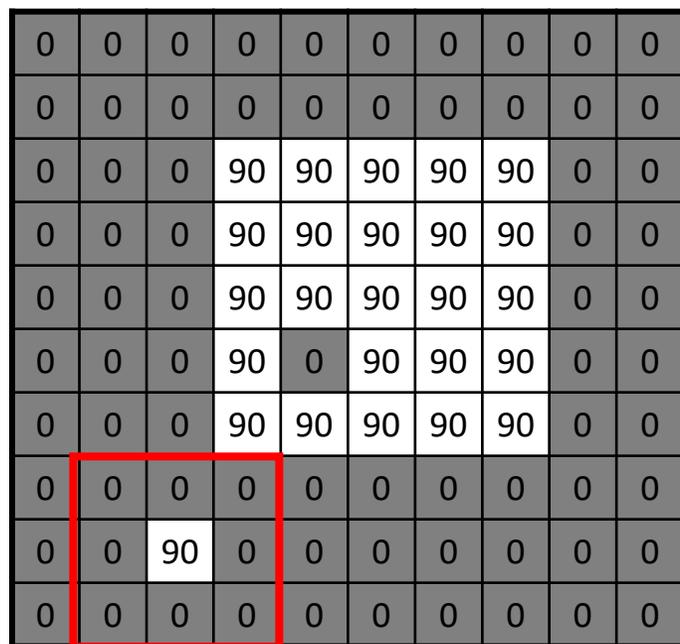


filtered

# Gaussian Filtering

# Gaussian Filter

- What if we want nearest neighboring pixels to have the most influence on the output?



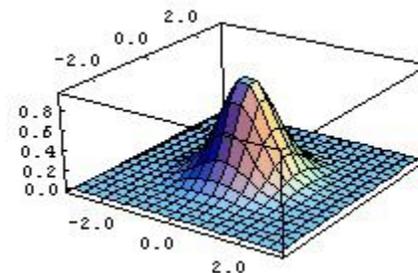
$F[x, y]$

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$H[u, v]$

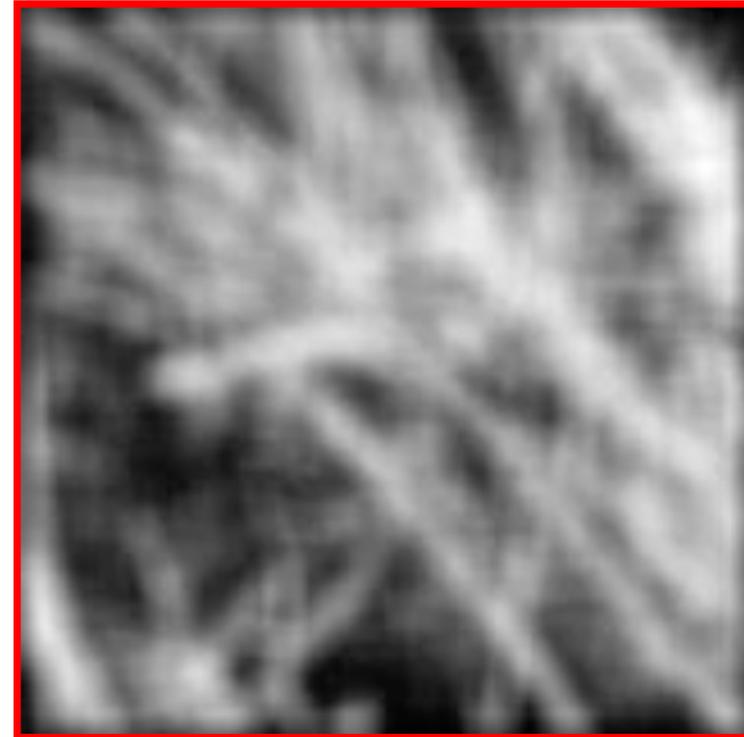
This kernel is an approximation of a 2d Gaussian function:

$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



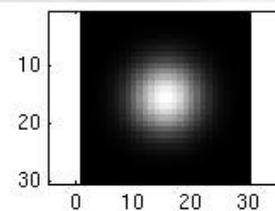
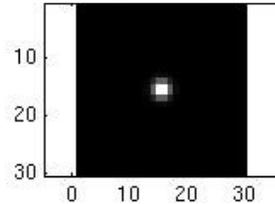
- Removes high-frequency components from the image ("low-pass filter").

# Smoothing with a Gaussian

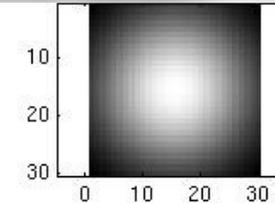


# Smoothing with a Gaussian

Parameter  $\sigma$  is the "scale" / "width" / "spread" of the Gaussian kernel, and controls the amount of smoothing.



...



# Strategy for Smoothing Images

- Images are not smooth because adjacent pixels are different.
- Smoothing = making adjacent pixels look more similar.
- Smoothing strategy  
    pixel  $\sim$  average of its neighbors

# Sharpening

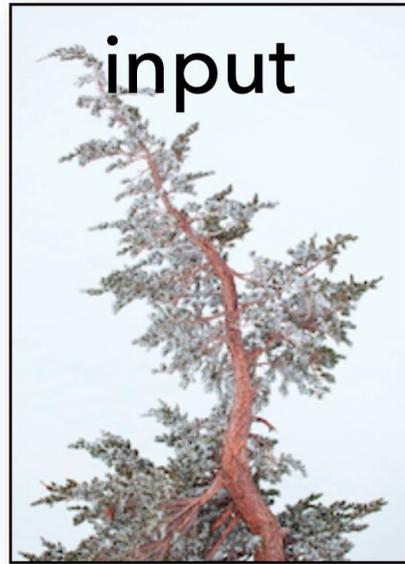
# How can we sharpen?

- Blurring was easy
- Sharpening is not as obvious

# How can we sharpen?

- Blurring was easy
- Sharpening is not as obvious
- Idea: amplify the stuff not in the blurry image
- $\text{output} = \text{input} + k * (\text{input} - \text{blur}(\text{input}))$

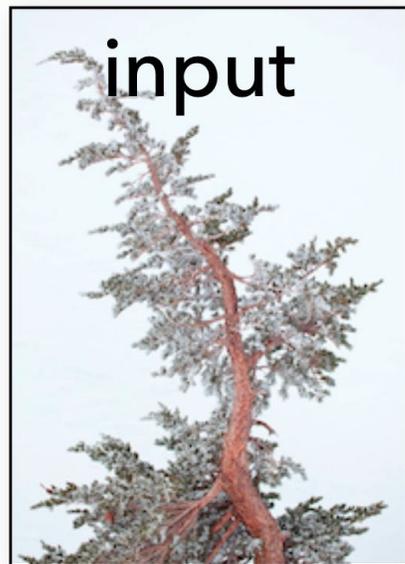
# Sharpening



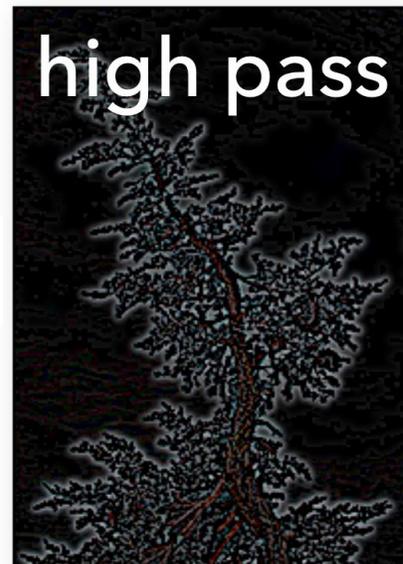
-



=



+k\*



=



# Sharpening: kernel view

- Recall

$$f' = f + k * (f - f \otimes g)$$

$f$  is the input

$f'$  is a sharpened image

$g$  is a blurring kernel

$k$  is a scalar controlling the strength of sharpening

# Sharpening: kernel view

- Recall

$$f' = f + k * (f - f \otimes g)$$

- Denote  $\delta$  the Dirac kernel (pure impulse)

$$f = f \otimes \delta$$

# Sharpening: kernel view

- Recall

$$f' = f + k * (f - f \otimes g)$$

$$f' = f \otimes \delta + k * (f \otimes \delta - f \otimes g)$$

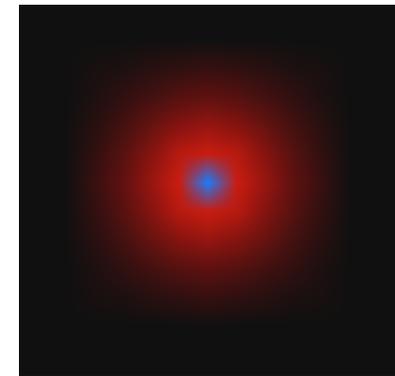
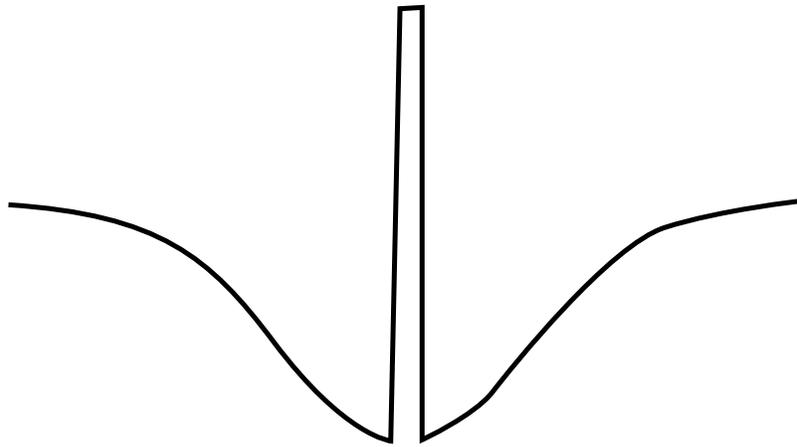
$$f' = f \otimes ((k + 1)\delta - g)$$

- Sharpening is also a convolution

# Sharpening kernel

- Note: many other sharpening kernels exist (just like we saw multiple blurring kernels)
- Amplify the difference between a pixel and its neighbors

$$f' = f \otimes ((k + 1)\delta - g)$$



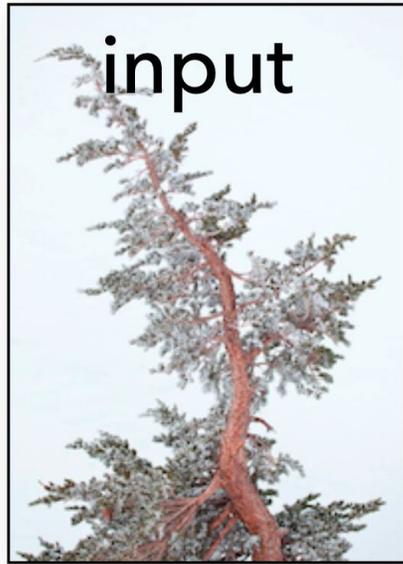
blue: positive  
red: negative

# Alternate interpretation

- $out = input + k * (input - blur(input))$
- $out = (1 + k) * input - k * blur(input)$
- $out = lerp(blur(input), input, 1+k)$

linearly extrapolate from the blurred image “past” the original input image

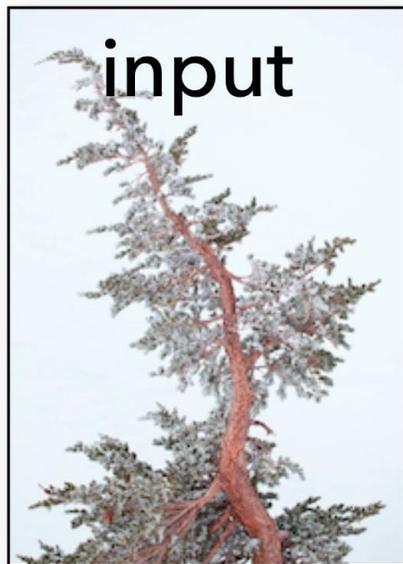
# Sharpening



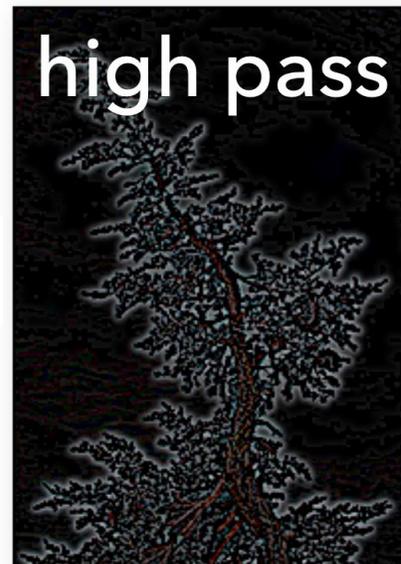
-



=



+k\*

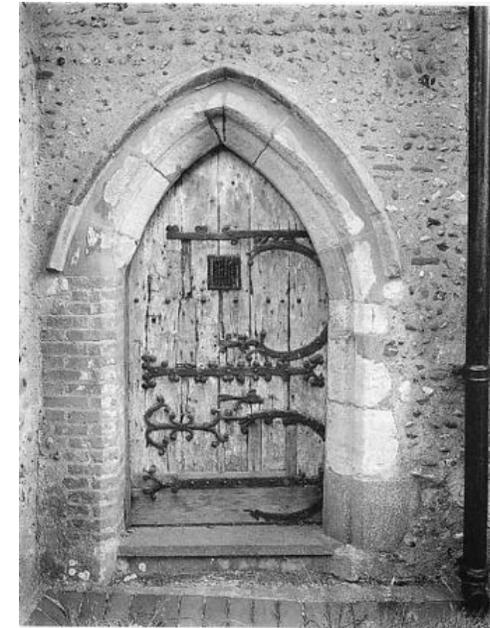
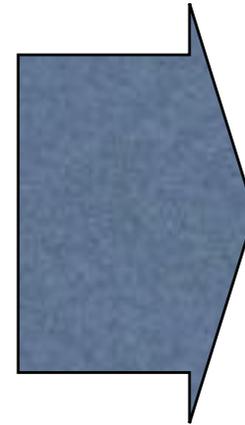
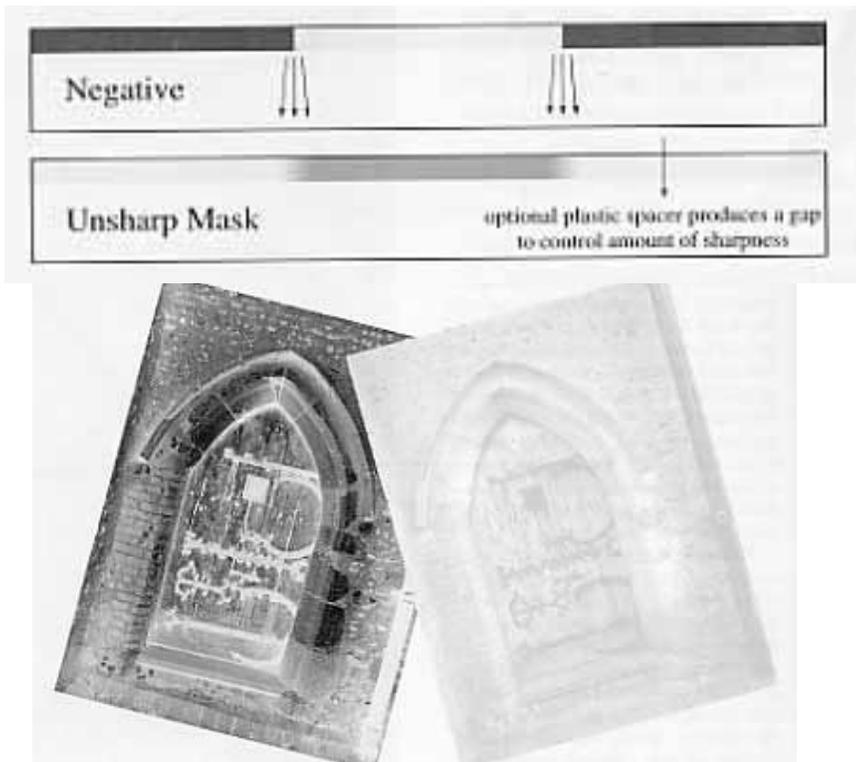


=



# Unsharp mask

- Sharpening is often called “unsharp mask” because photographers used to sandwich a negative with a blurry positive film in order to sharpen

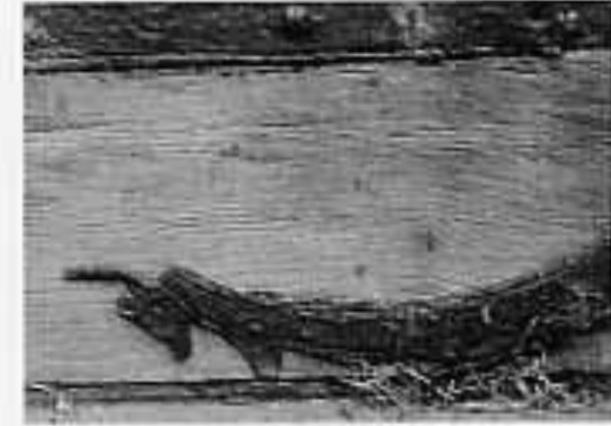
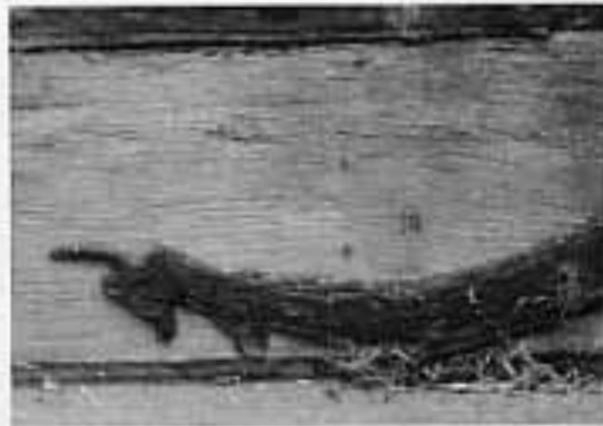


# Unsharp mask

Fig.4: The two examples here show a detail of the brickwork to the left of the church door. The one on the left was printed with the negative alone – the one on the right was printed with both negative and mask as a sandwich. The increase in local contrast and edge sharpness is minute, but clearly visible. Grade 2.5 was used for the straight print but increased to 4.5 for the sandwiched image to compensate for the reduced contrast.



Fig.5: These two examples show a detail of the lower right hand side of the church door. Here the difference in sharpness is clearly visible between the (left) negative and (right) sandwich prints.



# Problem with excess

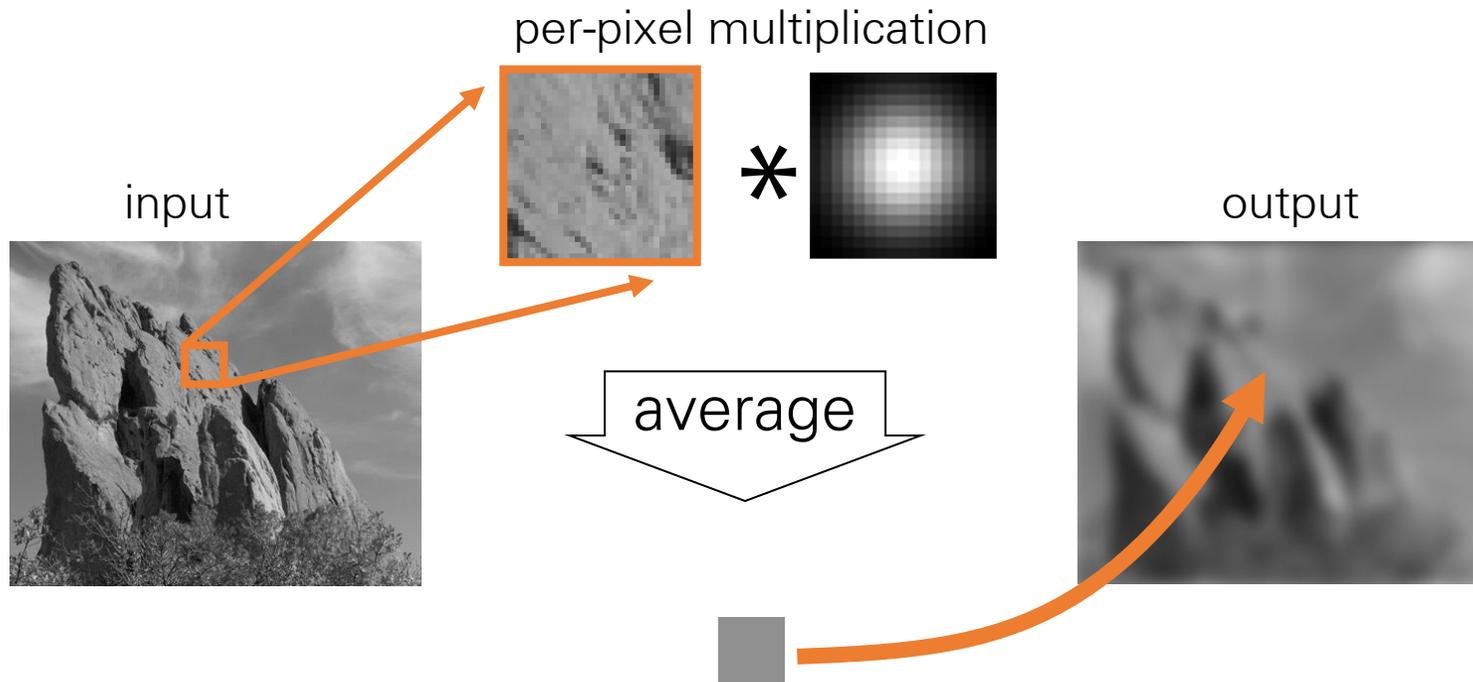
- Haloes around strong edges



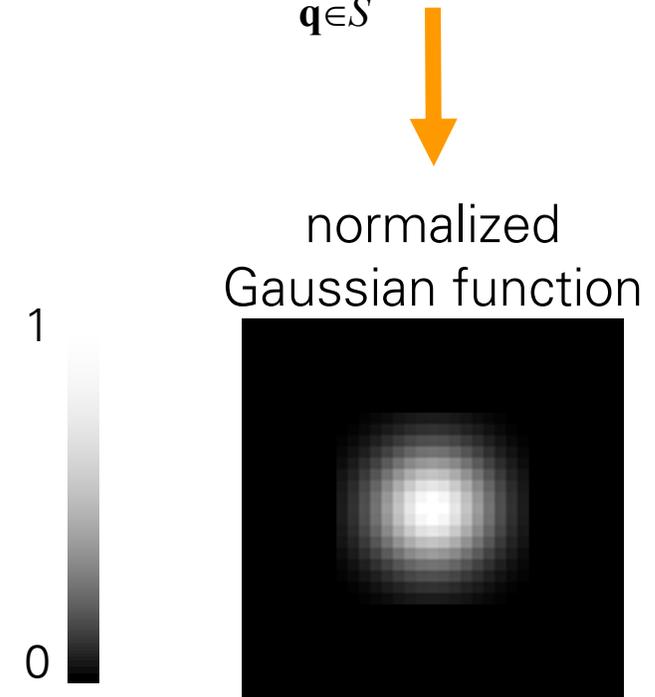
# Bilateral Filter

# Gaussian Filter

Idea: weighted average of pixels.



