Fractional Order PID Control of a Radar Guided Missile Under Disturbances

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Abstract—**This research demonstrates the use of Fractional Order Proportional Integral Derivative (FOPID) system for radar guided missile in order to accurately guide the missile toward its target in the presence of multi noise and error sources such as system delay, radome aberration error and receiver noise. The proposed system is intended to filter out the noises and increase the accuracy of the missile during its flight until it hits its target with minimum miss distance. Genetic Algorithm is used to tune the parameters of the proposed FOPID system. The simulation results demonstrate the effectiveness of the proposed system compared to the conventional system in guiding the missile accurately in the presence receiver noise, system delay and radome aberration error.**

Keywords—FOPID; fractional order; missile guidance; control; miss distance; radar; noise; radome aberration; system delay; receiver noise.

I. INTRODUCTION

Accurate missile targeting is one of the most interesting research topics studied many times in the past. Miss Distance (MD) is considered to be one of the fundamental performance metrics to measure the missile accuracy. However, accuracy based on MD can be deteriorated by many factors. Some of these factors are related to communication issues as investigated in this study. In order to overcome the problems arising during the communication side, a number of methods have been proposed and applied in the literature. In most of the methods applied, noise filtering is adopted as a solution to avoid the adverse effects of disturbances. Using filtering approach for radar signals is an alternative especially when there are delays and refraction based false alarm situations. Using special sorts of controllers could decrease the effect of the noise, increase the tracking ability of the radar system as well as increasing the tracking performance. This leads to better hitting accuracy and lower MD value. In the literature, there are several research reports on this, such as, in [1], the author presented an analysis of three error factors that affects accuracy of the missile seeker and the mathematical model of the seeker. The parameters that have been investigated by this research are information latency, proportional guidance coeffecient deviation and measurement noise. The simulation results demonstrated that the effects of these parameters on the performance of the radar tracking accuracy is substantial. In [2], the author presented comparisons between three guidance systems, namely, BP, VP and PN in term of MD. The comparison has been made when these guidance systems are subjected to error factors such as angular noise, target glint and heading error. The author also analysed the models of these systems and the effects of these error factors on MD of the missile. The simulation results showed that the BP and VP systems have lower MD than PN only when the system is affected by angular noise and target glint. In [3], the glint noise has been considered as one of the main noises effecting the radar system, it has been also stated that the avereging procedure is not effective for filtering the considered type of noise, therefore the authors introduced a new algorithm depending on weighted average. Using this approach, large fluctuations produced by complex targets could be reduced and filtered. In conjunction with this, the usage of robust controllers could be very effective in filtering the noise, as introduced by [4], which present a new controller design based on PID scheme augmented with fuzzy controller applied for a radar. This hybrid system showed how fuzzy controller increases the robustness of a PID controller in control a radar system that is exposed to noise. In [5], Kalman filter was used in order to develop tracking algorithms for both slow and aggressive targets. Scanning radar was used and minimum tracking error was aimed. The results in [6] showed a novel intelligent approch that utilizes neural networks in order to guide a missile that is affected by multiple error factors such as target manuver, glint and fading noises. The results have been compared with the traditional proportional navigation guidance law. It has been proved by simulation results that the proposed intillegent method produces less miss distance value in all cases. Also in a previous work [7], a dual fractional order control system has been presented to control the position of guided missile. This controlling method proved to have good efficiency in tracking the target, but the controlling procedure was based on the final positions of the missile and target, and didn't focuse on controlling the radar itself. Also it didn't consider the effect of noises and errors which are very important for real applications.

The rest of the paper is organized as follows: the second section inroduces the principle of fractional calculus, the third setion discusses the main noise and error sources investigated by this research, the fourth section introduces the proposed fractional order PID system applied for missile radar system, the

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fifth section contains the simulation results of the proposed system compared with the conventional one. Finally the conclusions that have been deduced from the proposed system are discussed and future work issues are mentioned at the end of the paper.

