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Intelligent mixed H_2/H_∞ FOPID controller optimized for radar guided missile

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***Abstract*—A mixed weighted H_2/H_∞ tuning algorithm for fractional order PID controller is introduced and optimized for missile guidance system. The missile dynamics are highly nonlinear and the performance of the controller is usually degraded near the impact region which could cause the missile to miss the target. Therefore, a new tuning strategy is introduced to put weights for H_2/H_∞ which depends on the range between the missile and the target. The introduced tuning procedure makes the controller more optimized in tracking the target near the impact region and eventually hit the target with more accuracy and less miss distance. Genetic algorithm is used to search the space for minimum H_2/H_∞ value especially near the impact region. The control system is compared with Zeigler-Nichols tuning method applied on the standard PID controller, and the simulation results demonstrated the accuracy and superiority of the proposed controller over the conventional one.**

***Keywords*— $H_2/H\infty$, intelligent control, genetic algorithm, GA, FOPID, fractional order, PID, missile guidance, radar guided missile.**

I. INTRODUCTION

In this research, a mixed H_2/H_∞ tuning method is introduced and optimized for radar guided missiles. Proportional navigation guidance system is the most used system for guiding the missile and tracking moving targets. One of the most interesting problems found in these systems is the near impact controlling problem, which causes the missile to miss its target. In the literature, there are a lot of solutions for this problem, one of them is to use PID controller, which improved the controlling performance near the impact region, yet the dynamics of the missiles are highly non-linear, and using PID controller for nonlinear systems needs to linearize the system at a set of operating points, and customize a specialized PID controller for each of these points. Fractional order proportional integral derivative controller (FOPID) proved to have better controlling performance for such situations, especially when the plant is nonlinear, and using one FOPID is usually sufficient for such kind of problems, [1]. Tuning controllers for tracking missions such as missile guidance or UAV control, could have the following aspects: either the controller will be optimized for smooth tracking, which will degrade its agility, or the controller will be optimized for fast and better agility but with oscillations, [2]. Smooth tracking is preferred during flight time, which will help the missile to reach the target faster with least distance. Optimizing for better agility will help the controlled object to have better performance in tracking fast maneuvering targets and have better performance near the impact region. Mixed H_2/H_∞ based tuning method is getting very high interest in control systems due to its prominent features on the closed loop performance. H_2 is based on averaging and finding the overall controlling performance of the system, while H_∞ is about finding a guaranteed performance value for the entire set of uncertainties available in the system. In the current problem setting, applying H_2 or H_∞ will not focus or solve the near impact controlling problem, therefore in this research a new tuning procedure based on producing range dependent weights, which will increase the weights applied on the mixed H_2/H_∞ as long as the missile gets closer to the target. By that the controller will be tuned for the whole flight path, while requiring more optimization for the controller near the impact region. In the literature, there are many research outcomes that optimize the mixed H_2/H_∞ tuning procedure. In [3], a mixed H_2/H_∞ robust tuning method is applied on a feedback controller is introduced for controlling a quadrotor UAV. The measured controlling signals are exposed to noises and error sources. Linear Matrix Inequalities (LMI) is used to optimize the mixed $H_2/H\infty$ control procedure since the system is based on multi-objective convex problem. The control method proved to have good robustness in the presence of noised and disturbances. In [4], an intelligent adaptive control scheme based on H_2/H_∞ optimization for controlling a two-axis motion. A three set of controllers are introduced which are a self-organizing recurrent fuzzy-wavelet-neural-network controller (SORFWNNC) an H_2/H_∞ and a third robust controller. The SORFWNNC is used as the main controller to estimate the dynamical parameters alongside with noise and disturbances, the H_2/H_∞ is used to minimize the quadratic error and the robust controller is designed to deal with the approximation error. The experiment results showed the effectiveness of the controller in tracking the reference in the presence of noises and external disturbances. In [5], the H_2/H_∞ mixed control method is applied on 3-DOF helicopter. The H_2/H_∞ sub optimal control method is used with multiple-weighted control method to adapt the constantly changing elevation angle. By applying these two control methods, the dynamic performance and the control precision are both proved to be improved. In [6], particle swarm optimization (PSO) technique is used to tune a PID controller applied to a UAV quadcopter. The effort of the controlling system is tested and compared with and without disturbances. The results showed the accuracy and error minimization of the control system during flight time. In [7] the well-known gradient decent methodology is used to optimize the parameters of a PID controller online, which is applied on drone quadrotor. The presented have been tested with waypoint navigation, and with leader follower formation control. In [8], the author replaced the proportional navigation guidance (PNG) in missile guidance system with a PID controller, and improved the miss distance accuracy alongside with the finite time stability.

