COMPUTER-AIDED NONLINEAR CONTROL EXPERIMENTS FOR GRADUATE LEVEL CONTROL ENGINEERING EDUCATION

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ABSTRACT

Teaching nonlinear control is an art depending on the instructor and the availability of experimental facilities in the laboratory. Nonlinearity of the closed loop system sometimes stems from the plant dynamics and sometimes the controller is nonlinear as well. In all such cases the students are taught the analysis of closed loop stability in the lectures yet the implementation in real time is generally omitted. This paper aims at filling this gap to some extent through describing several experiments for those mastering on the broad field of nonlinear control systems. A twin rotor helicopter system, a rotary inverted pendulum, a flexible link system, and finally a gear and belt equipped DC motor control setup are utilized to describe the experiments.

KEYWORDS

Control experiments, MRAC, PBC, SMC and helicopter, DC motor, flexible link

1 INTRODUCTION

Use of computers in closed loop control systems has greatly influenced the control engineering education due to the amazing progress in computer and instrumentation technology. The advances in software and hardware have made it possible to develop interesting computer based learning and prototyping platforms so that they have thoroughly changed the approaches and strategies adopted in engineering education so far. The usage of computer aided design allows the students to do more complicated experiments which are encountered frequently in the real life such as in robotics, aerospace, chemistry, fluids and networks. Therefore, from a pedagogical point of view, activities in the laboratory help the students grasp the theory well via the real time examples. Although it is possible to describe alternative methods to teach control engineering such as telelab, virtual lab, or web based interactive lab in Duan et al (2005); Casini et al (2003); Ugur and Savas (2010), the diversity of the possibilities offered by the computer centric technologies motivate the professors to propose novel teaching sets with novel experimental compositions. This study focuses on the teaching of essential poplinear control strategies such as sliding mode

nonlinear control strategies such as sliding mode control (Young et al (1999); Hung et al (1993); Slotine (1991); Khallil (2002)), feedback linearization, passivity based control (Slotine (1991); Khallil (2002)), singular perturbation theory (Khallil (2002)) and Kokotovic et al (1986)) and model reference adaptive control (Ioannou and Sun (1996)). The described experiments need the Matlab/Simulink® based real time client software and Wincon[®] real time server is used as a software realization platform to generate the machine code. Such an experimental infrastructure gives the students considerable degrees of freedom to prototype alternative methods on a selected test setup easily. Some of the studied systems are underactuated systems making them further appealing. Until recently, designing global controllers for underactuated systems was a challenge, yet, to overcome this problem, the control engineering framework now offers several tools based on nonlinear techniques such as hard boundary switching laws considered under the subtitle sliding mode control here, or energy based control considered under the subtitle passivity based control in the sequel. The problems chosen for prototyping the nonlinear control strategies are also widely used benchmark problems and this aspect of the paper makes it a beneficial guide to a large number of readers. The contribution of the current paper is to describe a set of nonlinear control experiments to teach the fundamental concepts of nonlinear control systems.

This paper is organized as follows. The second section gives the description of the nonlinear control experiments together with the model of the plant under investigation. The experiments are arranged as SMC of rotary inverted pendulum (RIP), passivity based control of a helicopter plant, sliding mode control based on singular perturbation theory to flexible link (Kokotovic, (1986); Extebarria et al (2005); Solis et al (2009)) and finally model reference adaptive control for a gear-belt equipped DC motor setup. All the experiments are discussed with real time results. The concluding remarks are given at the end of the paper.

2 SLIDING MODE CONTROL OF RIP

Rotary inverted pendulum (RIP) is one of the popular benchmark problems of control engineering. The system can be used to teach a number of control techniques. A photo of the apparatus is illustrated in Fig. 1 and the dynamics is described by (1).



