Acknowledgement: The course slides are adapted from the slides prepared by Steve Marschner of Cornell University
What light is

- Light is electromagnetic radiation
  - exists as oscillations of different frequency (or, wavelength)
Measuring light

- Salient property is the *spectral power distribution (SPD)*
  - the amount of light present at each wavelength
  - units: Watts per nanometer (tells you how much power you’ll find in a narrow range of wavelengths)
  - for color, often use ‘‘relative units’’ when overall intensity is not important

\[
\text{amount of light} = 180 \, d\lambda
\]
(relative units)

wavelength band (width \(d\lambda\))
What color is

- Colors are the sensations that arise from light energy of different wavelengths
  - we are sensitive from about 380 to 760 nm—one “octave”
- Color is a phenomenon of human perception; it is not a universal property of light
- Roughly speaking, things appear “colored” when they depend on wavelength and “gray” when they do not.
The problem of color science

• Build a model for human color perception
• That is, map a *Physical light description* to a *Perceptual color sensation*

![Diagram of fluorescent light spectrum with arrows pointing from physical to perceptual](attachment:image.png)
The eye as a measurement device

- We can model the low-level behavior of the eye by thinking of it as a light-measuring machine
  - its optics are much like a camera
  - its detection mechanism is also much like a camera

- Light is measured by the photoreceptors in the retina
  - they respond to visible light
  - different types respond to different wavelengths
A simple light detector

• Produces a scalar value (a number) when photons land on it
  – this value depends strictly on the number of photons detected
  – each photon has a probability of being detected that depends on the wavelength
  – there is no way to tell the difference between signals caused by light of different wavelengths: there is just a number

• This model works for many detectors:
  – based on semiconductors (such as in a digital camera)
  – based on visual photopigments (such as in human eyes)
A simple light detector

\[ X = \int n(\lambda)p(\lambda) \, d\lambda \]
Light detection math

• Same math carries over to power distributions
  – spectrum entering the detector has its spectral power distribution (SPD), \( s(\lambda) \)
  – detector has its \textit{spectral sensitivity} or \textit{spectral response}, \( r(\lambda) \)

\[
X = \int s(\lambda)r(\lambda) \, d\lambda
\]

- measured signal
- detector’s sensitivity
- input spectrum
Light detection math

\[ X = \int s(\lambda) r(\lambda) \, d\lambda \quad \text{or} \quad X = s \cdot r \]

• If we think of \( s \) and \( r \) as vectors, this operation is a dot product (aka inner product)
  – in fact, the computation is done exactly this way, using sampled representations of the spectra.

  • let \( \lambda_i \) be regularly spaced sample points \( \Delta\lambda \) apart; then:
    \[ \tilde{s}[i] = s(\lambda_i); \tilde{r}[i] = r(\lambda_i) \]
    \[ \int s(\lambda) r(\lambda) \, d\lambda \approx \sum_i \tilde{s}[i] \tilde{r}[i] \Delta\lambda \]

  • this sum is very clearly a dot product
Cone Responses

- S, M, L cones have broadband spectral sensitivity
- S, M, L neural response is integrated w.r.t. $\lambda$
  - we’ll call the response functions $r_S, r_M, r_L$
- Results in a trichromatic visual system
- S, M, and L are tristimulus values
Cone responses to a spectrum $s$

\[
S = \int r_S(\lambda)s(\lambda) \, d\lambda = r_S \cdot s
\]

\[
M = \int r_M(\lambda)s(\lambda) \, d\lambda = r_M \cdot s
\]

\[
L = \int r_L(\lambda)s(\lambda) \, d\lambda = r_L \cdot s
\]
Colorimetry: an answer to the problem

• Wanted to map a *Physical light description* to a *Perceptual color sensation*

• Basic solution was known and standardized by 1930
  – Though not quite in this form—more on that in a bit

$$S = r_s \cdot s$$

$$M = r_m \cdot s$$

$$L = r_l \cdot s$$
Basic fact of colorimetry

• Take a spectrum (which is a function)
• Eye produces three numbers
• This throws away a lot of information!
  – Quite possible to have two different spectra that have the same S, M, L tristimulus values
  – Two such spectra are metamers
Pseudo-geometric interpretation

