BIL 717
Image Processing
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Linear Filtering
Edge Detection

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Today

• Linear Filtering
  – Review
  – Gauss filter
  – Linear diffusion

• Edge Detection
  – Review
  – Derivative filters
  – Laplacian of Gaussian
  – Canny edge detector
Today

• Linear Filtering
  – Review
  – Gauss filter
  – Linear diffusion

• Edge Detection
  – Review
  – Derivative filters
  – Laplacian of Gaussian
  – Canny edge detector
Filtering

• The name “filter” is borrowed from frequency domain processing

• Accept or reject certain frequency components

• Fourier (1807): Periodic functions could be represented as a weighted sum of sines and cosines

Image courtesy of Technology Review
Signals

- A signal is composed of low and high frequency components

  - **Low frequency components:** smooth / piecewise smooth
    - Neighboring pixels have similar brightness values
    - You’re within a region

  - **High frequency components:** oscillatory
    - Neighboring pixels have different brightness values
    - You’re either at the edges or noise points
Signals – Examples

[Diagram of a ring and its signal examples]
Motivation: noise reduction

• Assume image is degraded with an additive model.
• Then,

\[
\text{Observation} = \text{True signal} + \text{noise}
\]
\[
\text{Observed image} = \text{Actual image} + \text{noise}
\]

\[\text{low-pass filters} \quad \text{high-pass filters}\]

\[\downarrow\]

\text{smooth the image}
Common types of noise

- **Salt and pepper noise**: random occurrences of black and white pixels
- **Impulse noise**: random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Slide credit: S. Seitz
Gaussian noise

\[ f(x, y) = \overline{f(x, y)} + \overline{\eta(x, y)} \]

Gaussian i.i.d. (“white”) noise:
\[ \eta(x, y) \sim \mathcal{N}(\mu, \sigma) \]

>> noise = randn(size(im)).*sigma;
>> output = im + noise;

What is the impact of the sigma?

Slide credit: M. Hebert
Motivation: noise reduction

- Make multiple observations of the same static scene
- Take the average
- Even multiple images of the same static scene will not be identical.

Adapted from: K. Grauman
Motivation: noise reduction

- Make multiple observations of the same static scene
- Take the average
- Even multiple images of the same static scene will not be identical.
- What if we can’t make multiple observations? What if there’s only one image?

Adapted from: K. Grauman
Image Filtering

• **Idea:** Use the information coming from the neighboring pixels for processing

• Design a transformation function of the local neighborhood at each pixel in the image
  – Function specified by a “filter” or mask saying how to combine values from neighbors.

• Various uses of filtering:
  – Enhance an image (denoise, resize, etc)
  – Extract information (texture, edges, etc)
  – Detect patterns (template matching)

Adapted from: K. Grauman
Filtering

• Processing done on a function
  – can be executed in continuous form (e.g. analog circuit)
  – but can also be executed using sampled representation

• Simple example: smoothing by averaging
Linear filtering

• Filtered value is the linear combination of neighboring pixel values.

• Key properties
  – linearity: $\text{filter}(f + g) = \text{filter}(f) + \text{filter}(g)$
  – shift invariance: behavior invariant to shifting the input
    • delaying an audio signal
    • sliding an image around

• Can be modeled mathematically by convolution

Adapted from: S. Marschner
First attempt at a solution

• Let’s replace each pixel with an average of all the values in its neighborhood

• Assumptions:
  – Expect pixels to be like their neighbors (spatial regularity in images)
  – Expect noise processes to be independent from pixel to pixel

Slide credit: S. Marschner, K. Grauman
First attempt at a solution

- Let’s replace each pixel with an average of all the values in its neighborhood
- Moving average in 1D:

Slide credit: S. Marschner
Discrete convolution

• Simple averaging:

\[
b_{\text{smooth}}[i] = \frac{1}{2r + 1} \sum_{j=i-r}^{i+r} b[j]
\]

– every sample gets the same weight

• Convolution: same idea but with \textit{weighted} average

\[
(a \ast b)[i] = \sum_{j} a[j] b[i - j]
\]

– each sample gets its own weight (normally zero far away)

• This is all convolution is: it is a \textit{moving weighted average}

Slide credit: S. Marschner
Filters

- Sequence of weights $a[j]$ is called a *filter*
- Filter is nonzero over its *region of support*
  - usually centered on zero: support radius $r$
- Filter is *normalized* so that it sums to 1.0
  - this makes for a weighted average, not just any old weighted sum
- Most filters are symmetric about 0
  - since for images we usually want to treat left and right the same