The rest of the paper is organized as follows: the second section introduces the principles of fractional calculus. The third section shows the conventional and the proposed controller with the tuning process. The fourth section presents the simulation results and comparison for both control systems. Concluding remarks with suggested future work are given in the last part of the paper.

II. FRACTIONAL CALCULUS

It is known that the history of fractional calculus started at 1695 when G. de L'Hospital asked G. Leibniz about the meaning of a derivative with fractional order e.g. $d^{1/2}/dx^{1/2}$, and G. Leibniz answered "it will lead to a paradox, from which one day useful consequences will be drawn" [9]. After that, a number of mathematicians started to investigate on this filed, such as Euler and Liouville, who brought important ideas to this field. However, the work that has been done to the fractional calculus field is infrequent until recently where the theory of chaos and fractals showed the relationships between the fractional order derivatives and integrals, [10].

A. Definitions

Fractional calculus is a generalization of integer order calculus in which a new operator has been introduced. This new operator is called differintegration and denoted by ${}_aD_t^\alpha$ which allows the integration or derivation to be presented with fractional order. In this representation, α is the order of differintegration, t is the variable over which the differintagration is operated, and a is the initial value. The definition of this operator with respect to integration or derivation is shown in the following equation:

$$ {}_aD_t^\alpha = \begin{cases} \frac{d}{dt^\alpha}, \alpha > 1 \\ 1, \alpha = 1 \\ \int_a^t (d\tau)^\alpha, \alpha < 1 \end{cases} \quad (1) $$

There are several definitions presented in the literature in the purpose of approximating the fractional order operator, three of them have been widely used until today by many researchers around the world. The first one is the approximation presented by Riemann-Liouville (RL) as given below:

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-a)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (2) $$

Another one, which is a discrete time approximation to differintegration operator, that is commonly used is presented by Grünwald-Letnikov (GL) as shown in the following equation:

$$ {}_aD_t^\alpha f(t) = \lim_{h\to 0} \left(\frac{1}{h^\alpha}\right) \sum_{m=0}^{\left[\frac{t-a}{h}\right]} (-1)^m \frac{1^\eta(\alpha+1)}{m!\Gamma(\alpha-m+1)} f(t-mh) \quad (3) $$

While both (RL) and (GL) received good acceptance but recently the third one (Caputo definition) is considered to be better because it is more easy to be integrated into engineering applications with its initial conditions that are of the integer orders, [10]. The Caputo approximation is defined by the following equation:

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-a)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (4) $$

In control systems, the standard PID controller can be generalized to fractional order PID by using integration and differentiation with fractional orders as shown in the following equation:

$$ C(s) = \frac{D(s)}{U(s)} = k_p + \frac{k_i}{s^\lambda} + k_d s^\mu \quad (5) $$

where λ and μ is the order of integration and derivation respectively. Contrary to classical PID controller design problems, fractional order case generalizes the parameter space and that is five dimensional.

III. DESIGNING AND TUNING THE CONTROLLER

In this research, a fractional order proportional integral derivative control system is designed and tuned using weighted mixed H_2/H_∞ with miss distance constraint. The intelligent tuning procedure is performed using Genetic Algorithm (GA) in order to obtain the controller parameters, which lead to minimum H_2/H_∞ and miss distance values.

The proposed controller is tested and compared against the standard PID controller, which is tuned by Zeigler-Nichols tuning method, which is commonly accepted as a tuning scheme for PID based control systems.