• A dot product is a projection
• We are projecting a high dimensional vector (a spectrum) onto three vectors
  – differences that are perpendicular to all 3 vectors are not detectable
• For intuition, we can imagine a 3D analog
  – 3D stands in for high-D vectors
  – 2D stands in for 3D
  – Then vision is just projection onto a plane
Pseudo-geometric interpretation

- The information available to the visual system about a spectrum is three values
  - this amounts to a loss of information analogous to projection on a plane
- Two spectra that produce the same response are metamers
Basic colorimetric concepts

• Luminance
  – the overall magnitude of the visual response to a spectrum (independent of its color)
    • corresponds to the everyday concept “brightness”
  – determined by product of SPD with the luminous efficiency function $V_\lambda$ that describes the eye’s overall ability to detect light at each wavelength
  – e.g. lamps are optimized to improve their luminous efficiency (tungsten vs. fluorescent vs. sodium vapor)
Luminance, mathematically

• Y just has another response curve (like S, M, and L)

\[ Y = r_Y \cdot s \]

– \( r_Y \) is really called “\( V_\lambda \)”

• \( V_\lambda \) is a linear combination of S, M, and L
  – Has to be, since it’s derived from cone outputs
More basic colorimetric concepts

• Chromaticity
  – what’s left after luminance is factored out (the color without regard for overall brightness)
  – scaling a spectrum up or down leaves chromaticity alone

• Dominant wavelength
  – many colors can be matched by white plus a spectral color
  – correlates to everyday concept “hue”

• Purity
  – ratio of pure color to white in matching mixture
  – correlates to everyday concept “colorfulness” or “saturation”
Color reproduction

• Have a spectrum $s$; want to match on RGB monitor
  – “match” means it looks the same
  – any spectrum that projects to the same point in the visual color space is a good reproduction
• Must find a spectrum that the monitor can produce that is a metamer of $s$
Additive Color
CRT display primaries

- Curves determined by phosphor emission properties
LCD display primaries

- Curves determined by (fluorescent) backlight and filters
Combining Monitor Phosphors with Spatial Integration
Color reproduction

• Say we have a spectrum \( s \) we want to match on an RGB monitor
  – “match” means it looks the same
  – any spectrum that projects to the same point in the visual color space is a good reproduction

• So, we want to find a spectrum that the monitor can produce that matches \( s \)
  – that is, we want to display a metamer of \( s \) on the screen
Color reproduction

- We want to compute the combination of r, g, b that will project to the same visual response as s.
Color reproduction as linear algebra

• The projection onto the three response functions can be written in matrix form:

\[
\begin{bmatrix}
S \\
M \\
L
\end{bmatrix} =
\begin{bmatrix}
- r_S \\
- r_M \\
- r_L
\end{bmatrix}
\begin{bmatrix}
s
\end{bmatrix}
\]

or,

\[
V = M_{SML} s.
\]
Color reproduction as linear algebra

• The spectrum that is produced by the monitor for the color signals R, G, and B is:

\[ s_a(\lambda) = R s_r(\lambda) + G s_g(\lambda) + B s_b(\lambda). \]

• Again the discrete form can be written as a matrix:

\[
\begin{bmatrix}
\vdots \\
\vdots
\end{bmatrix}
= \begin{bmatrix}
\vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots \\
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} =
\]

or,

\[ s_a = M_{RGB} C. \]
Color reproduction as linear algebra

• What color do we see when we look at the display?
  – Feed $C$ to display
  – Display produces $s_a$
  – Eye looks at $s_a$ and produces $V$

\[
V = M_{SML}M_{RGB}C
\]

\[
\begin{bmatrix}
S \\
M \\
L
\end{bmatrix} = \begin{bmatrix}
r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\
r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\
r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B
\end{bmatrix} \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]
Color reproduction as linear algebra

• Goal of reproduction: visual response to $s$ and $s_a$ is the same:

\[ M_{SML} \tilde{s} = M_{SML} \tilde{s}_a. \]

• Substituting in the expression for $s_a$:

\[ M_{SML} \tilde{s} = M_{SML} M_{RGB} C \]

\[ C = (M_{SML} M_{RGB})^{-1} M_{SML} \tilde{s} \]

*color matching matrix for RGB*
Subtractive Color
Reflection from colored surface

[Graphs showing reflectance and intensity relationships for different light sources and objects.]