II. FRACTIONAL CALCULUS

Fractional calculus is a branch of calculus that generalizes the integration and differentiation operation with integer order ${}_{a}\mathrm{D}_{t}$ to a non-integer order operation ${}_{a}\mathrm{D}_{t}^{\alpha}$, where *t* and *a* are the limits of the integration or differentiation and $\alpha \in$ □ is the differentiation or integration order. One can define the fractional operator ${}_{a}\mathrm{D}_{t}^{\alpha}$ as

$$ {}_{a}\mathrm{D}_{t}^{\alpha} = \begin{cases} \frac{d}{dt^{\alpha}}, \alpha > 1 \\ 1, \alpha = 1 \\ \int_{a}^{t} (d\tau)^{\alpha}, \alpha < 1 \end{cases} \tag{1} $$

$$ {}_{a}\mathrm{D}_{t}^{\alpha}\mathrm{f(t)} = \lim_{h\to 0} \left(\frac{1}{h^{\alpha}}\right) \sum_{m=0}^{\left[\frac{t-a}{h}\right]} (-1)^{m} \frac{1^{\eta}(\alpha+1)}{m!\Gamma(\alpha-\mathrm{m}+1)} f(t - mh) \tag{2} $$

$$ {}_{a}\mathrm{D}_{t}^{\alpha}\mathrm{f(t)} = \frac{1}{\Gamma(n-a)} \left(\frac{d}{dt}\right)^{n} \int_{a}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-\mathrm{n}+1}} d\tau \tag{3} $$

$$ {}_{a}\mathrm{D}_{t}^{\alpha}\mathrm{f(t)} = \frac{1}{\Gamma(n-a)} \int_{a}^{t} \frac{f(n)}{(t-\tau)^{\alpha-\mathrm{n}+1}} d\tau \tag{4} $$

$$ \mathrm{L}\{D^{\alpha}\mathrm{f(t)}\} = \mathrm{s}^{\alpha}\mathrm{F(s)} \tag{5} $$

It has been proved that using fractional order systems is much better than integer order counterparts in many applications which proved to have better robustness and stability properties [8]. Also FOPID can deal with variations, uncertainties and load disturbances that are available in any control system. FOPID is more effective in filtering noises that could affect the overall performance of the system [9], therefore it is found to be very useful to use them in filtering and control as applied here. Also in systems that have time delay, the performance of FOPID is better than integer PID approach. Moreover, in control applications that deal with nonlinear systems, the system is usually linearized at multiple points, and then a specialized PID is designed for each point, but when using FOPID it's usually sufficient to use one FOPID with the whole nonlinear system [10].

III. THE NOISE AND ERROR SOURCES

A. *Radome Aberature Error*

In radar guided missiles, a radome is usually applied to protect the missile from the strong air flow. The electromagnetic waves of the radar pass through this radome. If the shape of this radome is hemispheric, the electromagnetic waves will face no refraction. But the hemispheric shape of the radome will make the drag force of the airflow at a very high level. Therefore, the shape of the radome is usually non-hemispheric, and the electromagnetic waves that is entering through the radome is refracted. This phenomenon could be a serious problem to the radar guided missile, which will affect the measurement of the true line of sight angle, and therefore will cause the missile to miss its target [11].

B. *Time Delay*

Time delay or computational delay is one of the main factors that could affect the performance and accuracy of real time systems such as missiles, it also determines the responsiveness of the system. Time delay should always be kept within acceptable range so the system could make use of the information it receives before timeout. The time delay could be useful sometimes as the existence of time delay make the system more robust against noises, therefore time delays sometimes added intentionally to the system with specific value to reject noises but it should be kept within acceptable range.

C. *Receiver Noise*

This type of noise usually contains many subdivisions such as glint noise, receiver active noise, receiver passive noise and fading noise. Some of them could be considered as range dependent, and some others as range independent. In this research the receiver noise will be considered as range independent angular white noise with power density of 6.5×10^{-8} rad^{2} / Hz [12].

