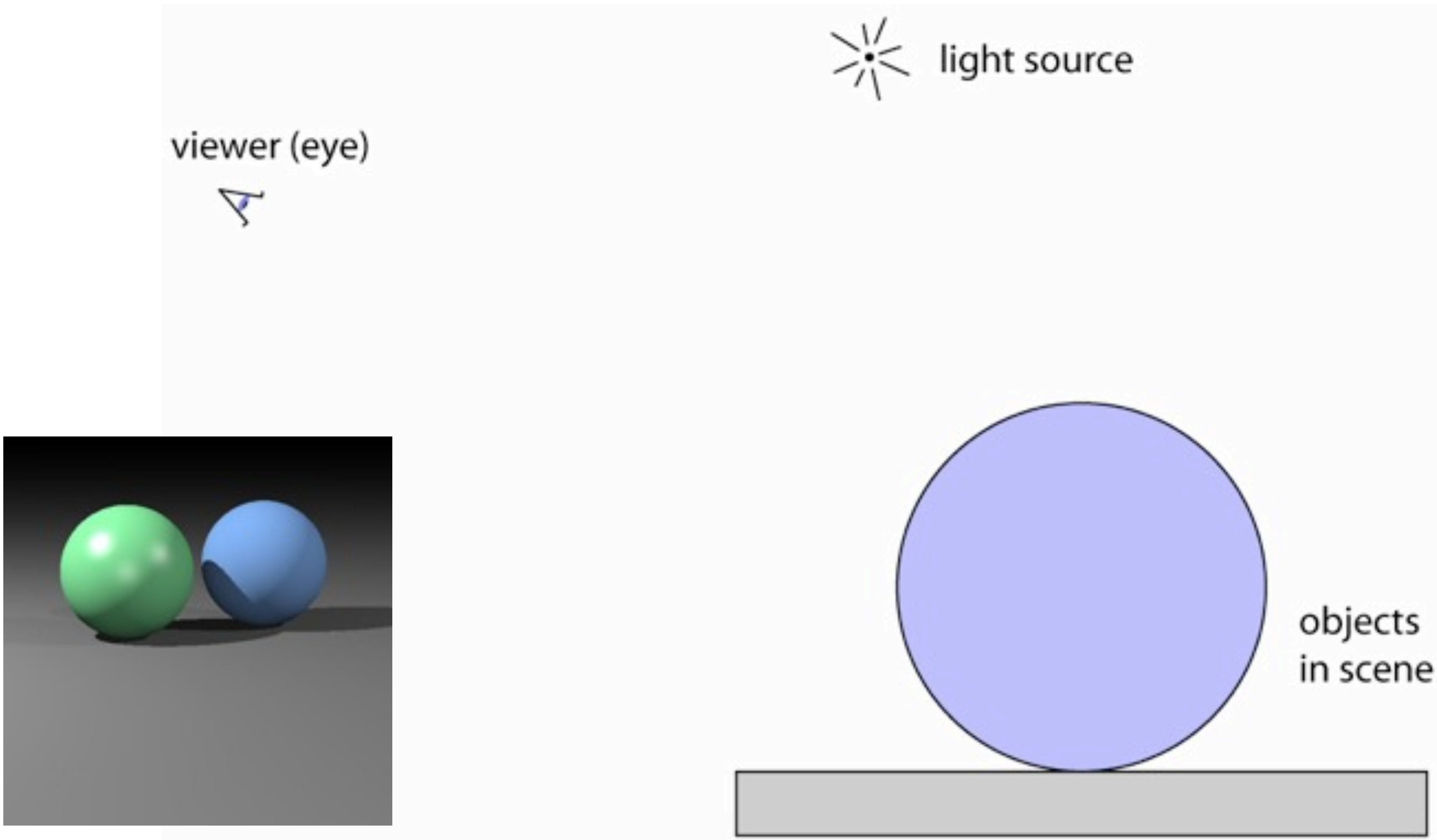
Ray Tracing

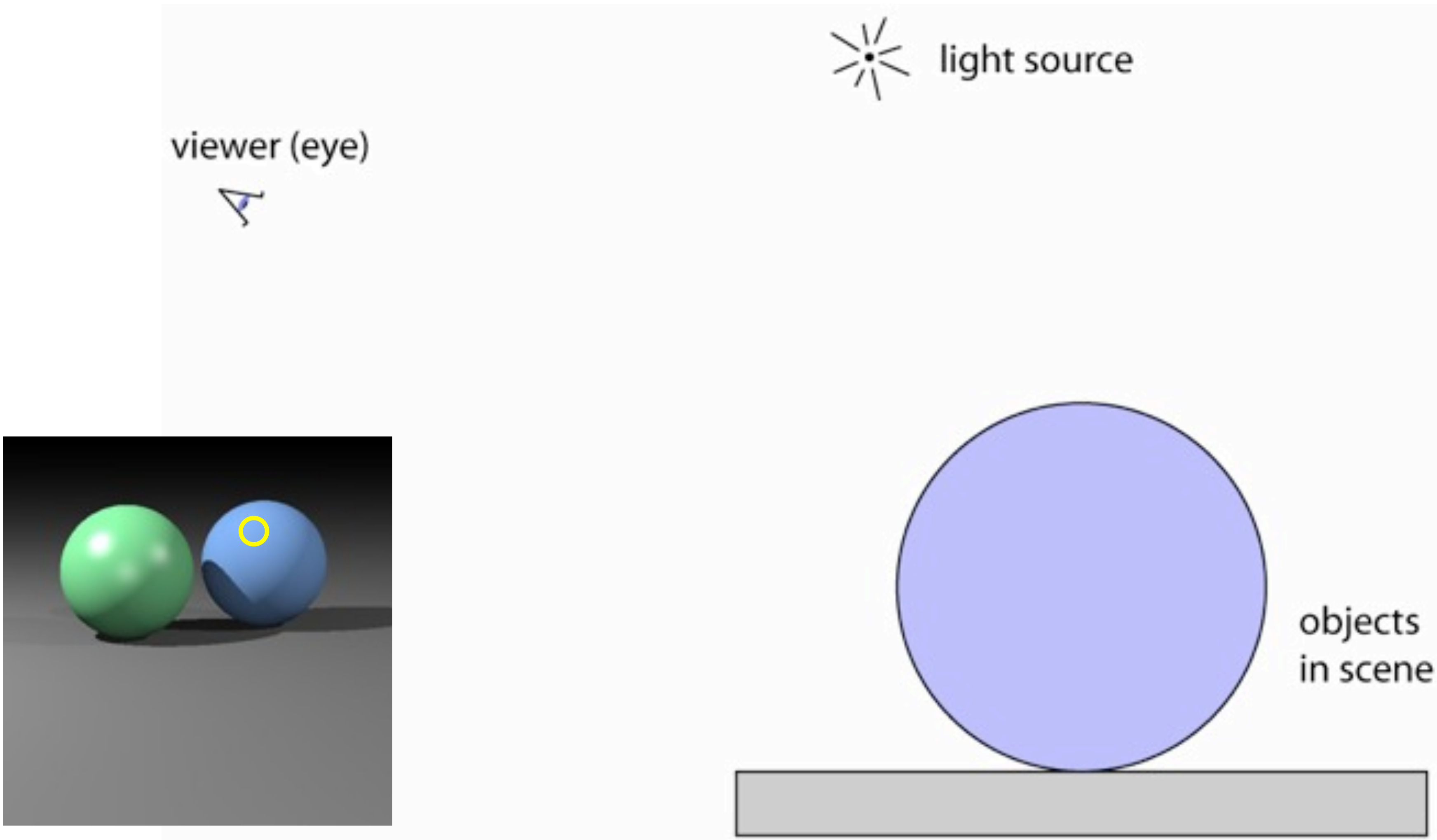
Week 3

Acknowledgement: The course slides are adapted from the slides prepared by Steve Marschner of Cornell University

Ray tracing idea



Ray tracing idea



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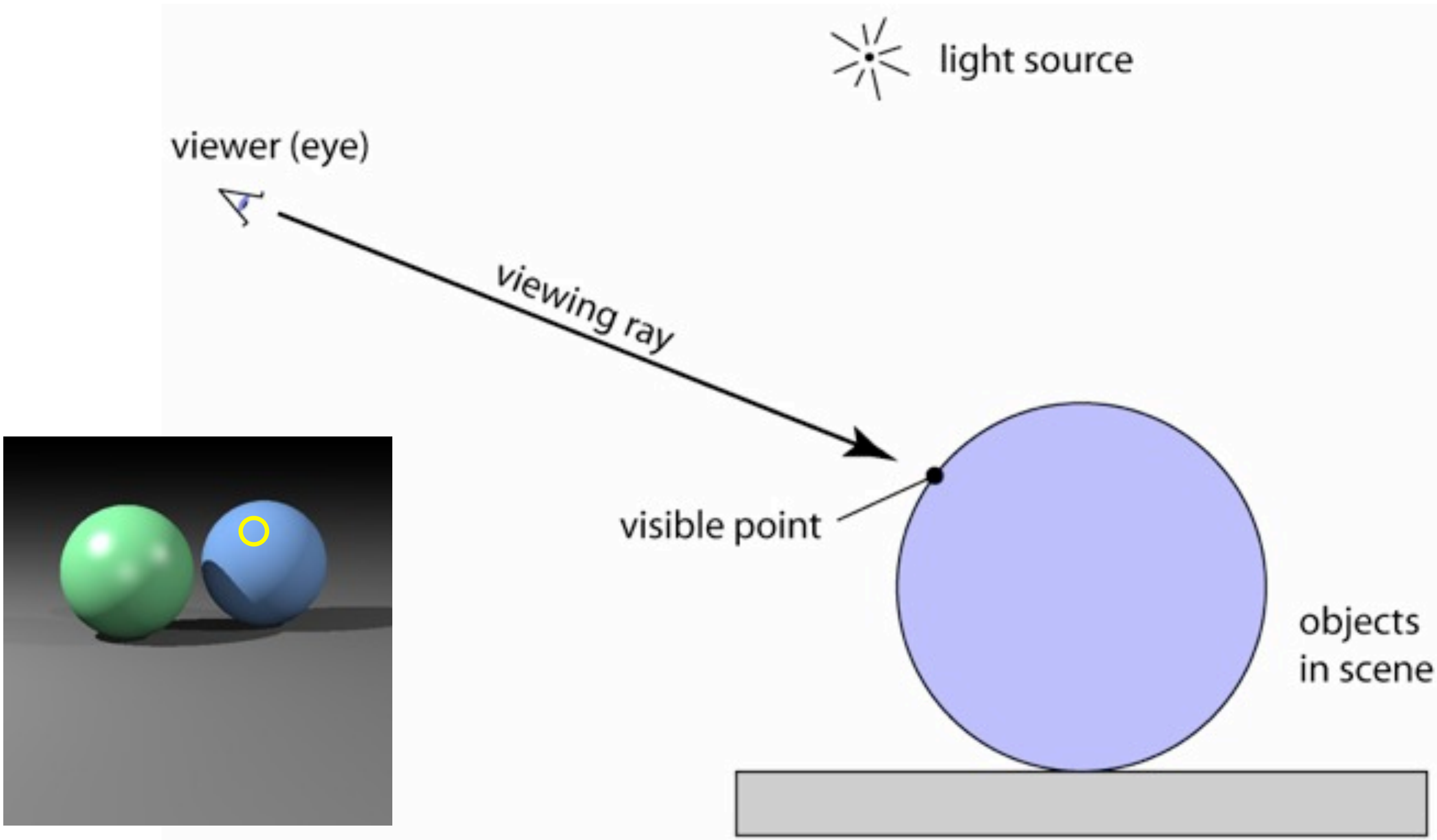

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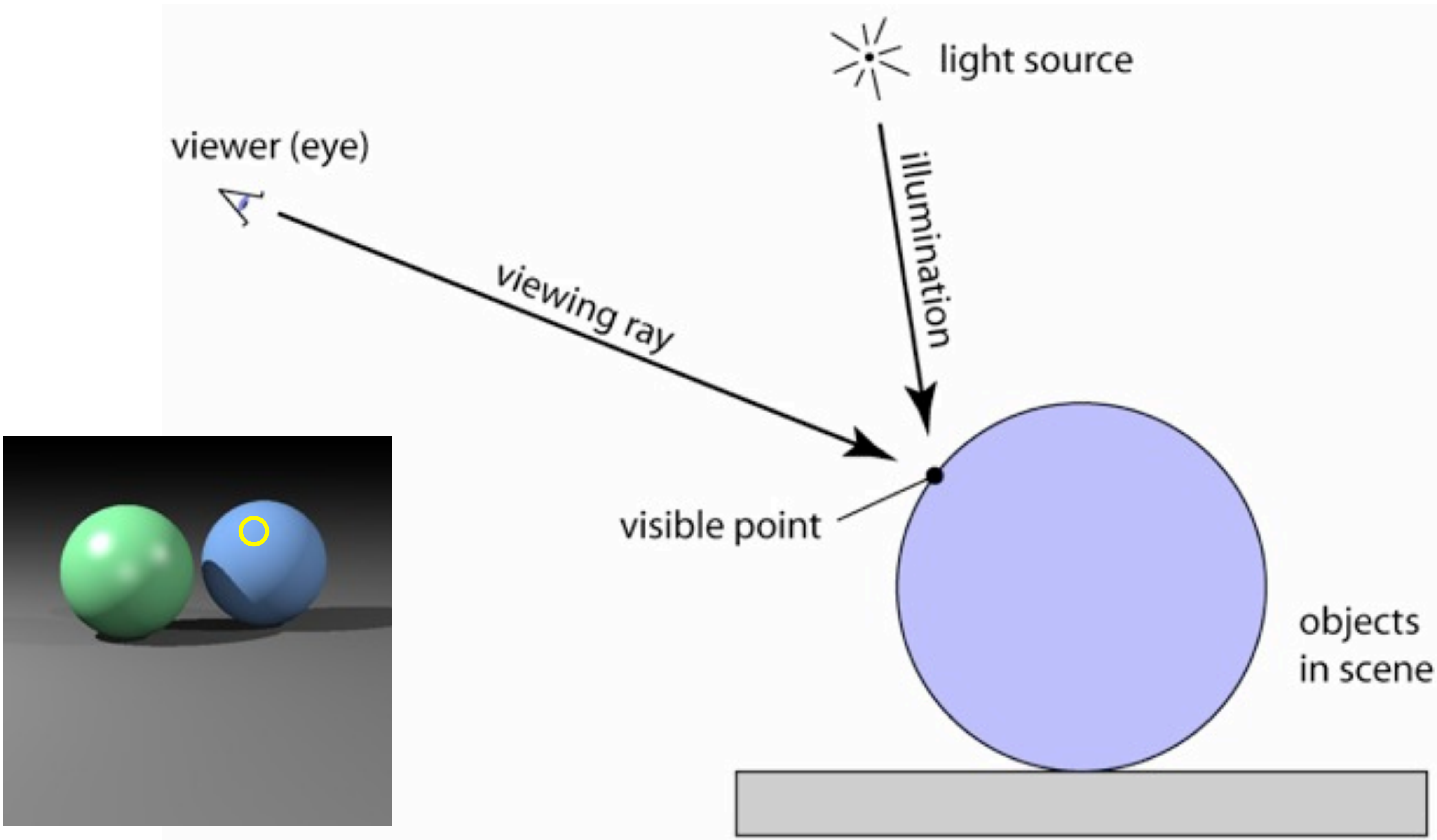
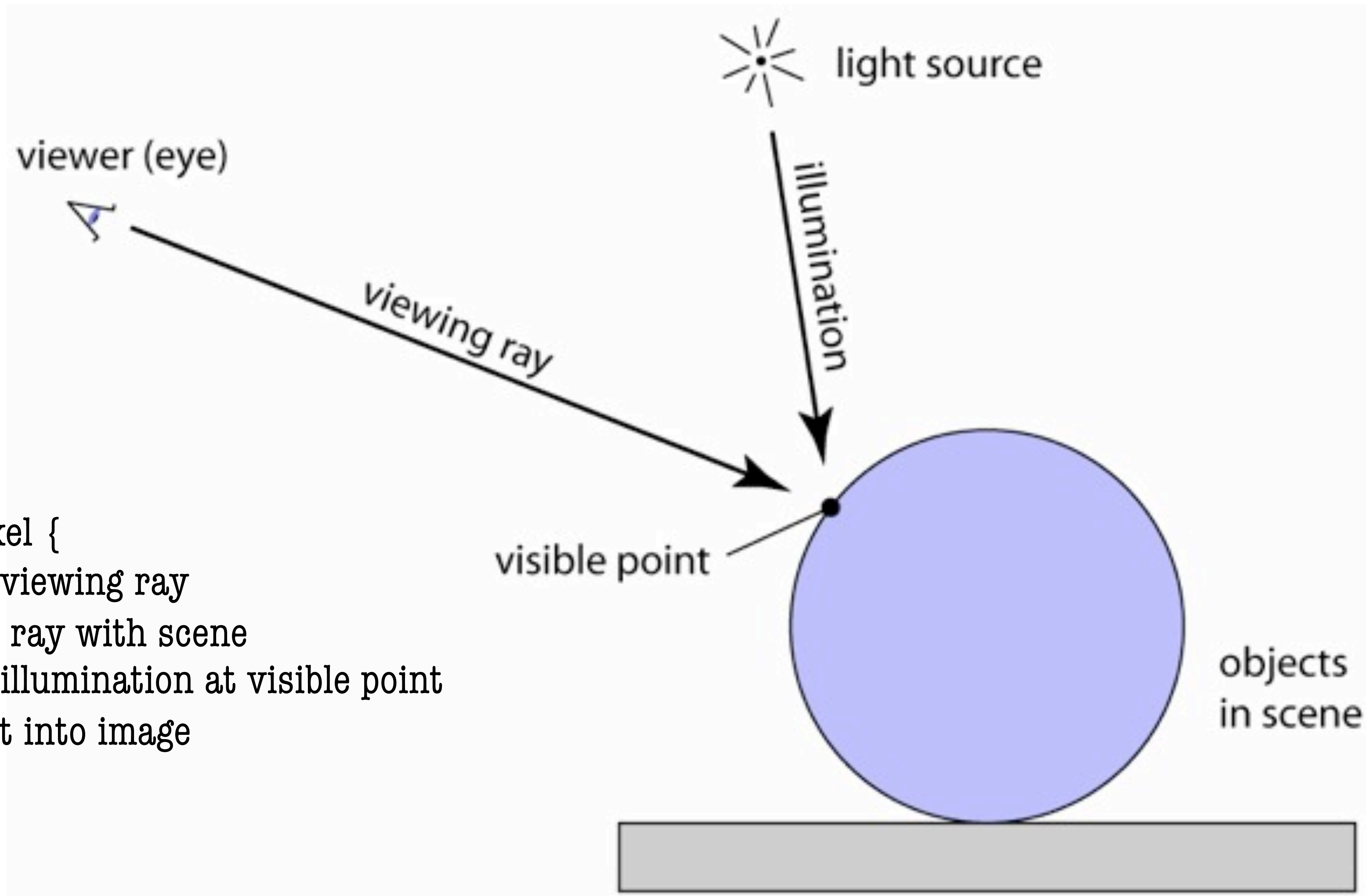


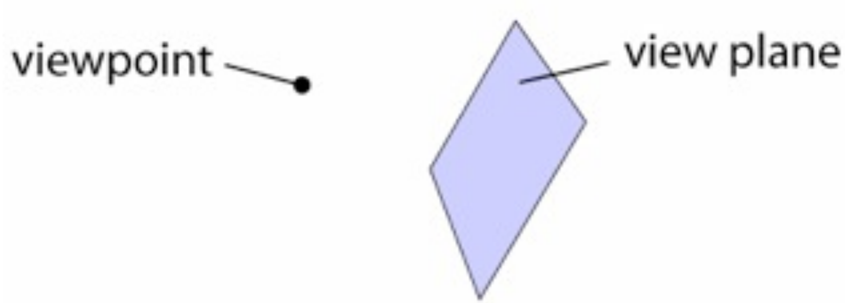
Ray tracing algorithm




```
for each pixel {
    compute viewing ray
    intersect ray with scene
    compute illumination at visible point
    put result into image
    }
```

