Shape Analysis

Erkut Erdem

BIL717, April 2012

Introduction

- Shape is the primary source of visual information
  - Objects can be immediately recognized and classified based on their shapes
- Other visual clues are color, texture, spatial and temporal information, etc.




Slide: A. Erdem

What is a shape?

- A general theory of shape does not exist to date.

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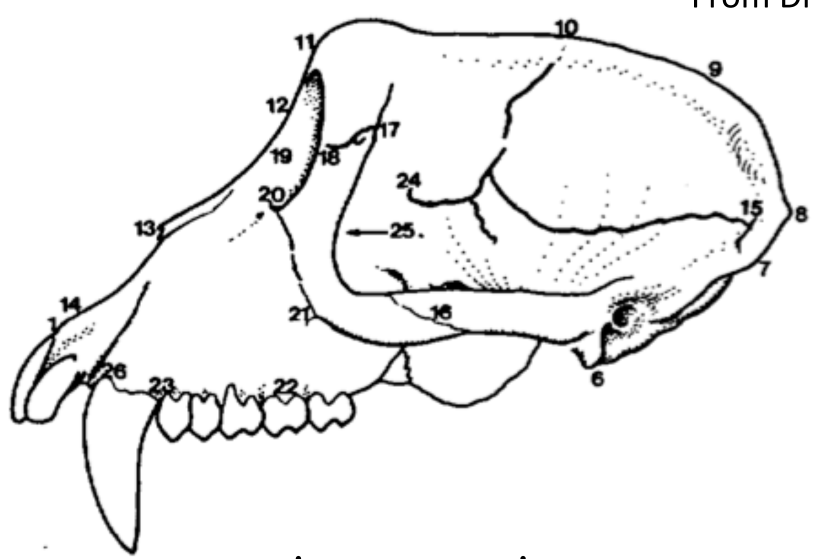
- Common shape representations:
  - Landmarks

From Dryden and Mardia



interest points

What is a shape?

- A general theory of shape does not exist to date.

- Common shape representations:
  - Landmarks
  - Shape Boundary

points, splines, level sets, etc.

What is a shape?

- A general theory of shape does not exist to date.
- Common shape representations:
  - Landmarks
  - Shape Boundary
  - Shape Skeleton

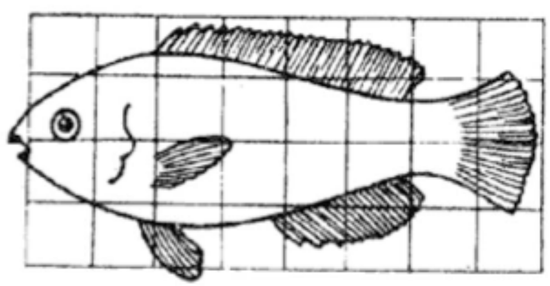
local symmetry axis

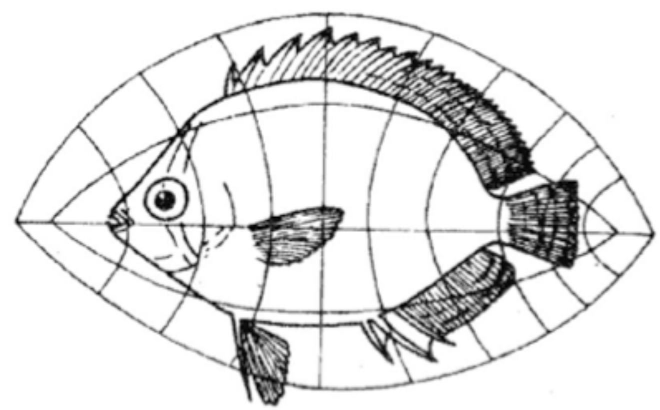
From “Skeletons and segmentation of shapes”, Shah, 2005

What is a shape?

- A general theory of shape does not exist to date.
- Common shape representations:
  - Landmarks
  - Shape Boundary
  - Shape Skeleton
  - Transformation models

From D'Arcy Thomson's On Growth and Form





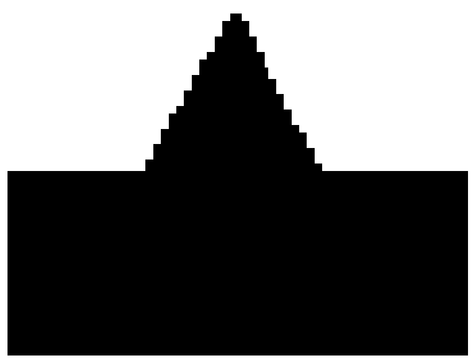
diffeomorphisms

What is a shape?

- A general theory of shape does not exist to date.
- Common shape representations:
  - Landmarks
  - Shape Boundary
  - Shape Skeleton
  - Transformation models
  - **Implicit representations**
- Each representation has its own strengths and weaknesses

Implicit Shape Representations

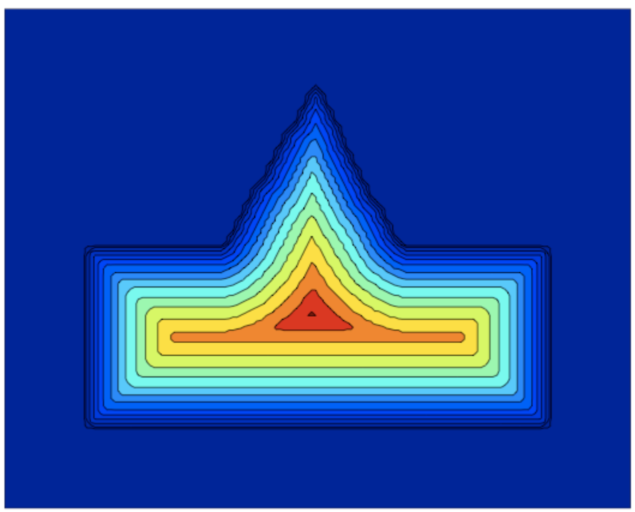
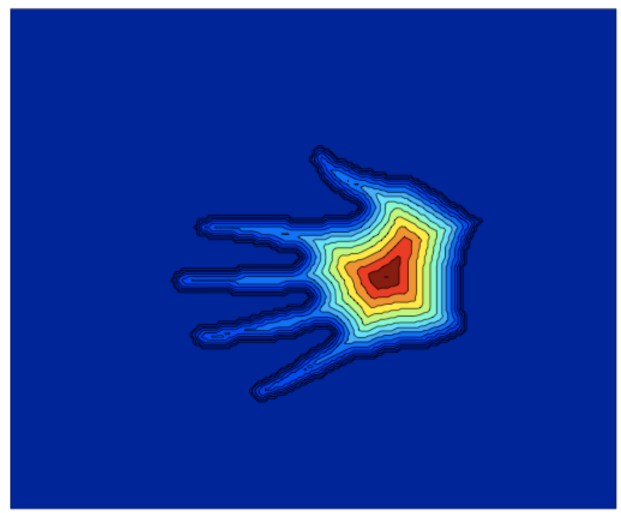
- Distance transform
- TSP Surfaces [Tari, Shah and Pien, 1997]
- Poisson Transform [Gorelick et al., 2006]
- Integral Kernels [Hong et al., 2006]



Distance Transform

- Estimate a surface whose value at each internal point is the minimum distance of the point to the shape boundary

$$\phi(x) = \min_{y \in \Gamma} \; dist(x, y) \qquad\qquad |\nabla \phi| = 1$$

TSP Surfaces

- depends on the Ambrosio-Tortorelli model
- A sufficiently large value of ρ, instead of a small one
- For a binary silhouette, a TSP surface is estimated by solving:

