Path Planning using Model Predictive Controller based on Potential Field for Autonomous Vehicles

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***Abstract*— In recent decades, one of the challenging problems is path planning for autonomous vehicle in dynamic environments with along static or moving obstacles. The main aim of these researches is to reduce congestion, accidents and improve safety. We propose an optimal path planning using model predictive controller (MPC) which automatically decides about the mode of maneuvers such as lane keeping and lane changing. For ensuring safety, we have additionally used two different potential field functions for road boundary and obstacles where the road potential field keeps the vehicle for going out of the road boundary and the obstacle potential field keep the vehicle away from obstacles. We have tested the proposed path planning on the different scenarios. The obtained results represent that the proposed method is effective and makes reasonable decision for different maneuvers by observing road regulations while it ensures the safety of autonomous vehicle.**

Keywords—Path Planning, Autonomous Vehicles, Model Predictive Control, Sequential Quadratic Programming, Artificial Potential Field.

I. INTRODUCTION

Nowadays, the autonomous vehicle has been becoming an important research in vehicle engineering. In autonomous vehicle or intelligent unmanned vehicle, the safety of vehicle is a critical factor. The development of autonomous vehicle technology has a significant effect on reducing traffic, accidents and relieving the stress of motorists. The automobile manufacturers such as Tesla Motors and several internet companies have decided to develop fully autonomous vehicles in order to avoid collides in the roads, but these vehicles are very complex systems and include sensing systems [1], path planning systems, trajectory tracking systems and suspension systems [2], [3]. Although, there are many researches on the path planning and tracking to avoid collision for unmanned vehicle and other robots, it is not easy to utilize these approaches directly to avoid collision scenarios. Because, these vehicles are only able to move in its stable limitations and handling capability in a constrained environment. Therefore, to solve collision avoidance problem in the road, it is necessary to consider the other moving vehicles that have their own motion properties.

Early researches on the path planning for autonomous vehicle date back to the 1980s that focus on the computation of optimal time and collision-free path from a given point to another. Many approaches are presented for path planning that can be categorized in two classifications: conventional and heuristic approaches. Conventional approaches are referred to common methods such as sampling based algorithms and grid-based methods. The sampling based algorithms such as rapidly exploring random trees algorithm and its variants, perform by sampling of configuration to connect the initial configuration with the target [4]. Grid-based algorithms have mapped free space to set of cells and then try to solve a problem of graph search. The major shortcoming of the classic approaches is that they do not consider the dynamics of the vehicle. In contrast, the controller approaches such as Model Predictive Control (MPC) consider the model of system to predict the evaluation of system states and has been widely used in a variety of systems [5]. Due to its capabilities to overcome physical constraints, MPC has been used to path planning and tracking for the autonomous vehicle. MPC based approaches are a problem of optimization and solve it for finding an optimal open-loop control sequence which minimizes the objective function by satisfying all the state and input constraints.

Many conventional approaches are proposed for path planning methods such as A* heuristic search, Visibility graph, Voronoi diagram and Potential field [6]. The motion of the autonomous vehicle and the sum of used forces are combined by the potential field method [7]. This method is consisting of two forces, one is the attractive force to move the vehicle towards goal or destination and the other is repulsive force to avoid collisions with obstacles. However, an inherent problem of traditional potential field methods is the formation of local minimum that prevents from arriving the vehicle to target. In this paper, we developed a new potential field method based on boundary road and the vehicle's kinematic model to generate a collision free path for autonomous vehicle. The proposed algorithm can be updated to obtain a collision free path along with static and dynamic obstacles in real time.

The reminder of paper is organized as follows. In section II, the structure of an autonomous vehicle system and its equations, the proposed potential field functions for road and obstacles and also model predictive controller for path planning is formulated. The results of proposed path planning for several scenarios are evaluated and discussed in section III. Finally, section IV concludes the paper.

II. Problem Statement

In this section, we describe the used vehicle model for simulation and control design. The main aim of this paper is driving to a given destination with regulated speed while the vehicle avoids the collision with obstacle or surrounding vehicles and presents a comfortable driving experience.