Generating eye rays

- Use window analogy directly



Generating eye rays

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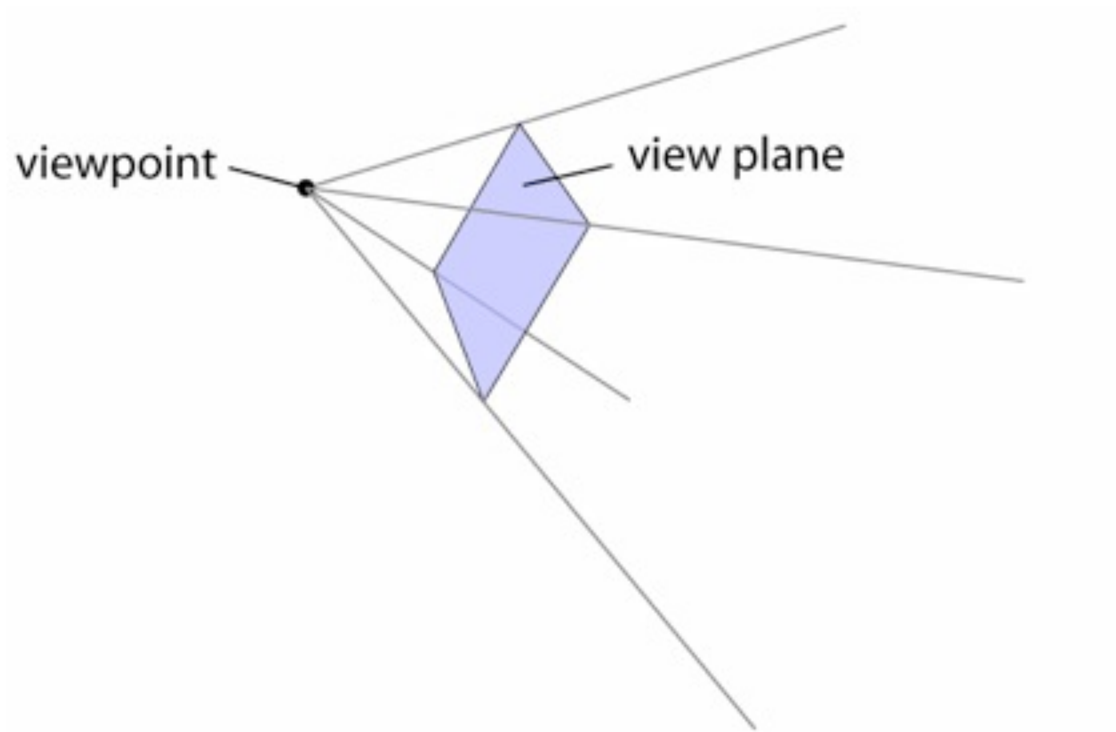

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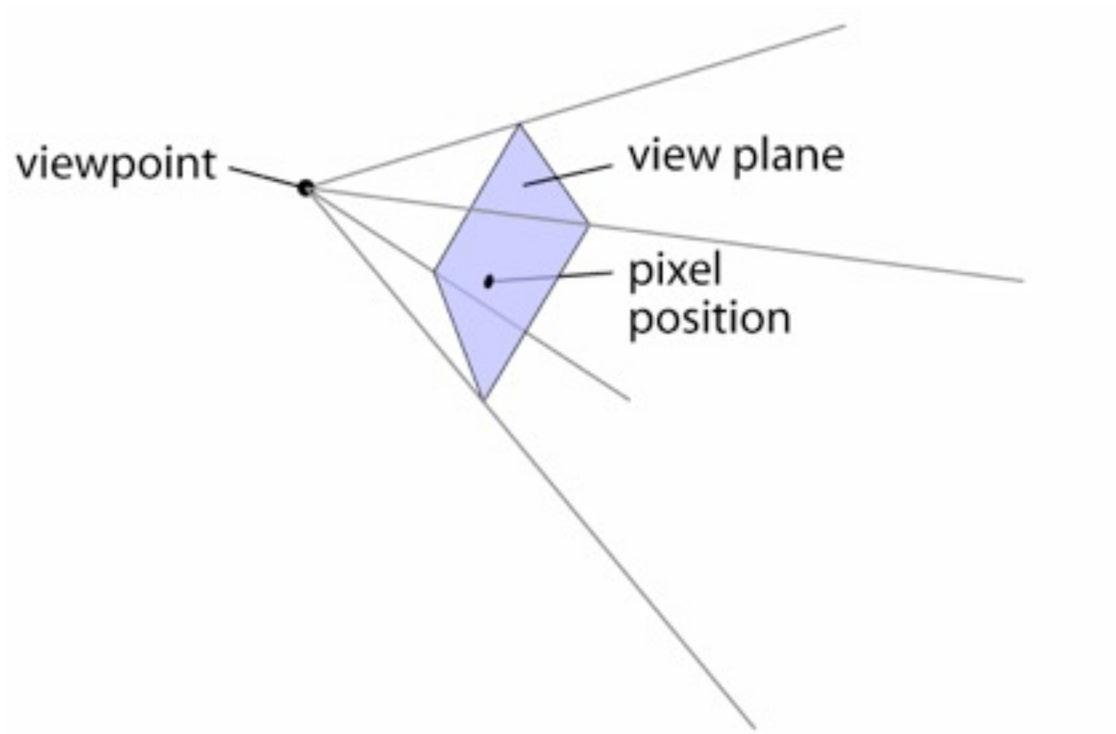

Generating eye rays

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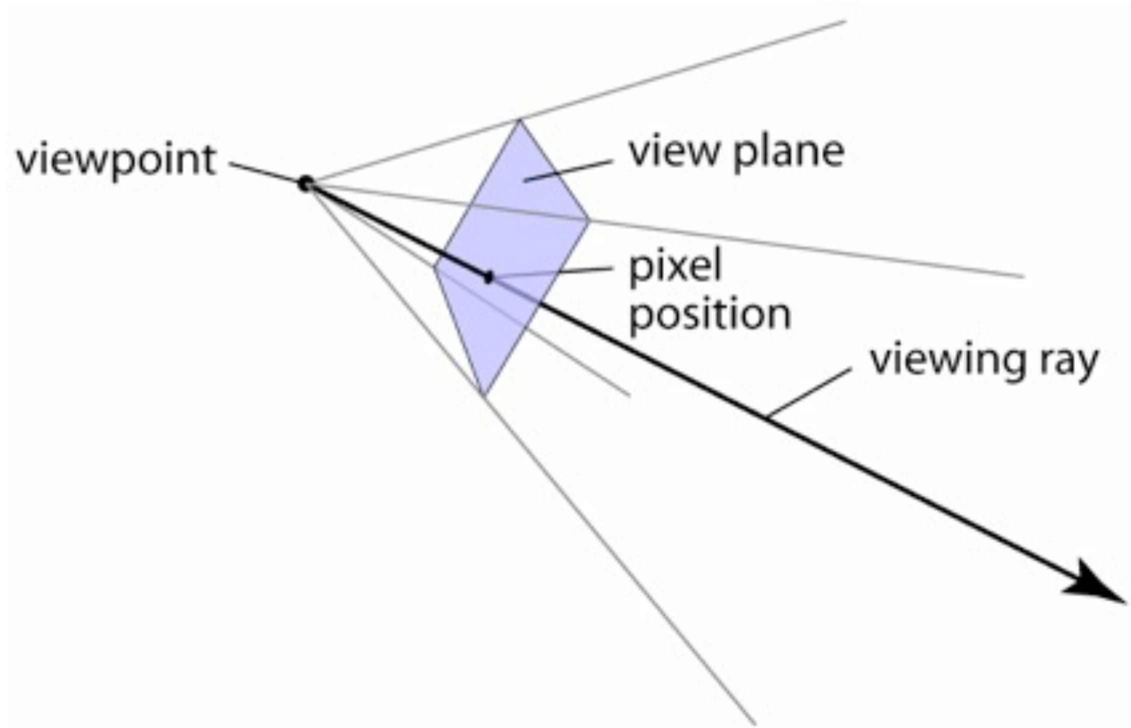
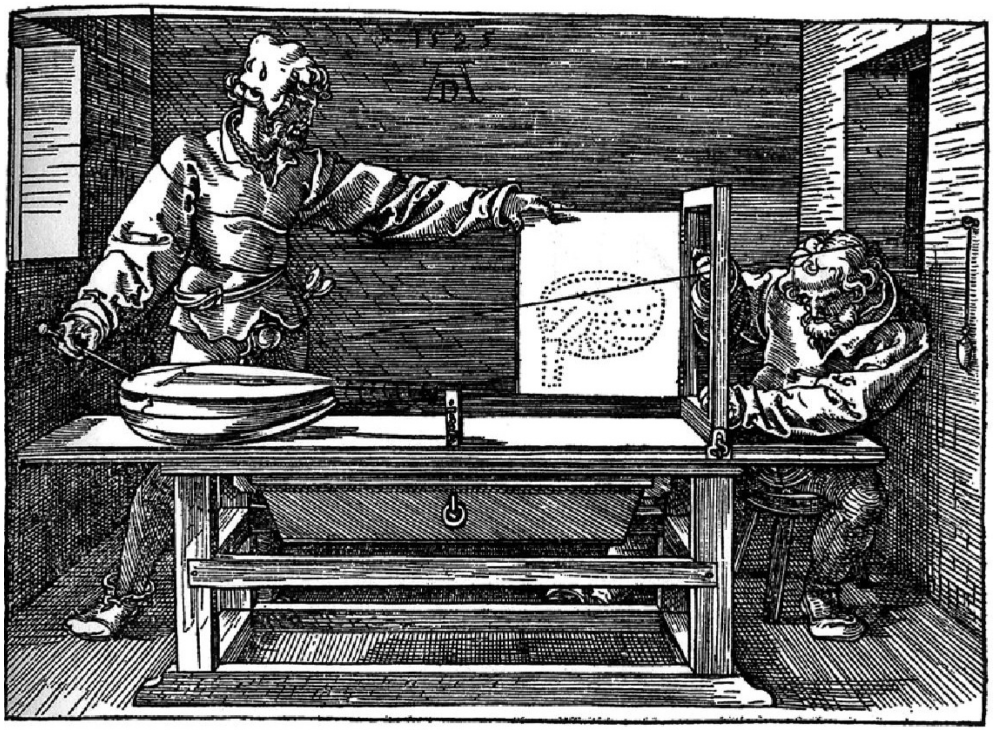
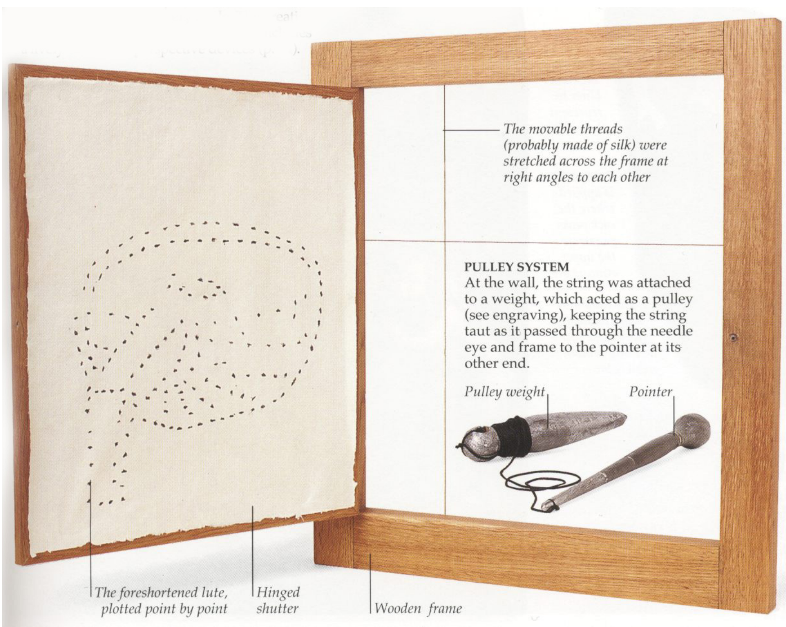


Durer’s Ray casting machine

- Albrecht Durer, 16th century





Source: F. Durand

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Generating eye rays

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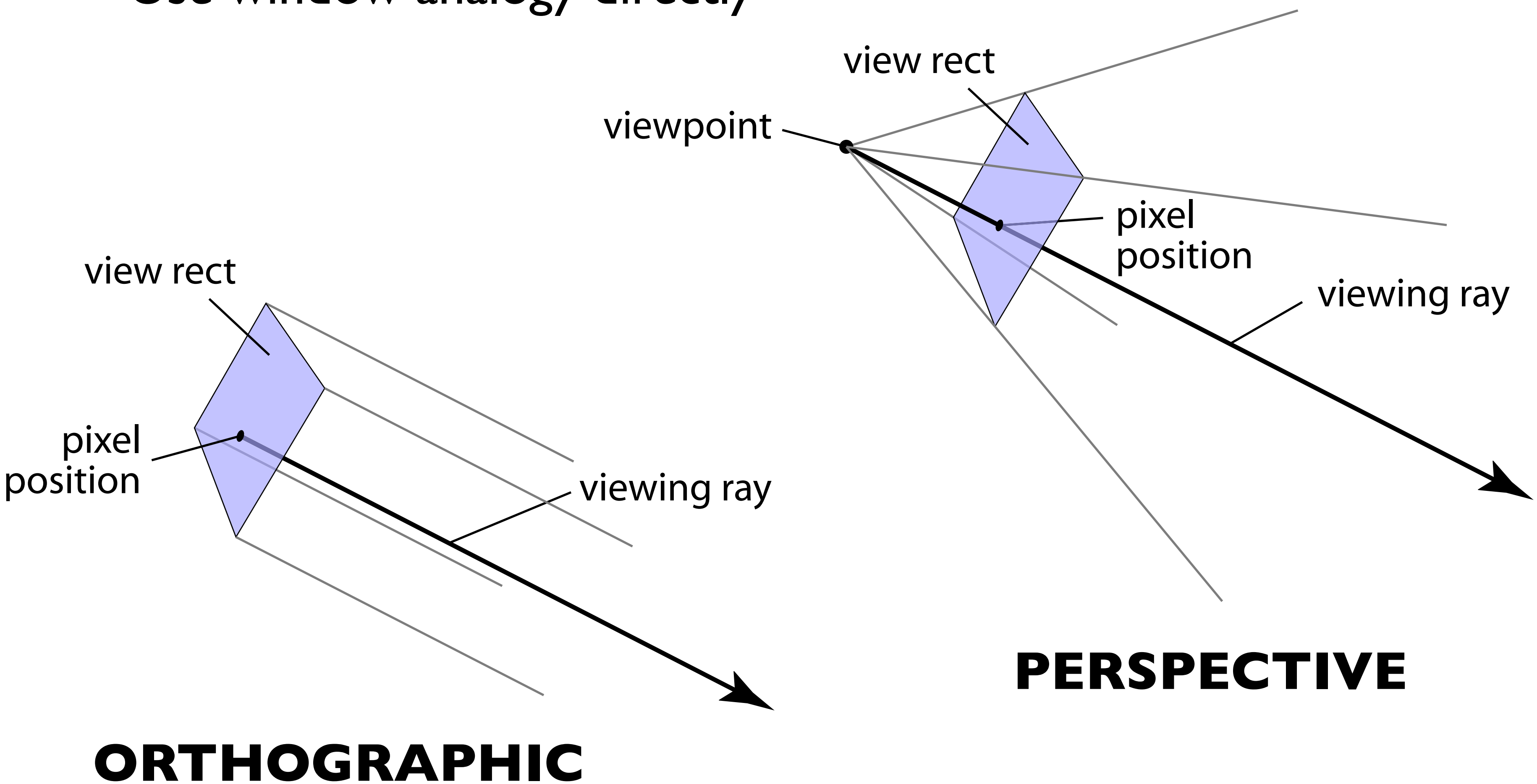
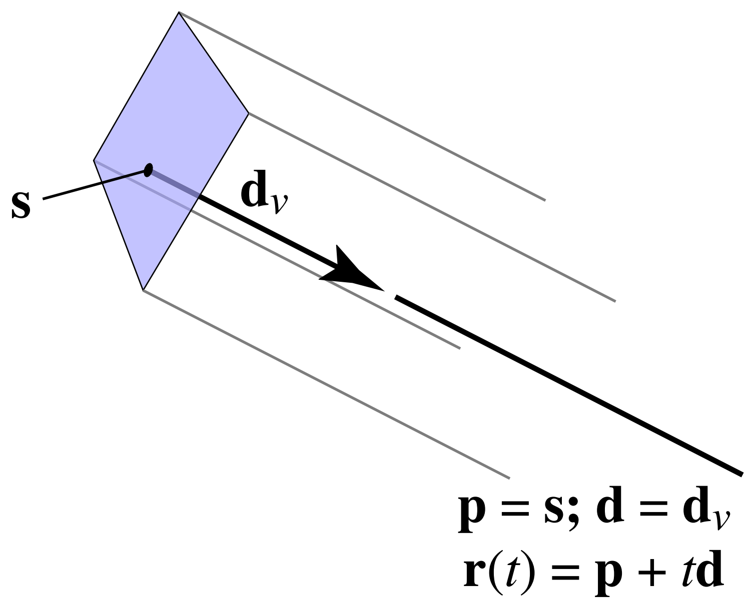


Vector math review

- Vectors and points
- Vector operations
  - addition
  - scalar product
- More products
  - dot product
  - cross product
- Bases and orthogonality

Generating eye rays—orthographic

- Just need to compute the view plane point **s**:



  – but where exactly is the view rectangle?

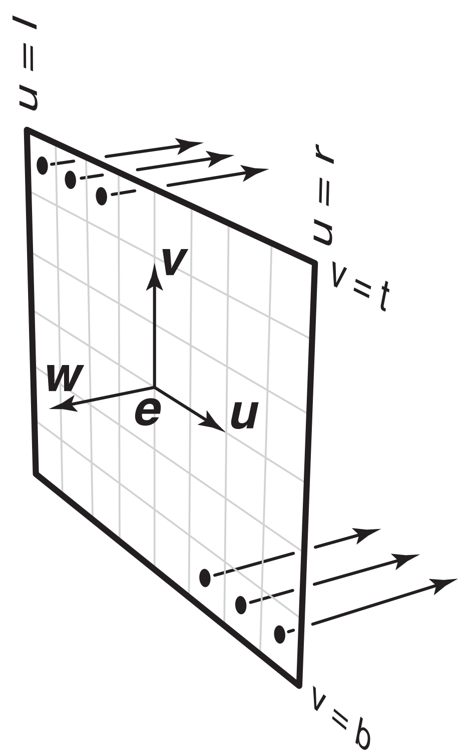
Generating eye rays—orthographic

- Positioning the view rectangle
  - establish three vectors to be *camera basis:* **u**, **v**, **w**
  - view rectangle is in **u**–**v** plane, specified by l, r, t, b
  - now ray generation is easy:

$$\mathbf{s} = \mathbf{e} + u\mathbf{u} + v\mathbf{v}$$

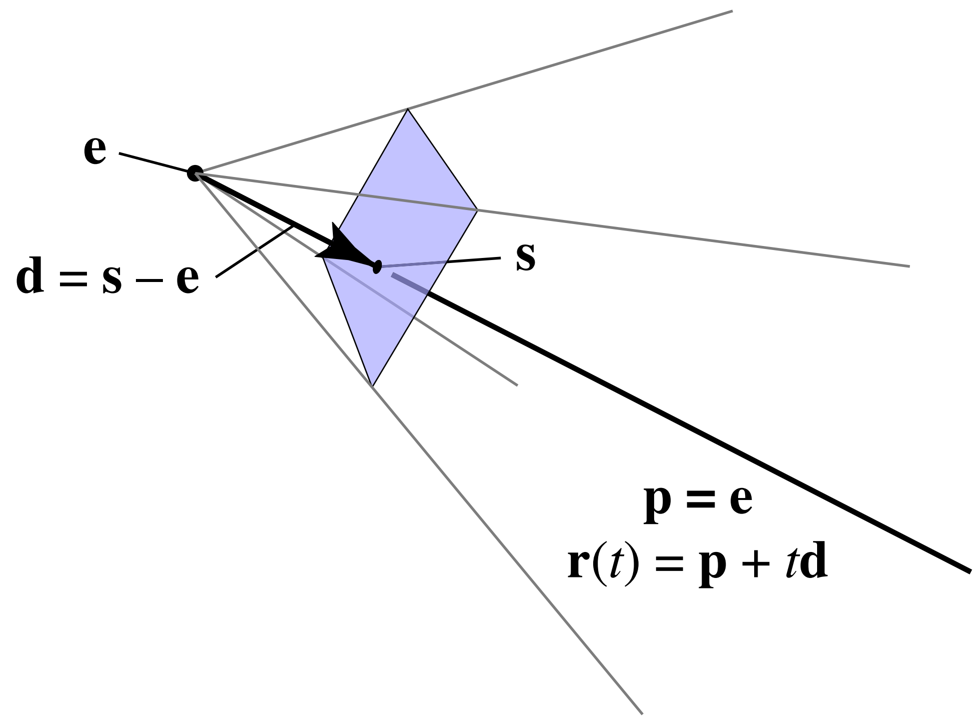
$$\mathbf{p} = \mathbf{s};\ \mathbf{d} = -\mathbf{w}$$

$$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$



Generating eye rays—perspective

- View rectangle needs to be away from viewpoint
- Distance is important: "focal length" of camera
  - still use camera frame but position view rect away from viewpoint
  - ray origin always **e**
  - ray direction now controlled by **s**