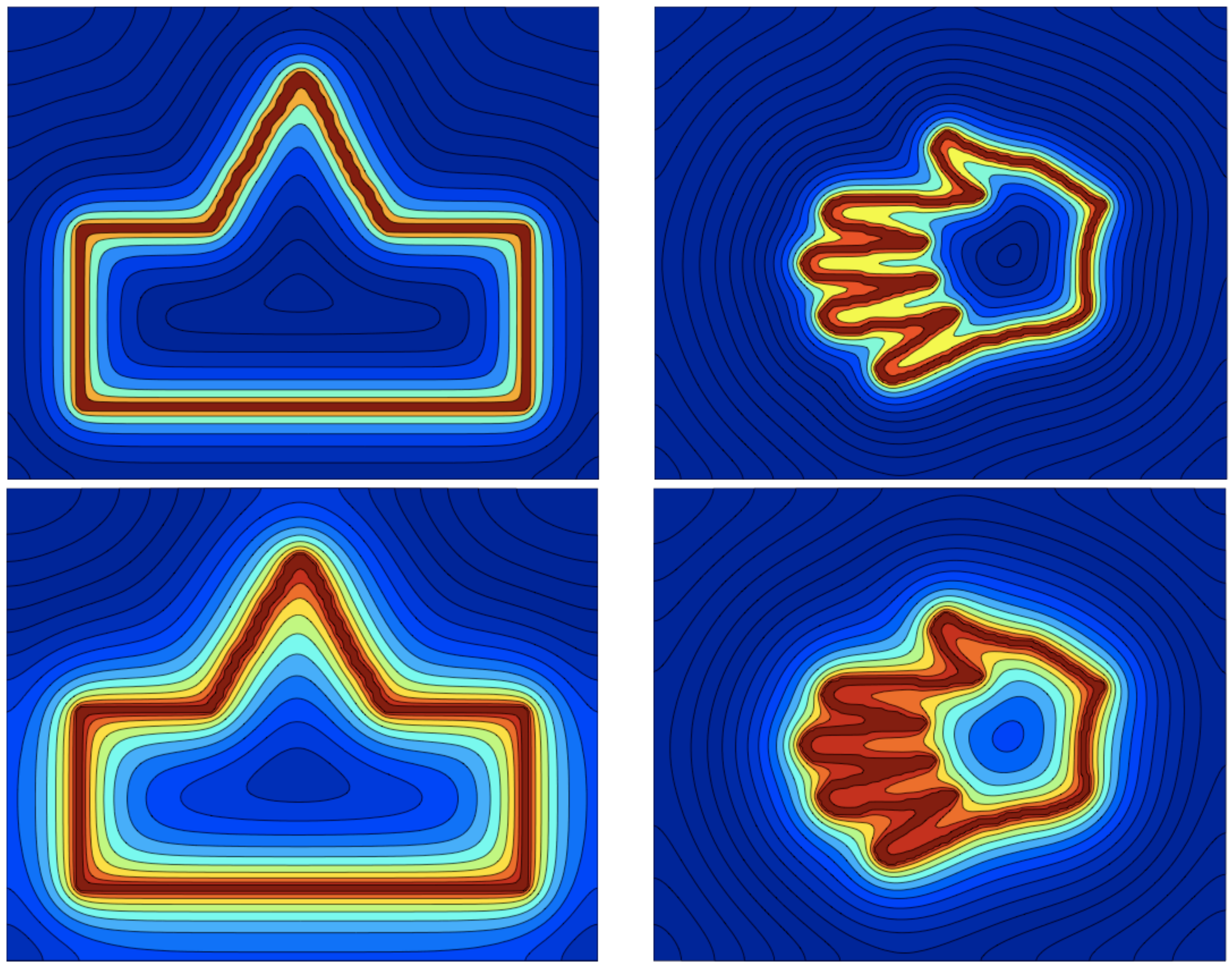
$$\frac{\partial v}{\partial t} = \nabla^2 v - \frac{v}{\rho^2}, \qquad v|_\Gamma = 1$$

Tari, Shah and Pien, Extraction of shape skeletons from grayscale images, *CVIU, 66(2), 1997*

TSP Surfaces

$\rho = 4$

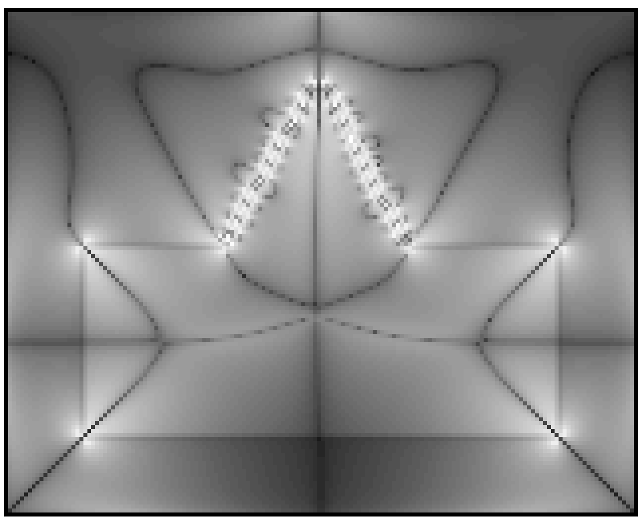
$\rho = 8$

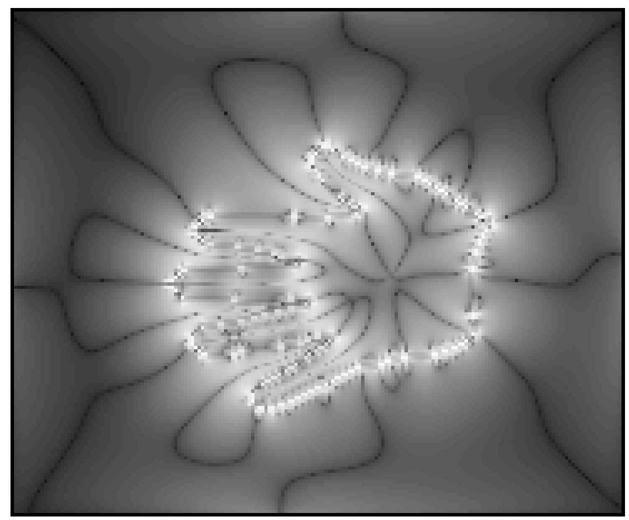


Tari, Shah and Pien, Extraction of shape skeletons from grayscale images, *CVIU, 66(2), 1997*

TSP Surfaces

- The TSP method can be easily extended to grayscale images.
- The TSP surfaces encode the skeleton information.





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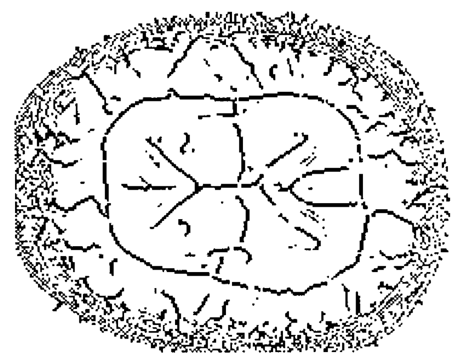
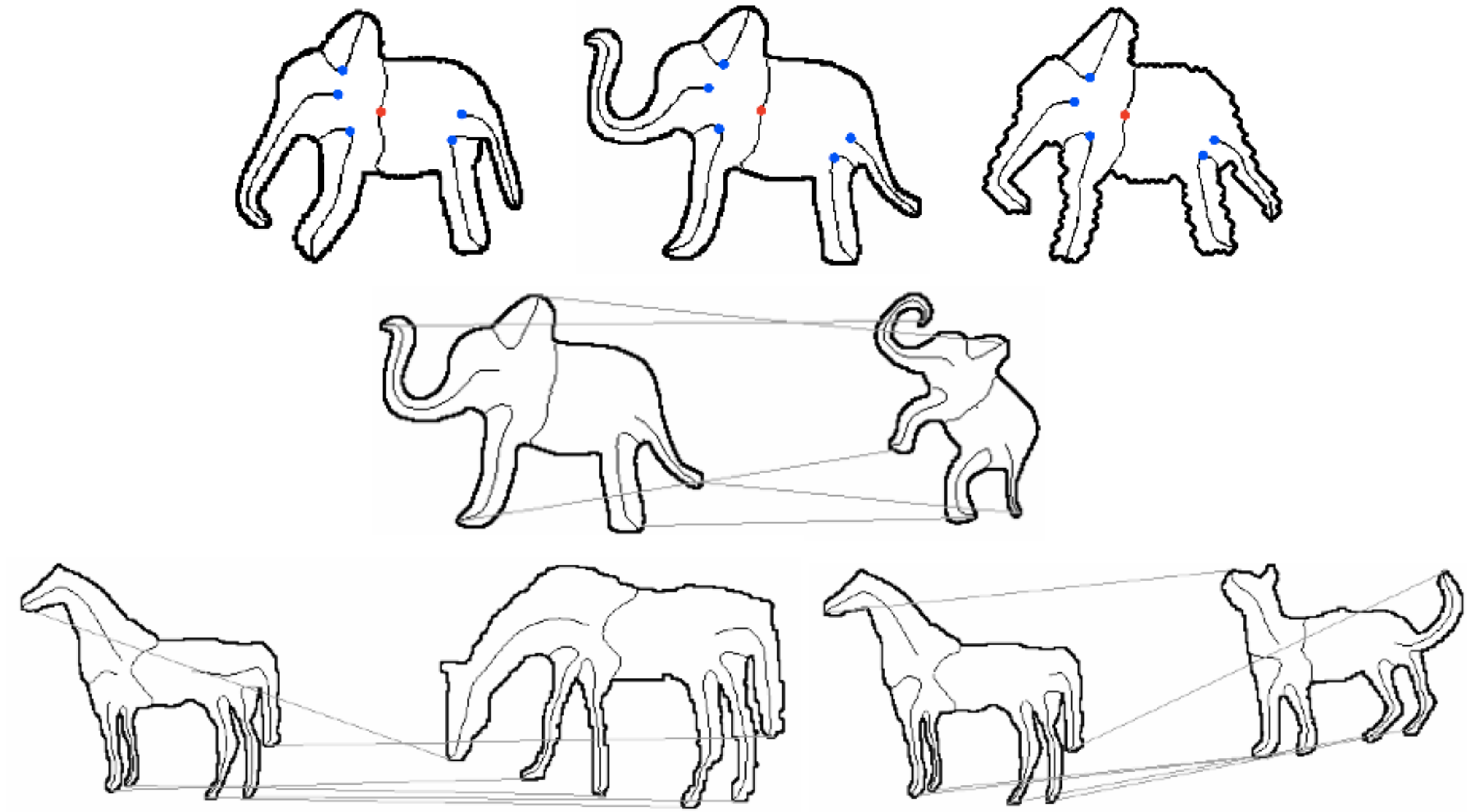