As mentioned above, the methods based on mathematical optimization are interesting in current years. For considering dynamics of vehicle and safety constraints, these methods offer a symmetric and precise method and consequently generate the control optimal inputs. A mathematical optimization method is either in open-loop form if the environment is fully pre-identified or in closed-loop form if the environment is unidentified therefore, a feedback controller is used to identify it [8-9]. Model Predictive Control (MPC) is one of the mathematical optimization methods that is used for online path planning in most researches.

A. *The model of the vehicle*

To model the autonomous vehicle, a bicycle model is used. In this model, the two front wheels of the vehicle are lumped into a unique wheel in the center of front axle and the two rear wheels are located in the center of rear axle. The kinematic model is used for modeling the ego vehicle and obstacle or surrounding vehicles. Meanwhile, the vehicle model is illustrated on Figure 1. The equations of motion of the bicycle model are as follows:

$$m(\dot{v}_x - v_y r) = F_{x_T} \tag{1}$$

$$m(\dot{v}_y + v_x r) = F_{y_f} + F_{y_r} \tag{2}$$

$$I_z \dot{r} = l_f F_{y_f} - l_r F_{y_r} \tag{3}$$

$$\dot{\theta} = r \tag{4}$$

$$\dot{X} = v_x \cos\theta - v_y \sin\theta \tag{5}$$

$$\dot{Y} = v_y \cos\theta + v_x \sin\theta \tag{6}$$

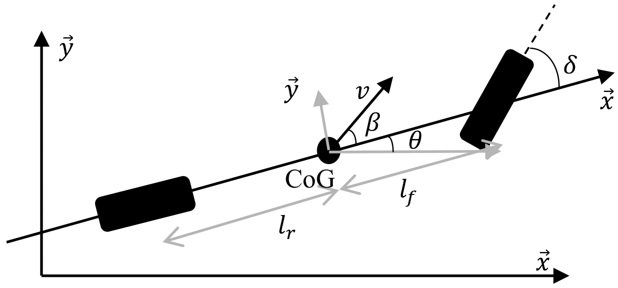

Fig. 1. The model of bicycle vehicle.

where m is the mass of vehicle. v_x, v_y and r are longitudinal velocity, lateral velocity and yaw rate of the vehicle in its center of gravity, respectively. X and Y are the longitudinal and lateral positions and θ is heading angle of the vehicle. I_z is the vehicle momentum of inertia around its vertical axis. F_{x_T} is the total longitudinal force of tire. Also, F_{y_f} and F_{y_r} are the total lateral forces of the front and rear tires. A linear model of tire is used for the lateral forces of tire when the vehicle has a front steering system [10].

$$F_{y_f} = c_f \alpha_f = c_f\left(\delta - \frac{v_y + l_f r}{v_x}\right) \tag{7}$$

$$F_{y_r} = c_r \alpha_r = c_r\left(-\frac{v_y + l_r r}{v_x}\right) \tag{8}$$

where α_f and α_r are the sideslip angles of the front and rear tires and δ is the angle of steering. Also, c_f and c_r show the cornering stiffness values of the front and rear tires. These values are calculated similar to [10]. By linearizing equations (1)-(8), the linear dynamics of vehicle can be calculated as follows:

$$\dot{x} = Ax + Bu_c \tag{9}$$

$$x = [X \;\; v_x \;\; Y \;\; v_y \;\; \theta \;\; r]^T \tag{10}$$

$$u_c = [F_{x_T} \quad \delta]^T \tag{11}$$

where x is the state vector, u_c is the input vector, A is the state matrix, and B is the input matrix. To use as a model predictive controller, it is required that the model is discretized by zero order hold method.

B. *Potential Field Function (PF)*

The potential field method is based on attractive and repulsive functions that the attractive function causes vehicle to move towards the goal while repulsive function avoids from collision of the vehicle with obstacles. The goal potential field has a minimum value in the target position then it attracts the vehicle, the obstacle PF has a maximum value in the obstacle positions that repulses the vehicle from obstacle [11]. In this paper, the main aim is to navigate vehicle to the target position without any collision that is performed by tracking term of objective function controller. Therefore, we consider only repulsive function as PF. For this purpose, two different PF such as lane marker PF (U_R) and obstacle PF (U_O) are defined. The potential field is the sum of the PFs:

$$U_{tot} = \lambda_r U_R + \lambda_o U_O \tag{12}$$

where λ_r and λ_o are PF weights for road and obstacle. Also, the other functions for modeling of road regulation and obstacles can be used.