IV. THE PROPOSED FOPID SYSTEM

In addition to the approximation methods for fractional order derivatives and integrations, there are also many other methods that have its own specialties and properties, Oustaloup's approximation is considered to be one of these method which is known for its accuracy, therefore the system used in this proposed design is dependent on Oustaloup's approximation. By using this approximation, the definition of the fractional order operator (s^{γ}) can be expressed as the following [13]:

$$ s^{\gamma} = \omega_{h}^{\gamma} \prod_{k=-N}^{N} \frac{s+\omega'_{k}}{s+\omega_{k}} \tag{6} $$

$$ \omega'_{k} = \omega_{b} \left(\frac{\omega_{h}}{\omega_{b}}\right)^{\frac{k+N+\frac{1}{2}(1-\gamma)}{2N+1}} \tag{7} $$

$$ \omega_{k} = \omega_{b} \left(\frac{\omega_{h}}{\omega_{b}}\right)^{\frac{k+N+\frac{1}{2}(1+\gamma)}{2N+1}} \tag{8} $$

where $0<\gamma<1$. Here, N is the order of approximation that lies between the frequency ranges of ω_b and ω_h. Fig. 1. Shows the conventional system, while The proposed system is depicted in Fig. 2.

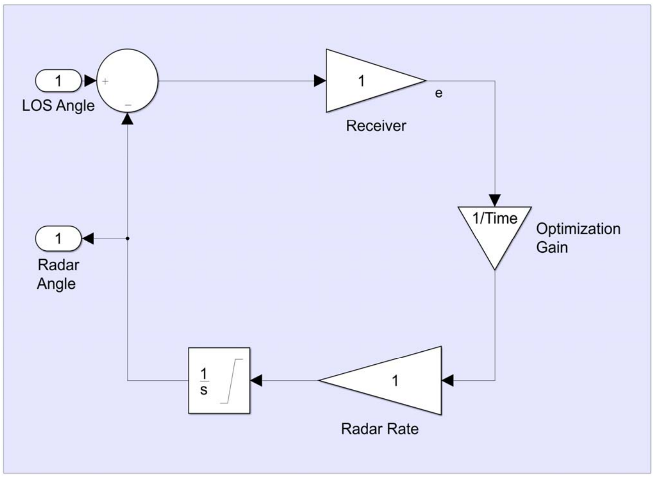

Fig. 1. The conventional radar system.

By the proposed FOPID system shown in Fig. 2, the error signal representing the difference between the line of sight angle and the radar angle is applied to the FOPID as the input. The output signal is sent to the receiver. The noise signal is a band limited one and it contaminates the observations. These received information is divided by an appropriate time constant for feedback control signal optimization, and then integrating this signal to calculate the final radar gimbal angle. But according to our model, the signal is affected by time delay, which affects the calculated gimbal angle. The radome aberration error is approximated as a linear function of the gimbal angle, and is calculated by multiplying the gimbal angle by the value of -0.04.

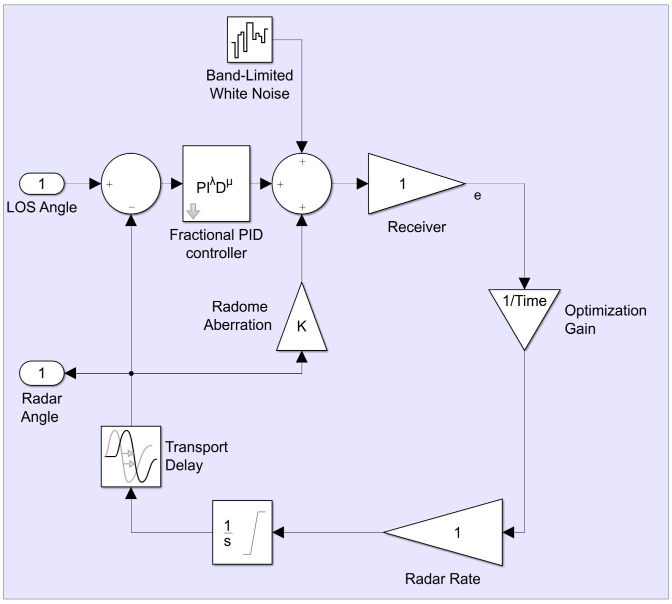

Fig. 2. The proposed FOPID model.