Slide credit: S. Marschner
Convolution and filtering

- Can express sliding average as convolution with a *box filter*
- \( a_{\text{box}} = [..., 0, 1, 1, 1, 1, 1, 0, ...] \)
Example: box and step

\[ a_{ij} \]

\[ b_{ij} \]

Slide credit: S. Marschner
Convolution and filtering

- Convolution applies with any sequence of weights
- Example: bell curve (gaussian-like) $[\ldots, 1, 4, 6, 4, 1, \ldots]/16$
And in pseudocode...

```plaintext
function convolve(sequence a, sequence b, int r, int i)
    s = 0
    for j = -r to r
        s = s + a[j] * b[i - j]
    return s
```
Key properties

• **Linearity:** \(\text{filter}(f_1 + f_2) = \text{filter}(f_1) + \text{filter}(f_2)\)

• **Shift invariance:** \(\text{filter}(\text{shift}(f)) = \text{shift}(\text{filter}(f))\)
  - same behavior regardless of pixel location, i.e. the value of the output depends on the pattern in the image neighborhood, not the position of the neighborhood.

• Theoretical result: any linear shift-invariant operator can be represented as a convolution

Slide credit: S. Lazebnik
Properties in more detail

• Commutative: $a \ast b = b \ast a$
  – Conceptually no difference between filter and signal

• Associative: $a \ast (b \ast c) = (a \ast b) \ast c$
  – Often apply several filters one after another: $(((a \ast b_1) \ast b_2) \ast b_3)$
  – This is equivalent to applying one filter: $a \ast (b_1 \ast b_2 \ast b_3)$

• Distributes over addition: $a \ast (b + c) = (a \ast b) + (a \ast c)$

• Scalars factor out: $ka \ast b = a \ast kb = k(a \ast b)$

• Identity: unit impulse $e = [..., 0, 0, 1, 0, 0, ...]$, $a \ast e = a$
Discrete filtering in 2D

• Same equation, one more index

\[(a \ast b)[i, j] = \sum_{i', j'} a[i', j'] b[i - i', j - j']\]

– now the filter is a rectangle you slide around over a grid of numbers

• Usefulness of associativity

  – often apply several filters one after another: \(((a \ast b_1) \ast b_2) \ast b_3)\)
  – this is equivalent to applying one filter: \(a \ast (b_1 \ast b_2 \ast b_3)\)
And in pseudocode...

```
function convolve2d(filter2d a, filter2d b, int i, int j)
    s = 0
    r = a.radius
    for i' = -r to r do
        for j' = -r to r do
            s = s + a[i'][j']b[i - i'][j - j']
    return s
```
Moving Average In 2D

$F[x, y]$  

$G[x, y]$  

Slide credit: S. Seitz
Moving Average In 2D

\[ F[x, y] \quad G[x, y] \]
Moving Average In 2D

\[ F[x, y] \]

\[ G[x, y] \]

Slide credit: S. Seitz
Averaging filter

- What values belong in the kernel $H$ for the moving average example?

$$G = H \otimes F$$

Slide credit: K. Grauman
Smoothing by averaging

What if the filter size was 5 x 5 instead of 3 x 3?

Slide credit: K. Grauman
Boundary issues

• What is the size of the output?

• MATLAB: output size / “shape” options
  – shape = ‘full’: output size is sum of sizes of f and g
  – shape = ‘same’: output size is same as f
  – shape = ‘valid’: output size is difference of sizes of f and g
Boundary issues

• What about near the edge?
  – the filter window falls off the edge of the image
  – need to extrapolate
  – methods:
    • clip filter (black)
    • wrap around
    • copy edge
    • reflect across edge
Boundary issues

• What about near the edge?
  – the filter window falls off the edge of the image
  – need to extrapolate
  – methods (MATLAB):
    • clip filter (black): \( \text{imfilter}(f, g, 0) \)
    • wrap around: \( \text{imfilter}(f, g, 'circular') \)
    • copy edge: \( \text{imfilter}(f, g, 'replicate') \)
    • reflect across edge: \( \text{imfilter}(f, g, 'symmetric') \)
Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?

- Removes high-frequency components from the image ("low-pass filter").
Smoothing with a Gaussian

Slide credit: K. Grauman
Gaussian filters

• What parameters matter here?

