Nominal Model Based Switching Control of a Twin Rotor System

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***Abstract*—This paper considers a novel model based switching control scheme. The philosophy of the approach is to design a conventional linear or nonlinear feedback control scheme for a nominal plant model and to force the true system states to that of the nominal model by introducing a switching term. In the demonstrated example, a twin rotor system is considered. Feedback controller for the nominal system is designed using the backstepping method and the results show that the proposed technique is successful.**

***Keywords*—Backstepping technique, sliding mode control, switching control**

I. Introduction

Control of dynamic systems is a mature field offering the engineers a large number of design possibilities. Depending on the operating conditions, available information, restrictions and performance expectations, one's choice may change. Backstepping control technique is one such algorithm developed by Petar Kokotovic, [1]. The algorithm runs a nested structure and stabilizing laws for every stage is determined successively. At the final stage, the control input to the whole system is postulated and the overall system becomes Lyapunov stable. One should note that the design procedure assumes the availability of the functions embodying the dynamics of the true system and this is practically a challenge as a model typically approaches the true system and there is always a mismatch between the nominal plant model and the true model.

Typical next step of the design in the presence of uncertainties is to reconsider the feedback system with the uncertain plant model and prove the stability using the available information about the uncertainties and disturbances. Nevertheless, at the very initial stage, we obtain a feedback control law for the nominal system. The question put forth in this paper is the following: Can we force the uncertain system response to that of the nominal model by minor modifications to the original feedback scheme and obtain closed loop stability and performance?

The contribution of the current work is to enhance the backstepping control law with a discontinuous term and to derive the condition its magnitude must satisfy.

In the literature, backstepping control technique has successfully been implemented in the field of robotics and several other processes as one of the state variables is of type position and the other is of type velocity, [2-5].

The paper considers a twin rotor system to demonstrate the prominent features of the proposed scheme. The system dynamics fit the assumptions of the paper. In the literature, such tethered helicopter models have been used by feedback control experts many times. In [6], the process is kept under sliding mode control, in [7] the control scheme is modified to two-sliding mode control, which is known as a high order sliding mode control. Use of feedforward neural networks for the control of twin rotor system is discussed in [8] and a comparison with PID loop is presented. In [9], nonlinear predictive control scheme is implemented, and in [10], neural network aided backstepping scheme is chosen for feedback control. The twin rotor model is experimented with feedback linearization technique in [11], multivariable model predictive control in [12] and adaptive super-twisting algorithms in [13]. This short literature search shows that the twin rotor system is a good test bed to demonstrate the performance of novel control schemes. Some of the works presented are critically dependent to the process model, which we would like to override to some extent.

Current work differs from the cited literature in that the present study is nominal model based and the goal is to enforce the nominal system response. The discrepancies between the process response and the nominal system response are handled by a switching term, whose argument is the switching variable. When the argument is forced to zero, the error exponentially converges zero and the true process, containing imprecision and uncertainty, is forced to behave exactly as the nominal model does. The approach is different from internal model control or high gain control in architecture and to our best knowledge, the approach is a contribution to the field.

This paper is organized as follows: The second section introduces the backstepping control design for the nominal system dynamics. In the third section, we present the design approach for the true model (or the plant model containing uncertainties). The fourth section introduces the plant dynamics, the fifth section discusses the simulation results and the last part is devoted to the concluding remarks.

II. BACKSTEPPING CONTROL METHOD

Consider the nominal system, where x_1, x_2, $f(.,.)$, $y \in \Re^{n\times 1}$, $u\in\Re^{m\times 1}$, $g(.,.)\in\Re^{n\times m}$,

$$\dot{x}_1 = x_2 \tag{1}$$

$$\dot{x}_2 = f_n(x_1,x_2) + g_n(x_1,x_2)u \tag{2}$$

$$y_n = x_1 \tag{3}$$

The functions embodying the nominal system are known. The goal of the control system is to force the system output toward a reference signal denoted by

$$\dot{r}_1 = r_2, \quad \dot{r}_2 \in \Re^{n\times 1} \tag{4}$$

where r_1 and r_2 are the reference signals for x_1 and x_2, respectively. Let's define the following internal variables.