Figure 1: Rotary inverted pendulum system

The method adopted is sliding mode control that is known for its robustness against disturbances and uncertainties. As mentioned earlier RIP is a nonminimum phase underactuated system thus to control such a system is not a straightforward task for students. The objective is to make the students understand effects of sliding line parameter, chattering phenomena, robustness concept and boundary layer thickness. Parameters to study in this experience are c_1 , c_2 , c_3 , k_1 , ε and their roles in sliding mode control. The mathematical model of the system is obtained by using Lagrange formulation as in (1).

$$\begin{array}{l} 0.0094\ddot{\theta} - 0.0045\cos(\alpha)\ddot{\alpha} + 0.0045\sin(\alpha)\dot{\alpha}^2 + 0.073\dot{\theta} = 0.1285V_m \\ 0.0047\ddot{\alpha} - 0.0045\cos(\alpha)\ddot{\theta} - 0.2054\sin(\alpha) = 0 \end{array}$$
(1)

This control experiment aims at stabilizing the rotary inverted pendulum around the upright position, which is unstable. The sliding mode controller has coupled surfaces as in (2) to achieve this goal.

$$s_{\alpha} = \dot{\alpha} + c_1 \alpha = 0, \quad s_{\theta} = \dot{\theta} + c_2 \theta = 0$$
 (2)

The students are asked to investigate the effects of slope parameters denoted by c_1 and c_2 . The two surfaces above are merged into one as given in (3).

$$S = s_{\alpha} + c_3 s_{\theta} \tag{3}$$

Now consider a Lyapunov function as given in (4).

$$V_s = s^2 / 2 \tag{4}$$

Taking the time derivative of *S* and forcing its time derivative to $-k_1S$ yields the control law in (5).

$$u = -\left(\frac{f_1 + f_2}{g_1 + g_2}\right)$$

$$-\frac{D}{g_1 + g_2}\left(k_1 \frac{S}{|S| + \varepsilon} + c_2 \dot{\theta} + c_1 \dot{\alpha}\right)$$
(5)

where

$$f_{1} = \frac{ad \sin(\alpha) - b \cos(\alpha) \left(b \sin(\alpha) \dot{\alpha}^{2} + G \dot{\theta} \right)}{ac - b^{2} \cos^{2}(\alpha)}$$

$$g_{1} = \frac{ak}{ac - b^{2} \cos^{2}(\alpha)}$$

$$f_{2} = \frac{bd \cos(\alpha) \sin(\alpha) - bc \sin(\alpha) \dot{\alpha}^{2} - Gc \dot{\theta}}{ac - b^{2} \cos^{2}(\alpha)}$$

$$g_{2} = \frac{kb \cos(\alpha)}{ac - b^{2} \cos^{2}(\alpha)}$$
(6)

Students in this experiment study the effects of changing the design parameters (c_i) and the reaching law parameter k_1 . Figs.2-3 with the data in Table 1 illustrates an exemplar set of results.



Figure 2: Stabilization results with SMC





Table 1: Parameters for SMC

Parameter	Value	Parameter	Value	Parameter	Value
K_{l}	1.7	c_I	13	<i>C</i> ₃	1.9
Ε	0.05	<i>c</i> ₂	4		

3 PASSIVITY BASED CONTROL OF A TWIN ROTOR HELICOPTER

Passivity based control is a direct application of passivity theorems. In order to realize an experimental study about PBC in the lab, the two degree of freedom (2-dof) twin rotor helicopter setup shown in Fig. 4 is utilized.



Figure 4: 2-dof twin rotor helicopter

A Lagrangian model of the 2-dof helicopter is used for the PBC experiment and the equations of the motions are given in the following standard form containing inertia, damping, friction, gravitational and applied torque matrices, respectively.

$$\mathbf{D}(q)\ddot{\mathbf{\eta}} + \mathbf{C}(q,\dot{q})\dot{q} + \mathbf{B}\dot{q} + \mathbf{G}(q) =$$
(7)
where

