

Adaptive Neural FOPID Controller Applied for Missile Guidance System

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Abstract—Adaptive Neural Fractional Order Proportional Integral Derivative (FOPID) controller is designed to control a missile using Proportional Navigation Guidance PNG system. The proposed FOPID controller is intended to improve the performance of PNG system in terms of miss distance accuracy and stability. A new tuning procedure has been proposed by applying hybrid neural genetic algorithm in order to align the missile attitude with the Line of Sight (LOS) angle. Genetic algorithm is used first in which it has fast convergence speed at the initial tuning stages, but near the global optimum values the tuning process becomes very slow, therefore neural technique is used to train a neural network which has faster tuning speed near the optimal values, and the tuning accuracy becomes much better. The tuning method has been compared with Ziegler–Nichols which is applied on PID controller and the results showed the superiority of the proposed tuning method. The need for fractional order type of feedback control system is justified by the nonlinear nature of the proportional navigation system and the dynamics of the missiles, to which many alternatives were applied in the literature using less accurate controllers while the proposed control system proved to have more accuracy hitting the target with less value of miss distance and more stability in the angle of attack and the motion during flight as well as the performance of the controller in terms of second and infinity norms.

Keywords—missile guidance; proportional navigation; fractional order; FOPID; Neural Tuning; miss distance.

I. INTRODUCTION

Missile's interception accuracy is considered to be one of the most critical control problems that researchers are trying to improve around the world. Scientists in the field of defense systems have put many performance tests for this purpose, one of the most significant performance tests that measures the accuracy of missiles is called miss distance estimation. Guo defined the miss distance as "the minimum distance between a guided flying object and its intended target site during their intersection" [1]. In literature there are many systems used for guiding missiles toward a specific target, Proportional Navigation Guidance PNG system is one of them that is used for directing a missile and keeping it on the right course during navigation in order to track a moving target until it hits that target. It has been confirmed by many researchers that PNG is the most commonly used method for missile guidance [2]. In the literature, there are many researches on applying controllers for the Proportional Navigation System. In [3], a flight path angle

guidance law is proposed for intercepting a highly maneuvering target by using Relative Impact Angle Control (RIAC), which is a modified version of the conventional optimal Impact Angle Control (IAC) that is used for non-maneuvering target, this relative impact angle indicates the clearness of viewing the target by the seeker and the proposed controlling law was tested and simulated for anti-air missiles, which is used against ballistic targets. In [4], the effect of noise on missile guidance system is investigated, and in order to improve the clearance of the signal they introduced the digital fading memory filters, which is considered to be more advanced than Kalman filters for such application. In [5], research on using the LOS angular rate as well as relative distance between missile and target in order to be used with missile guidance system is proposed by formulating nonlinear equations for PNG. In that work, the authors assumed that the target acceleration is unknown and it has to be estimated by a specialized observer, the stability of the control system has been proved by measuring the miss distance, which proved to have a miss distance value near zero. In [6], a solution for highly maneuvering target is proposed, in which the used missile is a short-range type, and the target is assumed to have the same maximum acceleration of the missile; by this study, an adaptive control procedure is applied to estimate tangential part of the target acceleration, the proposed guidance law has been compared against other guidance laws by applying and cancelling noise effects and the simulations produced more accurate results compared to other methods. In [7], a guidance law, which considers the dynamics of the control loop, is introduced for intercepting maneuvering targets, the fractional order showed better robustness and performance properties over the integer one. In that research, the autopilot dynamics have been considered as a first order transfer function because of its influence on the stability of the guidance law, and the effectiveness of this controller has been proved by simulating it against highly maneuvering targets. In [8], a new two-step strategy is introduced in order to deal with simultaneous cooperative attack, for that, a cooperative guidance procedure is used with more than one missile that allows them to interact with each other, after that, the missiles disconnect with each other and each one of them reach the target independent of each other. It is proved that by a communication network among the missiles, the missiles can cooperate with each other and attack a target autonomously. The simulation results showed the effectiveness of the proposed strategy. In [9], a new guidance law is designed in order to find acceleration requirements using true PNG, and