$$z_1 := x_1 - r_1 := e_1 \tag{5}$$

$$z_2 := x_2 - r_2 - A := e_2 - A \tag{6}$$

where A is a variable to be selected in the sequel. Now we choose the first stage Lyapunov function as

$$V_1 = \frac{1}{2} z_1^T z_1 \tag{7}$$

Taking the time derivative, one gets

$$\dot{V}_1 = z_1^T \dot{z}_1 = z_1^T (z_2 + A) \tag{8}$$

With k_1>0, setting $A := -k_1 z_1$

$$\dot{V}_1 = z_1^T \dot{z}_1 = -k_1 z_1^T z_1 + z_1^T z_2 \tag{9}$$

For the second stage, we choose a Lyapunov function of the form

$$V_2 = V_1 + \frac{1}{2} z_2^T z_2 \tag{10}$$

Taking the time derivative and inserting (9) with a k_2>0 yields

$$\dot{V}_2 = -k_1 z_1^T z_1 - k_2 z_2^T z_2 \tag{11}$$

if and only if the control input (u) is selected as $u = u_{nom}$, where

$$u_{nom} := g_n^{-1}\left(\dot{r}_2 - f_n(x_1,x_2) - (1-k_1^2)z_1 - (k_1+k_2)z_2\right) \tag{12}$$

or

$$u_{nom} := g_n^{-1}\left(\dot{r}_2 - f_n(x_1,x_2) - (1+k_1k_2)e_1 - (k_1+k_2)e_2\right) \tag{13}$$

Obviously, if there were no uncertainties, this control signal would force the system states to follow the reference signals and the time derivative of the Lyapunov function in (10) would be negative.

III. PROPOSED APPROACH

Consider the true system with uncertainties, where x_{1p}, x_{2p}, $f_p(.,.)$, $y_p \in\Re^{n\times 1}$, $u_p\in\Re^{m\times 1}$, $g_p(.,.)\in\Re^{n\times m}$,

$$\dot{x}_{1p} = x_{2p} \tag{14}$$

$$\dot{x}_{2p} = f_p(x_{1p},x_{2p}) + g_p(x_{1p},x_{2p})u_p \tag{15}$$

$$y_p = x_{1p} \tag{16}$$

Following boundedness assumptions are imposed on the true system model.

$$f_p(x_1,x_2) = f_n(x_1,x_2) + \Delta \tag{17}$$

$$g_p(x_1,x_2) = \Lambda g_n(x_1,x_2) \tag{18}$$

$$\left|\Delta_{ij}\right| < L_{\Delta u} \quad \forall i,j \tag{19}$$

$$0 < L_{\Lambda l} < \left|\Lambda_{ij}\right| < L_{\Lambda u} \quad \forall i,j \tag{20}$$

The control signal for the true system is constructed as follows.

$$u_p = u_{nom} + M\,\mathrm{sgn}(\sigma) \tag{21}$$

where

$$\sigma := \dot{e}_m + \lambda e_m \tag{22}$$

$$e_m := x_1 - x_{1p} \tag{23}$$

and M is a matrix to be determined.

Theorem: For the nominal system in (1)-(3), the nominal control signal ensures the Lyapunov stability of nominal system. Considering the true system in the same structure and assuming its input as in (21) with $v := \dot{r}_2 - (1-k_1^2)z_1 - (k_1+k_2)z_2$ and

$$M > \left(L_{\Lambda l} 1_{n\times n} g_n\right)^{-1}\left\{\left(I + L_{\Lambda u}1_{n\times n}\right)\left|v - f_n\right| + L_{\Delta u}1_{n\times n} + \left|\lambda \dot{e}_m\right|\right\} \tag{24}$$

ensures $\sigma = 0$ and the true system states follow the nominal system states in the sliding mode.

Proof: Choose the following Lyapunov function

$$V_3 = V_2 + \frac{1}{2}\sigma^T\sigma \tag{25}$$

and taking the time derivative entails the analysis of the last term only. This is given as below. First we evaluate $\ddot{e}_m$ as in (26), construct the time derivative of σ in (27), and rearrange to get $\dot{\sigma}^{\mathrm{T}}\sigma$ in (28) and combine the results in (28).