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$$\mathbf{q} = \begin{bmatrix} \theta \\ \psi \end{bmatrix}, \ \mathbf{g}(\theta) = \begin{bmatrix} K_{pp}(\theta) & 0.0068 \\ 0.0219 & 0.072 \end{bmatrix}, \ \mathbf{u} = \begin{bmatrix} V_p \\ V_y \end{bmatrix}$$
(8)

$$\mathbf{D}(q) = \begin{bmatrix} 0.0862 & 0 \\ 0 & 0.0432 + 0.0478 \cos \theta^2 \end{bmatrix}$$
(9)

$$\mathbf{C}(q,\dot{q}) = \begin{bmatrix} 0 & 0.0478\sin\theta\cos\theta\dot{\psi} \\ -0.0956\sin\theta\cos\theta\dot{\psi} & 0 \end{bmatrix}$$
(10)

$$\mathbf{B} = \begin{bmatrix} 0.0478 & 0\\ 0 & 0.318 \end{bmatrix}, \quad \mathbf{G}(q) = \begin{bmatrix} 2.5271\cos\theta\\ 0 \end{bmatrix} \quad (11), (12)$$

$$\mathbf{\tau} = \mathbf{g}(\theta)\mathbf{u} \tag{13}$$

Due to its significant nonlinearity, K_{pp} parameter is approximated by the following function.

$$K_{vv}(\theta) = -9.535 \cdot 10^{-6} \theta^2 - 7.281 \cdot 10^{-4} \theta + 0.1624$$
 (14)

The vector of tracking error \mathbf{q}_{e} is defined as

$$\mathbf{q}_e = \mathbf{q} - \mathbf{q}_d \tag{15}$$

where $\mathbf{q}_{d} \coloneqq [\mathbf{q}_{d,\theta} \mathbf{q}_{d,\psi}]$ designates the desired position of the helicopter. With this error term, the dynamic equation of the system given in (7) is reorganized as follows with an appropriate choice for $\boldsymbol{\tau}$ given in (17).

$$\mathbf{D}(q)\ddot{\mathbf{q}}_{e} \notin \mathbf{C}(q,\dot{q})\dot{\mathbf{q}}_{e} + \mathbf{B}\dot{\mathbf{q}}_{e} + \mathbf{K}_{p}\mathbf{q}_{e} = (16)$$

 $\boldsymbol{\tau} := \mathbf{G}(q) + \mathbf{D}(q)\ddot{\mathbf{q}}_{d} + \mathbf{C}(q,\dot{q})\dot{\mathbf{q}}_{d} + \mathbf{B}\dot{\mathbf{q}}_{d} - \mathbf{K}_{p}\mathbf{q}_{e} + \boldsymbol{\xi} \qquad (17)$

where \mathbf{K}_p is a 2×2 positive definite constant matrix and $\boldsymbol{\xi}$ is a control component that is defined in (18) to ensure the passivity and stability of the system with the positive definite storage function given in (19). The variables used in the above equations are listed and explained in Table 2.

Table 2: Definitions of the variables

- $\begin{array}{lll} \theta & \text{Pitch angle} & [-0.7, 0.6] & \text{rad} \\ \psi & \text{Yaw angle} & (-\infty, \infty) & \text{rad} \\ K_{pp} & \text{Relation between pitch motor} & \text{See text} & \text{Nm/V} \\ & \text{voltage and the torque acting} \\ & \text{on pitch axis} \end{array}$
- V_p Input voltage of pitch angle [0, 24] V motor (main motor)
- *V_y* Input voltage of yaw angle [-15, 0] V motor (tail motor)

$$\boldsymbol{\xi} \coloneqq \mathbf{C}\dot{\mathbf{q}}_{e} + \mathbf{B}\dot{\mathbf{q}}_{e} - \frac{1}{2}\dot{\mathbf{D}}\dot{\mathbf{q}}_{e} + \mathbf{v}$$
(18)

$$V := \frac{1}{2} \dot{\mathbf{q}}_{e}^{\mathrm{T}} \mathbf{D} \dot{\mathbf{q}}_{e} + \frac{1}{2} \mathbf{q}_{e}^{\mathrm{T}} \mathbf{K}_{p} \mathbf{q}_{e}$$
(19)