1) *Lane Marker PF*

Lane marker PF is used to prevent the vehicle from departing of the main road and driving too close to the borders that increase the risk of accident. Therefore, the lane marker should have a maximum value in the road boundaries. Additionally, the slope of arriving to this maximum value is maximum and provides a restoring force with maximum value. Meanwhile, a peak at the location of driving lane is operated to resist changing lanes. Therefore, the vehicle makes attempt to keep maintaining its current lane in order to prevent relevant cost. For this purpose, the PF is zero and locally symmetric in the center of lane which is the preferred position when the vehicle does not face traffic or obstacles. When changing lane is necessary, the vehicle is able to overcome this resistance. The PF used for lane marker (U_R) is as follows:

$$U_R = A_r(1 - e^{-b_r(Y_h - Y_r)})^2 + A_r(1 - e^{b_r(Y_h - Y_l)})^2 \quad (13)$$

where A_r is the depth of road PF and is equal to 0.5, b_r is controlling parameter for road PF width and is equal to 1. Y_h is lateral position of ego vehicle from CoG in local road frame and Y_r, Y_l are lateral positions in right and left of the center of straight road, respectively. Figure 2(a) shows the 3D plot of the total PF on the straight road.

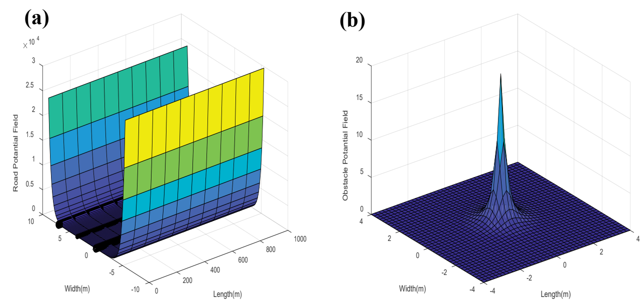

Fig. 2. (a). The road potential field function, (b) The obstacle potential field function

2) Obstacle PF

The structure of obstacle PF (U_O) is more complex and important than the structure of the road PF. According to the obstacle PF, the lane change maneuver is performed if the obstacle or surrounding vehicle is approaching the ego vehicle. This is based on the structure and protocol of highway-driving. Also, the vehicle could change to the left side to overtake slower preceding vehicles. To achieve this, obstacle PF is modeled as function of the measured position of the obstacle vehicle, the relative and absolute velocity of ego and obstacle vehicles and the road curvature. The position of obstacle PF is obtained by the available sensor measurements of obstacle. The longitudinal and lateral distances, given by x_O and y_O, between the obstacle and ego vehicle are the gained information and do not include the heading angle of the obstacle vehicle.

The shape of obstacle vehicle is considered rectangular because it provides a better approximation of the outline of an obstacle. Also, in order to avoid slope discontinuities in PF, the continuous functions are required to represent the obstacle value such as hyperbolic function. This function is used as the distance between the ego and obstacle vehicles that generates the desired potential field. When the distance between the ego vehicle and obstacle is too small, the change rate of function strictly increases and therefore its value tends to infinity that prevents the collision of ego vehicle with obstacle. The used pointwise repulsive potential function is as follows:

$$U_o = \frac{1}{(x - x_O)^2 + (y - y_O)^2 + \varepsilon} \quad (14)$$

where (x, y) is the current location of the vehicle and (x_O, y_O) is the location of the obstacle. ε is a small and positive value which is used for preventing singularity. The formulation is logical and provides the ideal response for the given limitations of computational complexity, besides it will avoid actual obstacle and keep a specific distance from obstacle.

If the ego and obstacle are closing together in each direction, the approaching velocity is equal to the difference of velocity between them and the otherwise is set to zero. The potential field of obstacle is located in $(x, y) = (0,0)$ that is shown in Figure 2(b).

3) MPC Formulation

The model predictive controller is a combination of optimal and adaptive controllers. The method uses a model based on controller that is involved in the optimization step of the predicted states of the model for generating the optimal control input. The model predictive controller is able to adjust itself to change conditions, for this reason, it is similar to adaptive controller. At each control interval, it handles input and output constraints to solve optimization problem. According to these properties, MPC is a suitable candidate for path planning and tracking based on potential field.