the effectiveness of the proposed law is proved by simulation results. Byungjun in [10] proposes the use of Proportional Integral Derivative (PID) controller in navigation guidance system in order to improve the accuracy of the PNG system in terms of miss distance, but the PNG system is a nonlinear control problem as stated by [11]. Integer order PID controller may not get satisfactory results in such an application while using Fractional Order Proportional Integral Derivative (FOPID) controller is proved to yield better results in the control of nonlinear systems. In this paper, a FOPID controller is designed and integrated into the PNG system in order to improve its accuracy in terms of miss distance. The FOPID controller is designed to minimize the error between LOS angle and the attitude of the missile, in which the attitude angle should be aligned with LOS angle in order to allow the missile to catch the target accurately.

The rest of the paper is organized as follows: Section 2 describes the concept of PNG system with some modifications, Section 3 shows the plant and missile dynamics and equation of motions, Section 4 describes the mathematical model of FOPID controller, Section 5 explains the procedure for applying and tuning the proposed controller, and finally Section 6 shows and compares the effectiveness of the proposed controller with the conventional one and the final section contains conclusions as well as suggestion for future work to be carried out.

II. PROPORTIONAL NAVIGATION GUIDANCE SYSTEM

The main concept of proportional navigation is about finding a normal acceleration (a_m) which is proportional to the deviation of the missile from the LOS, that is, the missile will be directed toward the target, in other words, the missile should always be on the LOS and the rotation rate of the missile and LOS are the same. Fig. 1. shows the kinematic model in which the missile is positioned away from the LOS. In order to align the missile with the LOS, a normal acceleration a_m should be applied on the missile, this normal acceleration is proportional to the distance between the missile and the LOS.

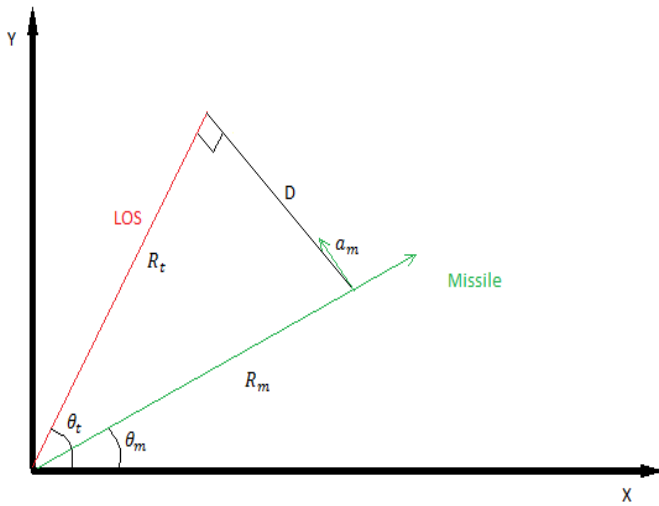


Fig. 1. Kinematic model for missile positioned away from LOS.

The value of normal acceleration a_m could be found using (1) and (2) as follows:

$$a_m = KD \quad (1)$$

$$a_m = KR_m \sin(\theta_t - \theta_m) \quad (2)$$

where K is constant, D is the deviation of the missile from LOS, θ_m is the angle between the missile and the reference axis, θ_t is the angle between the target and the reference axis.

Fig. 2 shows kinematic model for missile and target with respect to a stationary tracker.

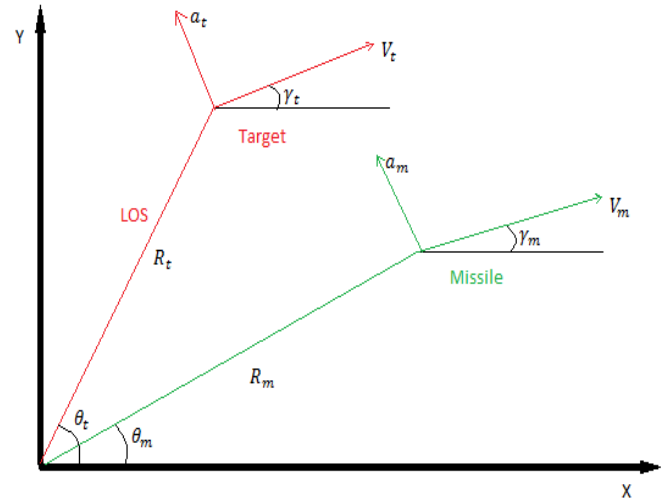


Fig. 2. Kinematic model for missile and target with stationary tracker.