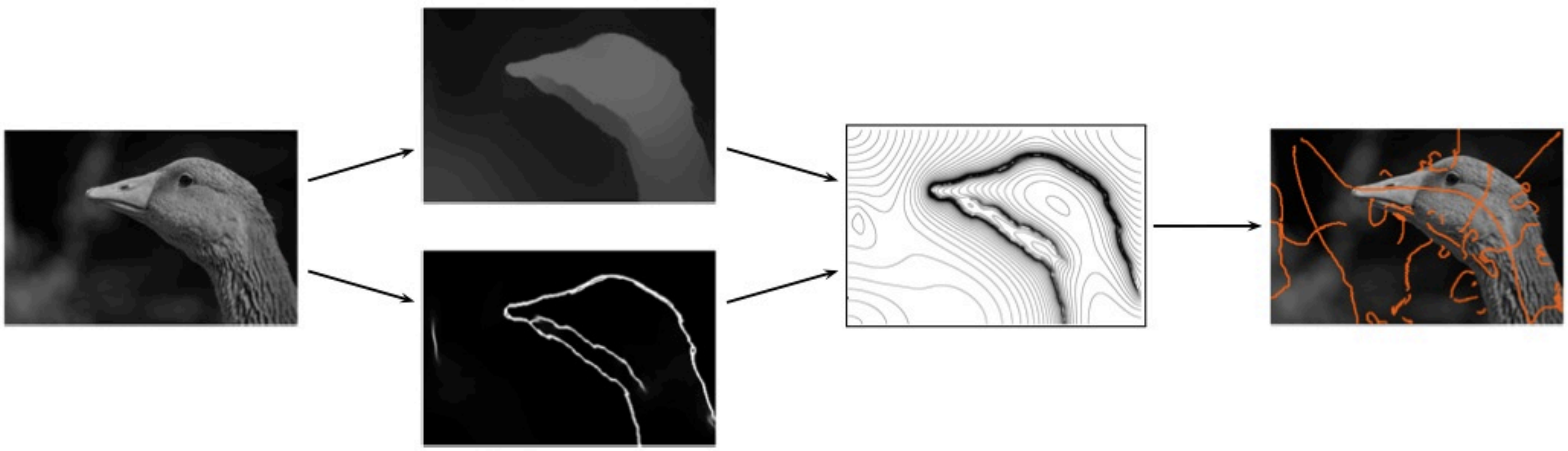
Tari, Shah and Pien, Extraction of shape skeletons from grayscale images, *CVIU, 66(2), 1997*

Disconnected Skeleton



C. Aslan and S. Tari, An Axis-Based Representation for Recognition, ICCV, 2005
C. Aslan, A. Erdem, E. Erdem and S. Tari, Disconnected Skeleton: Shape at its Absolute Scale. IEEE Trans. Pattern Anal. Mach. Intel., 2008

Skeleton Extraction from Natural Images



input image g

smoothed image u and function v after first step

level curves of the medialness function v after the second step

extracted skeletons

E. Erdem and S. Tari, unpublished work

Skeleton Extraction from Natural Images

sample input images

E. Erdem and S. Tari, unpublished work

Skeleton Extraction from Natural Images

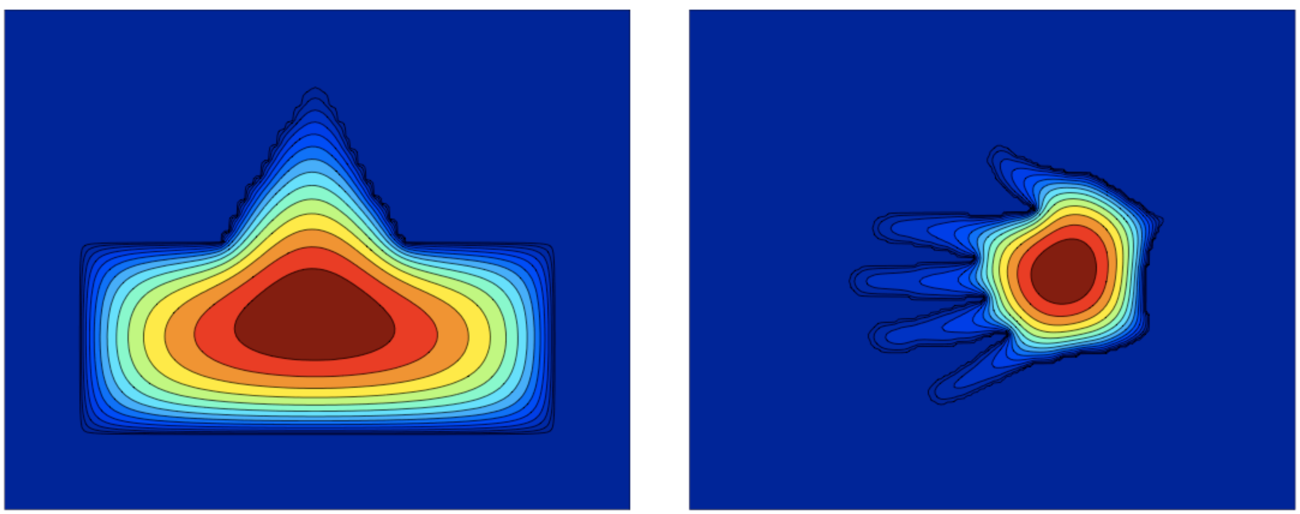
proposed method

E. Erdem and S. Tari, unpublished work

Poisson Transform

- Estimate a surface from a given silhouette by solving a Poisson equation

$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -1 \qquad U(x,y)\Big|_{\partial S} = 0$$



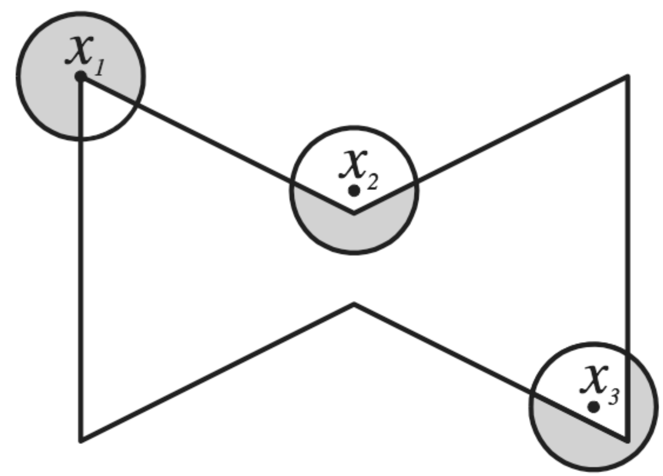
Gorelick et al., Shape representation and classification using the poisson equation, *IEEE TPAMI, 28(12),2006*