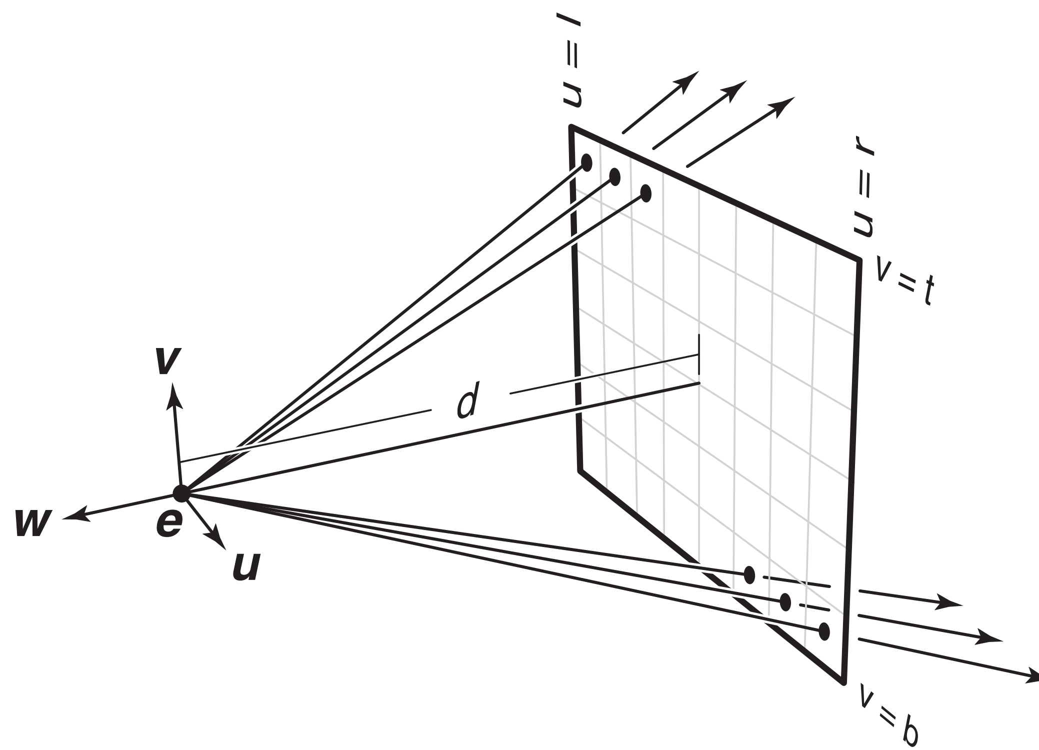
Generating eye rays—perspective

- Compute **s** in the same way; just subtract *d***w**
  - coordinates of **s** are (*u*, *v*, –*d*)

$$\mathbf{s} = \mathbf{e} + u\mathbf{u} + v\mathbf{v} - d\mathbf{w}$$

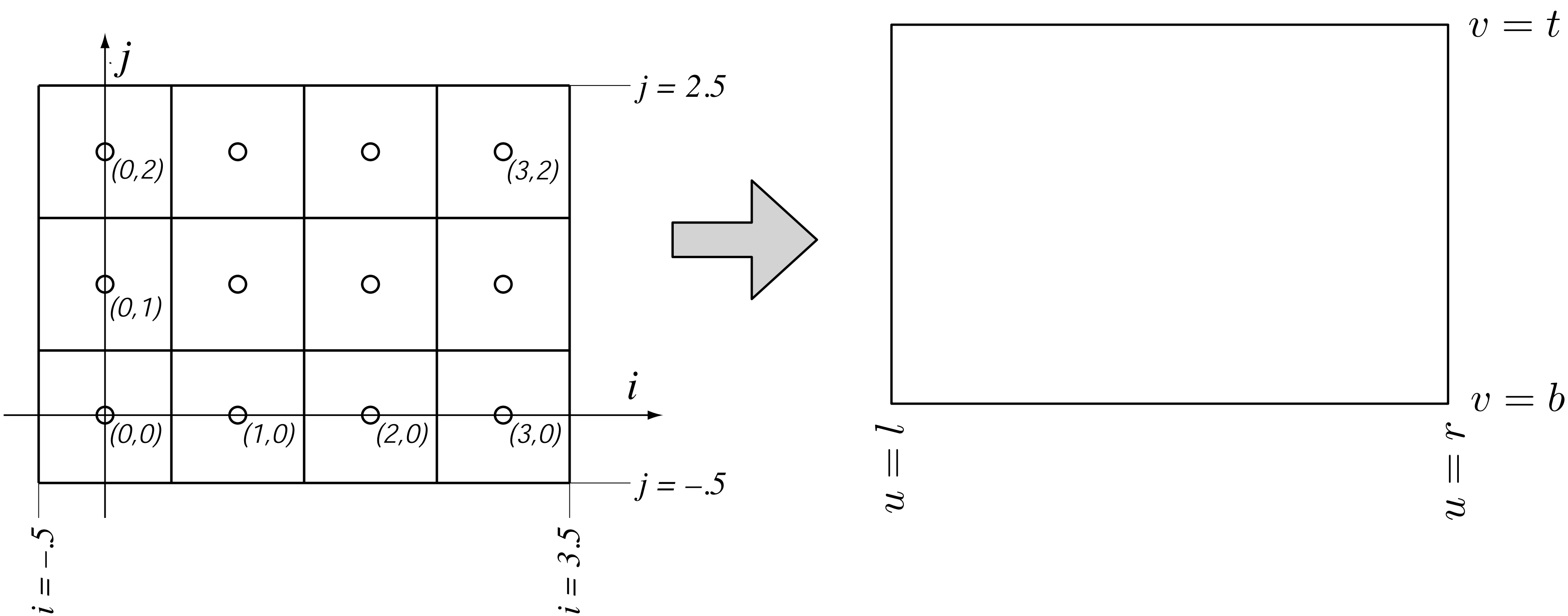
$$\mathbf{p} = \mathbf{e};\ \mathbf{d} = \mathbf{s} - \mathbf{e}$$

$$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$



Pixel-to-image mapping

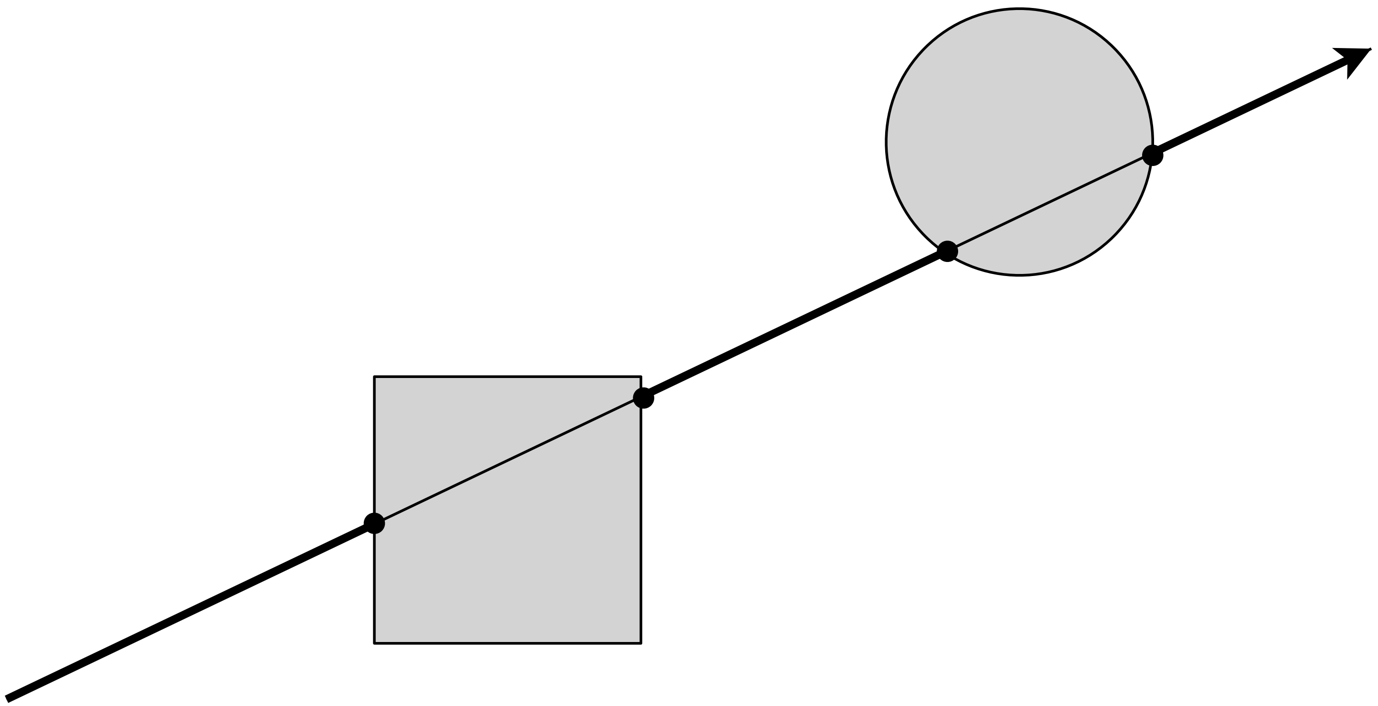
- One last detail: (*u*, *v*) coords of a pixel



$$u = l + (r - l)(i + 0.5)/n_x$$

$$v = b + (t - b)(j + 0.5)/n_y$$

Ray intersection

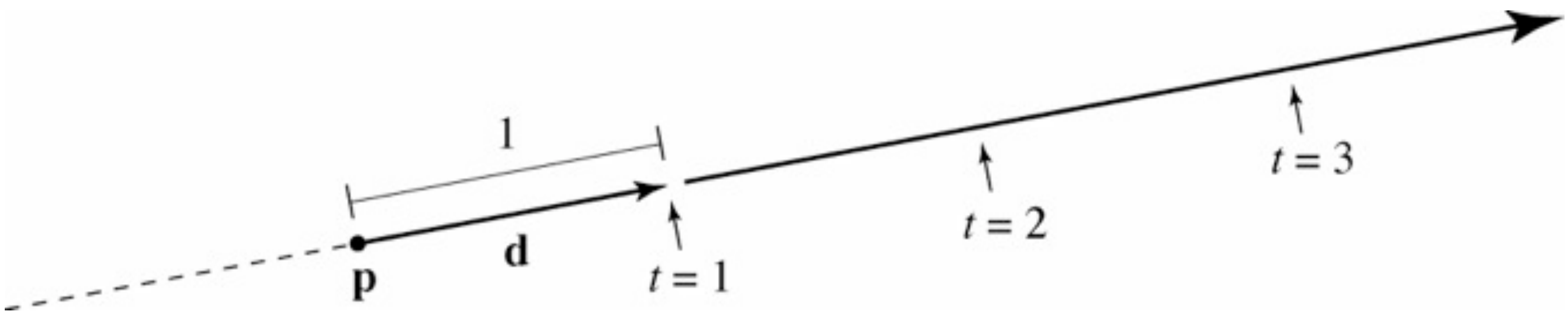


Ray: a half line

- Standard representation: point **p** and direction **d**

  $$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$

  - this is a *parametric equation* for the line
  - lets us directly generate the points on the line
  - if we restrict to $t > 0$ then we have a ray
  - note replacing **d** with a**d** doesn't change ray ($a > 0$)



Ray-sphere intersection: algebraic

- Condition 1: point is on ray

$$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$

- Condition 2: point is on sphere
  - assume unit sphere; see Shirley or notes for general

$$\|\mathbf{x}\| = 1 \Leftrightarrow \|\mathbf{x}\|^2 = 1$$
$$f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{x} - 1 = 0$$

- Substitute:

$$(\mathbf{p} + t\mathbf{d}) \cdot (\mathbf{p} + t\mathbf{d}) - 1 = 0$$

  - this is a quadratic equation in *t*

Ray-sphere intersection: algebraic

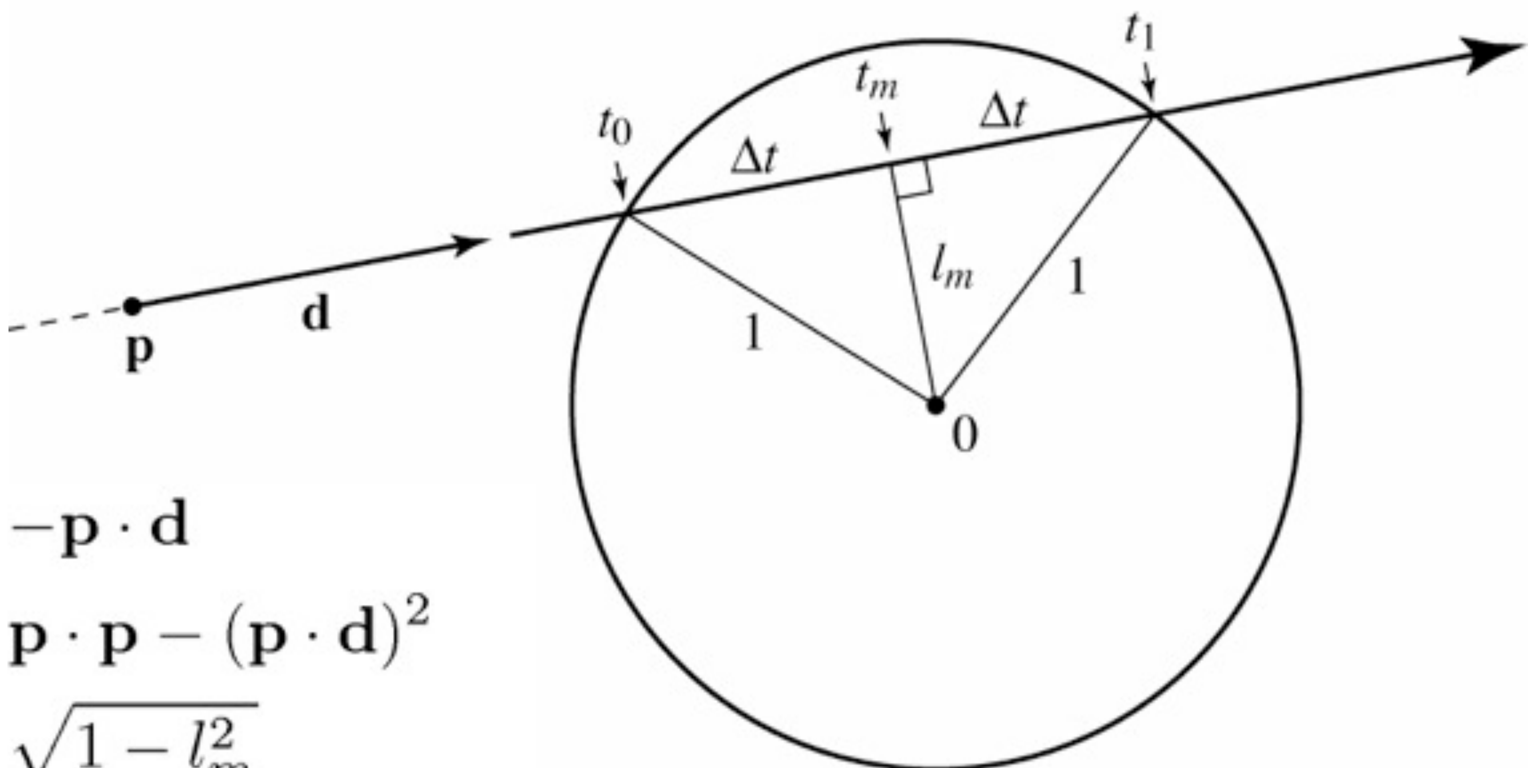
- Solution for *t* by quadratic formula:

$$t = \frac{-\mathbf{d}\cdot\mathbf{p} \pm \sqrt{(\mathbf{d}\cdot\mathbf{p})^2 - (\mathbf{d}\cdot\mathbf{d})(\mathbf{p}\cdot\mathbf{p}-1)}}{\mathbf{d}\cdot\mathbf{d}}$$

$$t = -\mathbf{d}\cdot\mathbf{p} \pm \sqrt{(\mathbf{d}\cdot\mathbf{p})^2 - \mathbf{p}\cdot\mathbf{p} + 1}$$

  - simpler form holds when **d** is a unit vector
    but we won't assume this in practice (reason later)
  - I'll use the unit-vector form to make the geometric interpretation

Ray-sphere intersection: geometric



$$t_m = -\mathbf{p} \cdot \mathbf{d}$$

$$l_m^2 = \mathbf{p} \cdot \mathbf{p} - (\mathbf{p} \cdot \mathbf{d})^2$$

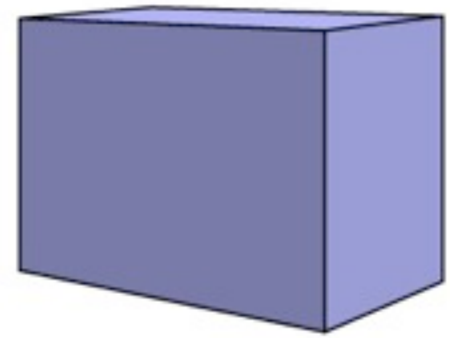
$$\Delta t = \sqrt{1 - l_m^2}$$

$$= \sqrt{(\mathbf{p} \cdot \mathbf{d})^2 - \mathbf{p} \cdot \mathbf{p} + 1}$$

$$t_{0,1} = t_m \pm \Delta t = -\mathbf{p} \cdot \mathbf{d} \pm \sqrt{(\mathbf{p} \cdot \mathbf{d})^2 - \mathbf{p} \cdot \mathbf{p} + 1}$$

Ray-box intersection

- Could intersect with 6 faces individually
- Better way: box is the intersection of 3 slabs



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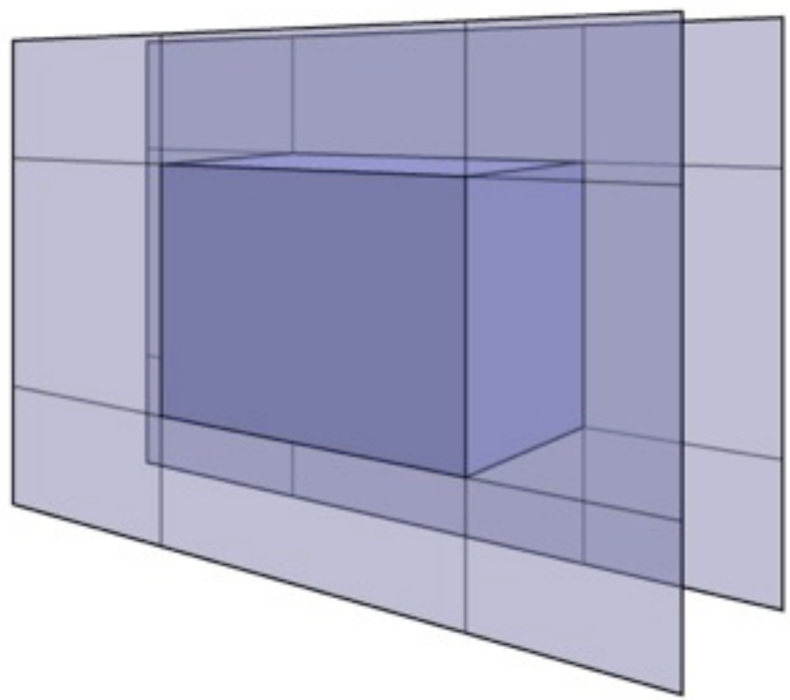