[Stone 2003]
Subtractive color

• Produce desired spectrum by *subtracting* from white light (usually via absorption by pigments)
• Photographic media (slides, prints) work this way
• Leads to C, M, Y as primaries
• Approximately, I – R, I – G, I – B
Color spaces

• Need three numbers to specify a color
  – but what three numbers?
  – a *color space* is an answer to this question
• Common example: monitor RGB
  – define colors by what R, G, B signals will produce them on your monitor
    (in math, $s = RR + GG + BB$ for some spectra $R, G, B$)
  – device dependent (depends on gamma, phosphors, gains, …)
    • therefore if I choose RGB by looking at my monitor and send it to you, you may not see the same color
  – also leaves out some colors (limited *gamut*), e.g. vivid yellow
Standard color spaces

- Standardized RGB (sRGB)
  - makes a particular monitor RGB standard
  - other color devices simulate that monitor by calibration
  - sRGB is usable as an interchange space; widely adopted today
  - gamut is still limited
A universal color space: XYZ

- Standardized by CIE (Commission Internationale de l’Eclairage, the standards organization for color science)
- Based on three “imaginary” primaries $X$, $Y$, and $Z$
  (in math, $s = XX + YY + ZZ$)
  - imaginary = only realizable by spectra that are negative at some wavelengths
  - key properties
    - any stimulus can be matched with positive $X$, $Y$, and $Z$
    - separates out luminance: $X$, $Z$ have zero luminance, so $Y$ tells you the luminance by itself
Separating luminance, chromaticity

- Luminance: $Y$
- Chromaticity: $x, y, z$, defined as

\[
x = \frac{X}{X + Y + Z}
\]
\[
y = \frac{Y}{X + Y + Z}
\]
\[
z = \frac{Z}{X + Y + Z}
\]

- since $x + y + z = 1$, we only need to record two of the three
  - usually choose $x$ and $y$, leading to $(x, y, Y)$ coords
Chromaticity Diagram

spectral locus

purple line
Chromaticity Diagram
Color Gamuts

Monitors/printers can’t produce all visible colors

Reproduction is limited to a particular domain

For additive color (e.g. monitor) gamut is the triangle defined by the chromaticities of the three primaries.
Perceptually organized color spaces

- Artists often refer to colors as *tints*, *shades*, and *tones* of pure pigments
  - tint: mixture with white
  - shade: mixture with black
  - tones: mixture with black and white
  - gray: no color at all (aka. neutral)
- This seems intuitive
  - tints and shades are inherently related to the pure color
  - “same” color but lighter, darker, paler, etc.
Perceptual dimensions of color

- **Hue**
  - the “kind” of color, regardless of attributes
  - colorimetric correlate: dominant wavelength
  - artist’s correlate: the chosen pigment color

- **Saturation**
  - the “colorfulness”
  - colorimetric correlate: purity
  - artist’s correlate: fraction of paint from the colored tube

- **Lightness (or value)**
  - the overall amount of light
  - colorimetric correlate: luminance
  - artist’s correlate: tints are lighter, shades are darker
Perceptual dimensions: chromaticity

- In x, y, Y (or another luminance/chromaticity space), Y corresponds to lightness
- hue and saturation are then like polar coordinates for chromaticity (starting at white, which way did you go and how far?)
**Perceptual dimensions of color**

- There’s good evidence ("opponent color theory") for a neurological basis for these dimensions
  - the brain seems to encode color early on using three axes:
    - white — black, red — green, yellow — blue
  - the white—black axis is lightness; the others determine hue and saturation
  - one piece of evidence: you can have a light green, a dark green, a yellow-green, or a blue-green, but you can’t have a reddish green (just doesn’t make sense)
    - thus red is the *opponent* to green
  - another piece of evidence: afterimages (next slide)
RGB as a 3D space

• A cube:

(demo of RGB cube)
Perceptual organization for RGB: HSV

- Uses hue (an angle, 0 to 360), saturation (0 to 1), and value (0 to 1) as the three coordinates for a color
  - the brightest available RGB colors are those with one of R, G, B equal to 1 (top surface)
  - each horizontal slice is the surface of a sub-cube of the RGB cube

(demo of HSV color pickers)
Perceptually uniform spaces

- Two major spaces standardized by CIE
  - designed so that equal differences in coordinates produce equally visible differences in color
  - LUV: earlier, simpler space; $L^*, u^*, v^*$
  - LAB: more complex but more uniform: $L^*, a^*, b^*$
  - both separate luminance from chromaticity
  - including a gamma-like nonlinear component is important