A. Tuning the Order of Approximation and Frequencies

Angular frequency ranges and the order of approximation N could determine the accuracy of approximation in which a higher value of N could produce more accurate results at the cost of higher computational complexity, which may cause larger system delays in real time applications. Also, the frequency ranges ω_b and ω_h should be chosen to include all the operating frequencies of the system. Here, in the proposed system, FOMCON toolbox was used in order to simulate the Oustaloup's approximation method, and the default values for the frequencies are 0.001 rad/s for ω_b and 1000 rad/s for ω_h and the order of approximation (N) is equal to 5. These default values have been used in the tests and good accuracy is observed [14].

B. Tuning the FOPID Parameters using Genetic Algorithm

Genetic Algorithms (GAs) have been used to tune the main FOPID parameters which are K_i, K_p, K_d, λ and μ. This process has been set up by using the miss distance as the fitness function for the genetic algorithm, in which the GA will search the space for the combination values of parameters that produce the less value of miss distance, so the accuracy of the missile will be as high as possible. The values of the five parameters after applying genetic algorithm are shown in Table I.

TABLE I. Tuned Parameters After Applying Genetic Algorithm.

Parameter	Value
K_i	0.2579
K_p	0.8599
K_d	0.0985
λ	0.6434
μ	0.0753

V. SIMULATION RESULTS

The proposed FOPID system was tested against the conventional system, in which noises and error sources have been applied. The performance comparisons have been made in terms of miss distance, normal acceleration and incidence angle of the missile. Miss distance is the shortest distance between the missile and its target during the course of a flight. Normal acceleration is proportional to the force applied to the missile body, and this force is responsible for rotating the missile toward the target. Incidence angle is the angle between the missile velocity direction and the reference x axis.

A. Effect of Time Delay

A time delay is existed due to the gimbals motor reaction time. In simulation, the time delay is produced before calculating the final value of the radar gimbal, as shown in Fig. 2. The simulated time delay was set to 0.005 s, using conventional system, the miss distance was 10.52 m, while using FOPID the miss distance of the missile was 0.8809 m. When the time delay increases, the miss distance also increases. Fig. 3 and 4 show comparison between FOPID and conventional system in terms of normal acceleration and incidence angle respectively. it appears from the figure that the presence of time delay higly affected the conventional system, as it ossilates every where after the 1st second, where the target have been detected by the radar. But using FOPID, the effect of time delay has been decreased significantly, the behavior of FOPID is much more smooth, and the normal acceleration applied on the missile is less, which indicates that FOPID system needs much less energy to direct the missile in the presence of time delay.

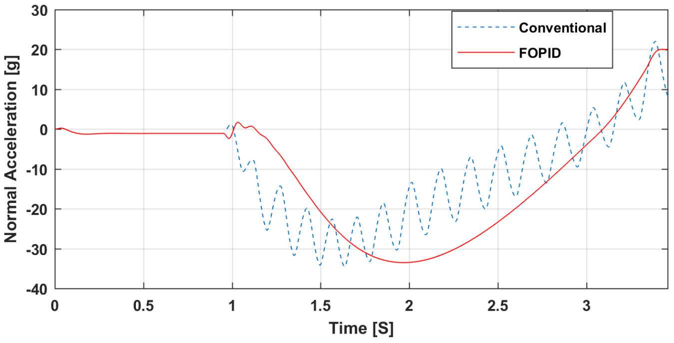

Fig. 3. Comparison of normal accelerations due to time delay.

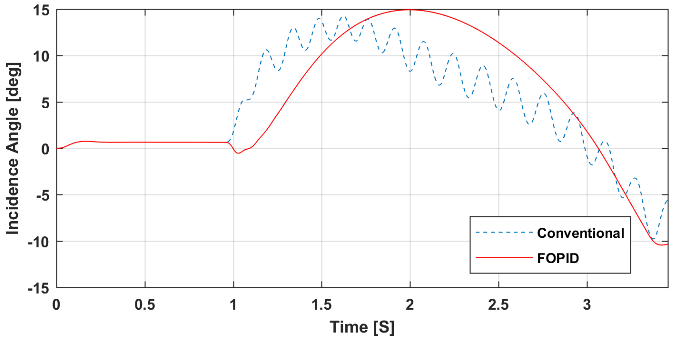

Fig. 4. Comparison of Incidence angles due to time delay.