Integral Kernels

- Describe the local structure of a shape by using a kernel representation

$$\begin{aligned}\mathcal{R}_\sigma(\chi; x) = \chi(x)(G_\sigma(x) * (1 - \chi(x))) \\ + (1 - \chi(x))(G_\sigma(x) * \chi(x))\end{aligned}$$

$$G_\sigma(x) = \frac{1}{2\pi\sigma^2} exp\left(-\frac{|x|^2}{2\sigma^2}\right)$$

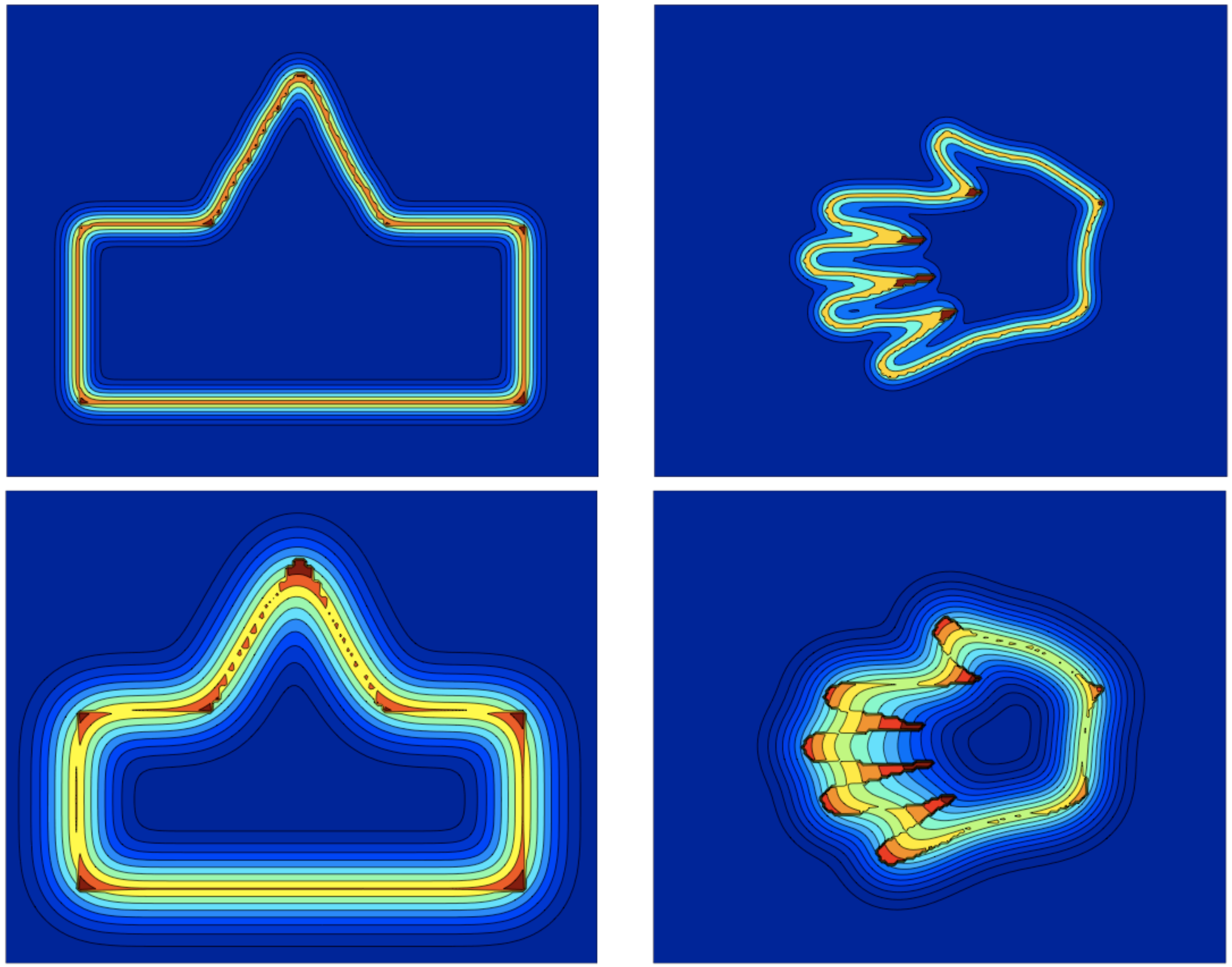


Hong et al., Shape representation based on integral kernels: Application to image matching and segmentation, *CVPR, 2006*

Integral Kernels

$\sigma = 3$

$\sigma = 7$



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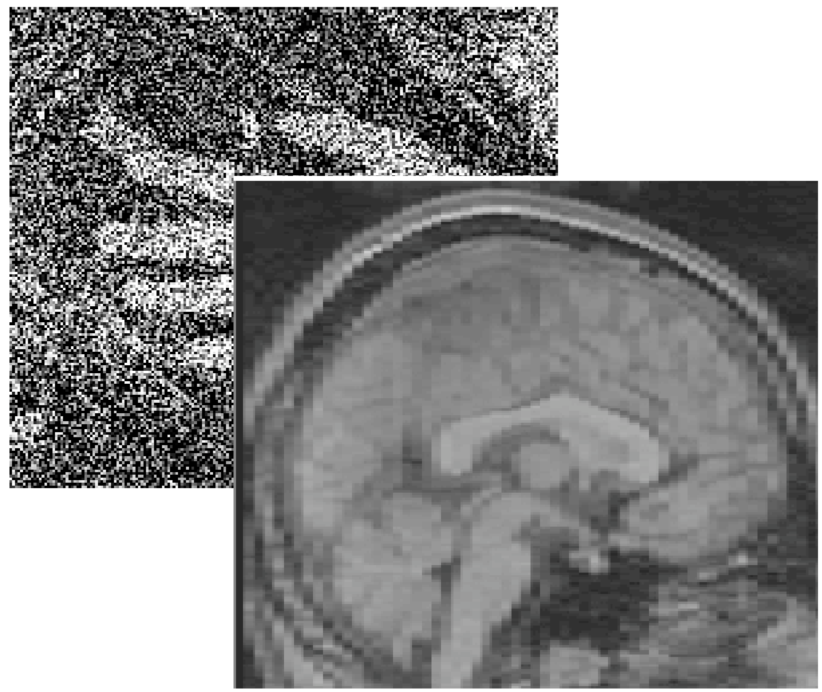
Segmentation

- Partition an image into meaningful regions that are likely to correspond to objects exist in the image



Prior-Guided Segmentation

- Incorporate prior shape information into the segmentation process
- Early 2000, -
  - Leventon et al.'00, Rousson and Paragios'02, Cremers et al.'02, Tsai et al.'03, Riklin-Raviv et al.'04, Hong et al.'06, …
  - Borenstein and Ullman'02, Leibe et al.'04, Shotton et al.'05, Opelt et al.'06, …