$$\begin{aligned}
\ddot{e}_m &= \dot{x}_2 - \dot{x}_{2p} \\
&= v - f_p - g_p\left(u_{nom} + u_p\right) \\
&= v - f_p - g_p\left(g_n^{-1}(v - f_n) + M\,\mathrm{sgn}(\sigma)\right) \\
&= v - f_p - g_p g_n^{-1}\left(v - f_n + g_n M\,\mathrm{sgn}(\sigma)\right) \\
&= \left(I - g_p g_n^{-1}\right)v - \left(f_p - g_p g_n^{-1} f_n\right) - g_p M\,\mathrm{sgn}(\sigma) \\
&= \left(I - g_p g_n^{-1}\right)v - \left(f_n + \Delta - g_p g_n^{-1} f_n\right) - g_p M\,\mathrm{sgn}(\sigma) \\
&= \left(I - g_p g_n^{-1}\right)v - \left(I - g_p g_n^{-1}\right) f_n - \Delta - g_p M\,\mathrm{sgn}(\sigma) \\
&= (I - \Lambda)v - (I - \Lambda) f_n - \Delta - \Lambda g_n M\,\mathrm{sgn}(\sigma) \\
&= (I - \Lambda)(v - f_n) - \Delta - \Lambda g_n M\,\mathrm{sgn}(\sigma)
\end{aligned} \tag{26}$$

This lets us have the following derivative.

$$\begin{aligned}
\dot{\sigma} &= \ddot{e}_m + \lambda \dot{e}_m \\
&= (I - \Lambda)(v - f_n) - \Delta - \Lambda g_n M\,\mathrm{sgn}(\sigma) + \lambda \dot{e}_m
\end{aligned} \tag{27}$$

The relevant term of V_3 now can be written as

$$\dot{\sigma}^T \sigma = \left\{(I - \Lambda)(v - f_n) - \Delta - \Lambda g_n M\,\mathrm{sgn}(\sigma) + \lambda \dot{e}_m\right\}^T \sigma \tag{28}$$

Using the bound conditions given in (19)-(20), the following result can be derived.

$$\begin{aligned}
\dot{\sigma}^T \sigma &= \left\{(I - \Lambda)(v - f_n) - \Delta + \lambda \dot{e}_m\right\}^T \sigma \\
&\quad - \left\{\Lambda g_n M\,\mathrm{sgn}(\sigma)\right\}^T \sigma \\
&< \left|(I - \Lambda)(v - f_n) - \Delta + \lambda \dot{e}_m\right|^T |\sigma| \\
&\quad - \left(\Lambda g_n M\right)^T |\sigma| \\
&< \left((I + |\Lambda|)|v - f_n| + |\Delta| + |\lambda \dot{e}_m|\right)^T |\sigma| \\
&\quad - \left(\Lambda g_n M\right)^T |\sigma| \\
&< \left((I + L_{\Delta u} 1_{n\times n})|v - f_n| + L_{\Delta u} 1_{n\times 1} + |\lambda \dot{e}_m|\right)^T |\sigma| \\
&\quad - \left(L_{\Lambda l} 1_{n\times n} g_n M\right)^T |\sigma|
\end{aligned} \tag{29}$$

The choice of M in (24) ensures the negativity of the Lyapunov function in (25).

IV. Twin Rotor System Dynamics

The physical appearance of the twin rotor system is shown in Fig. 1, where it is seen that the system has two degrees of freedom (2-dof) and the positional variables are θ (pitch) and ψ (yaw) as shown.

A Lagrangian model of the 2-dof helicopter is used for the simulations, [14]. The equations of motions for pitch and yaw angles of the helicopter are given in (30) and (31), respectively.

$$\ddot{\theta} = \frac{K_{pp}V_p + K_{py}V_y - B_p\dot{\theta} - m_h l_{cm}^2 \sin(\theta)\cos(\theta)\dot{\psi}^2 - m_h g l_{cm}\cos(\theta)}{J_p + m_h l_{cm}^2} \tag{30}$$

$$\ddot{\psi} = \frac{K_{yp}V_p + K_{yy}V_y - B_y\dot{\psi} - 2 m_h l_{cm}^2 \sin(\theta)\cos(\theta)\dot{\theta}\dot{\psi}}{J_y + m_h l_{cm}^2\left(\cos(\theta)\right)^2} \tag{31}$$

Among the variables seen above, K_{pp} is a function of pitch angle and is given by the parabolic equation in (32). The other parameters are constants and their values are given in Table I.