The kinematic equations for the missile are expressed as follows:

$$\frac{dR_m}{dt} = V_m \cos(\gamma_m - \theta_m) \quad (3)$$

$$R_m \left(\frac{d\theta_m}{dt} \right) = V_m \sin(\gamma_m - \theta_m) \quad (4)$$

$$V_m \left(\frac{d\gamma_m}{dt} \right) = a_m \quad (5)$$

where R_m is the distance between missile and tracker, V_m is the speed of the missile, θ_m is the angle between the missile and the reference axis, γ_m is the angle between moving direction of the missile and the reference axis, a_m is the normal acceleration applied on the missile. The kinematic equations for target are expressed as follows:

$$\frac{dR_t}{dt} = V_t \cos(\gamma_t - \theta_t) \quad (6)$$

$$R_t \left(\frac{d\theta_t}{dt} \right) = V_t \sin(\gamma_t - \theta_t) \quad (7)$$

$$V_t \left(\frac{d\gamma_t}{dt} \right) = a_t \quad (8)$$

Where R_t is the distance between target and tracker, V_t is the speed of the target, θ_t is the angle between the target and the reference axis, γ_t is the angle between moving direction of the target and the reference axis, a_t is the normal acceleration applied on the target.

In order to keep the missile aligned with LOS, the normal acceleration a_m could be applied on the missile as shown in the following equations:

$$a_m = (d(R_m(\frac{d(\theta_t - \theta_m)}{dt}))/dt) + (\frac{dR_m}{dt})(\frac{d(\theta_t - \theta_m)}{dt}) \quad (9)$$

$$a_m = R_m \left(\frac{d^2(\theta_t - \theta_m)}{dt^2} \right) + \left(\frac{dR_m}{dt} \right) \left(\frac{d(\theta_t - \theta_m)}{dt} \right) + \left(\frac{dR_m}{dt} \right) \left(\frac{d(\theta_t - \theta_m)}{dt} \right) \quad (10)$$

$$a_m = R_m \left(\frac{d^2(\theta_t - \theta_m)}{dt^2} \right) + 2 \left(\frac{dR_m}{dt} \right) \left(\frac{d(\theta_t - \theta_m)}{dt} \right) \quad (11)$$

By substituting $V_m = \frac{dR_m}{dt}$ in (11) we get,

$$a_m = 2V_m \left(\frac{d(\theta_t - \theta_m)}{dt} \right) + R_m \left(\frac{d^2(\theta_t - \theta_m)}{dt^2} \right) \quad (12)$$

III. MISSILE DYNAMICS AND EQUATIONS OF MOTION

The plant dynamics and equations of motions are implemented in Simulink in order to generate the motion and position of the missile, the following equations are used first to find the forces and moment applied on the missile:

$$F_x = C_x(0.5\rho V^2 S_{ref}) \quad (13)$$

$$F_z = C_z(0.5\rho V^2 S_{ref}) \quad (14)$$

$$M = (C_m + q)(0.5\rho V^2 S_{ref} * D_{ref}) \quad (15)$$

Where F_x is the force applied on the missile in the x-axis, F_z is the force applied on the missile in the z-axis, V is the velocity of the missile, M is the pitch moment of the missile, C_x and C_z are coefficients stored in a lookup table and its relative to the missile's velocity and pitch angle, D_{ref} is the reference circular body diameter of the missile, D_{ref} is the reference cross-sectional area of the missile. After that, the values of the forces and moment are used in 3DOF block in Simulink in order to generate the position and movement of the missile according to the following equations:

$$A_x = \frac{F_x}{m} - qv_x - g\sin(\theta) \quad (16)$$

$$A_z = \frac{F_z}{m} - qv_z - g\sin(\theta) \quad (17)$$

$$\dot{q} = \frac{M}{I} \quad (18)$$

$$\dot{\theta} = q \quad (19)$$

Where A_x is the acceleration of the missile in the x-axis, A_z is the acceleration of the missile in the z-axis, θ is the missile attitude, q is the missile rotation rate, m is the mass of the missile, v_x is the velocity of the missile relative to the x-axis of the missile's body, v_z is the velocity of the missile relative to the z-axis of the missile's body, I is the inertia of the missile and g is the gravity force.