B. *Effect of Radome Aberration Error*

Radome aberration noise has been added as a linear function of radar gimbal angle with a gain value of -0.04 degree, the miss distance using conventional system was 38.8 m, while using FOPID system it was 0.8703 m. by this it has been proved that FOPID effective in decreasing the radome aberration error. Figures 5 and 6 show the normal acceleration and incidence angle of the missile subjected to radome aberration error. For the conventional system, there are many oscillation points after the first second where the target has been detected. While using FOPID the system is more smooth and the difference between highest and lowest values are less in the FOPID. By these results, the graphs prove the ability of the FOPID to stabilize the system and decrease the effect of radome aberration error.

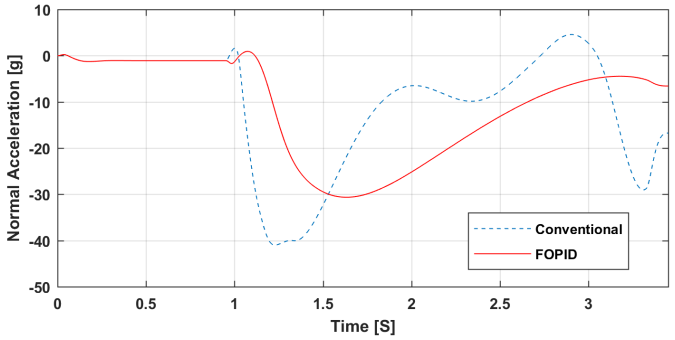

Fig. 5. Comparison of normal accelerations due to radome aberration error.

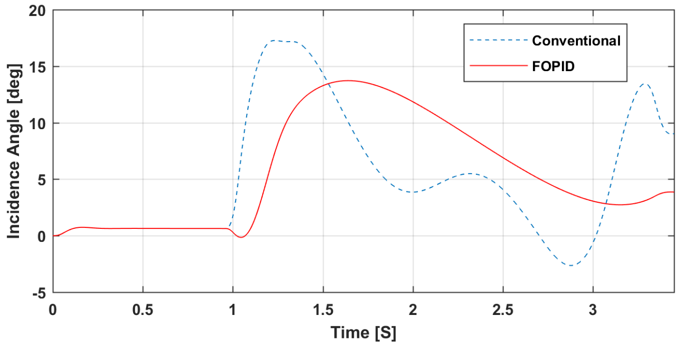

Fig. 6. Comparison of Incidence angles due to radome aberration error.

C. *Receiver Noise*

In the presence of receiver noise with power of $6.5*10^{-8}$ rad^2/Hz [12] as shown in Fig. 7. The miss distance for the conventional system was 20.59 m while for the FOPID system the miss distance was 3.761 m.

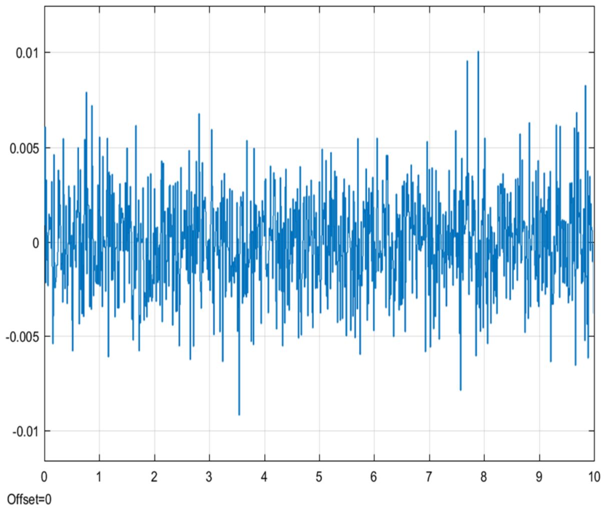

Fig. 7. White noise applied for the radar guided missile.