A model predictive controller is presented based on dynamic model of vehicle, potential field and road regulations. By using these objectives, an optimization problem of conflicting demands can be defined. The model predictive controller predicts the response of ego vehicle on a horizon that named the prediction horizon (N) and optimizes the dynamics of vehicle, obstacle avoidance, road regulation and command following based on this value. The desired lane and speed are predefined. Therefore, the desired lateral position (the center of the desired lane) and longitudinal velocity are the outputs of system that should be tracked:

$$y = [Y \quad v_x]^T \quad (15)$$

$$y_{des} = [Y_{des} \quad v_{x_{des}}]^T \quad (16)$$

$$Y_{des} = \left(l_{des} - \frac{1}{2}\right) L_w + \Delta Y_R \quad (17)$$

where y is the output matrix tracking, y_{des} is the desired lateral position, $v_{x_{des}}$ is the desired speed, l_{des} is the index number of the desired lane from the right, L_w is the lane width and ΔY_R is the lateral offset of road compared to a straight road. The nonlinear optimization problem for the path planning can be formulated as follows:

$$\min_{u_c, \varepsilon} \sum_{i=1}^{N} \|y(k+i|k) - y_{des}(k+i|k)\|_Q^2 + \|u_c(k+i-1|k) - u_c(k+i-2|k)\|_R^2 + \|u_c(k+i-1|k)\|_S^2 + U_R(k+i|k) + U_O(k+i|k) + \|\varepsilon_k\|^2 \quad (18)$$

$$s.t. \quad x(k+i|k) = Ax(k+i-1|k) + Bu_c(k+i-1|k) \quad (19)$$

$$y(k+i|k) = Cx(k+i-1|k) + Du_c(k+i-1|k) \quad (20)$$

$$v_{x_{min}} < v_x < v_{x_{max}} \quad (21)$$

$$\left(\frac{F_{x_T}}{F_{x_T-max}}\right)^2 + \left(\frac{F_{y_{f,r}}}{F_{y_{f,r}-max}}\right)^2 < 1 \quad (22)$$

$$\varepsilon_k \geq 0 \quad (23)$$

$$\varepsilon_{k+1} = \varepsilon_k \quad N_{rs}+1, \quad c_1 = 1,2,\dots,N/N_{rs} \quad (24)$$

$$u_{c-min} < u_c(k+i-1|k) < u_{c-max} \quad (25)$$

$$\Delta u_{c-min} < u_c(k+i-1|k) - u_c(k+i-2|k) < \Delta u_{c-max} \quad (26)$$

$$u_c(k+i|k) = u_c(k+i-1|k), \quad k > N_c, \quad k \neq c_2 N_{rc} + N_c, \quad c_2 = 1,2,\dots,(N-N_c)/N_{rc} \quad (27)$$

where *(k+i|k)* index indicate the values at future time $k+i$ and predicted at current time *k*. ε_k is the vector of slack variables at *k* time. The objective function consists of the quadric term of tracking, changes in inputs, inputs, potential field functions and slack variables. This variable allows some violation and makes a penalty term in the objective which can be used to penalize the violation. *Q* and *R* are the tuning matrices of the controller. The predicted states are obtained by (19). *A, B* are discrete state and input matrices that is resulted by discretizing (9). The tracking output is calculated by (20) and *C, D* are output and feedforward matrices. The speed constraint and constraint of octagon approximation are applied as soft constraints represented in (21), (22) equations, respectively. These constraints are considered due to some road regulations on the limit of minimum and maximum speed and the longitudinal and lateral forces of tire that cannot exceed the friction ellipse. By reducing the number of slack variable and control inputs in (24) and (27), the cost of computation can be reduced. The vector of slack variable changes in every N_{rs} prediction steps, and after the first N_c prediction steps, the inputs of control change in every N_{rc} steps. The inputs of control and their changes are constrained in (25) and (26) for satisfying the limitations of actuator, where u_{c-min} and u_{c-max} are the matrices of lower and upper bounds of control input, and Δu_{c-min} and Δu_{c-max} are the matrices of lower and upper bound of the control inputs changes.