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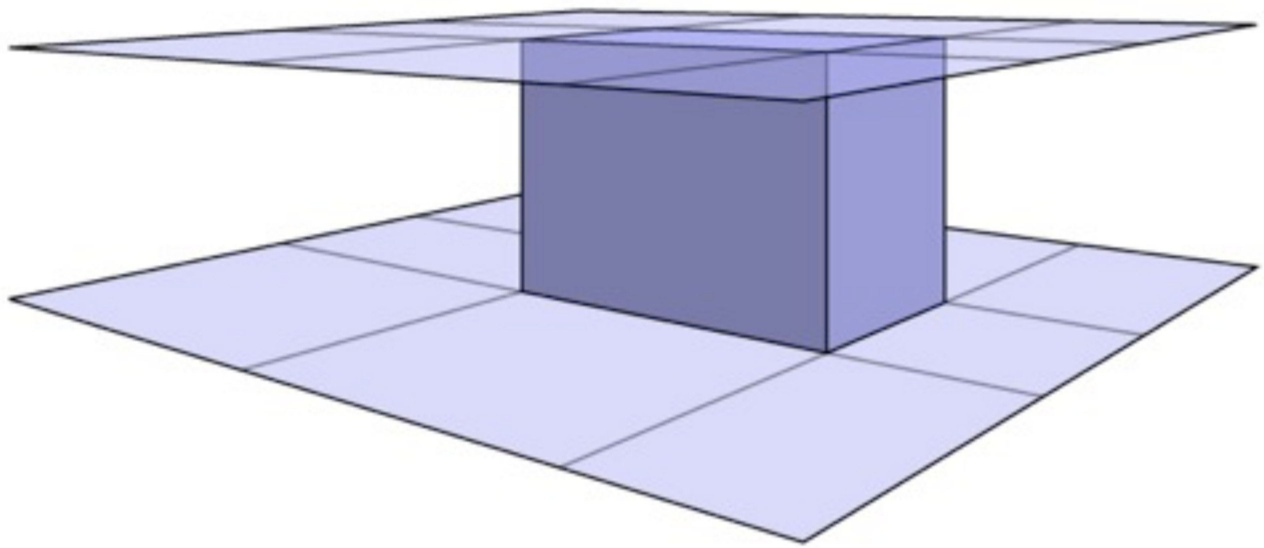

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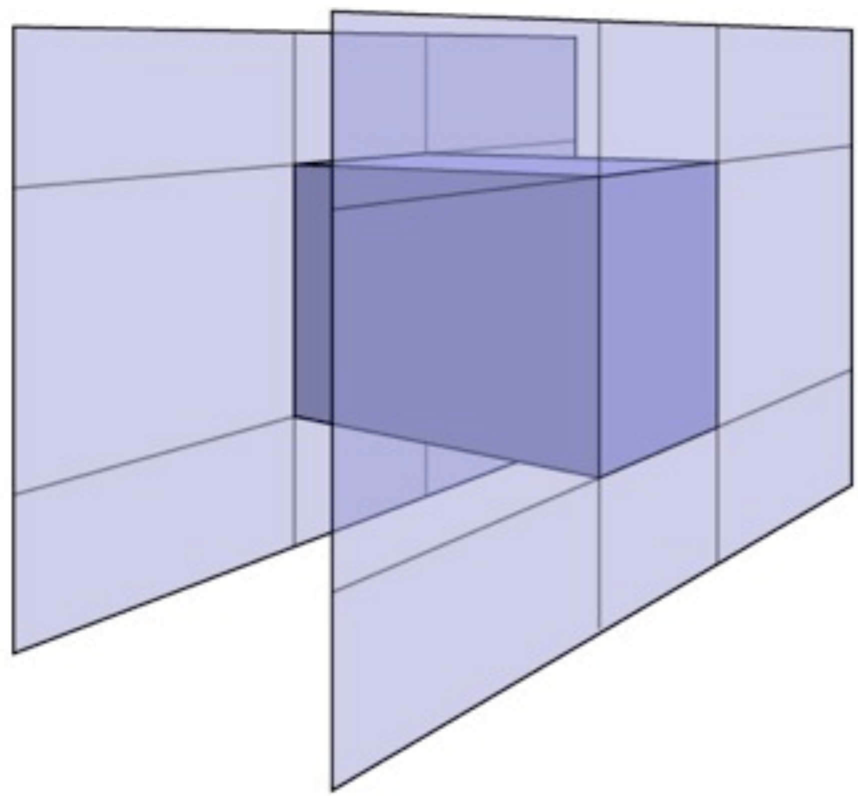
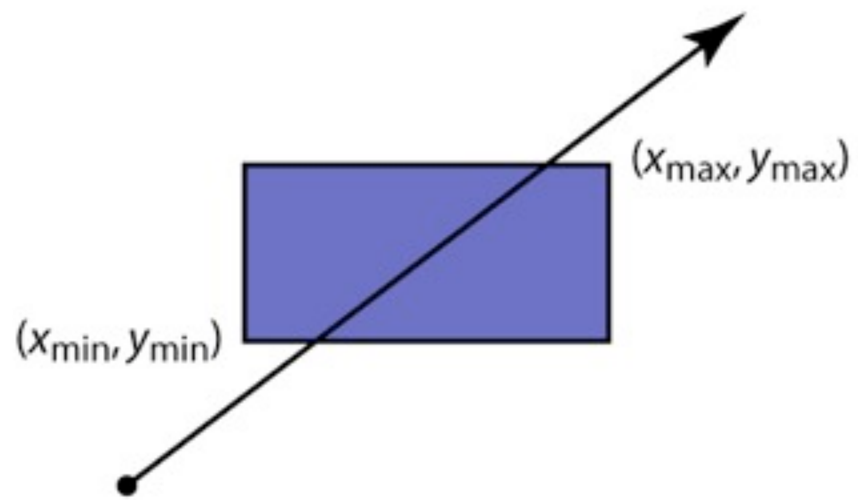


Ray-slab intersection

- 2D example
- 3D is the same!



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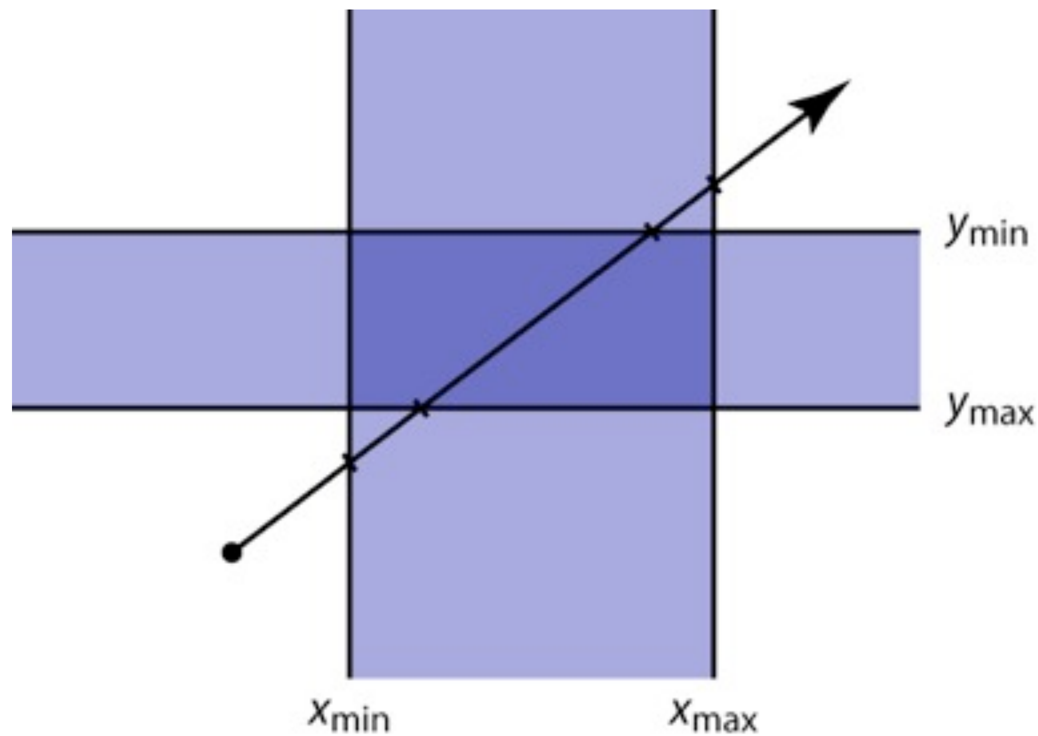
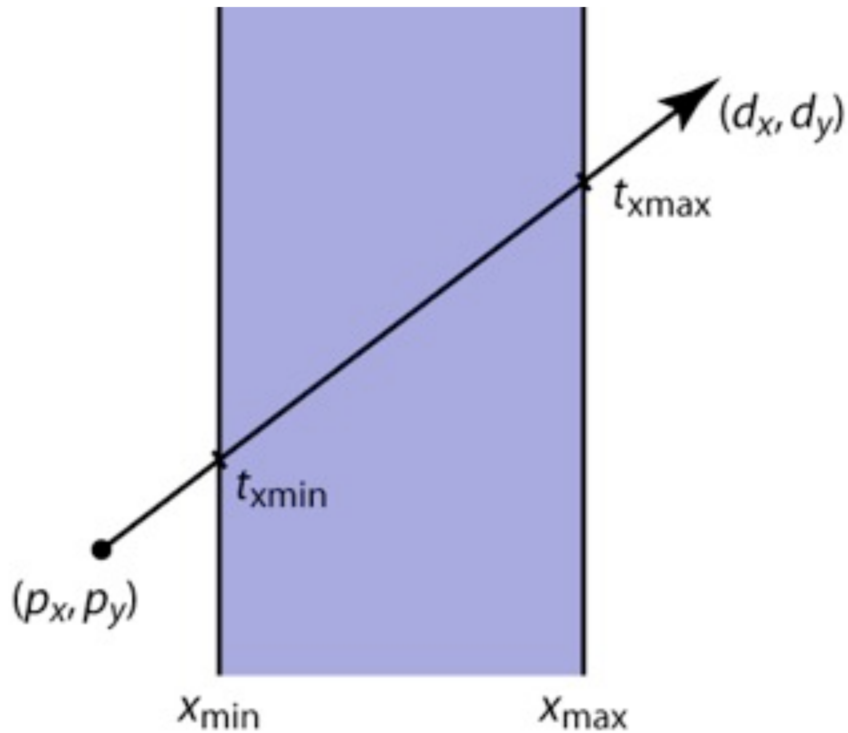


Ray-slab intersection

- 2D example
- 3D is the same!

$$p_x + t_{x\min}\, d_x = x_{\min}$$
$$t_{x\min} = (x_{\min} - p_x)/d_x$$

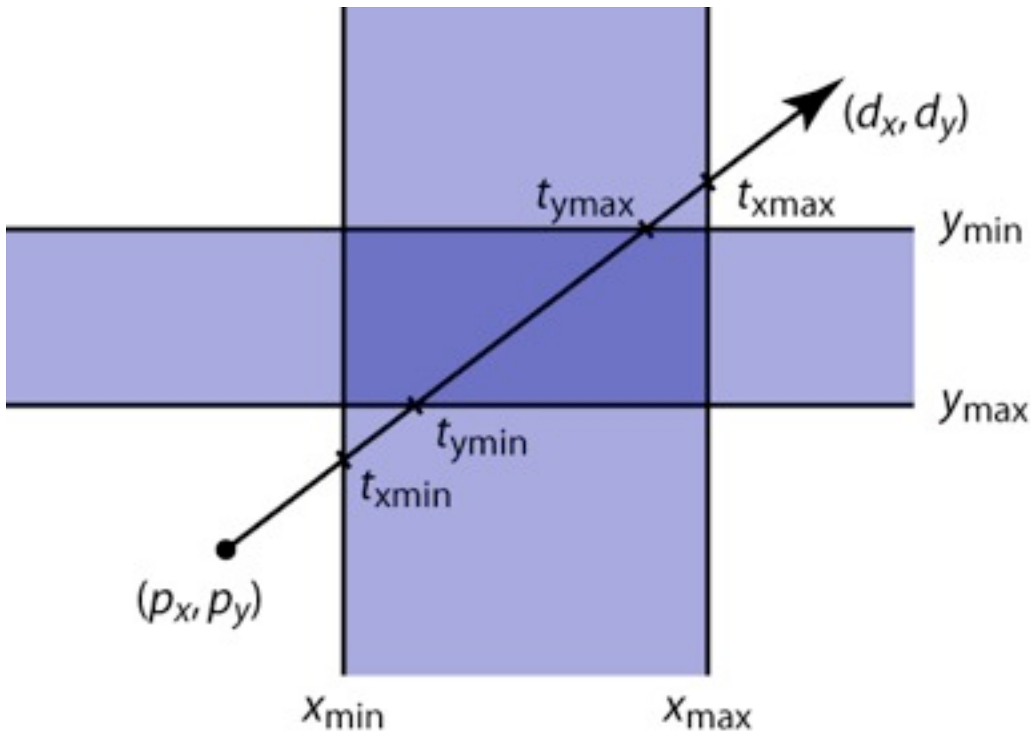


Ray-slab intersection

- 2D example
- 3D is the same!

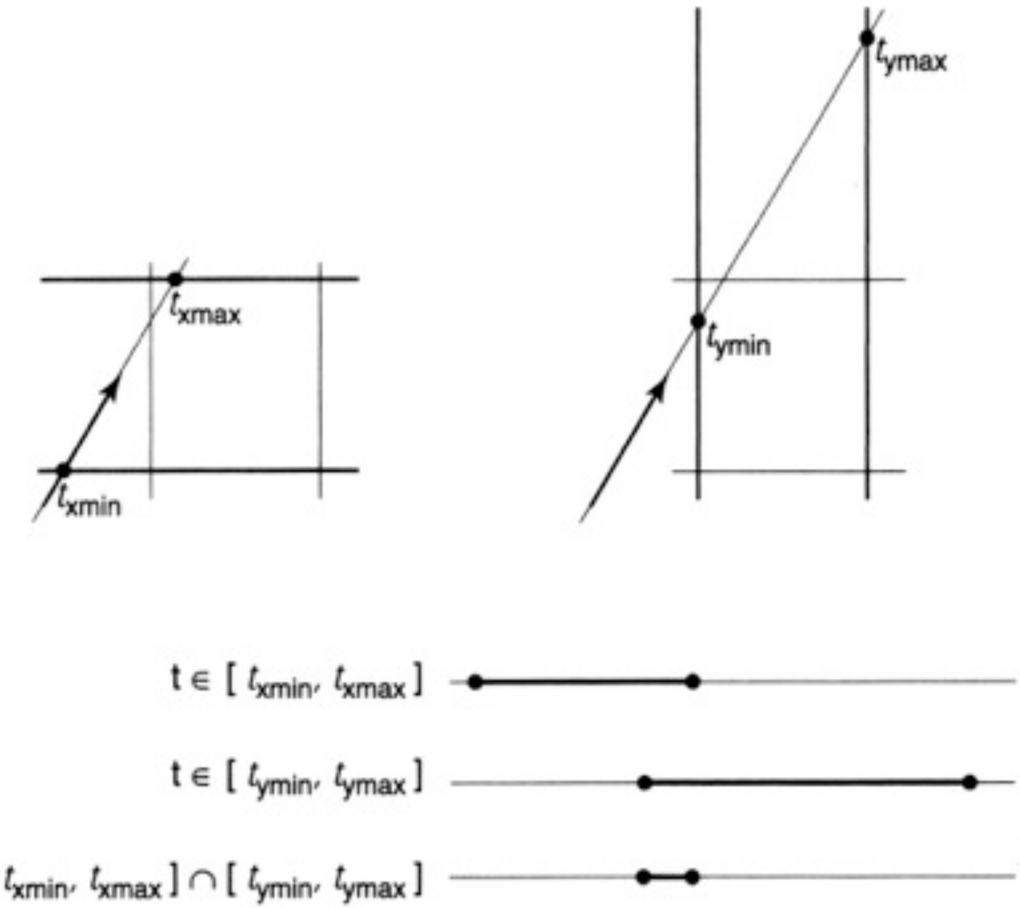
$$p_x + t_{x\min}\, d_x = x_{\min}$$
$$t_{x\min} = (x_{\min} - p_x)/d_x$$

$$p_y + t_{y\min}\, d_y = y_{\min}$$
$$t_{y\min} = (y_{\min} - p_y)/d_y$$



Intersecting intersections

- Each intersection is an interval
- Want last entry point and first exit point



$$t_{\min} = \max(t_{x\min}, t_{y\min})$$
$$t_{\max} = \min(t_{x\max}, t_{y\max})$$

Shirley fig. 10.16

Ray-triangle intersection

- Condition 1: point is on ray

$$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$

- Condition 2: point is on plane

$$(\mathbf{x} - \mathbf{a}) \cdot \mathbf{n} = 0$$

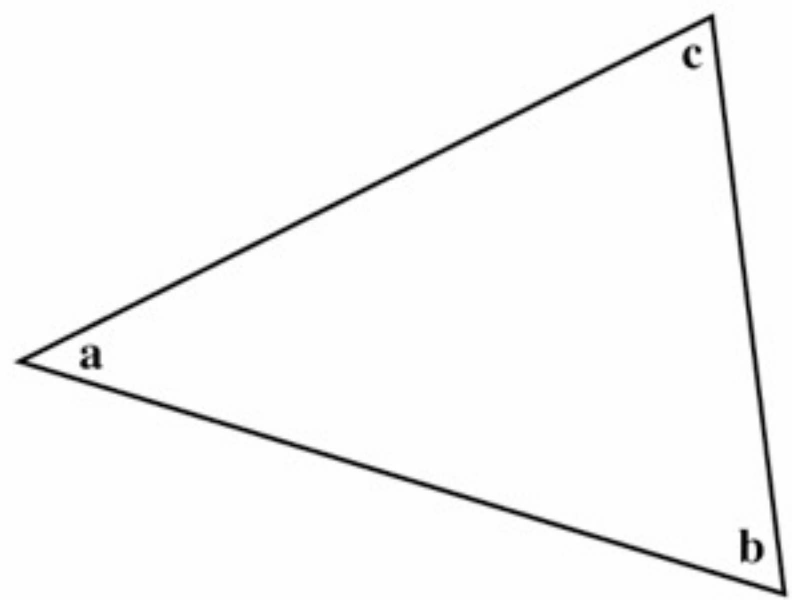
- Condition 3: point is on the inside of all three edges
- First solve 1&2 (ray–plane intersection)
  - substitute and solve for *t*:

$$(\mathbf{p} + t\mathbf{d} - \mathbf{a}) \cdot \mathbf{n} = 0$$

$$t = \frac{(\mathbf{a} - \mathbf{p}) \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}$$

Ray-triangle intersection

- In plane, triangle is the intersection of 3 half spaces



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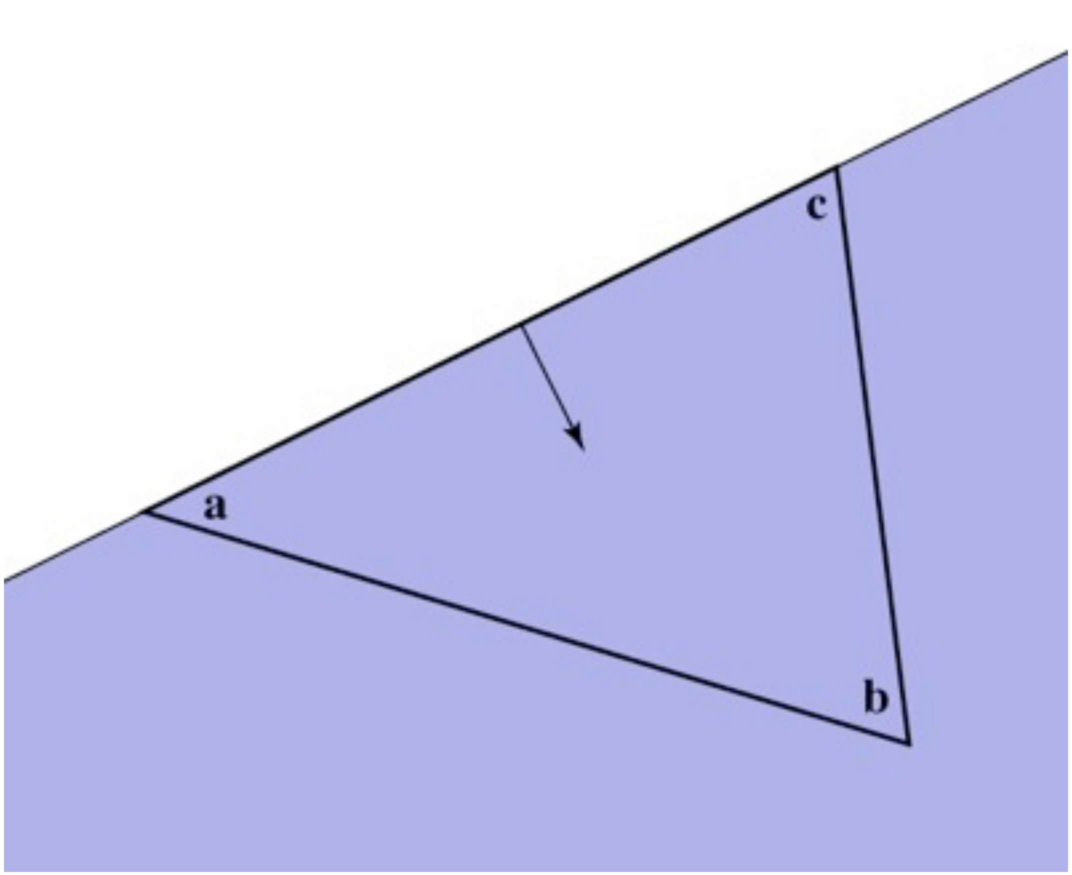