The variable **v** is defined as the fictitious control input of the system and the derivative of the storage function V is obtained as $\dot{v} = \dot{\mathbf{q}}_{c}^{\mathsf{T}}\mathbf{v}$. Consequently, the system given in (16) is passive and can be globally asymptotically stabilized with the storage function Vfor the system output $y = \dot{\mathbf{q}}_{e}$ and the control input $v = -\phi(y)$ such that $\phi(0) = 0$. Under these circumstances, the choice for **v** is made as

$$\mathbf{v} = -\mathbf{K}_{d} \dot{\mathbf{q}}_{e} \; ; \; \mathbf{K}_{d} = \operatorname{diag}\left(K_{d,\theta}, K_{d,\psi}\right), \; \mathbf{K}_{d} > 0 \qquad (20)$$

The designed controller works as a PD controller in conjunction with necessary nonlinearity canceling terms. The controller is realized in Matlab/Simulink[®] environment and several tracking tasks are performed for different values of \mathbf{K}_p and \mathbf{K}_d . In the experiment, the effects of \mathbf{K}_p and \mathbf{K}_d are studied and different choices of students for the control input \mathbf{v} are tested. Real time results of a tracking action for the values of $\mathbf{K}_p = \text{diag}(5,10)$, $\mathbf{K}_d = \text{diag}(5,3)$ are given in Fig. 5 and Fig. 6 showing the pitch and yaw angles of the

145

helicopter, respectively. As seen in the results, the tracking precision is achieved appropriately and the 2-dof helicopter follows the given path with an admissible level of error. The aim of this experiment was to apply the passivity theorems and the students acquire how a robust control law is implemented through a storage function.



Figure 5: Yaw angle tracking (top), tracking error (middle) and control signal generated by the PBC (bottom).

4 CONTROL OF A FLEXIBLE LINK

Flexible link system is another setup available in many control engineering labs. The control problem is to force the link follow a predefined trajectory without displaying oscillations.

The method adopted in this experiment is sliding mode control based on singular perturbation theory. The control objective is to teach the students how to design a nonlinear controller for the flexible link that overcomes the oscillations while maintaining a rapid response, effects of the sliding line parameter, concept of robustness, singular perturbation theory and optimal control issues. Parameters to study are μ , β , λ . The generalized representation of flexible link system is shown in (21).

$$\mathbf{D}(q)\ddot{\mathbf{q}} + \mathbf{C}(q,\dot{q}) + \mathbf{K}q + \mathbf{F}(q,\dot{q}) = \mathbf{B}(q)\mathbf{u}$$
(21)

$$0.0023\theta + 0.0031(\theta + \hat{\alpha}) = T_{out} - 0.004\theta$$
(22)

$$0.0031(\ddot{\theta} + \ddot{\alpha}) + 1.1175\alpha = 0$$

D represents the total inertia matrix, C is the Coriolis vector, K is the stiffness matrix, F is the friction vector and B is the input matrix as given in (23).

$$\mathbf{D} = \begin{bmatrix} 0.0054 & 0.0031 \\ 0.0031 & 0.0031 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0.2107 & 0 \\ 0 & 0 \end{bmatrix}, K = \begin{bmatrix} 0 & 0 \\ 0 & 1.115 \end{bmatrix}$$
$$\mathbf{H}(q) = \mathbf{D}^{-1}(q), \mathbf{q} = \begin{bmatrix} q_r \\ q_f \end{bmatrix} = \begin{bmatrix} \theta \\ \alpha \end{bmatrix}, \mathbf{B} = \begin{bmatrix} B_r \\ B_f \end{bmatrix}$$
(23)

The rigid modes are symbolized by θ and flexible modes are symbolized by α . The dynamic analysis and control of flexible link systems is more complicated than the rigid systems as nonlinear rigid body motions are strongly coupled with the distributed effects of flexibility, and to overcome this drawback, singular perturbation theory can be applied. A composite control law as in (24) is postulated in this design approach.



Figure 6: Yaw angle tracking (top), tracking error (middle) and control signal generated by the PBC (bottom).