$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma}(\|\mathbf{p} - \mathbf{q}\|) I_{\mathbf{q}}$$



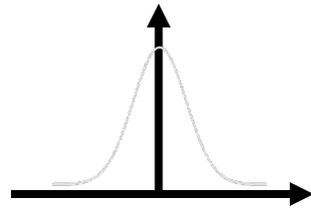
# Spatial Parameter



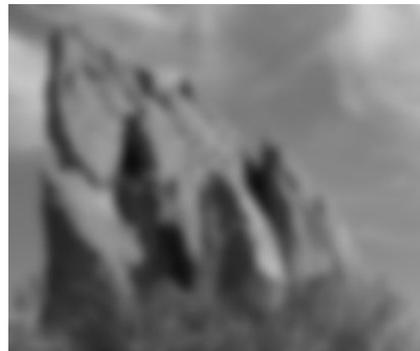
input

$$GB[I]_p = \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma}(\|\mathbf{p} - \mathbf{q}\|) I_{\mathbf{q}}$$

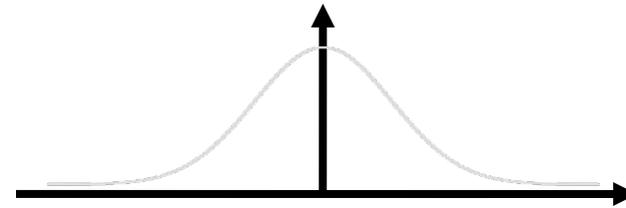
size of the window



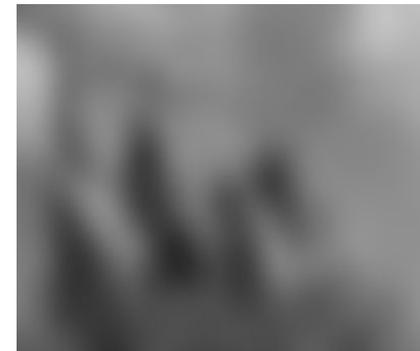
small  $\sigma$



limited smoothing



large  $\sigma$



strong smoothing

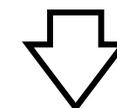
# Properties of Gaussian Blur

- Weights independent of spatial location
  - linear convolution
  - well-known operation
  - efficient computation (recursive algorithm, FFT...)
- Does smooth images
- But smooths too much:  
edges are blurred.
  - Only spatial distance matters
  - No edge term

$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma}(\|\mathbf{p} - \mathbf{q}\|) I_{\mathbf{q}}$$

space

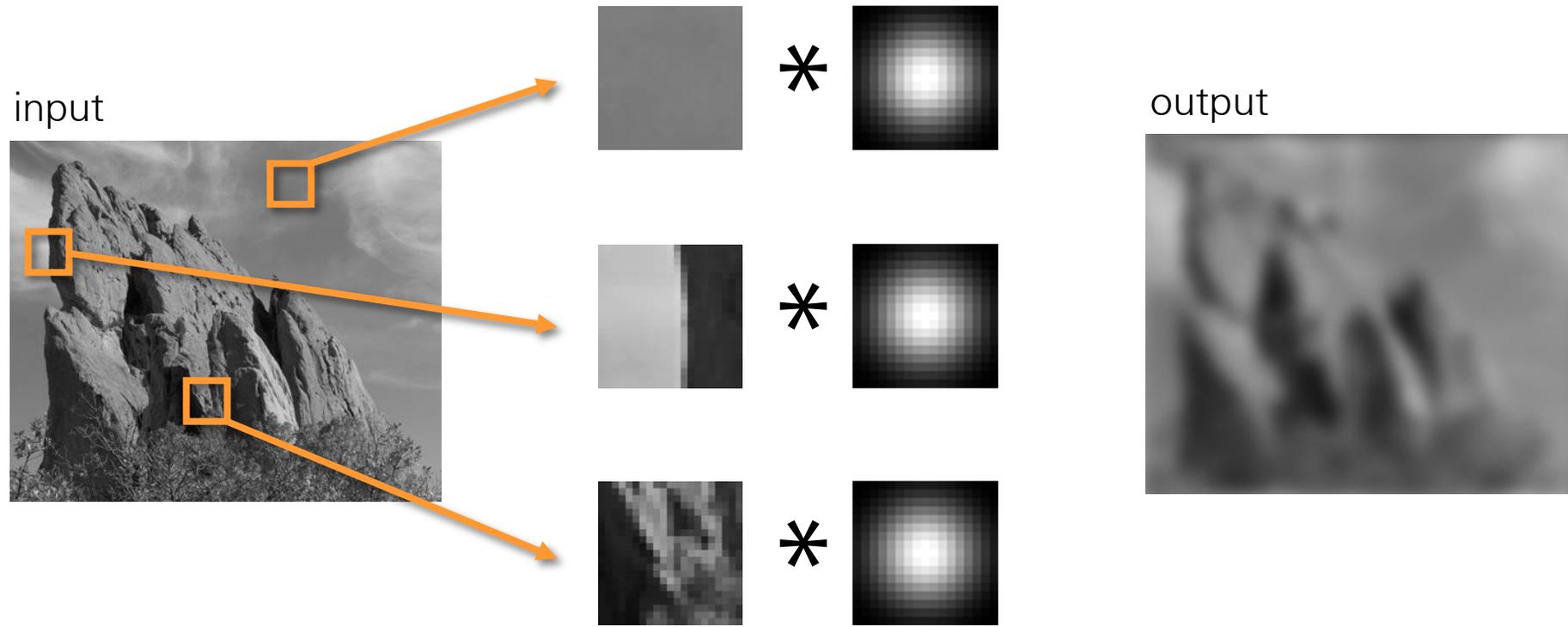
input



output

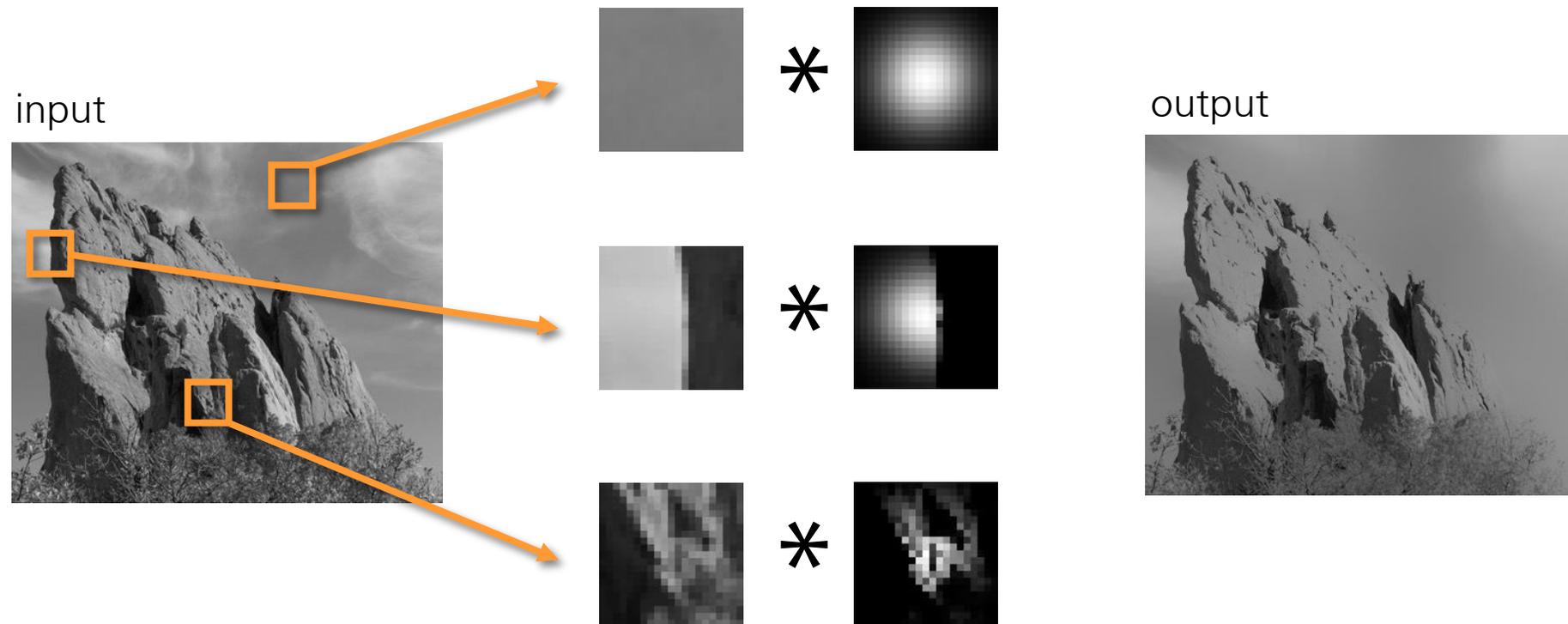


# Blur Comes from Averaging across Edges



Same Gaussian kernel everywhere.

# Bilateral Filter: No Averaging across Edges



The kernel shape depends on the image content.

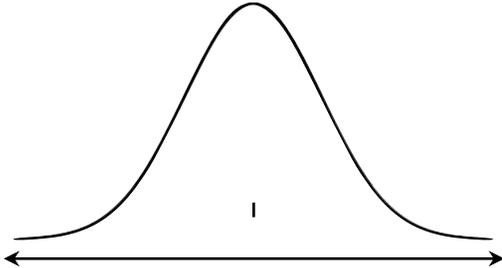
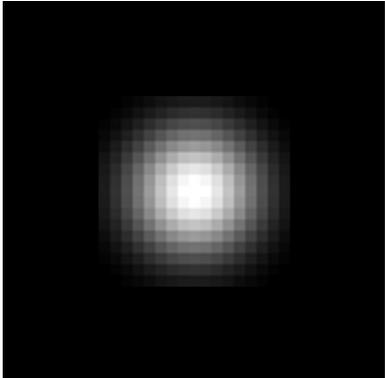
# Bilateral Filter: An Additional Edge Term

Same idea: weighted average of pixels.

$$BF[I]_p = \frac{1}{W_p} \sum_{q \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

new  
not new  
new

normalization factor      space weight      range weight



# Bilateral Filter: An Additional Edge Term

Same idea: weighted average of pixels.

$$BF[I]_p = \frac{1}{W_p} \sum_{q \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

new

not new

new

normalization factor

space weight

range weight

favor nearby pixels

favor similar pixels

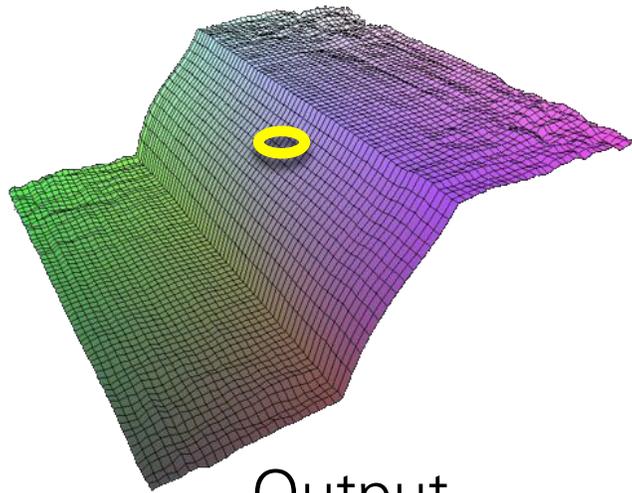
# Space and Range Parameters

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) I_{\mathbf{q}}$$

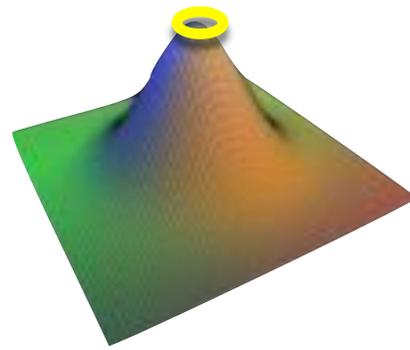

- space  $\sigma_s$  : spatial extent of the kernel, size of the considered neighborhood.
- range  $\sigma_r$  : “minimum” amplitude of an edge

# Gaussian filtering visualization

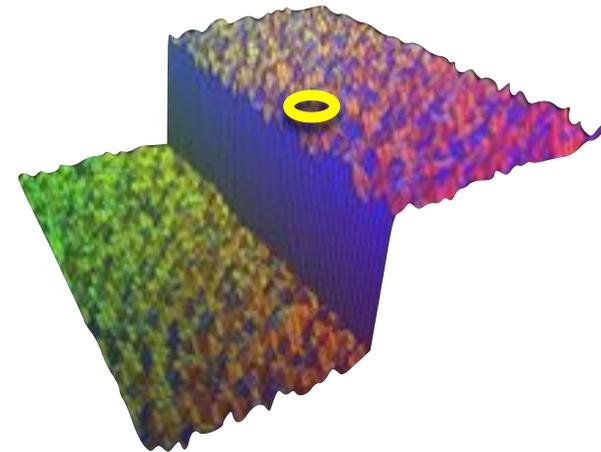
$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$



Output



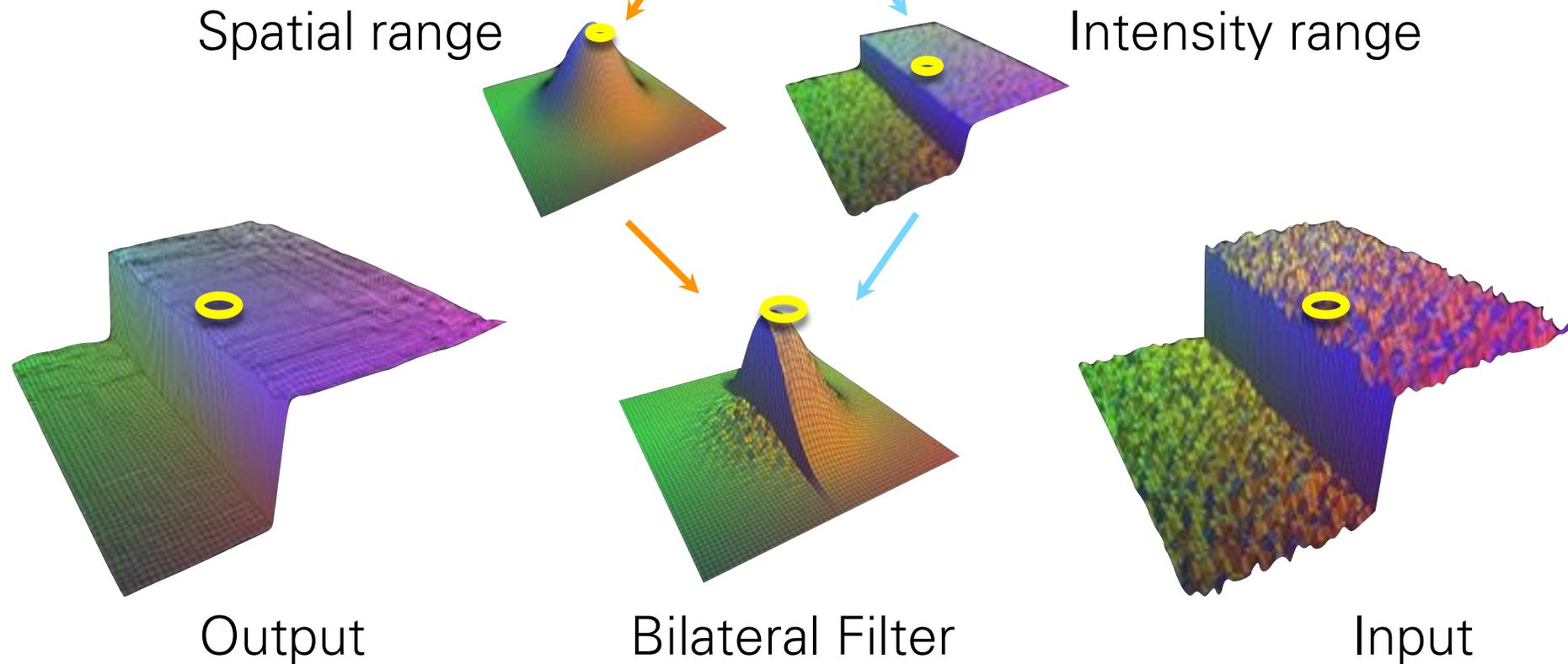
Gaussian Filter



Input

# Bilateral filtering visualization

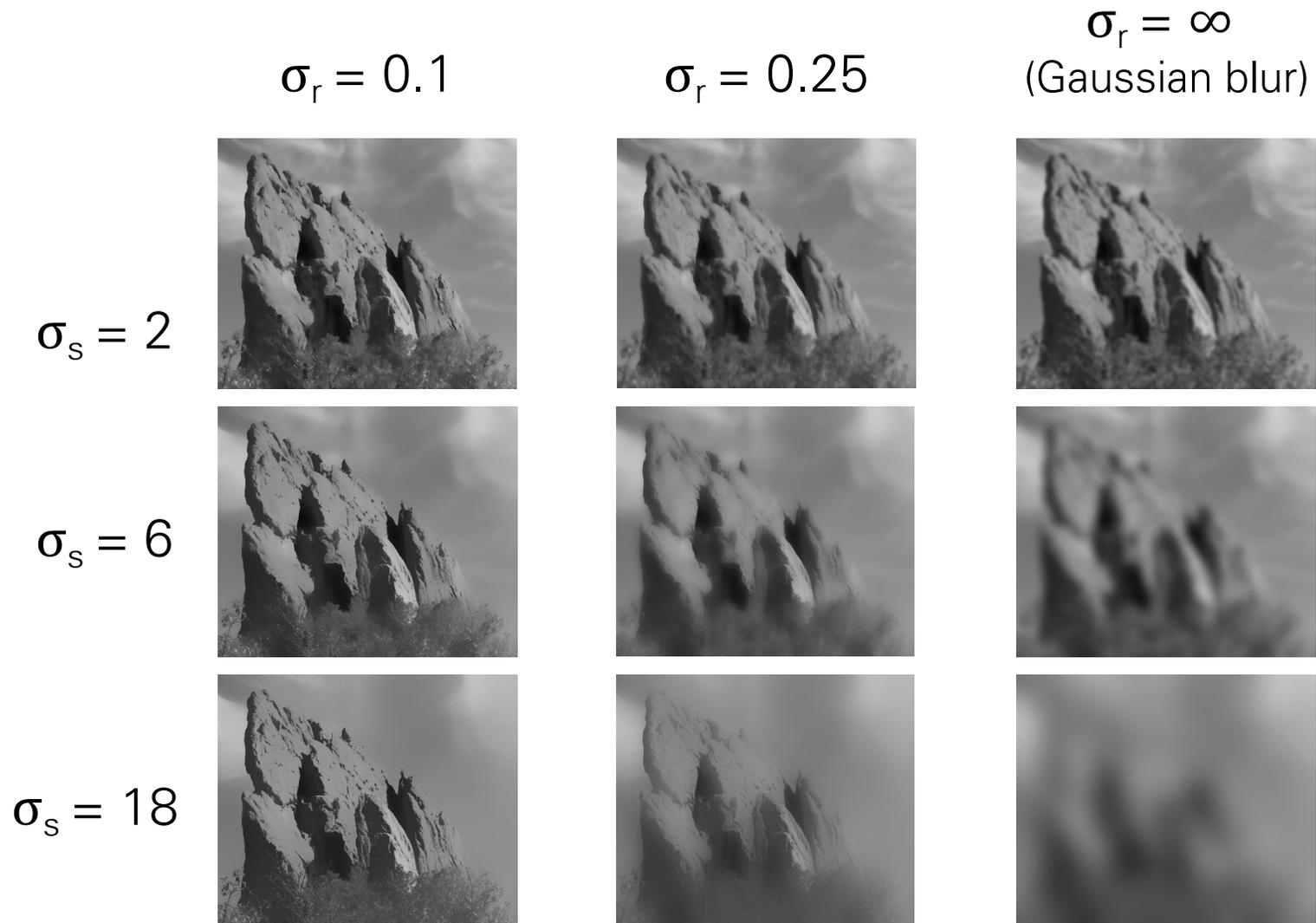
$$h[m, n] = \frac{1}{W_{mn}} \sum_{k, l} g[k, l] r_{mn}[k, l] f[m + k, n + l]$$



# Exploring the Parameter Space



input



# Bilateral Filtering Color Images

For gray-level images

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(\|I_{\mathbf{p}} - I_{\mathbf{q}}\|) I_{\mathbf{q}}$$

intensity difference  
scalar

For color images

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(\|C_{\mathbf{p}} - C_{\mathbf{q}}\|) C_{\mathbf{q}}$$

color difference  
3D vector  
(RGB, Lab)

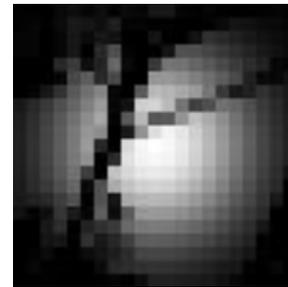
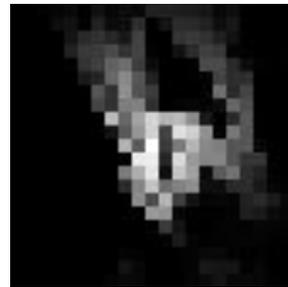
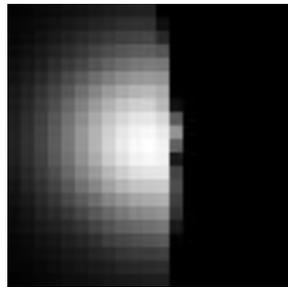
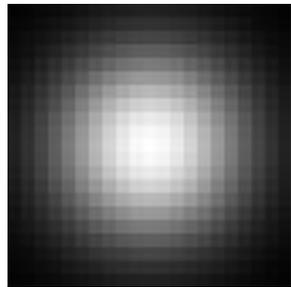


# Hard to Compute

- Nonlinear

$$BF[I]_p = \frac{1}{W_p} \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

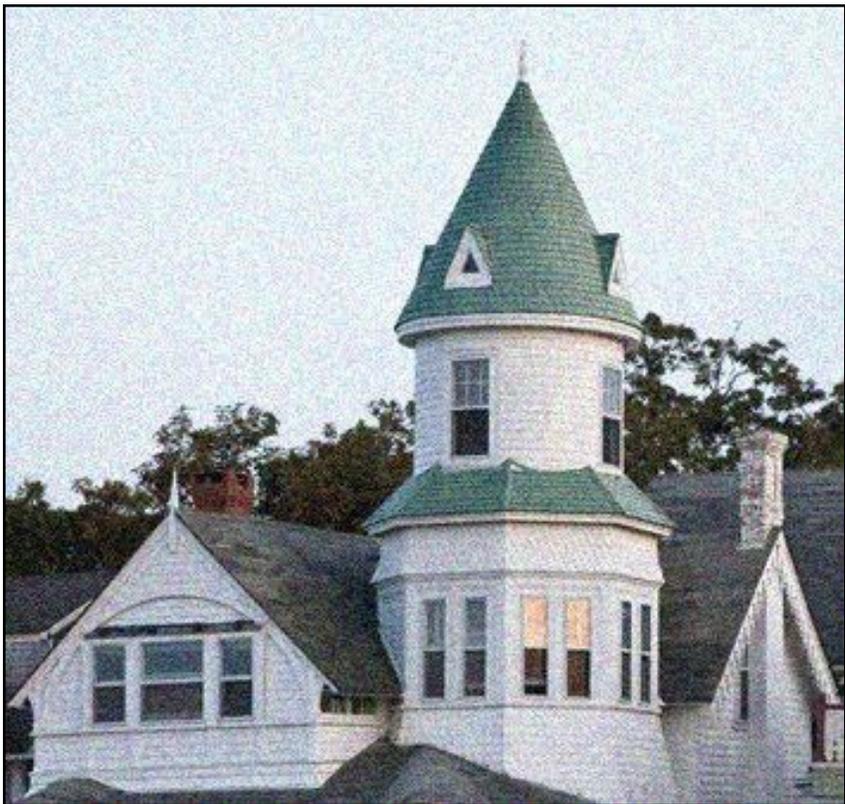
- Complex, spatially varying kernels
- Cannot be precomputed, no FFT...



- Brute-force implementation is slow > 10min

Additional Reading: S. Paris and F. Durand, A Fast Approximation of the Bilateral Filter using a Signal Processing Approach, In Proc. ECCV, 2006

# Denoising



noisy input



bilateral filtering



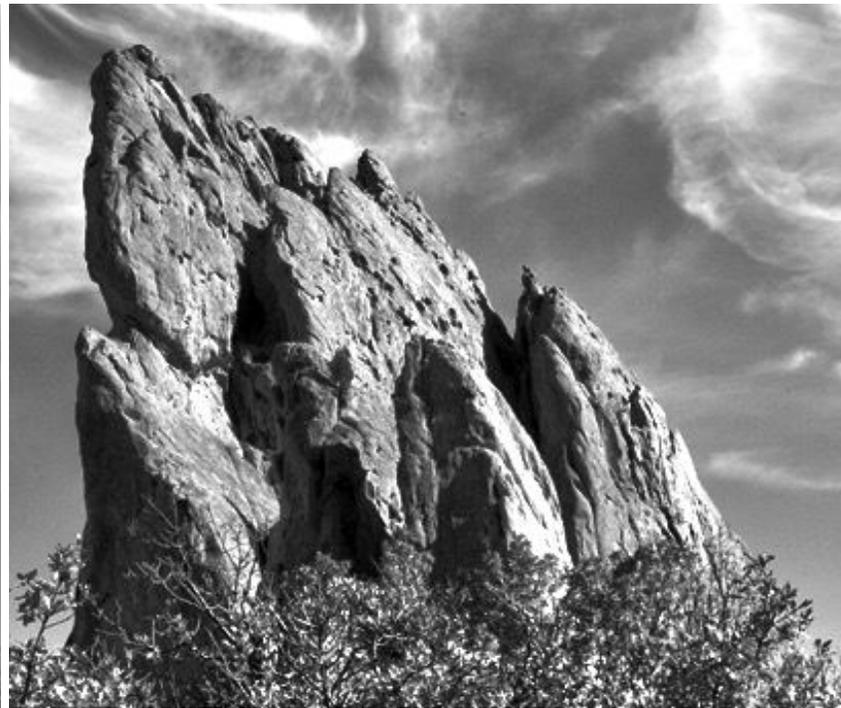
median filtering

# Contrast enhancement

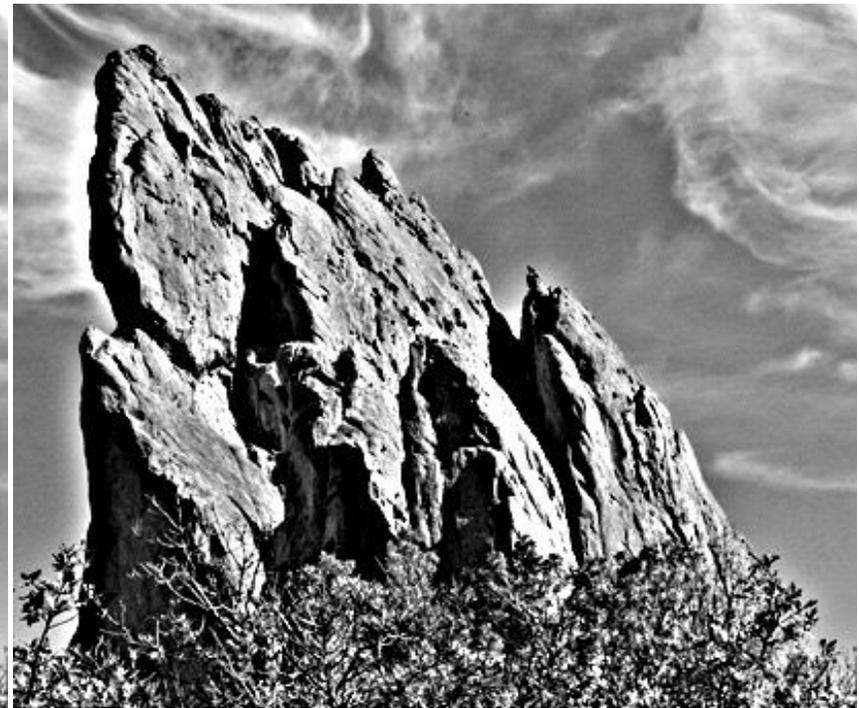
How would you use bilateral filtering for sharpening?



input



sharpening based on  
bilateral filtering



sharpening based on  
Gaussian filtering

# Photo retouching



# Photo retouching



original



digital pore removal (aka bilateral filtering)

# Before



# After



# Close-up comparison



original



digital pore removal (aka bilateral filtering)

# Cartoonization



input



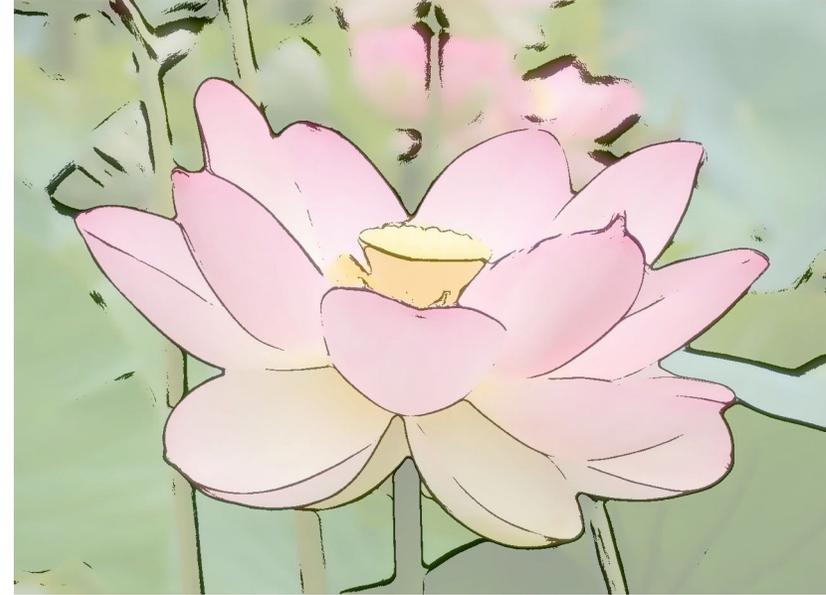
cartoon rendition

# Cartoonization



How would you create this effect?

