Active Contours Without Edges

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1 Active Contours Without Edges

Chan and Vese [1] proposed an approximation for the MS segmentation model by following the level-set based curve evolution formulation [2, 3]. Level sets provide an implicit contour representation where an evolving curve is represented with the zero-level line of a level set function (Figure 1). The basic aim of Chan and Vese (CV) model is to partition a given image into two regions that are likely to correspond object and background regions by embedding the object boundary by the zero-level curve of a 3D level set function.

Let ϕ be a level set function. Then, the Chan-Vese functional is

$$\begin{aligned} E_{CV}(c_1, c_2, \phi) &= \lambda_1 \int_\Omega (f - c_1)^2 H(\phi) dx + \lambda_2 \int_\Omega (f - c_2)^2 (1 - H(\phi)) dx \\ &+ \mu \int_\Omega |\nabla H(\phi)| dx \end{aligned} \tag{1}$$

where λ_1, $\lambda_2 > 0$ and $\mu \geq 0$ are fixed parameters. The length parameter μ can be interpreted as a scale parameter since it determines the relative importance of the length term. The possibility of detecting smaller objects/regions increases with decreasing μ.

The model represents the segmented image with the variables c_1, c_2 and $H(\phi)$, where $H(\phi)$ denotes the Heaviside function of the level set function ϕ defined by

$$H(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0\,. \end{cases} \tag{2}$$

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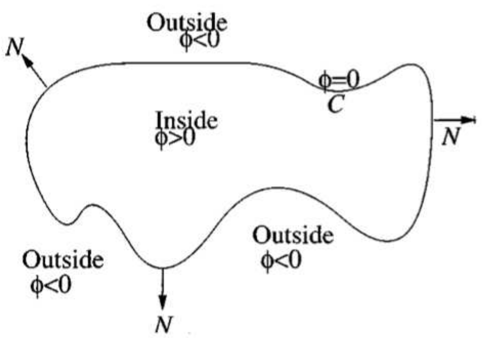

Figure 1: A curve can be represented as the zero-level line of a level set function (image taken from [1]).

The Heaviside function of the level set function, $H(\phi)$, specifies object and background regions in the observed image f, while the last term in (1), $\int_\Omega |\nabla H(\phi)|$, expresses the length of the object boundary. On top of that, the scalars c_1 and c_2 denote the average gray values of object and background regions indicated by $\phi \geq 0$ and $\phi < 0$, respectively. Thus, the CV model can be seen as a two-phase piecewise constant approximation of the MS model, which can theoretically be obtained by letting the weight α of the smoothness term tend to infinity, and forcing a two-region segmentation.

To segment a given image, the functional (1) needs to be minimized with respect to c_1, c_2, and ϕ. Keeping ϕ fixed, the average gray values c_1 and c_2 can be easily estimated by

$$c_1 = \frac{\int_\Omega f(x)H(\phi(x))dx}{\int_\Omega H(\phi(x))dx}, \tag{3}$$

$$c_2 = \frac{\int_\Omega f(x)(1-H(\phi(x)))dx}{\int_\Omega (1-H(\phi(x)))dx}. \tag{4}$$

Keeping c_1 and c_2 fixed and using the calculus of variations for the functional (1), the gradient descent equation for the evolution of ϕ is derived as

$$\frac{\partial \phi}{\partial t} = \delta(\phi)\left[\mu\, \nabla \cdot \left(\frac{\nabla\phi}{|\nabla\phi|}\right) - \lambda_1(f-c_1)^2 + \lambda_2(f-c_2)^2\right]. \tag{5}$$

In Figure 2, we illustrate segmentation of a sample noisy image which contains several objects of different shapes and sizes. We initialiaze the level set function ϕ with $\phi_0 = -\sqrt{(x-100)^2+(y-100)^2}+90$. As the zero-level line of the evolving level set function ϕ is attracted to object boundaries, a more accurate piecewise constant approximations of the original image f is recovered. Although some of the objects in the image have holes, they can be automatically detected by the CV model without considering additional curves since the level set formulation allows change of topology.

