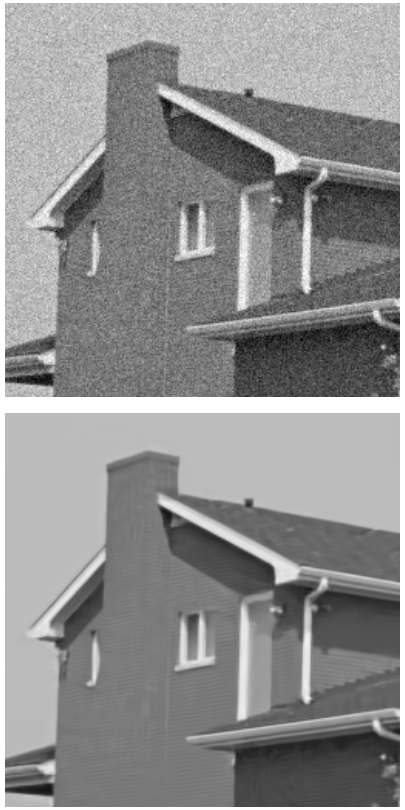
IMAGE DENOISING



Non-local Sparse Models for Image Restoration
ICCV2009

Julien Mairal
Francis Bach
Jean Ponce
Guillermo Sapiro
Andrew Zisserman

Simultaneous Sparse Coding

- Unify two different approaches to image restoration

  1. Learning a basis set (dictionary) adapted to sparse signal descriptions
  2. Non-local means approach

- Decompose groups of similar signals
on subsets of the learned dictionary.

- Outperforms the state of the art image denoising and restorations works

Denoising and Restoration Problem

- To reduce noise and/or extract useful image structures
- Eliminating undesirable characteristics

Different approaches

- anisotropic filtering
- total variation
- *image decompositions on fixed bases such as wavelets*
- **non-local means filtering**
- **learned sparse models**
- block matching with 3D filtering (BM3D)

Inroduction to Simultaneous Sparse Coding

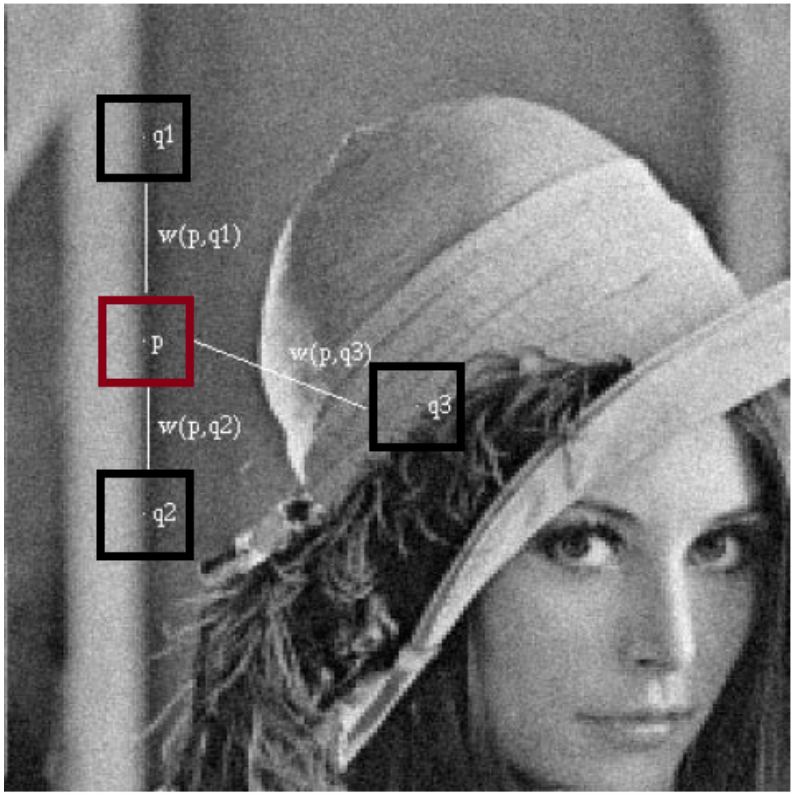
- Different devices produce different kinds of noise, and introduce different types of artefacts and spatial correlations in the noise.
- Operate directly on the raw sensor output, that suffers from non-homogeneous noise, but is less spatially correlated and not corrupted by postprocessing artefacts
- View both denoising and demosaicking as image reconstruction problems