It's clear from the miss distance values that FOPID has the ability to reduce the effect of the noise compared with the conventional system.

VI. CONCLUSION AND FUTURE WORK

Radar guided missile has been introduced with multiple noise and error sources. Three main types of noises and errors effecting radar guided missile which are: time delay, radome error aberration and receiver noise. FOPID system has been used and compared with conventional system in order to decrease the effect of noises and errors. The comparisons were based on miss distance, incidence angle and normal acceleration values. The simulation results show the superiority of the proposed FOPID for decreasing the effect of time delay compared with conventional system as well as increase the stability and the accuracy of the missile. Also it showed good ability in smoothing and decreasing the effect of radome aberration error. The effect of correcting the receiver white noise were not that obvious in terms of normal acceleration and incidence angle values, but it showed better miss distance value for FOPID compared with conventional system. and errors compared with the conventional one.

As a future work, the accuracy of the missile could be increased by applying second and infinity norm tuning methods, which stabilizes the radar during the whole course of the missile rather than a single point. Also using digital FOPID could be used as digital systems have better noise rejecting capabilities than analogue one.

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REFERENCES

[1] J. Feng, D. Wang, T. Liu and C. Cai, "A study on the effect of radar seeker performance parameters on control and guide precision," in *Advanced Information Technology, Electronic and Automation Control Conference (IAEAC), IEEE*, Yantai, China, 2017.

[2] H. WANG, D.-f. LIN, J. WANG and Z.-x. CHENG, "Study on homing guidance systems based on different source errors," in *International Conference on Computer, Mechatronics, Control and Electronic Engineering (CMCE), IEEE*, Beijing, China, 2010.

[3] C. Hartzstein, "Weighted filtering of monopulse signals," in *IEEE Radar Conference (RadarConf)*, Philadelphia, PA, USA, 2016.

[4] D. Zhe, C. Jiabin, S. Chunlei and C. Hongye, "Design of longitudinal control system for target missiles based on fuzzy adaptive PID control," in *2017 29th Chinese Control And Decision Conference (CCDC), IEEE*, Chongqing, China, 2017.

[5] K. D. S. Raj and I. M. Krishna, "Kalman filter based target tracking for track while Scan data processing," in *International Conference on Electronics and Communication Systems (ICECS 2015), IEEE*, Coimbatore, India, 2015.

[6] J.-M. Lin and C.-H. Lin, "A novel intelligent neural guidance law design by using adjoint method," in *Proceedings of the 2014 International Conference on Machine Learning and Cybernetics, IEEE*, Lanzhou, 2014.

[7] M. Yaghi and M. Ö. Efe, "Dual fractional order feedback control system applied for missile guidance," in *9th Ankara International Aerospace Conference*, Ankara, Turkey, 2017.

[8] R. Pradhan, P. Patra and a. B. Pati, "Comparative studies on design of fractional order proportional integral differential controller," in *Conference on Advances in Computing, Communications and Informatics (ICACCI), IEEE*, Jaipur, India, 2016.

[9] E. Edet and R. Katebi, "Design and tuning of fractional-order PID controllers for time-delayed processes," in *2016 UKACC 11th International Conference on Control (CONTROL), IEEE*, Belfast, United Kingdom, 2016.

[10] P. Shah and S. Agashe, "Review of Fractional PID controller," *Elsevier,* vol. 38, p. 29–41, 2016.

[11] M.-G. Seo, M.-J. Tahk and C.-K. Ryoo, "Observability analysis on radome aberration estimation," in *Proceedings of the 19th World Congress The International Federation of Automatic Control*, Cape Town, South Africa, 2014.

[12] R. Yanushevsky, Modern Missile Guidance, New York: CRC Press, 2007.

[13] S. Das and I. Pan, "Basics of fractional order signals and systems," in *SpringerBriefs in Applied Sciences and Technology*, S. Das and I. Pan, 2012, pp. 18-19.

[14] A. Tepljakov, E. Petlenkov, J. Belikov and M. Halás, "Design and implementation of fractional-order PID controllers for a fluid tank system," in *2013 American Control Conference (ACC)*, Washington DC, USA, 2013.