# Cartoonization



edges from bilaterally filtered image

bilaterally filtered image

cartoon rendition



+



=



Note: image cartoonization and abstraction are very active research areas.

# Is the bilateral filter:

Linear?

Shift-invariant?

# Is the bilateral filter:

Linear?

- No.

Shift-invariant?

- No.

Does this have any bad implications?

# The bilateral grid

## Real-time Edge-Aware Image Processing with the Bilateral Grid

Jiawen Chen      Sylvain Paris      Frédo Durand

Computer Science and Artificial Intelligence Laboratory  
Massachusetts Institute of Technology



**Figure 1:** The bilateral grid enables edge-aware image manipulations such as local tone mapping on high resolution images in real time. This 15 megapixel HDR panorama was tone mapped and locally refined using an edge-aware brush at 50 Hz. The process used about 1 MB of texture memory. The inset shows the original input.

Data structure for fast  
edge-aware image  
processing.

# Modern edge-aware filtering: domain transform

## Domain Transform for Edge-Aware Image and Video Processing

Eduardo S. L. Gastal\*

Manuel M. Oliveira†

Instituto de Informática – UFRGS



(a) Photograph



(b) Edge-aware smoothing



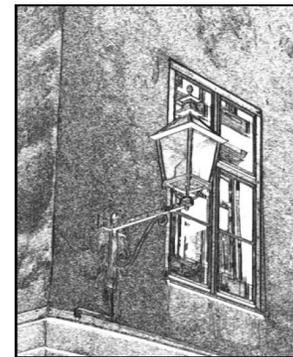
(c) Detail enhancement



(d) Stylization



(e) Recoloring



(f) Pencil drawing



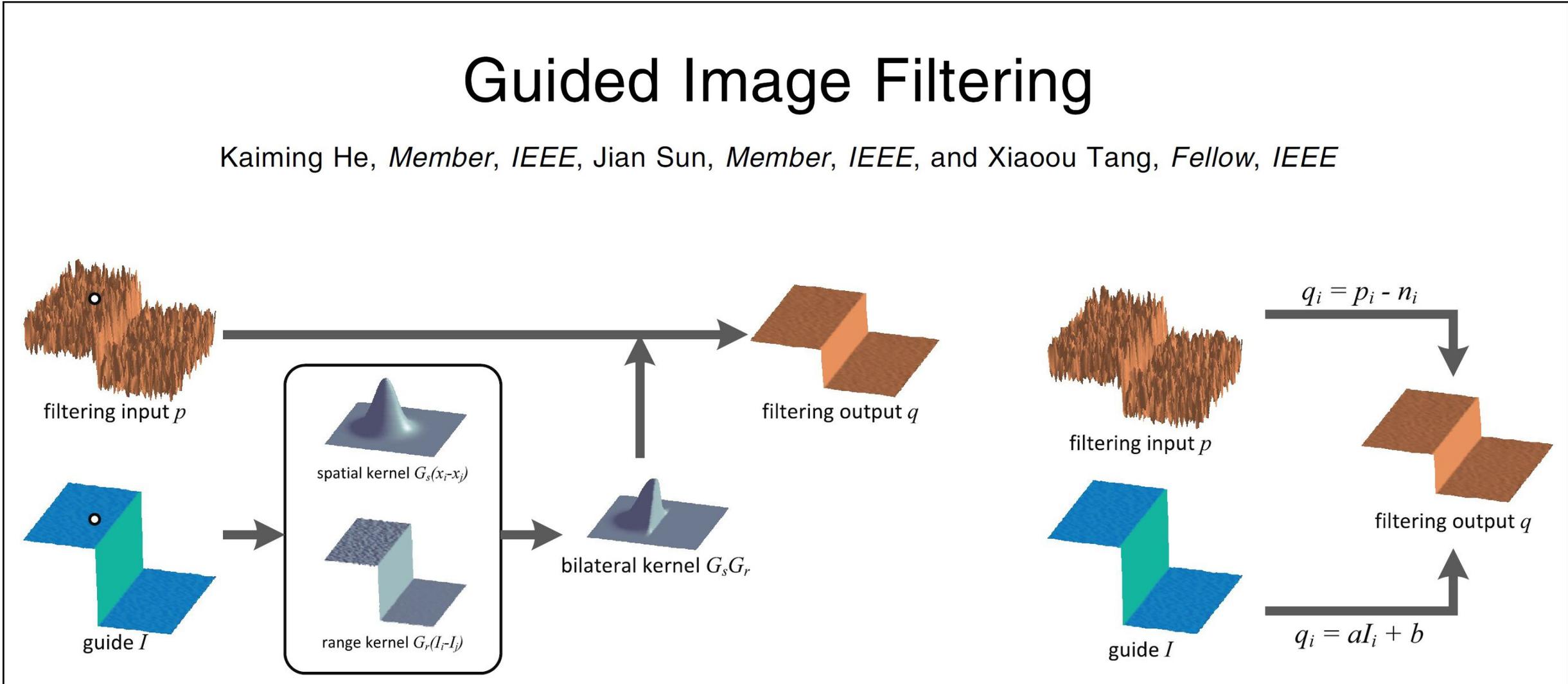
(g) Depth-of-field

Lots of great examples at: <https://www.inf.ufrgs.br/~eslgastal/DomainTransform/>

# Modern edge-aware filtering: guided filter

## Guided Image Filtering

Kaiming He, *Member, IEEE*, Jian Sun, *Member, IEEE*, and Xiaoou Tang, *Fellow, IEEE*



# Flash/no-flash photography



Red Eye



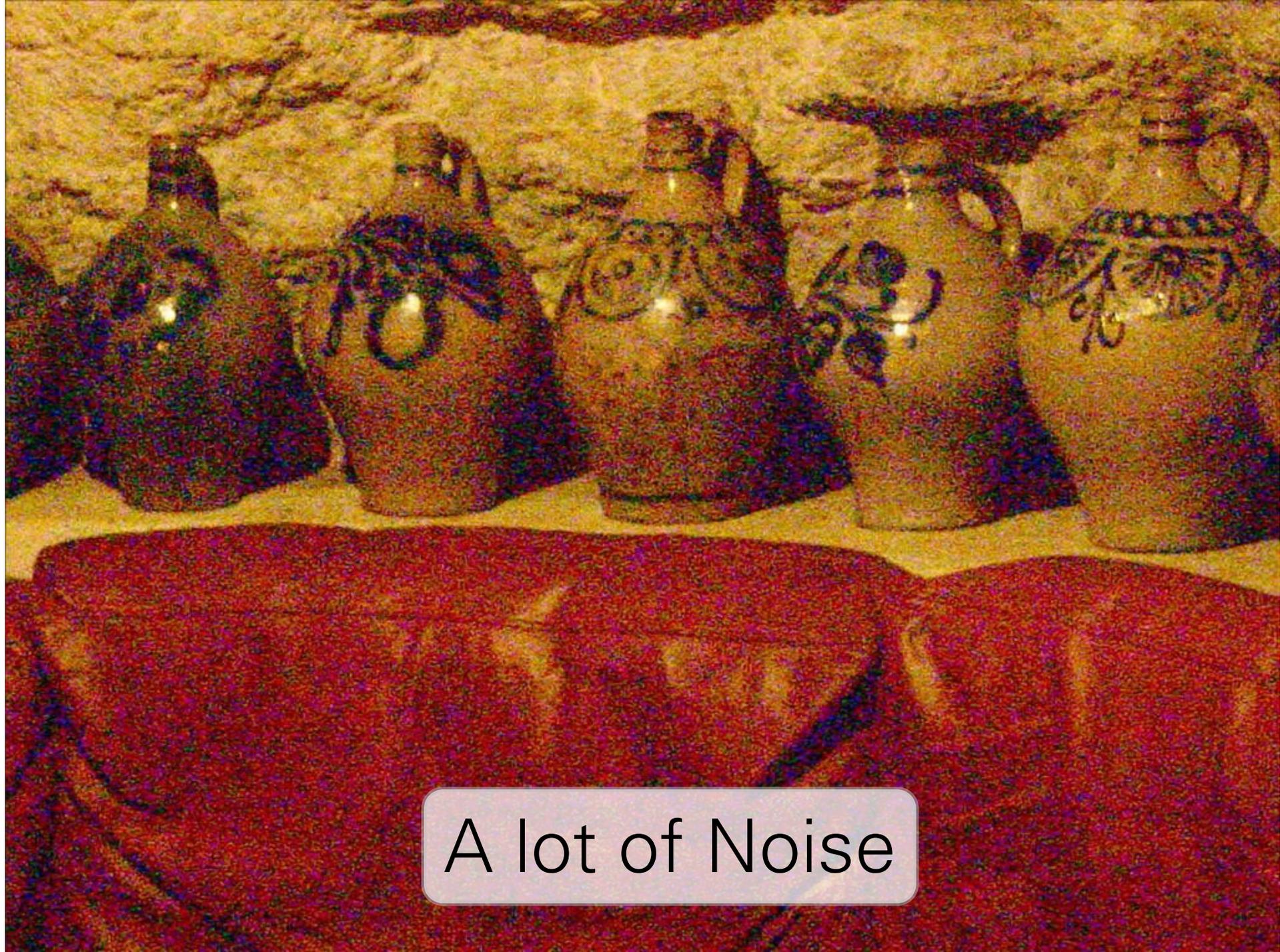
Unflattering Lighting



Motion Blur



Noise



A lot of Noise



Ruined Ambiance

Flash

No-Flash

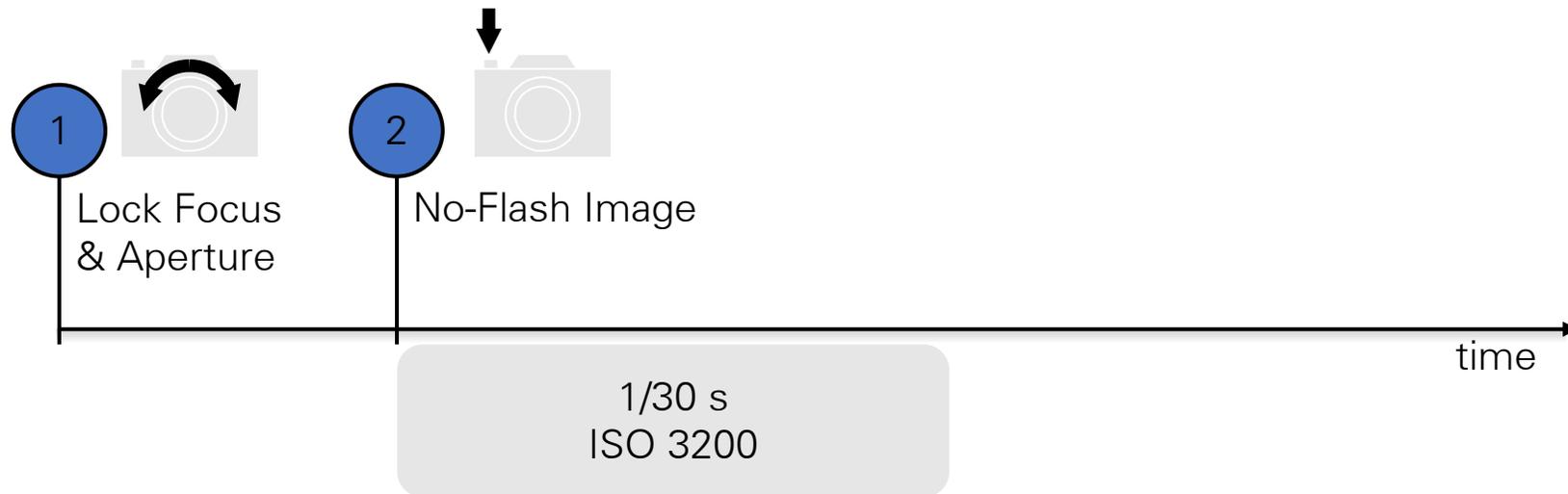
- + Low Noise
- + Sharp
- Artificial Light
- Jarring Look

- High Noise
- Lacks Detail
- + Ambient Light
- + Natural Look

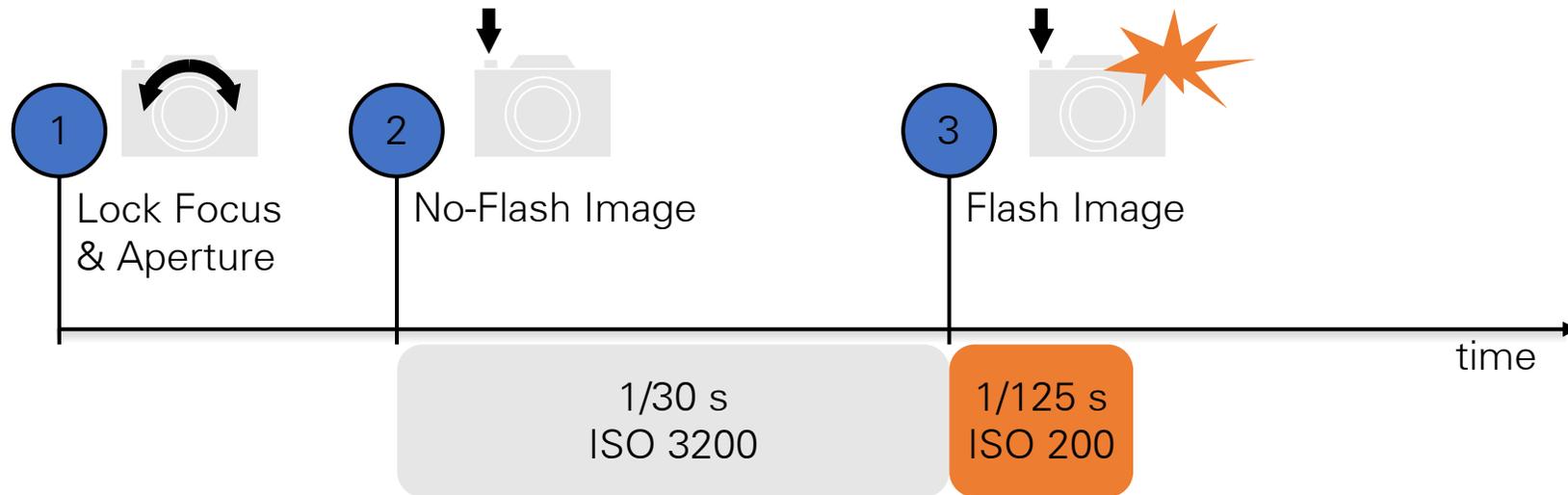
# Image acquisition



# Image acquisition



# Image acquisition





Denoising Result



No-Flash



Denoising Result

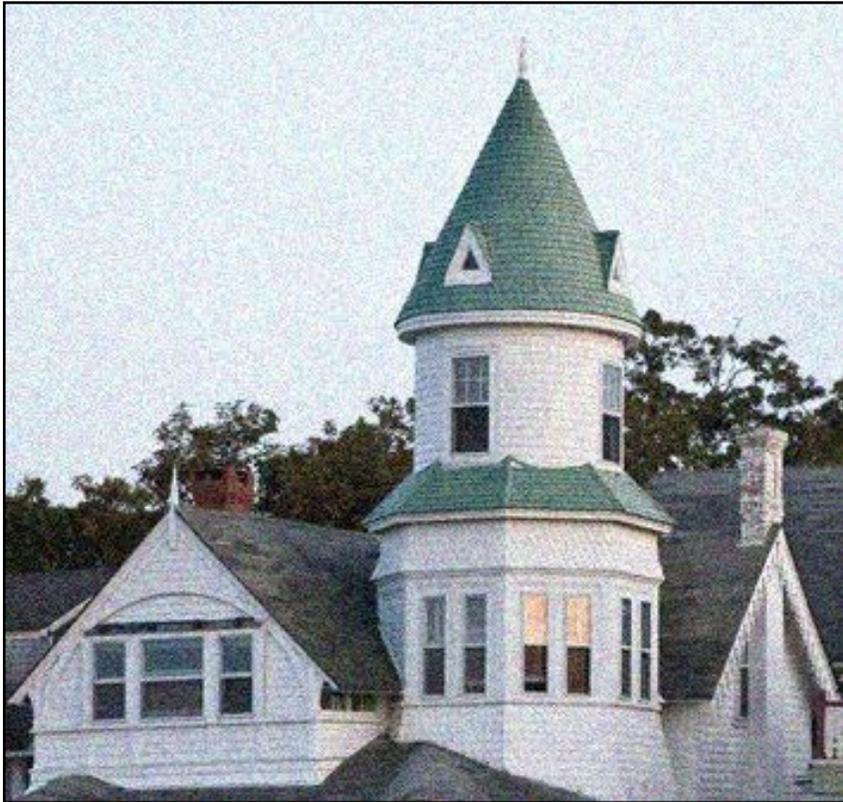
# Key idea

Denoise the no-flash image while maintaining the edge structure of the flash image

- How would you do this using the image editing techniques we've learned about?

# Joint bilateral filtering

# Denoising with bilateral filtering



noisy input



bilateral filtering



median filtering

# Denoising with bilateral filtering

$$A_{p(col)}^{Base} = \frac{1}{k(p(col))} \sum_{p' \in \Omega} \overset{\text{spatial kernel}}{g_d(|p - p'|)} \underset{\text{intensity kernel}}{g_r(A_{p(col)} - A_{p'(col)})} A_{p'(col)}$$

- However, results still have noise or blur (or both)



# Denoising with joint bilateral filtering

$$A_{p(col)}^{NR} = \frac{1}{k(p(col))} \sum_{p' \in \Omega} g_d(|p - p'|) g_r(\mathbf{F}_{p(col)} - \mathbf{F}_{p'(col)}) A_{p'(col)}$$

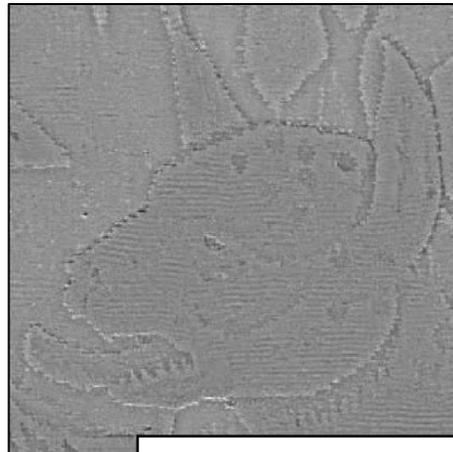
- In the flash image there are many more details
- Use the flash image  $F$  to find edges

# Denoising with joint bilateral filtering

$$A_{p(col)}^{NR} = \frac{1}{k(p(col))} \sum_{p' \in \Omega} g_d(|p - p'|) g_r(\mathbf{F}_{p(col)} - \mathbf{F}_{p'(col)}) A_{p'(col)}$$



Bilateral filter



The difference

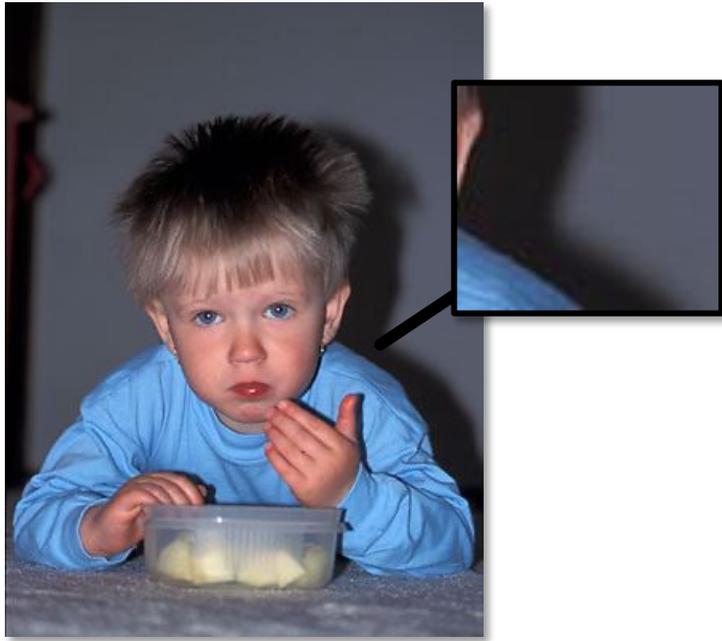


Joint Bilateral filter

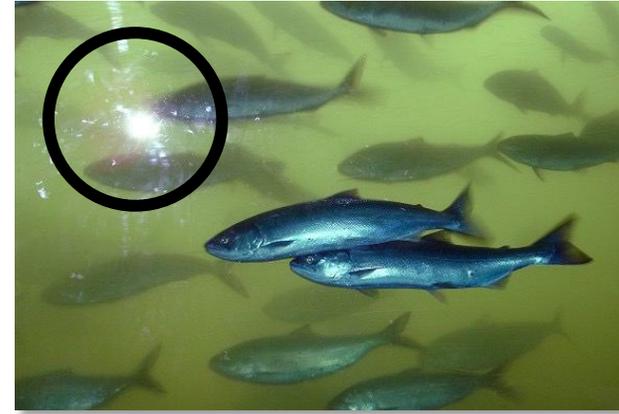
# Not all edges in the flash image are real

Can you think of any types of edges that may exist in the flash image but not the ambient one?

# Not all edges in the flash image are real



shadows



specularities

- May cause over- or under-blur in joint bilateral filter
- We need to eliminate their effect

# Detecting shadows

- Observation: the pixels in the flash shadow should be similar to the ambient image.
- Not identical:
  1. Noise.
  2. Inter-reflected flash.
- Compute a shadow mask.
- Take pixel  $p$  if  $F_{p(col)}^{Lin} - A_{p(col)}^{Lin} \leq \tau_{Shadow}$
- $\tau_{Shadow}$  is manually adjusted
- Mask is smoothed and dilated

# Detecting specularities

- Take pixels where sensor input is close to maximum (very bright).
  - Over fixed threshold  $\tau_{spec}$
- Create a specularity mask.
- Also smoothed.
- M – the combination of shadow and specularity masks:

Where  $M_p=1$ , we use  $A^{Base}$ . For other pixels we use  $A^{NR}$ .

# Detail transfer

- Denoising cannot add details missing in the ambient image
- Exist in flash image because of high SNR
- We use a quotient image:

$$F_{p(col)}^{Detail} = \frac{F_{p(col)} + \varepsilon}{F_{p(col)}^{Base} + \varepsilon}$$

Reduces the effect of noise in F

Bilateral filtered

- Multiply with  $A^{NR}$  to add the details
- Masked in the same way

Why does this quotient image make sense for detail?

# Detail transfer

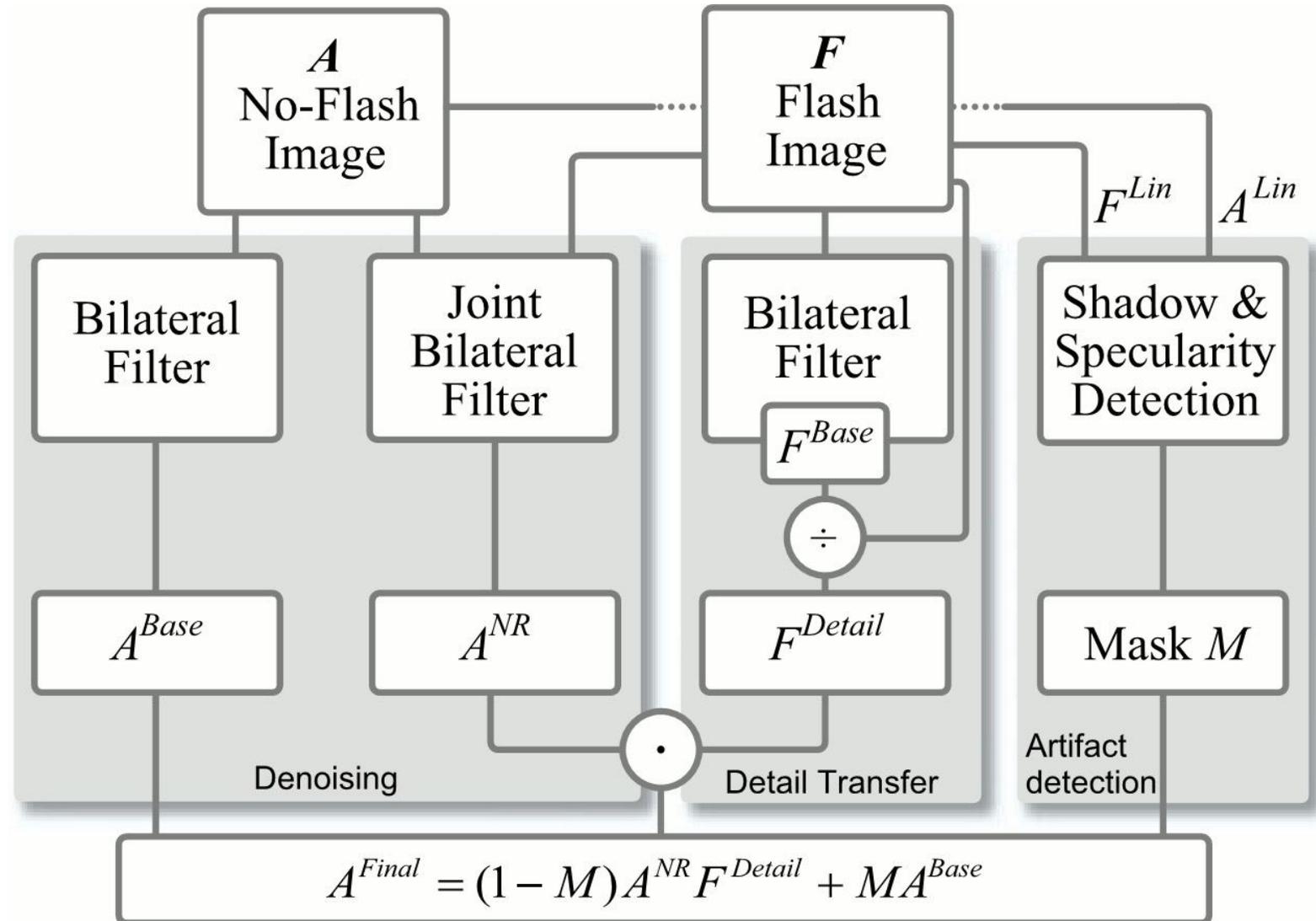
- Denoising cannot add details missing in the ambient image
- Exist in flash image because of high SNR
- We use a quotient image:

$$F_{p(col)}^{Detail} = \frac{F_{p(col)} + \varepsilon}{F_{p(col)}^{Base} + \varepsilon}$$

Reduces the effect of noise in F



# Full pipeline



# Demonstration



ambient-only



joint bilateral and detail transfer



Flash



No-Flash



No-Flash



Result



Flash



No-Flash



No-Flash



Result



Flash



No-Flash



Flash



No-Flash

Result

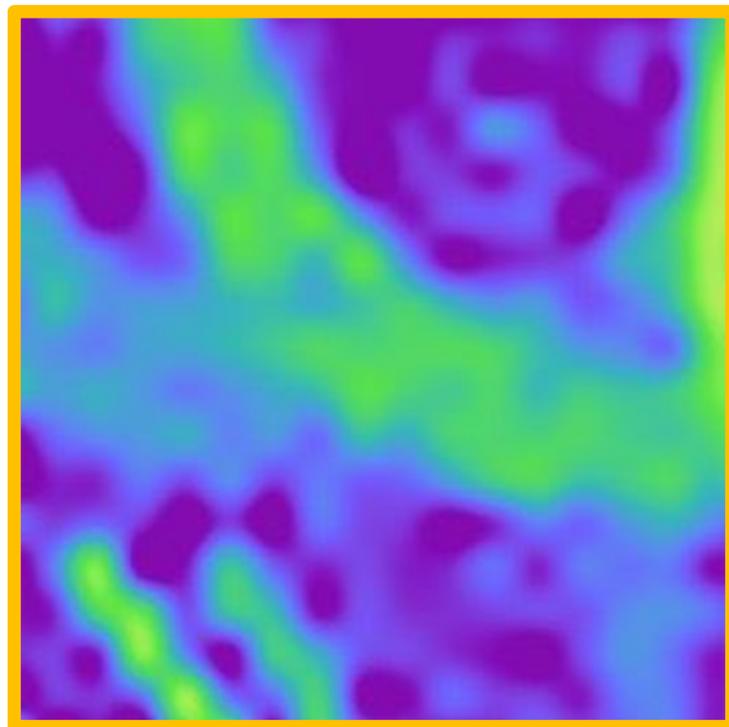
# Edge-aware depth denoising

$$A_{p(col)} = \frac{1}{k(p(col))} \sum_{p' \in \Omega} g_d(|p - p'|) g_r(F_{p(col)} - F_{p'(col)}) A_{p'(col)}$$

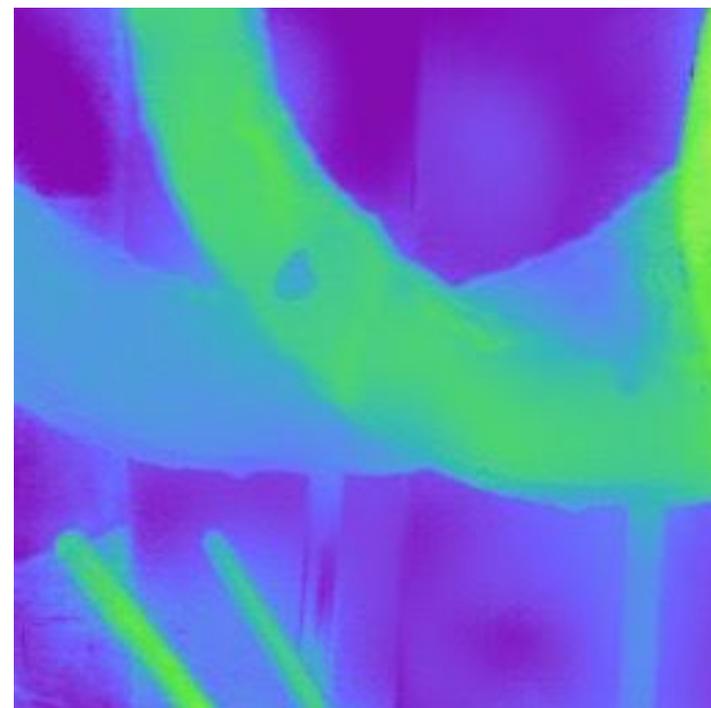
Use joint bilateral filtering, with the input image as guide.