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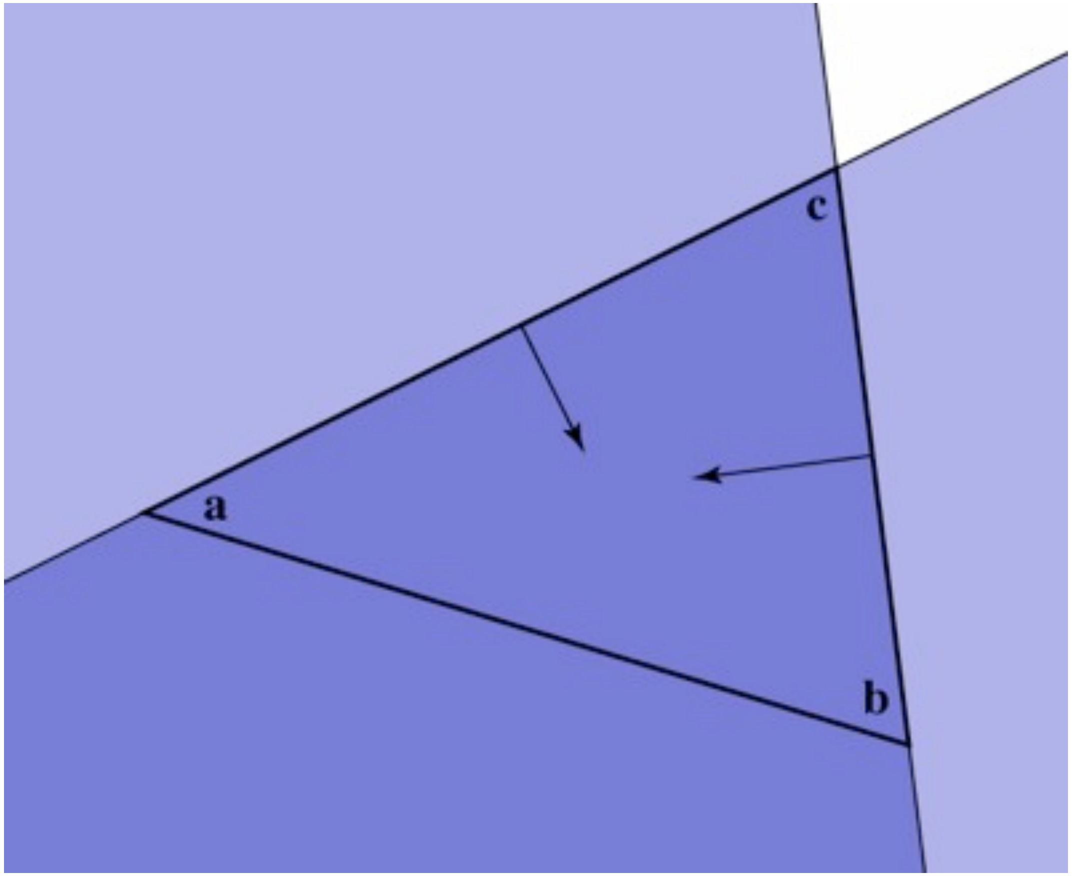

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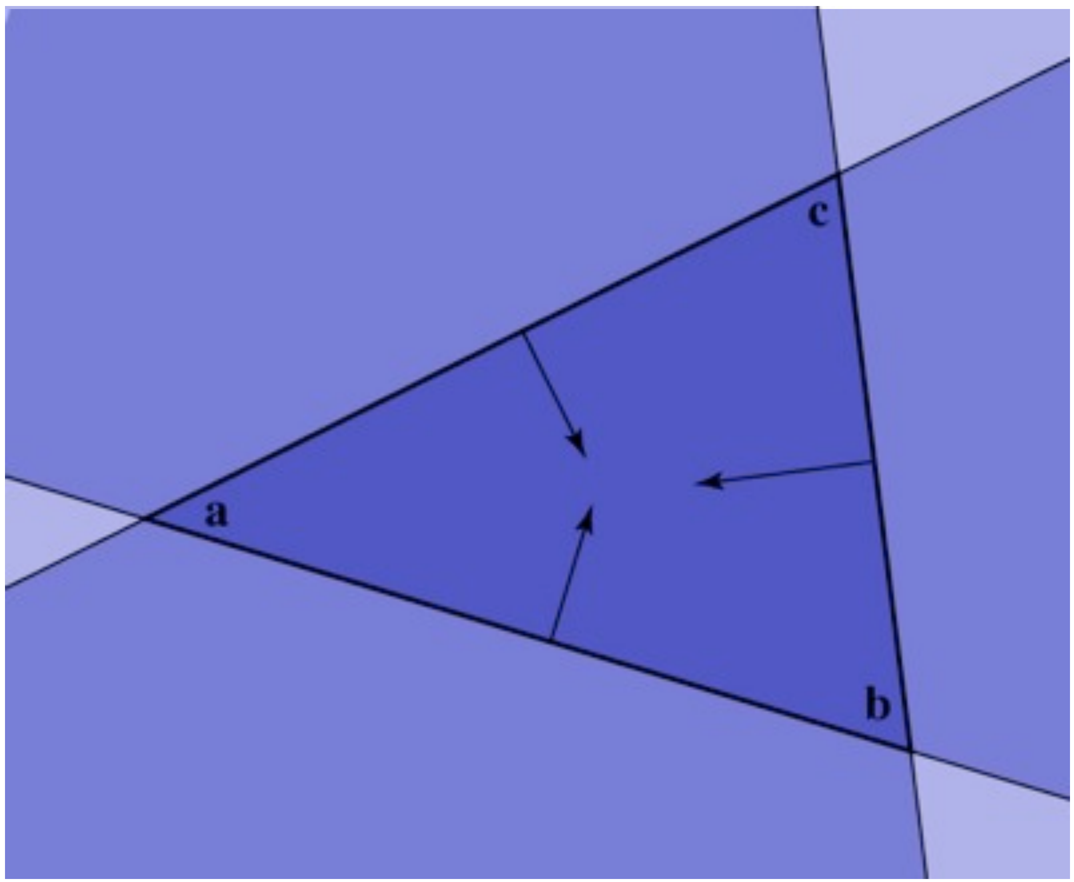
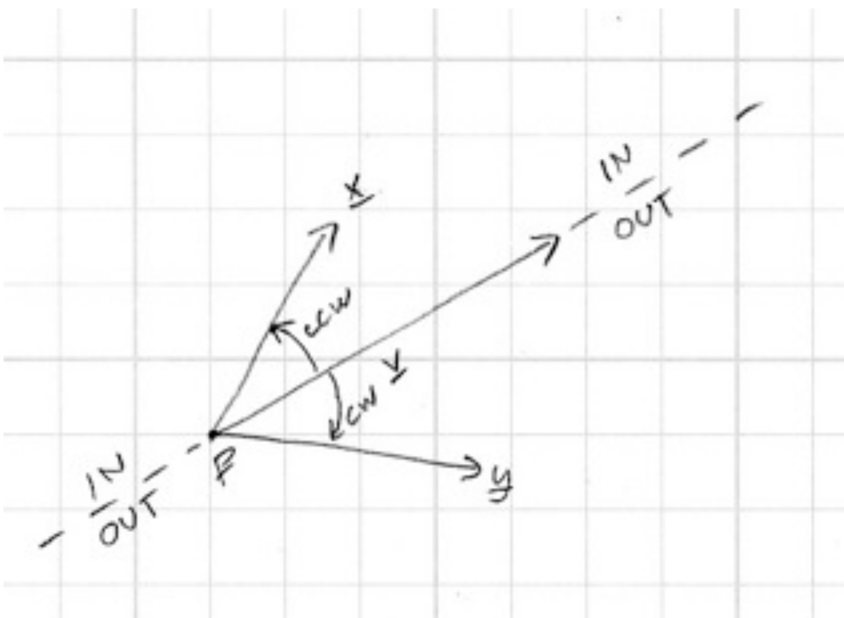
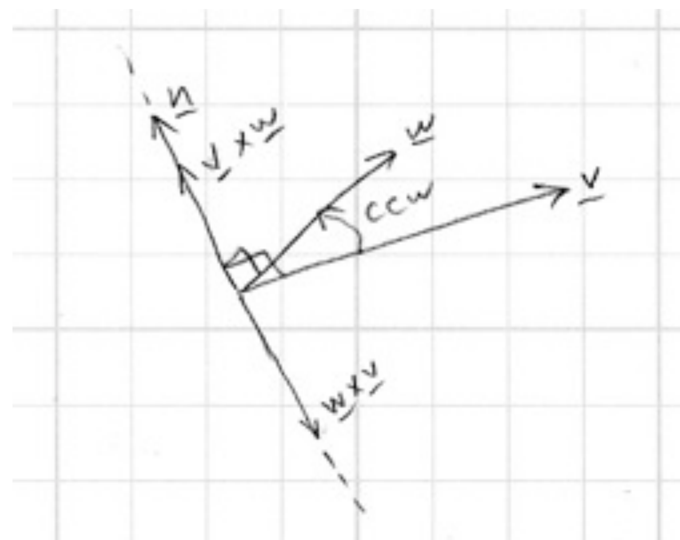


Inside-edge test

- Need outside vs. inside
- Reduce to clockwise vs. counterclockwise
  - vector of edge to vector to **x**
- Use cross product to decide



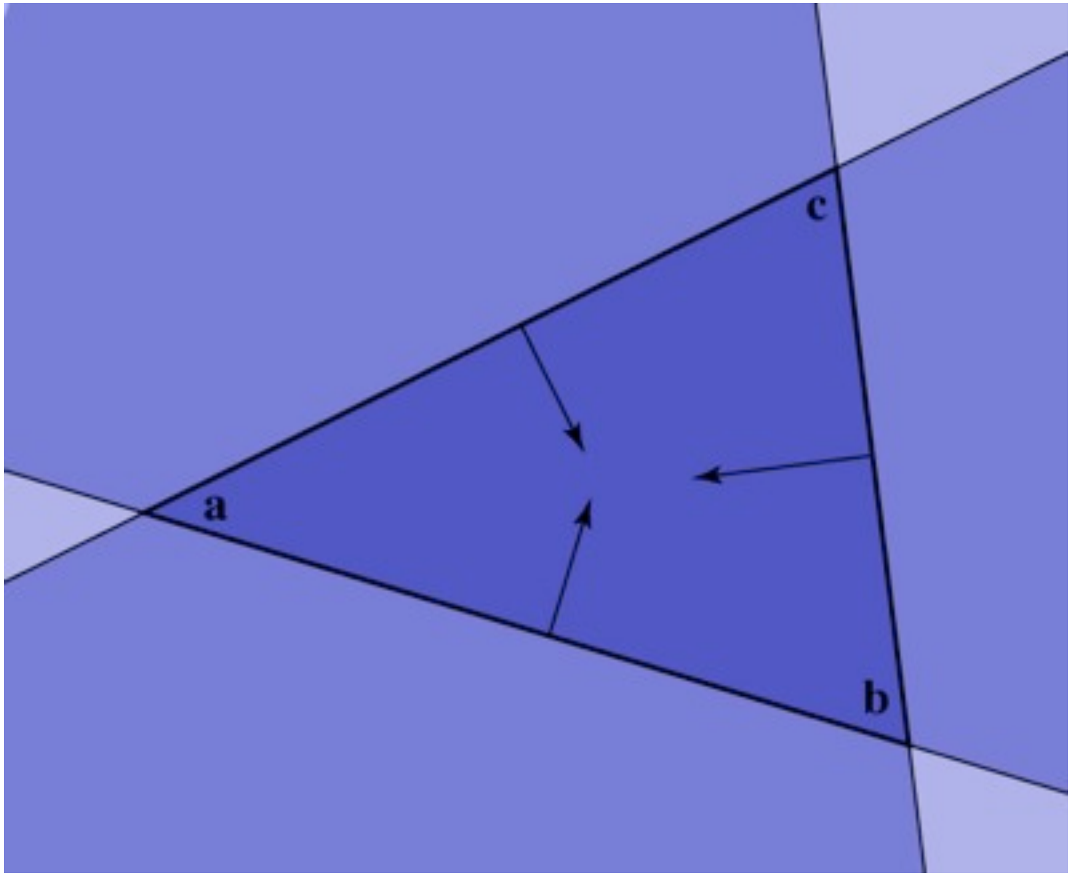


Ray-triangle intersection

$$(\mathbf{b}-\mathbf{a}) \times (\mathbf{x}-\mathbf{a}) \cdot \mathbf{n} > 0$$

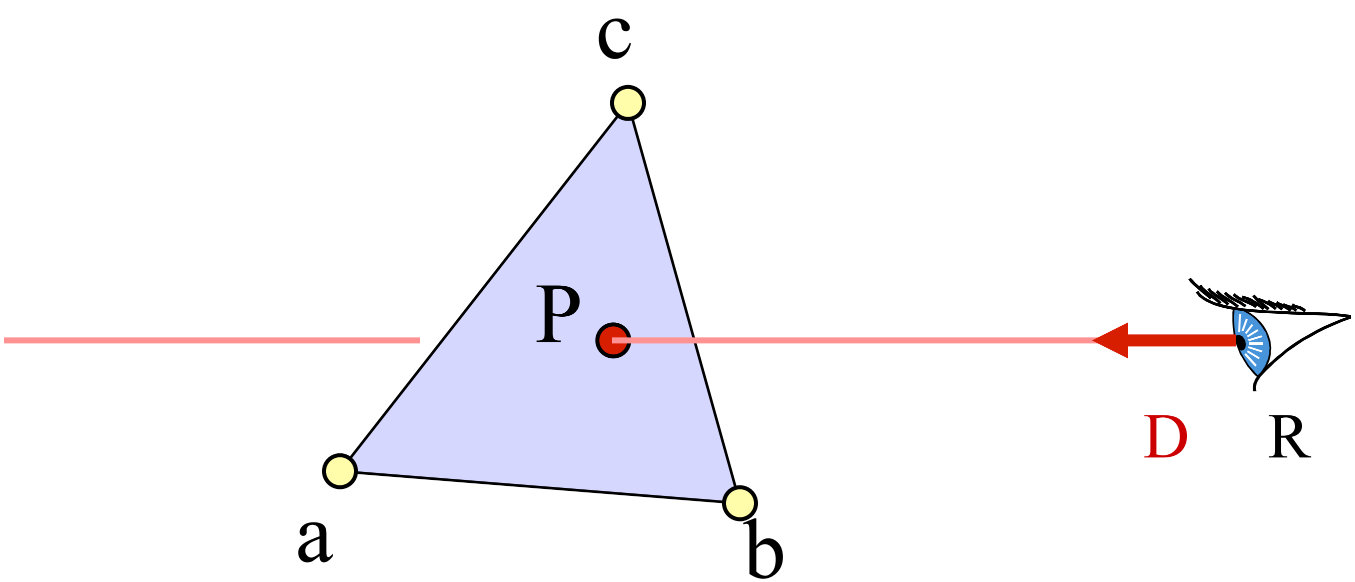
$$(\mathbf{c}-\mathbf{b}) \times (\mathbf{x}-\mathbf{b}) \cdot \mathbf{n} > 0$$

$$(\mathbf{a}-\mathbf{c}) \times (\mathbf{x}-\mathbf{c}) \cdot \mathbf{n} > 0$$



Ray-triangle intersection

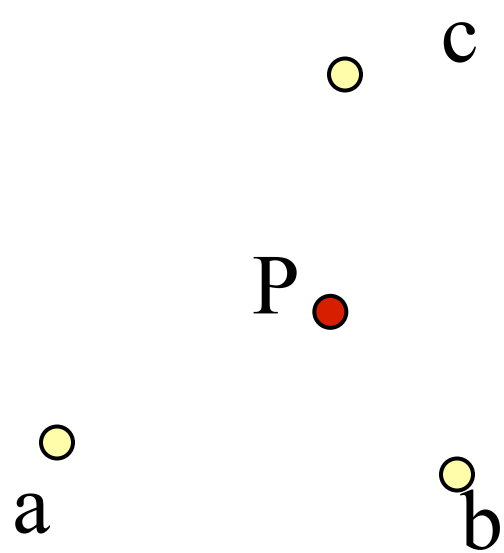
- a more efficient method
  - use barycentric coordinates



Source: F. Durand

Barycentric definition of a plane

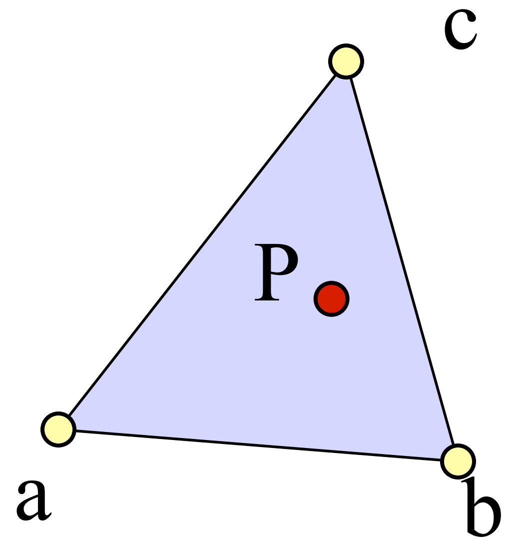
- $P(\alpha, \beta, \gamma)=\alpha a+\beta b+\gamma c$
  with $\alpha + \beta + \gamma = 1$



Source: F. Durand

Barycentric definition of a triangle

- $P(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c$
  with $\alpha + \beta + \gamma = 1$
- $0 < \alpha < 1$
  $0 < \beta < 1$
  $0 < \gamma < 1$



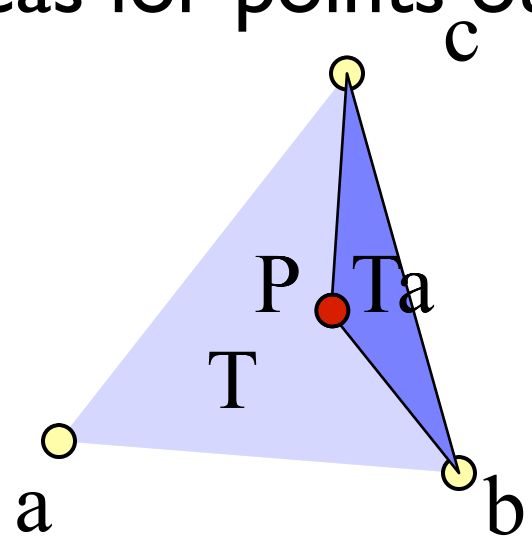
Source: F. Durand

Given P, how can we compute α, β, γ ?