IV. FRACTIONAL ORDER PID CONTROLLER

Fractional calculus is a branch of mathematics that demonstrates the definition and approximation of integration and derivation with non-integer orders. Proportional Integral Derivative (PID) controller is one of the most used controllers in industry due to the existence of many tuning methods for its parameters, and it is easy to integrate this controller in a wide range of industrial applications. FOPID controller is considered as a generalization of the integer order PID controller, this controller is referred to as (PI $^\lambda$ D $^\mu$). The transfer function of FOPID controller can be expressed as the following:

$$C(s) = \frac{D(s)}{U(s)} = k_p + \frac{k_i}{s^\lambda} + k_d s^\mu \quad (20)$$

where λ and μ are the integration and derivative orders which are both positive real numbers. The existence of fractional calculus dates back to 1695 when Leibniz received a letter from L'Hôpital asking about the definition of the derivation with fractional order of 1/2. But the real presence of fractional order calculus has begun just few decades ago in some automatic control applications. In fractional calculus. In the literature, the operator ${}_a D_t^q$ is referred to as differintegration and it has the following definition:

$${}_a D_t^q \begin{cases} \frac{d^q}{dt^q}, & q > 0 \\ 1, & q = 0 \\ \int_a^t (d\tau)^{-q}, & q < 0 \end{cases} \quad (21)$$

In theory, there are several definitions for this operator, and the most famous of them are Riemann–Liouville and Caputo definitions, which are given as follows:

$$D^\alpha y = y^{(\alpha)} := \frac{1}{\Gamma(r-\alpha)} \left(\frac{d}{dt} \right)^r \int_0^t \frac{y(\xi)}{(t-\xi)^{\alpha+1-r}} d\xi \quad (22)$$

$$D^\alpha y = y^{(\alpha)} := \frac{1}{\Gamma(r-\alpha)} \int_0^t \frac{y^{(r)}(\xi)}{(t-\xi)^{\alpha+1-r}} d\xi \quad (23)$$

The Riemann–Liouville technique is not suitable to deal with Laplace transform because it needs to know the non-integer order derivative at ($t = 0$), while this problem is solved in Caputo approximation technique. The Laplace transform could be used to find fractional order transfer functions as $L(D^\beta) = s^{-\beta}$, where L is the expression for the Laplace transform operation while s is the Laplace variable [12]. There are also many other approximation techniques exists in the literature, one of them is Oustaloup's approximation which is known for its accuracy, and this method has been used in order to design our FOPID controller as explained in the next section. In many applications it's recommended to use FOPID controller instead of integer order PID due to the superiority of FOPID in performance, stability and robustness [13]. FOPID also handles the variations in gains and uncertainties that can be produced by the plant model, it can also handle load disturbances produced by the plant model. FOPID also has better ability to reject the noises

that affects the system [14]. FOPID also shows better performance than integer order PID in dealing with systems that has time delay. Moreover, when dealing with nonlinear systems, the system is usually linearized at different points, therefore for each point a specific PID controller is assigned to work at that point, while usually using one FOPID is sufficient for such situation [15].

V. CONTROLLER DESIGN AND TUNING

In this research, a system of Neural Fractional Order Proportional Integral Derivative controller is designed and integrated into PNG system, the proposed controller has been simulated and compared with the conventional PID controller as shown in Fig. 3.

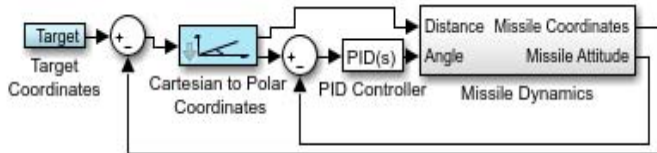


Fig. 3. Conventional PNG System.