One of two input images



Depth from disparity



Guided filtering

# Other applications of joint bilateral filtering

## Deep Bilateral Learning for Real-Time Image Enhancement

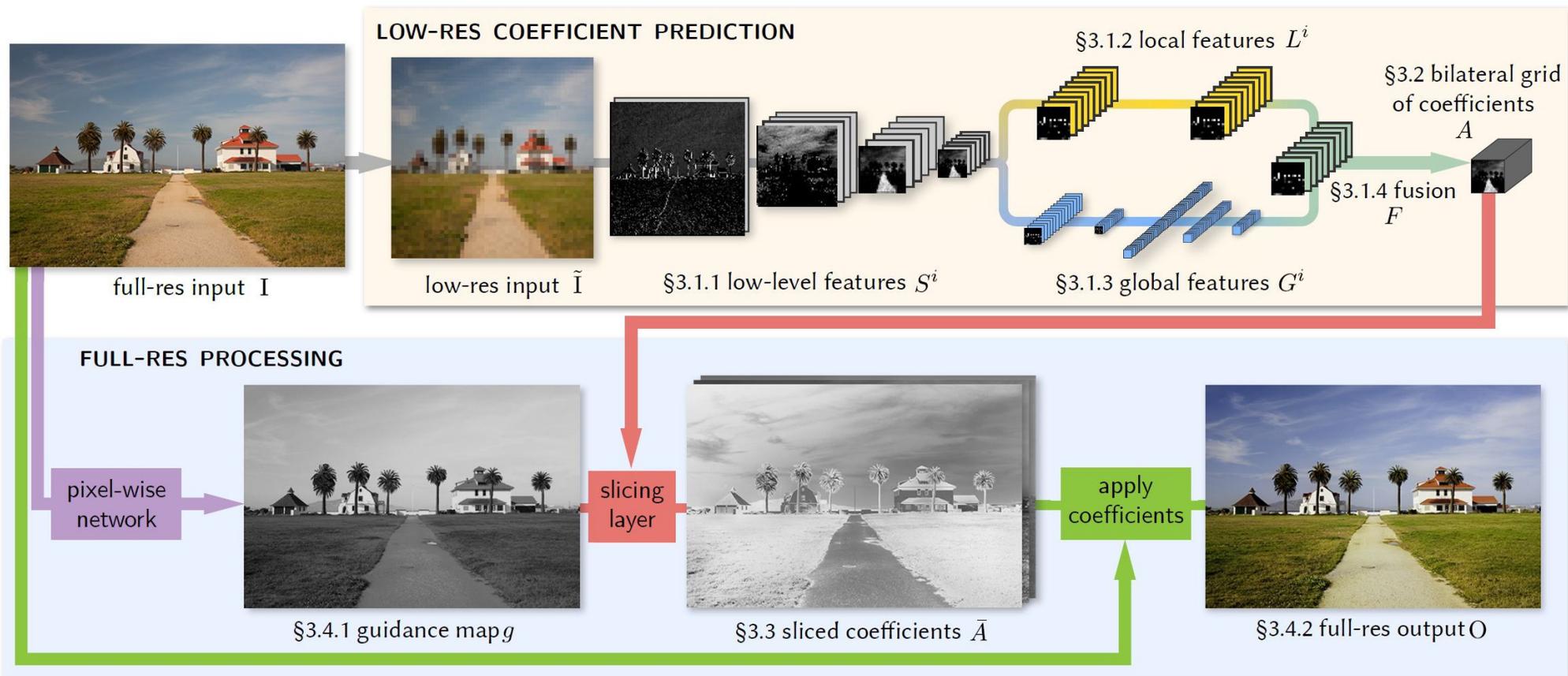
MICHAËL GHARBI, MIT CSAIL

JIAWEN CHEN, Google Research

JONATHAN T. BARRON, Google Research

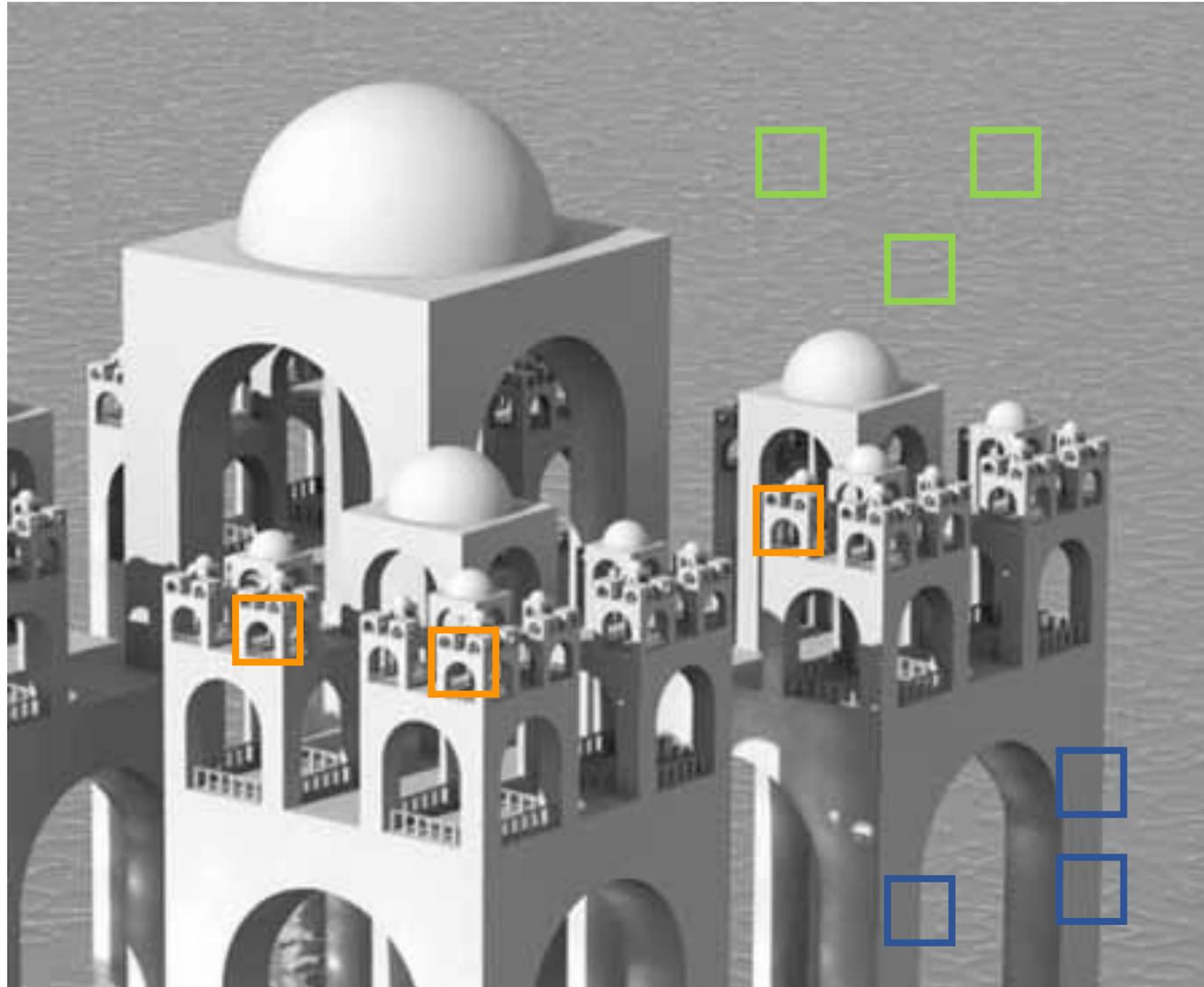
SAMUEL W. HASINOFF, Google Research

FRÉDO DURAND, MIT CSAIL / Inria, Université Côte d'Azur



# Non-Local Means Filter

# Redundancy in natural images

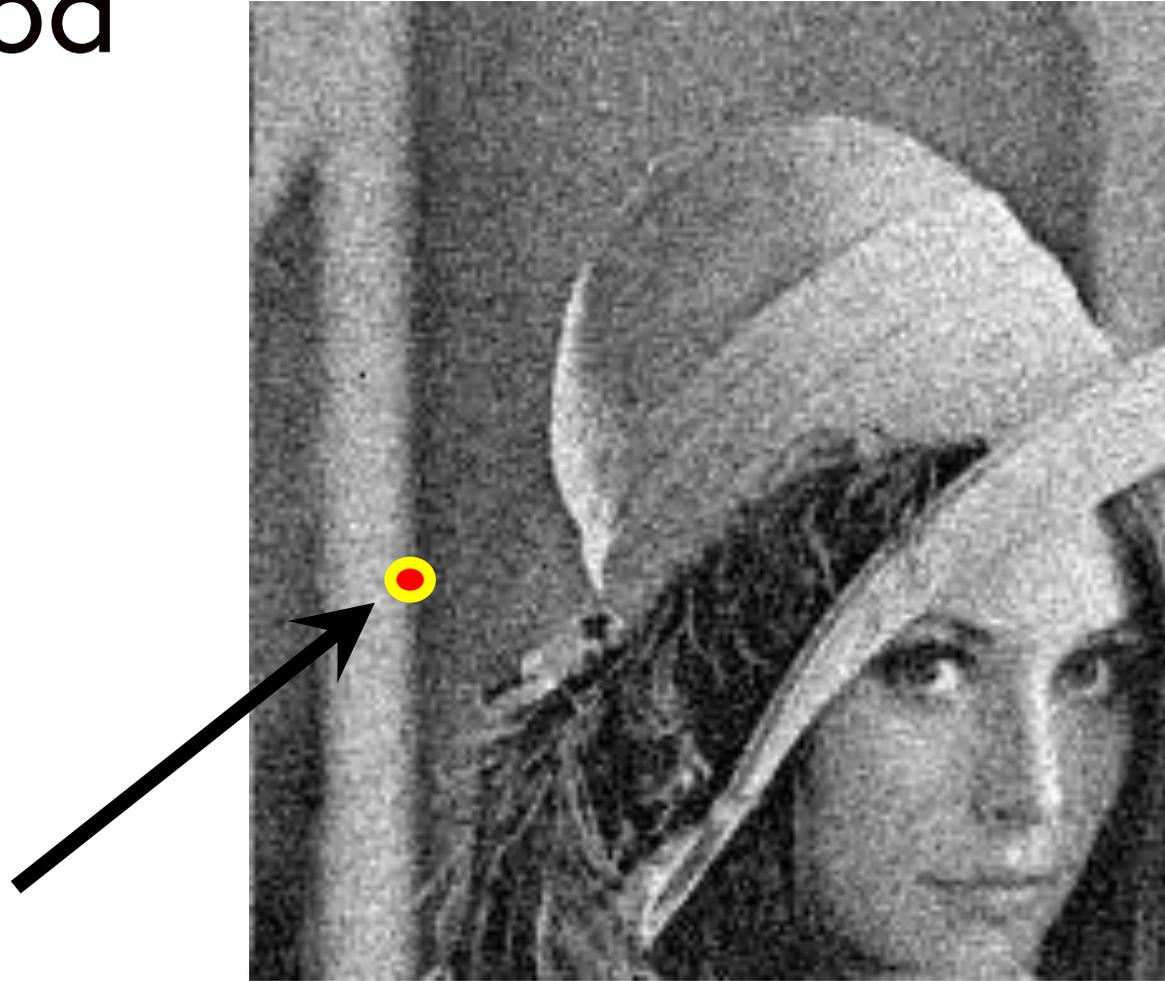


# NL-Means Filter (Buades 2005)

- Same goals: 'Smooth within Similar Regions'
- KEY INSIGHT: Generalize, extend 'Similarity'
  - Bilateral:  
Averages neighbors with similar intensities;
  - NL-Means:  
Averages neighbors with similar neighborhoods!

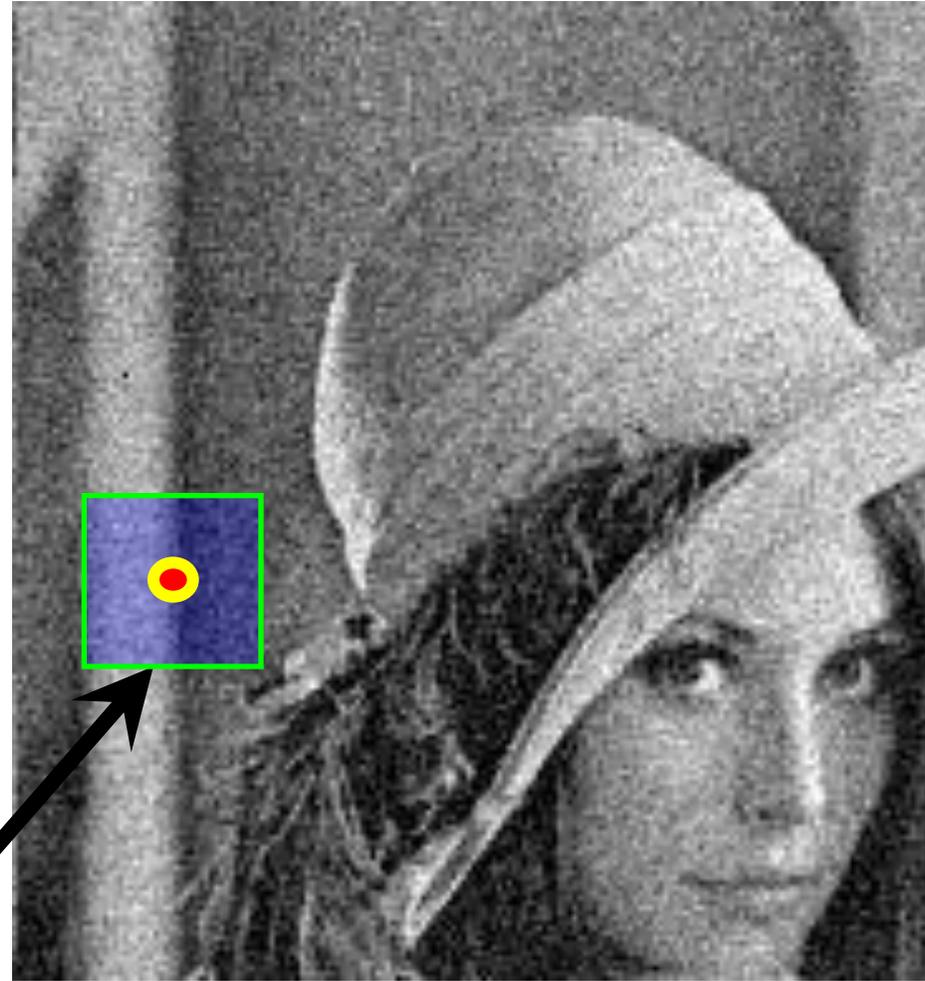
# NL-Means Method

- For each and every pixel  $p$ :



# NL-Means Method

- For each and every pixel  $p$ :
  - Define a small, simple fixed size neighborhood;



# NL-Means Method

$$V_p = \begin{bmatrix} 0.74 \\ 0.32 \\ 0.41 \\ 0.55 \\ \dots \\ \dots \\ \dots \end{bmatrix}$$

- For each and every pixel  $p$ :

- Define a small, simple fixed size neighborhood;
- Define vector  $V_p$ : a list of neighboring pixel values.



# NL-Means Method

'Similar' pixels  $p, q$

→ SMALL

vector distance;

$$\|V_p - V_q\|^2$$



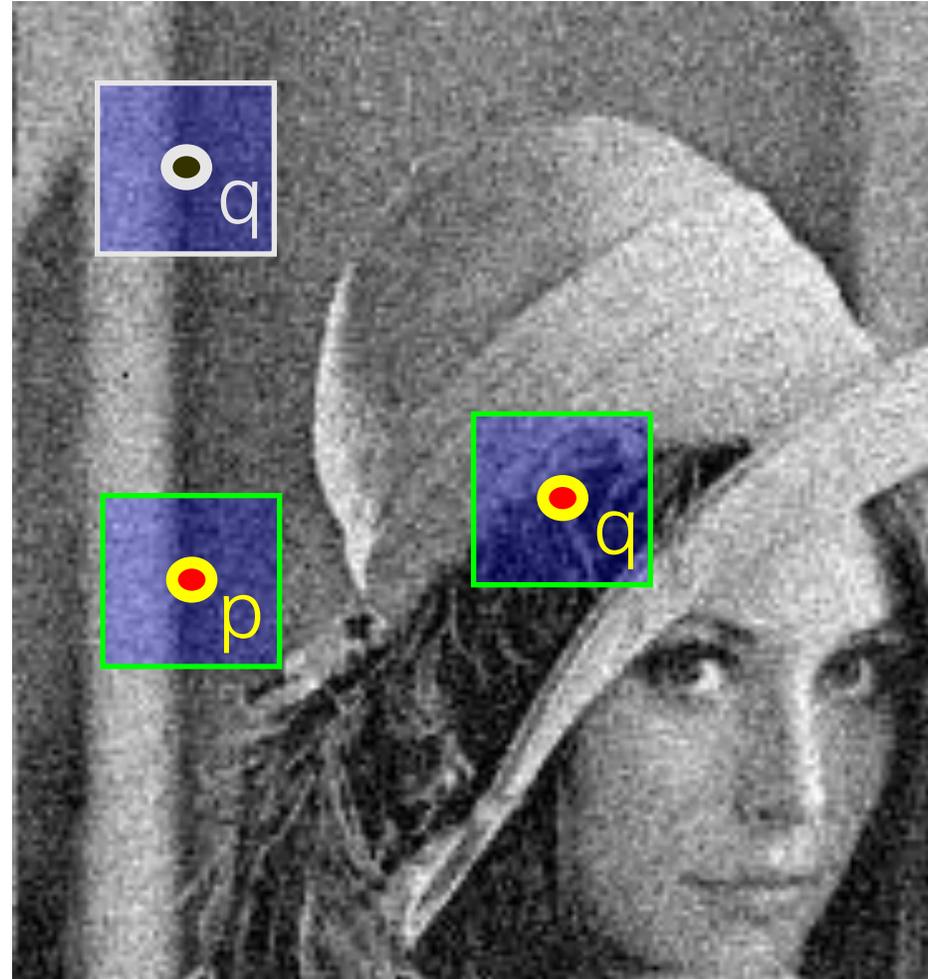
# NL-Means Method

'Dissimilar' pixels  $p, q$

→ LARGE

vector distance;

$$\|V_p - V_q\|^2$$



# NL-Means Method

'Dissimilar' pixels  $p, q$

→ LARGE  
vector distance;

$$\|V_p - V_q\|^2$$

Filter with this!



# NL-Means Method

$p, q$  neighbors define  
a vector distance;

$$\|V_p - V_q\|^2$$

Filter with this:

No spatial term!



$$NLMF[I]_p = \frac{1}{W_p} \sum_{q \in S} \cancel{G_{\sigma_s}(\|p - q\|)} G_{\sigma_r}(\| \vec{V}_p - \vec{V}_q \|^2) I_q$$

# NL-Means Method

pixels  $p, q$  neighbors  
Set a vector distance;

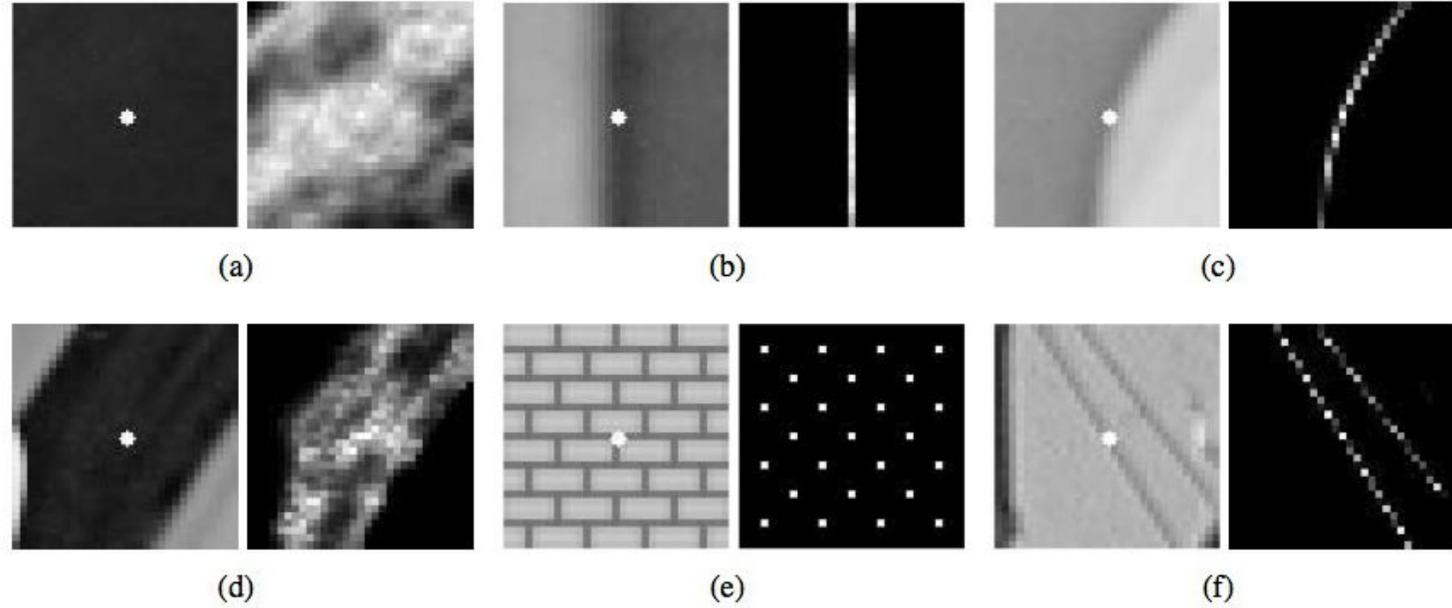
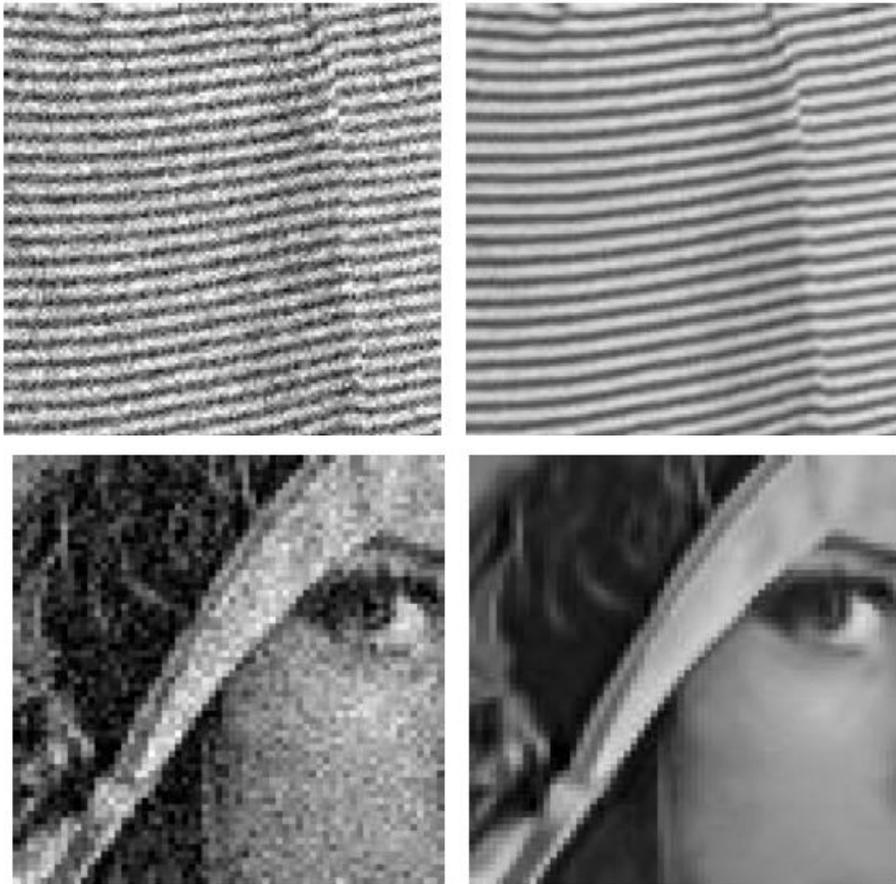
$$\|V_p - V_q\|^2$$