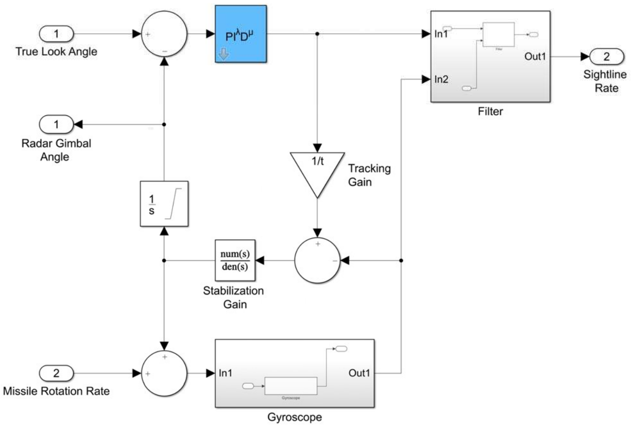

Fig. 1. The radar control system.

A. *The Conventional PID Controller*

The conventional PID controller is designed based on Zeigler-Nichols tuning method. The tuning process is done by obtaining the open loop response of the system between the True Look Angle as an input, and the Gimbal Angle of the radar as an output. Then the transfer function (TF) for the radar guided missile is computed which is shown in the following equation:

$$T(s) = \frac{2513s^2+2.211\times10^6 s+9.922\times10^8}{s^4+879.6s^3+3.973\times10^5 s^2+5.187\times10^7 s+1.012\times10^9} \qquad (6)$$

Then the PID parameters are calculated from the transfer function using Zeigler-Nichols method. The calculated values for the PID parameters are shown in Table I.

TABLE I. PID Controller Parameters Obtained by Ziegler-Nichols Method

Parameter	PID Controller
K_p	78.6901
T_i	0.0050
T_d	0.0012

B. *The Proposed FOPID controller*

The proposed control system is based on Fractional Order PID control system. The controller was simulated on Matlab using FOMCON toolbox, [11]. The approximation process of the fractional order operator is done using Oustaloup's approximation. Fractional order operator (s^γ) can be approximated by Oustaloup's approximation using the following equations:

$$s^\gamma \approx \omega_h^\gamma \prod_{k=-N}^{N} \frac{s+\omega'_k}{s+\omega_k} \qquad (7)$$

$$\omega'_k = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+1/(1-\gamma)}{2N+1}} \qquad (8)$$

$$\omega_k = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+1/(1+\gamma)}{2N+1}} \qquad (9)$$

where ω_b and ω_h are the angular frequency ranges and N is the approximation order. The approximation quality can be increased by increasing the approximation order N, but in the cost of computation time. The values of ω_b and ω_h should be chosen to contain all the possible angular frequencies that could be generated by the plant. The default values for these values in FOMCON Toolbox were between 0.001 and 1000, and the approximation order N was 5, which showed good results through the simulation. Genetic Algorithm (GA) is used to tune the FOPID parameters (K_p, K_i, K_d, λ and μ). The fitness function of the GA is optimized to produce the minimum weighted mixed H_2/H_∞. As a result of this process, the error vector of the feedback process will be divided by the range, which will give high weights for the small error values. Based on that, the controller will be tuned and optimized for the region near the target. The generated parameters for the controller is shown in Table II.

TABLE II. FOPID Parameters using mixed H_2/H_∞ tuning method

Parameter	FOPID Controller
K_i	0.0519
K_p	0.7597
K_d	0.2701
λ	0.4388
μ	0.0962

IV. SIMULATION RESULTS

The proposed control system is simulated and compared against the conventional PID controller in terms of Miss Distance, Incidence Angle, Trajectories motion, acceleration demands and the error between the true look angle and the gimbal angle.

A. *Miss Distance and H_2/H_∞*

Miss Distance (MD) is the least distance between the missile and the target during the flight time. The miss distance for the conventional PID controller was 6.278 m which indicates that the missile will miss the target. The other performance metrics were: H_∞= 0.7973 & H_2= 11.2946. But using the proposed FOPID control system, the performance metrics were: MD= 0.0024. H_∞= 0.6005, H_2= 0.0015. According to these results, the accuracy of the proposed FOPID controller in hitting the target over the conventional one is proved.

B. *Trajectories of Missile and Target*

Fig. 2. Shows the trajectories of missile and target for the conventional system. It can be seen from the figure that the path

that the missile tries to follow is not straight especially near the impact region where it has some oscillation.