The tuning process of the conventional controller is done using Ziegler–Nichols tuning method by obtaining a linearized approximation of the plant at a suitable operating point, and then the controller parameters using Ziegler-Nichols tuning method has been obtained as shown in Table I.

TABLE I. PID Controller Parameters Obtained by Ziegler-Nichols Method.

Parameter	Value
K_i	1.6796876
K_p	0.1075
K_d	0.0016555

The proposed Neural FOPID is designed in MATLAB using FOMCON toolbox, which is based on Oustaloup's approximation. By this method a fractional-order operator (s^γ) for $0 < \gamma < 1$, can be approximated using the following equations:

$$s^\gamma \approx \omega_h^\gamma \prod_{k=-N}^N \frac{s + \omega'_k}{s + \omega_k} \quad (24)$$

$$\omega'_k = \omega_b \left(\frac{\omega_h}{\omega_b} \right)^{\frac{k+N+1/(1-\gamma)}{2N+1}} \quad (25)$$

$$\omega_k = \omega_b \left(\frac{\omega_h}{\omega_b} \right)^{\frac{k+N+1/(1+\gamma)}{2N+1}} \quad (26)$$

where N is the order of approximation within the frequency range between ω_b and ω_h .

In order to have a good controlling performance, the accuracy of the controller is dependent on the order of approximation N, and the angular frequency range ω_b and ω_h . A higher value of approximation order N results in higher approximation quality

for the same frequency range, and the frequency range between ω_b and ω_h should be chosen to include the operating frequencies of the plant. Here in our controller the default value of N was 5 and the frequency range was between [0.001 and 1000], and the experiment results shows good performance for these default values. This controller has 5 parameters to be tuned (K_i , K_p , K_d , λ and μ) as shown in Fig. 4.

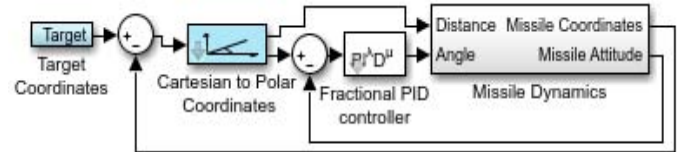


Fig. 4. Neural Fractional Order PID Guidance System.

The tuning process is done using genetic algorithm, which is considered to be fast at initial stages, but near the optimal values the genetic algorithm is becoming slow, therefore the genetic algorithm will work to achieve an initial miss distance value of less than 0.1, which could be done in seconds by genetic algorithm, in the simulation results, the miss distance of value less than 0.1 occurred after the 7th generation. The parameters of the controller that are generated using genetic algorithm is shown in Table II.

TABLE II. Tuned Parameters After Applying Genetic Algorithm.

Parameter	Value
K_i	0.2046
K_p	0.7094
K_d	0.2048
λ	0.4165
μ	0.1531

By using these parameters in the neural FOPID controller, the resulting miss distance was observed as 0.0168, which is much better than using the conventional PID, which has a miss distance of 25.48. After that, the genetic algorithm will stop and the neural technique starts to search for more accurate results compared to the genetic algorithm. The training process of the neural network is much faster than using genetic algorithm. The proposed neural tuning technique is started by taking training samples, which are the combination of the FOPID controller parameters around the values generated by the genetic algorithm. The inputs for these samples are the FOPID controller parameters (K_i , K_p , K_d , λ and μ), and the output is the miss distance value. The training process is started by reversing the inputs with the outputs in order to train the network to find the relative parameters for each miss distance value as shown in Fig. 5. The initial weights and biases are generated using Nguyen-Widrow algorithm, which has its advantages such as speeding the learning process as well as making use of all the neurons in the network. Nguyen-Widrow algorithm has some random element which results in different initial weight each time, but the final measured miss distance is slightly different in each experiment and all of them has good final result after training the network. The training process is done by entering a small miss distance value (near to zero), and then the neural

network will guess the relative controller parameters corresponding to that miss distance value.