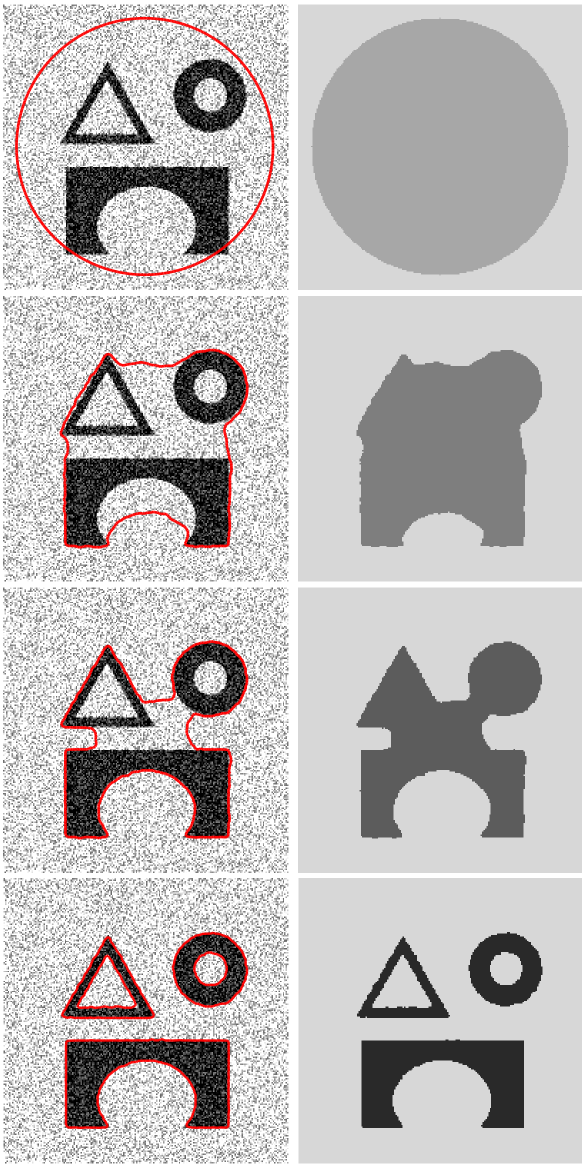

Figure 2: Example segmentation results (evolving contour ϕ superimposed on the original image f and the corresponding piecewise constant approximations of f). The parameters and the initial level set function are chosen as $\lambda_1 = \lambda_2 = 1$, $\mu = 0.5 \cdot 255^2$, $\varepsilon = 1$, and $\phi_0 = -\sqrt{(x-100)^2 + (y-100)^2} + 90$.

Numerical Implementation

In the numerical approximation of the CV model, generally, a regularized Heaviside function is used. For the remainder of this thesis, the following regularization is considered:

$$H_\varepsilon(z) = \frac{1}{2}\left(1 + \frac{2}{\pi} arctan\left(\frac{z}{\varepsilon}\right)\right), \tag{6}$$

$$\delta_\varepsilon(z) = \frac{dH_\varepsilon(z)}{dz} = \frac{1}{\pi}\frac{\varepsilon}{\varepsilon^2 + z^2}. \tag{7}$$

The evolution equation of ϕ (5) can be discretized by using standard finite differences as

$$\begin{aligned} \frac{\phi_{i,j}^{k+1} - \phi_{i,j}^{k}}{\Delta t} = \quad \delta(\phi_{i,j}^{k}) \quad & \Bigg[\mu \Delta_-^x \cdot \left(\frac{\Delta_+^x \phi_{i,j}^{k+1}}{\sqrt{(\Delta_+^x \phi_{i,j}^{k})^2 + (\phi_{i,j+1}^{k} - \phi_{i,j-1}^{k})^2/4}} \right) \\ & + \mu \Delta_-^y \cdot \left(\frac{\Delta_+^y \phi_{i,j}^{k+1}}{\sqrt{(\phi_{i+1,j}^{k} - \phi_{i-1,j}^{k})^2/4 + (\Delta_+^y \phi_{i,j}^{k})^2}} \right) \\ & - \lambda_1 \left(f_{i,j} - c_1(\phi^k) \right)^2 + \lambda_2 \left(f_{i,j} - c_2(\phi^k) \right)^2 \Bigg] \end{aligned} \tag{8}$$

where (i, j) denotes the pixel position, Δt is the time step, and forward and backward differences are defined as

$$\begin{aligned} \Delta_-^x \phi_{i,j} = \phi_{i,j} - \phi_{i-1,j}, \quad & \Delta_+^x \phi_{i,j} = \phi_{i+1,j} - \phi_{i,j}, \\ \Delta_-^y \phi_{i,j} = \phi_{i,j} - \phi_{i,j-1}, \quad & \Delta_+^y \phi_{i,j} = \phi_{i,j+1} - \phi_{i,j}. \end{aligned}$$

The minimization procedure is summarized in Algorithm 1. Keeping ϕ fixed, first the average gray values of object and background regions c_1 and c_2 are estimated. Next, the level set function ϕ is evolved according to (8). A numerical stopping criteria can be defined in the sense that the rate of change of ϕ or the overall energy (1) is less than a threshold.

Algorithm 1 Minimization of the Chan-Vese Model

Initialize the level set function with $\phi^0 = \phi_0$
for $k = 0$ to $kmax$ **do**
  Estimate $c_1(\phi^k)$ and $c_2(\phi^k)$ using (3) and (4), respectively
  Solve (8) for ϕ^{k+1}
  Check whether a numerical stopping criteria on ϕ is reached
  if it is reached **then**
    stop iterations
  end if
end for

References

[1] T. F. Chan and L. A. Vese. Active contours without edges. *IEEE Trans. Image Processing*, 10(2):266–277, 2001.

[2] S. Osher and J. A. Sethian. Fronts propagating with curvature-dependent speed: Algorithms based on Hamilton-Jacobi formulation. *J. Comput. Phys.*, 79:12–49, 1988.

[3] J. Sethian. *Level Set Methods.* Cambridge University Press, 1996.