Shape Matching Using A Local Deformation Model

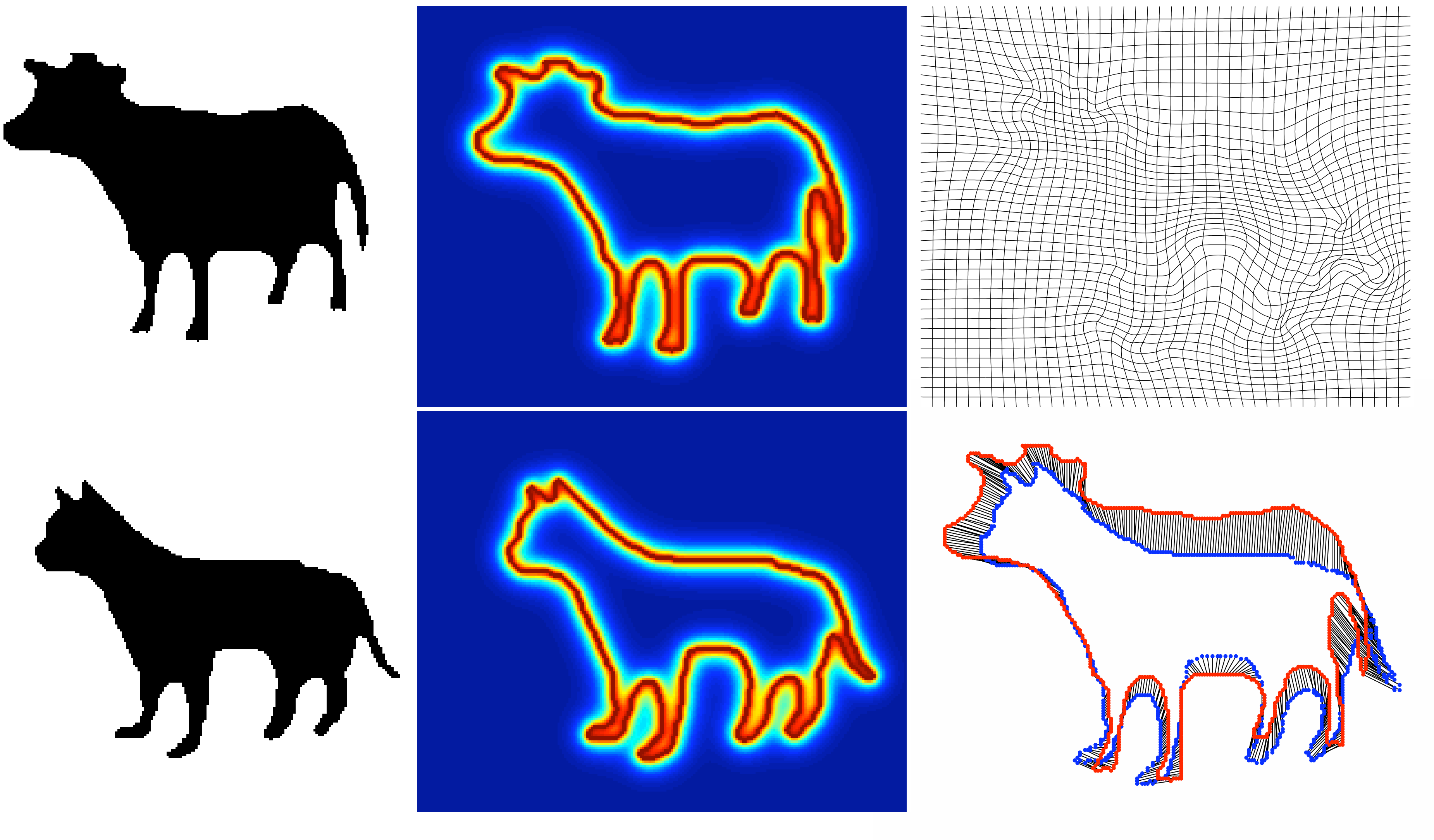
- Determine correspondences between two shapes
- Matching as a registration problem [Hong et al'06]: Estimate a transformation function between two shapes

$$E_{match}(h) = E_{fid}(h) + \beta E_{reg}(h)$$

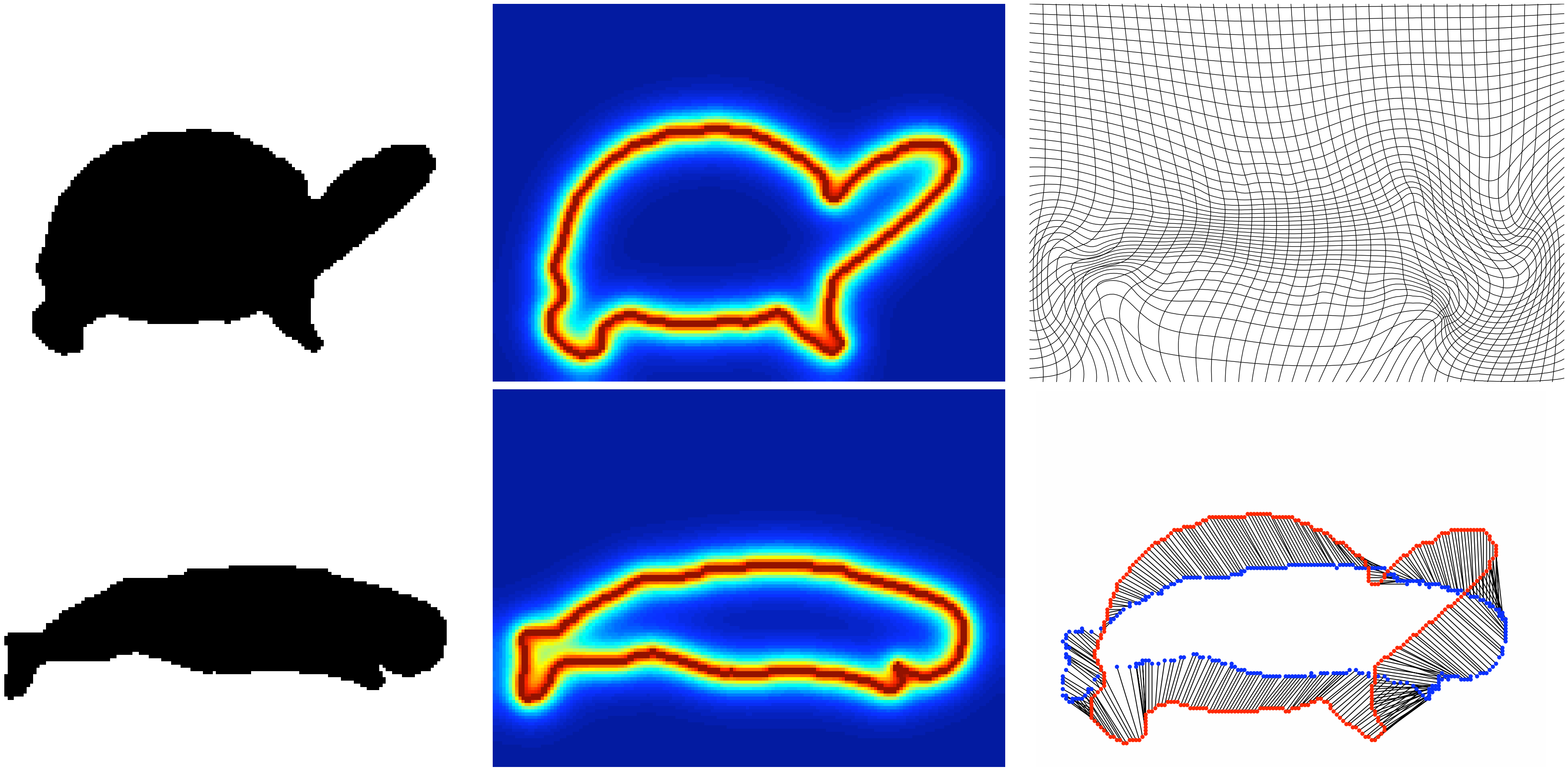
$$E_{fid}(h) = \frac{1}{2}\int_{\Omega} \left(v_2(x + h(x)) - v_1(x)\right)^2 dx$$

$$E_{reg}(h) = \int_{\Omega} \left(\frac{\bar{\mu}}{4} \sum_{i,j=1}^{2} \left(\partial_{x_i} h_j + \partial_{x_j} h_i\right)^2 + \frac{\lambda}{2} (\nabla \cdot h)^2 \right) dx$$

Matching Examples



Matching Examples



- Shapes are registered accurately, yet the correspondences are not meaningful
- Shapes to be matched should be locally similar