• Size of kernel or mask
  – Note, Gaussian function has infinite support, but discrete filters use finite kernels

\[ \sigma = 5 \text{ with } 10 \times 10 \text{ kernel} \]

\[ \sigma = 5 \text{ with } 30 \times 30 \text{ kernel} \]
Gaussian filters

- What parameters matter here?
- **Variance** of Gaussian: determines extent of smoothing

\[
\sigma = 2 \text{ with } 30 \times 30 \text{ kernel}
\]

\[
\sigma = 5 \text{ with } 30 \times 30 \text{ kernel}
\]

Slide credit: K. Grauman
Matlab

```matlab
>> hsize = 10;
>> sigma = 5;
>> h = fspecial('gaussian' hsize, sigma);

>> mesh(h);

>> imagesc(h);

>> outim = imfilter(im, h); % correlation
>> imshow(outim);
```

Slide credit: K. Grauman
Smoothing with a Gaussian

Parameter $\sigma$ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.

for $\sigma=1:3:10$
   h = fspecial('gaussian', fsize, $\sigma$);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
Properties of smoothing filters

- **Smoothing**
  - Values positive
  - Sum to 1 → constant regions same as input
  - Amount of smoothing proportional to mask size
  - Remove “high-frequency” components; “low-pass” filter
Linear Diffusion

• Let \( f(x) \) denote a grayscale (noisy) input image and \( u(x, t) \) be initialized with \( u(x,0) = u^0(x) = f(x) \).

• The linear diffusion process can be defined by the equation:

\[
\frac{\partial u}{\partial t} = \nabla \cdot (\nabla u) = \nabla^2 u
\]

where \( \nabla \cdot \) denotes the divergence operator. Thus,

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}
\]
Linear Diffusion (cont’d.)

- Diffusion process as an evolution process.
- Artificial time variable $t$ denotes the diffusion time.
- Input image is smoothed at a constant rate in all directions.
  - $u^0(x)$: initial image,
  - $u(x, t)$: the evolving images under the governed equation representing the successively smoothed versions of the initial input image $f(x)$.
- Diffusion process creates a scale space representation of the given image $f$, with $t > 0$ being the scale.
Linear Diffusion (cont’d.)

\[
\frac{\partial u}{\partial t} = \nabla \cdot (\nabla u) = \nabla^2 u
\]

Heat equation: 0

Gray-level image

Intensity

Diffusion

Influence of the central pixel on the other pixels (red: high, blue: low)

Credit: S. Paris
Linear Diffusion (cont’d.)

As we move to coarser scales,
- Evolving images become more and more simplified
- Diffusion process removes the image structures at finer scales.

![Linear diffusion results for different diffusion times.](image)

It is shown that the solution of the linear diffusion equation with initial condition

\[ u(x,0) = f(x) \]

for a specific diffusion time \( T \) is equivalent to the convolution of the input image \( f(x) \) with the Gaussian kernel \( G_\sigma(x) \) with standard deviation \( \sigma = \sqrt{2T} \,[2, 3, 6] \). Thus, linear diffusion can be regarded as a low-pass filter. The correspondence between the diffusion time variable \( t \) and the standard deviation \( \sigma \) clearly depicts the effect of \( t \) on the evolving images. The higher the value of \( t \), the higher the value of \( \sigma \), and the more the image becomes. This relation provides the following explicit solution to (1):

\[ u(x, T) = (G_\sqrt{2T} \ast f)(x) \]

with

\[ G_\sigma(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{|x|^2}{2\sigma^2}\right). \]

Numerical Implementation

Since we deal with digital images, solving the linear diffusion equation requires discretization in both spatial and time coordinates. Central differences are the typical choices for the spatial derivatives:

\[
\begin{align*}
\frac{d^2 u_{i,j}}{dx^2} &\approx u_{i+h,x,j} - 2u_{i,x,j} + u_{i-h,x,j} \\
\frac{d^2 u_{i,j}}{dy^2} &\approx u_{i,x,j+h} - 2u_{i,x,j} + u_{i,x,j-h}
\end{align*}
\]

where \( u_{i,j} \) denotes the gray value or the brightness of the evolving image at pixel location \((i, j)\).

The values of \( h_x \) and \( h_y \) are generally set to 1 as digital images are discretized on a regular pixel grid. For the remainder of this thesis, we take \( h_x = h_y = 1 \). This leads to

\[ T = 0 \]
\[ T = 1.25 \]
\[ T = 2.5 \]
\[ T = 5 \]
\[ T = 10 \]
\[ T = 20 \]
Linear Diffusion (cont’d.)