Vector Distance to  $p$  sets  
weight for each pixel  $q$

$$NLMF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_r} \left( \| \vec{V}_p - \vec{V}_q \|^2 \right) I_q$$



# NL-Means Method



# NL-Means Method

- Noisy source image:



# NL-Means Method

- Gaussian Filter

Low noise,  
Low detail



# NL-Means Method

- Anisotropic Diffusion

Note 'stairsteps':  
~ piecewise constant



# NL-Means Method

- Bilateral Filter

Better, but similar  
'stairsteps':



# NL-Means Method

- NL-Means:

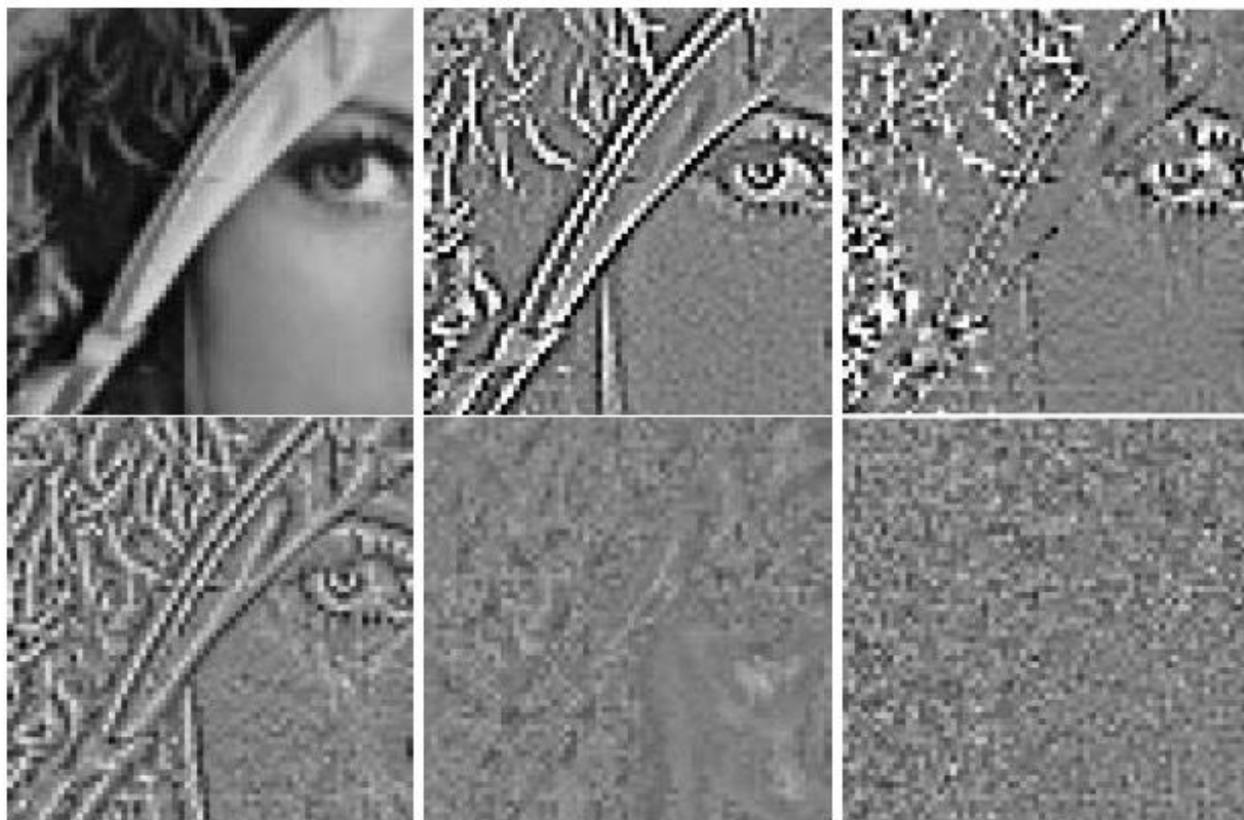
Sharp,

Low noise,

Few artifacts.



# NL-Means Method



**Figure 4. Method noise experience on a natural image. Displaying of the image difference  $u - D_h(u)$ . From left to right and from top to bottom: original image, Gauss filtering, anisotropic filtering, Total variation minimization, Neighborhood filtering and NL-means algorithm. The visual experiments corroborate the formulas of section 2.**

# NL-Means Method

*original*



*noisy, standard deviation 15*



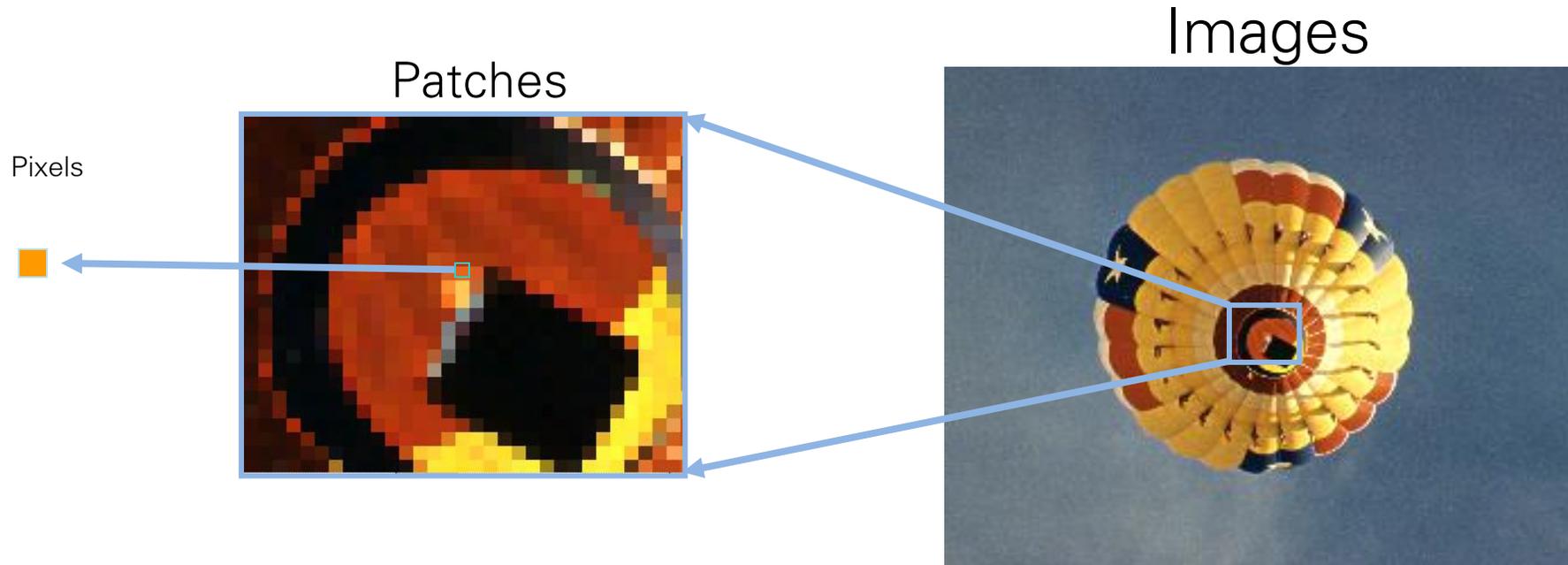
*denoised*



[http://www.ipol.im/pub/algo/bcm\\_non\\_local\\_means\\_denoising/](http://www.ipol.im/pub/algo/bcm_non_local_means_denoising/)

# RegCov Smoothing

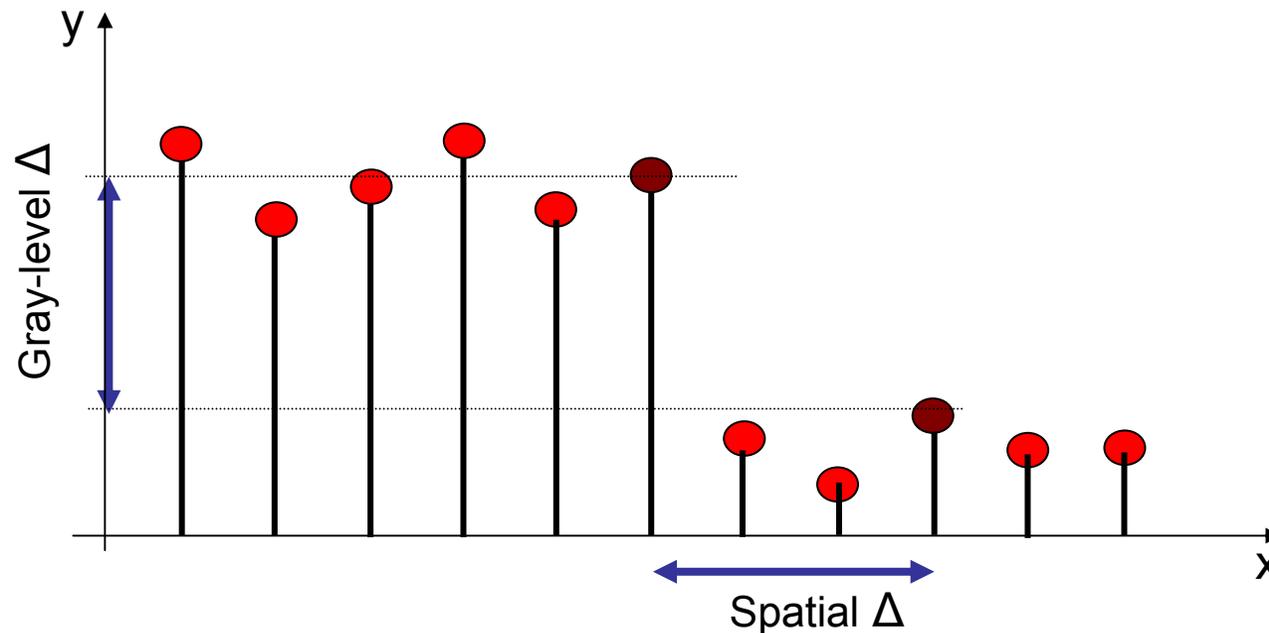
# From pixels to patches and to images



Similarities can be defined at different scales..

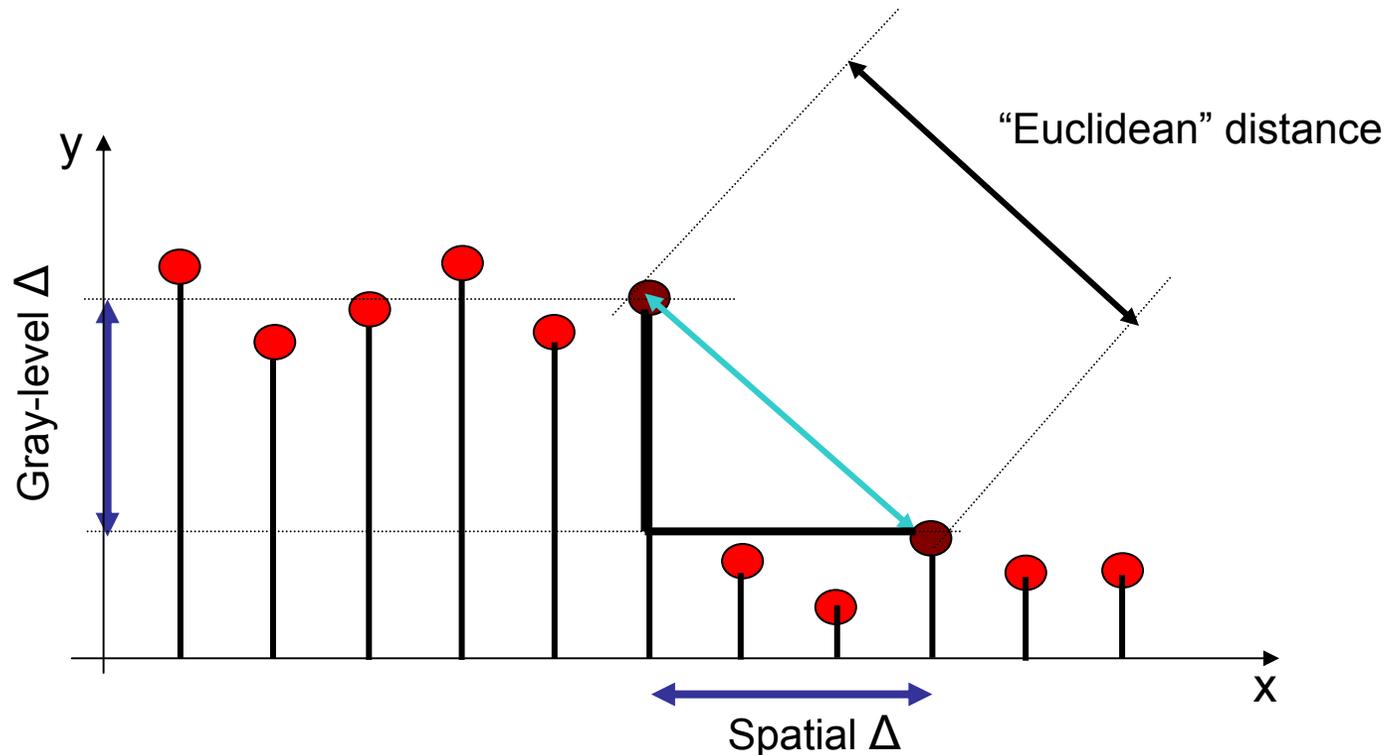
# Pixelwise similarity metrics

- To measure the similarity of two pixels, we can consider
  - Spatial distance
  - Gray-level distance



# Euclidean metrics

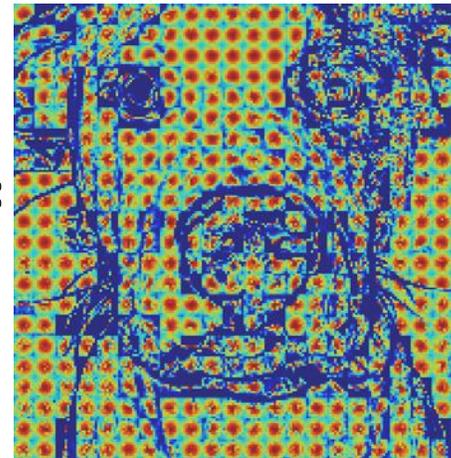
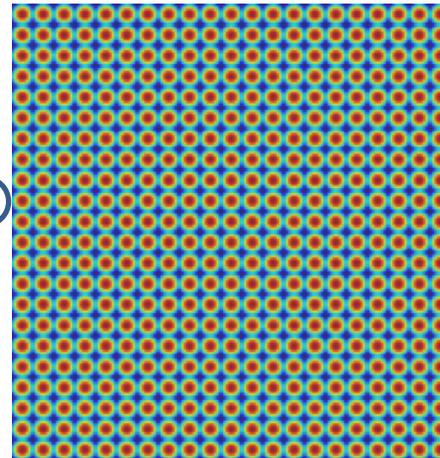
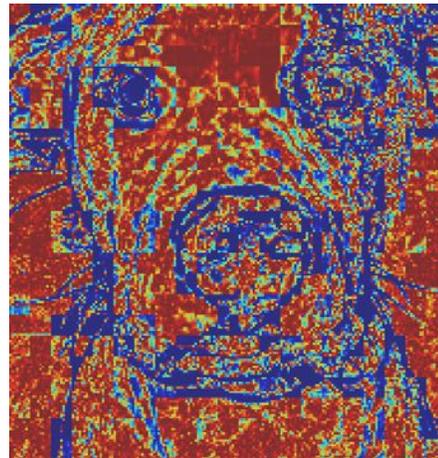
- Natural ways to incorporate the two  $\Delta$ s:
  - Bilateral Kernel [Tomasi, Manduchi, '98] (pixelwise)
  - Non-Local Means Kernel [Buades, et al. '05] (patchwise)



# Bilateral Kernel (BL) [Tomasi et al. '98]

$$K(\mathbf{x}_l, \mathbf{x}, y_l, y) = \exp \left\{ - \frac{\|y_l - y\|^2}{h_r^2} - \frac{\|\mathbf{x}_l - \mathbf{x}\|^2}{h_d^2} \right\}$$

↓ Pixel similarity      ↓ Spatial similarity





# Structure-Texture Decomposition

- Decomposing an image into structure and texture components

Input Image



# Structure-Texture Decomposition

- Decomposing an image into structure and texture components

Structure Component



# Structure-Texture Decomposition

- Decomposing an image into structure and texture components

Texture Component



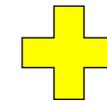
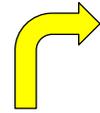
# Structure-Texture Decomposition

- Decomposing an image into structure and texture components

Input Image



Structure



Texture



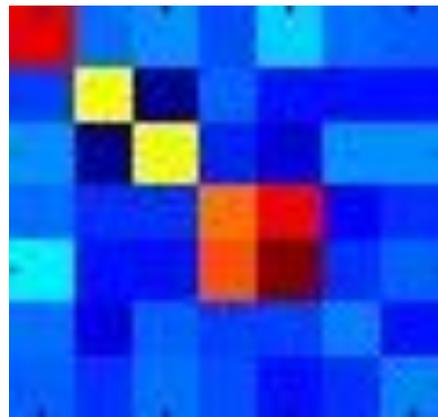
# Structure-Texture Decomposition



$$F(x, y) = \phi(I, x, y)$$



$$F(x, y) = \left[ I(x, y) \quad \left| \frac{\partial I}{\partial x} \right| \quad \left| \frac{\partial I}{\partial y} \right| \quad \left| \frac{\partial^2 I}{\partial x^2} \right| \quad \left| \frac{\partial^2 I}{\partial y^2} \right| \quad x \quad y \right]^T$$



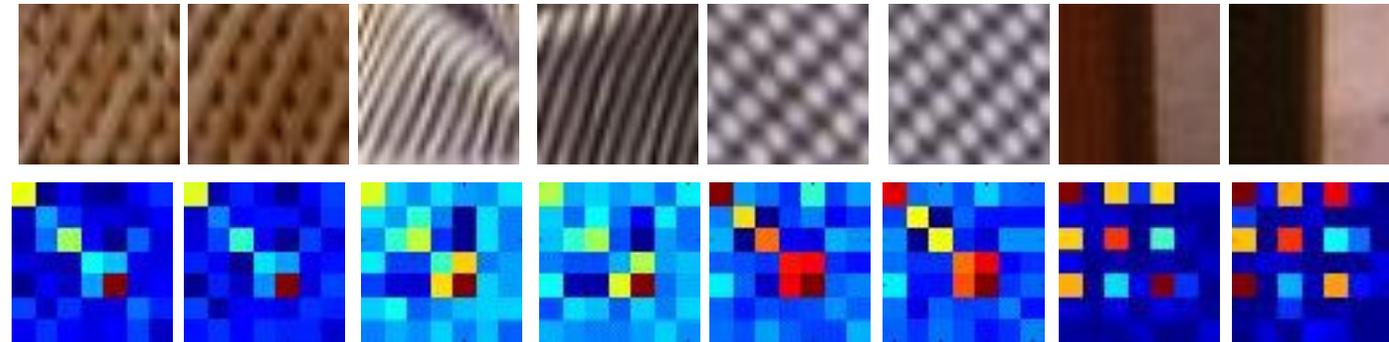
$$\mathbf{C}_R = \frac{1}{n-1} \sum_{i=0}^n (\mathbf{z}_k - \mu)(\mathbf{z}_k - \mu)^T$$

Tuzel et al., ECCV 2006

# Structure-Texture Decomposition



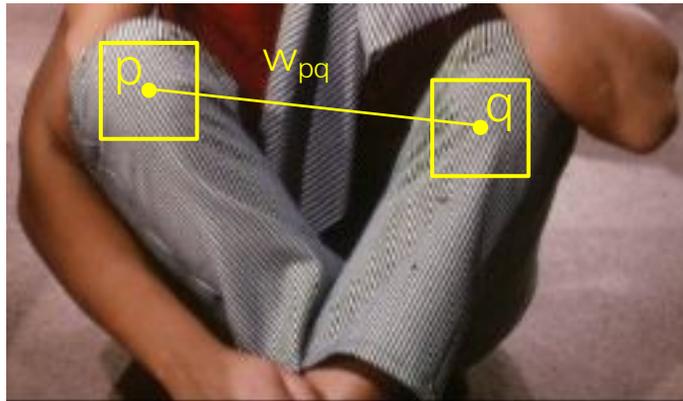
- Region covariances capture local structure and texture information.
- Similar regions have similar statistics.



# RegCov Smoothing - Formulation

$$I = S + T$$

$$S(\mathbf{p}) = \frac{1}{Z_{\mathbf{p}}} \sum_{\mathbf{q} \in N(\mathbf{p}, r)} w_{\mathbf{p}\mathbf{q}} I(\mathbf{q})$$



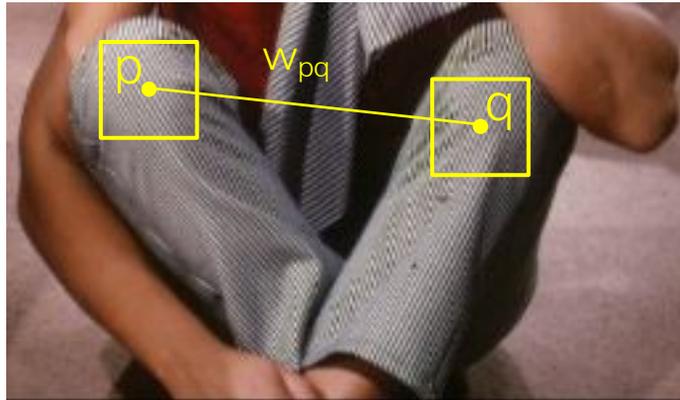
- Structure-texture decomposition via smoothing
- Smoothing as weighted averaging
- Different kernels ( $w_{\mathbf{p}\mathbf{q}}$ ) result in different types of filters.
- Three novel patch-based kernels for structure texture decomposition.
  