$$K_{pp}(\theta) = -9.535\cdot 10^{-6}\theta^2 - 7.281\cdot 10^{-4}\theta + 0.1624 \tag{32}$$

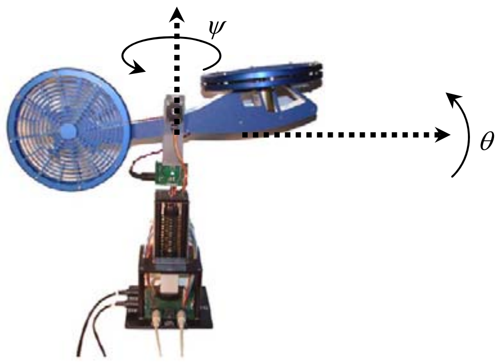

Fig. 1. Picture of the twin rotor system

TABLE I. PARAMETERS OF THE DYNAMICAL MODEL

Symbol	**Description**	**Value**	
θ	Pitch angle	[-0.7, 0.6]	rad
ψ	Yaw angle	(-∞,∞)	rad
B_p	Viscous friction on pitch motion	0.8000	*N/V*
B_y	Viscous friction on yaw motion	0.3180	*N/V*
g	Acceleration due to gravity	9.8100	m/s^2
J_p	Mom. of inertia about pitch axis	0.0384	kgm^2
J_y	Mom. of inertia about yaw axis	0.0432	kgm^2
m_h	Mass of helicopter	1.3872	*kg*
l_{cm}	Dist. from pivot to center of mass	0.1857	*m*
K_{pp}	Input gain	See text	*Nm/V*
K_{py}	Imput gain	0.0068	*Nm/V*
K_{yp}	Input gain	0.0219	*Nm/V*
K_{yy}	Input gain	0.0720	*Nm/V*
V_p	Input voltage	[0, 24]	*V*
V_y	Input voltage	[-15, 0]	*V*

V. Simulation Studies

During the simulations we chose

$$\tilde{r}_{1\theta} = 0.9\sin(2\pi t/10) \tag{33}$$

$$\tilde{r}_{1\psi} = 0.8\sin(2\pi t/7) \tag{34}$$

and filter these signals to get a smooth positional references as follows.

$$r_{1\theta} = \left(\frac{10}{p+10}\right)^2 \tilde{r}_{1\theta} \tag{35}$$

$$r_{1\psi} = \left(\frac{10}{p+10}\right)^2 \tilde{r}_{1\psi} \tag{36}$$

where p is the derivative operator, i.e. p:=d/dt. We simulate the system for 40 seconds with a time step of 10 msec and consider the uncertainties as

$$\Delta = 0.1\left[\sin(\psi) \quad \cos(\theta)\right]^T \tag{37}$$

$$\Lambda = \begin{bmatrix} 1+0.01\sin(\dot{\psi}) & 1+0.01\cos(\dot{\theta}) \\ 1+0.01\cos(\psi) & 1+0.01\sin(\theta) \end{bmatrix} \tag{38}$$

With these definitions, we infer the following.

$$\left|\Delta_{ij}\right| \le 0.1 < L_{\Delta u} \quad \forall i,j \tag{39}$$

$$0 < L_{\Lambda l} < 0.99 \le \left|\Lambda_{ij}\right| \le 1.01 < L_{\Lambda u} \quad \forall i,j \tag{40}$$

Regarding the backstepping design, the parameters are chosen as $k_1 = k_2 = 5$ and the observed uncertain system states are corrupted with a random noise changing in between −0.001 and 0.001. The simulation results are seen in Figs. 2-6, where the proposed scheme is active throughout the simulation.

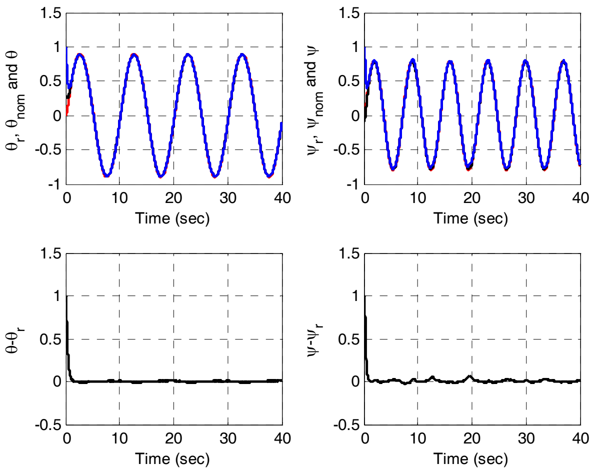

Fig. 2. Tracking of position references with errors

In Fig. 2, the reference positional signals are shown for both axes and the response of the nominal system is shown together with the response of the uncertain system. The three signals, except during the early transient phase, are almost indistinguishable. The errors shown in the bottom row of Fig. 2 support this claim.