- Compute the areas of the opposite subtriangle
  - Ratio with complete area

$\alpha = A_a/A$, $\beta = A_b/A$ $\gamma = A_c/A$

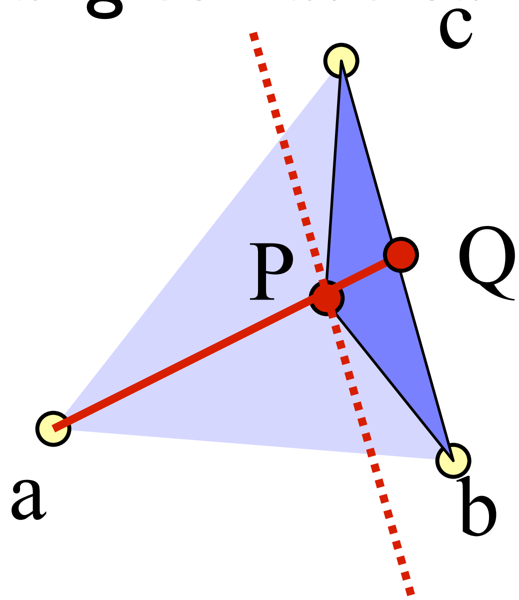
Use signed areas for points outside the triangle



Source: F. Durand

Intuition behind area formula

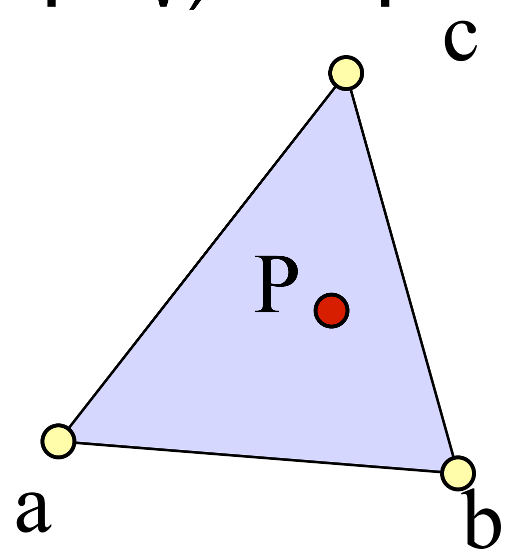
- P is barycenter of a and Q
- A is the interpolation coefficient on aQ
- All points on line parallel to bc have the same α
- All such Ta triangles have same height/area



Source: F. Durand

Simplify

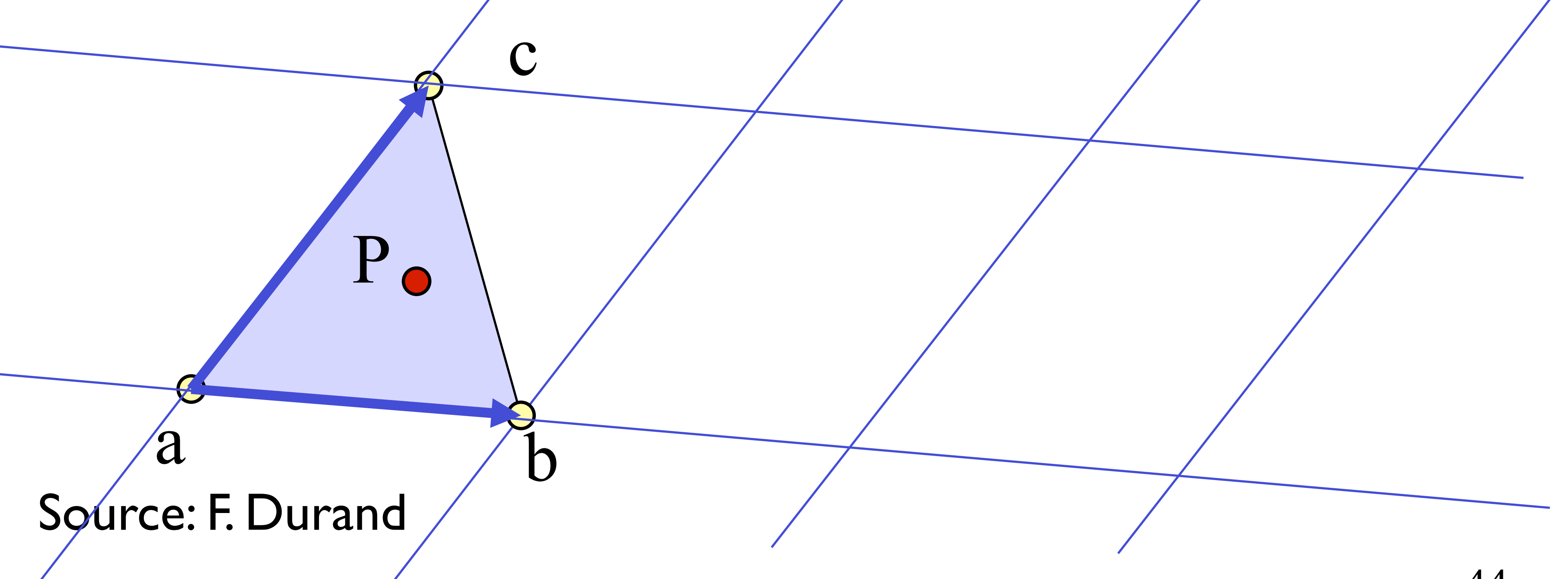
- Since $\alpha + \beta + \gamma = 1$
  we can write $\alpha = 1 - \beta - \gamma$
- $P(\beta, \gamma) = (1 - \beta - \gamma)\, a + \beta b + \gamma c$



Source: F. Durand

Simplify

- $P(\beta, \gamma) = (1 - \beta - \gamma)\, a + \beta b + \gamma c$
- $P(\beta, \gamma) = a + \beta(b-a) + \gamma(c-a)$
- Non-orthogonal coordinate system of the plane



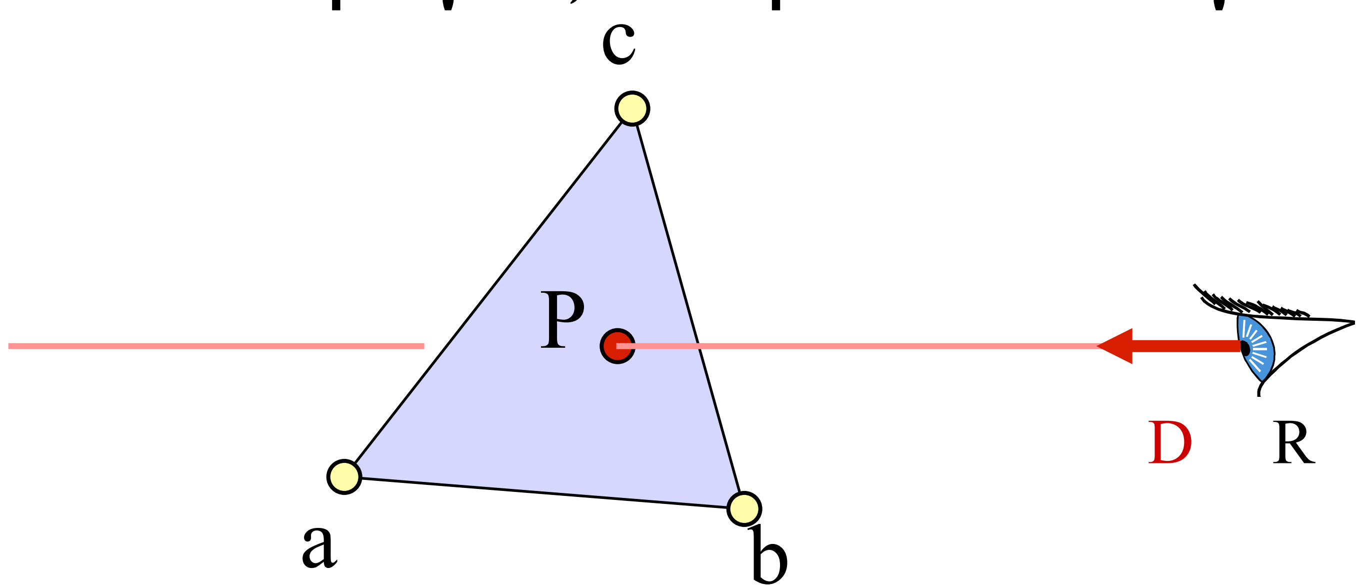
Source: F. Durand

How do we use it for intersection?

Insert ray equation into barycentric expression of triangle

$P(t)= a+\beta\ (b-a)+\gamma\ (c-a)$

Intersection if $\beta+\gamma<1$; $0<\beta$ and $0<\gamma$



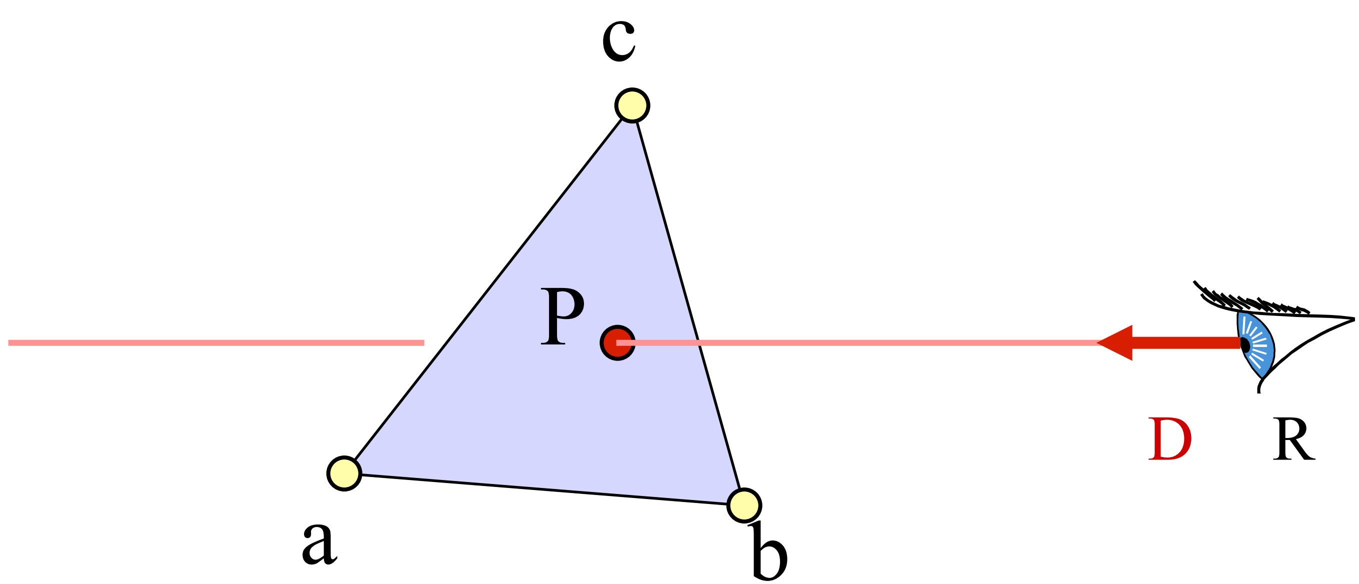
Source: F. Durand

Intersection

$$R_x + tD_x = a_x + \beta\,(b_x - a_x) + \gamma\,(c_x - a_x)$$

$$R_y + tD_y = a_y + \beta\,(b_y - a_y) + \gamma\,(c_y - a_y)$$

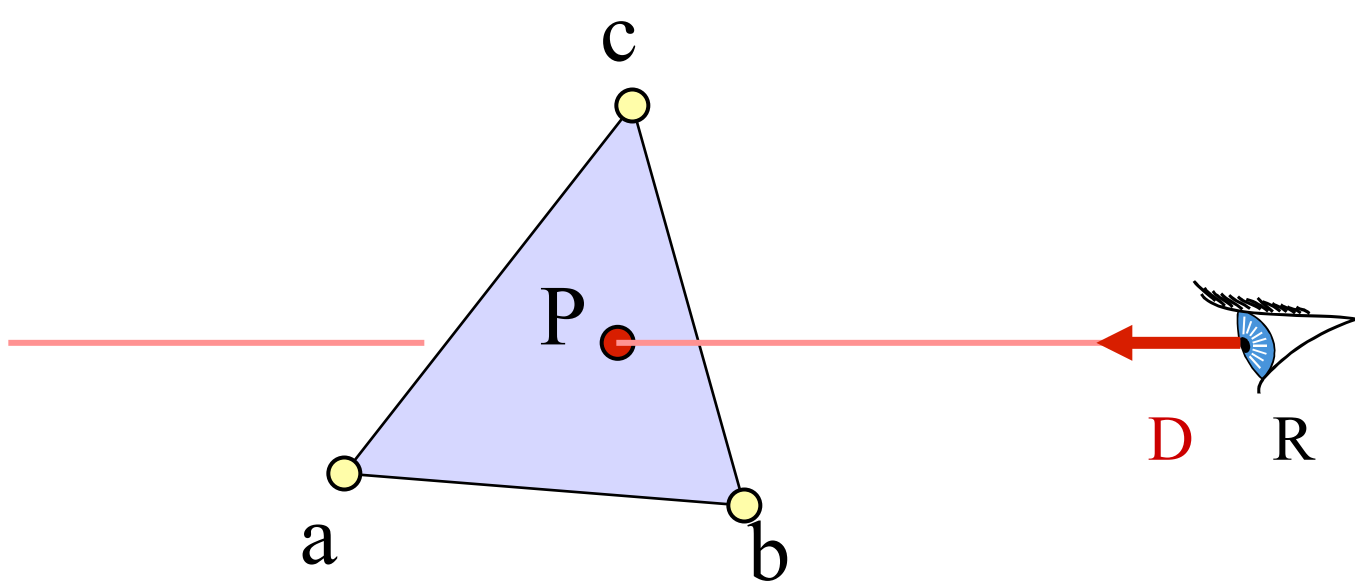
$$R_z + tD_z = a_z + \beta\,(b_z - a_z) + \gamma\,(c_z - a_z)$$



Source: F. Durand

Matrix form

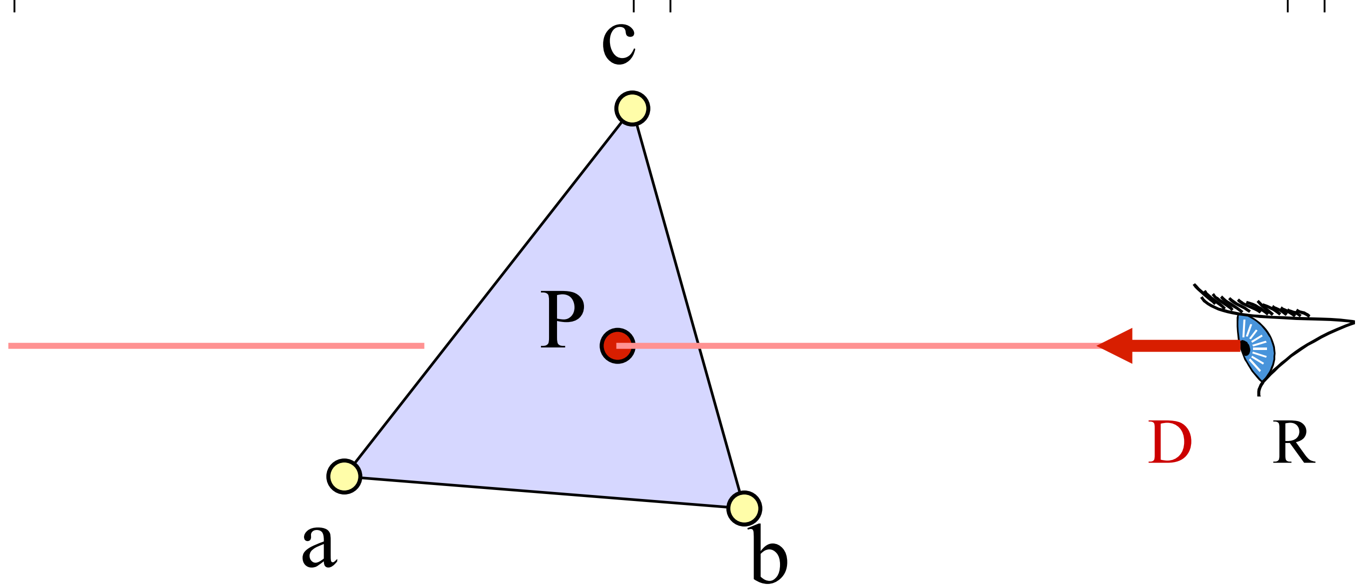
$$\begin{bmatrix} a_x - b_x & a_x - c_x & D_x \\ a_y - b_y & a_y - c_y & D_y \\ a_z - b_z & a_z - c_z & D_z \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} a_x - R_x \\ a_y - R_y \\ a_z - R_z \end{bmatrix}$$



Source: F. Durand

Cramer's rule

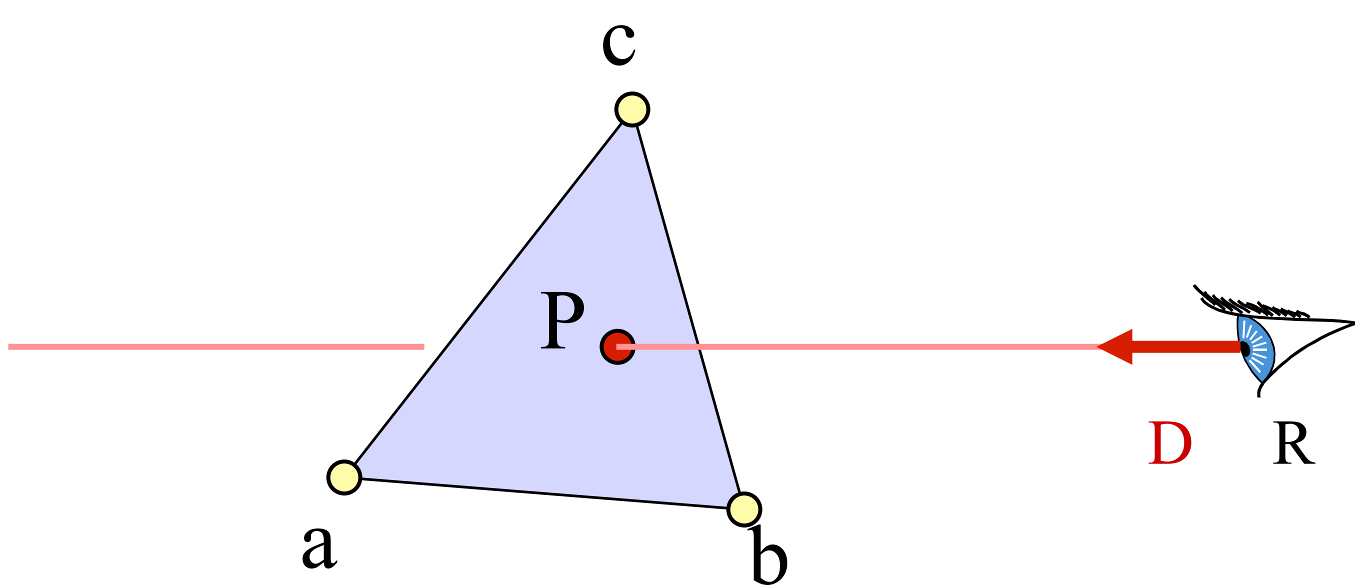
$$\beta = \frac{\begin{vmatrix} a_x - R_x & a_x - c_x & D_x \\ a_y - R_y & a_y - c_y & D_y \\ a_z - R_z & a_z - c_z & D_z \end{vmatrix}}{|A|} \qquad \gamma = \frac{\begin{vmatrix} a_x - b_x & a_x - R_x & D_x \\ a_y - b_y & a_y - R_y & D_y \\ a_z - b_z & a_z - R_z & D_z \end{vmatrix}}{|A|} \qquad t = \frac{\begin{vmatrix} a_x - b_x & a_x - c_x & a_x - R_x \\ a_y - b_y & a_y - c_y & a_y - R_y \\ a_z - b_z & a_z - c_z & a_z - R_z \end{vmatrix}}{|A|}$$



Source: F. Durand

Advantage

Efficient
Store no plane equation
Get the barycentric coordinates for free
    Useful for interpolation, texture mapping

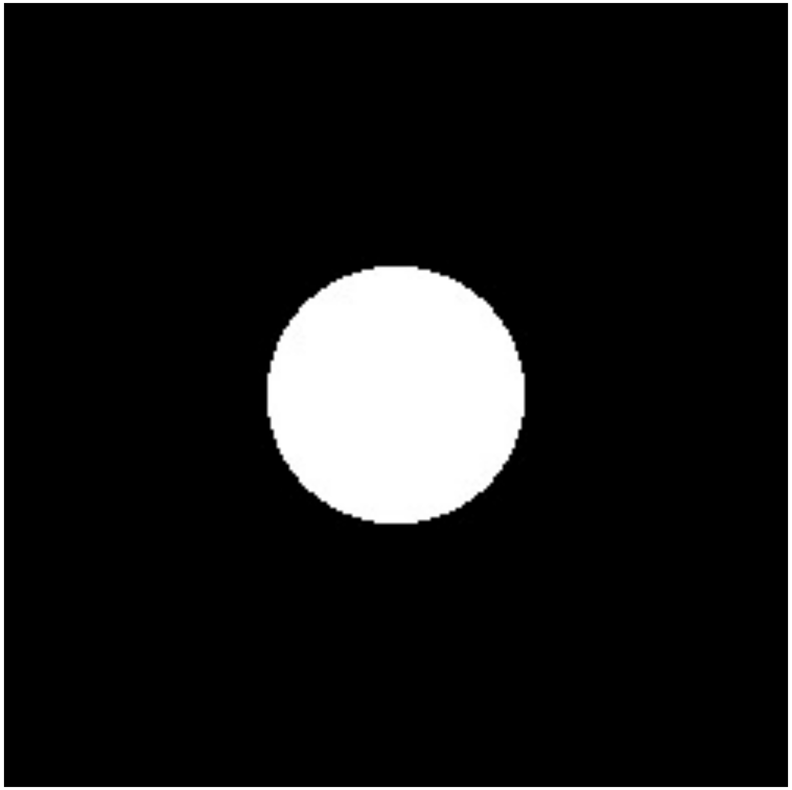


Source: F. Durand

Image so far

- With eye ray generation and sphere intersection

```
Surface s = new Sphere((0.0, 0.0, 0.0), 1.0);
for 0 <= iy < ny
    for 0 <= ix < nx {
        ray = camera.getRay(ix, iy);
        hitSurface, t = s.intersect(ray, 0, +inf)
        if hitSurface is not null
            image.set(ix, iy, white);
    }
```