Figure 7: Flexible link system

The mathematical model of the flexible link system is obtained using Lagrange formulation as in (22)

$$u = \hat{u}(\theta, \dot{\theta}) + \tilde{u}(\tilde{\phi}_1, \tilde{\phi}_2)$$
(24)

The control law for slow (rigid) subsystem is \hat{u} and the control law for fast subsystem is \tilde{u} . To apply this strategy the elastic forces are introduced as new state

variables. The discontinuity of the solutions caused by singular perturbations can be analyzed in separate time scales. The slow subsystem has the same properties of the rigid arm in (25) and slow manifold can be rewritten as in (26). Let γ denote the smallest stiffness constant and μ denote the small scale factor used for obtaining the singular perturbed model the system. The singular perturbation model of a dynamical system is a state model where the derivatives of some of the states multiplied by a small positive parameter (μ^2).

$$\ddot{\theta} = D_{11}^{-1} [B_1 \hat{\mu} - C_{11} \dot{\theta} - F_1]$$
(25)

$$\hat{\psi} = K_{22}^{-1} H_{22}^{-1} (B_2^{-1} \hat{u} - V_{21} \dot{\theta} - H_{21} F_1)$$
(26)

The fast subsystem can be derived with a time-scale change $T = t / \mu$ and we obtain the representation in (27).

$$\frac{\mathrm{d}\phi_1}{\mathrm{d}T} = \phi_2, \quad \frac{\mathrm{d}\phi_2}{\mathrm{d}T} = -H_{22}K_{22}\phi_1 - C_{22}^{-1}\mu\phi_2 - B_2^1\tilde{u}$$
(27)

 $\phi_1 = \psi - \hat{\psi}, \ \phi_2 = \mu \dot{\psi}, \ \mu^2 = \gamma_m^{-1}$

The sliding surface is chosen as in (28), in which q_d stands for the desired trajectory.

 $\dot{e}_f = \dot{q}_d - \lambda e \quad e = \hat{q}_r - q_d, \quad s = \dot{e} + \lambda e \tag{28}$

The sliding mode control law for the slow subsystem is given in (29).

$$\hat{u} = B_1^{-1} [D_{11} \ddot{e}_f + C_{11} \dot{e}_f + F_1] - K \ sat(s \ \beta)$$
(29)



Figure 8: Tracking performance of flexible link via a composite controller

For the fast subsystem, in order to stabilize the deflections and damp out the vibrations, a LQR controller is designed. The control objective is to minimize the cost function given by (30).

$$T = \int (\phi^T Q \phi + \tilde{u} R \tilde{u}) dt$$
(30)

where, Q and R stand for the weighting matrices. The weighting matrices and gain matrix for LQR are chosen as in (31).

$$Q = \text{diag}(100, 500, 1, 0), R = 2, K = [2.35 \ 0.5]$$
 (31)

 Table 3: The parameters of singular perturbed system sliding mode controller

$\lambda = 6$	$\beta = 1/3$
µ=0.048	K=1

As shown in Figure 8 the composite controller with LQR, the tracking performance is satisfactory.

5 MODEL REFERENCE ADAPTIVE CONTROL OF A DC MOTOR

Model reference control is based on minimizing the error between the responses of a system under control and a desired reference model following the reference signal, Ioannou and Sun (1996). In the cases where the system parameters are not known, the task is achieved by defining an adaptive control law updating the parameters seen in the control law online. In this experiment, the DC motor control setup shown in Figure 9 is utilized. This control setup is composed of two modules called mechanical unit and digital unit.



Figure 9: DC motor control setup.

Mechanical unit is equipped with a DC motor, a tachogenerator and angle measuring wheels connected to motor with straps which introduce backlash type nonlinearity. Digital control unit enables implementing the control algorithms designed on Matlab/Simulink platform. State space model of this single input single output linear time invariant system is as given below.

