Written Assignment 1

due on Wednesday, March 23rd, 2011

1. *(from Cormen, Ex.3-2)* Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω or Θ of B. Assume that $k \geq 1, \epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with *yes* or *no* written in each box.

	A	B	O	o	Ω	ω	Θ
a.	$\log^k n$	n^ϵ					
b.	n^k	c^n					
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\log c}$	$c^{\log n}$					
f.	$\log(n!)$	$\log(n^n)$					

2. Let A denote a set with n elements and B denote a set with m elements. You are asked to find an algorithm that determines whether A and B are disjoint (i.e., $A \cap B = \emptyset$). Specify your algorithm very briefly (do not provide an implementation), and analyze its complexity. Assume that n is a sufficiently large value with respect to m, i.e. $m \ll n$.

3. *(from Cormen, Ex.3-4)* Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove each of the following conjectures.
   (a) $f(n) = O(g(n))$ implies $g(n) = O(f(n))$.
   (b) $f(n) + g(n) = \Theta(\min(f(n), g(n)))$.
   (c) $f(n) = O(g(n))$ implies $\log(f(n)) = O(\log(g(n)))$, where $\log(g(n)) \geq 1$ and $f(n) \geq 1$ for all sufficiently large n.
   (d) $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$.
   (e) $f(n) = O((f(n))^2)$.
   (f) $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$.
   (g) $f(n) = \Theta(f(n/2))$.
   (h) $f(n) + o(f(n)) = \Theta(f(n))$.

4. Using the substitution method, show that the solution of $T(n) = 2T(\lfloor n/2 \rfloor) + n$ is $\Theta(n \log n)$.

5. *(from Cormen, Ex.4.2-5)* Use a recursion tree to give an asymptotically tight solution to the recurrence $T(n) = T(\alpha n) + T((1-\alpha)n) + cn$, where α is a constant in the range $0 < \alpha < 1$ and $c > 0$ is also a constant.