Related Works

- non-local means filtering
- learned sparse models
- block matching with 3D filtering (BM3D)

- *These methods assume white Gaussian noise*
- This work demonstrate empirically that their approach is effective at restoring real images corrupted by non-Gaussian, non-uniform noise

Non Local Means Filtering



- The prominence of selfsimilarities is used as a prior on natural images.

$$x[i] = \sum_{j=1}^{n} \frac{K_h(y_i - y_j)}{\sum_{l=1}^{n} K_h(y_i - y_l)} y[j].$$

y in $R\uparrow n$,*y[i] the i−th pixel,*
patch of size m centered on this pixel,
Kh is a Gaussian kernel of bandwidth h

- Two pixels associated with similar patches yi and yj should have similar values y[i] and y[j]

Learned Sparse Coding

- The clean signal can be approximated by a *sparse* linear combination of elements from a basis set called dictionary

$$\min_{\boldsymbol{\alpha}_i \in \mathbb{R}^k} ||\boldsymbol{\alpha}_i||_p \ \text{ s.t. } \ ||\boldsymbol{y}_i - \mathbf{D}\boldsymbol{\alpha}||_2^2 \leq \varepsilon,$$

- To learn a dictionary D adapted to the image at hand, and demonstrated that learned dictionaries lead to better empirical performance than off-the- shelf ones

- For an image of size n, a dictionary in $R\uparrow mxk$ adapted to the n overlapping patches of size m

$$\min_{\mathbf{D}\in\mathcal{C},\mathbf{A}} \sum_{i=1}^{n} ||\boldsymbol{\alpha}_i||_p \ \text{ s.t. } \ ||\boldsymbol{y}_i - \mathbf{D}\boldsymbol{\alpha}_i||_2^2 \leq \varepsilon \qquad \boldsymbol{x} = \frac{1}{m}\sum_{i=1}^{n} \mathbf{R}_i \mathbf{D}\boldsymbol{\alpha}_i$$

C is the set of matrices in $R\uparrow mxk$, A = [$\alpha\downarrow 1$, . . . , $\alpha\downarrow n$] is a matrix in $R\uparrow mxk$,

Ri in $R\uparrow mxk$ is the binary matrix which places patch number i at its proper position in the image.

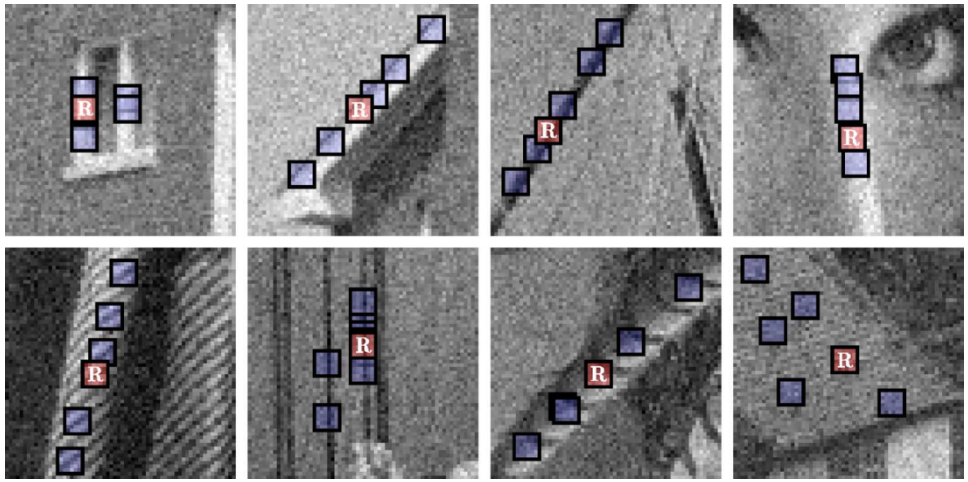
Block Matching 3D (BM3D)