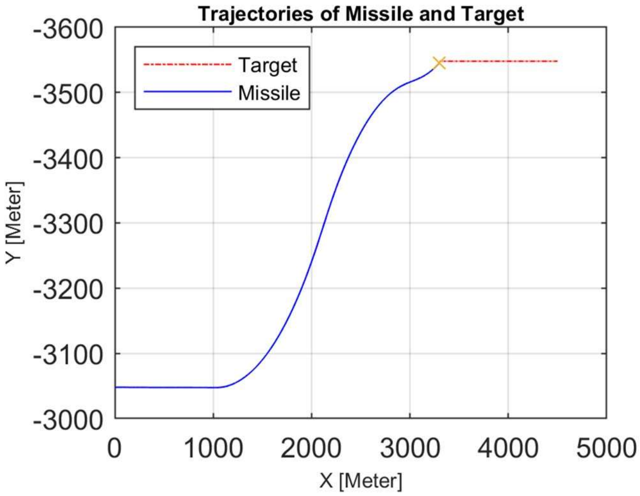

Fig. 2. Trajectories of Missile and Target for PID.

Fig. 3. shows the Trajectories for the proposed FOPID control system. It's obvious that the path the missile follows is more straight and its more stable especially near the impact region compared with the conventional controller.

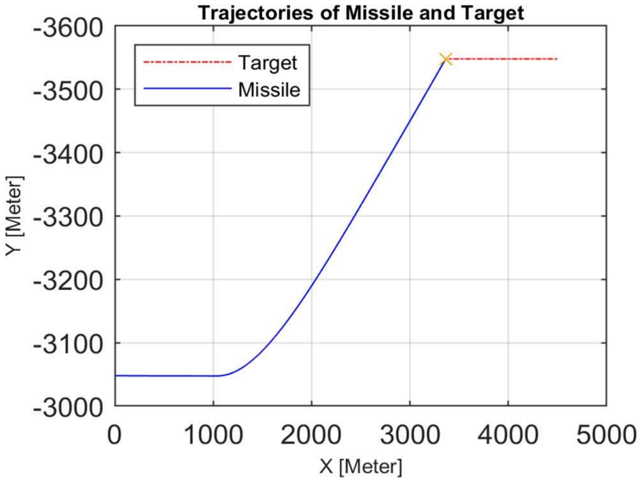

Fig. 3. Trajectories of Missile and Target for FOPID.

C. *Incidence Angle*

Incidence angle or angle of attack is the angle between the reference line on the missile body and the motion direction of the missile. As seen in Fig. 4. The target has been found by the radar at about 0.7 second and the missile started to change its incidence angle toward the target at about 0.9 second. The incidence angle for the conventional system is oscillating and it is not stable during flight time, while it is more smooth and stable for the proposed FOPID control system.

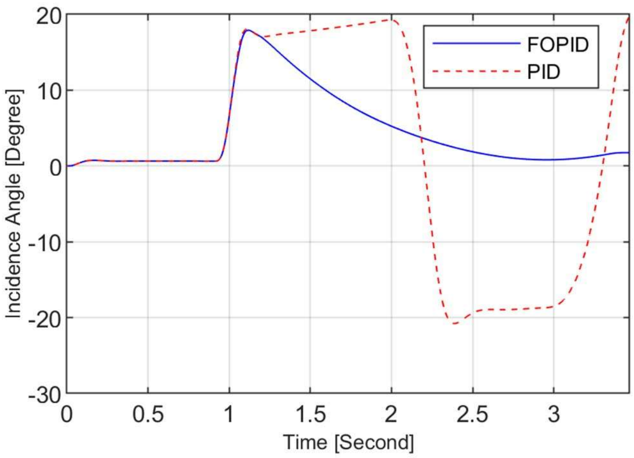

Fig. 4. Incidence Angle for PID and FOPID.

D. *Acceleration Demands*

Acceleration Demand is the normal acceleration needed to be produced on the missile body in order to rotate the missile to align the missile with the Line of Sight, which will lead the missile towards the target and eventually collide with it. The less the acceleration demands, the less energy the missile needs to be rotated towards the target. It is seen from Fig. 5 that the radar gimbals are oscillating away from the true look angle, before it could align with it, also the true look angle is oscillating due to the missiles body oscillation using the conventional controller.