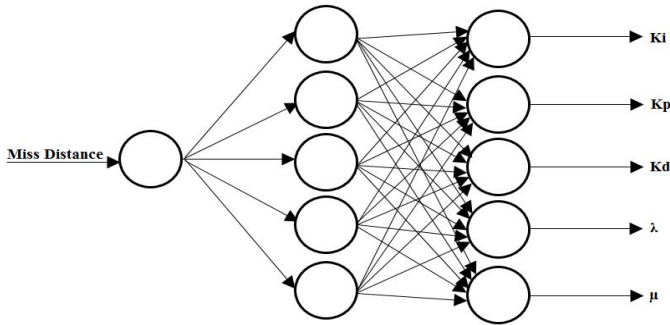


Fig. 5. Proposed Neural Tuning Technique for FOPID Parameters.

The resulting parameters using neural network are listed in Table III.

TABLE III. Tuned Parameters Generated by the Neural Network.

Parameter	Value
K_i	0.0112
K_p	0.9368
K_d	0.0419
λ	0.1738
μ	0.0588

By applying these parameters to the neural FOPID controller, the resulted miss distance was 0.0047, which is a significant improvement of the genetic algorithm technique as well as the conventional controlling system.

VI. PERFORMANCE ANALYSIS

A. Missile Trajectory

Fig. 6 shows the missile and target trajectories using the conventional PID controller, as shown in the figure, the course of the missile is changing and oscillating during flight time while tracking the target, but using the Neural-FOPID controller, the missile was stable with linear course as shown in Fig. 7.

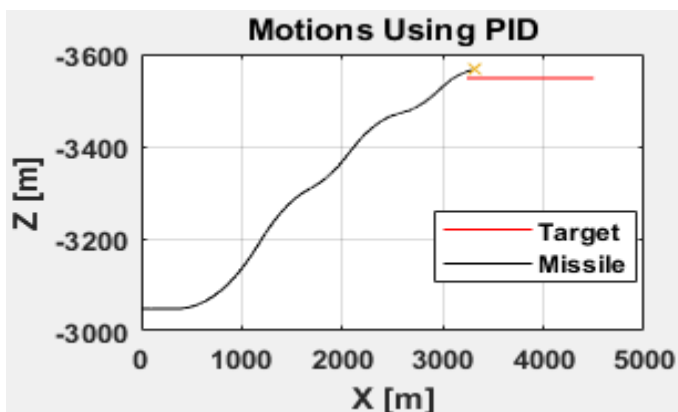


Fig. 6. Missile and target trajectories using conventional PID controller.

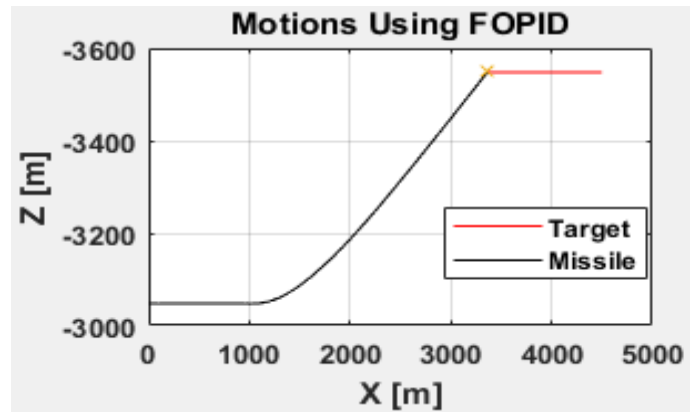


Fig. 7. Missile and target trajectories using Neural-FOPID controller.

B. Angle of Attack

Fig. 8 shows the angle of attack for the conventional PID controller. By this figure, the change in the angle is unstable and it's oscillating around the line of sight (LOS) between the missile and the target.

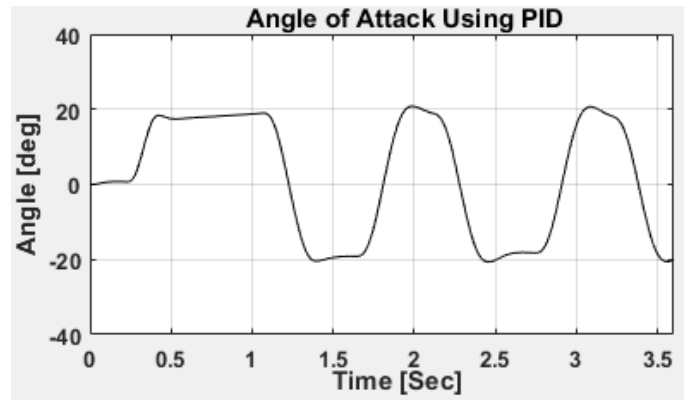


Fig. 8. Angle of attack for the missile using conventional PID controller.