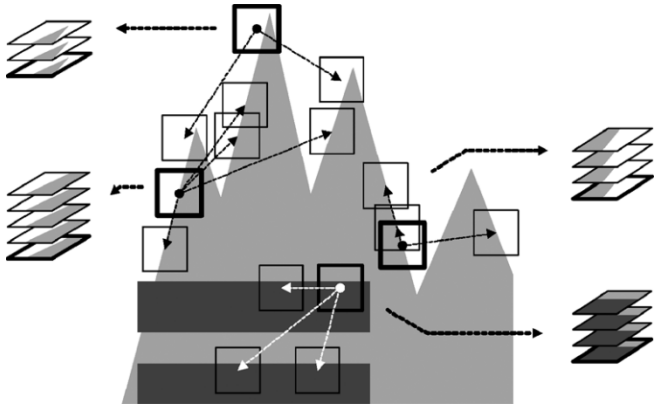
- A patch-based procedure that exploits image self-similarities and gives state-of-the-art results.

- estimate the codes of overlapping patches and average the estimates

- they reconstruct patches by finding similar ones in the image (*block matching*), stacking them together into a 3D signal block

- denoising the block using hard or soft thresholding with a 3D orthogonal dictionary





Simultenaous Sparse Coding

- Non-local means filtering has proven very effective in general, but it fails in some cases
- A patch does not look like any other one in the image, it is impossible to exploit self-similarities to denoise the corresponding pixel value
- Sparse image models can handle such situations by exploiting the redundancy between overlapping patches, but they suffer from another drawback
- Similar patches sometimes admit very different estimates due to the potential instability of sparse decompositions
- In this paper address this problem by forcing similar patches to admit similar decompositions.

$$||\mathbf{A}||_{p,q} \triangleq \sum_{i=1}^{k} ||\boldsymbol{\alpha}^i||_q^p,|$$

Simultenaous Sparse Coding

$$S_i \triangleq \{j = 1, \ldots, n \text{ s.t. } ||\boldsymbol{y}_i - \boldsymbol{y}_j||_2^2 \leq \xi\}$$

A fixed dictionary D in $R\uparrow mxk$

- Decomposing the patch yi with a grouped-sparsity regularizer on the set Si amounts to solving:

$$\min_{\mathbf{A}_i} ||\mathbf{A}_i||_{p,q} \text{ s.t. } \sum_{j \in S_i} ||\boldsymbol{y}_j - \mathbf{D}\boldsymbol{\alpha}_{ij}||_2^2 \leq \varepsilon_i, \quad \mathbf{A}_i = [\boldsymbol{\alpha}_{ij}]_{j \in \mathcal{S}_i} \in \mathbb{R}^{k \times |S_i|}$$

Optimization problem:

$$\min_{(\mathbf{A}_i)_{i=1}^n, \mathbf{D} \in \mathcal{C}} \sum_{i=1}^{n} \frac{||\mathbf{A}_i||_{p,q}}{|S_i|^p} \text{ s.t. } \forall i \sum_{j \in S_i} ||\boldsymbol{y}_j - \mathbf{D}\boldsymbol{\alpha}_{ij}||_2^2 \leq \varepsilon_i$$

D is in Rm×k with unit ℓ2-norm columns,

- The normalization by |Si|p is used to ensure equal weights for all groups

Simultenaous Sparse Coding

- use the convex $\ell 1,2$ norm for learning the dictionary
- use the $\ell 0,\infty$ pseudo-norm for the final reconstruction
- this formulation allows all the image patches to be processed as if they were *independent* of each other.
- To reconstruct the final image, average the estimates of each pixel:

$$\boldsymbol{x} = \mathrm{diag}\left(\sum_{i=1}^{n} \sum_{j \in S_i} \mathbf{R}_j \mathbf{1}_m\right)^{-1} \sum_{i=1}^{n} \sum_{j \in S_i} \mathbf{R}_j \mathbf{D} \boldsymbol{\alpha}_{ij}.$$