Prior-Guided Segmentation Framework

- Segmentation by minimizing coupled energies

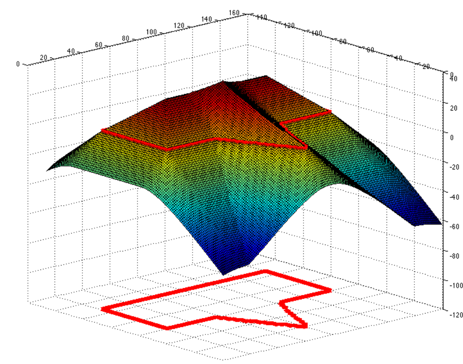
$$E_1(\phi|h) = E_{CV}(\phi) + \mu_1 E_{shape}(\phi|h)$$

local deformation

image energy

level set function

shape energy

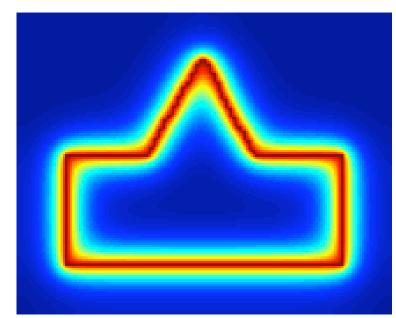


$$E_2(v, h|\phi) = E_{TSP}(v|\phi) + \mu_2 E_{match}(h)$$

TSP surface estimated from the evolving object region

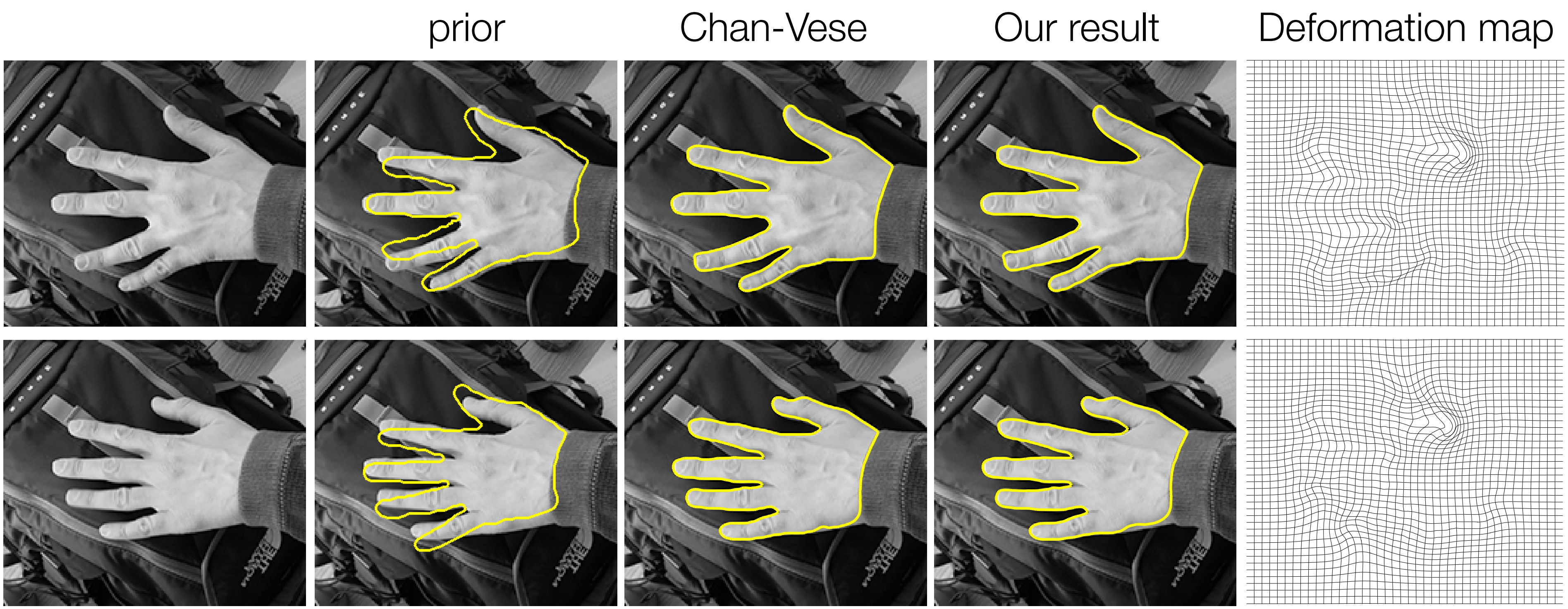
TSP energy

matching energy



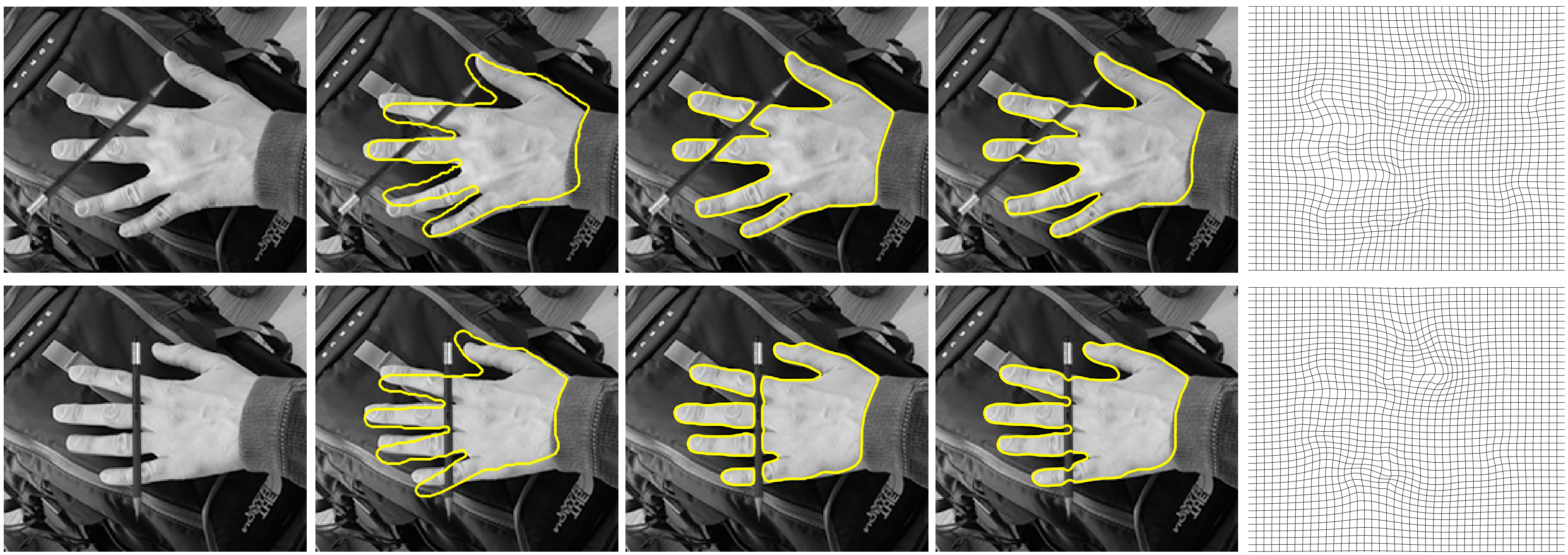
Erdem, Tari and Vese, *Segmentation Using the Edge Strength Function as a Shape Prior within a Local Deformation Model*, ICIP, 2009