The given potential field is a non-convex and nonlinear function, for this reason, the optimization problem is non-convex and nonlinear and its solution is expensive. Thus, to reduce the computational time, the problem is converted into a quadratic and convex problem. For this purpose, PFs are first approximated by convex functions. Then, the resulted convex function is approximated by a quadratic function through the second order Taylor series. The obtained function is close convex quadratic approximation of the original function around the nominal point. The obtained gradient is equal to the gradient of original function and also Hessian matrix of approximated function is the closest positive definite matrix to the Hessian matrix of original function in term of Frobenius norm. Although the quadric approximation of the PFs increases the calculation time, the added time is negligible compared to the time needed for solving a nonlinear optimization problem [12].

Using these PFs, the problem of optimal control is a convex quadratic optimization problem. This problem is similar to a nonlinear problem solved by Sequential Quadratic Programming (SQP) in one sequence. An upper bound for the optimization error of each sequence of SQP is derived by Bogges et al. [13], where this error is difference between the sequence result and local minimum of the nonlinear problem in the neighborhood of the initial value of problem. According to this upper bound, if the initial value of problem is closer to minimum, the error of optimization will be smaller and the predicted position of vehicle is equal to the position of vehicle at minimum point as well. Moreover, in the case of Hessian matrix the closer calculated ones of PFs to their values at the minimum will reduce the optimization error. So, a PF with a smaller convex quadratic approximation error and a smaller variation of Hessian matrix in the neighborhood of the problem's initial value result in a smaller optimization error.

III. Simulation and Results

A. Scenarios

Path planning and control designing are the most challenging tasks in autonomous vehicles. Path planning in dynamic or structured environments such as road includes global and local path planning in which a global path planning is used along with a local path planning. A global path planning is a slow and deliberative process which is used for long distance paths for reaching the target. While a local path planning is a faster process and is used for short distance paths and deals with tasks such as vehicle stability, obstacle avoidance, comfortable and safety. This planner is more reactive and runs in real time.

Driving in structured road can be simplified into two basic maneuvers of vehicle, namely lane keeping and lane changing. The main aim of lane keeping is to follow vehicle and stay in its current position by continuously adjusting its orientation and distance to the lane center. Lane changing is the most common maneuver which vehicle changes its current lane to overtaking, obstacle avoidance and road departure. According to road, given lane and obstacles on the road, the maneuver might be different. These are the several samples of maneuvers that happen on the road. In below, to evaluate the performance of autonomous vehicle, several scenarios are presented:

- Lane keeping on the straight and curved road
- Keeping a desired distance from the vehicle in front of the ego vehicle
- Lane changing with moving obstacle on the curved road

B. Simulation

In this section, the performance of the proposed MPC is evaluated on the autonomous vehicle according to road regulation, obstacle avoidance and maneuverability. The MPC formulation is solved by the **fmincon** solver Sequential Quadratic Programming (SQP) via the YALMIP toolbox in MATLAB/Simulink. The parameters of controller for a dry road

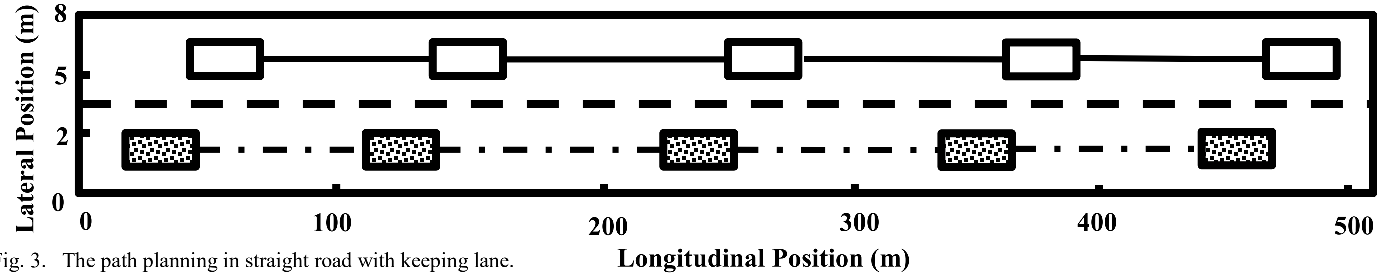

Fig. 3. The path planning in straight road with keeping lane.

are presented in Table 1. The speed of vehicle is 100 *km/h* and the controller time step is 80 *ms*.

