IMAGE DENOISING

Non-local Sparse Models for Image Restoration
ICCV2009

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Simultaneous Sparse Coding

• Unify two different approaches to image restoration
  1. Learning a basis set (dictionary) adapted to sparse signal descriptions
  2. Non-local means approach

• Decompose groups of similar signals on subsets of the learned dictionary.

• Outperforms the state of the art image denoising and restorations works
Denoising and Restoration Problem

- To reduce noise and/or extract useful image structures
- Eliminating undesirable characteristics

Different approaches

- anisotropic filtering
- total variation
- *image decompositions on fixed bases such as wavelets*
- non-local means filtering
- learned sparse models
- block matching with 3D filtering (BM3D)
Inroduction to Simultaneous Sparse Coding

- Different devices produce different kinds of noise, and introduce different types of artefacts and spatial correlations in the noise.

- Operate directly on the raw sensor output, that suffers from non-homogeneous noise, but is less spatially correlated and not corrupted by postprocessing artefacts.

- View both denoising and demosaicking as image reconstruction problems.
Related Works

- non-local means filtering
- learned sparse models
- block matching with 3D filtering (BM3D)

- *These methods assume white Gaussian noise*
- This work demonstrate empirically that their approach is effective at restoring real images corrupted by non-Gaussian, non-uniform noise
Non Local Means Filtering

- The prominence of selfsimilarities is used as a prior on natural images.

\[
x[i] = \sum_{j=1}^{n} \sum_{l=1}^{n} \frac{K_h(y_i - y_j)}{K_h(y_i - y_l)} y[j].
\]

- Two pixels associated with similar patches \(y_i\) and \(y_j\) should have similar values \(y[i]\) and \(y[j]\)
Learned Sparse Coding

- The clean signal can be approximated by a \( \text{sparse} \) linear combination of elements from a basis set called dictionary

\[
\min_{\alpha_i \in \mathbb{R}^k} \| \alpha_i \|_p \quad \text{s.t.} \quad \| y_i - D\alpha \|_2^2 \leq \varepsilon.
\]

- To learn a dictionary \( D \) adapted to the image at hand, and demonstrated that learned dictionaries lead to better empirical performance than off-the-shelf ones

- For an image of size \( n \), a dictionary in \( \mathbb{R}^{mxk} \) adapted to the \( n \) overlapping patches of size \( m \)

\[
\min_{D \in \mathcal{C}, A} \sum_{i=1}^{n} \| \alpha_i \|_p \quad \text{s.t.} \quad \| y_i - D\alpha_i \|_2^2 \leq \varepsilon \quad x = \frac{1}{m} \sum_{i=1}^{n} R_i D\alpha_i
\]

\( \mathcal{C} \) is the set of matrices in \( \mathbb{R}^{mxk} \), \( A = [\alpha \downarrow 1, \ldots, \alpha \downarrow n] \) is a matrix in \( \mathbb{R}^{mxk} \), 

\( R_i \) in \( \mathbb{R}^{mxk} \) is the binary matrix which places patch number \( i \) at its proper position in the image.
Block Matching 3D (BM3D)

- A patch-based procedure that exploits image self-similarities and gives state-of-the-art results.

- estimate the codes of overlapping patches and average the estimates

- they reconstruct patches by finding similar ones in the image \((\text{block matching})\), stacking them together into a 3D signal block

- denoising the block using hard or soft thresholding with a 3D orthogonal dictionary
Simultaneous Sparse Coding

- Non-local means filtering has proven very effective in general, but it fails in some cases.

- A patch does not look like any other one in the image, it is impossible to exploit self-similarities to denoise the corresponding pixel value.

- Sparse image models can handle such situations by exploiting the redundancy between overlapping patches, but they suffer from another drawback.

- Similar patches sometimes admit very different estimates due to the potential instability of sparse decompositions.

- In this paper we address this problem by forcing similar patches to admit similar decompositions.

\[ \| A \|_{p,q} \triangleq \sum_{i=1}^{k} \| \alpha_i \|_q^p \]
Simultaneous Sparse Coding

\[ S_i \triangleq \{ j \in \{1, \ldots, n\} \text{ s.t. } \| y_i - y_j \|_2^2 \leq \xi \} \]

A fixed dictionary D in \( R^{m \times k} \)
- Decomposing the patch \( y_i \) with a grouped-sparsity regularizer on the set \( S_i \) amounts to solving:

\[
\min_{\mathbf{A}_i} \| \mathbf{A}_i \|_{p,q} \quad \text{s.t.} \quad \sum_{j \in S_i} \| \mathbf{y}_j - \mathbf{D} \alpha_{ij} \|_2^2 \leq \varepsilon_i, \quad \mathbf{A}_i = [\alpha_{ij}]_{j \in S_i} \in \mathbb{R}^{k \times |S_i|}
\]

Optimization problem:

\[
\min_{(\mathbf{A}_i)_{i=1}^n, \mathbf{D} \in \mathcal{C}} \sum_{i=1}^n \frac{\| \mathbf{A}_i \|_{p,q}}{|S_i|^p} \quad \text{s.t.} \quad \forall i \sum_{j \in S_i} \| \mathbf{y}_j - \mathbf{D} \alpha_{ij} \|_2^2 \leq \varepsilon_i
\]

D is in \( R^{m \times k} \) with unit \( \ell_2 \)-norm columns,
- The normalization by \( |S_i|^p \) is used to ensure equal weights for all groups
Simultaneous Sparse Coding

- use the convex $\ell_{1,2}$ norm for learning the dictionary
- use the $\ell_{0,\infty}$ pseudo-norm for the final reconstruction
- this formulation allows all the image patches to be processed as if they were independent of each other.
- To reconstruct the final image, average the estimates of each pixel:

$$x = \text{diag}(\sum_{i=1}^{n} \sum_{j \in S_i} R_{ij} 1_m)^{-1} \sum_{i=1}^{n} \sum_{j \in S_i} R_{ij} D \alpha_{ij}.$$
Results

Demosaicking

LSC

LSSC
## Results

### Demosaicking

#### PSNR results

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Results

Denoising
Results

Denoising
Thank you.

THE END