Intersection against many shapes

- The basic idea is:

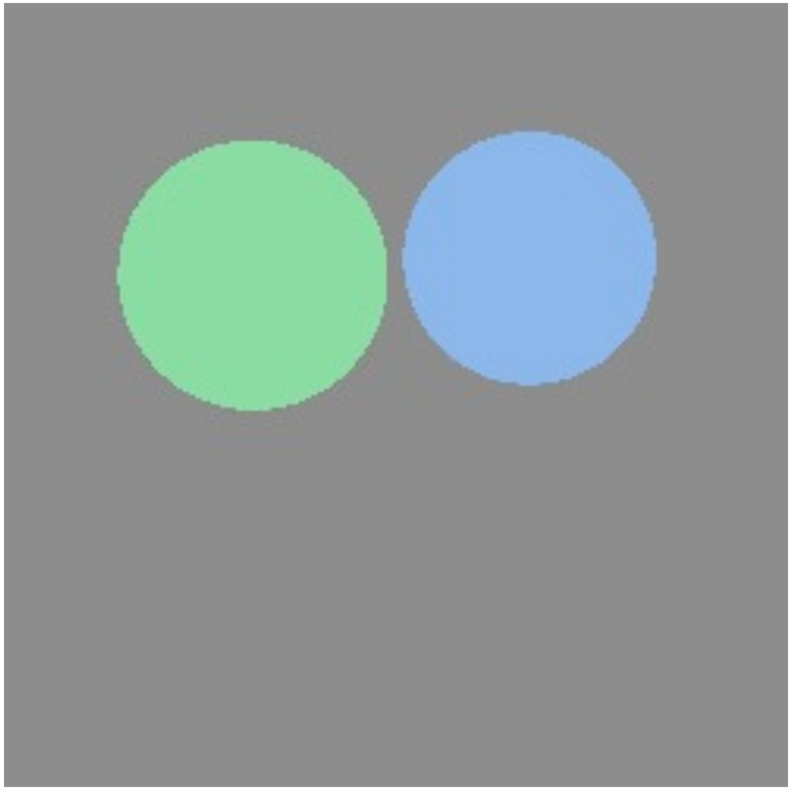
```
Group.intersect (ray, tMin, tMax) {
    tBest = +inf; firstSurface = null;
    for surface in surfaceList {
        hitSurface, t = surface.intersect(ray, tMin, tBest);
        if hitSurface is not null {
            tBest = t;
            firstSurface = hitSurface;
        }
    }
return hitSurface, tBest;
}
```

  - this is linear in the number of shapes
    but there are sublinear methods (acceleration structures)

Image so far

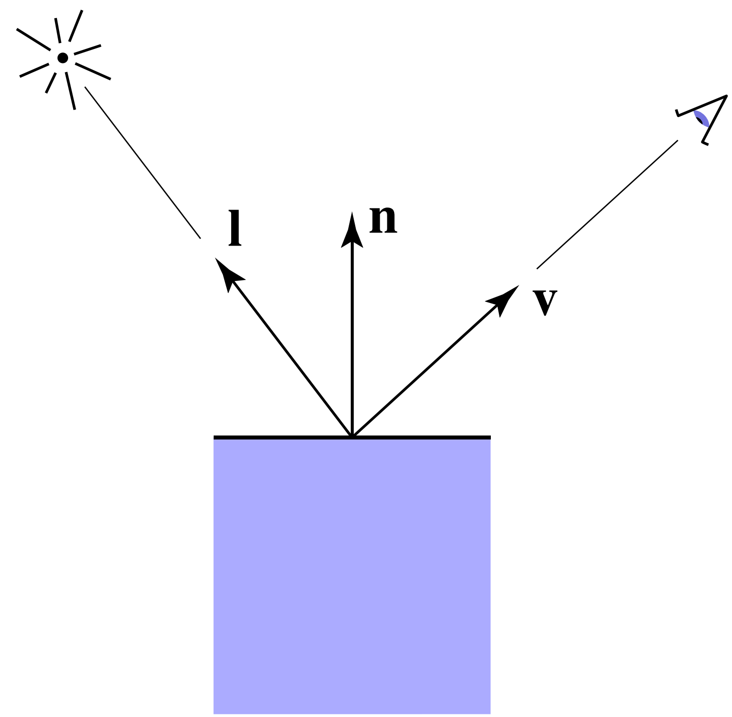
- With eye ray generation and scene intersection

```
for 0 <= iy < ny
    for 0 <= ix < nx {
        ray = camera.getRay(ix, iy);
        c = scene.trace(ray, 0, +inf);
        image.set(ix, iy, c);
    }

...

Scene.trace(ray, tMin, tMax) {
    surface, t = surfs.intersect(ray, tMin, tMax);
    if (surface != null) return surface.color();
    else return black;
}
```



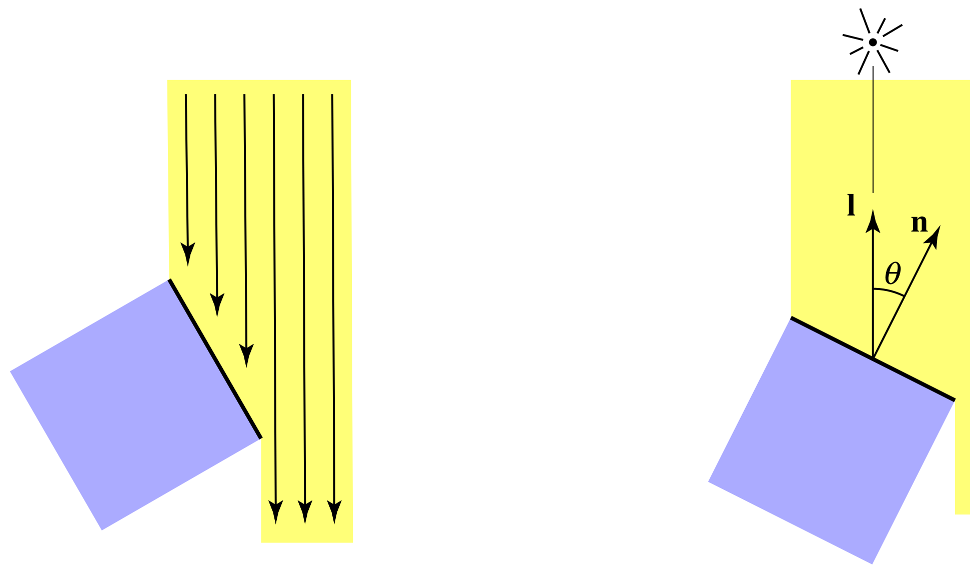
Shading

- Compute light reflected toward camera
- Inputs:
  - eye direction
  - light direction
    (for each of many lights)
  - surface normal
  - surface parameters
    (color, shininess, …)



Diffuse reflection

- Light is scattered uniformly in all directions
  - the surface color is the same for all viewing directions
- Lambert's cosine law



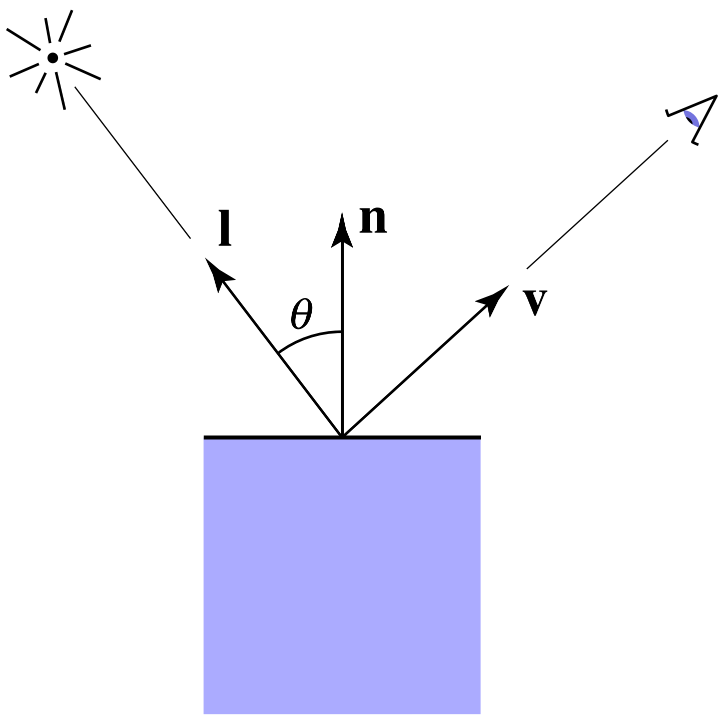
Top face of cube receives a certain amount of light

Top face of 60° rotated cube intercepts half the light

In general, light per unit area is proportional to $\cos\theta = \mathbf{l} \cdot \mathbf{n}$

Lambertian shading

- Shading independent of view direction



illumination
from source

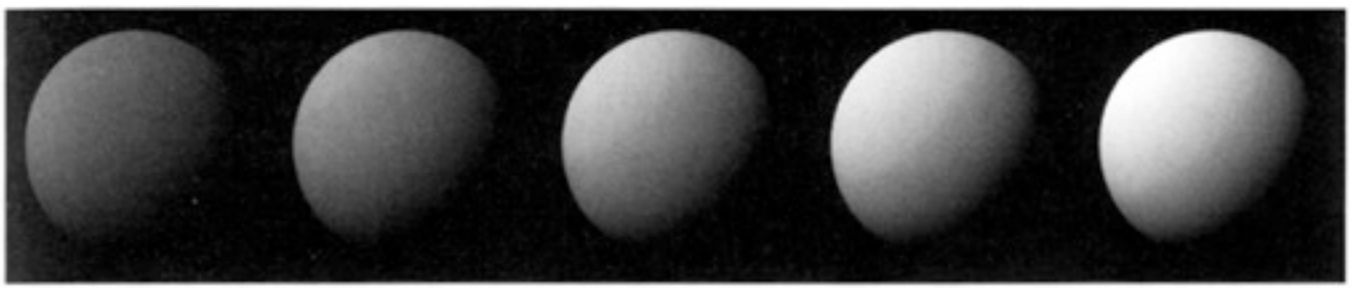
$$L_d = k_d \, I \max(0, \mathbf{n} \cdot \mathbf{l})$$

diffuse
coefficient

diffusely
reflected
light

Lambertian shading

- Produces matte appearance



$k_d \longrightarrow$

[Foley et al.]

Diffuse shading

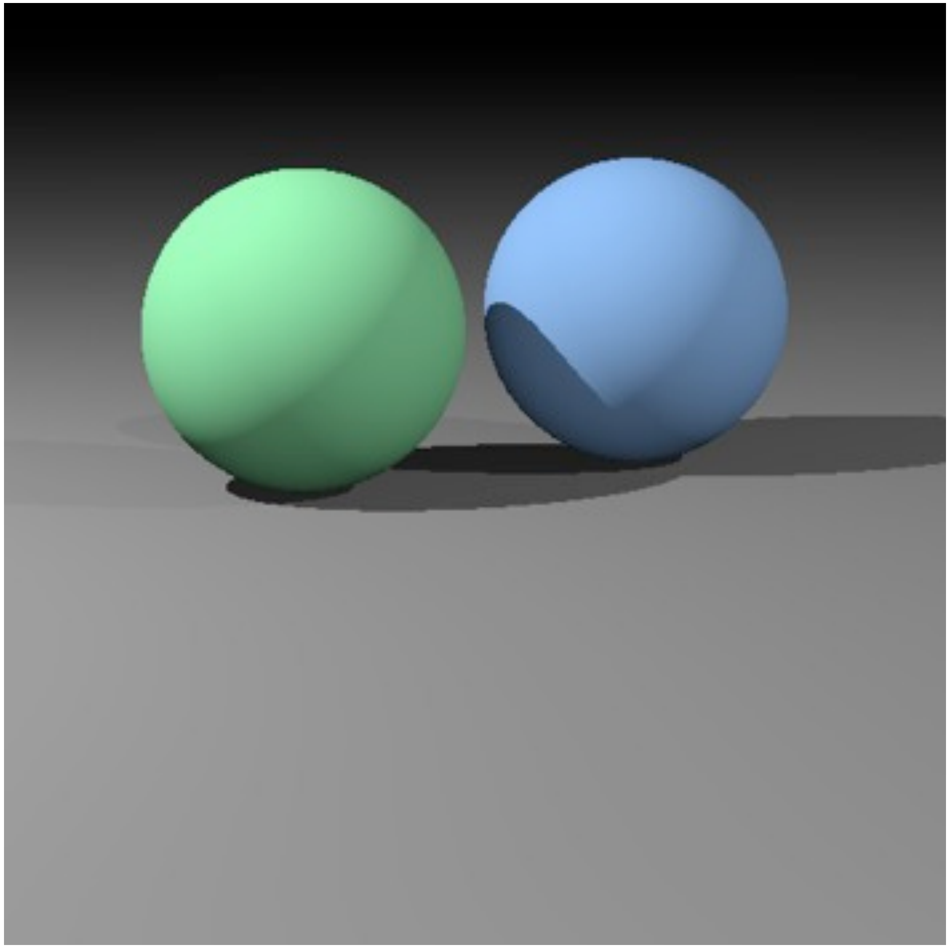
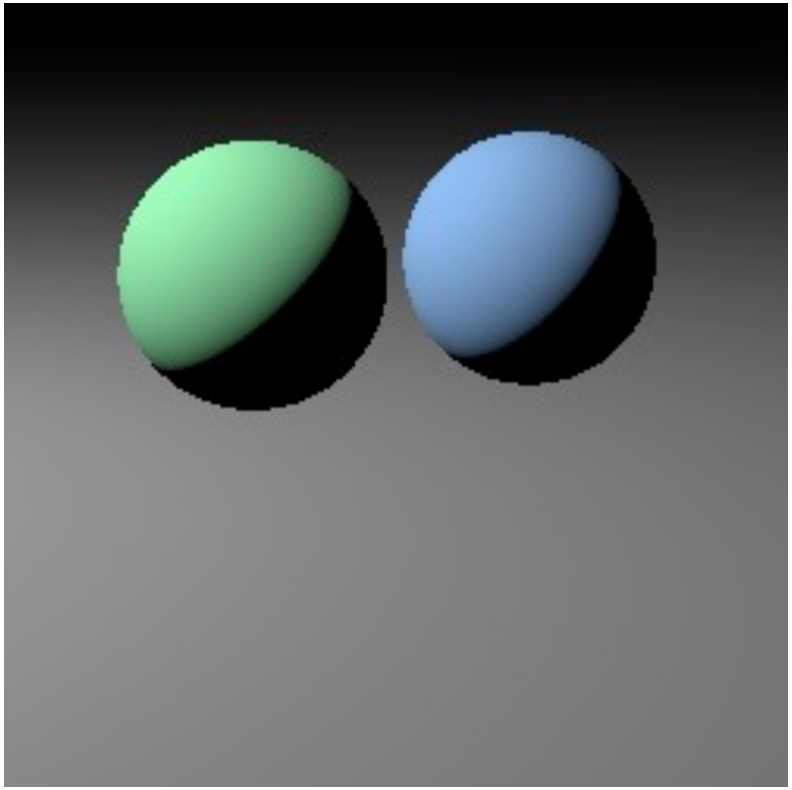


Image so far

```
Scene.trace(Ray ray, tMin, tMax) {
    surface, t = hit(ray, tMin, tMax);
    if surface is not null {
        point = ray.evaluate(t);
        normal = surface.getNormal(point);
        return surface.shade(ray, point,
            normal, light);
    }
    else return backgroundColor;
}

...

Surface.shade(ray, point, normal, light) {
    v = −normalize(ray.direction);
    l = normalize(light.pos − point);
    // compute shading
}
```

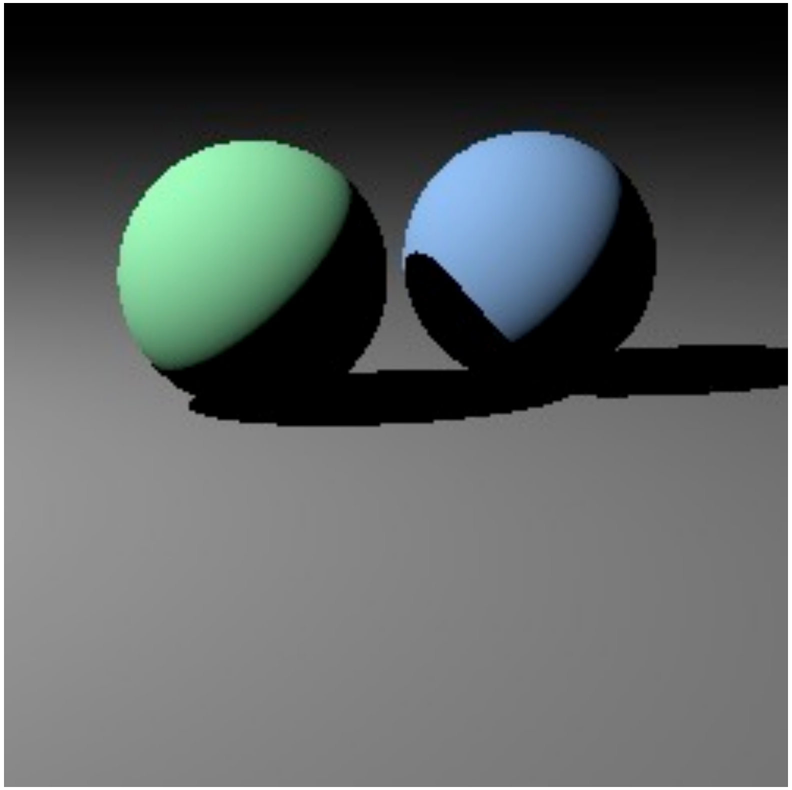


Shadows

- Surface is only illuminated if nothing blocks its view of the light.
- With ray tracing it's easy to check
  - just intersect a ray with the scene!