TABLE I. THE CONTROLLER PARAMETERS

Parameter	Value	Unit	Parameter	Value	Unit
m	1625	*kg*	N	20	-
I_z	2865.6	$kg.m^2$	N_c	5	-
l_f	1.108	*m*	N_{rc}	5	-
l_r	1.502	*m*	N_{rs}	10	-
C_{α_f}	98389	*Nm/rad*	u_{min}	-[24800 0.2]	-
C_{α_r}	198142	*Nm/rad*	u_{max}	[13000 0.2]	-
L_w	3.5	*m*	Δu_{min}	-[1600 0.02]	-
λ_r	1	-	Δu_{max}	[1600 0.02]	-
λ_0	25	-	Q	[0.2 0.01]	-
F_{x_T-max}	24800	*N*	R	[5e-8 500]	-
F_{y_f-max}	10400	*N*	S	[2e-9 100]	-
F_{y_r-max}	10600	*N*			

First scenario that is shown in Figure 3 is related to a path planned in a normal highway. The road geometry is approximated using a 4th order polynomial based on offline lane marking and mapped waypoints. It is one-way road with two lanes. The ego vehicle is exhibited by dashed rectangle and is moving on the lane 1, the surrounding or obstacle vehicle is shown as empty rectangle and is moving on the other lane. Each of rectangles is used to demonstrate the position of ego and surrounding vehicles or obstacles. The crux of this scenario is lane keeping on the straight road. The future trajectories of the ego and obstacle vehicle are represented as a sequence of rectangles. The desired speed of the ego vehicle is higher than obstacle vehicle in the other lane.

When the road is also curved, which is demonstrated in Figure 4, the ego vehicle tries to keep its lane. In these two scenarios, the obstacle vehicle is moving on the side of ego vehicle and the potential field of obstacles keeps the ego vehicle away from the other lane.

In the second scenario, the ego vehicle starts on the lane 1 and it is commanded that it should keep its lane. In lane 2, there is a moving obstacle vehicle with the same longitudinal position and speed as the ego vehicle. It carelessly changes its lane from the center of lane 2 towards lane 1 with a constant lateral velocity in the specific time interval. The main goal of this scenario is to keep an enough distance from the vehicle in front of the ego vehicle until avoiding possible collisions. The result of simulation is shown in Figure 5. Due to the obstacle potential field, the ego vehicle reduces its speed to make enough space to obstacle and moves to right in order to keep its lateral distance from obstacle and avoid collision. The road potential field guides the vehicle towards the center of lane and keeps the vehicle in the intended lane. The moving obstacle is on the middle lane marker and the vehicle has made a longitudinal distance around 8 meters to have a safe distance with the obstacle. Due to the potential field of right road and after making an enough longitudinal distance to the obstacle, the vehicle goes back towards the center of lane.

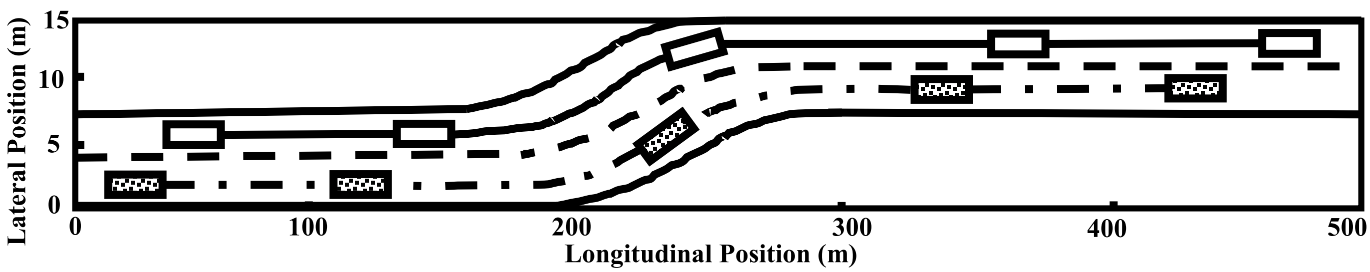

Fig. 4. The path planning in curved road with keeping lane.