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \tag{32}$$

where the parameters of the system are not known, but it is assumed that the pair (\mathbf{A}, \mathbf{B}) is controllable. In above, **x** is the state vector containing the angular position and speed of motor and *u* is the control input. The reference model is defined as

$$\dot{\mathbf{x}}_{m}(t) = \mathbf{A}_{m}\mathbf{x}_{m}(t) + \mathbf{B}_{m}r(t)$$
(33)

The tracking error is defined as $\mathbf{e} := \mathbf{x} - \mathbf{x}_m$ and the control signal has the following form,

$$u = -\mathbf{K}(t)\mathbf{x} + L(t)r \tag{34}$$

where the control gains \mathbf{K} and L are updated with the rules given as below.

$$\dot{\mathbf{K}} = \mathbf{B}_{m}^{\mathsf{T}} \mathbf{P} \mathbf{e} \mathbf{x}^{\mathsf{T}} \operatorname{sgn}(L), \ \dot{L} = -\mathbf{B}_{m}^{\mathsf{T}} \mathbf{P} \mathbf{e} r^{\mathsf{T}} \operatorname{sgn}(L)$$
(35)

The matrix **P** is obtained by solving the Lyapunov equation $\mathbf{PA}_m + \mathbf{A}_m^T \mathbf{P} = -\mathbf{Q}$ with the choice of a positive definite symmetric **Q**. Thus, as stated by Casini et al (2003), the stability of the system is ensured in the sense of Lyapunov and the variables $\mathbf{K}(t)$, L(t), $\mathbf{e}(t)$ evolve bounded, furthermore $\mathbf{e}(t) \rightarrow 0$ as $t \rightarrow \infty$. The control signal proposed in (34) is realized in Matlab/Simulink environment for a number of different reference models.

$$\mathbf{A}_{m} = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix}, \quad \mathbf{B}_{m} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
(36)

The results of real time implementation are illustrated in Figure 10. A step signal is used as the reference input and a reference model displaying overdamped response is chosen. Results show that the system settles at the desired position in a finite time and the control parameters $\mathbf{K}(t)$ and L(t) converge to steady values when the command signal is constant.

4 CONCLUSIONS

This paper describes four experiments to students mastering on nonlinear control. Despite undergraduate laboratories in control engineering have some degree of maturity; courses like nonlinear control still necessitate particular attention when designing for the graduate level students. The paper presents SMC and a variant of it, PBC and MRAC. Standard experimental setups have been used and repeatability of the experiments has been ensured.

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Figure 10: Closed loop control results

REFERENCES

- Casini, M., Prattichizzo, D. and Vicino, A., 2003. The Automatic Control Telelab: A User Friendly Interface for Distance Learning, *IEEE Tran. on Education*, v.46, no.2, pp.252-257.
- Duan, B., Ling, K-V., Mir, H., Hosseini, M. and Gay, R.K.L., 2005. An Online Laboratory Framework for Control Engineering Courses, *Int.l J. of Eng. Educ.*, v.21, no.6, pp.1068-1075.
- Extebarria, V., Sanz, A. and Lizarraga, I., 2005. Control of a Ligthweight Flexible Robotic Arm Using Sliding Modes. *Int. J.l of Advanced Robotic Sys.*, v.2, no.2, pp.103-110.
- Hung, J.Y., Gao, W. and Hung, J.C. 1993. Variable Structure Control: ASurvey, *IEEE TIE*, v.40, no.1, pp. 2-22.
- Ioannou P. and Sun, J., 1996. *Robust Adaptive Control*, Prentice Hall, Inc.
- Khallil, H.K., 2002. *Nonlinear systems*, Upper Saddle River, N.J., Prentice Hall.
- Kokotovic, P.V., Khalil, H.K. and O'Reilly, J., 1986. Singular Perturbation Methods in Control: Analysis and Design, Academic Press.
- Slotine, J.J., Li, W., 1991, *Applied Nonlinear Control*. Prentice Hall.
- Solis, J.F.P., Navarro, G.S., Linares, R.C., 2009. Modelling and Tip Position Control of a Flexible Link Robot: Experimental Results. *Computation Systems*, v.12, no.4, pp.421-435.
- Ugur, M., Savas, K., 2010. An internet-based real-time remote control laboratory for control education, *Procedia-Social* and Behavioral Sciences, v.2, pp.5271-5275.
- Young, K.D., Utkin, V.I. and Özgüner, Ü. 1999. A Control Engineer's Guide to Sliding Mode Control, *IEEE TCST*, v.7, no.3, pp.328-342.