Fig. 9 shows the angle of attack for the proposed Neural-FOPID control system. With this controller, the angle of attack is stable and it has taken its highest value at the moment when the target has been found, then it has started to decrease until the missile intercepts the target.

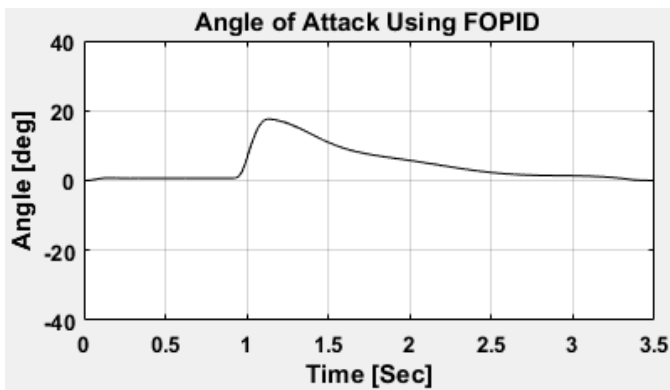


Fig. 9. Angle of attack for the missile using Neural-FOPID controller.

C. Miss Distance

The proposed controller has been tuned and the miss distance value has been compared with the conventional PNG system. The miss distance value for the conventional PNG system is 25.48. The tuning process of the Neural-FOPID controller is started by applying genetic algorithm, which is faster in getting near optimal values at initial stages compared to neural network techniques. In this stage, using the resulting controller parameter values, the miss distance was 0.0168. The second stage is accomplished using neural networks. The training process was fast and more accurate than genetic algorithm. The final miss distance after this stage is observed as 0.0047. These values demonstrate the accuracy of the proposed controller over the conventional one in terms of miss distance.

D. H_2 -Norm and H_∞ -Norm

The H_2 -Norm is defined as the energy of the output of the controller and it's used to stabilize the system by the indication of the overall performance, while the H_∞ -norm is the maximal possible amplification in the output of the controller and it's used in control systems in order to stabilize the system with guaranteed performance for all frequency values. The second norm of the conventional PID controller has been calculated and it was 24.1781 while it was 4.1760 using Neural-FOPID control system. The infinity-norm of the PID controller was 1.1058 while using Neural-FOPID controller, the infinity-norm was 0.1921. These values indicate the performance and stability of the Neural-FOPID controller which outperform the conventional PID tuned by Ziegler–Nichols method.

CONCLUSION

In this paper, a neural FOPID controller has been presented and applied for the problem of missile guidance. The goal is to align the line of sight angle with missile attitude angle. The controller was tuned in two stages. The first stage employs a genetic algorithm, which has a quick convergence at initial stages, and slow convergence near optimal values. Then neural network approach has been used, which is more accurate and has a fast training near the optimal values. The controller has been simulated and tested against the conventional PID controller

tuned using Ziegler–Nichols method. The simulation results illustrated that the proposed controller outperforms the conventional controller in terms of stability and miss distance accuracy. Using genetic algorithm, the proposed controller reached a value of 0.0168 for miss distance, after using neural networks techniques, the controller reached a value of 0.0047. Comparing these results with a miss distance value of 25.48 yielded by the conventional PID controller, we can confirm that the proposed controller has much better accuracy than the conventional one. These controllers also have been compared in terms of the overall missile course, the angle of attack, H_2 -Norm and H_∞ -Norm and all of the simulations results showed that Neural-FOPID controller outperform the conventional one. These results could provide the basis for future work by applying the Neural-FOPID controller to deal with highly maneuvering targets, as well as working in noisy conditions which FOPID can show higher robustness compared to many other types of controllers.

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