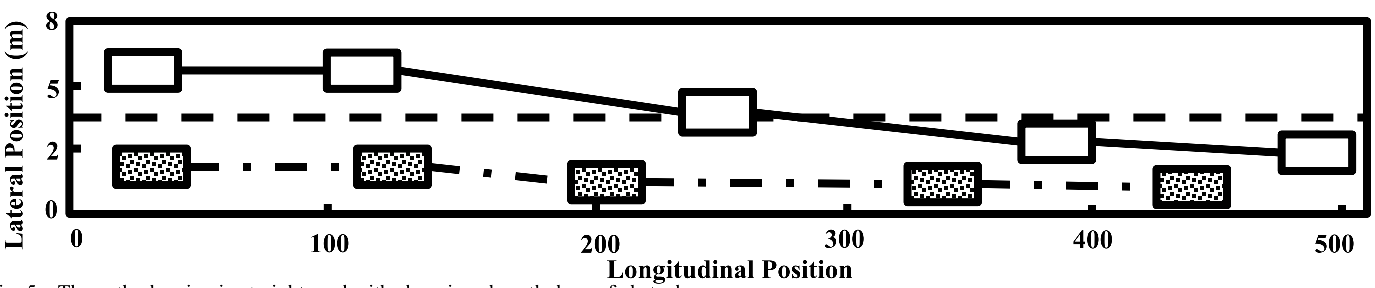

Fig. 5. The path planning in straight road with changing abruptly lane of obstacle.

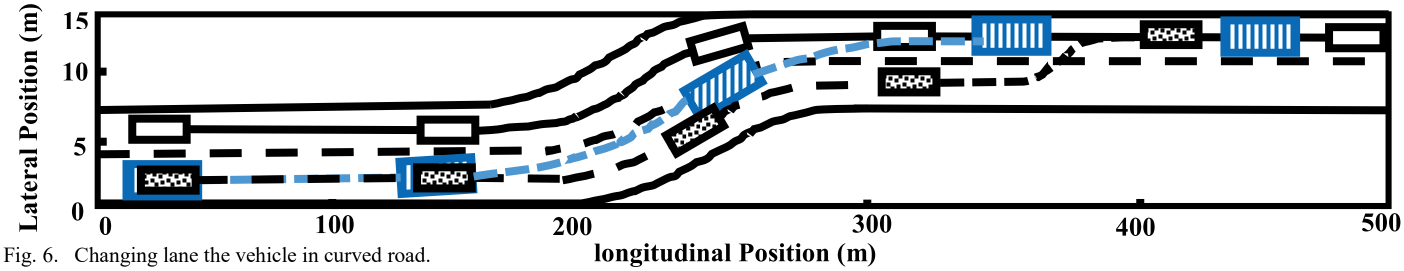

Fig. 6. Changing lane the vehicle in curved road.

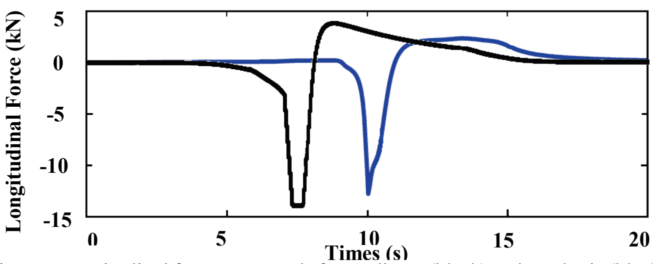

Fig. 7. Longitudinal force commands for nonlinear (black) and quadratic (blue) problems.

In the third scenario, while there are moving obstacles on the lane 2, the vehicle changes its lane. This scenario is simulated for the nonlinear (which is illustrated by vertical line in blue rectangle) and quadratic path planning, the result of simulation is represented in the Figure 6. Since there is a moving obstacle on the side of vehicle, the ego vehicle cannot immediately proceed to change its lane; and the given obstacle potential field keeps the ego away from lane 2. The ego vehicle reduces its speed then approaches to the middle lane and waits to pass the obstacle from its side. When there is enough distance between the obstacles in its front and behind, the vehicle moves to the lane 2 by adjusting its speed while it keeps its distance between the obstacles. The changes of speed and lateral movements of the vehicle are based on the intended potential field functions which keep the vehicle away from the obstacles and road. Also to evaluate, last scenario is simulated for nonlinear problem which the result of longitudinal force commands for nonlinear and quadratic problems can be seen in Figure 7. The performance of the quadratic problem is comparable with the nonlinear. Additionally, the average run time for the nonlinear problem for a time step is 18.26s while it is 0.0089s for the quadratic. Because the given step time is 0.5s, the quadratic problem can be feasible in real time.