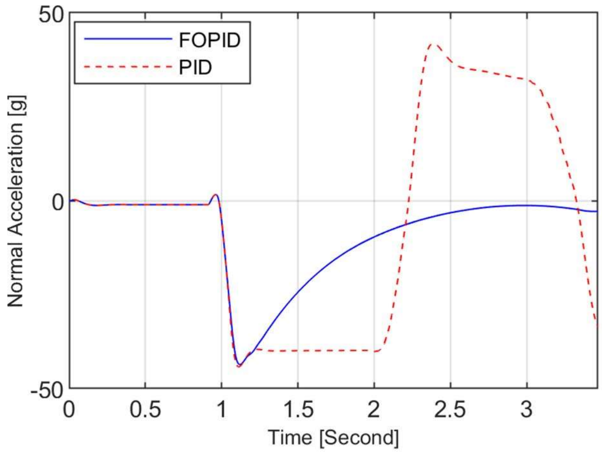

Fig. 5. Normal Acceleration for PID and FOPID.

E. *True Look and Gimbal Angles*

True look angle is the angle between a reference axis on the missile body and the target, while the gimbal angle, is the angle between that reference and radar direction. The radar direction (gimbal angle) should be aligned with the true look angle, which will eventually lead the missile towards the target. Fig. 6 shows that using the conventional controller, the gimbal angle had many oscillations before it could align with the true look angle. Even the true look angle is not stable missile body oscillation that cause the true look angle to be changed.

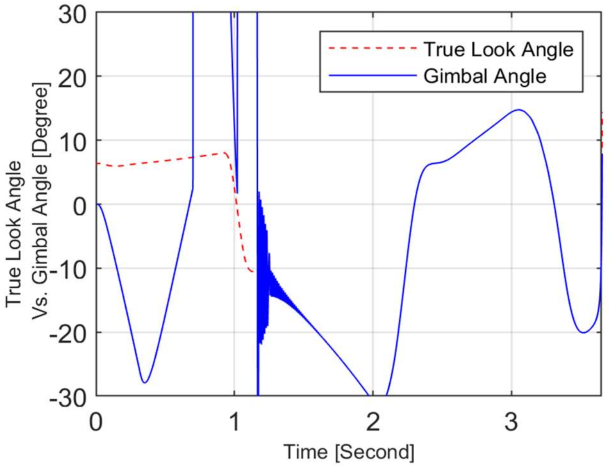

Fig. 6. True Look Angle Vs. Gimbal Angle for PID.

Fig. 7 shows the performance of the proposed FOPID system. By the figure, after the radar found the target at about 0.7 second, the gimbal angle tracked the true look angle smoothly and efficiently without oscillations during the whole flight time.

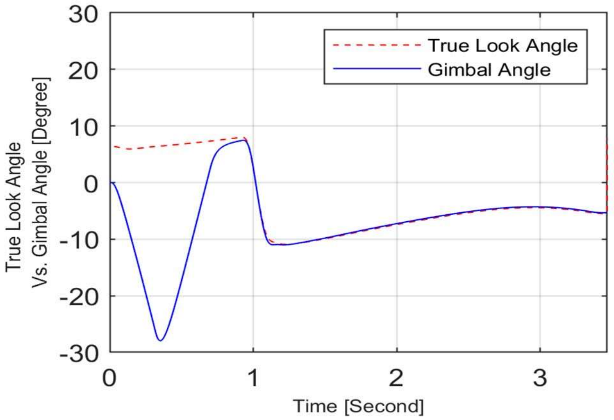

Fig. 7. True Look Angle versus Gimbal Angle for FOPID.

V. CONCLUSION AND FUTURE WORK

In this paper, an intelligent weighted H_2/H_∞ tuning method is introduced for tuning fractional order PID controller. The weighted method for H_2/H_∞ is intended for solving the near impact control problem where the control performance becomes unstable near the impact region. Genetic algorithm is used for finding the best combination of the FOPID parameters that makes the miss distance and H_2 value as small as possible with H_∞ constraint. The proposed controller is compared against the standard PID control system tuned by Zeigler-Nichols tuning method in terms of miss distance, H_2/H_∞, true look angle vs. gimbal angle, incidence Angle, missile and target trajectories and Acceleration demands. The simulation results showed the superiority of the proposed FOPID system in guiding the missile accurately toward the target with minimum miss distance.

For future work, sliding mode FOPID controller could be used which is supposed to produce promising results during flight time especially when the air density changes during flight and when the aerodynamic laws change when the missile crosses different atmosphere layers.

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