- L. Karacan, A. Erdem, E. Erdem, "Structure Preserving Image Smoothing via Region Covariances", ACM TOG 2013 (SIGGRAPH Asia 2013)

# RegCov Smoothing – Model 1

- Depends on sigma-points representation of covariance matrices (Hong et al., CVPR'09)

$\mathbf{C} = \mathbf{L}\mathbf{L}^T$  Cholesky Decomposition

$\mathcal{S} = \{\mathbf{s}_i\}$  Sigma Points  $\mathbf{s}_i = \begin{cases} \alpha\sqrt{d}\mathbf{L}_i & \text{if } 1 \leq i \leq d \\ -\alpha\sqrt{d}\mathbf{L}_i & \text{if } d+1 \leq i \leq 2d \end{cases}$



Final representation

$$\Psi(\mathbf{C}) = (\mu, \mathbf{s}_1, \dots, \mathbf{s}_d, \mathbf{s}_{d+1}, \dots, \mathbf{s}_{2d})^T$$

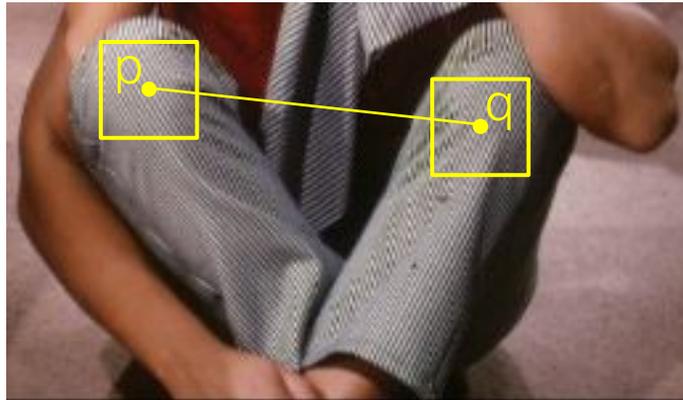
Resulting kernel function

$$w_{\mathbf{p}\mathbf{q}} \propto \exp\left(-\frac{\|\Psi(\mathbf{C}_{\mathbf{p}}) - \Psi(\mathbf{C}_{\mathbf{q}})\|^2}{2\sigma^2}\right)$$

# RegCov Smoothing – Model 2

- An alternative way is to use statistical similarity measures.
- A Mahalanobis-like distance measure to compare to image patches.

$$d(\mathbf{p}, \mathbf{q}) = \sqrt{(\mu_{\mathbf{p}} - \mu_{\mathbf{q}}) \mathbf{C}^{-1} (\mu_{\mathbf{p}} - \mu_{\mathbf{q}})^T}$$



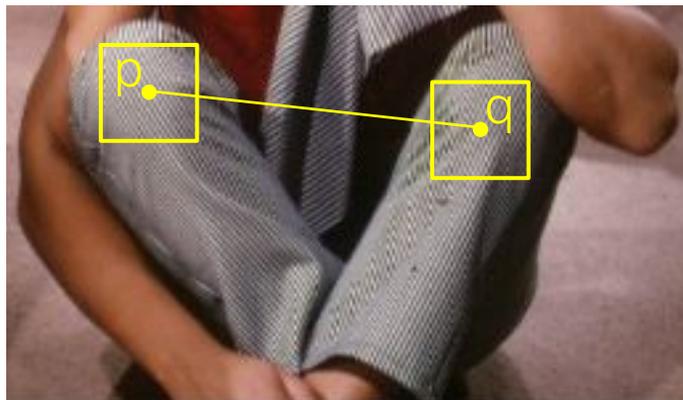
$$\mathbf{C} = \mathbf{C}_{\mathbf{p}} + \mathbf{C}_{\mathbf{q}}$$

Resulting kernel

$$w_{\mathbf{p}\mathbf{q}} \propto \exp\left(-\frac{d(\mathbf{p}, \mathbf{q})^2}{2\sigma^2}\right)$$

# RegCov Smoothing – Model 3

- We use Kullback-Leibler(KL)-Divergence measure from probability theory.
- A KL-Divergence form is used to calculate statistical distance between two multivariate normal distribution



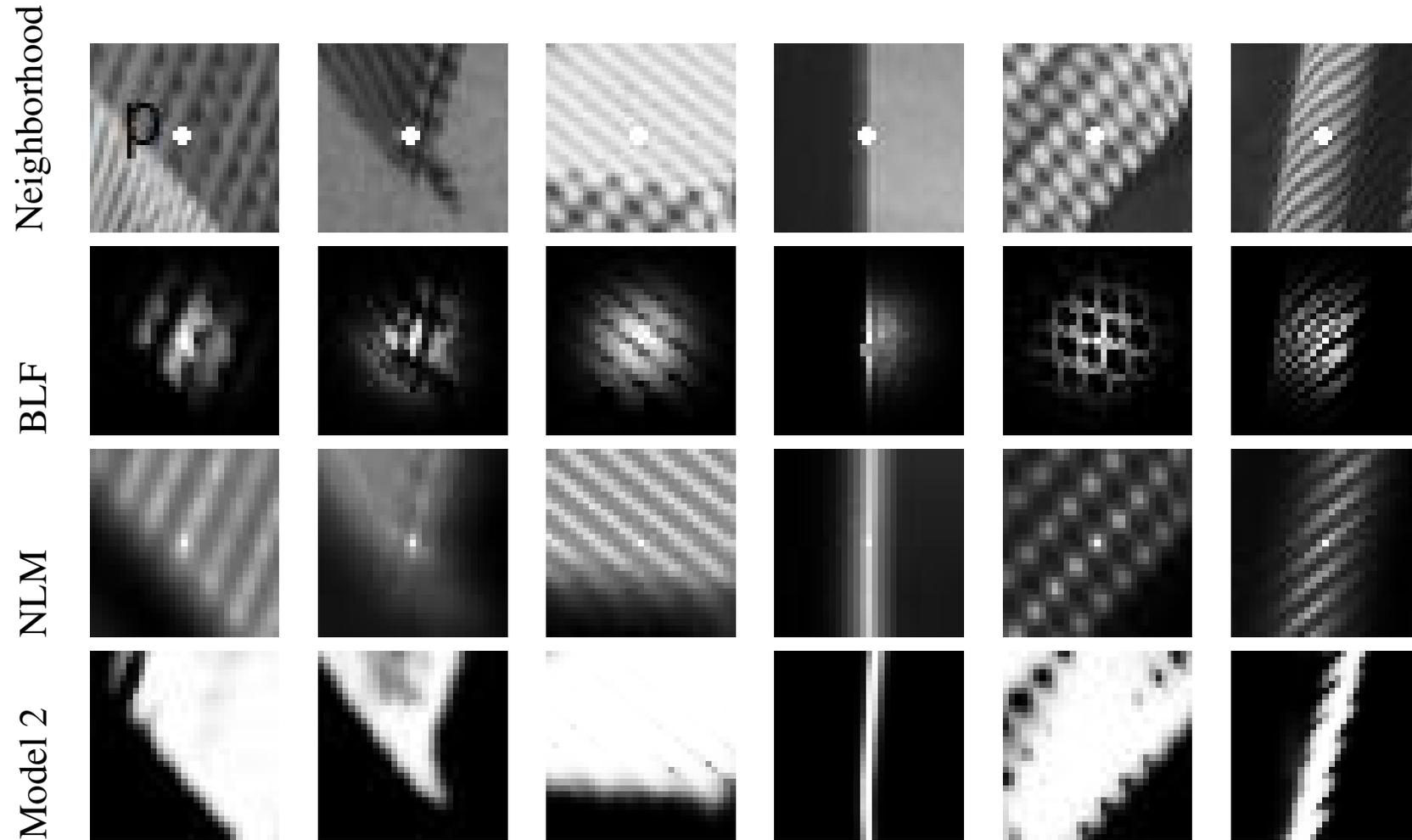
$$d_{KL}(\mathbf{p}, \mathbf{q}) = \frac{1}{2} \left( \text{tr}(\mathbf{C}_{\mathbf{q}}^{-1} \mathbf{C}_{\mathbf{p}}) + (\mu_{\mathbf{p}} - \mu_{\mathbf{q}})^T \mathbf{C}_{\mathbf{q}}^{-1} (\mu_{\mathbf{p}} - \mu_{\mathbf{q}}) - k - \ln \left( \frac{\det \mathbf{C}_{\mathbf{p}}}{\det \mathbf{C}_{\mathbf{q}}} \right) \right)$$

Resulting kernel

$$w_{pq} \propto \frac{d_{KL}(\mathbf{p}, \mathbf{q})}{2\sigma^2}$$

resulted from a discussion with Rahul Narain (Berkeley University)

# RegCov Smoothing – Smoothing Kernels



# Results



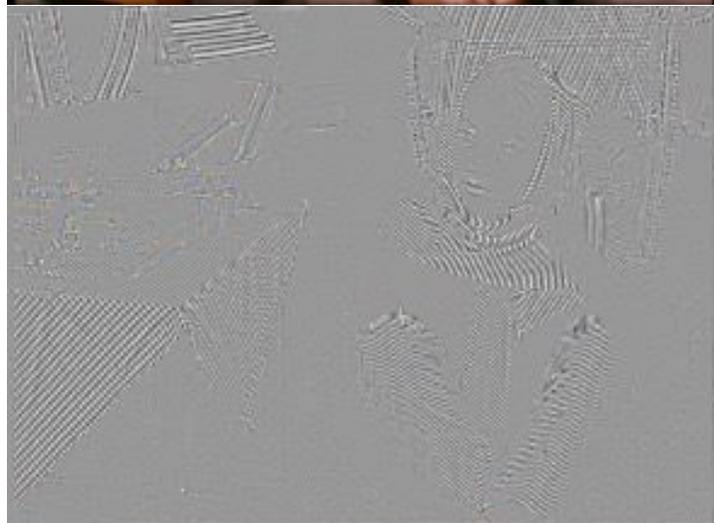
Input

# Results

Model1



Model2

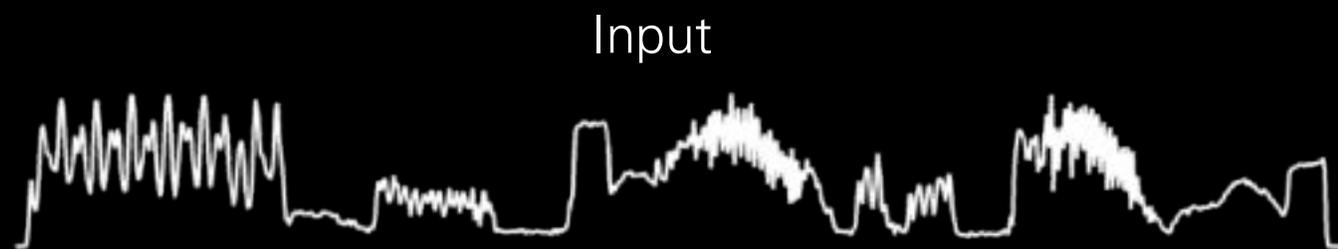


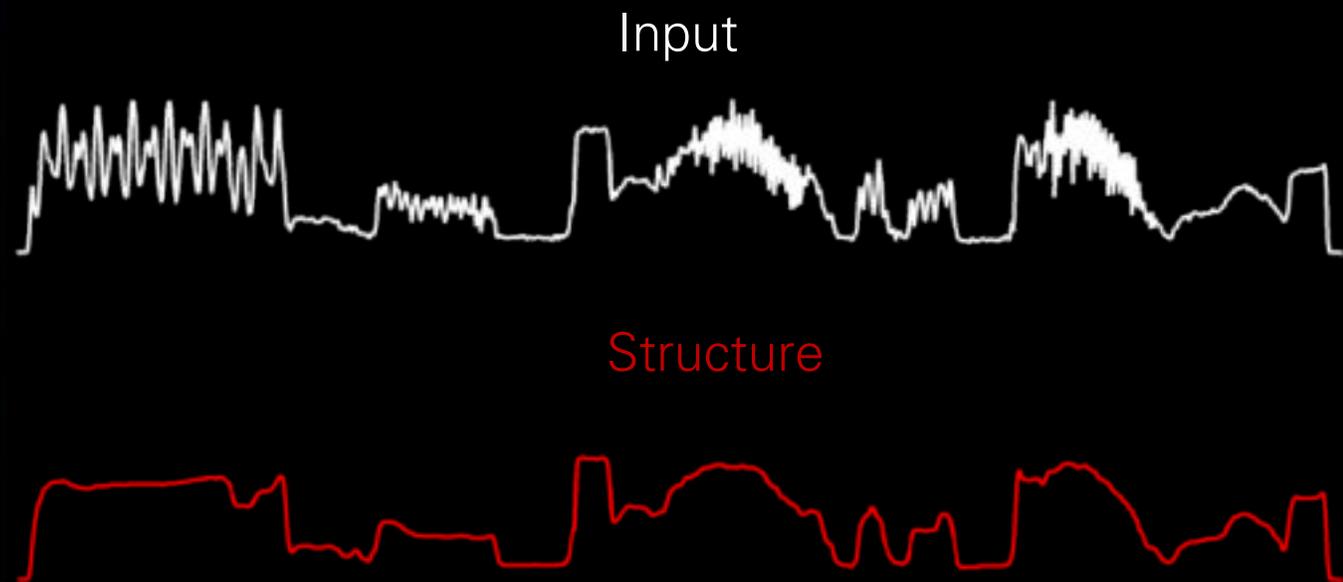
Model3





Input

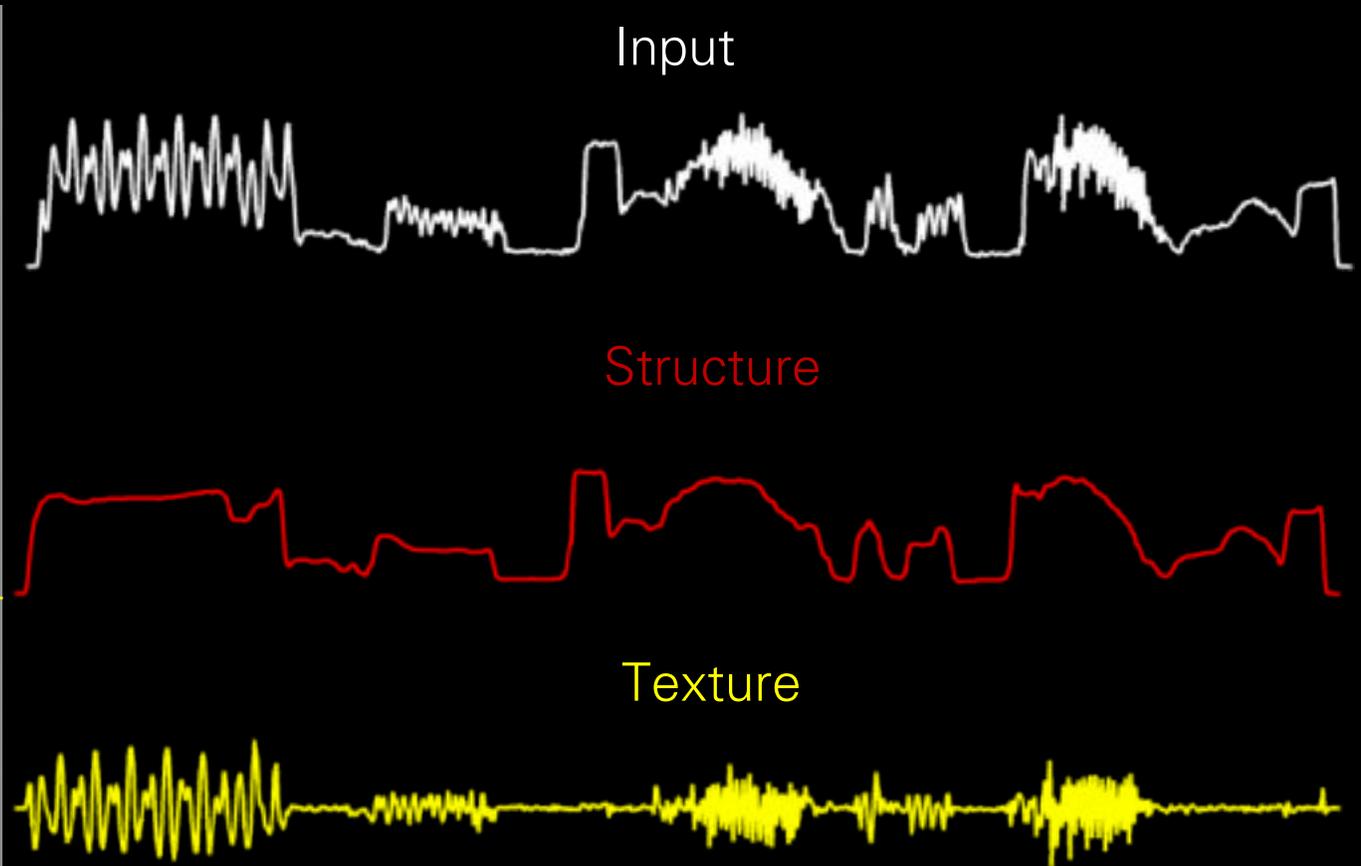




Model2 Structure



Model2 Texture

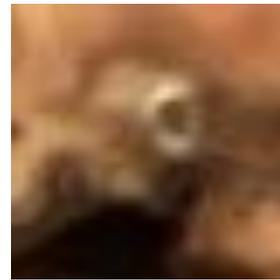
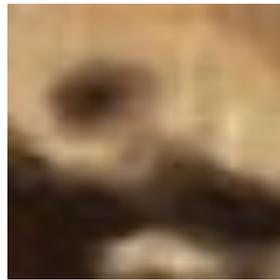
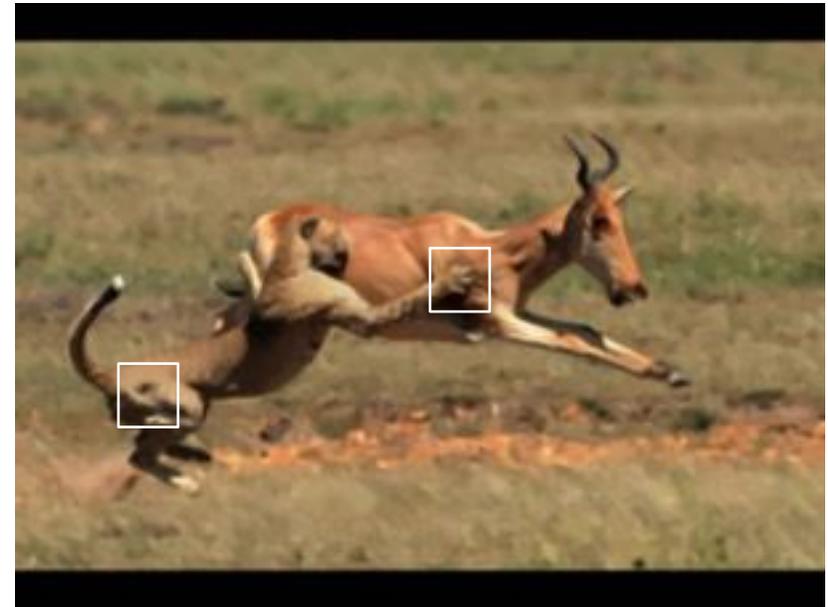
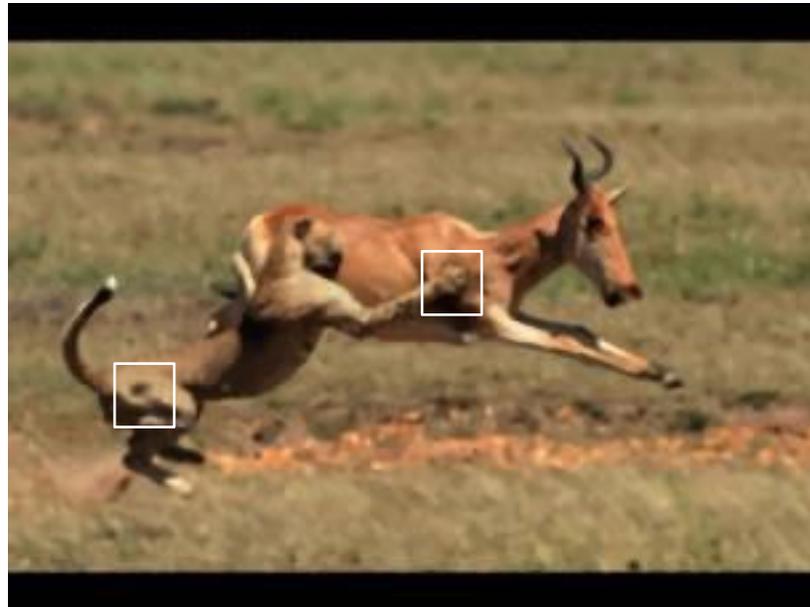
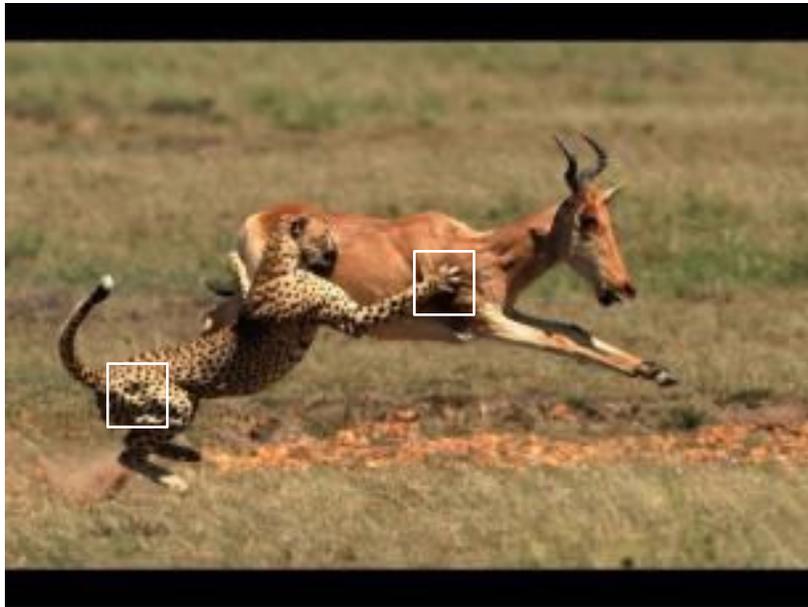