IV. Conclusion

In this paper, we propose a path planning based on model predictive controller for autonomous vehicles in the dynamic environments. To avoid collision and ensure the safety of the vehicle, we propose two different potential field functions for the road and the obstacles or surrounding vehicles. Problem of optimal control is a nonlinear therefore to reduce computational burden, the problem is approximated by a convex quadratic problem. This function has obtained by the easy integration of the vehicle dynamics and constraints of system into the MPC optimization that is used to determine lane keeping and lane changing maneuvers. The computation time and performance of nonlinear and quadric problems are compared by simulation. According to errors in approximation, the performance of the quadratic problem is better than nonlinear. In additional, an advantage of the proposed path planning is related to representing different behaviors for different obstacles. To evaluate different scenarios, the several simulations are conducted. The results illustrate that the proposed path planning algorithm is effective to generate safe and comfortable path for autonomous vehicles.

References

[1] E. Shang, X. An, T. Wu, T. Hu, Q. Yuan and H. He, “Lidar based negative obstacle detection for field autonomous land vehicles”, Journal of Field Robotics, vol. *33*(5), pp. 591-617, 2016.

[2] H. Zhang, Q. Hong, H. Yan, F. Yang and G. Guo, “Event-based distributed H filtering networks of 2DOF quarter-car suspension systems”, IEEE Transactions on Industrial Informatics, vol. 13(1), pp. 312-321, 2017.

[3] H. Zhang, X. Zheng, H. Yan, Z. Wang and Q. Chen, “Codesign of eventtriggered and distributed H filtering for active semi-vehicle suspension systems”, IEEE Transactions on Mechatronics, vol. 22(2), pp. 1047-1058, 2017.

[4] J. Kuffner and S. M. LaValle, “RRT-Connect: An Efficient Approach to Single-Query Path Planning”, Proceedings of the 2000 IEEE International Conference on Robotics and Automation, pp. 995-1001, 2000.

[5] H. Zhang, G. Feng, H. Yan and Q. Chen,” Sampled-data control of nonlinear networked systems with time-delay and quantization”. International Journal of Robust and Nonlinear Control, vol. 26(5), pp. 919-933, 2016.

[6] V. Kunchev, L. Jain, V. Ivancevic, and A. Finn, “Path planning and obstacle avoidance for autonomous mobile robots: A review”, In International Conference on Knowledge-Based and Intelligent Information and Engineering Systems, pp. 537-544. October 2006.

[7] A. K. Pamosoaji, and K. S. Hong, “A Path-Planning Algorithm Using Vector Potential Functions in Triangular Regions”, IEEE Transactions on Systems, Man, and Cybernetics: Systems, 43(4), pp. 832-842, 2013.

[8] Y. Gao, A. Gray, J. V. Frasch, T. Lin, E. Tseng, J. K. Hedrick, and F. Borrelli, “Spatial predictive control for agile semi-autonomous ground vehicles”, In Proceedings of the 11th International Symposium on Advanced Vehicle Control, September 2012.

[9] A. Carvalho, Y. Gao, A. Gray, H. E. Tseng and F. Borrelli, “Predictive control of an autonomous ground vehicle using an iterative linearization approach”. 16th International IEEE Conference on Intelligent Transportation Systems-(ITSC), pp. 2335-2340, October 2013.

[10] Y. Gao, A. Gray, H. E. Tseng and F. Borrelli, “A tube-based robust nonlinear predictive control approach to semiautonomous ground vehicles”, Vehicle System Dynamics, vol. 52(6), pp. 802-823, 2014.

[11] R. Silva-Ortigoza, C. Márquez-Sánchez, F. Carrizosa-Corral, V. M. Hernández-Guzmán, J. R. García-Sánchez, H. Taud, M. Marciano-Melchor and J. A. Álvarez-Cedillo, “Obstacle avoidance task for a wheeled mobile robot–a matlab-simulink-based didactic application”, In Matlab Applications for the Practical Engineer, InTech, 2014.

[12] M. Tanaka and K. Nakata, “Positive definite matrix approximation with condition number constraint”, Optimization Letters, vol. 8(3), pp. 939-947, 2014.

[13] P. T. Boggs and J. W. Tolle, “Sequential quadratic programming”, Acta numerica, vol. *4*, pp. 1-51, 1995.