Image so far

```
Surface.shade(ray, point, normal, light) {
    shadRay = (point, light.pos – point);
    if (shadRay not blocked) {
        v = –normalize(ray.direction);
        l = normalize(light.pos – point);
        // compute shading
    }
    return black;
}
```

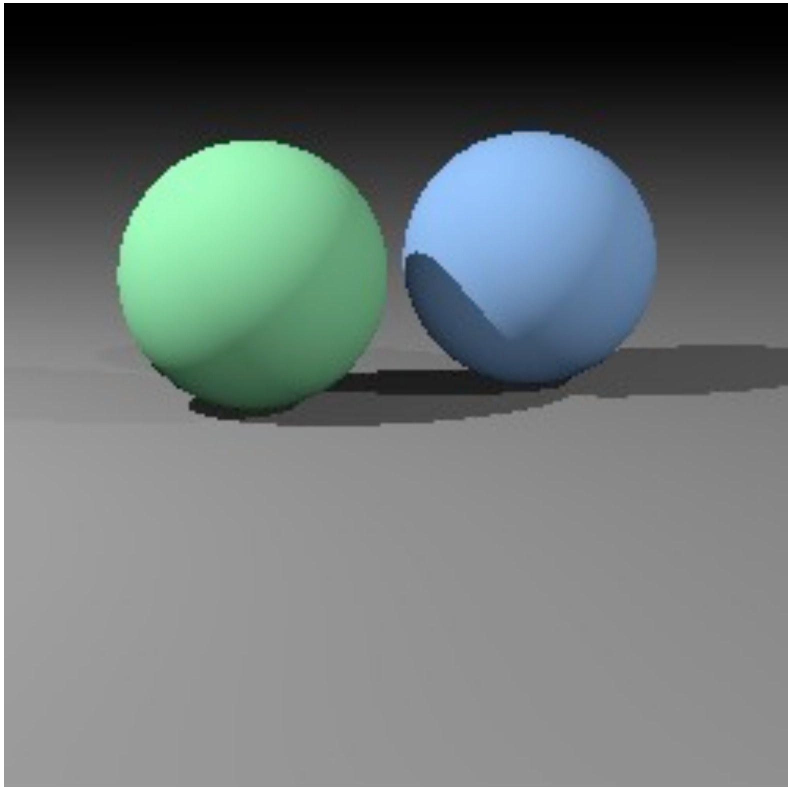


Multiple lights

- Important to fill in black shadows
- Just loop over lights, add contributions
- Ambient shading
  - black shadows are not really right
  - one solution: dim light at camera
  - alternative: add a constant "ambient" color to the shading…

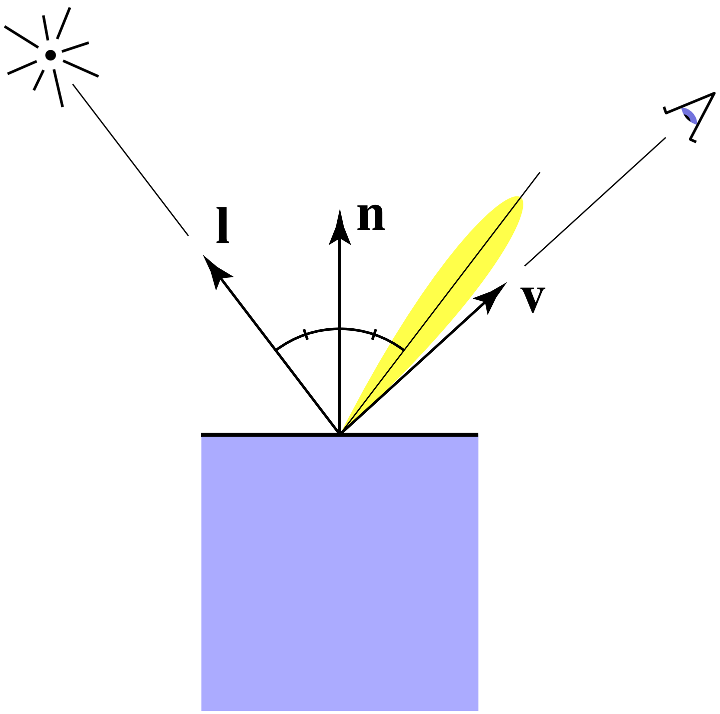
Image so far

```
shade(ray, point, normal, lights) {
    result = ambient;
    for light in lights {
        if (shadow ray not blocked) {
            result += shading contribution;
        }
    }
    return result;
}
```



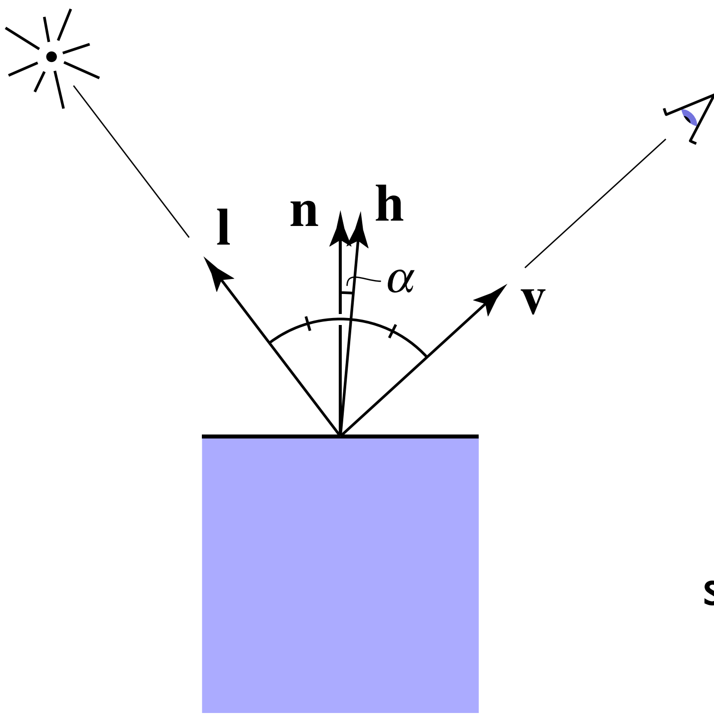
Specular shading (Blinn-Phong)

- Intensity depends on view direction
  - bright near mirror configuration



Specular shading (Blinn-Phong)

- Close to mirror ⇔ half vector near normal
  - Measure "near" by dot product of unit vectors



$$\mathbf{h} = \mathrm{bisector}(\mathbf{v}, \mathbf{l}) = \frac{\mathbf{v} + \mathbf{l}}{\|\mathbf{v} + \mathbf{l}\|}$$

$$L_s = k_s\, I \max(0, \cos\alpha)^p = k_s\, I \max(0, \mathbf{n} \cdot \mathbf{h})^p$$

specularly reflected light

specular coefficient

Phong model—plots

- Increasing *n* narrows the lobe

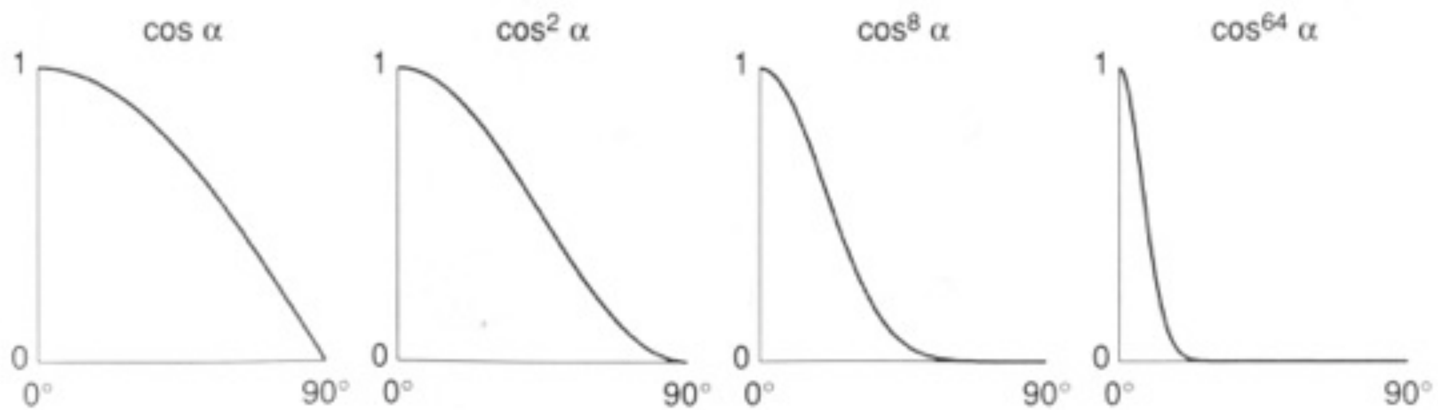
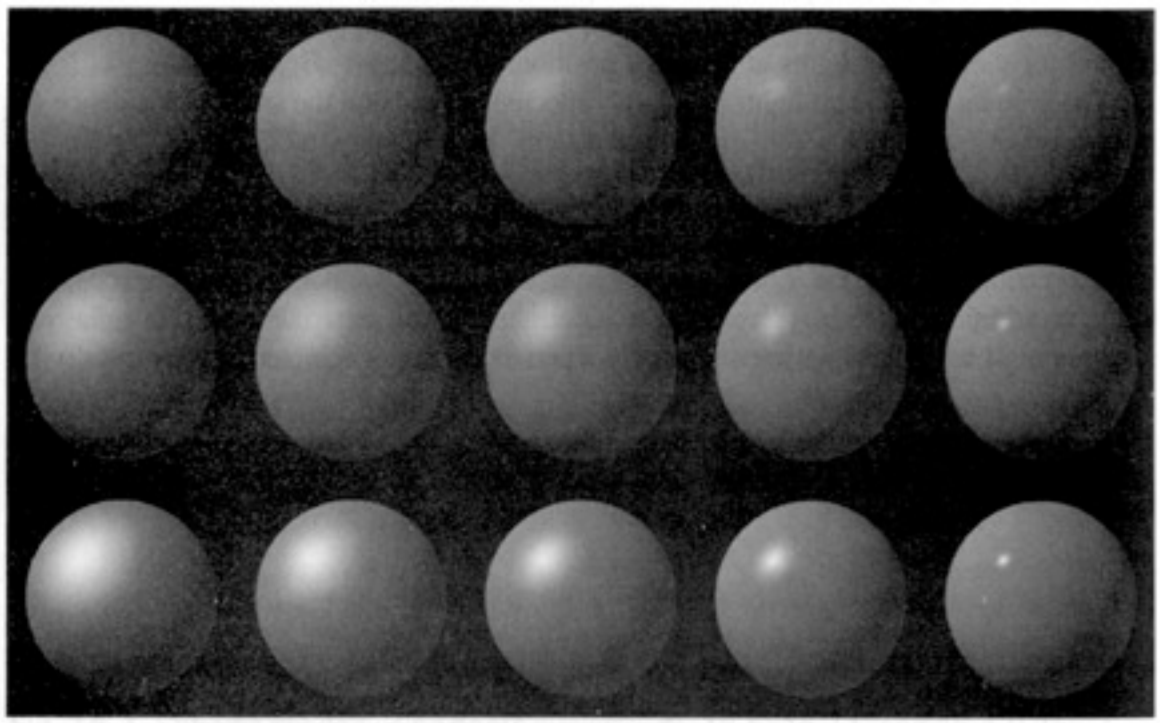


Fig. 16.9 Different values of $\cos^n \alpha$ used in the Phong illumination model.

[Foley et al.]

Specular shading

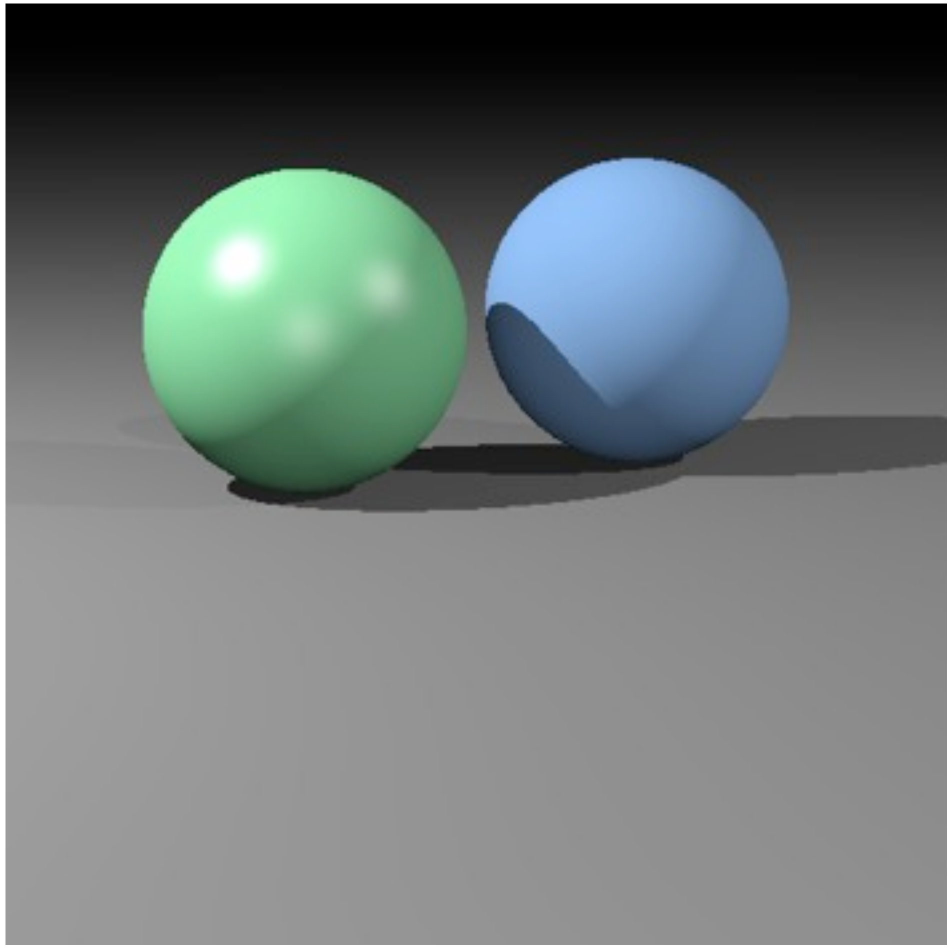


k_s ↓

p →

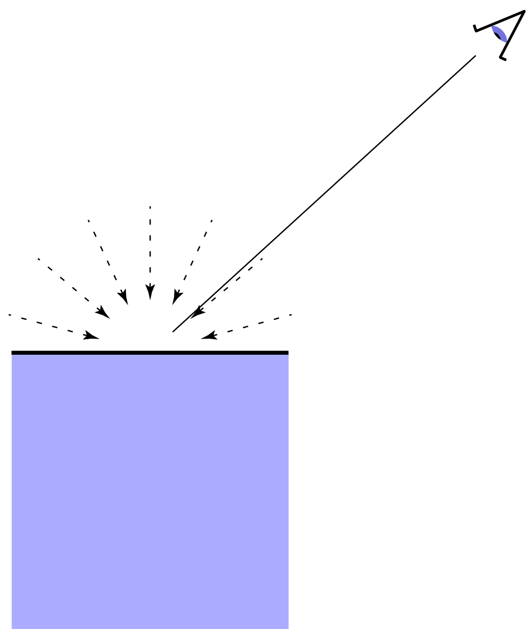
[Foley et al.]

Diffuse + Phong shading



Ambient shading

- Shading that does not depend on anything
  - add constant color to account for disregarded illumination and fill in black shadows



$$L_a = k_a \, I_a$$

ambient coefficient

reflected ambient light

Putting it together

- Usually include ambient, diffuse, Phong in one model

$$\begin{aligned} L &= L_a + L_d + L_s \\ &= k_a\, I_a + k_d\, I \max(0, \mathbf{n} \cdot \mathbf{l}) + k_s\, I \max(0, \mathbf{n} \cdot \mathbf{h})^p \end{aligned}$$

- The final result is the sum over many lights

$$L = L_a + \sum_{i=1}^{N} \left[(L_d)_i + (L_s)_i \right]$$

$$L = k_a\, I_a + \sum_{i=1}^{N} \left[k_d\, I_i \max(0, \mathbf{n} \cdot \mathbf{l}_i) + k_s\, I_i \max(0, \mathbf{n} \cdot \mathbf{h}_i)^p \right]$$

Mirror reflection

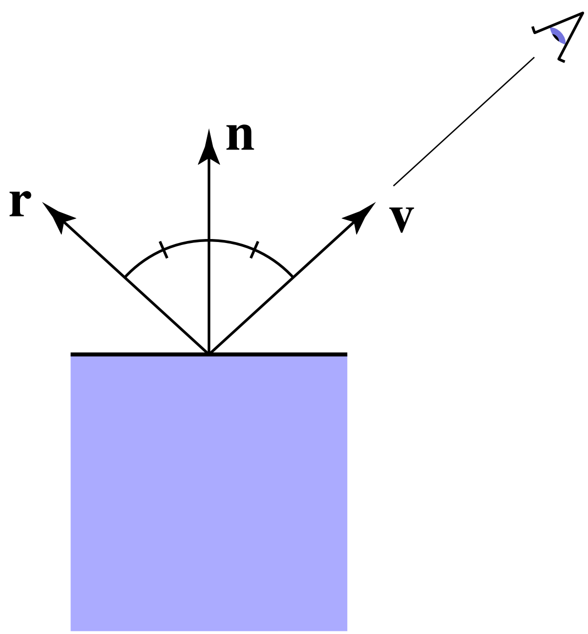
- Consider perfectly shiny surface
  - there isn't a highlight
  - instead there's a reflection of other objects
- Can render this using recursive ray tracing
  - to find out mirror reflection color, ask what color is seen from surface point in reflection direction
  - already computing reflection direction for Phong…
- "Glazed" material has mirror reflection and diffuse

$$L = L_a + L_d + L_m$$

  - where L_m is evaluated by tracing a new ray

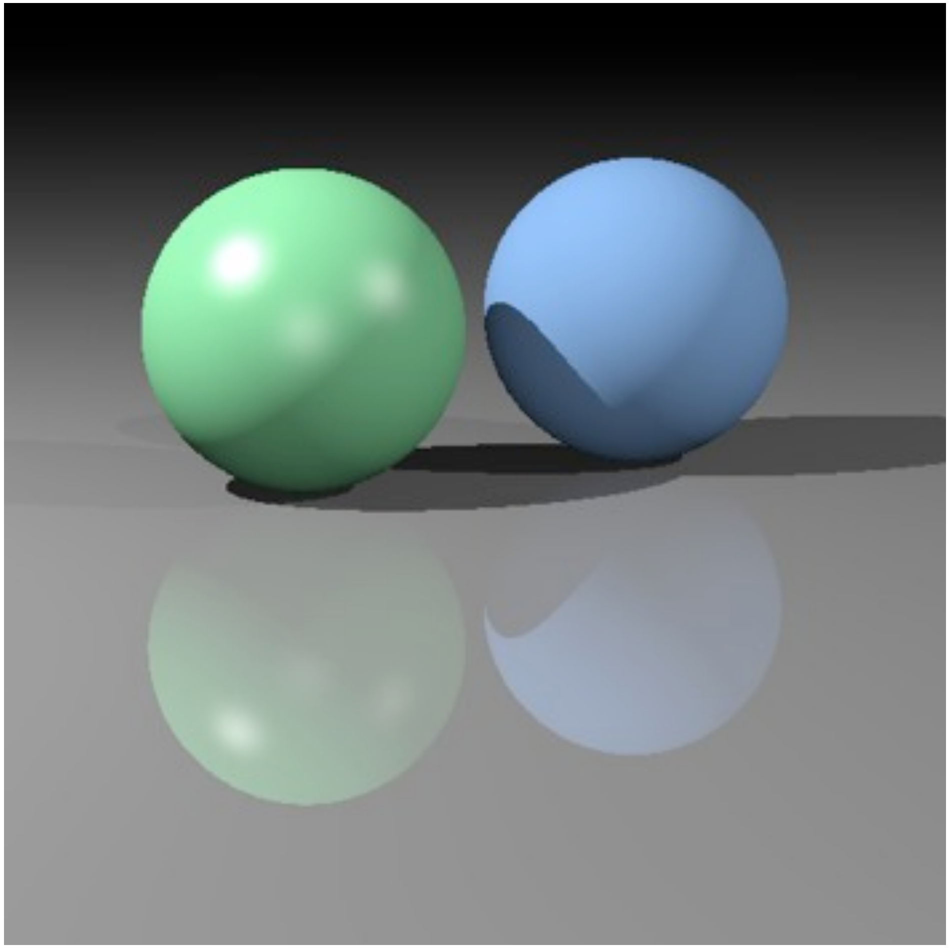
Mirror reflection

- Intensity depends on view direction
  - reflects incident light from mirror direction



$$\begin{aligned}\mathbf{r} &= \mathbf{v} + 2((\mathbf{n}\cdot\mathbf{v})\mathbf{n} - \mathbf{v}) \\ &= 2(\mathbf{n}\cdot\mathbf{v})\mathbf{n} - \mathbf{v}\end{aligned}$$

Diffuse + mirror reflection (glazed)



(glazed material on floor)

Ray tracer architecture 101

- You want a class called Ray
  - point and direction; evaluate(t)
  - possible: tMin, tMax
- Some things can be intersected with rays
  - individual surfaces
  - groups of surfaces (acceleration goes here)
  - the whole scene
  - make these all subclasses of Surface
  - limit the range of valid *t* values (*e.g.* shadow rays)
- Once you have the visible intersection, compute the color
  - may want to separate shading code from geometry
  - separate class: Material (each Surface holds a reference to one)
  - its job is to compute the color

Architectural practicalities

- Return values
  - surface intersection tends to want to return multiple values
    - *t*, surface or shader, normal vector, maybe surface point
  - in many programming languages (e.g. Java) this is a pain
  - typical solution: an *intersection record*
    - a class with fields for all these things
    - keep track of the intersection record for the closest intersection
    - be careful of accidental aliasing (which is very easy if you're new to Java)
- Efficiency
  - what objects are created for every ray? try to find a place for them where you can reuse them.
  - Shadow rays can be cheaper (any intersection will do, don't need closest)
  - but: "First Get it Right, Then Make it Fast"