Experimental Results
no corrupting influence

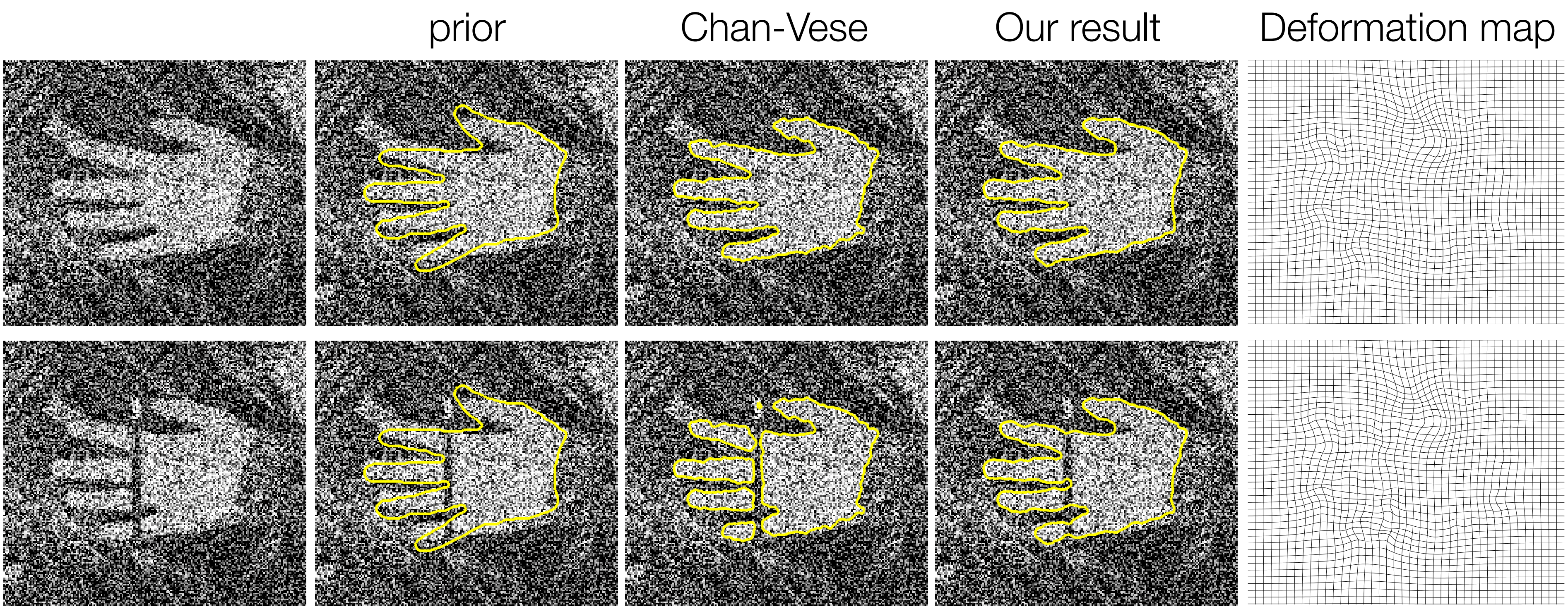


Experimental Results

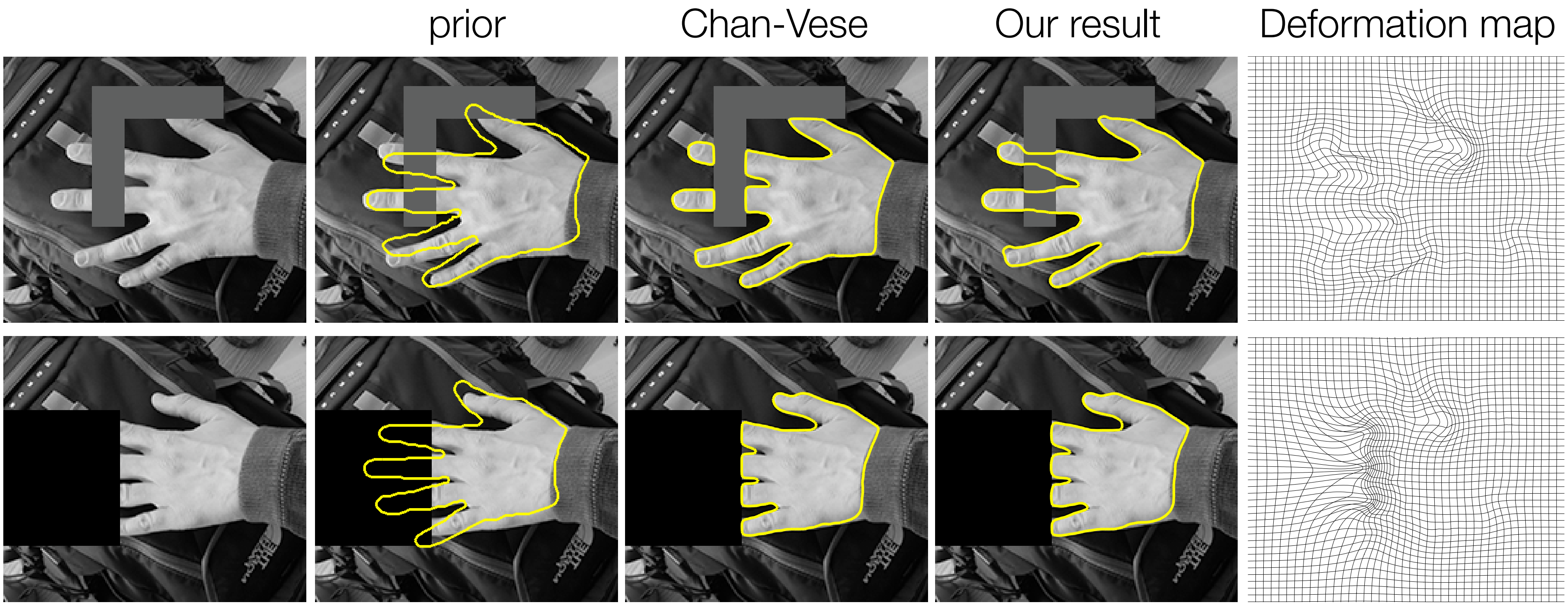
partial occlusion



Experimental Results
significant amount of noise

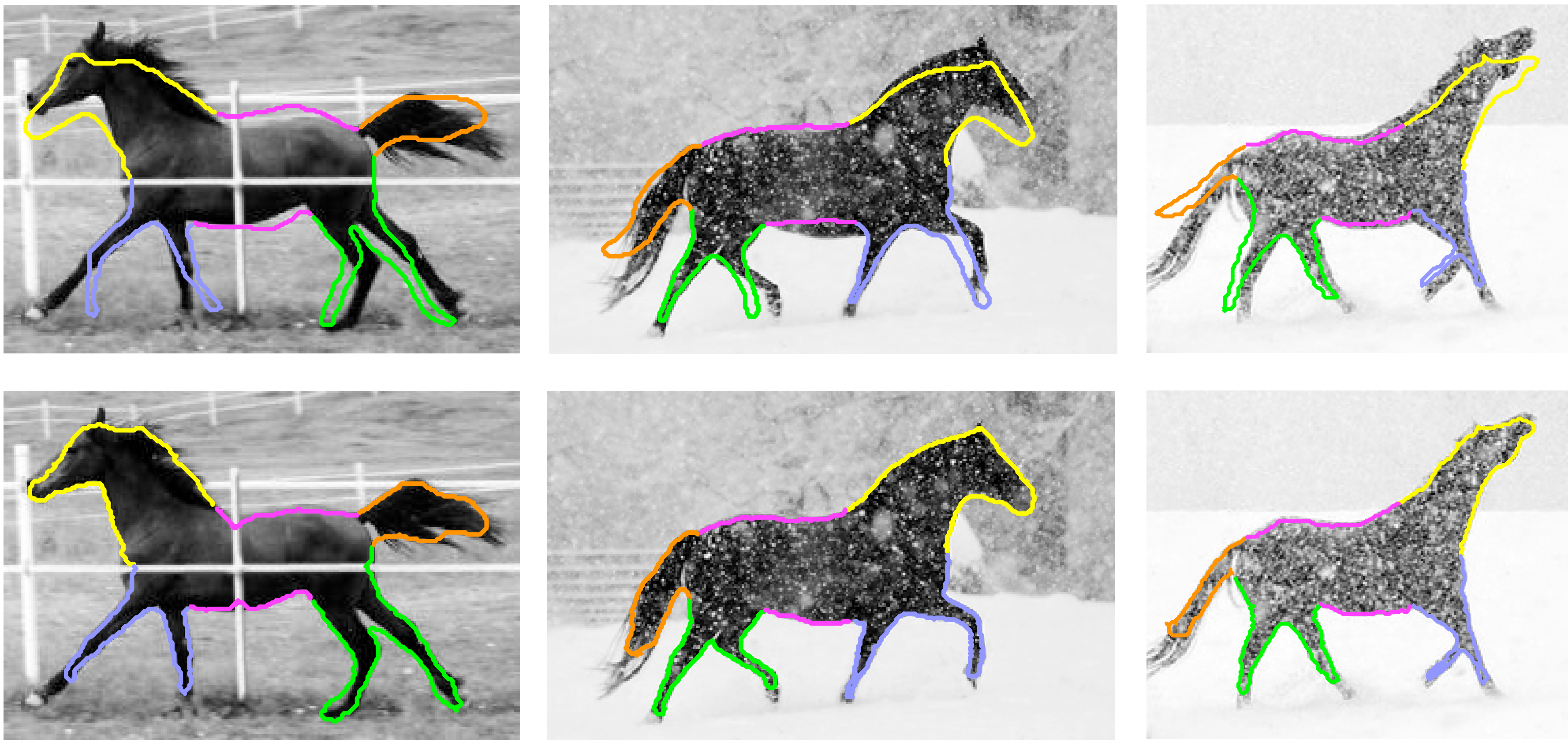


Experimental Results
heavy occlusion



Experimental Results

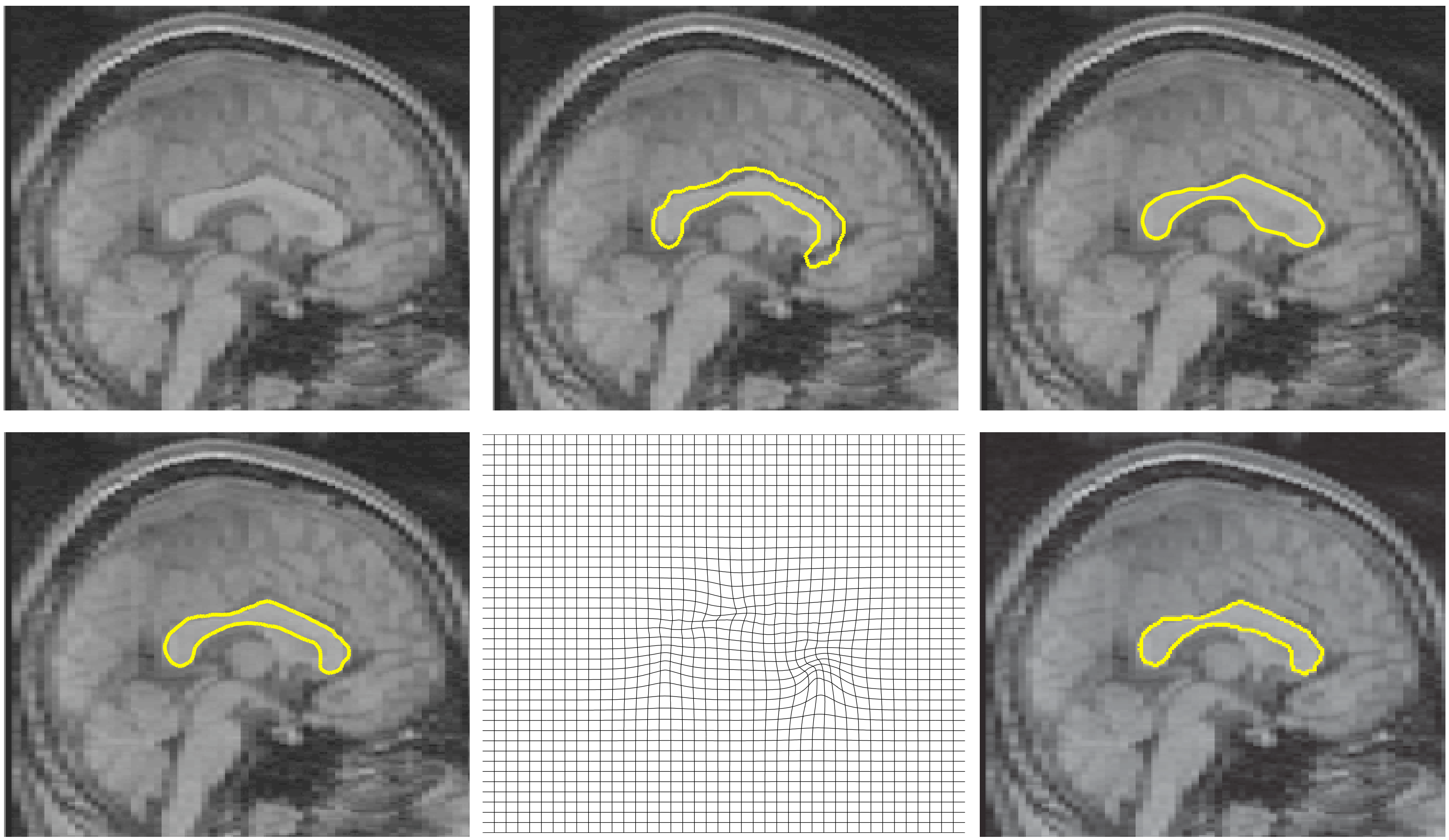