- As we move to coarser scales,
  - Evolving images become more and more simplified
  - Diffusion process removes the image structures at finer scales.

Figure 2: Linear diffusion results for different diffusion times.

\[
\begin{align*}
T &= 0 \\
T &= 5 \\
T &= 10 \\
T &= 20 \\
T &= 40 \\
T &= 80
\end{align*}
\]

The straightforward approach to solve (5) is to consider an iterative scheme with an explicit time discretization, where homogeneous Neuman boundary conditions are imposed along the image boundary:

\[
\Delta t = u_{k+1}(i,j) - u_k(i,j) = \frac{1}{4} \left( u_{k+1}(i+1,j) + u_{k+1}(i-1,j) + u_{k+1}(i,j+1) + u_{k+1}(i,j-1) \right) - u_k(i,j).
\]

Numerical stability condition for the discrete scheme requires \( \Delta t \leq 0.25 \).

Relation Between Variational Regularization and Diffusion Equations

Interestingly, there is a strong relation between variational regularization methods and diffusion equations [4]. The variational regularization methods formulate smoothing process as a functional minimization via which a noise-free approximation of a given image is to be estimated. Most of these formulations assume a additive noise model:

\[
f(x) = u(x) + n(x)
\]

where \( f(x) \) and \( u(x) \) respectively denote the given noisy image and the desired denoised image, and \( n(x) \) represents the additive noise.
Linear Diffusion and Gaussian Filtering

- The solution of the linear diffusion can be explicitly estimated as:

\[
u(x, T) = \left( G_{\sqrt{2T}} * f \right)(x)
\]

with

\[
G_{\sigma}(x) = \frac{1}{2\pi\sigma^2} \exp \left( -\frac{|x|^2}{2\sigma^2} \right)
\]

- Solution of the linear diffusion equation is equivalent to a proper convolution of the input image with the Gaussian kernel \(G_{\sigma}(x)\) with standard deviation \(\sigma = \sqrt{2T}\)

- The higher the value of \(T\), the higher the value of \(\sigma\), and the more smooth the image becomes.
Numerical Implementation

- Solving the linear diffusion equation requires discretization in both spatial and time coordinates.
- Central differences for the spatial derivatives:

\[
\frac{d^2 u_{i,j}}{dx^2} \approx \frac{u_{i+h_x,j} - 2u_{i,j} + u_{i-h_x,j}}{h_x^2}
\]

\[
\frac{d^2 u_{i,j}}{dy^2} \approx \frac{u_{i,j+h_y} - 2u_{i,j} + u_{i,j-h_y}}{h_y^2}
\]

where \( u_{i,j} \) denotes the gray value or the brightness of the evolving image at pixel location \((i, j)\).
- We take \( h_x = h_y = 1 \) for a regular grid.
Numerical Implementation (cont’d.)

- Original model:
  \[
  \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}
  \]

- Space discrete version:
  \[
  \frac{du_{i,j}}{dt} = u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j}
  \]

- Space-time discrete version:
  \[
  \frac{u_{i,j}^{k+1} - u_{i,j}^k}{\Delta t} = u_{i+1,j}^k + u_{i-1,j}^k + u_{i,j+1}^k + u_{i,j-1}^k - 4u_{i,j}^k
  \]

homogeneous Neumann boundary condition
\[\Delta t \leq 0.25\] is required for
along the image boundary
numerical stability
Variational Regularization

- Variational regularization models formulate smoothing process as a functional minimization via which a noise-free approximation of a given image is to be estimated.

- With an additive model, \( f(x) = u(x) + n(x) \)
  - \( f(x) \): original image
  - \( u(x) \): smoothed image
  - \( n(x) \): noise component

- **An example**: Tikhonov energy functional

\[
E(u) = \int_{\Omega} \left( (u - f)^2 + \alpha |\nabla u|^2 \right) \, dx
\]
Tikhonov energy functional

\[
E(u) = \int_{\Omega} \left( (u - f)^2 + \alpha |\nabla u|^2 \right) dx
\]

where

- \( \Omega \subset \mathbb{R}^2 \) is connected, bounded, open subset representing the image domain,
- \( f \) is an image defined on \( \Omega \),
- \( u \) is the smooth approximation of \( f \),
- \( \alpha > 0 \) is the scale parameter.
Variational Regularization and Diffusion Equations

- A strong relation between variational regularization methods and diffusion equations.