# Results

Input

Model2

Model3



# Experimental evaluation



Input

# Experimental evaluation



TV

Rudin et al. 1992

# Experimental evaluation



Bilateral  
Filter

# Experimental evaluation



Envelope  
Extraction

Subr et al. 2009

# Experimental evaluation



RTV

Xu et al. 2012

# Experimental evaluation



Model 1

# Experimental evaluation



Model 2

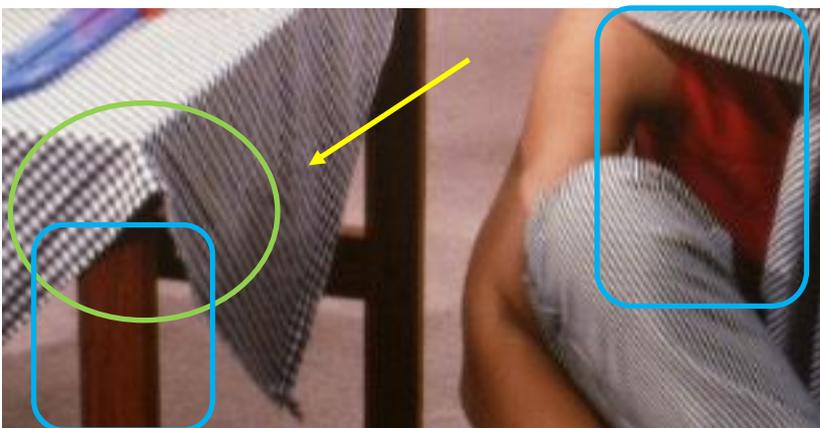
# Experimental evaluation



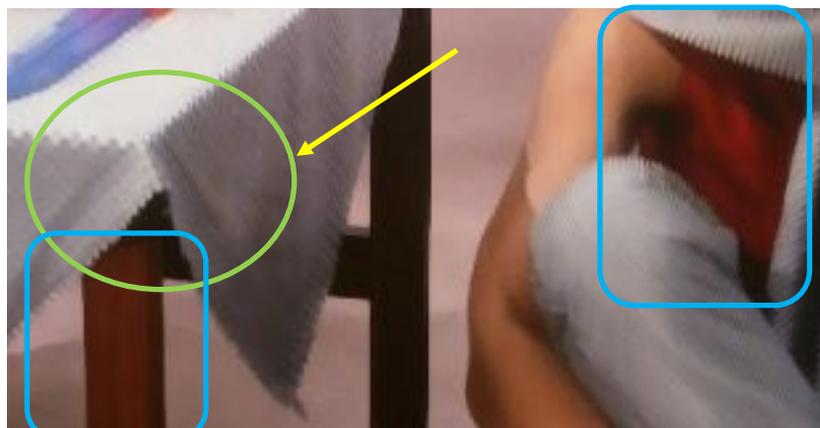
Model 3

# Experimental evaluation

Input



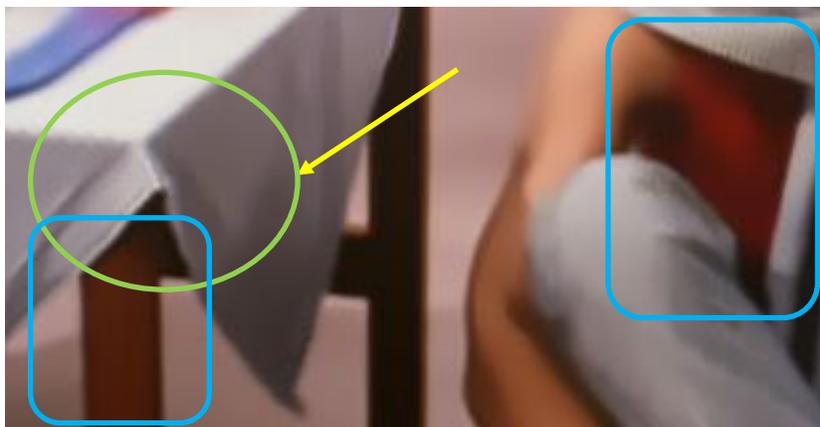
Local Exrema



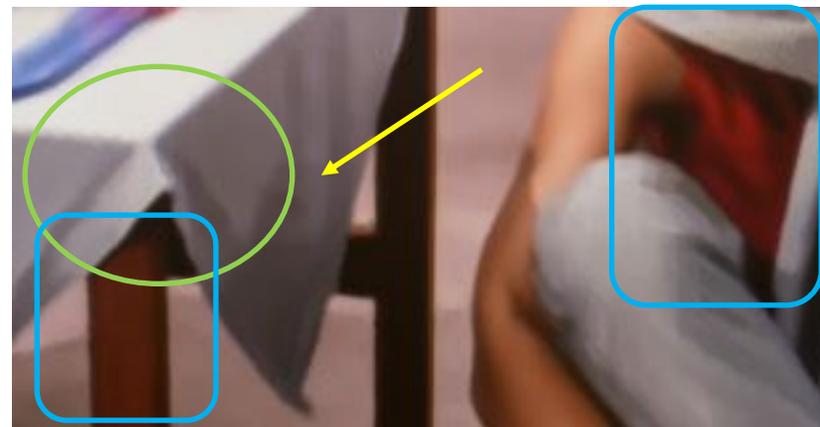
RTV



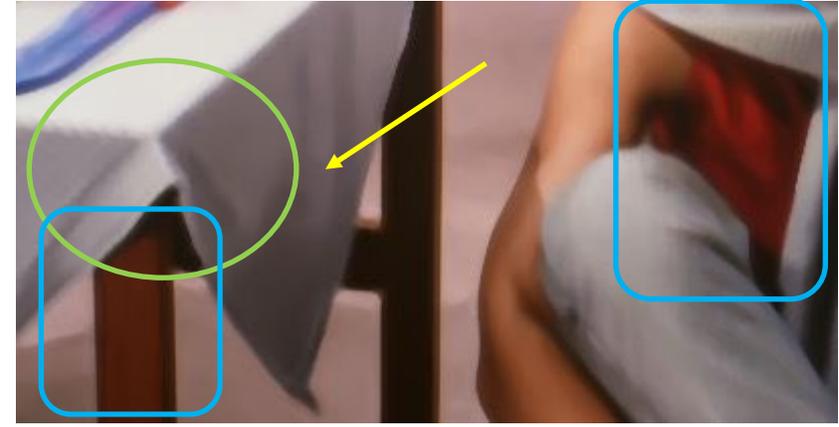
Model1



Model2



Model3

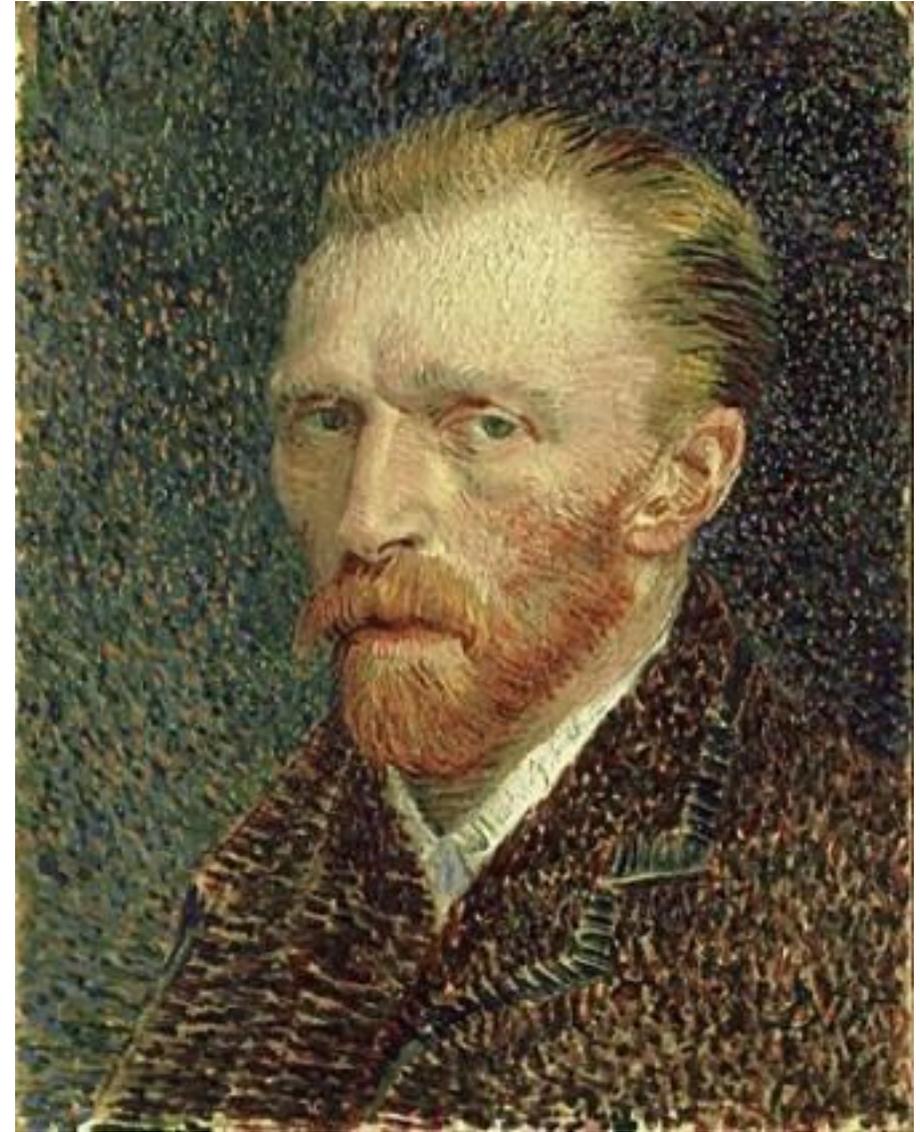


Shading preserved

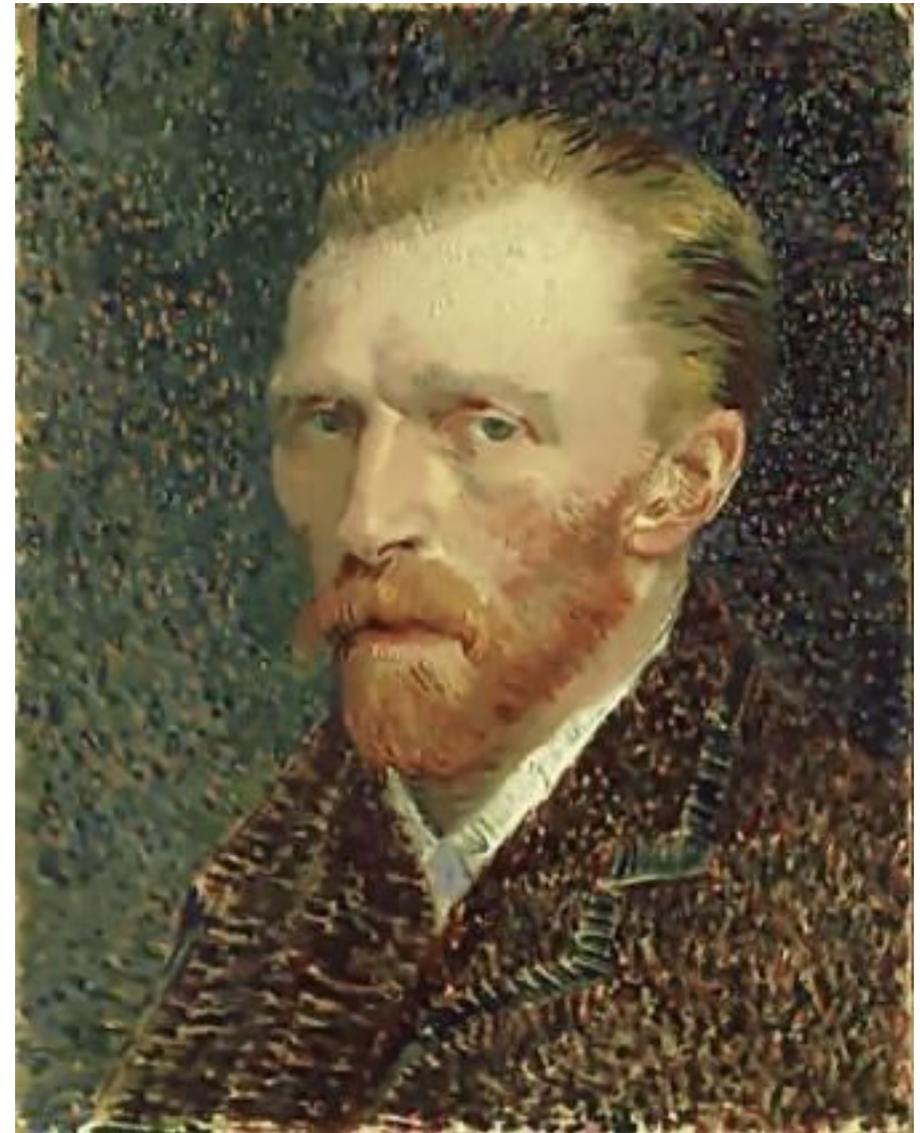
Structure preserved

No unintuitive edge

# Multiscale decomposition



# Multiscale decomposition



$S_1(k = 5)$

# Multiscale decomposition



$S_2(k = 7)$

# Multiscale decomposition



$S_3(k = 9)$

# Challenging cases

Input



Model2

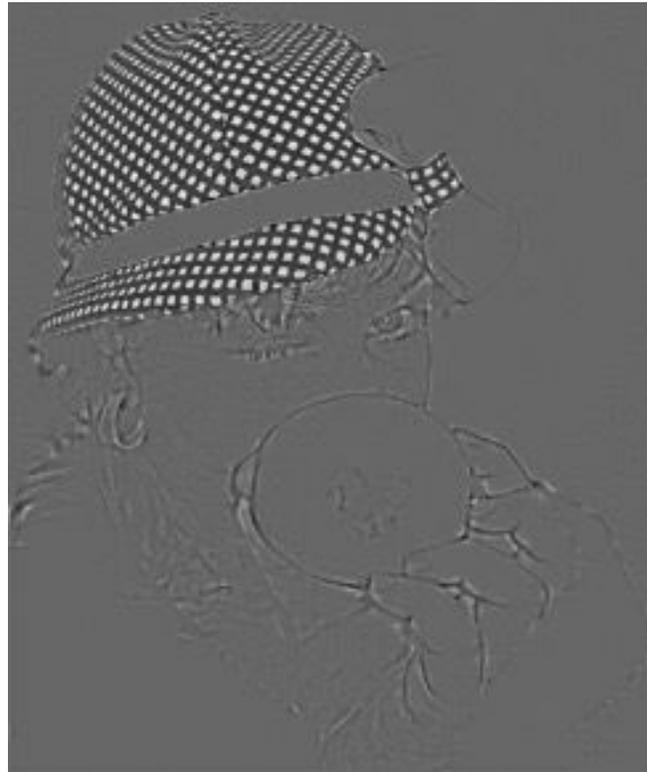


# Challenging cases

Input



Model2 Texture



Model2+Model1



# Edge detection



# Edge detection



# Edge detection

Canny edges of original image



Canny edges of smoothed image



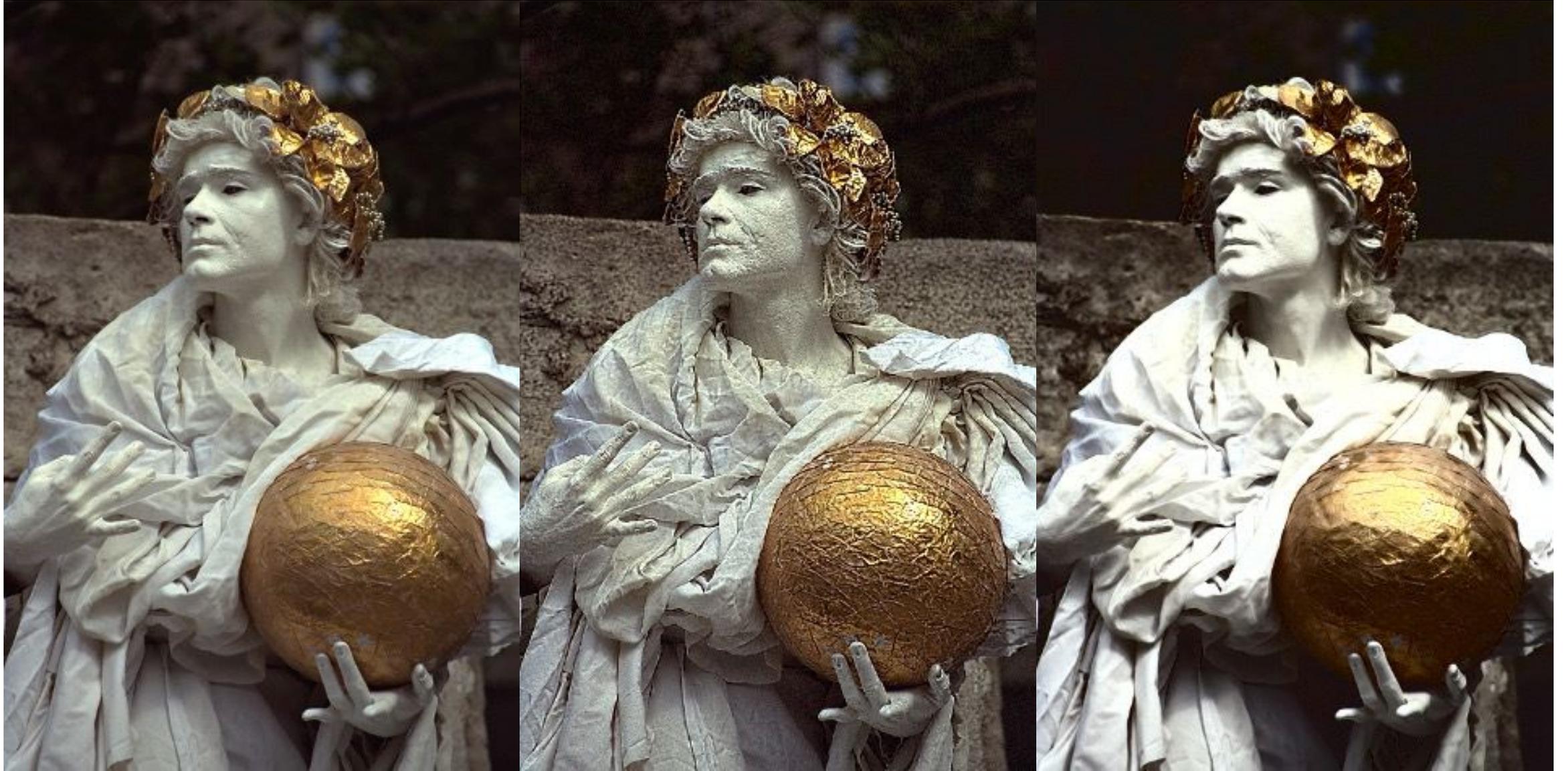
# Image abstraction



# Image abstraction



# Detail boosting

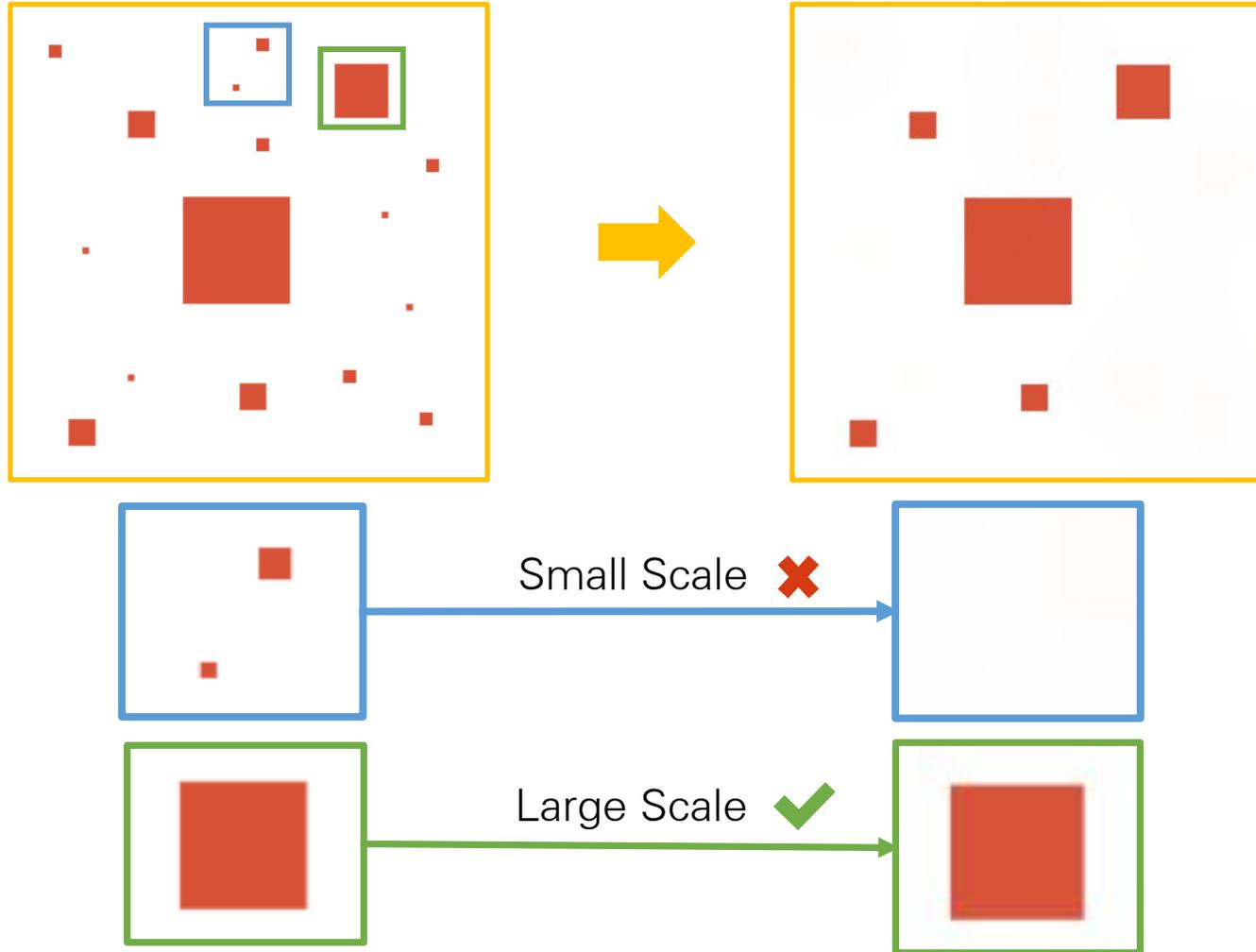


# Image composition



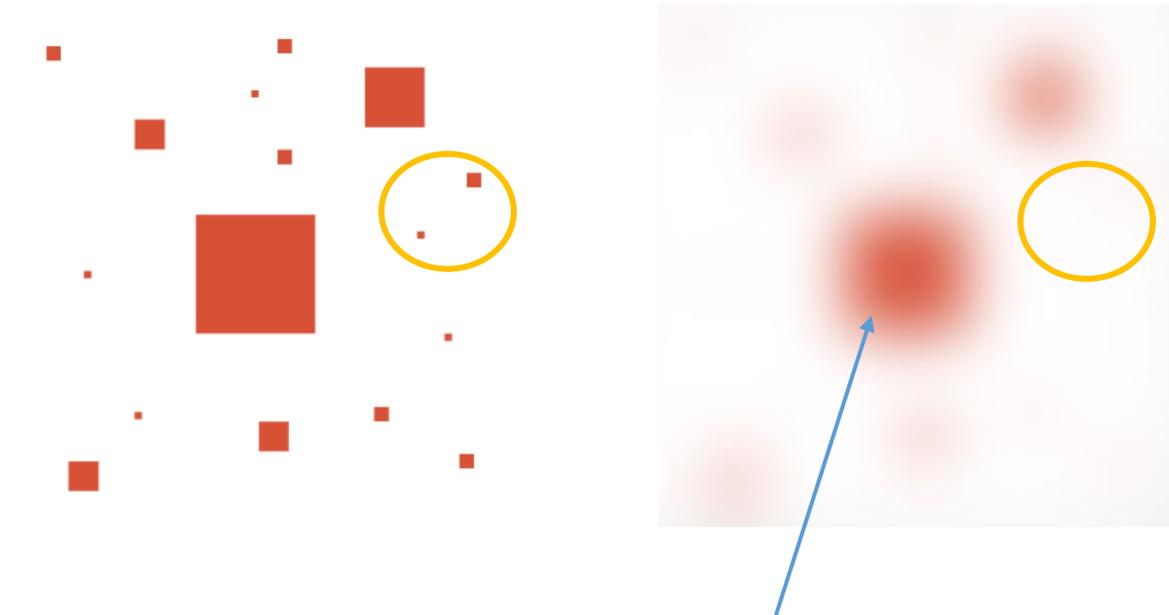
# Rolling Guidance Filter

# Scale-Aware Filtering

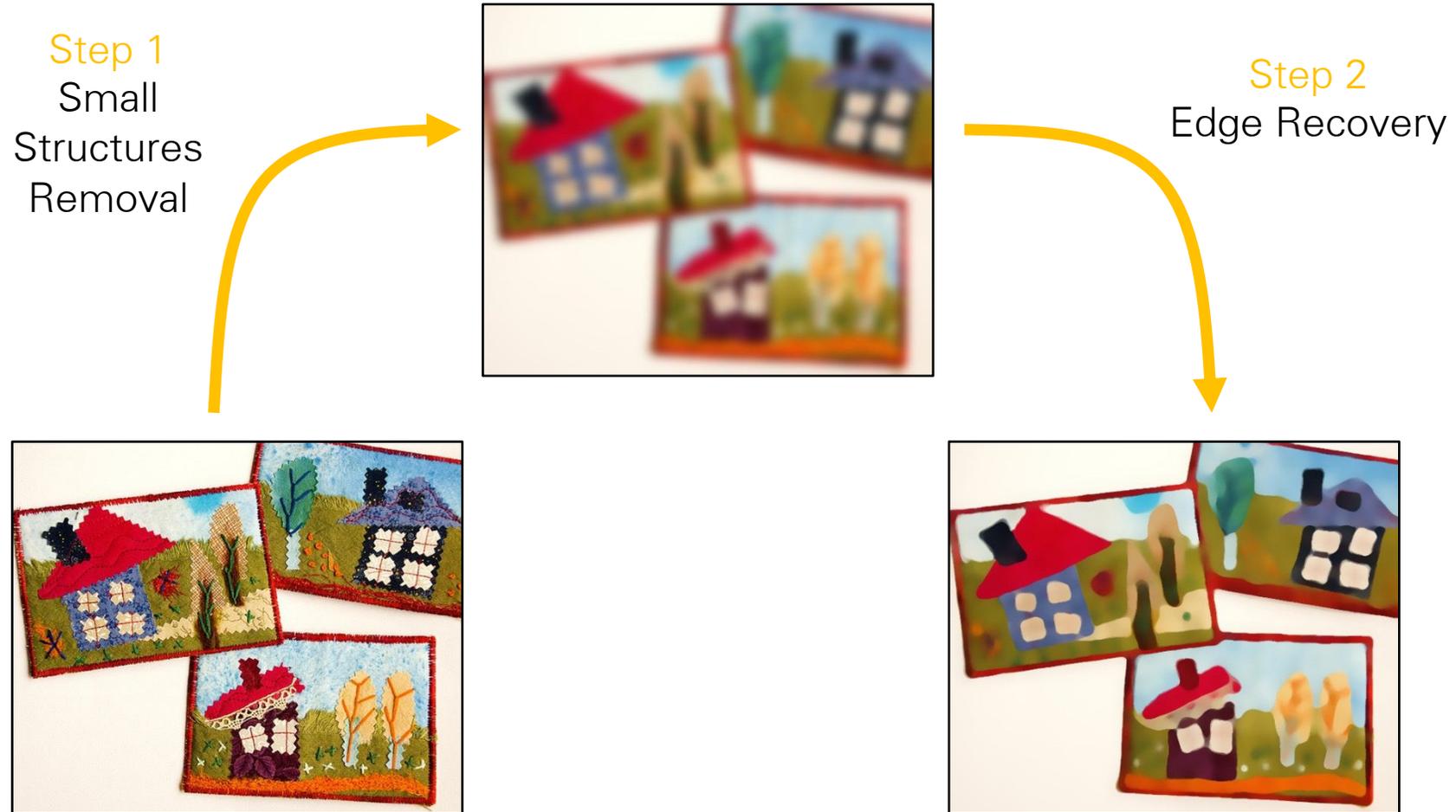


# Main Idea

- Scale Space Theory [Lindeberg, 1994]:
  - An object of size  $t$ , will be largely smoothed away with Gaussian filter of variance  $t^2$ .



# RGF: A scale-aware Filter



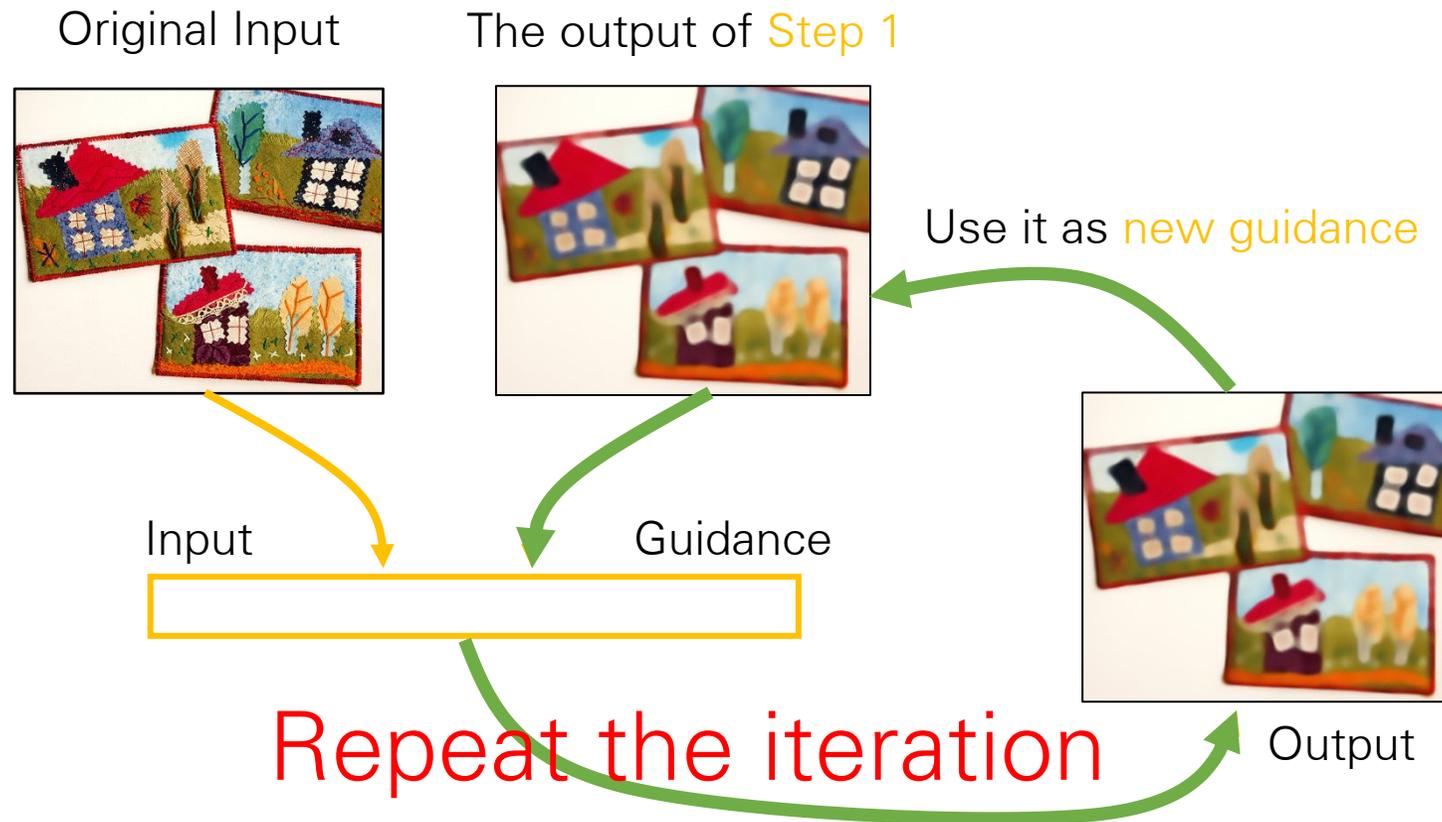
# Step 1: Small Structures Removal

Gaussian Filter

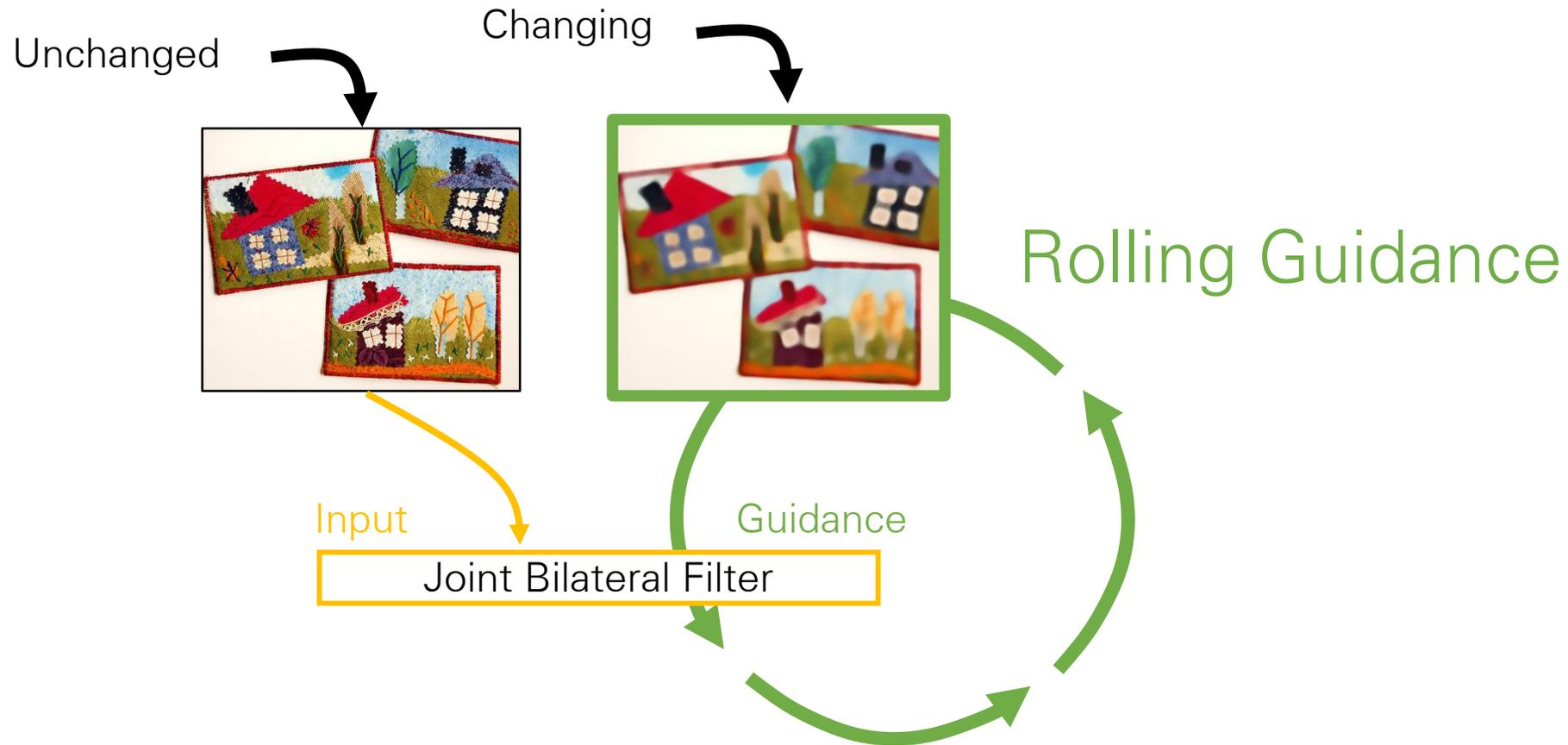


# Step 2: Edge Recovery

- A rolling guidance



# Rolling Guidance



# Rolling Guidance



Guidance for the 1st iteration

# Rolling Guidance



Guidance for the 2nd  
iteration

# Rolling Guidance



Guidance for the 3rd iteration

# Rolling Guidance



Guidance for the 5th iteration

# Rolling Guidance



Input



Output



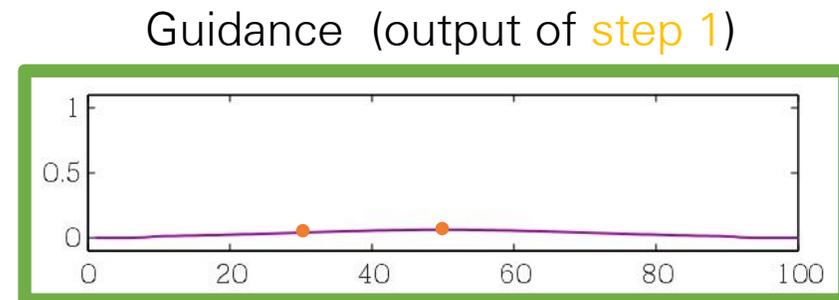
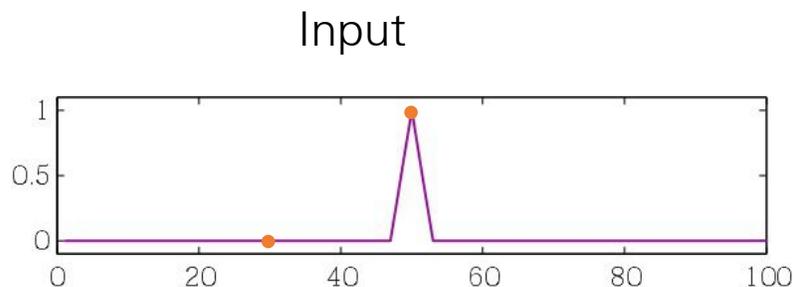
Small structures are **removed**.  
Large structure are **NOT** blurred.