Results

Demosaicking

LSC

LSSC

Results

Demosaicking

PSNR results

Im.	AP	DL	LPA	SC	LSC	LSSC
1	37.84	38.46	40.47	40.84	40.92	**41.36**
2	39.64	40.89	41.36	41.76	42.03	**42.24**
3	41.40	42.66	43.47	43.15	43.92	**44.24**
4	39.92	40.49	40.84	41.99	42.14	**42.45**
5	37.28	38.07	37.51	38.72	39.15	**39.45**
6	38.69	40.19	40.92	41.29	41.36	**41.71**
7	41.75	42.35	43.06	43.30	43.59	**44.06**
8	35.58	36.02	37.13	37.42	37.38	**37.57**
9	41.84	43.05	43.50	43.17	43.74	**43.83**
10	41.93	42.54	42.77	43.01	43.17	**43.33**
11	39.25	40.01	40.51	41.19	41.29	**41.51**
12	42.62	43.45	44.01	44.29	44.49	**44.90**
13	34.28	34.75	36.08	36.16	36.29	**36.35**
14	35.66	36.91	36.86	37.64	38.48	**38.77**
15	39.17	39.82	40.09	41.04	41.24	**41.74**
16	42.10	43.75	44.02	44.36	44.42	**44.91**
17	41.23	41.68	41.75	41.75	41.86	**41.98**
18	37.31	37.64	37.59	38.05	38.27	**38.38**
19	39.99	41.01	41.55	41.58	41.71	**42.31**
20	40.63	41.24	41.48	41.95	42.25	**42.27**
21	38.72	39.10	39.61	40.55	40.59	**40.65**
22	37.63	38.37	38.44	38.73	38.97	**39.24**
23	41.93	43.22	43.92	43.47	43.93	**44.34**
24	34.74	35.55	35.44	35.59	35.85	**35.89**
Av.	39.21	40.05	40.52	40.88	41.13	**41.39**

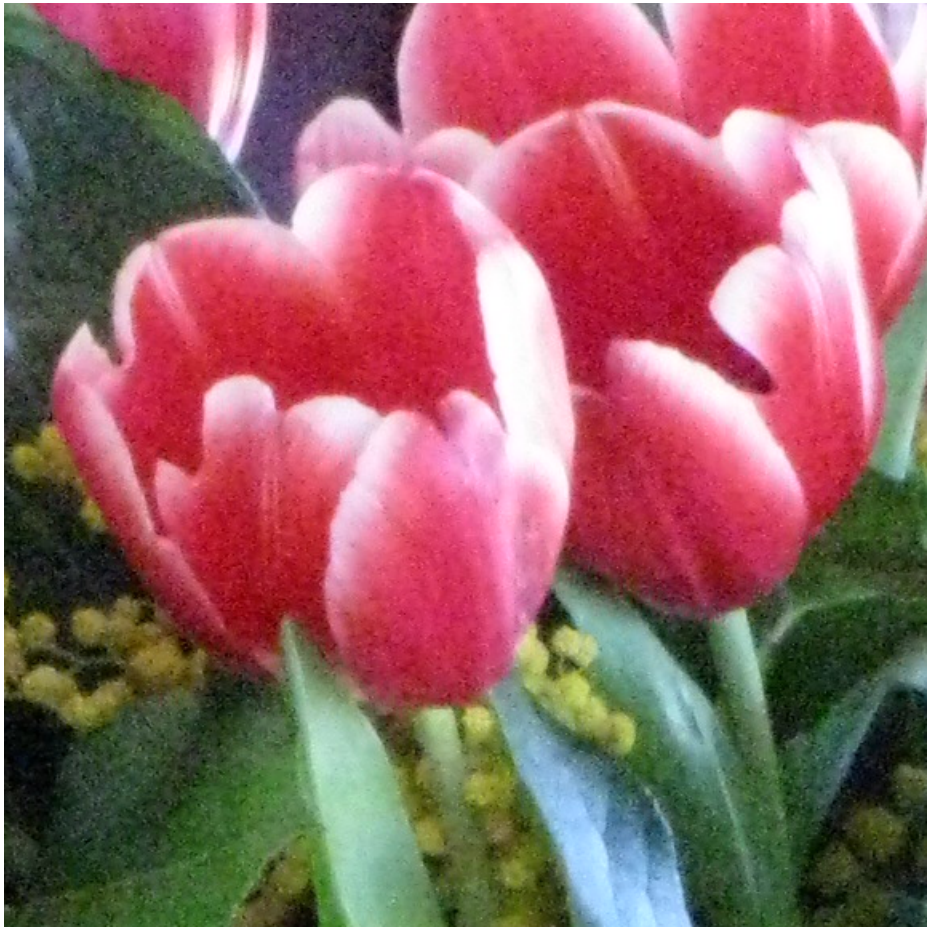
Results

Denoising

Results

Denoising

THE END



Thank you.