- The minimizing function $u$ of the Tikhonov energy functional formally satisfies the Euler-Lagrange equation:

\[
(u - f) - \alpha \nabla^2 u = 0
\]

with the Neumann boundary condition $\frac{\partial u}{\partial n} |_{\partial \Omega} = 0$

- can be rewritten as:

\[
\frac{u - u^0}{\alpha} = \nabla^2 u \quad \text{with} \quad u^0 = f.
\]

implicit time discretization of the linear diffusion equation with a single time step ($T = \alpha$)
Today

• Linear Filtering
  – Review
  – Gauss filter
  – Linear diffusion

• Edge Detection
  – Review
  – Derivative filters
  – Laplacian of Gaussian
  – Canny edge detector
Signals and Images

- A signal is composed of low and high frequency components

  low frequency components: smooth /
  piecewise smooth
  Neighboring pixels have similar brightness values
  You’re within a region

  high frequency components: oscillatory
  Neighboring pixels have different brightness values
  You’re either at the edges or noise points
Edge detection

• **Goal:** Identify sudden changes (discontinuities) in an image
  – Intuitively, most semantic and shape information from the image can be encoded in the edges
  – More compact than pixels

• **Ideal:** artist’s line drawing (but artist is also using object-level knowledge)
Why do we care about edges?

• Extract information, recognize objects

• Recover geometry and viewpoint

Source: J. Hays
Closeup of edges

Slide credit: D. Hoiem
What causes an edge?

- Reflectance change: appearance information, texture
- Change in surface orientation: shape
- Depth discontinuity: object boundary
- Cast shadows

Slide credit: K. Grauman
Characterizing edges

- An edge is a place of rapid change in the image intensity function

Slide credit: K. Grauman
Derivatives with convolution

For 2D function $f(x,y)$, the partial derivative is:

$$\frac{\partial f(x, y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x, y)}{\partial x} \approx \frac{f(x+1, y) - f(x, y)}{1}$$

To implement above as convolution, what would be the associated filter?

Slide credit: K. Grauman
Partial derivatives of an image

\[ \frac{\partial f(x, y)}{\partial x}, \quad \frac{\partial f(x, y)}{\partial y} \]

Which shows changes with respect to x?

Slide credit: K. Grauman
Assorted finite difference filters

Prewitt: \[ M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}; \quad M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \]

Sobel: \[ M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}; \quad M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \]

Roberts: \[ M_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}; \quad M_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 0 & 0 \\
-1 & -1 & -1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & 1 \\
0 & 0 & 0 \\
-1 & -2 & -1
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\]
The gradient points in the direction of most rapid increase in intensity.

- How does this direction relate to the direction of the edge?

The gradient direction is given by

\[ \theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right) \]

The edge strength is given by the gradient magnitude

\[ \| \nabla f \| = \sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2} \]

Image gradient

- The gradient of an image: \( \nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \)

\[ \nabla f = \left[ \frac{\partial f}{\partial x}, 0 \right] \]

\[ \nabla f = \left[ 0, \frac{\partial f}{\partial y} \right] \]
Gradient magnitude image

Slide credit: K. Grauman
Thresholding gradient with a lower threshold

Slide credit: K. Grauman
Thresholding gradient with a higher threshold

Slide credit: K. Grauman
Intensity profile

Slide credit: D. Hoiem
With a little Gaussian noise
Effects of noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal

\[ f(x) \]

\[ \frac{d}{dx} f(x) \]

Where is the edge?

Slide credit: S. Seitz
Effects of noise

• Difference filters respond strongly to noise
  – Image noise results in pixels that look very different from their neighbors
  – Generally, the larger the noise the stronger the response

• What can we do about it?
Solution: smooth first

- To find edges, look for peaks in $\frac{d}{dx} (f \ast g)$

Slide credit: S. Seitz
Smoothing with a Gaussian

Recall: parameter $\sigma$ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.
Effect of $\sigma$ on derivatives

The apparent structures differ depending on Gaussian’s scale parameter.

Larger values: larger scale edges detected
Smaller values: finer features detected

Slide credit: K. Grauman
So, what scale to choose?

It depends what we’re looking for.

Slide credit: K. Grauman
Smoothing and Edge Detection

• While eliminating noise via smoothing, we also lose some of the (important) image details.
  – Fine details
  – Image edges
  – etc.