# Implementation

Rolling Guidance Filter (RGF) has only **1 line** of code

```
1 Mat rollingGuidanceFilter(Mat im, float scale, int iter){  
2     Mat res = im.mul(0);  
3     while(iter-->0) res = bilateralFilter(im, res, scale, SIGMA_R);  
4     return res;  
5 }
```

# Small Structure

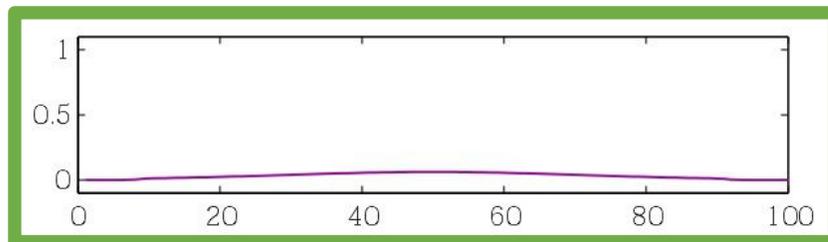


$$J^{t+1}(p) = \frac{1}{K_p} \sum_{q \in N(p)} \exp\left(-\frac{\|p - q\|^2}{2\sigma_s^2}\right)$$

It becomes a Gaussian filter

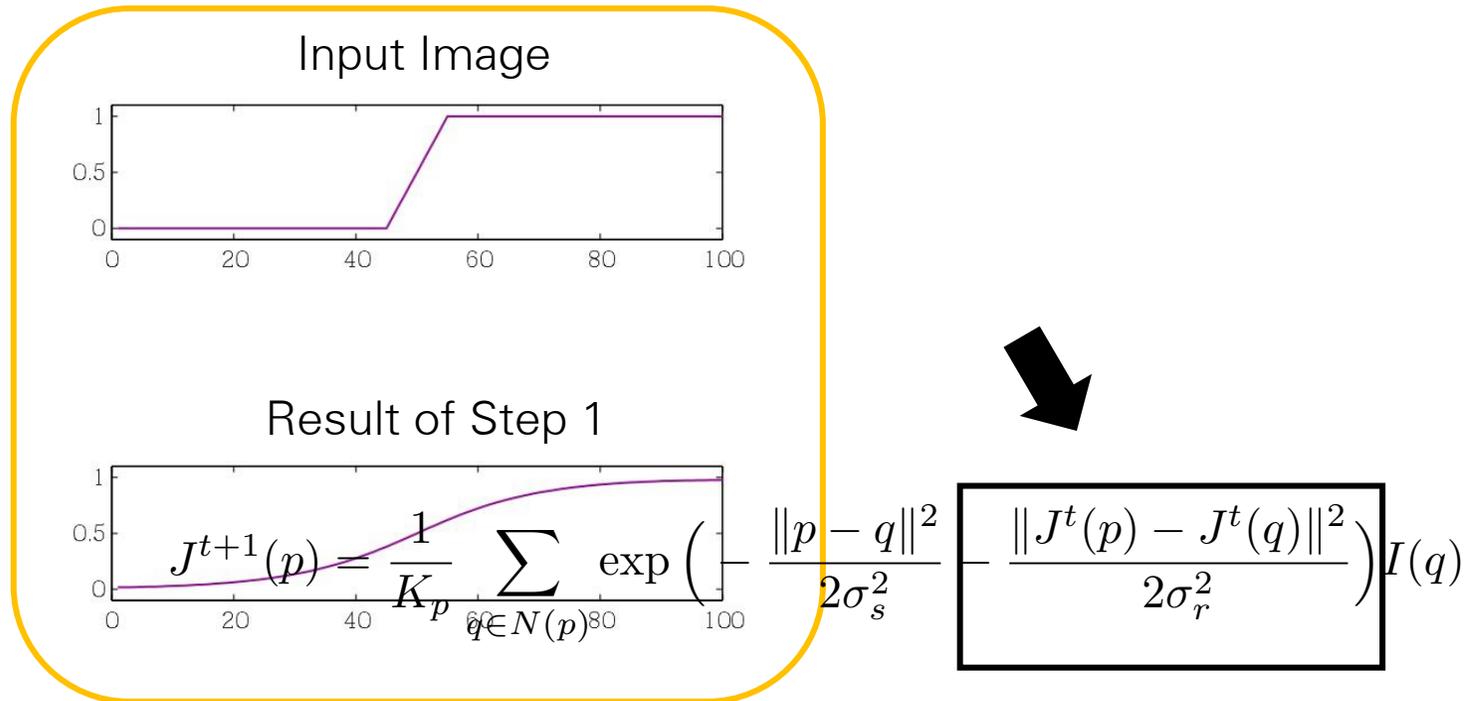


Joint Bilateral Filter



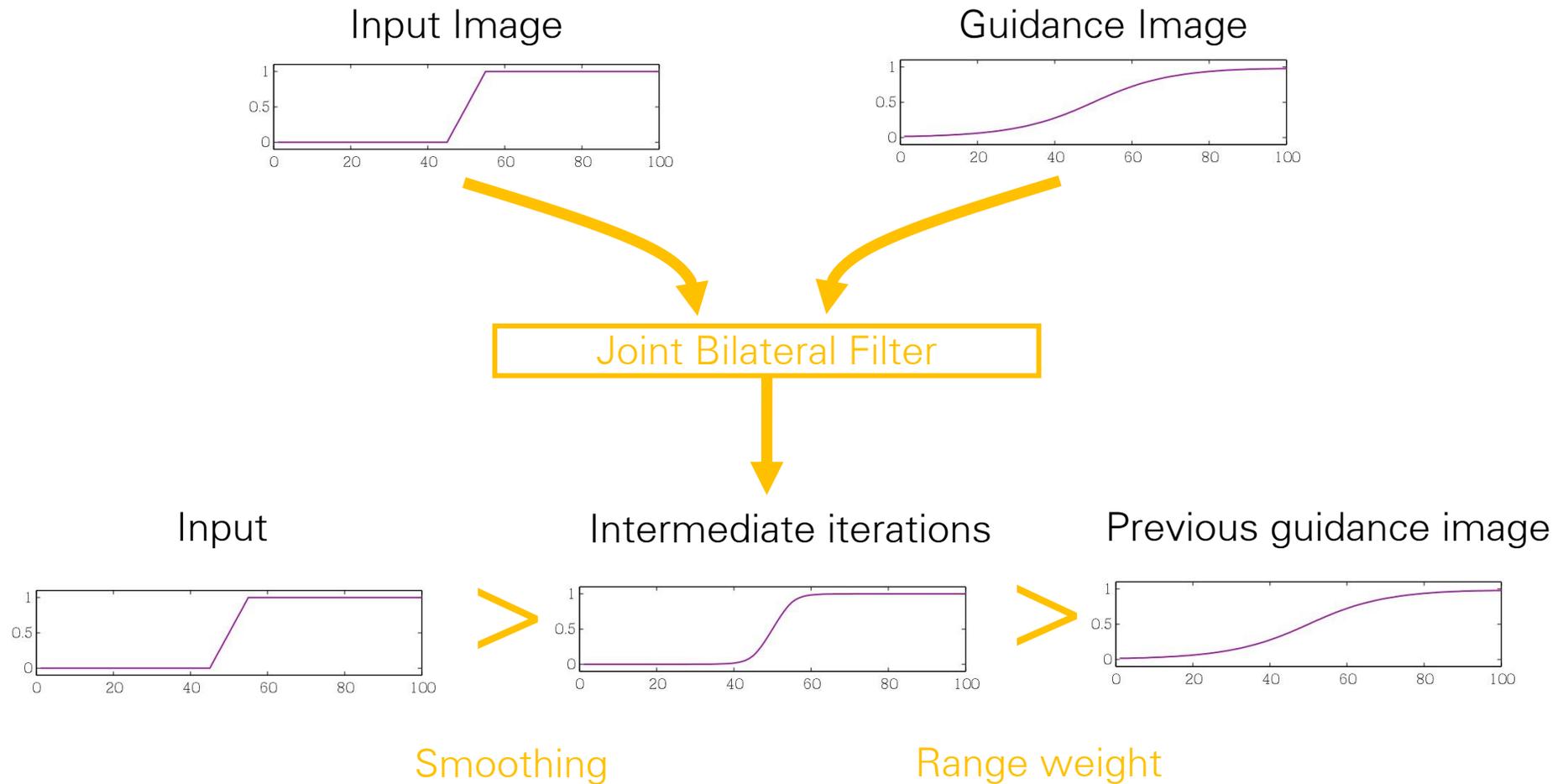
Same

# Large Structure



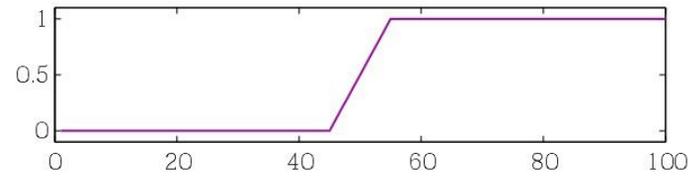
Due to this range weight  
It generates sharper results than Gaussian!

# Processing

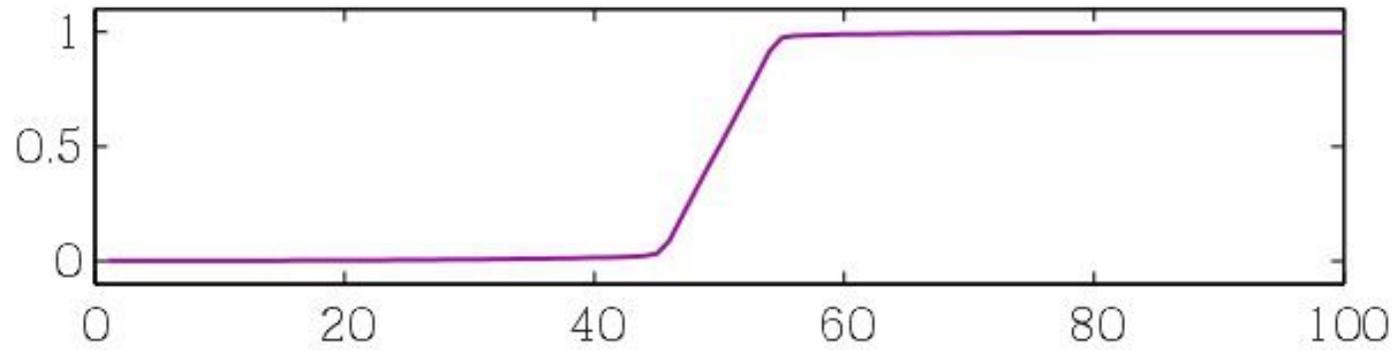


# Processing

Input Image



Guidance Image



**3<sup>rd</sup> Iteration**

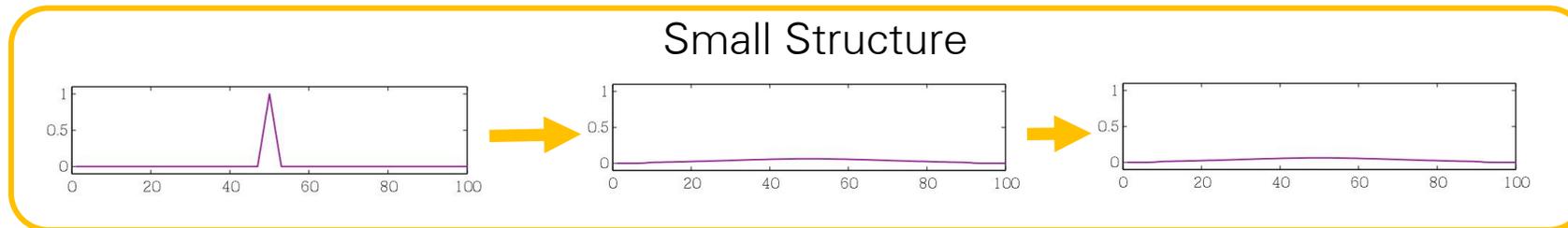
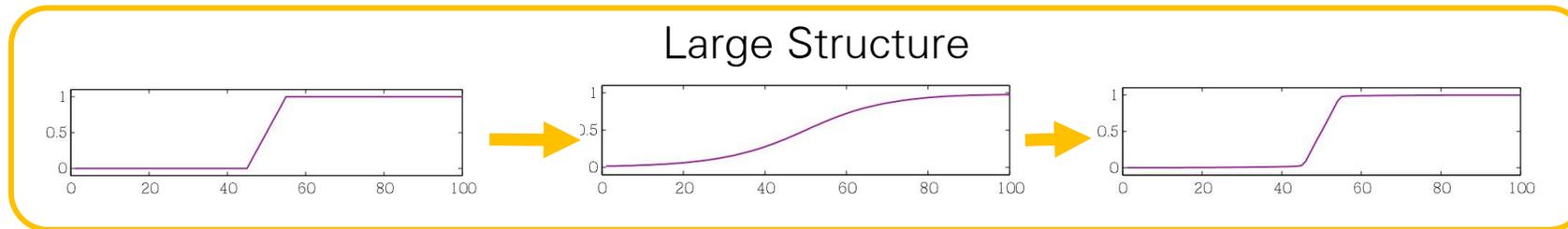
# Large Structure



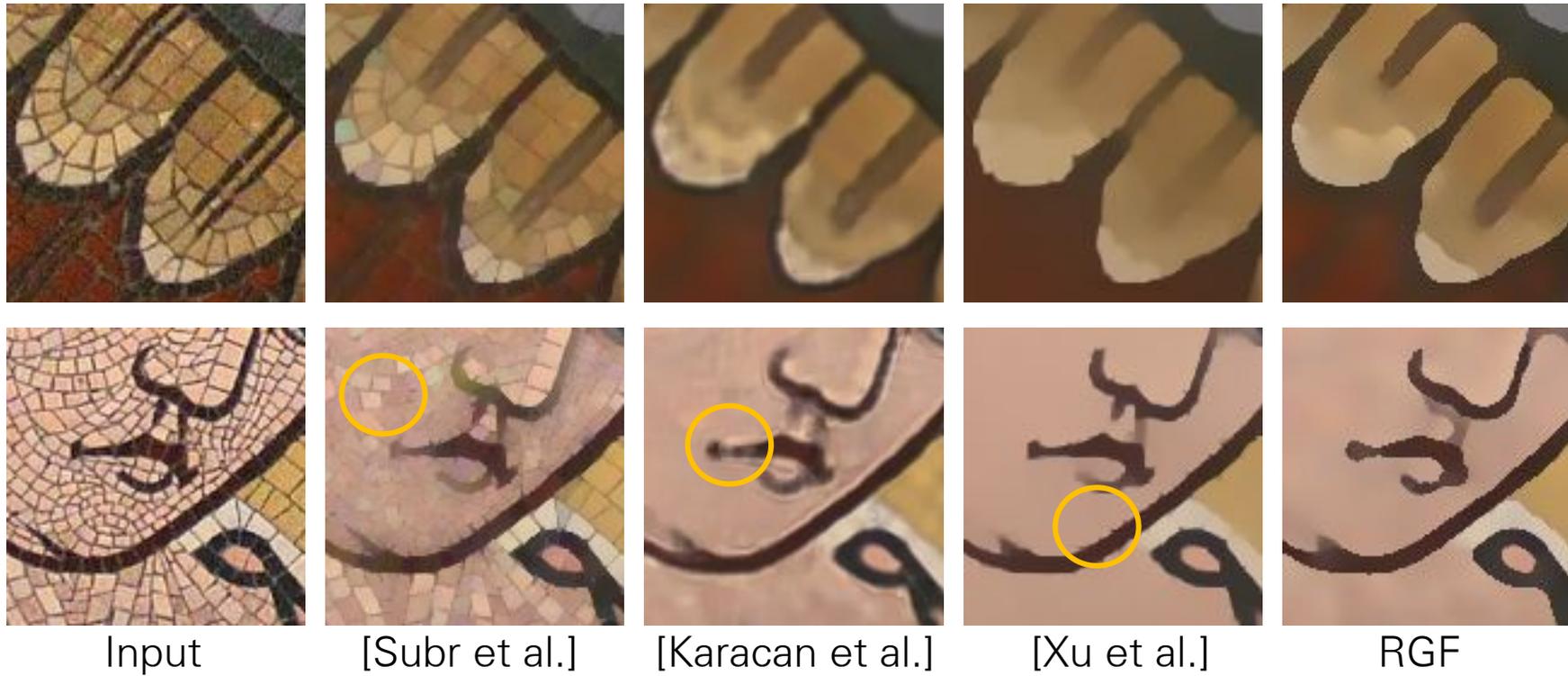
Take-home message

Rolling guidance recovers an edge as long as it still exists in the blurred image after Gaussian smoothing.

# Rolling Guidance Filter



# Result Comparison



# Performance Comparison



Input

RGF  
2013]

For 4 Megapixel Image

2

seconds

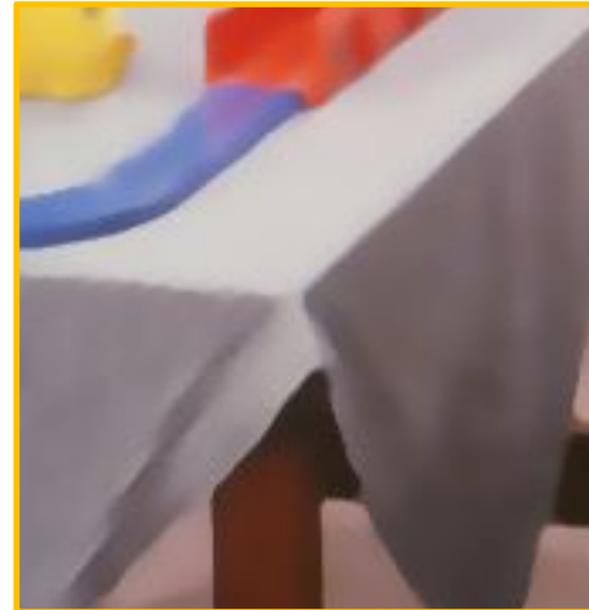
# Performance Comparison

Algorithms	Time (seconds/Megapixel)
Local Extrema [Subr et al., 2009]	95
RTV [Xu et al., 2012]	14
Region Covariance [Karacan et al., 2013]	240
RGF	0.05(Real-time)

# Texture Removal



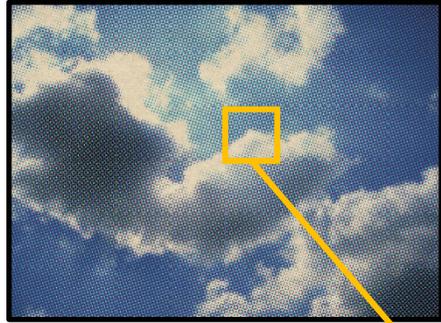
# Texture Removal



# Halftone Image

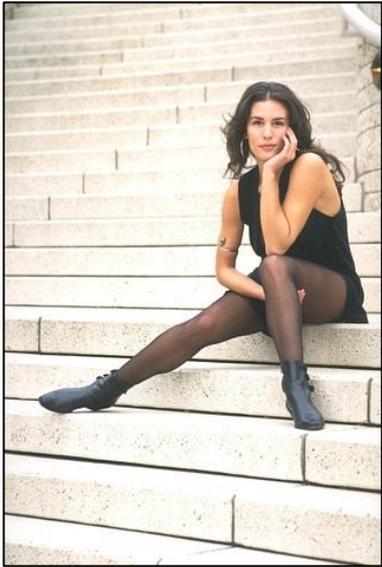


# Halftone Image



# Boundary detection

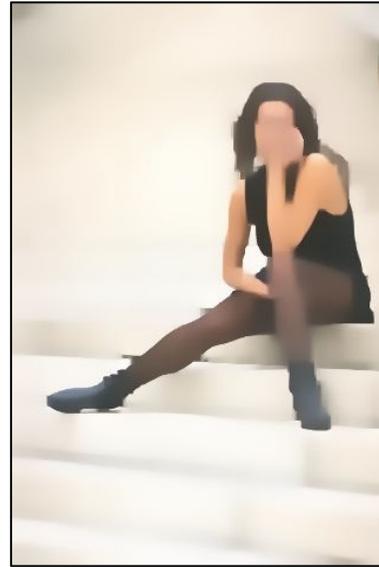
Input



Boundary Detection



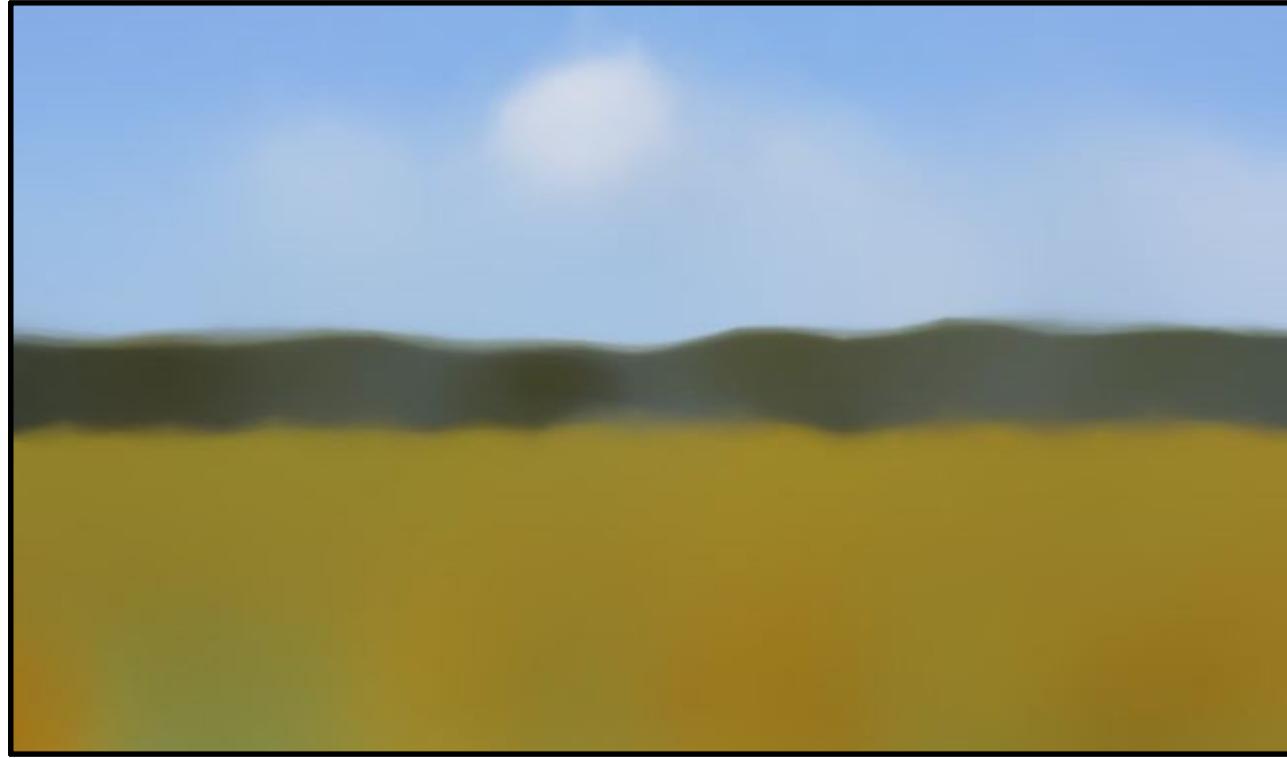
Filtered Input



Boundary Detection



# Multi-Scale Filtering



= 30

determine the scale.

# Limitations

- Sharp corners could be rounded
  - It is because sharp corner presents high frequency change.
  - In other words, sharp corners are small-scale structures.

# Recap

- Filtering plays a key role for many applications.
- Filtering by taking into account image content generally gives better results.

**Next Lecture:**  
Edge-aware filtering,  
Gradient-domain image  
processing