In Fig. 3, the velocities are shown. The reference signal together with the response of the nominal system and the uncertain system demonstrate that the reference tracking performance of the proposed scheme is good.

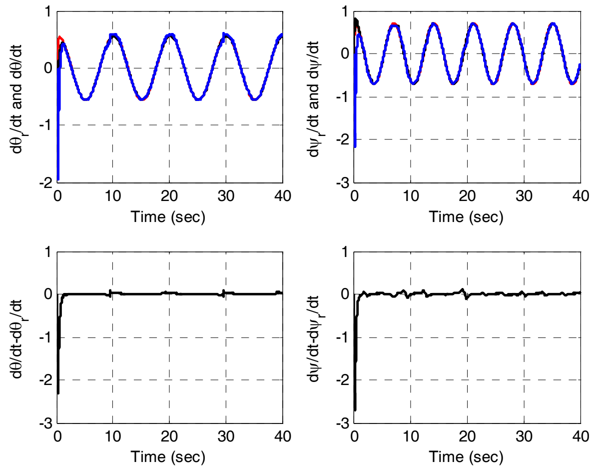

Fig. 3. Tracking of velocity references with errors

The behavior in the space of z variables are shown in Fig. 4, which demonstrates that the nominal system is under a stabilizing control law that keeps z_1 and z_2 around origin after a quick transient.

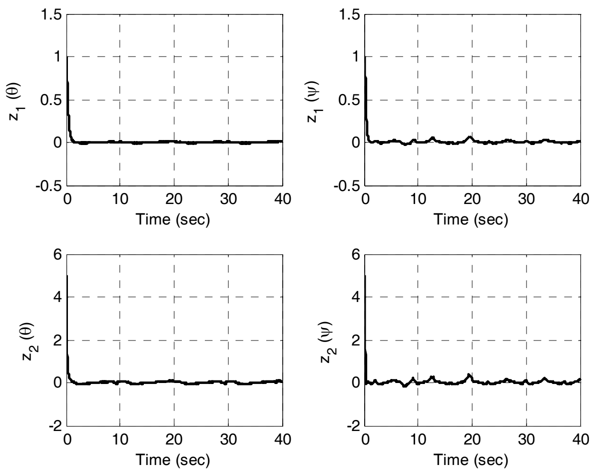

Fig. 4. Evolution of backstepping design parameters

Fig. 5 demonstrates the control signals in the top (V_p) and middle (V_y) subplots. Since the proposed scheme computes the discontinuous term by using the error between the uncertain model and the nominal model, the signals are not dominated by the high frequency switching action. Sign function is smoothed by utilizing

$$\operatorname{sgn}(\sigma) \cong \frac{\sigma}{0.05+|\sigma|} \tag{41}$$

and this introduces a thin boundary layer as in the classical sliding mode control systems. One good observation is the fact that the allowable control signal limits, which are given in Table I, have never been violated after the initial transient. This should be considered as a prominent feature of the proposed scheme. The last row of Fig. 5 demonstrates the time derivative of the Lyapunov function in (25). The negativitiy of the function is maintained during the whole course of the simulation.