• What can we do to preserve such details?
  – Use edge information during denoising!
  – This requires a definition for image edges.

  Chicken-and-egg dilemma!

• Edge preserving image smoothing (Next week’s topic!)
Differentiation is convolution, and convolution is associative:

This saves us one operation:

\[
\frac{d}{dx} (f * g) = f * \frac{d}{dx} g
\]
Derivative of Gaussian filter

\[
\begin{bmatrix}
1 & -1
\end{bmatrix}
\]
Derivative of Gaussian filter

- Which one finds horizontal/vertical edges?

Slide credit: S. Lazebnik
Smoothing vs. derivative filters

• **Smoothing filters**
  – Gaussian: remove “high-frequency” components; “low-pass” filter
  – Can the values of a smoothing filter be negative?
  – What should the values sum to?
    • **One:** constant regions are not affected by the filter

• **Derivative filters**
  – Derivatives of Gaussian
  – Can the values of a derivative filter be negative?
  – What should the values sum to?
    • **Zero:** no response in constant regions
    – High absolute value at points of high contrast

Slide credit: S. Lazebnik
Laplacian of Gaussian

Consider \( \frac{\partial^2}{\partial x^2} (h \ast f) \)

\( \frac{\partial^2}{\partial x^2} h \)

\( (\frac{\partial^2}{\partial x^2} h) \ast f \)

Where is the edge? Zero-crossings of bottom graph

Slide credit: K. Grauman
2D edge detection filters

- The Laplacian operator:

\[ \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \]

Slide credit: K. Grauman
Laplacian of Gaussian

original image

Source: D. Marr and E. Hildreth (1980)
Laplacian of Gaussian

\[ \nabla^2 h_\sigma(u, v) \]

Source: D. Marr and E. Hildreth (1980)
Laplacian of Gaussian

\[ \nabla^2 h_\sigma(u, v) \]
(pos. values – white, neg. values – black)

Source: D. Marr and E. Hildreth (1980)
Laplacian of Gaussian

zero-crossings

Source: D. Marr and E. Hildreth (1980)
Designing an edge detector

• Criteria for a good edge detector:
  – **Good detection**: the optimal detector should find all real edges, ignoring noise or other artifacts
  – **Good localization**
    • the edges detected must be as close as possible to the true edges
    • the detector must return one point only for each true edge point

• Cues of edge detection
  – Differences in color, intensity, or texture across the boundary
  – Continuity and closure
  – High-level knowledge

Slide credit: L. Fei-Fei
The Canny edge detector

original image (Lena)

Slide credit: K. Grauman
The Canny edge detector

thresholding

Slide credit: K. Grauman
The Canny edge detector

How to turn these thick regions of the gradient into curves?

Slide credit: K. Grauman
Non-maximum suppression

Check if pixel is local maximum along gradient direction, select single max across width of the edge
- requires checking interpolated pixels p and r
The Canny Edge Detector

Problem: pixels along this edge didn’t survive the thresholding

thinning
(non-maximum suppression)

Slide credit: K. Grauman
Hysteresis thresholding

• Threshold at low/high levels to get weak/strong edge pixels
• Do connected components, starting from strong edge pixels

Slide credit: J. Hays
Hysteresis thresholding

• Check that maximum value of gradient value is sufficiently large
  – drop-outs? use hysteresis
    • use a high threshold to start edge curves and a low threshold to continue them.
Hysteresis thresholding

- **High threshold** (strong edges)
- **Low threshold** (weak edges)
- **Hysteresis threshold**

Slide credit: L. Fei-Fei
Hysteresis thresholding

high threshold (strong edges)

low threshold (weak edges)

hysteresis threshold

Slide credit: L. Fei-Fei
Recap: Canny edge detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   - Thin wide “ridges” down to single pixel width
4. Linking and thresholding (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB: `edge(image, 'canny');`
Effect of $\sigma$ (Gaussian kernel spread/size)

The choice of $\sigma$ depends on desired behavior

- large $\sigma$ detects large scale edges
- small $\sigma$ detects fine features

Slide credit: S. Seitz
Low-level edges vs. perceived contours

Background

Texture

Shadows

Slide credit: K. Grauman
Edge detection is just the beginning…

- Berkeley segmentation database:
  http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/

Source: S. Lazebnik
Learn from humans which combination of features is most indicative of a “good” contour? [D. Martin et al. PAMI 2004]