joint segmentation and registration



Experimental Results

Comparison with the Method of Hong et al.'06

prior Chan-Vese



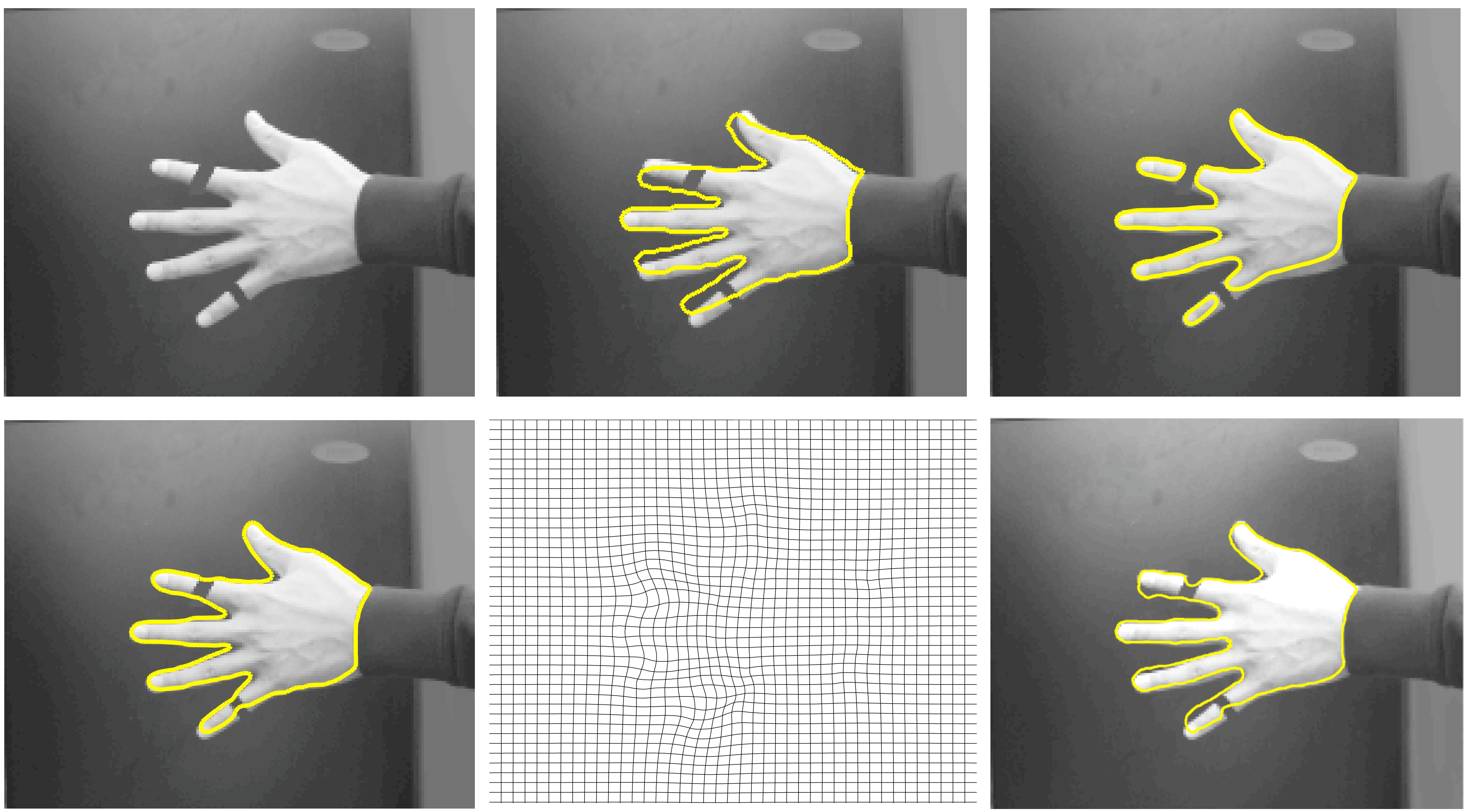
Our result Deformation map Hong et al.'06

Experimental Results

Comparison with the Method of Hong et al.’06

prior

Chan-Vese



Our result

Deformation map

Hong et al.’06