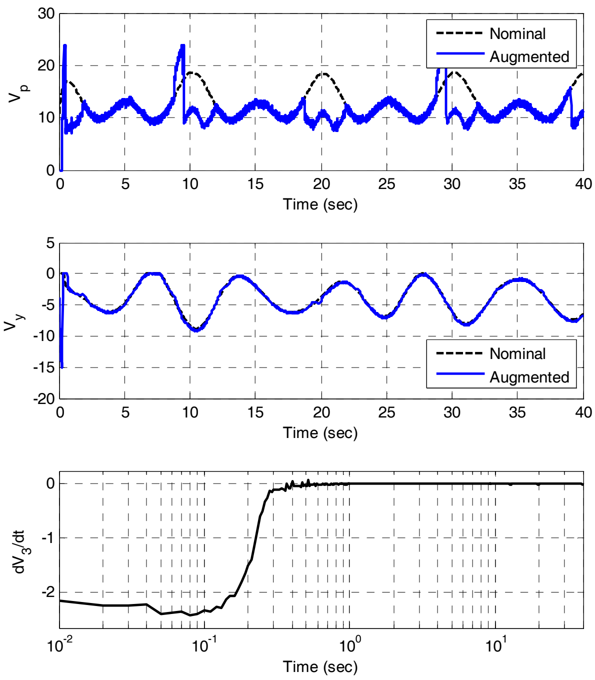

Fig. 5. Control signals and the time derivative of V_3

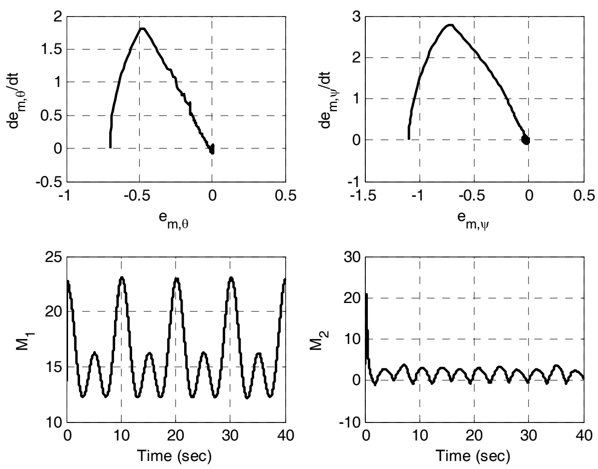

Fig. 6. Phase space behaviors and computed M values.

Lastly, we consider the phase space behavior and the produced M values as shown in Fig. 6. The top subplots illustrate the phase space trajectories in both variables and we see that the error values defined in (23) for θ and ψ axes are forced toward a line having slope –4, which corresponds to $\lambda = 4$. The errors eventually converge the origin and the uncertain plant is forced to follow the nominal system, which follows the reference signal.

The bottom row of Fig. 6 show the time evolution of the M gains generated to build the control signal in (21). Clearly the gains are bounded and periodic.

When the proposed scheme is turned off, the results are shown in Fig. 7, where we see that the backstepping based controller that is designed for the nominal system is unable to control the plant having uncertainties.

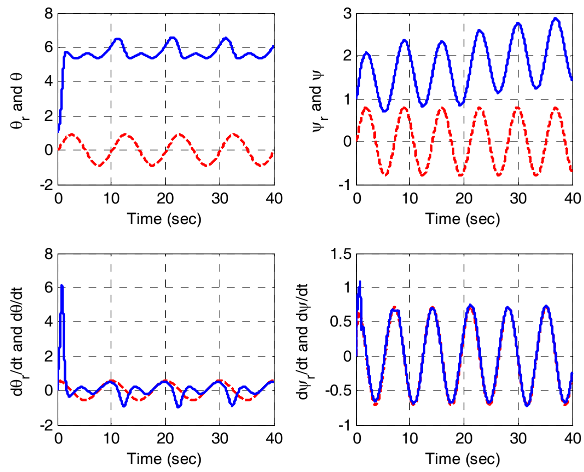

Fig. 7. The results without the proposed term. The controller is the one designed for the nominal system.

VI. Conclusions

This paper proposes a method for robust control of nonlinear and uncertain systems. The approach is based on the availability of a nominal system model and a stabilizing controller for the nominal system. For this pair, we considered backstepping design method to obtain the nominal control system.

The paper shows that the nominal controller is unable to meet the performance specifications when it is applied to the uncertain system. As a remedy to this, a discontinous term is introduced. The error between the nominal model and the uncertain system outputs are used to define a switching variable, which is forced toward zero and sliding mode characteristics are observed. The control signal to the uncertain system is the additive combination of the nominal signal and the derived discontinuous term.

The contribution of this work is to demonstrate that a discontinuous term can be introduced and its gain can be computed utilizing the available variables and a good tracking performance with no excessive control effort can be observed with mildly corrupted observations from the uncertain system outputs.

VII. ACKNOWLEDGEMENTS

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